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Keskusteluaiheita - Discussion papers

No. 575

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OPTIMAL DESIGN OF FOREST TAXATION WITH MULTIPLE-USE CHARACTERISTICS OF FOREST STANDS**

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** An earlier version of this paper was presented at the 6th Annual Conference of the European Association of Resource and Environmental Economists (EAERE) in Umeå, Sweden, June 17-20, 1995. We are indebted to two anonymous referees as well as participants for helpful comments. This paper is a part of the research program of the Research Unit on Economic Structures and Growth (RUESG). The unit is funded by the University of Helsinki, the Academy of Finland and the Yrjö Jahnsson Foundation. Financial Support from the Kyösti Haataja Foundation is also gratefully acknowledged. *Forthcoming in Environmental and Resource Economics.*

ISSN 0781-6847

11.12.1996

KOSKELA, Erkki - **OLLIKAINEN**, Markku, **OPTIMAL DESIGN OF FOREST TAXATION WITH MULTIPLE-USE CHARACTERISTICS OF FOREST STANDS**. Helsinki: ETLA Elinkeinoelämän Tutkimuslaitos, The Research Institute of the Finnish Economy, 1996, 31 p. (Keskusteluaiheita, Discussion Papers ISSN 0781-6847; no. 575).

ABSTRACT: The paper studies optimal forest taxation under uncertainty about future timber price when private forest owners value amenity services of forest stands; forest stands have public goods characteristics. It is assumed that preferences of forest owners can be described by a quasi-linear, intertemporal utility function which reflects risk aversion in terms of consumption and constant marginal utility in terms of amenity services. The comparative statics of current and future harvesting in terms of timber price risk, site productivity tax and yield tax are first developed. It is shown that, given the optimal site productivity tax, which is independent of the timber harvested and thus non-distortionary, it is desirable to introduce the yield tax at the margin; it both corrects externality due to the public goods characteristic of forest stands and serves as an social insurance device. The optimal yield tax is less than 100 % and depends on the social value of forest stands, timber price risk and properties of compensated timber supply. In the general case the "inverse elasticity rule" -- according to which the optimal yield tax is negatively related to the size of the substitution effects-- may not hold. Under certainty, the desirability of the yield tax, given the optimal site productivity tax, depends only on the existence of public goods characteristic and is thus a pure Pigouvian tax.

KEYWORDS: multiple-use forests, timber supply, optimal forest taxation.
JEL classification: D62, H21, Q23

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TIIVISTELMÄ: Paperissa tarkastellaan yhteiskunnan kannalta optimaalista metsäverotusta olosuhteissa, joissa metsänomistajat arvostavat metsiensä monikäyttöä, metsät ovat monikäytön suhteen julkishyödyke ja puun tulevaan hintaan liittyy epävarmuutta. Tarkasteltavina veroina ovat pinta-alavero ja myyntitulovero. Aluksi karakterisoidaan nykyisten ja tulevien hakkuiden komparatiivista statiikkaa puun hintaan liittyvän riskin ja veroparametrien suhteen. Tämän jälkeen tutkitaan yhteiskunnan kannalta optimaalista metsäverojärjestelmää. Annettuna optimaalinen pinta-alavero, joka ei riipu hakkuista ja on siis ei-vääristävä, on haluttavaa ottaa käyttöön myös myyntitulovero. Sen käyttö pienentää puun tulevaan hintaan liittyvää riskiä, mikä kasvattaa metsänomistajien hyvinvointia. Lisäksi myyntituloverojen käyttöönotto pienentää hakkuita ja kasvattaa monikäyttöä. Jos metsät ovat monikäytön suhteen julkishyödyke, niin myös yhteiskunnan hyvinvointi kasvaa. Optimaalinen myyntituloveroaste riippuu metsien julkishyödyke-ominaisuudesta, hintariskistä ja puun (kompensoidun) tarjonnan herkkyydestä myyntituloveron suhteen. Vaikka puun hintaan ei liittyisi epävarmuutta, niin molempia veroja tarvitaan silloinkin. Myyntitulovero on nyt vain puhdas Pigou-tyyppinen vero, jolla korjataan metsien liian vähäistä monikäyttöä ja liian suuria hakkuita.

ASIASANAT: metsien monikäyttö, puun tarjonta, optimaalinen metsäverotus
JEL-luokitus: D26, H21, Q23

1. INTRODUCTION

Besides timber and timber revenues, forests provide a large variety of tangible and nontangible services. These nontimber services are produced jointly with timber - nontimber services vanish with the standing stock. Therefore, the decision to harvest timber affects automatically the flow of nontimber services one can get from standing stock of forests. This joint production property is not, however, reflected in the property rights over forests. Forest owners typically own timber stock and forest land, but most of nontimber services belong to the class of public goods or common property resources. This creates a problem: as the trees are owned privately, the harvesting decisions tend to neglect the amenities of forests causing an external effect to the society as a whole even though the amenity services of forest stock would have some private value. These ecological relationships are further complicated by an underlying uncertainty associated at least with the future timber prices. These features raise several interesting issues: (i) what are the qualitative properties of timber supply when standing stock has private value?, (ii) what is the role of risk associated with the future timber prices in this context?, (iii) an finally, what is the nature of optimal policy from the point of view of the society when amenity services are public goods? The purpose of this paper is to analyze these largely unexplored issues. Before getting down to business we present a brief review of the relevant existing literature which provides an additional motivation for our analysis.

Hartman (1976) was first to analyze the implications of the public goods characteristics of amenity services for the optimal rotation in the so-called Faustmann rotation framework. He showed that accounting for amenity services would lengthen the rotation period under the assumption that amenity services are homogenous and increase with the age of the forest stand. This analysis was clarified and extended in various ways by Strang (1983). The role of nontimber services was empirically evaluated in Calish, Fight and Teeguarden (1978). The production of amenities in many land sites and stands has been analyzed in Bowes and Krutilla (1989), Swallow and Wear (1993), Vincent and Binkley (1993), Knapp (1981) and, Snyder and Bhattacharyya (1990).

Production of amenities has also been studied in the utility maximization framework starting from Binkley's (1981) static model, where the forest owner is assumed to get utility from timber revenues and non-timber services. Max and Lehman (1988) used a two-period consumption cutting model, where standing values of the forest are fully captured by forest owners, to develop comparative statics of various forest taxes. Hyberg and Holthausen (1989) combined utility maximization with an analysis of aforestation and amenity services. While these models differ from the rotation models in details, their basic results are similar to the Hartman solution; accounting for nontimber services either individually or socially decreases harvesting compared with the case where only timber revenue matters.

The divergence of private and social valuation of nontimber services raises a question of how the externality caused by the private harvesting decision could be eliminated via taxation so that the flow of amenities would reach the socially efficient level. Using the rotation framework, Englin and Klan (1990) define the socially optimal Pigouvian rates for property, severance and yield taxes and offer a case study for Douglas fir for finding out how the taxes would work in practice. Neutral taxes, such as an unmodified property tax, do not change the rotation period and cannot increase the production of amenities. A modified property tax, yield and severance taxes affect all the length of rotation period and, therefore, can change the mix of timber and amenities produced from the forests to the socially desired level. Their analysis, however, abstracts from uncertainty and government tax revenue considerations. Thus the relative efficiency of various taxes, when government faces budget constraint in setting taxes cannot be studied.[1]

This paper explores various issues associated with taxation and harvesting behavior of nonindustrial forest owners when they value amenity services of forest stands, when forest stands have a public goods characteristic and when there is uncertainty about future timber price. It is assumed that preferences of forest owners can be described by a quasi-linear, additively separable intertemporal utility function, which reflects risk aversion in terms of consumption and

[1] The optimal design of forest taxes with multiple-use and public goods characteristic of forest stands, when government faces a budget constraint in setting taxes, has recently been analyzed in Amacher and Brazee (1995). They, however, abstract from uncertainty.

constant marginal utility in terms of amenity services. The paper extends the earlier analyses in several important respects. First, it develops the comparative statics of harvesting in terms of timber price risk and forest taxes by decomposing the total effect into various subcomponents in what is a relatively complex set-up. Second, in complement to the earlier literature, which has analyzed forest taxes in isolation, we study the optimal forest taxation from the public finance point of view. This means that we pose the question: given that government has to acquire a certain amount of tax revenue from forests, how the tax structure should be designed so as to maximize social welfare? The forest taxes to be compared are site productivity tax and yield tax. The site productivity tax has no effect on relative prices and is thus non-distortionary so that it is natural to regard it as the benchmark case to which other taxes are compared. The yield tax, on the other hand, is a forest tax which is commonly used in various countries.

