ETLA

Keskusteluaiheita - Discussion papers

No. 577

Erkki Koskela* - Markku Ollikainen*

TAX INCIDENCE AND

OPTIMAL FOREST TAXATION

UNDER STOCHASTIC DEMAND**

*   Department of Economics, P.O.Box 54 (Unioninkatu 37),
        FIN-00014 University of Helsinki, Finland. E-mail:
        Erkki.Koskela@Helsinki.fi, Markku.Ollikainen@Helsinki.fi.

07.01.1997

ABSTRACT: The paper examines the incidence and welfare effects of forest taxation in the competitive roundwood markets when future demand for timber is uncertain, thus making the future timber price stochastic. It turns out that the nature of timber price risk is crucial for the behavioral, incidence and welfare effects of forest taxes. Under private risk, when some do well and others poorly, the land site tax is like a pure profit tax and is fully borne by forest owners, while the burden of the yield tax is generally shared by both sides of the market. The incidence of the yield tax does not, however, matter qualitatively for the optimal structure of taxation. It is desirable to supplement the land site tax by a distortionary yield tax, because it provides social insurance by decreasing the after-tax timber price risk, which overweights its distortionary effect. The level of the optimal yield tax is determined by the trade-off between its social insurance and distortionary effects. Under aggregate risk, when all either gain or lo-se, government budget constraint is stochastic. Given the optimal land site tax, optimal yield tax is non-distortionary and thus fully borne by forest owners. The neutrality of the optimal yield tax is due to the fact that it has no social insurance role in the presence of aggregate risk. The level of the yield tax depends on the risk attitudes towards variability of after-tax timber revenue and government consumption.
Additional keywords: private and aggregate risk, welfare effects of taxation.
JEL classification: D81, H21,H22, Q23.

TIIVISTELMÄ: Tutkimusraportissa tarkastellaan metsäverotuksen aiheuttamia käyttäyty-mis-, kohtaanto- ja hyvinvointivaikutuksia kilpailullisilla raakapuumarkkinoilla. Raakapuun tulevaan kysyntään liittyy joko yksityistä tai aggregaattitason epävarmuutta, mistä johtuen tu-leva puun hinta on stokastinen. Yksityisen hintariskin tapauksessa pinta-alavero ei riipu hak-kuista ja on puhtaan voittoveron kaltainen ei-vääristävä vero. Sen kantavat kokonaisuudes-saan metsänomistajat, kun taas myyntituloverotuksen rasitus jakaantuu yleensä markkinoiden molempien osapuolten kannettavaksi. Jos valtio haluaa kerätä metsäveroilla jonkin annetun määrän verotuloja, valtion budjettirajoitus on yksityisen riskin tapauksessa deterministinen. Valtio kerää verotulot pinta-alaveron avulla, mutta samalla on haluttavaa ottaa käyttöön myös myyntitulovero. Se toimii sosiaalivakuutuksena pienentämällä puun tulevaan hintaan liittyvää yksityistä riskiä. Tämä dominoi myyntituloveron vääristävää vaikutusta. Optimaali-sen myyntituloveron suuruus riippuu sen sosiaalivakuutusominaisuudesta ja vääristävyydestä. Aggregaattiriskin vallitessa valtion budjettirajoitus on stokastinen. Optimaalisen pinta-alave-ron ohella on jälleen haluttavaa ottaa käyttöön myyntivero, joka on nyt ei-vääristävä ja jonka metsänomistajat kantavat kokonaisuudessaan. Optimaalisen myyntiveron suuruus riippuu sii-tä, kuinka painotetaan keskenään metsänomistajien veronjälkeisiin tuloihin ja julkiseen menoi-hin liittyvää epävarmuutta.
Asiasanat: yksityinen ja aggregaattiriski, optimaalinen metsäverotus.
JEL-luokitus: D81, H21,H22, Q23.

Introduction

The effects of forest taxation on timber supply has been studied extensively since the 1970s, mostly using the so-called rotation framework under the circumstances of perfect capital markets and certainty (Chang 1982, 1983). The main focus of interest has been in two broad classes of forest taxes, namely annual property taxes (levied on the value of timber and/or land) and yield taxes (levied on the stumpage income). The general flavor of results has been that yield taxes tend to increase rotation periods, while property taxes tend to shorten rotation periods or leave them unchanged. More specifically, it has been shown that a lump sum type land site tax has no effect on the rotation period. Relaxing the assumptions of perfect capital markets and certainty leads to some qualification of these results. Under uncertainty, the timber supply effect of the land site tax among others is sensitive to what is assumed about the behavior of absolute risk-aversion (Koskela 1989a, 1989b).

All these taxation results are, however, derived under three restrictive assumptions. First, it has been assumed that forest owners fully bear the incidence of taxes.[1] This is not necessarily the case, however. In fact, the standard theory of tax incidence implies that forest owners bear forest taxes fully only if either timber supply is totally insensitive to timber price and/or demand for timber is infinitely elastic with respect to timber price (Kotlikoff and Summers 1984). These extreme cases are hard to defend; most empirical studies show that both the demand for and the supply of timber are sensitive to timber prices, but not infinitely so (Johansson and Löfgren 1985, Hetemäki and Kuuluvainen 1992). Analyzing only the supply side effects of forest taxes cannot take due account of other potentially important channels of the influence of taxes. A general purpose of this paper is to rectify this omission in the literature of forest taxation by analyzing forest taxes in a simple partial equilibrium framework, where forest taxes affect not only timber supply, but also timber prices and thereby indirectly the income of forest owners and profits of firms in the forest industry.

Second, using a market equilibrium framework enables us to relax the assumption of the exogenous future timber price uncertainty. Allowing for endogenous determination of timber prices means that the timber price uncertainty is also determined as a part of market equilibrium. This necessitates some assumptions about the underlying cause of timber price

---

[1] As for the earlier literature on the incidence of forest taxation, one should mention Stier and Chang (1983), who looked at the implications of various assumptions about tax incidence, and Johansson and Löfgren (1985), who suggested a static framework for analysis, but did not develop the incidence implications of forest taxes. Aronsson (1990) deals with the incidence of forest taxation within the framework of a roundwood market model and evaluates it empirically using time-series data from Sweden. He did not, however, account for uncertainty or deal with optimal taxation.

uncertainty at the market level. Naturally, there are various possibilities here. Uncertainty may enter the roundwood markets through the demand side as a random shock in the demand for final product and/or in the production technology and/or through the supply side as a random shock in forest growth. It is assumed in what follows that the source of uncertainty is a technological shock in the future production function, which shows up in the stochastic future demand for timber, thus making the future timber price random.

Third, and finally, the earlier literature has analyzed the positive question of what the comparative static effects of forest taxes are. While this kind of analysis is an important element in the analysis of tax reforms, it does not come to grips with welfare aspects of forest taxation when governments face a budget constraint.[2]

The purpose of our paper is to extend the existing literature in all these respects -- by analyzing forest taxation in a market equilibrium context under uncertainty with endogenous prices and by deriving the optimal structure of forest taxes when the social welfare function consists of the expected utility of forest owners and the expected profits of the firm in the forest industry. The forest taxes to be compared are land site tax and gross yield tax. The land site tax has no effect on relative prices and is thus non-distortionary. It is thus natural to regard it as a benchmark. The gross yield tax, on the other hand, is a tax levied on the timber selling revenue and is wide used in various countries.

As for the general features of the framework, there is a stochastic component in the future production function, which makes future demand for timber stochastic. Firms in the forest industry are assumed to be risk-neutral and to maximize their expected profits, while private forest owners are risk-averse and make harvesting decisions so as to maximize the expected utility of the present value of harvest revenue. The agents on both sides of the market are assumed to have rational expectations over uncertain future timber prices. The analysis of market equilibrium under uncertainty easily becomes intractable so that some simplifying assumptions are needed. We use two such assumptions. First, the production function is assumed to be quadratic in terms of timber input. This gives rise to linear demands for timber. Second, in most analyses the forest growth function is assumed to be linear. Though restrictive, the linear growth function is very convenient and carries the message in the sense that the qualitative properties of timber supply with linear growth are similar to those with more realistic logistic growth.[3] Since the forestry sector may give a rise to various kinds of

---

[2] The optimal design of forest taxes when a government faces a budget constraint in setting tax rates was recently analyzed by Amacher and Brazee (1996) and Koskela and Ollikainen (1996). Neither of the papers considers tax incidence issues, however.

[3] Moreover, the combination of uncertainty and logistic growth yields a third-order polynomial for timber supply. This is intractable in market equilibrium analysis, and gives a rise to existence and uniqueness problems (see McCafferty and Driskill 1980). A complete set of results is available from the authors upon request.

risks, we analyze two cases separately. The first is private risk. This means that risk is independent across forest owners implying that some do well and others poorly. As demonstrated in an empirical study by Tilli and Uusivuori (1994) in Finland, the volatility of timber prices may be private risk in at least two ways. First, regional timber prices have varied considerably in a given year independently of their volatility over time. Second, in a given year there have been differences in the prices of different timber assortments which, together with different species distribution on a particular site, causes private risk.[4] In what follows we analyze the private risk separately and in a pure form in order to see its implications in a sharp focus. Under private risk, the law of large numbers guarantees that government tax revenue is deterministic. The second type of risk is aggregate risk or business cycle risk, common to all forest owners, implying that all either gain or lose. Aggregate risk is caused by the volatility of the general timber price level over time. Under aggregate risk government tax revenue is stochastic and private agents ultimately bear all the risk in one way or another so that taxes do have no insurance role.

