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### **INCOME TAX INDEXATION IN AN OPEN ECONOMY**

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**ABSTRACT:** In an economy with progressive taxation, income tax indexation provides a way of cushioning against various shocks. It provides an alternative as well as a complement to wage indexation. We nevertheless show that there is much more ambiguity about income tax indexation and its effects compared to wage indexation. The specification of the money demand equation is important, and especially crucial is the point put forward by e.g. Holmes and Smyth (1972) and Mankiw and Summers (1986) that if money demand depends on disposable income or consumer spending, then it may well be that tax cuts are contractionary. We show that this may change the results of Bruce (1981) concerning tax indexation completely. We also show that the presence of progressive taxation makes Aizenman's (1985) results concerning optimal wage indexation more complex. An increase in openness, e.g., does not necessarily increase the optimal degree of wage indexation.

**KEY WORDS:** tax and wage indexation, demand and supply disturbances, automatic stabilization, exchange rate expectations.



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The model</b>	<b>2</b>
2.1	Modelling income tax indexation . . . . .	2
2.2	The goods and money markets . . . . .	2
<b>3</b>	<b>Short-run equilibrium</b>	<b>5</b>
3.1	The role of expectations . . . . .	5
3.2	Taxation and the demand for money . . . . .	7
<b>4</b>	<b>Optimal income tax indexation</b>	<b>9</b>
<b>5</b>	<b>Temporary shocks vs. permanent shifts</b>	<b>11</b>
<b>6</b>	<b>Taxation and optimal wage indexation</b>	<b>13</b>
<b>7</b>	<b>The shock sensitivity of prices and output</b>	<b>14</b>
<b>8</b>	<b>Concluding remarks</b>	<b>15</b>
<b>A</b>	<b>Appendix: Optimizing the tax parameters</b>	<b>16</b>

# 1 Introduction

The vast literature on wage indexation seems to have one rather robust result: compared to complete nominal wage rigidity, wage indexation stabilizes output with respect to demand shocks but makes it more sensitive to supply shocks. On the other hand, wage indexation destabilizes prices with respect to both kind of shocks.

Bruce (1981) studies income tax indexation in connection with wage indexation in a closed economy. Bruce defines income taxes as indexed if the real amount of income taxes depends only on real income and not on the price level. Bruce claims that income tax indexation makes output more sensitive to demand shocks but less sensitive to supply shocks, which are precisely the opposite results as under wage indexation. Prices, however, become more sensitive to both kind of shocks.

Unfortunately, in his analysis Bruce allows only one transmission mechanism of taxation — through disposable income to expenditure. This restriction makes it possible to achieve unambiguous results. But even in a very aggregated model, taxation may have other important effects. We explore here the view put forth by Holmes and Smyth (1972) and Mankiw and Summers (1986) that disposable income affects the demand for money.

If taxation affects directly the demand for money, then tax cuts may not be expansionary. Mankiw and Summers (1986) argue that consumer demand affects the demand for money more than other components of demand do. This makes the sign of the effect of a tax reduction to output unclear a priori. If the interest elasticity of the demand for money is sufficiently low, tax reductions are contractionary. Mankiw and Summers present empirical evidence concerning the U.S. economy that supports the view that both consumer demand and disposable income are better explanatory variables in a money demand equation than total GNP.

This article studies the effects of income tax indexation in an open economy context. Our starting point is Aizenman (1985), who studied how optimal wage indexation is affected by the degree to which domestic output is exposed to the prices of internationally traded goods. His main finding is that, under flexible exchange rates, the more open the economy is, the higher is the optimal degree of wage indexation. We combine progressive taxation into his model in a way similar to Bruce, and study the effects of income tax indexation.

## 2 The model

### 2.1 Modelling income tax indexation

With constant real income, price level changes may change the share of taxes relative to total income basically for two reasons. First, marginal tax rates are often graduated with respect to nominal income, not real income. Second, deduction limits are usually fixed in nominal terms.

We follow Bruce (1981) in the modelling of income tax indexation and define  $T$  as the ratio of gross to net real income. We assume that  $\ln T$ , the logarithm of  $T$ , is given by

$$(1) \quad \ln T = \tau + uy + (1 - r)up$$

where

$$\tau > 0, 0 < u < 1.$$

and  $y$  and  $p$  are the logarithmic deviations of real output  $Y$  and price level  $P$  of their equilibrium values, respectively. Various shocks cause output and prices to deviate from their equilibrium values, thus changing the share of taxes changes relative to income. Without any shocks the share of taxes would be  $1 - e^{-\tau}$ . The degree of tax progressivity is described by  $u$ , assumed to be positive<sup>1</sup>.  $u$  is also assumed to be less than one, otherwise the marginal tax rate from real income would exceed unity. The parameter  $r$  measures the degree of tax indexation. If taxes are fully indexed,  $r$  is equal to one and the average tax rate depends only on the real income. If taxation is completely unindexed,  $r$  is equal to zero and the average tax rate depends only on the nominal income.