It is shown that timber price risk affects current harvesting positively, while it has an a priori ambiguous effect on future harvesting. A rise in the site productivity tax increases harvesting, while the effect of the yield tax is a priori ambiguous both on current and future harvesting. As for the optimal taxation, it is shown that given the optimal site productivity tax it is desirable to introduce the yield tax at the margin; it both corrects the externality due to the public goods characteristic of forest stands and serves as an insurance device. The optimal yield tax, which is less than 100% for incentive reasons, is the Ramsey-Pigou tax with social insurance; it depends on the public goods characteristics of forest stands, the social insurance role of the yield tax and properties of compensated timber supply. The "inverse elasticity rule", according to which the optimal yield tax is negatively related to the size of the substitution effects, may not hold. Under certainty, the desirability of the yield tax, given the optimal site productivity tax, depends only on the existence of public goods characteristic and is thus a pure Pigouvian tax.

The paper is organized as follows. Section 2 presents first the model of timber supply and forest taxation when standing forest has private value and there is an uncertainty about future timber price. The remaining part of section 2 derives results about the qualitative properties of timber supply. The optimal forest taxation with and without timber price risk, when forest stand has public goods characteristic, is analyzed in section 3. Finally, there are some concluding remarks.

2. TIMBER SUPPLY AND FOREST TAXATION UNDER TIMBER PRICE RISK WHEN STANDING FOREST HAS PRIVATE AMENITY SERVICES

2.1. A Model of Timber Supply under Uncertainty with Amenity Services

The forest owner is assumed to have a preference ordering over present and future consumption (c_1 and c_2) and over present and future amenity services provided by the forest stands (k_1 and k_1) respectively[2]. This is represented by a utility function which is assumed to be both additively separable and additive across periods and concave in each argument so that

(1) $$U = u(c_1) + \beta u(c_2) + v(k_1) + \beta v(k_2)$$

where $\beta = (1+\rho)^{-1}$ describes the time preference factor. Thus U describes the discounted utility from consumption and private amenity services in both periods. In what follows the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. E.g. $u'(c_1) = \partial u(c_1)/\partial c_1$, $A_x(x,y) = \partial A(x,y)/\partial x$ etc. The joint production of timber and amenities is given in equations (2a-2b).

(2)
(a) $$k_1 = Q - h_1$$
(b) $$k_2 = (Q - h_1) + F(Q - h_1) - h_2$$

By harvesting a part h_1 of the initial forest stand Q the owner chooses also k_1 according to (2a). This remaining stock will grow according to a concave growth function $F(Q-h_1)$ with $F'(\cdot) \succ 0, F''(\cdot) \prec 0$. By choosing the size of future harvesting h_2 the owner decides also the future

2 See Montgomery and Adams (1995) for a basic certainty version of the two-period multiple-use model. This specification has been originally presented in Ovaskainen (1992).

forest stand k_2 which gives future amenity services. Notice that $(dk_2 + dh_2)/dh_1 = -(1+F') \prec 0$. Thus a rise in current harvesting means that the sum of future harvesting and future forest stand decreases by the amount which depends on the growth function of forest.

The government is assumed to levy two forest taxes on forest owners, namely the site productivity tax T and the yield tax τ. The site productivity tax is a lump-sum tax, which is independent of harvesting. The yield tax is a proportional tax imposed upon timber revenues. If the timber price is denoted by $p_i, i = 1,2$, then the post-tax price is $p_i^* = p_i(1-\tau)$. During the first period the forest owner allocates the net revenue from harvesting between consumption (c_1), saving (s) and site productivity tax (T) so that

(3) $$c_1 = p_1^* h_1 - T - s$$

where we have abstracted from other incomes for simplicity. The stochastic future consumption is defined by the sum of the future net revenue from harvesting and capital income plus savings minus the site productivity tax so that we have

(4) $$\tilde{c}_2 = \tilde{p}_2^* h_2 - T + Rs,$$

where $R = (1+r)$, r denotes the interest rate on the capital market and tilde for stochasticity of future timber price and future consumption.

Combining the flow-of-funds equations (3) and (4) yields the stochastic intertemporal budget constraint for the forest owner

(5) $$\tilde{c}_2 = \tilde{p}_2^* h_2 - T + R\left[p_1^* h_1 - T - c_1\right].$$

In the spirit of traditional public finance we have assumed that both the site productivity tax T and the yield tax τ are the same now and in the future, but their levels are determined by maximizing the social welfare function under the government tax revenue requirement. This means that the policy maker is assumed to commit future policy so that before any private decisions are made, government announces a tax policy to which it commits.[3]

Assuming that the forest owners behave according to the expected utility maximization hypothesis and are risk averse so that $u''(\tilde{c}_2) < 0$. The decision problem can now be posed as maximizing the expected utility EU with respect to $c_1, h_1$and h_2 subject to (5) (2a) and (2b), where E denotes the expectations operator. The first-order conditions for the expected utility maximization are

$$\text{(a)}\quad EU_{c_1} = u'(c_1) - \beta R E u'(\tilde{c}_2) = 0$$

$$\text{(6)}\qquad \text{(b)}\quad EU_{h_1} = \beta R p_1^* E u'(\tilde{c}_2) - v'(k_1) - \beta(1+F')v'(k_2) = 0$$

$$\text{(c)}\quad EU_{h_2} = \beta E\left[\tilde{p}_2^* u'(\tilde{c}_2)\right] - \beta v'(k_2) = 0 .$$

Equation (6a) describes the consumption-saving decision, while the privately optimal current and future harvesting are defined by equations (6b) and (6c). What kind of harvesting rule emerges from the set of first-order conditions (6b) and (6c)? Substituting (6c) into (6b) yields the generalized harvesting rule

$$\text{(7)}\qquad R p_1^* - \bar{p}_2^*(1+F') = \frac{\operatorname{cov}(u'(\tilde{c}_2), \tilde{p}_2^*)(1+F')}{E u'(\tilde{c}_2)} + \frac{v'(k_1)}{\beta E u'(\tilde{c}_2)}$$

where $\bar{p}_2^*$ denotes the expected post-tazx future timber price and $\operatorname{cov}(u'(\tilde{c}_2), \tilde{p}_2^*) < 0$ because of

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[3] In the terminology of game theory we study a Stackelberg equilibrium with the government as the dominant player. If the government cannot enter into binding commitments, but instead reoptimizes at the beginning of each period, then we have the Nash equilibrium without commitment. In the model once the harvesting decision in the first period has been made, its tax base becomes predetermined and it is ex post optimal to tax it as much as needed (or as much as possible). If forest owners see that possibility, they anticipate it and change their behavior beforehand. The analysis of tax policy without commitment, however, lies beyond the scope of this paper. See, e.g., Persson and Tabellini (1990) for an survey on these issues.

risk aversion.