To anticipate results, it is shown that the nature of timber price risk is crucial for the behavioral, incidence and welfare effects of forest taxes. *Under private risk*, the land site tax is like a pure profit tax and fully borne by forest owners, while the burden of the yield tax is generally shared by both sides of the market. The incidence of the yield tax does not, however, matter qualitatively for the optimal structure of taxation. It is optimal to supplement the neutral land site tax by a distortionary yield tax, because welfare gains are achieved through the yield tax, which provides social insurance by decreasing the after-tax timber price risk. Its level is determined by the trade-off between its social insurance and distortionary properties. *Under aggregate risk*, the optimal yield tax is non-distortionary -- fully borne by forest owners -- and depends solely on the risk attitudes of forest owners towards the variability of private income vis-à-vis the variability of public consumption. The yield tax is neutral, because it cannot provide any social insurance in the presence of aggregate risk.

The paper is organized as follows: we start with a simple market equilibrium model of harvesting decisions and equilibrium determination of timber prices in circumstances of uncertainty about future production technology. Optimal forest taxation with and without endogenous timber prices and with private and aggregate risk is then analyzed.

---

[4] This risk is found to vary quite dramatically in some regions of Finland (the percentage of the change being from -5% in the Helsinki region up to +154% in the Kainuu region).

A model of harvesting decisions with stochastic demand and exogenous timber prices

Demand for timber under stochastic production function

Firms in the forest industry produce final products (pulp, paper) by using roundwood as an input over two periods, now and the future. Current and future timber inputs are denoted by x and z respectively. Production functions are assumed to be identical for both periods and quadratic in terms of roundwood input. The current production function is known, while the future production function is subject to a technological shock, which affects the demand for timber additively. These assumptions can be written as $Q_1 = g(x) = [ba - (1/2)bx]x$ for current and $Q_2 = g(z) = [b\hat{a} - (1/2)bz]z$ for future production respectively, where $\hat{a}$ is assumed to be normally distributed by expectation $\bar{a}$ and variance σ_a^2 so that $\bar{a} = a$.[5] The forest industry firm pays p_1 for the current roundwood and p_2 for the future roundwood. Making the small open economy assumption that the price of the final product is exogenous and normalizing it to one, the decision problem of the risk-neutral firms is to choose x and z so as to maximize the present value of its expected profits as in [1].

[1]
$$\underset{\{x,z\}}{Max} \ \bar{\pi} = [ba - (1/2)bx]x - p_1 x + R^{-1}\{[b\bar{a} - (1/2)bz]z - \bar{p}_2 z\},$$

where $R = 1 + r$ is the interest rate factor in the capital market. This yields the current and future demand for timber as functions of the parameters of the production function and the timber prices as follows

a) $x^d = a - \dfrac{1}{b} p_1$

[2]

b) $z^d = \bar{a} - \dfrac{1}{b} \bar{p}_2$.

A notable feature of demand functions which depend negatively on (expected) prices is their separability. This results from a lack of interrelatedness between current and future production functions. Not surprisingly, timber demands depend negatively on (expected) timber prices.[6]

---

[5] To facilitate the analysis of market equilibrium, the parameters of the production function were chosen so that b will describe the price sensitivity and a the shift parameter of demand for timber function (the demand functions under risk-neutrality are solved in equation [2].
[6] Note that here we have used the expected value for $\bar{p}_2$, which we later solve for rational expectations market equilibrium.

Timber Supply Behavior under Stochastic Timber Price

Forest owners are assumed to be risk-averse and to maximize the expected utility of the present value of harvest revenue. The decision problem is to determine how much to cut today and how much timber to allocate for the future given the current timber price and the probability distribution of the future timber price. The harvesting possibilities - which determine the biological trade-off between current and future harvesting - are given in equation [3]. Forest growth is assumed to follow a modified version of a familiar Lotka-Volterra logistic growth function, which is a valid presentation of growth for an even-aged stand given by $(Q-x)\left[f-\dfrac{k}{2}(Q-x)\right]$, where f is the linear growth component, k is the concavity factor and Q is the original stock of timber (see e.g. Clark 1990, pp. 10-16). According to equation [3] one can harvest in the future an amount z left from the current harvesting $(Q-x)$ plus the growth of the remaining stock.

[3] $$z = (Q-x) + (Q-x)\left[f - \frac{k}{2}(Q-x)\right]$$

The cost of harvesting timber (c) is proportional to the amount of felling.[7] The future timber price $\hat{p}_2$, which is solved later on as a part of market equilibrium, is uncertain and normally distributed by $\hat{p}_2 \approx N(\bar{p}_2, \sigma_p^2)$. The government levies a land site tax T, which the forest owner has to pay regardless cutting or silvicultural activities, so that T is a lump-sum tax. The yield tax τ is a proportional tax and levied on the gross timber selling revenue. In what follows we denote the after-tax timber prices by $p_i^* = p_i(1-\tau)$, $i = 1,2$.

These assumptions lead to the following stochastic present value of harvest revenue V.

[4] $$\hat{V} = (p_1^* - c)x + R^{-1}(\hat{p}_2^* - c)z - (1 + R^{-1})T$$

In what follows, the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. To sharpen the analysis the preferences are described by an exponential utility function $u(V) = -\exp(-AV)$, where $A = -u''(V)/u'(V)$ is the Arrow-Pratt measure of constant absolute risk-aversion (see e.g. Hirschleifer and Riley (1992)). Now the twin assumptions that $\hat{p}_2$ is normally distributed and the utility function is exponential have the major advantage that the forest owner's expected utility maximizing problem can be formulated not in terms of the whole

---

[7] In different contexts Clarke and Reed (1989) assume that harvesting costs are zero, while Thomson (1992) postulates positive harvesting costs.

distribution but in terms of the mean $\overline{V}$ and variance σ_V^2, namely, choosing x (and z via the growth function [3]) so as to maximize $EU = -\exp(v)$, where $v = -A\overline{V} + (1/2)A^2 R^{-2}(1-\tau)^2 z^2 \sigma_p^2$. This is equivalent to[8]

[5] $\qquad Max_{\{x\}} \ M = \overline{V} - \frac{1}{2}A\sigma_V^2 .$

and leads to the following harvesting rule [6].

[6] $\qquad Rp_1 - \overline{p}_2(1 + f - ky) + d(1 + f - ky)z - (1-\tau)^{-1}\psi = 0 ,$

where $\psi = [(r - (f - ky)]c$ and $d = A(1-\tau)R^{-1}\sigma_p^2 > 0$. One should notice that here we have implicitly assumed that uncertainty does not feed back to the forest owner via government expenditures, i.e., we have the case of private risk. Harvesting rule [6] includes many interesting special cases depending on the assumptions concerning the role of growth function, harvesting cost and the term d including uncertainty and risk aversion.

In the case of certainty ($d = 0$), one can distinguish between subcases of zero and positive harvesting costs, and logistic and constant forest growth. Under zero harvesting cost one gets the familiar benchmark case $Rp_1 - p_2(1 + f - ky) = 0$, according to which the marginal return on harvesting is equated to the opportunity cost of harvesting at the margin. Moreover, if $k = 0$, one obtains the case of constant forest growth with the harvesting rule $Rp_1 - p_2(1 + f) = 0$. Under these conditions, the yield tax works like a pure profit tax and has no effect on harvesting irrespective of the assumptions concerning the growth function. With positive harvesting costs, the cutting rule reduces to $Rp_1 - p_2(1 + f - ky) - (1-\tau)^{-1}\psi = 0$, so that now the gross yield tax matters unless $r = f - ky$. In particular, the comparative statics of the gross yield tax in terms of timber supply gives $x_\tau^s < 0$ as $r \geq f$ and is ambiguous otherwise.[9]

---

[8] In general the expected utility depends on the entire probability distribution of p_2. Given the exponential utility and normally distributed p_2 one gets $EU(\hat{V}) = -\int e^{-AV}f(V)dV = -e^{-A(\overline{V} - \frac{A}{2}\sigma_V^2)}$.

Since the expected utility is increasing in $\overline{V} - \frac{1}{2}A\sigma_V^2$ we can take a monotonic transformation and evaluate the exponential utility function by using $\overline{V} - \frac{1}{2}A\sigma_V^2$. This has the very convenient property of being linear in the mean $\overline{V}$ and variance σ_V^2.

[9] Under certainty, the net yield tax, i.e., a yield tax levied on the net timber price $p_i - c$, i=1,2, has no effect on harvesting regardless of the values of f and r.

Allowing for timber price uncertainty $(d > 0)$ with risk-aversion increases current harvesting relative to that under certainty. Current and future supply of roundwood can be solved from the first-order condition [6]. In the general logistic growth case of $k > 0$, current and future timber supply is described by a third-order polynomial, which is highly nonlinear in terms of parameters of the model, but the constant growth function $(k = 0)$ makes it possible to express current and future harvesting under constant growth explicitly as

[7a] $$x^s = Q + \frac{C}{B},$$

[7b] $$z^s = Q(1 + f) - x^s(1 + f),$$

where $B = A(1 + f)^2 R^{-1}(1 - \tau)\sigma_p^2 > 0$, $C = Rp_1 - \bar{p}_2(1 + f) - (1 - \tau)^{-1}\psi < 0$ and $\psi = c(r - f)$. The advantage of this formulation is that x^s and z^s depend linearly on current and expected future prices so that the formulation is convenient in market equilibrium analysis.[10]

The comparative statics of current harvesting under constant growth are straightforward, given by

[8a] $$x^s = x^s(Q, p_1, \bar{p}_2, \sigma_p^2, A, T, \tau)$$
$$+ \quad + \quad - \quad + \quad + \quad 0 \quad ?$$

For the yield tax one gets

[8b] $$x_\tau^s = \frac{C - \psi(1 - \tau)^{-1}}{B(1 - \tau)} < 0 \text{ as } r \gtreqless f.$$

The original stock of timber and the current (expected future) timber price affect current harvesting positively (negatively). Increases in timber price risk and risk-aversion boost current harvesting. As for forest taxes, the land site tax has no effect on the timing of harvesting. The gross yield tax has an a priori ambiguous effect on current harvesting, since it depends on the relationship between the interest rate r and the growth rate of forest f. A sufficient but not a necessary condition for $x_\tau^s < 0$ is that $r \geq f$. Note also that the

---

[10] One should stress that the qualitative properties of timber supply with constant growth are similar to those with logistic growth so that there is no loss of generality in assuming constant growth. A complete set of results is available from the authors upon request.

comparative statics of future timber supply is simply $-(1+f)$ times the comparative statics of current supply in terms of variables other than Q.[11]

Market Equilibrium under Rational Expectations

After developing the properties of the demand for and supply of timber we consider the market equilibrium in the roundwood markets and the role of forests taxes in its determination. Here we stick to the assumption of competitive roundwood markets and assume that the equilibrium prices are determined by equality of demand and supply when private agents are price-takers.

To get the expectation and the variance of the future price, one has to take a stand on how expectations are formed. We make use of the rational expectations hypothesis, which can be defined as a situation in which agents do not make systematic mistakes in forecasting. More precisely, since under rational expectations, agents' subjective beliefs about probability distributions correspond to the objective probability distributions, their expectations are the same as the conditional expectations of the model used to describe the behavior of agents.[12] The next step is to solve for p_1 and $\hat{p}_2$ and develop the properties of a simultaneous equilibrium, where $x^d = x^s$ and $z^d = z^s$. This gives for $\bar{p}_2$ and σ_p^2

[9a]
$$\bar{p}_2 = \frac{b\{\bar{a}[B+bR]+bR(1+f)(a-Q)+(1+f)(1-\tau)^{-1}\psi\}}{S}$$

[9b]
$$\sigma_p^2 = \frac{b^2(B+bR)^2}{S}\sigma_a^2,$$

where $S = B+bR+b(1+f)^2$.[13]

The expected future timber price is expressed in terms of underlying parameters, the original amount of timber, its growth function, interest rate and timber price risk according to [9a]. The timber price risk in turn is determined both by uncertainty associated with the future technology of firms (σ_a^2) and by the parameters of the model including the yield tax according to [9b].

---

[11] Koskela (1989a) contains an analysis of timber supply under more general assumptions concerning risk and preferences than here but excludes harvesting costs, while Ollikainen (1993) analyzes timber supply under assumptions of normality and exponential utility when there are two sources of uncertainty.

[12] See Bray (1985) for an introduction to the issues involved in modeling rational expectations equilibrium under uncertainty.

[13] A complete set of results is available from the authors upon request.

Incidence of Forest Taxes and Their Underlying Determinants

The final step is to develop the properties of the resulting simultaneous equilibrium. This is a straightforward exercise. Before deriving the comparative statics of forest taxes it is useful to develop results for other important parameters of the model.

It can be shown that risk-aversion, the shift parameter reflecting a general level of demand, price sensitivity and timber price risk all affect both current and expected timber price positively. A rise in risk-aversion tends to decrease (increase) the equilibrium current (expected future) timber price. This is because, given the demand for timber, higher risk-aversion leads to higher (lower) current (future) harvesting and thus 'lower' current (higher expected future) timber prices. Changes in the pure risk, given by the timber price variance, affects up to a scale factor like changes in risk-aversion. In fact, the relationship can be expressed as follows: $p_{i\sigma_p^2} = \dfrac{\sigma_p^2}{A} p_{iA}$ $i = 1,2$.[14]

As for the effects of forest taxes, it is obvious that the land site tax T has no price effects because it works like a non-distortionary tax so that $p_{1T} = \bar{p}_{2T} = 0$. The land site tax is thus fully borne by forest owners. An increase in the gross yield tax usually raises the current timber price and decreases expected future timber price, as equations [10] and [11] indicate

[10] $\qquad p_{1\tau} = \Delta^{-1}\left[z^d_{p_2} x^s_\tau\right] = -\dfrac{b\left\{[Rp_1 - \bar{p}_2(1+f)] - 2\psi(1-\tau)^{-1}\right\}}{(1-\tau)S} > 0 \ \text{ as } r \geq f$

[11] $\qquad \bar{p}_{2\tau} = -(1+f)p_{1\tau}$,

where $\Delta > 0$ is the determinant of the system $x^d - x^s = 0$ and $z^d - z^s = 0$, and $z^d_{p_2}, x^s_\tau$ refer to partial derivatives of z^d and x^s respectively. In the case of $r \geq f$ the current timber supply is decreased by a rise in τ so that given the timber demand p_1 increases and $\bar{p}_2$ decreases. If $r < f$, then x^s_τ is ambiguous making also the price effects ambiguous.

Before discussing some special cases we define the current demand elasticity $\eta = \dfrac{x^d_{p_1} p_1}{x^d}$, and the current supply elasticity $\varepsilon = \dfrac{x^s_{p_1} p_1}{x^s}$ using equations [2a] and [7a] as follows

---

[14] This relationship is due to the fact that $x^s_{\sigma_p^2} = \dfrac{\sigma_p^2}{A} x^s_A$ and similarly for z (see equations [7a] and [7b]). A complete set of incidence results is available from the authors upon request.

[12a] $\qquad \eta = \dfrac{p_1}{ab - p_1} > 0$

[12b] $\qquad \varepsilon = \dfrac{Rp_1}{BQ + C} > 0$.

In the extreme case of a linear production function ($b = 0$), the price elasticity of the demand for timber η is infinite. Then $p_{1\tau} = \bar{p}_{2\tau} = 0$ from equations [10] and [11], so that the gross yield tax has no effect on timber prices; the tax burden is fully borne by forest owners. On the other hand, if B increases, e.g. due to a rise in risk-aversion or in the timber price risk, then the price elasticity of timber supply ε decreases. On the other hand, if the production function becomes more concave (b increases), then, by applying L'Hospital's rule, expression [10] reduces in the limit, when $\eta = 0$, to

[10'] $\qquad p_{1\tau|b=\infty} = -\dfrac{Rp_1 - \bar{p}_2(1+f) - 2\psi(1-\tau)^{-1}}{(1-\tau)\left[R + (1+f)^2\right]} > 0$ as $r \geq f$.

Finally, when risk-aversion and/or timber price risk fall, B and C approach zero in [12b] and the price elasticity of supply approaches infinity; one again ends up with equation [10']. In both cases ($b \to \infty$, or $B, C \to 0$) the net price $p_{1\tau}(1 - \tau)$ remains unchanged so that forest owners shift the yield tax burden onto firms in the forest industry fully.

Complete tax shifting in the case of zero demand elasticity occurs simply because the supply response does not lead to a change in the quantity, so that the timber price increases by the amount the yield tax increases. In the case of infinite supply elasticity, the timber supply shifts upwards by the amount of the change in the yield tax so that irrespective of demand elasticity the timber price increases by the same amount.

Having thus pointed out some special cases about tax incidence, we can now summarize the findings in

Proposition 1: *In terms of incidence, (a) land site tax is like a pure profits tax and is fully borne by forest owners, while (b) the burden of the yield tax is generally shared by both sides of the market. More specifically, (c) forest owners pay a high share of the yield tax if the price elasticity of timber demand is high and/or the risk-aversion of forest owners and/or the timber demand volatility are very high so that the price elasticity of timber supply is very low, while (d) the yield tax is largely passed onto firms in the forest industry if the price*

*elasticity of timber demand is low and/or risk-aversion or timber demand
volatility are very low so that price elasticity of timber supply is very high.*

An intuitive explanation goes as follows. If timber demand volatility is high and/or forest owners are very risk-averse, then the timber supply tends to be relatively inelastic. On the other hand, if the production function is close to linear, the demand elasticity tends to be high. In both of these cases, the forest owners bear a major part of the burden of the yield tax, because yield tax passes onto firms via higher prices only to a small extent. The opposite happens if uncertainty is low, forest owners are not very risk-averse and/or timber demand is relatively inelastic. In these circumstances the timber prices faced by firms are "strongly" affected by the gross yield tax. In the unlikely case of $x^s_\tau > 0$, the yield tax, however, overshifts to the forest owners, because current timber price decreases as a result of a rise in τ. Note that $r < f$ does not imply $x^s_\tau > 0$.