Several interesting observations can be presented on the basis of the equation (7). First, if there is no uncertainty and no valuation of amenity services, then the RHS of (7) would be zero and the harvesting decisions would be separable from the preferences of the forest owner. Harvesting would be carried out to the point, where the post-tax marginal returns from harvesting Rp_1^* are equal to the post-tax marginal costs of harvesting $p_2^*(1+F')$.[4] Second, under uncertainty with no valuation of amenity services the first RHS term is negative and the second zero so that the LHS of (7) is negative. Thus allowing for future timber price uncertainty with risk aversion will have the effect of increasing harvesting today compared with the certainty situation. Moreover, harvesting is no longer separable from the preferences of the forest owner (see e.g. Koskela (1989a))[5]. Third, under certainty when the forest stand has private value, the first term in the RHS is zero, while the second term is positive so that the LHS of (7) is positive. Thus allowing for the valuation of amenity services will have the effect of decreasing harvesting today. This is another channel via which the harvesting decisions become dependent on preferences of the forest owner.[6]

2.2. Analytics of Timber Supply under Timber Price Risk and Private Amenity Services

In general terms comparative statics under timber price risk becomes messy and not very illuminating. In order to get sharper insights we proceed by simplifying the analysis in three respects. First, we assume that the marginal private valuation of amenity services as a function of

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[4] Thus under certainty with no amenity valuation, yield tax is neutral in terms of harvesting decision. Because site productivity tax is also neutral, it does not matter which of those taxes is used from the welfare point of view. Note also that under constant timber prices one gets the Jevons-Wicksell rule for one rotation period $r = F'$.

[5] It is possible to extend the analysis to deal with multiple sources of uncertainty. See, e.g., Ollikainen (1993) for an analysis of forest taxation with two sources of uncertainty.

[6] In fact, credit rationing in the form of a binding borrowing constraint provides the third channel, through which harvesting decisions may depend on the preferences of forest owner (see, e.g., Koskela 1989b).

the forest stand is constant so that we have $v(k_i)=mk_i$, for i=1,2. This is of course a simplification. One can think of amenity services provided by the forest, for which this is incorrect, e.g., biodiversity. Yet it is possible to think of many amenity services, where it does apply. For instance if amenity services, like campsites or recreational facilities are offered under the circumstances, in which congestion is not an issue over the relevant range, then the assumption seems to hold. Second, the future timber price is assumed to be normally distributed $\tilde{p}_2 \approx N(\bar{p}_2, \sigma_p^2)$, where $\bar{p}_2$ is the expected future timber price and σ_p^2 its variance. Finally, the utility function in terms of consumption is described as $u(c_i) = -\exp(-Ac_i)$, where $A = -u''(c_i)/u'(c_i)$ is the Arrow-Pratt constant absolute risk-aversion (see, e.g., Hirschleifer and Riley 1992). Now the twin assumptions that $\tilde{p}_2$ is normally distributed and the utility function is exponential have the major advantage that the forest owner's expected utility maximization problem for consumption components can be formulated not in terms of the whole probability distribution but in terms of the mean and variance of future consumption. Under these assumptions the forest owner's decision problem can be rephrased as maximizing

(8) $$EU = -\exp(-Ac_1) - \beta\exp(x) + m(k_1 + \beta k_2),$$

where $x = -A(\bar{c}_2 - \frac{1}{2}A\sigma_{c_2}^2) = -A(\bar{c}_2 + \frac{1}{2}A(1-\tau)^2 h_2^2 \sigma_p^2)$ and m is the constant marginal valuation of amenity services. These simplifications make the analysis much more transparent without throuwing the baby with the water.[7]

It is easy to show that maximizing (8) with respect to c_1, h_1 and h_2 produces a cutting rule similar to (7), as can be seen in (7.')

(7.') $$Rp_1^* - \bar{p}_2^*(1+F') = -A(1-\tau)^2 h_2 \sigma_p^2 (1+F') + \frac{m}{\beta A \exp(x)}$$

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[7] In general the expected utility $E[u(\tilde{c}_2)]$ depends on the entire probability distribution of $\tilde{p}_2$. Given the exponential utility and normally distributed $\tilde{p}_2$, one gets
$E[u(\tilde{c}_2)] = -\int e^{-Ac_2} f(c_2)dc_2 = -e^{-A(\bar{c}_2 - \frac{1}{2}A\sigma_{c_2}^2)}$.

Given that the second-order conditions for the expected utility maximization hold, the first-order conditions define implicitly the optimal consumption and harvesting in terms of exogenous parameters, especially in terms of timber price risk and forest taxes so that $c_1 = c_1(\sigma_p^2, T, \tau, \ldots)$, $h_1 = h_1(\sigma_p^2, T, \tau, \ldots)$ and $h_2 = h_2(\sigma_p^2, T, \tau, \ldots)$. Substituting these for the respective variables in (8) makes it possible to express the expected utility indirectly in terms of the same parameters. Utilizing the envelope theorem to the expected indirect utility function EU* gives

(9)

(a) $EU^*_{\sigma_p^2} = -\frac{1}{2}\beta A^2(1-\tau)^2 h_2^2 \exp(x) < 0$

(b) $EU^*_T = -\beta(1+R)A\exp(x) < 0$

(c) $EU^*_\tau = (1+R)^{-1} z EU^*_T < 0$

where $z = \left[\bar{p}_2 - A(1-\tau)h_2\sigma_p^2\right]h_2 + Rp_1h_1 > 0$ and the risk-adjusted expected future timber price $\bar{p}_2 - A(1-\tau)h_2\sigma_p^2 > 0$. An increase in timber price risk, site productivity tax and yield tax decrease the maximum expected utility attainable to the forest-owner.

Given that $EU^*_T \prec 0$, $EU^* = u^0$ can be inverted for T in terms of timber price risk, yield tax and maximum expected utility so that $T = g(\sigma_p^2, \tau, u^0)$. Substituting this expression for T in EU* gives the compensated indirect utility function $EU*\left[g(\sigma_p^2, \tau, u^0), \tau\right]$.[8] The expected compensated indirect utility function answers the following question: if e.g. the yield tax rate τ is increased, how much the site productivity tax T has to be changed so as to keep the expected utility of the forest owner unchanged? Differentiating with respect to τ produces $EU^*_T g_\tau + EU^*_\tau = 0$ so that $g_\tau = -EU^*_\tau EU_T^{*-1} = -(1+R)^{-1}z < 0$ from (9c). This expression is useful later on and indicates the required compensation necessary to keep the level of expected utility unchanged as the yield tax changes.

[8] See e.g. Diamond and Yaari (1972).

It is known that at the expected utility maximization point the following holds

(10) $$h_i(\sigma_p^2, \tau, T) = h_i^c(\sigma_p^2, \tau, u^0) \quad \text{for } i = 1,2,$$

where h_i is the uncompensated timber supply implicitly defined by the expected utility maximization and h_i^c is the compensated timber supply, which is the timber supply, when yield tax is changed and the forest owner is compensated by a change in site productivity tax so as to keep the expected utility unchanged. Substituting the g-function, defined above, for T in the uncompensated timber supply function h_i and differentiating the equation (10) with respect to τ gives $h_{i\tau} + h_{iT} g_\tau = h_{i\tau}^c$ for $i = 1,2$. Thus one gets the Slutsky decomposition for current and future harvesting in terms of the yield tax

(11) $$h_{i\tau} = h_{i\tau}^c + (1+R)^{-1} z h_{iT} \quad \text{for } i = 1,2,$$

where the total effect of the yield tax consists of the substitution effect ($h_{i\tau}^c$) and the income effect ($(1+R)^{-1} z h_{iT}$). This decomposition of the effect of taxes is both useful and important. The substitution effect reflects the distortionary effect of taxes at the margin. The tax is neutral if the substitution effect is zero and distortionary otherwise. The income effect reflects the fact that a change in the tax rate affects like a change in the forest owner's income. The total effect of taxes reflects both these aspects.

2.2.1 Comparative Statics under Certainty

It is useful to look first at the simpler case of certainty. The Slutsky decomposition (11) reduces to

(11') $$h_{i\tau}^0 = h_{i\tau}^{0c} + (1+R)^{-1} y h_{iT}^0, \quad \text{for } i = 1,2$$

where h_i^0 denotes timber supply under certainty and $y = p_2 h_2 + R p_1 h_1 \succ 0$. As for the income effects one can show

(12)

(a) $h_{1T}^{0} = 0$

(b) $h_{2T}^{0} = mF''p_2^*\Phi > 0$.

where $\Phi = \Delta^{0^{-1}}[\beta^2 A^4(1+R)\exp(-2Ac_2 - Ac_1)] < 0$ with $\Delta^0 < 0$ due to the second-order conditions for utility maximization (see Appendix 1 for details).

Site productivity tax is neutral at the margin so that it causes only an income effects. A rise in site productivity tax has no effect on current harvesting, while it increases future harvesting. The first-order condition (6b) determines h_1 by equating the marginal utility of future consumption ($\beta R p_1^* u'(c_2)$) and the marginal utility of current amenity services ($m(1+\beta(1+F'))$) from h_1. A rise in T increases the marginal utility of future consumption, which tends to increase current harvesting. But according to (6a) a fall in current consumption due to a rise in T decreases the marginal utility of current consumption so that there is no need for current harvesting to adjust. And h_2 is determined by equating the marginal utility of future consumption ($p_2^* u'(c_2)$) and the marginal utility of future amenity services (m) from h_2 as in (6c). A rise in T increases the marginal utility of future consumption, but leaves the marginal utility of future amenity services unchanged. Hence, it is optimal for forest owners to increase h_2 and decrease k_2. Thus future amenity services are normal goods.

Yield tax is distortionary at the margin, and one gets as the substitution effects

(13)

(a) $h_{1\tau}^{0c} = 0$

(b) $h_{2\tau}^{0c} = mF''p_2 N^0 < 0$

where $N^0 = \Delta^{-1}[\beta^2 A U_{c_1c_1} \exp(-2Ac_2)] \succ 0$. The substitution effect of yield tax is zero for current harvesting, but negative for future harvesting. This is because the changes in current consumption take care of the need to change current harvesting, while a change in the yield tax τ tends to change the marginal utility of future consumption relative to the constant marginal utility of future amenity services. Therefore future amenity services (future harvesting) changes.

These can be summarized in

Result 1: Under certainty and private valuation of amenity services (a) site productivity tax has no effect on current harvesting, but affects future harvesting positively, (b) the total effect of the yield tax on current harvesting is zero, while its effect on future harvesting is a priori ambiguous, (c) the substitution effect of yield tax is proportional to the income effect and zero (negative) for current (future) harvesting.

2.2.2 Comparative Statics under Uncertainty

Let us now turn back to the general Slutsky equation (11) which decomposes the total effect of yield tax into the substitution and income effects respectively, the latter expressed in terms of site productivity tax. The effect of site productivity tax under uncertainty can be shown to be

$$\text{(a)}\quad h_{1T} = h_{1T}^{00} - \beta R p_1^* A^2 (1-\tau)^2 \sigma_p^2 \tilde{\Phi} \succ 0$$

(12')

$$\text{(b)}\quad h_{2T} = h_{2T}^{00}\left[1 - A(1-\tau)^2 h_2 \sigma_p^2 \bar{p}_2^{*-1}\right] \succ 0$$

where $\tilde{\Phi} = \Delta^{-1}\left[\beta^2 A^4 (1+R)\exp(2x - Ac_1)\right] \prec 0$ with $\Delta \prec 0$ due to the second-order conditions for the expected utility maximization, $h_{1T}^{00} = h_{1T}^{0} = 0$, and h_{2T}^{00} is almost equal to h_{2T}^{0}, in which just the variance term has vanished from the exponent of Φ (see Appendix 1 for details). Thus allowing for uncertainty makes also current harvesting positively dependent on site productivity tax. This is due to the fact that a rise in site productivity tax increases precautionary saving which can be done by harvesting more today. Under uncertainty both current and future amenity services are normal goods.

Before developing signs and economic interpretation of the substitution effects of yield tax we look at the relationship between harvesting and future timber price risk. Differentiating the compensated indirect utility function $EU^{*}\left[g(\sigma_p^2, \tau, u^0), \sigma_p^2, \tau\right]$ with respect to σ_p^2 gives the compensation in terms of T required to keep the owner's utility constant as σ_p^2 changes, $g_{\sigma_p^2} = -(1/2)(1+R)^{-1}A(1-\tau)^2 h_2^2 < 0$. Utilizing the equality (10) between compensated and uncompensated harvesting at the expected utility maximization point and differentiating it with respect to T and σ_p^2 gives the Slutsky decomposition for the effect of timber price risk

(14) $$h_{i\sigma_p^2} = h_{i\sigma_p^2}^c + \frac{1}{2}(1+R)^{-1}A(1-\tau)^2 h_2^2 h_{iT} \quad \text{for } i = 1,2,$$

where the toal effect is decomposed into the substitution ($h_{i\sigma_p^2}^c$) and income ($\frac{1}{2}(1+R)^{-1}A(1-\tau)^2 h_2^2 h_{iT}$) effects.

It can be shown that (see Appendix 1 for details)

Result 2: Under private valuation of amenity services a) the total effect of timber price risk is positive for current harvesting, while a priori ambiguous for future harvesting, b) the income effect of timber price risk is positive for both current and future harvesting and c) the substitution effect is positive (negative) for current (future) harvesting.

The economic intuition is the following. A rise in timber price risk makes the forest owner worse off; it affects like a decrease in the expected timber price. Hence forest owners tend to harvest more and use amenity services less due to the income effect. At the same time, a rise in the timber price risk makes future harvesting less attractive at the margin. Hence the substitution effect is to increase current and decrease future harvesting.

The next and final step is to use the substitution effect in (14) to re-express the substitution effects of the yield tax. These substitution effects under timber price risk can be decomposed into two components as follows.

$$\text{(15)}\quad \begin{aligned} &\text{(a)}\quad h^c_{1\tau} = S_{1\tau} - (1-\tau)^{-1}\sigma_p^2 h^c_{1\sigma_p^2} \prec 0 \\ &\text{(b)}\quad h^c_{2\tau} = S_{2\tau} - (1-\tau)^{-1}\sigma_p^2 h^c_{2\sigma_p^2} = ? \end{aligned}$$

where

$$\text{(16)}\quad \begin{aligned} &\text{(a)}\quad S_{1\tau} = -(1-\tau)^{-1}\left[p_1 h^c_{1p_1} + \overline{p}_2 h^c_{1p_2}\right] \prec 0 \\ &\text{(b)}\quad S_{2\tau} = -(1-\tau)^{-1}\left[p_1 h^c_{2p_1} + \overline{p}_2 h^c_{2p_2}\right] \prec 0 \end{aligned}$$

The total substitution effects of the yield tax ($h^c_{i\tau}$) under uncertainty can be decomposed into the substitution effects due to a change in after-tax timber prices ($S^c_{i\tau}$) on the one hand and into the substitution effects due to a change in timber price risk ($-(1-\tau)^{-1}\sigma_p^2 h^c_{i\sigma_p^2}$) on the other hand.