Stier and Chang (1983) have looked at the implications of ad valorem property tax under three assumptions concerning tax incidence: forest owners i) fully bear the burden of taxation, ii) partially or fully shift the burden to the demand side. In our paper, the incidence of the land site tax and the yield tax results from the properties of demand for and supply of timber. Our proposition 1 defines precisely when each of these tax incidence assumptions holds. One should also note that Stier and Chang did not allow for the possibility of overshifting, which comes out from our model.

As we noted earlier, the timber price risk in equilibrium also depends on the parameters of the model. The partial derivative of [9b] with respect to the yield tax can be expressed in terms of timber price variance as follows

[13]
$$\sigma^2_{p\tau} = 2\sigma^2_p \frac{b^2 B_\tau (1+f)^2}{S^2} < 0$$

where $B_\tau = -A(1+f)^2 R^{-1}\sigma^2_p < 0$. Naturally, $\sigma^2_{pT} = 0$.

Thus we have from [9b] and [13]

Corollary 1: *Under private stochastic demand, the equilibrium timber price risk is
independent of the land site tax but is a decreasing function of the yield tax.*

An increase in the yield tax makes the future timber supply curve flatter in terms of future price. Therefore, a given demand fluctuation gives rise to a smaller fluctuation in prices and a larger fluctuation in quantities. Figure 1 provides a simple geometric illustration of the effect of yield tax on the expected timber price and timber price variance (see also Newbery

and Stiglitz (1981), p. 134). A downward-sloping future demand function is stochastic and fluctuates according to the normally distributed density function σ_a^2, the expected demand being $\bar{z}^d$. An upward sloping future supply function is z^s and expected equilibrium price is determined by the intersection of demand and supply as $\bar{p}_2$, while the variance for price is σ_p^2 at point A. A rise in the yield tax will increase future timber supply so that the supply function z^s shifts downwards. On the other hand, the timber price variance decreases to because the z^s curve becomes flatter. In the new equilibrium the supply curve is $z^{s'}$, the expected timber price $\bar{p}_2'$, the equilibrium amount of timber used $\bar{z}'$ and the variance $\sigma_p^{2'}$.

Figure 1. A rise in yield tax, expected timber price and timber price risk
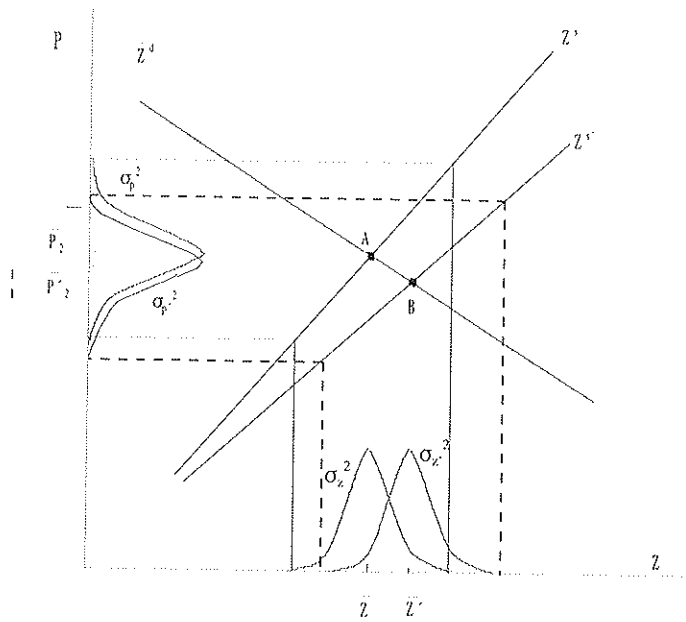
Higher yield tax flattens the supply curve resulting to new market equilibrium B.
Given the normally distributed demand fluctuation, the timber price fluctuation
decreases ($\sigma_p^2 > \sigma_{p'}^2$) while the quantity fluctuation increases ($\sigma_z^2 > \sigma_{z'}^2$).

Optimal forest taxation under private and aggregate risk

Private risk

The above analysis of roundwood market equilibrium and its comparative static properties provides a basis on which to consider the issue of optimal forest taxation from the viewpoint of society. Before doing this one has to clear up few things. First, in the line with the optimal taxation literature, it is assumed that forest taxes are chosen so as to keep the government

tax revenue given.[15] Second, some assumptions concerning the nature of technological uncertainty have to be made. We assume first that the government budget constraint is deterministic. This means either that the risk is private, i.e., that it is identically and independently distributed among individual forest owners or that the government is risk-neutral, interested only in the expected value of tax revenues and that the possible stochasticity does not feed back to the private sector.[16] Any of these assumptions mean that the government tax revenue requirement remains deterministic. The present value of government forest tax revenues can then be written as

[14] $$\bar{T} = (1 + R^{-1})T + \tau\left[p_1 x + R^{-1}\bar{p}_2 z\right].$$

In what follows the tax revenues (the RHS of equation 14) are denoted by G.

The social planner's problem - acting as a "benevolent dictator" - is to choose the land site tax T and yield tax rate τ so as to maximize the social welfare function subject to both the government budget constraint [14] and the behavioral and market constraints analyzed in section 2.

The social welfare function consists of the sum of the expected indirect objective function M^* of the representative forest owner and the expected indirect profit function of the representative firm in the forest industry $\bar{\pi}^*(\cdot)$ in terms of tax parameters and timber prices[17] so that

[15] $$W = M^*(T, \tau, p_1, \bar{p}_2, \sigma_p^2) + \bar{\pi}^*(p_1, \bar{p}_2).$$

---

[15] See e.g. Atkinson and Stiglitz (1980).

[16] The third justification for risk neutrality on the part of government can be developed by utilizing Arrow and Lind (1970). Provided that the number of tax payers is sufficiently large, the sector (here the forest sector) is not large relative to the economy as a whole, and that the (random) cost to any tax payer of the sector is not correlated with any existing uncertainty in their income stream, then the risk premium of each tax payer and the total risk premium to the society tends to zero. Given this, the public sector should act as if it were risk-neutral. The private risk case has been analyzed in an optimal income taxation framework e.g. by Varian (1980). If risk is aggregate, then the government tax revenues are stochastic and in one way or another private agents must ultimately bear all the risk, whether through random taxes, random government expenditures or random government deficits. We come back to this later on. Recall also the discussion in the introduction about the empirical relevance of private risk.

[17] M^* and $\bar{\pi}^*$ are defined as the maximum expected utility and the maximum expected profit respectively and can be obtained by substituting equations [2] and [7] for x and z in the objective functions of forest owners and firms.

Thus we abstract from the differences between forest owners and firms mainly because we are interested in the trade-off between the efficiency and insurance aspects of taxation (see e.g. Starrett (1988) Ch. 8, for an account of how an extension to allow for the heterogeneity of agents can be done). Before any private decisions are made, the government is assumed to announce a tax policy and to commit itself to it. The first-order conditions for the social welfare maximization under the binding tax revenue requirement in a Stackelberg equilibrium with the government as the dominant player can be obtained by setting the partial derivatives of the Lagrangian function $\Omega = W - \lambda(\overline{T} - G)$ with respect to T and τ zero.[18]

As the land site tax does not affect timber supply and thereby equilibrium prices, its optimum is expressed in [16].

[16] $\qquad \Omega_T = -(1 + R^{-1}) + \lambda(1 + R^{-1}) = 0 \Leftrightarrow \lambda = 1$.

At the optimum, land site tax is chosen so that the marginal utility $(1 + R^{-1})$ is equal to the marginal cost in terms of tax revenues at the value of the Lagrangian multiplier $\lambda > 0$ $\lambda(1 + R^{-1})$.

The optimal condition for the gross yield tax is much more complicated as equation [17] suggests. The yield tax rate affects the welfare of the forest owners and firms in the forest industry not only directly, but also indirectly by changing the equilibrium prices and the equilibrium variance of the future timber price. Finally, one has also to account for the timber supply and timber price effects of the yield tax via the government budget constraint [14]. Allowing for these effects gives

[17] $\qquad \Omega_\tau = M_\tau^* + \lambda G_\tau + \left[M_{p_1}^* + \overline{\pi}_{p_1}^* + \lambda G_{p_1} \right] p_{1\tau} + \left[M_{\overline{p}_2}^* + \overline{\pi}_{\overline{p}_2}^* + \lambda G_{\overline{p}_2} \right] \overline{p}_{2\tau} + M_{\sigma_p^2}^* \sigma_{p\tau}^2 = 0$,

which implicitly defines the optimal value of the gross yield tax τ^*.