The substitution effect is negative (a priori ambiguous) for current (future) harvesting. A rise in the yield tax works like a fall in timber price risk, which tends to increase (decrease) current (future) harvesting. But it also decreases the after-tax timber prices, which tends to decrease current and future harvesting at the margin. Thus the "after-tax price" and "risk substitution" effects reinforce each other for current harvesting, while run counter to each other for future harvesting.[9]

One can summarize these in

Result 3: Under uncertainty and private valuation of amenity services (a) timber price risk affects current harvesting positively, but has an a priori ambiguous effect on future

9 See Koskela (1984) for a similar decomposition and interpretation in a different context.

harvesting, (b) amenity services are normal goods so that a rise in site productivity tax increases both current and future harvesting, (c) the effect of yield tax is a priori ambiguous both on current and future harvesting, while (d) the substitution effect is negative (a priori ambiguous) for current (future) harvesting.

One might be tempted to believe that in the presence of sign ambiguities, there is no hope to characterize the optimal forest design of taxation at all. This belief is not, however, justified.

3. OPTIMAL DESIGN OF FOREST TAXATION UNDER UNCERTAINTY WHEN AMENITY SERVICES ARE PUBLIC GOODS

After having developed comparative statics of timber supply under uncertainty about future timber price when the forest stand has private value, we turn to consider the issue of optimal forest taxation from the point of view of the society. Before doing that we have to clear up some things. First, in the line with the optimal taxation literature, it is assumed that forest taxes are chosen so as to keep the government tax revenue given. Second, we tract the government tax revenue requirement as deterministic. This assumption can be justified along two independent lines. If the government is risk-neutral, then it is interested in the expected valua of tax revenue and the stochasticity of timber price need not be taken into account in the desing of tax policies. If risk is idiosyncratic, i.e., independent across individual forest owners, then the government tax revenue at the aggregate can be regarded as deterministic[10] (for analysis of optimal income taxation in the idiosyncratic case, see Varian 1980). The discounted value of government tax revenues can be written

(17) $$G = T(1+R^{-1}) + \tau\left[p_1 h_1 + R^{-1}\overline{p}_2 h_2\right]$$

[10]As demonstrated in an empirical study from Finland by Tilli and Uusivuori (1994), timber price risk may be idiosyncratic in at least two ways. First, regional timber prices have varied considerably in a given year independently of their volatility over time. Second, in a given year there have been differences in the prices of various timber assortments which, together with different tree species assortments in an average plot, causes idiosyncratic risk.

Third, we allow for the possibility that amenities of forest stands are public goods.[11] The simpliest way to do this is to assume that people, who are not forest owners, can benefit from the amenity services of forest stands without depleting their availability to others. If n people benefit from amenity services, then the extra component of the social welfare due the public goods-characteristic of forest stand is $(n-1)m(k_1+\beta k_2)$, when the private valuation of amenity services of non-forest owners is the same than that of forest owners.

The social planner's problem is now to choose the site productivity tax T and the yield tax τ so as to maximize the social welfare function with multiple-use characteristic of forest stands

(18) $$W = EU^*(T,\tau,\ldots)+(n-1)m(k_1+\beta k_2)$$

subject to the government tax revenue requirement (17).[12] In the expression (18) the first RHS component describes the maximum expected utility of a representative forest owner in terms of forest taxes T and τ, while the second RHS component is just the external benefit to the society from the assumption that amenity services of the forest stand have public goods-characteristic.

3.1. Ramsey-Pigou Taxation with Social Insurance

Before any private decisions are made, the government is assumed to announce a tax policy and to commit itself to it. The first-order conditions for the social welfare maximization under the tax revenue requirement can be obtained by setting the partial derivatives of the Lagrangian function $\Omega = W+\lambda G$ with respect to T and τ zero so that

(19) $$\Omega_T = EU_T^* - (n-1)\left[B_1 h_{1T} + B_2 h_{2T}\right] + \lambda G_T = 0$$

[11] According to the conventional terminology a public good is a commodity for which the use of one unit by an agent does not preclude its use by other agents. See Mas-Colell, Whinston and Green (1995, Ch. 11) for a modern account of some basic issues associated with public goods.
[12] See e.g. Atkinson and Stiglitz (1980) for an advanced textbook on the optimal taxation from the public finance point of view.

(20) $$\Omega_\tau = EU_\tau^* - (n-1)[B_1 h_{1\tau} + B_2 h_{2\tau}] + \lambda G_\tau = 0$$

where $\lambda > 0$ is the Lagrangian multiplier associated with the government tax revenue requirement (17), $B_1 = m(1+\beta(1+F')) > 0$ and $B_2 = \beta m > 0$. The equations (19) and (20) implicitly define the optimal values T^* and τ^*.[13]

According to the equation (19) the optimal site productivity tax T is determined by equating the loss of marginal social utility due to the site productivity tax ($W_T = EU_T^* - (n-1)[B_1 h_{1T} + B_2 h_{2T}] < 0$) to the increase in tax revenues evaluated at the value of the Lagrangian multiplier λG_T, where $G_T = (1+R^{-1}) + \tau(p_1 h_{1T} + R^{-1}\overline{p}_2 h_{2T}) > 0$. The loss of marginal social utility results from two effects: the maximum expected utility decreases directly and the social welfare indirectly via the fact that current and future harvesting increase thus decreasing social value of amenity services.

Utilizing the envelope results and the Slutsky decompositions for timber supply the expression (20) can be written in terms of (19) as

(20') $$\Omega_\tau = (1+R)^{-1} y\Omega_T - (n-1)[B_1 h_{1\tau}^c + B_2 h_{2\tau}^c] + \beta A^2(1-\tau)h_2^2\sigma_p^2 \exp(x) + \lambda\tau[p_1 h_{1\tau}^c + R^{-1}\overline{p}_2 h_{2\tau}^c] = 0$$

Given the optimal choice of $T = T^*$ defined by (19) the equation (20') is reduced to

(21) $$\Omega_\tau(T = T^*) = -(n-1)[B_1 h_{1\tau}^c + B_2 h_{2\tau}^c] + \beta A^2(1-\tau)h_2^2\sigma_p^2 \exp(x) + \lambda\tau[p_1 h_{1\tau}^c + R^{-1}\overline{p}_2 h_{2\tau}^c] = 0$$

where the first RHS term in (20') is zero according to (19). The expression (21) gives the optimal yield tax when the site productivity tax has been chosen optimally. In order to see, whether the yield tax is needed at all, one has to look at the corner solution. The partial derivative of the

[13] In the literature of dynamic games this kind of equilibrium with commitment would be described as "open-loop equilibrium", see e.g. Basar and Olsder 1982.

Lagrangian at the margin, when the yield tax is zero is

(22) $$\Omega_\tau(T=T^*,\tau=0)=-(n-1)\left[B_1h_{1\tau}^c+B_2h_{2\tau}^c\right]+\beta A^2h_2^2\sigma_p^2\exp(x)\succ 0$$

Thus given the optimal site productivity tax, it is welfare-increasing to introduce the yield tax at the margin. This is for two reasons. First, introducing the yield tax decreases harvesting via the substitution effects and thereby corrects for the externality due to the public goods characteristics of forest stand which makes timber supply too high from the viewpoint of social welfare maximization (the term $-(n-1)(B_1h_{1\tau}^c+B_2h_{2\tau}^c)>0$, see Appendix 2 for details). Second, introducing the yield tax at the margin decreases the risk due to the future timber price uncertainty which is also welfare-increasing from the point of view of risk-averse forest owners even though this creates a distortion (the term $\beta A^2h_2^2\sigma_p^2\exp(x)$ in (22)). Thus the social insurance value of the yield tax outweights it distortionary effect at the margin.

How far should one go of increasing the yield tax? The partial derivative of the Lagrangian (22) at the margin as $T=T^*$ and $\tau=1$ can be written as

(23) $$\Omega_\tau(T=T^*,\tau=1)=-(n-1)\left[B_1h_{1\tau}^c+B_2h_{2\tau}^c\right]+\lambda\left[p_1h_{1\tau}^c+R^{-1}\overline{p}_2h_{2\tau}^c\right]$$

The first-order conditions (6b) and (6c) are not feasible with $\tau=1$ since $EU_{h_i}\prec 0$ for i=1,2, so that $h_i\rightarrow 0$ as $\tau\rightarrow 1$. Under these circumstances the forest tax base goes down to zero and welfare can be increased by decreasing τ so that the optimal yield tax is less than 100 %.

The expression for the optimal interior yield tax can be obtained from (21)

(24) $$\tau^*=\frac{(n-1)K}{\lambda(M-L)}-\frac{L}{\lambda(M-L)}$$

where

(25)
(a) $K=B_1h_{1\tau}^c+B_2h_{2\tau}^c$
(b) $L=\beta A^2(1-\tau)h_2^2\sigma_p^2\exp(x)$

(c) $M = p_1 h_{1\tau}^c + R^{-1} \overline{p}_2 h_{2\tau}^c$

Equation (24) is not an explicit solution, since current and future harvesting depend on the yield tax, but it can be used to discuss the determinants of optimal τ. The optimal yield tax can be decomposed additively into the public goods or externality component (the first RHS term) and into the social insurance and efficiency components which interact (the second RHS term).[14] When private and social valuation of amenity services coincide, i.e., when amenity services of forest stands have no public goods charaxcteristics, only the second RHS term in (24) is relevant.

One can summarize the findings in

Proposition 1: Ramsey-Pigou taxation with social insurance

When the site productivity tax has seen set to the optimal level (a) it is desirable to introduce yield tax at the margin under the circumstances, where forest stands have public goods-characteristic and there is idiosyncratic uncertainty about the future timber price, (b) the optimal yield tax is less than 100 % and reflects three considerations; (i) the public goods characteristic of forest stands, (ii) the social insurance role of the yield tax with timber price risk and (iii) the distortionary effect of the yield tax, (d) the yield tax is still needed as a social insurance device even though amenity services would not be public goods.

3.1.1. On Comparative Statics of the Ramsey-Pigou Forest Taxation with Social Insurance

Can one say anything more precise about the comparative statics of the optimal yield tax in terms of underlying determinants on the basis of the equation (25)? It is immediate that the higher is the public goods characteristic of forest stand ($n = 1$), the higher is the optimal yield tax, ceteris paribus. As for timber price risk and risk aversion, the results are indeterminate. Both

[14] See Sandmo (1976) for a seminal analysis of optimal taxation with externalities.

of them affect τ^* directly via the L-term and indirectly via the compensated harvesting effects $(h_{i\tau}^c, i = 1,2)$.

What is the role of the substitution effects of yield tax for its optimal level? It is usually argued that tax rates should be negatively related to the strength of the substitution effects. According to this 'inverse elasticity'-rule the more sensitive the behavior is in terms of distortionary taxes, the lower the taxes should be, ceteris paribus.[15] Does the inverse elasticity rule hold in this case? Differentiating (24) with respect to $h_{i\tau}^c$ gives

(26) $$sgn\frac{\partial\tau^*}{\partial h_{1\tau}^c} = sgn\left[(n-1)X + p_1 L\right]$$

and

(27) $$sgn\frac{\partial\tau^*}{\partial h_{2\tau}^c} = sgn\left[(n-1)Y + \overline{p}_2 R^{-1} L\right]$$

where

(28)

(a) $$X = p_1(1-B_1)h_{1\tau}^c + (\overline{p}_2 R^{-1} - p_1 B_2)h_{2\tau}^c - B_1 L$$

(b) $$Y = (p_1 - B_1\overline{p}_2 R^{-1})h_{1\tau}^c + \overline{p}_2 R^{-1}(1-B_2)h_{2\tau}^c - B_2 L$$

If $h_{i\tau}^c < 0$, both derivatives are positive in the absence of public goods characteristic of forest stands $(n = 1)$, which means that yield tax decreases as compensated timber supply becomes more sensitive to yield tax. Generally the signs are indeterminate, however. Thus it is possible that, in contrast with the inverse elasticity rule, the yield tax increases with sensitivity of compensated timber supply.

Hence we have

Proposition 2: The optimal yield tax (a) increases with the public goods characteristic of forest stands, while the effect of timber price risk and risk aversion is a priori ambiguous,

15 See e.g. Varian 1993, 410-412.

(b) is negatively related to the substitution effects of the yield tax ly in line with the 'inverse elasticity'- rule if private and social valuation of amenity services coincide but (c) in the general case, the relationship between substitution effect and the level of the yield tax is a priori ambiguous; higher substitution effect may lead to higher yield tax.

The reason for the indeterminacy of the relationship between the optimal yield tax and the size of the substitution effect lies in the offsetting effects of the compensated timber supply which reflects the distortionary effect. A rise in the sensitivity of compensated timber supply tends to make a distortion higher, and the optimal yield tax lower on the one hand. But on the other hand, it makes the yield tax more effective in correcting the externality due to public goods property of amenity services. If the latter effect is strong enough, then in contrast with the 'inverse elasticity'-rule there might be a positive relationship between yield tax and compensated timber supply.

3.2. Pigouvian Taxation with Public Goods Characteristic of Forest Stand

What happens in the absence of future timber price risk? Under certainty the risk terms vanish and both the comparative statics and envelope results are slightly different. Now the equation (19) for the optimal site productivity tax T* can be written as

(29) $$\Omega_T = 0 = U_T^* - (n-1)B_2 h_{2T}^0 + \lambda G_T$$

where $G_T = (1+R)^{-1} + \tau R^{-1}\overline{p}_2 h_{2T}^0$. Again, the optimal site productivity tax equalizes the loss of marginal social utility $(U_T^* - (n-1)B_2)$ to the increase in tax revenues evaluated at the value of the Lagrangian multiplier (λG_T).

Given that $T = T^*$ the partial derivative of the Lagrangian at the margin, when the yield tax is zero is

(30) $$\Omega_\tau(T=T^*,\tau=0)=-(n-1)B_2h_{2\tau}^{oc}>0$$

The first-order conditions (6b) and (6c) are not feasible with $\tau=1$ so that the optimal yield tax is less than 100 %. Given $T=T^*$, the expression (20) can be written as

(31) $$\Omega_\tau(T=T^*)=\omega h_{2\tau}^{oc}$$

where $\omega=\lambda\tau p_2R^{-1}-(n-1)B_2$, $B_2=\beta m>0$ and $h_{2\tau}^{oc}<0$. Thus the optimal τ is determined from (28) by

(32) $$\tau^{**}=\frac{(n-1)B_2}{\lambda R^{-1}p_2}\geq 0 \text{ as } (n-1)\geq 0.$$

It is noticeable that it depends only on the public goods characteristic of forest stand, but has nothing to do with the properties of the substitution effect of the yield tax.