In order to see whether the yield tax is needed at all and how far it should be pushed when the land site tax has been set optimally, one needs evaluation of the partial derivative of the

---

[18] In the literature on dynamic games this kind of equilibrium would be described as "open-loop equilibrium", c.f., Basar and Olsder (1982). If government does not want or is not able to enter into binding commitment, but reoptimizes at the beginning of each period, then one ends up in a Nash equilibrium without commitment. Persson and Tabellini (1990) contains an excellent overview of main recent developments in this area.

Lagrangian at the corner solutions where $\tau = 0$ or $\tau = 1$ (see Appendix A for the derivation of equations [18a] and [18b]). This yields

[18a] $\Omega_\tau (T = T^*, \tau = 0) = AR^{-2} z^2 (\sigma_p^2 - \frac{1}{2} \sigma_{p\tau}^2) > 0$

[18b] $\Omega_\tau (T = T^*, \tau = 1) = [Rp_1 - \bar{p}_2(1+f)] R^{-1} x_\tau^s (1+\delta) \prec 0$ when $r \neq f$,

where $\delta = \dfrac{f z_{p_2}^d x_{p_1}^s}{\Delta} > 0$ is the correction factor due to the endogeneity of timber prices.

The interpretation of equation [18a] is that under private risk, it is welfare-increasing to introduce a distortionary yield tax at the margin. This affects the marginal return and costs of harvesting, thus decreasing timber price risk, which is beneficial for risk-averse forest owners, i.e., the social insurance value of the yield tax outweighs its distortionary effect at the margin. The yield tax has a direct negative effect on the risk (the term σ_p^2) but an indirect negative effect by decreasing σ_p^2 (the term $-\frac{1}{2} \sigma_{p\tau}^2$). If there is no uncertainty or forest owners are risk-neutral, then [18a] is zero and τ is not needed once $T = T^*$. How far should one go in increasing the yield tax rate as a risk-sharing device? The partial derivative of the Lagrangian [18b] at the margin suggests that with $r \neq f$ the optimal yield tax is less than 100% regardless of the sign of the effect of the yield tax on timber supply.

The optimal yield tax rate $0 < \tau^* < 1$ under endogenous prices is given by equation [19]

[19] $\tau^* = \dfrac{AR^{-1} z^2 (\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2)}{AR^{-1} z^2 (\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2) - [Rp_1 - \bar{p}_2(1+f)] R^{-1} x_\tau^s (1+\delta)} > 0$.

In the special case of exogenous prices, equation [19] reduces to

[20] $\tau^*_{exog} = \dfrac{Az^2 \sigma_p^2}{Az^2 \sigma_p^2 - [Rp_1 - \bar{p}_2(1+f)] x_\tau^s} > 0$.

Thus when $r \neq f$ the optimal yield tax with exogenous prices is affected by its social insurance role, the term $Az^2 \sigma_p^2$, and by its distortionary role, the term $[Rp_1 - \bar{p}_2(1+f)] x_\tau^s$.

In the presence of endogenous prices, these are qualitatively modified by incidence factors $-\frac{1}{2}(1-\tau)\sigma_{p\tau}^2 Az^2 > 0$ and $\delta > 0$.

Thus we have

Proposition 2: *If the land site tax has been set optimally under private risk, then regardless of the incidence of the yield tax (a) it is desirable to introduce the distortionary yield tax at the margin, (b) the optimal yield tax is less than 100% and is determined by the trade-off between its social insurance and distortionary properties, (c) the optimal yield tax is zero if there is no uncertainty or forest owners are risk-neutral.*

In the presence of private risk, the yield tax generally has both distortionary and social insurance effect. The former has to do with the question of how timber supply reacts to changes in τ, the latter with the fact that the yield tax affects the after-tax risk. The higher the effect of τ on timber supply is, the lower the optimal yield tax rate is, ceteris paribus. Moreover, the higher risk-aversion and/or timber price risk is, the higher the optimal yield tax is, ceteris paribus.

The first analysis of the excess burden of taxation in the forest economics literature is given by Gamponia and Mendelsohn (1987). They used the rotation framework with the assumptions of exogenous prices and perfect foresight to study the relative excess burden -- the present value of lost income from tax-induced distortions -- of the yield and property taxes. They were not, however, able to provide an analytical solution to the problem and were forced to rely on numerical simulations. According to their simulations property tax results in slightly larger excess burden than yield tax. Since the yield tax lengthens and property tax shortens rotation, a specific combination of both taxes could have a neutral effect. This property of neutrality is, however, a desirable goal only in the presence of certainty. Under private risk, the optimal tax scheme is a trade-off between (negative) distortionary effects and (positive) social insurance effects, as the proposition 2 suggests.

As for the role of the incidence of forest taxes, one derives from the comparison between [19] and [20]

Corollary 2a:*Allowing for the endogenous timber prices under private risk increases both the social insurance and distortionary effects of the yield tax and has thus an a priori ambiguous effect on its optimal level.*

The optimal yield tax is distortionary, reflecting the trade-off between its insurance and distortionary properties. The distortionary effect of the yield tax is higher under endogenous prices (the term $\delta > 0$) so that the optimal yield tax is lower, ceteris paribus. This is because the price changes reinforce the distortionary effect of the yield tax. But the yield tax also affects the timber price risk indirectly by decreasing the variance of future timber price (the term $-\frac{1}{2}(1-\tau)\sigma_{p\tau}^2 Az^2 > 0$). This tends to reinforce its social insurance role and thus lead to the higher yield tax, ceteris paribus. As for the incidence we have

Corollary 2b:*If price elasticity of timber demand is between zero and infinity and the yield tax effect on timber supply is negative, then the burden of the yield tax is shared at the optimum by forest owners and firms in forest industry.*

Aggregate risk

Thus far we have assumed that either risk affecting future demand for timber was private and thus did not exist at the aggregate level, or that the government was risk-neutral and that the potential stochasticity did not feed back to the private sector. Thus we could write the government tax revenue requirement [14] as deterministic. But to the extent that volatility of future timber demand is a business cycle phenomenon in the sense that all in the private sector gain or lose, the earlier justifications for treating government tax revenue as deterministic are not convincing, particularly where the forest sector is large relative to the scale of the economy. This raises the question of the structure of forest taxation under aggregate risk, where government tax revenue is stochastic and private agents must after all bear all the risk in one way or another.[19]

The welfare implications of aggregate risk depend on substitutability between the present value of private income V and the publicly provided consumption G on the one hand and the risk attitudes towards their variability on the other. If V and G are perfect substitutes and risk attitudes toward them are similar so that only their sum $V + G$ matters, then we have $V + G = (p_1 - c)x + R^{-1}(\hat{p}_2 - c)z$. Neither the land site tax T nor the gross yield tax τ enter the target function so that they do not matter at all. This is an example of the stochastic version of the Ricardian equivalence theorem (see e.g. Barro 1989). A simple way of postulating imperfect substitutability between private income and public consumption is to use an additively separable utility function.

---

[19] The implications of aggregate risk have been analyzed in a different context of risk-sharing between generations by Gordon and Varian (1988).

To sharpen the analysis the preferences associated with stochastic tax revenues G, distributed to forest owners as public consumption, are described by an exponential utility function $u(G) = -\exp(-A_g G)$, where $A_g = -u''(G)/u'(G)$ is the Arrow-Pratt measure of constant absolute risk-aversion associated with G. Assuming that the future timber price, which will be solved as a part of equilibrium, is normally distributed enables writing the public consumption part of the expected utility as $EY = -\exp(A_g N)$, where $N = -\overline{G} + (1/2)A_g R^{-2} z^2 \tau^2 \sigma_p^2$. When the private income and public consumption enters the utility function of forest owners in an additively separable way we have

[21] $$EU^0 = -\exp(AM) - \exp(A_g N),$$

where M is defined by [5].

Choosing x so as to maximize EU^0 gives

[22] $$EU_x^0 = -\exp(AM)AM_x - \exp(A_g N)A_g N_x = 0,$$

where

$$M_x = (p_1^* - c) - R^{-1}(\overline{p}_2^* - c)(1+f) + AR^{-2}(1+f)z(1-\tau)^2 \sigma_p^3$$
$$N_x = \tau p_1 - \tau R^{-1}\overline{p}_2(1+f) + A_g R^{-2}(1+f)z\tau^2 \sigma_p^3.$$

One cannot get anything clear-cut out of the properties of the harvesting rule under aggregate uncertainty from [22]. A useful way of simplifying the analysis of [22] is to assume that the land site tax has already been set optimally, which is equivalent to an optimal redistribution of uncertainty over private income and public consumption in this economy.[20] Choosing T so as to maximize EU^0 and noting that it has no indirect effect via timber supply on EU^0 because of the envelope theorem yields

[23] $$EU_T^* = -\exp(AM)A(1 + R^{-1}) + \exp(A_g N)A_g(1 + R^{-1}) = 0.$$

According to [23] the land site tax has been chosen optimally when the marginal utility from private income ($\exp(AM)A(1 + R^{-1})$) is equal to the marginal utility the forest owner derives from public consumption $\exp(A_g N)A_g(1 + R^{-1})$.