Proposition 3: Pigouvian taxation with public goods characteristics of forest stands

If the site productivity tax has been set to the optimal level under certainty, (a) it is desirable to use the yield tax as a pure Pigouvian tax to account for the externality when the social value of amenity services exceeds their private value, but (b) when private and social valuation of amenity services coincide, the yield tax is not needed once the site productivity tax has been set to the optimal level.

This reflect the priciple of an assignment of instruments to targets. Once the site productivity tax has been set at the optimum, the yield tax can be used as a pure Pigouvian corrective tax. The task of the site productivity tax is to collect tax revenue and the yield tax in this case accounts for the externality[16].

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16 Thus far we have argued that given the optimal site productivity tax, it is desirable on welfare grounds to introduce the yield tax, which is less than 100%. But what is the sign of the site productivity tax T? If forest taxation would be purely redistributive across forest owners, then the site productivity tax should be a subsidy. In that case forest taxation is linearly progressive in the sense that the average forest taxes increase with the tax base. But if there is a positive tax revenue requirement, the sign of T

4. CONCLUDING REMARKS

The purpose of the paper has been to explore various issues associated with harvesting behavior of nonindustrial forest owners, when forest stand provides private amenitys services, has multiple-use characteristics and there is an idiosyncratic uncertainty about the future timber price. The paper has extended the earlier analyses in several respects. First, we have developed the comparative statics of timber price risk and land site and yield taxes and decomposed them in a way that makes it possible to give an economic interpretation about what are relatively complex relationships. Second, in complement to the earlier literature, which has analyzed forest taxes in isolation, we have studied the optimal design of forest taxation from public finance point of view. Given that government has to acquire a certain amount of tax revenue from forests, what would be the optimal way of doing it?

As for the comparative statics of current and future harvesting, in the general uncertainty case timber price risk affects current harvesting positively, while has an a priori ambiguous effect on future harvesting. A rise in site productivity tax increases harvesting, while the effect of the yield tax is a priori ambiguous both on current and future harvesting. Under certainty, the behavioral effects of taxes are more determinate though still some ambiguities remain.

One might be tempted to believe that, in the presence of sign ambiguities of the comparative statics, the optimal forest taxation cannot be characterized at all. This belief is, however, is not justified. It is shown that given the optimal site productivity tax -- which is independent of the timber harvested and thus non-distortionary -- it is desirable to introduce the yield tax at the margin under uncertainty; it both corrects the externality due to public goods characteristic of forest stands and serves as an social insurance device for risk-averse forest owners. The optimal yield tax is less than 100 % and depends on the public goods characteristic of forest stands and on the substitution effects of the yield tax. In the general case the inverse elasticity rule --

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depends on how much the government collects tax revenues via the optimal yield taxation. To say more would necessitate the analysis of the optimal tax revenue requirement, which lies beyond the scope of this study.

according to which the optimal yield tax is negatively related to the size of the substitution effects -- may not hold. Under certainty, the desirability of the yield tax given the optimal site productivity tax depends only on the existence of the public goods characteristic of forest stands and is thus a pure Pigouvian tax.

There are several agendas for further research. This paper has developed the implications of the deterministic government tax revenue requirement which provides a social insurance role for taxation. If the risk happens to be aggregative then the insurance role of taxation vanishes, and one has to tradeoff between variability of private consumption and tax revenues, which may imply variability in public consumption. We have made a special assumption about the incidence of forest taxes, namely that forest owners bear them. But this is not so if the demand for timber is not infinitely elastic in terms of timber price. It is an area for further research to find out whether the incidence considerations modify the results.

APPENDIX 1: Comparative Statics of Timber Supply

This appendix derives the comparative statics of timber supply reported in the text both under timber price risk and under certainty. The expected utility maximization problem is reproduced here for convenience.

$$(1)\quad \underset{c_1,h_1,h_2}{MAX}\ EU = -\exp(-Ac_1) - \beta\exp(x) + m(k_1 + \beta k_2)$$

where $x = -A\bar{c}_2 + \frac{1}{2}A^2(1-\tau)^2 h_2^2\sigma_p^2$, subject to

$$(2)\quad \begin{aligned} &(a)\quad k_1 = Q - h_1 \\ &(b)\quad k_2 = (Q-h_1) + F(Q-h_1) - h_2 \\ &(c)\quad \bar{c}_2 = \bar{p}_2^* h_2 - T + (1+r)\left[p_1^* h_1 - T - c_1\right] \end{aligned}$$

The first-order conditions are

$$(3)\quad EU_{c_1} = A\exp(-Ac_1) - \beta RA\exp(x) = 0$$

$$(4)\quad EU_{h_1} = \beta ARp_1^*\exp(x) - m\left[1 + \beta(1+F')\right] = 0$$

$$(5)\quad EU_{h_2} = \beta A\left[\bar{p}_2^* - A(1-\tau)^2 h_2\sigma_p^2\right]\exp(x) - \beta m = 0$$

Utilizing $EU_{h_2} = 0$ in EU_{h_1} leads to the cutting rule given in equation (7.') of the text.

The second-order conditions in equation (6) hold due to the assumptions the concavity of the utility function and the forest growth function. These are

$$(6)\quad \begin{aligned} &a)\quad EU_{c_1c_1} = -A^2\exp(-Ac_1) - \beta A^2R^2\exp(x) \prec 0 \\ &b)\quad EU_{h_1h_1} = -\beta(ARp_1^*)^2\exp(x) + m\beta F'' \prec 0 \\ &c)\quad EU_{h_2h_2} = \beta A^2\left[\bar{p}_2^* - A(1-\tau)^2 h_2\sigma_p^2\right]^2\exp(x) - \beta A^2(1-\tau)^2\sigma_p^2\exp(x) \prec 0 \end{aligned}$$

$$d)\quad \Delta = \begin{vmatrix} EU_{c_1c_1} & EU_{c_1h_1} & EU_{c_1h_2} \\ EU_{h_1c_1} & EU_{h_1h_1} & EU_{h_1h_2} \\ EU_{h_2c_1} & EU_{h_2h_1} & EU_{h_2h_2} \end{vmatrix} \prec 0$$

where the cross-derivatives are

$$EU_{c_1h_1} = \beta A^2R^2p_1^*\exp(x) \succ 0$$

$$EU_{c_1h_2} = \beta A^2R(\bar{p}_2^* - A(1-\tau)^2 h_2\sigma_p^2)\exp(x) \succ 0$$

$$EU_{h_1h_2} = -\beta A^2Rp_1^*(\bar{p}_2^* - A(1-\tau)^2 h_2\sigma_p^2)\exp(x) \prec 0$$

To find how current and future harvesting change as the site productivity tax T, yield tax τ and timber price risk σ_p^2 changes we use Cramer's rule. First of all, we have

(7) $$\begin{bmatrix} EU_{c_1c_1} & EU_{c_1h_1} & EU_{c_1h_2} \\ EU_{h_1c_1} & EU_{h_1h_1} & EU_{h_1h_2} \\ EU_{h_2c_1} & EU_{h_2h_1} & EU_{h_2h_2} \end{bmatrix} \begin{bmatrix} dc_1 \\ dh_1 \\ dh_2 \end{bmatrix} = - \begin{bmatrix} EU_{c_1T} & EU_{c_1\tau} & EU_{c_1\sigma_p^2} \\ EU_{h_1T} & EU_{h_1\tau} & EU_{h_1\sigma_p^2} \\ EU_{h_2T} & EU_{h_2\tau} & EU_{h_2\sigma_p^2} \end{bmatrix} \begin{bmatrix} dT \\ d\tau \\ d\sigma_p^2 \end{bmatrix}$$

where the determinant Δ of the LHS matrix of (7) is negative by the second-order conditions.

Solving (7) for h_1 and h_2 in terms of dT gives

(8) $$h_{1T} = -\beta R p_1^* A^2 (1-\tau)^2 \sigma_p^2 \Phi \succ 0, \text{ where } \Phi = \Delta^{-1}\{\beta^2 A^4 (1+R)\exp(2x - Ac_1)\} \prec 0,$$

(9) $$h_{2T} = mF''\left[\overline{p}_2^* - A(1-\tau)^2 h_2 \sigma_p^2\right]\Phi \succ 0.$$

A change in the variance of the timber price leads to

(10)
$$\begin{aligned} &\text{(a)} \quad EU_{c_1\sigma_p^2} = EU^c_{c_1\sigma_p^2} - (1/2)(1-\tau)^2 h_2^2 (1+R)^{-1} EU_{c_1T} \\ &\text{(b)} \quad EU_{h_1\sigma_p^2} = EU^c_{h_1\sigma_p^2} - (1/2)(1-\tau)^2 h_2^2 (1+R)^{-1} EU_{h_1T} \\ &\text{(c)} \quad EU_{h_2\sigma_p^2} = EU^c_{h_2\sigma_p^2} - (1/2)(1-\tau)^2 h_2^2 (1+R)^{-1} EU_{h_2T} \end{aligned}$$

where $EU^c_{c_1\sigma_p^2}$ and $EU^c_{h_i\sigma_p^2}$, i=1,2, refer to the substitution effects. Solving (7) for h_1 and h_2 in terms of the substitution effects of σ_p^2 yields

(11) $$h^c_{1\sigma_p^2} = -\Delta^{-1}\left\{\beta^2 A^6 (1-\tau)\sigma_p^2 h_2 R p_1^* \left[\overline{p}_2^* - A(1-\tau)^2 h_2 \sigma_p^2\right] \exp(2x - Ac_1)\right\} \succ 0$$

(12) $$h^c_{2\sigma_p^2} = \Delta^{-1}\left\{\beta^2 A^6 (1-\tau)\sigma_p^2 h_2 (R p_1^*)^2 \exp(2x - Ac_1) - EU_{c_1c_1}\beta^2 A^2 (1-\tau)^2 h_2 m F'' \exp(x)\right\} \prec 0$$

The total effect of a change in the variance on harvesting is thus given by the Slutsky equation

(13) $$h_{i\sigma_p^2} = h^c_{i\sigma_p^2} - \frac{1}{2}(1-\tau)^2 h_2^2 (1+R)^{-1} h_{iT}, \quad \text{for } i = 1,2.$$

As for the effects of the yield tax note first that

(13)
$$\begin{aligned} &\text{(a)} \quad EU_{c_1\tau} = EU^c_{c_1\tau} - (1+R)^{-1} z EU_{c_1T} \\ &\text{(b)} \quad EU_{h_1\tau} = EU^c_{h_1\tau} - (1+R)^{-1} z EU_{h_1T} \\ &\text{(c)} \quad EU_{h_2\tau} = EU^c_{h_2\tau} - (1+R)^{-1} z EU_{h_2T} \end{aligned}$$

where $z = \left[\overline{p}_2 - A(1-\tau)^2 h_2 \sigma_p^2\right] h_2 + R p_1 h_1$, and $EU^c_{c_1\tau}$ and $EU^c_{h_i\tau}$, $i = 1,2$ refer to the substitution effects.

Solving (7) for h_1 and h_2 in terms of the substitution effects of τ and utilizing (11) and (12) gives

(14) $$h^c_{1\tau} = h^0_{1\tau} - (1-\tau)^{-1} \sigma_p^2 h^c_{1\sigma_p^2} \prec 0$$

(15) $$h^c_{2\tau} = h^0_{2\tau} - (1-\tau)^{-1}\sigma^2_p h^c_{2\sigma^2_p} = ?$$

where h^0_i, i=1,2 denote for the "conventional" substitution effects defined as follows

$$h^0_{1\tau} = (1-\tau)^{-1}\left[p_1 h^c_{1p_1} + \overline{p}_2 h^c_{1\overline{p}_2}\right] = -\Delta^{-1}\left[\beta A^3 R p^*_1 (1-\tau)\sigma^2_p EU_{c_1c_1}\exp(2x)\right] \prec 0$$
$$h^0_{2\tau} = -(1-\tau)^{-1}\left[p_1 h^c_{2p_1} + \overline{p}_2 h^c_{2\overline{p}_2}\right] = \Delta^{-1}\left[\beta^2 A m F''(\overline{p}^*_2 - A(1-\tau)h_2\sigma^2_p)EU_{c_1c_1}\exp(x)\right] \prec 0.$$

The total effect of a change in the yield tax can be obtained by utilizing the Slutsky decomposition and equations (14) and (15), and it is

(15) $$h_i = h^c_{i\tau} + (1+R)^{-1} z h_{iT},\ \text{i=1,2}.$$

* * *

APPENDIX 2: The sign of $(B_1 h^c_{1\tau} + B_2 h^c_{2\tau})$ as $\tau \to 0$

This appendix fixes the sign of $B_1 h^c_{1\tau} + B_2 h^c_{2\tau}$ in the equation (23) of the text as $\tau \to 0$. Recalling that $B_1 = m(1+\beta(1+F'))$and $B_2 = \beta m$ we have to determine the sign of

(1) $$\phi = \left[1+\beta(1+F')\right]h^c_{1\tau} + \beta h^c_{2\tau}.$$

Using the expressions of $h^c_{1\tau}$ and $h^c_{2\tau}$ and arranging the terms gives the following expression.

(2) $$\phi = -\sigma^2_p\left\{\left[1+\beta(1+F')\right]h^c_{1\sigma^2_p} + \beta h^c_{2\sigma^2_p}\right\} + \left[1+\beta(1+F')\right]h^0_{1\tau} + \beta h^0_{2\sigma^2_p}$$

The substitution effects $(h^0_{1\tau}, h^0_{2\tau})$ are negative at $\tau = 0$. As for the first RHS term, notice first that $EU_{h_2} = 0$ is equivalent to $(\overline{p}_2 - A h_2 \sigma^2_p) = m(\exp(x))^{-1}$ as $\tau = 0$. Utilizing equations (11) and (12) from appendix 1 and substituting $m(\exp(x))^{-1}$ for $(\overline{p}_2 - A h_2\sigma^2_p)$ yields

(3) $$-\sigma^2_p\Delta^{-1}\left\{\beta^2 A^6 h_2 R p_1\left[\beta R p_1 \exp(x) - (1+\beta(1+F'))m\right]\exp(x)^{-1}\right\}$$
$$-\sigma^2_p\Delta^{-1}\left\{\beta^2 A^2 h_2 m F'' EU_{c_1c_1}\exp(x)\right\}$$

The first term in (3) is zero by $EU_{h_1} = 0$. Hence what is left from ϕ is

(4) $$\phi = \left[1+\beta(1+F')\right]h^0_{1\tau} + \beta h^0_{2\sigma^2_p} - \sigma^2_p\Delta^{-1}\left\{\beta^2 A^2 h_2 m F'' EU_{c_1c_1}\exp(x)\right\}$$

This is equal to $\phi = \left[1+\beta(1+F')\right]h^0_{1\tau} + \beta\Delta^{-1}\left\{\beta^2 A h_2 m F''(p_2 - 2Ah_2\sigma^2_p)\exp(x)\right\}$, which is clearly negative so that $B_1 h^c_{1\tau} + B_2 h^c_{2\tau} \prec 0$ as $\tau \to 0$.

* * *

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