---

[20] This is the approach by Samuelson (1956) in his famous article *Social Indifference Curves*.

The optimal condition [23] implies that $\exp(AM)A = \exp(A_g N)A_g$. Assuming this to hold reduces the first-order condition [22] for timber supply to the relatively simple form

[24] $\qquad EU^0_{x|T=T^*} = M_x + N_x = 0$.

Utilizing the expressions for M_x and N_x produces the following harvesting rule [25] when $T = T^*$

[25] $\qquad Rp_1 - \bar{p}_2(1+f) + R^{-1}z(1+f)\sigma_p^2\Lambda - \psi = 0$,

where $\Lambda = A^2(1-\tau)^2 + A_g^2\tau^2 > 0$ and $\psi = c(r-f)$. This harvesting rule under aggregate risk differs slightly from the previously derived rule [6] for private risk. Now the risk consists of two components; the risk associated with private income $A^2(1-\tau)^2 R^{-1}z(1+f)$ and the risk associated with the value of government consumption $A_g^2\tau^2 R^{-1}z(1+f)$. This latter component reflects the fact that the stochastic government tax revenue feeds back to individual behavior.

The second-order condition is given in [26].

[26] $\qquad EU^0_{xx|T=T^*} = -R^{-1}(1+f)^2\sigma_p^2\Lambda < 0$

The comparative statics of the aggregate risk model for both current and future timber price, timber price risk, land site tax and risk-aversion associated with the owner's income are qualitatively similar to the private case,[21] but for the yield tax, we have

[27] $\qquad x_{\tau|T=T^*} = \dfrac{-\left[Rp_1 - \bar{p}_2(1+f) - \psi\right]\Lambda_\tau}{D\Lambda} \geq (<)0$,

where $\Lambda_\tau = -2\left[A(1-\tau) - A_g\tau\right] \geq (<)0$ as $\tau \geq (<)\dfrac{A}{A_g + A}$.

In the presence of aggregate risk the yield tax has an a priori ambiguous effect on current and thereby on future timber supply. While a rise in the yield tax decreases the variability of

---

[21] A complete set of results is available from the authors upon request.

after-tax timber revenues, which tends to decrease current timber supply, the variability of government consumption becomes higher, which tends to increase timber supply. The relative strength of these opposite effects depends both on the level of the yield tax, and on the risk attitudes towards the variability of after-tax timber revenue vis-à-vis the variability of government consumption. More precisely we have

Proposition 3: *A property of timber supply under aggregate risk:*

The effect of the yield tax on timber supply under aggregate risk a) depends on the level of the yield tax as well as the risk attitudes towards the variability of private after-tax timber revenues vis-à-vis the variability of government consumption, and is a priori ambiguous, b) is positive, negative or zero in the case of equal risk-attitudes if the yield tax is less, more or equal to 50%.

It is important to point out that under aggregate risk, which has not been analyzed previously in the literature, the relationship between the interest rate and the forest growth does not matter, while in the case of private risk it is crucial.

Given the timber demand, the properties of timber supply determine what is the incidence of the yield tax. One can immediately say

Proposition 4: *The burden of the yield tax under aggregate risk is generally a priori ambiguous, but is a) shared by both sides of the markets if* $\tau < \dfrac{A}{A_g + A}$,

b) fully borne by forest owners if $\tau = \dfrac{A}{A_g + A}$, *c) overshifted to the forest*

owners if $\tau > \dfrac{A}{A_g + A}$.

Intuition suggests that if $\tau < (>)\dfrac{A}{A_g + A}$ then the yield tax affects timber supply negatively (positively) so that the timber price increases (decreases). Hence the burden of the yield tax is shared between the firms in the forest industry and forest owners (overshifted to the forest owners because the current timber price decreases as a result of a rise in τ). In case $\tau = \dfrac{A}{A_g + A}$ the yield tax is fully borne by forest owners because timber prices do not react to the yield tax.

What is the effect of the yield tax on the timber price risk? By using the equilibrium condition $z^d = z^s$, one can solve the future timber price and its variance in terms of exogenous parameters to obtain

[28] $\qquad \sigma_p^2 = \dfrac{[b(D(1+f)+bR)]^2}{(DS')^2} \sigma_u^2,$

where $D = R^{-1}(1+f)\sigma_p^2 \Lambda$ and $S' = D(1+f)+bR+b(1+f)^2$. Thus the effect of yield tax on future timber price variance is

[29] $\qquad \sigma_{p\tau|T=T^*}^2 = 2\sigma_p^2 \dfrac{(1+f)b^2(1+f)^2}{S'^2} D_\tau \geq (<)0 \text{ as } \Lambda_\tau \geq (<)0$

where $D_\tau = R^{-1}(1+f)\sigma_p^2 \Lambda_\tau \geq (<)0 \text{ as } \tau \geq (<)\dfrac{A}{A_g+A}$.

Unlike the case of private risk, the effect of the yield tax on timber price risk is ambiguous a priori. Thus we are in a position to state

Corollary 3: *Under aggregate stochastic demand, the effect of yield tax on the equilibrium timber price risk is a) a priori ambiguous, b) positive (negative)*
$\quad if \tau < (>)\dfrac{A}{A_g+A}, \; c) \; zero \; if \; \tau = \dfrac{A}{A_g+A}.$

Intuition suggests that if $\tau < (>)\dfrac{A}{A_g+A}$, the slope of timber supply curve becomes flatter (steeper) so that given demand fluctuations, price fluctuations will be smaller (larger), but if $\tau = \dfrac{A}{A_g+A}$, then the slope of the timber demand curve remains unchanged and given demand fluctuations, the timber price risk does not change.

What about the optimal forest taxation under aggregate risk? First of all, one should notice that the usual separation between the concern with the tax structure and the concern with the level of taxation is no longer complete; varying tax rates will in general also affect also variability of government consumption in a way which has welfare implications, as we have already seen. Hence, in sharp contrast with the private risk, a simultaneous optimization of tax rates and the level of public consumption has to be carried out.

We derived the necessary condition for the optimal the land site tax above. Assume that the social welfare function consists of the forest owners' indirect utility function, public consumption and the forest industry's indirect profit function so that

[30] $SW_{|T=T^*} = M^*(\tau, p_1, \bar{p}_2, \sigma_p^2) + N^*(\tau, p_1, \bar{p}_2, \sigma_p^2) + \bar{\pi}^*(p_1, \bar{p}_2)$.

Choosing the yield tax rate so as to maximize [30] produces, after some manipulations,[22]

[31] $SW_{\tau|T=T^*} = M_\tau^* + N_\tau^* - \frac{1}{2}R^{-2}z^2\Lambda\sigma_{p\tau}^2 = -R^{-2}z^2(\sigma_p^2\Lambda_\tau + \frac{1}{2}\sigma_{p\tau}^2\Lambda) = 0$, as $\Lambda_\tau = 0$.

Now $\operatorname{sgn}\sigma_{p\tau}^2 = \operatorname{sgn}\Lambda_\tau$ from [29] so that $SW_\tau = 0$ holds since $\Lambda_\tau = 0$. Then $x_\tau^s = 0$ and [31] yields

[32] $\tau_{|T=T^*}^{**} = \frac{A}{A+A_g} > 0$.

Thus we have

Proposition 5: *If the land site tax has been set optimally under aggregate risk, and private after-tax timber revenue and government consumption are imperfect substitutes, it is desirable to use the yield tax as a risk-shifting device between after-tax timber revenue and government consumption. The optimal yield tax is non-distortionary and depends only on the risk attitudes towards variability of after-tax timber revenue vis-à-vis government consumption.*

Under aggregate risk but unlike private risk the yield tax has no social insurance role. It cannot redistribute from those who are lucky to those who are unlucky. The role of the yield tax is to affect the division of risk between after-tax timber revenues and government consumption. The optimal division of risk reflects risk attitudes towards variability in both components. At the optimum the yield tax is non-distortionary, its level depending on the

---

[22] Differentiating [30] with respect to τ first produces

$SW_{\tau|T=T^*} = M_\tau^* + N_\tau^* + \left(M_{p_1}^* + N_{p_1}^* + \bar{\pi}_{p_1}^*\right)p_{1\tau} - (1+f')\left(M_{\bar{p}_2}^* + N_{\bar{p}_2}^* + \bar{\pi}_{\bar{p}_2}^*\right)p_{1\tau} + \left(M_{\sigma_p^2}^* + N_{\sigma_p^2}^*\right)\sigma_p^2 = 0$,   where

the relationship $\bar{p}_{2\tau} = -(1+f)p_{1\tau}$ has been used. Applying the envelope results $\bar{\pi}_{p_1}^* = -x < 0$,

$\bar{\pi}_{\bar{p}_2}^* = -R^{-1}z < 0$, $M_{p_1}^* + N_{p_1}^* = x$ and $M_{\bar{p}_2}^* + N_{\bar{p}_2}^* = R^{-1}z$ and $M_{\sigma_p^2}^* + N_{\sigma_p^2}^* = -\frac{1}{2}R^{-2}z^2\Lambda < 0$ produces [31].

risk attitudes as follows. If forest owners are more (less) averse to private income risk than to public consumption risk, the optimal yield tax is higher (less) than 50%. This is natural; if agents are very worried about private income variability due to the volatility of the future timber price, then a tax system which lowers private income risk at the expense of public consumption risk is to be preferred. A 'high' yield tax rate does precisely this.

As for the incidence of the yield tax, [32] and [29] produce the novel finding that

Corollary 4: *Allowing for endogenous timber prices under aggregate risk has no effect on the optimal yield tax. It is non-distortionary and is fully borne by forest owners. At the optimum, timber price risk is independent of the yield tax.*

Concluding remarks

This paper has constructed a simple market equilibrium framework which incorporates stochastic demand for timber and timber price risk under rational expectations. This framework enabled consideration of the properties of timber supply, incidence and welfare effects of forest taxes under endogenous timber prices. For the first time in forest economic literature we have made the distinction between private and aggregate risk. It turned out that the nature of timber price risk is crucial for the behavioral effects of forest taxes and other properties as well.

In the presence of private risk the land site tax is like a pure profits tax and is fully borne by forest owners, while the yield tax is distortionary and its burden is generally shared by both sides of the market. At the social optimum, it is desirable to supplement the neutral land site tax by the yield tax, because it provides social insurance which overweighs its distortionary effect. If the risk is aggregate, then the government budget constraint is stochastic and private agents must ultimately bear all the risk involved in one way or another. The burden of the yield tax is a priori ambiguous and depends on the relationship between the level of the yield tax and risk attitudes towards variability of after-tax timber revenue vis-à-vis government consumption. In this case, it is desirable to use the yield tax rate as a risk-shifting device between after-tax timber revenue and government consumption. The optimal yield tax is non-distortionary -- fully borne by forest owners -- because taxation provides no social insurance.

What is the policy relevance of our analysis? We have repeatedly pointed out how important the nature of timber price risk is for the properties of timber supply function, tax incidence and for the issues of optimal forest taxation. Timber price risk concerns also forest industry: under private risk part of the yield tax passes on to firms in forest industry, while under

aggregate risk forest owners bear the whole tax. Thus it is clearly an important empirical question to find out which type of risk is more relevant in practice. There are several possibilities how this can be done. One could study the timber price volatility over time and in cross-sections. An important, supplementary agenda is to econometrically estimate timber supply functions and test for the alternative specifications arising from the hypotheses of private and aggregate risk.

Literature cited

Amacher, G.S., and Brazee, R.J. 1996. Designing Forest Taxes with Budget Constraints. J. Environ. Econ. Manage. In press.

Aronsson, T. 1990. The Incidence of Forest Taxation - A Study of the Swedish Roundwood Market. Scand. J. Econ. 92: 65-79.

Arrow, K.J., and Lind, R.C. 1970. Uncertainty and the Evaluation of Public Investment Decisions. Am. Econ. Rev. 60: 364-378.

Atkinson, A.B., and J.E. Stiglitz. 1980. Lectures on Public Economics. McGraw-Hill, New York.

Barro, R.J. 1989. The Ricardian Approach to Budget Deficits. J. Econ. Persp. 3: 37-54.

Basar, T., and G. Olsder. 1982. Dynamic Non-Cooperative Game Theory. Academic Press, London, United Kingdom.

Bray, M. 1985. Rational Expectations, Information and Asset Markets: An Introduction. Ox. Econ. Pap. 37: 161-195.

Chang, S.J. 1982.: An Economic Analysis of Forest Taxation's Impact on Optimal Rotation Age. Land Econ. 58: 310-323.

Chang, S.J. 1983. Rotation Age, Management Intensity, and the Economic Factors of Timber Production: Do Changes in Stumpage Price, Interest Rate, Regeneration Cost and Forest Taxation Matter? For. Sci. 29: 267-277.

Clark, C.W. 1990. Mathematical Bioeconomics. The Optimal Management of Renewable Resources. Second Edition. John Wiley and Sons, New York.

Clarke, H.R., and Reed, W.J. 1989. The Tree-Cutting Problem in a Stochastic Environment. J. Econ. Dyn. and Con. 13: 565-595.

Gamponia, V., and Mendelsohn, R. 1987. The Economic Efficiency of Forest Taxes. For. Sci. 33: 367-378.

Gordon, R.H., and Varian, H.R. 1988. Intergenerational Risk Sharing. J. Pub. Econ. 37: 185-202.

Hirschleifer, J., and J.G. Riley. 1992. The Analytics of Uncertainty and Information. Cambridge University Press, Cambridge, United Kingdom

Johansson, P.-O., and K.-G. Löfgren. 1985. The Economics of Forestry & Natural Resources. Basil Blackwell, Oxford, United Kingdom.

Koskela, E. 1989a. Forest Taxation and Timber Supply under Price Uncertainty: Perfect Capital Markets, For. Sci. 35: 137-159.

Koskela, E. 1989b. Forest Taxation and Timber Supply under Price Uncertainty: Credit Rationing in Capital Markets. For. Sci. 35: 160-172.

Koskela, E., and Ollikainen, M. 1996. Optimal Forest Taxation with Multiple-Use Characteristics of Forest Stand. Environ. Res. Econ. In press.

Kotlikoff, L., and Summers, L. 1984. Tax Incidence. P. 1043-1092 in Handbook of Public Economics, Vol II, Auerbach, A.J. and Feldstein, M.S. (eds). North Holland,

Amsterdam, The Netherlands.

McCafferty, S., and Driskill, R. 1980. Problems of Existence and Uniqueness in Nonlinear Rational Expectations Models. Econometrica 48: 1313-1317.

Newbery, D.M., and J.E. Stiglitz. 1981. The Theory of Commodity Price Stabilization: A Study in the Economics of Risk. Clarendon Press, Oxford, The Netherlands.

Ollikainen, M. 1993. A Mean-Variance Approach to Short-Term Timber Selling and Forest Taxation under Multiple Sources of Uncertainty. Can. J. For. Res. 23: 573-581.

Persson, T., and G. Tabellini. 1990. Macroeconomic Policy, Credibility and Politics. Harvester Academic Publishers, New York.

Samuelson, P.A. 1956. Social Indifference Curves. Quarterly J. Econ. 70: 1-22.

Starrett, D.A. 1988. Foundations of Public Economics. Cambridge University Press, New York.

Stier, J.C., and Chang, S.J. 1983. Land Use Implications of the Ad Valorem Property Tax: The Role of Tax Incidence. For.Sci. 29: 702-712.

Thomson, T.A. 1992. Optimal Forest Rotation When Stumpage Prices Follow a Diffusion Process. Land Econ. 68: 329-342.

Tilli, T., and Uusivuori, J. 1994. Short-Term Timber Price Variations in Finland during 1985-91 and 1991-93. Pellervo Economic Research Institute, Reports and Discussion Papers No. 132. (In Finnish with English summary.)

Varian, H.R. 1980. Redistributive Taxation as Social Insurance. J. Pub. Econ. 14: 49-68.

Varian, H.R. 1992. Microeconomic Analysis. W.W. Norton and Company, New York.

Appendix A. Derivation of optimal tax results for private risk

This appendix derives the optimal forest taxation results in the case of private uncertainty. The partial derivatives of the Lagrangian with respect to land site and yield taxes are

(1) $\quad \Omega_T = -(1+R^{-1}) + \lambda(1+R^{-1}) = 0 \Leftrightarrow \lambda = 1$

(2) $\quad \Omega_\tau = M_\tau^* + \lambda G_\tau + \left[M_{p_1}^* + \overline{\pi}_{p_1}^* + \lambda G_{p_1} \right] p_{1\tau} + \left[M_{\overline{p}_2}^* + \overline{\pi}_{\overline{p}_2}^* + \lambda G_{\overline{p}_2} \right] \overline{p}_{2\tau} + M_{\sigma_p^2}^* \sigma_{p\tau}^2 = 0$

When the land site tax has been chosen optimally, equation (2) can be reduced by applying (1) to

(3) $\quad \Omega_\tau(T = T^*) = AR^{-2}(1-\tau)z^2\sigma_p^2 + \tau\left[p_1 - \overline{p}_2 R^{-1}(1+f) \right]x_\tau^s + \left[M_{p_1}^* + \overline{\pi}_{p_1}^* + G_{p_1} \right] p_{1\tau}$
$\quad\quad + \left[M_{\overline{p}_2}^* + \overline{\pi}_{\overline{p}_2}^* + G_{\overline{p}_2} \right] \overline{p}_{2\tau} + M_{\sigma_p^2}^* \sigma_{p\tau}^2 = 0 ,$

where the following facts have been utilized:

$M_\tau^* = (p_1 x + R^{-1}\overline{p}_2 z)(1 + R^{-1})^{-1} M_T^* + AR^{-2}(1-\tau)z^2\sigma_p^2$

$G_\tau = (p_1 x + R^{-1}\overline{p}_2 z)(1 + R^{-1})^{-1} G_T + \tau\left[p_1 - \overline{p}_2 R^{-1}(1+f) \right]x_\tau^s .$

Utilizing the envelope theorem for the partial derivatives of M^* and π^* produces $M_{p_1}^* = (1-\tau)x > 0$, $M_{\overline{p}_2}^* = (1-\tau)zR^{-1} > 0$ and, $\overline{\pi}_{p_1}^* = -x < 0$ and $\overline{\pi}_{\overline{p}_2}^* = -R^{-1}z < 0$.[1] Noting that $G_{p_1} = \tau x + \tau\left[p_1 x_{p_1} + \overline{p}_2 R^{-1} z_{p_1} \right] > 0$, $G_{\overline{p}_2} = \tau R^{-1}z + \tau\left[p_1 x_{\overline{p}_2} + \overline{p}_2 R^{-1} z_{\overline{p}_2} \right] > 0$ and using the facts that $\overline{p}_{2\tau} = -(1+f)p_{1\tau}$, $z_{p_1}^s = -(1+f)x_{p_1}^s$ and $z_{\overline{p}_2}^s = -(1+f)x_{\overline{p}_2}^s$ make it possible to write equation (3) as follows

$\Omega_\tau(T = T^*) = AR^{-2}(1-\tau)z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2) + \tau\left[Rp_1 - \overline{p}_2(1+f) \right]R^{-1}(x_\tau^s - p_{1\tau}x_{p_1}f) ,$

where $M_{\sigma_p^2}^* = -AR^{-2}(1-\tau)^2 z^2 < 0$ has been used.

Utilizing the fact that $p_{1\tau} = \Delta^{-1}\left[z_{\overline{p}_2}^d x_\tau^s \right]$, one can express the term $p_{1\tau}x_{p_1}f$ as $\delta = \dfrac{fz_{\overline{p}_2}^d x_{p_1}^s}{\Delta} > 0$, which is the correction factor due to the endogeneity of timber prices. Thus one obtains

(4) $\quad \Omega_\tau(T = T^*) = AR^{-2}(1-\tau)z^2(\sigma_p^2 - \frac{1}{2}(1-\tau)\sigma_{p\tau}^2) + \tau\left[Rp_1 - \overline{p}_2(1+f) \right]R^{-1}x_\tau^s(1+\delta)$

---

[1] For the envelope theorem, see Varian (1992).

In order to see whether the yield tax is needed at all one needs evaluation of the partial derivative of the Lagrangian at the corner solution when yield tax is first zero and then equal to one. To provide the answer to how far the yield tax should be pushed the Lagrangian has to be evaluated at $\tau = 1$. This yields

(5) $\qquad \Omega_\tau(T = T^*, \tau = 0) = AR^{-2}z^2(\sigma_p^2 - \frac{1}{2}\sigma_{p\tau}^2) > 0$

(6) $\qquad \Omega_\tau(T = T^*, \tau = 1) = \left[Rp_1 - \bar{p}_2(1 + f) \right] R^{-1} x_\tau^s (1 + \delta)$.

As for the sign of (6), one can proceed as follows. When $r > f$, one gets $Rp_1 - \bar{p}_2(1 + f) > 0$ from [6] and $x_\tau^s < 0$ from [8f] at $\tau = 1$. In the case $r < f$, we have $Rp_1 - \bar{p}_2(1 + f) < 0$ from [6] and $x_\tau^s > 0$ from [8f] at $\tau = 1$. Thus $\Omega_\tau(T = T^*, \tau = 1) < 0$ with $r \neq f$ so that the optimal yield tax is less than 100% regardless of the effect of the yield tax on timber supply.

Finally, the optimal yield tax rate under endogenous prices (equation (19) in the text) can be solved directly from equation (4).

$* \quad * \quad * \quad *$

ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
LÖNNROTINKATU 4 B,   FIN-00120 HELSINKI

KESKUSTELUAIHEITA - DISCUSSION PAPERS ISSN 0781-6847

No 549   LAURA PAIJA - PEKKA YLÄ-ANTTILA, The Impact of Structure and Competition on Employment in the Telecommunications Cluster Case Finland. 12.02.1996. 34 p.

No 550   PENTTI VARTIA - PEKKA YLÄ-ANTTILA, Technology Policy and Industrial Clusters in a Small Open Economy - The Case of Finland. 16.02.1996. 15 p.

No 551   PONTUS BRAUNERHJELM - PER HEUM - PEKKA YLÄ-ANTTILA, Internationalization of Industrial Firms. Implications for growth and industrial structure in the Nordic countries. 16.02.1996. 33 p.

No 552   REIJO MANKINEN, Alkoholiveron alentamisen kansantaloudellisia vaikutuksia. 26.02.1996. 42 s.

No 553   RITA ASPLUND, Koulutus, työura ja palkkaerot. 22.03.1996. 13 s.

No 554   MARIANNE PAASI, The Absorptive Capacities of Estonian Firms. - Can a Technology-based Industrial Strategy Succeed? 22.03.1996. 17 p.

No 555   HANNU HERNESNIEMI, Barriers to Economic Cooperation of Baltic Rim Countries. 10.04.1996. 46 p.

No 556   ANNICK LARUELLE - MIKA WIDGRÉN, Is the Allocation of Voting Power among the EU States Fair? 17.04.1996. 19 p.

No 557   JARI HYVÄRINEN, A Survey of Corporate Governance - Which Model for Transition Countries? 13.05.1996. 32 p.

No 558   PASI KUOPPAMÄKI, Joint Implementation ilmastopolitiikan välineenä: Suomi ja lähialueiden kasvihuonekaasujen päästöjen rajoittaminen. 12.06.1996. 35 s.

No 559   MIKA PAJARINEN, Työnantajan kansaneläkemaksun porrastuksen toimivuus vuosina 1993 ja 1994. 20.6.1996. 34 s.

No 560   OKKO-PEKKA SALMIMIES, EU:n pakolais- ja maahanmuuttoasioita koskevan yhteistyön kehittäminen ja taloudellinen ulottuvuus. 10.07.1996. 32 s.

No 561   AIJA LEIPONEN, Education, Tenure and Innovation in Manufacturing firms. 16.08.1996. 26 p.

No 562   AIJA LEIPONEN, Education and Innovative Capabilities. 16.08.1996. 20 p.

No 563   AIJA LEIPONEN, Competences, Innovation and Profitability of Firms. 16.08.1996. 16 p.

No 564   JYRKI RUUTU, Suomen valuuttakurssijärjestelmä osana eurooppalaista järjestelmää. Historiallinen katsaus keskiajalta autonomian ajan loppuun. 29.08.1996. 22 s.

No 565   HEIKKI PALM, Eläkeuudistuksen vaikutukset Suomen kansantalouden numeerisessa limittäisten sukupolvien mallissa. 02.09.1996. 34 s.

No 566   JYRKI ALI-YRKKÖ, Teknologiaintensiivisten yritysten kansainvälistyminen - vaikutuksia Suomen talouteen. 05.09.1996. 53 s.

No 567   VENLA SIPILÄ, Suomen ja Venäjän välinen kauppa 1986-1995. 06.09.1996. 100 s.

No 568   EUGEN KOEV, Palkkadiskriminaatio teollisuuden toimihenkilöillä. 11.09.1996. 73 s.

No 569   MARIANNE PAASI, The Inherited and Emerging Absorptive Capacities of Firms - Results of a firms survey in the Estonian electronics industry. 24.09.1996. 26 p.

No 570   TANJA KIRJAVAINEN - HEIKKI A. LOIKKANEN, Efficiency Differences of Finnish Senior Secondary Schools: An Application of Dea and Tobit-analysis. 02.10.1996. 40 p.

No 571   ATRO MÄKILÄ, Teknologian työllisyysvaikutukset - Katsaus kirjallisuuteen. 21.10.1996. 19 s.

No 572   HEIKKI HELLA, On Outliers in Time Series Data. 25.10.1996. 11 p.

No 573   NINA HAUHIO - REIJA LILJA, The Evolution of Gender Wage Differentials Over the Career. 03.12.1996. 18 p.

No 574   HEIDI HAILI, The Forward Exchange Rate as a Predictor of the Spot Exchange Rate, An Empirical Study. 03.12.1996. 88 p.

No 575   ERKKI KOSKELA - MARKKU OLLIKAINEN, Optimal Design of Forest Taxation with Multiple-use Characteristics of Forest Stands. 11.12.1996. 31 p.

No 576   MIKA PAJARINEN, On Possibilities to Construct a Model for Trade Flows and Factors of Production Movements between the EU and Eastern Europe. 30.12.1996. 25 p.

No 577   ERKKI KOSKELA - MARKKU OLLIKAINEN, Tax Incidence and Optimal Forest Taxation under Stochastic Demand. 07.01.1997. 29 p.

d:\ratapalo\DP-julk.sam/07.01.1997