### 2.2 The goods and money markets

We take Aizenman (1985) as a starting point for our model. As in his model, we consider a two-sector economy where the country is small in the traded goods sector and large in the nontraded sector. Thus the relative price of traded to nontraded goods is determined endogenously. The more open the economy is, i.e. the larger the share of traded goods, the smaller is the

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<sup>1</sup>Income tax structure is progressive, proportional or regressive depending on whether  $u$  is positive, zero or negative.

relevance of this endogenous relative price. This affects also optimal wage and tax indexation.

Our model differs from Aizenman's model primarily with regard to taxation, which affects the demand in the goods market and may affect also the demand for money. Aizenman's model may be described as one with proportional taxation, which makes all tax parameters vanish from the analysis. Our model has progressive taxation.

Real output  $Y$  in period  $t$  is defined as

$$(2) \quad Y_t = (N_t P_{n,t} + Z_t P_{z,t}) / P_t$$

where  $N$  and  $Z$  represent the output of nontraded and traded goods.  $P_n$  and  $P_z$  are prices of nontraded and traded goods, respectively, and  $P$  is a price index:

$$(3) \quad P_t = (P_{n,t})^{\theta_n} (P_{z,t})^{\theta_z}$$

where

$$\theta_n + \theta_z = 1$$

The  $\theta$ -terms represent the shares of nontraded and traded goods.

Real disposable income is given by the identity:

$$(4) \quad Y_t^d = Y_t / T_t$$

For traded goods, the law of one price is assumed to hold:

$$(5) \quad P_{z,t} = S_t P_{z,t}^*$$

where  $S$  is the exchange rate and  $P_z^*$  is the international price of traded goods. The exchange rate is flexible.

The supply side of the economy is described by the labor supply and production functions. The labor supply is given by

$$(6) \quad L_{j,t}^s = Q_j (W_t / P_t)^\delta$$

where  $j$  denotes the sector (either  $n$  or  $z$ ).  $W$  is the money wage. Labor is the only mobile factor of production. The elasticity of labor supply with respect to real wage is assumed to be the same in both sectors.



Output is given by

$$(7) \quad N_t = Q_n(L_{n,t})^\alpha \exp(v_t)$$

and

$$(8) \quad Z_t = Q_z(L_{z,t})^\alpha \exp(v_t)$$

The term  $v_t$  is a multiplicative productivity shock.

Wages are thought to be set to equate the expected labor supply and expected labor demand. The contracts are made before any shocks occur, and the prices prevailing during the contract period  $t$  are therefore not known. Nominal wages are, however, partially indexed to prices. The actual real wage differs from the expected level because of shocks, and actual employment is assumed to be demand determined.

We could have formulated the supply of labor to depend on the net real wage instead of the gross real wage, following Blinder (1973). It would not make any qualitative difference in the short run, as employment is demand-determined. Whether the full equilibrium is affected is considered in a footnote in section 4.

Thus far the model has been identical to Aizenman's (1985) model. The demand side, however, is different. Instead of real output  $Y$  as a scaler, we use net real income as defined in equation (4).

The demand of the nontraded goods is given by

$$(9) \quad (P_{z,t}/P_{n,t})^a Y_t^d \exp(\bar{c} - c(i_t - \pi_t))$$

where  $a$  is the compensated demand elasticity,  $i$  is the money interest rate and  $\pi$  is the expected inflation:

$$(10) \quad \pi_t = (E_t P_{t+1} - P_t)/P_t$$

The term  $E_t$  denotes a conditional expectation operator, conditional on information available at time  $t$ .

The demand for money is specified in two different ways. First is the conventional one:

$$(11) \quad M_t^d = Y_t P_t \exp(-ki_t)$$

The second specification approximates the claim of Mankiw and Summers (1986) that consumer spending is the best scaler in money demand function; thus  $Y$  is replaced by  $Y^d$ :

$$(12) \quad M_t^d = Y_t^d P_t \exp(-ki_t)$$

The next equation is the arbitrage condition that connects domestic and foreign interest rates under perfect capital mobility:

$$(13) \quad i_t - i_t^* = (E_t S_{t+1} - S_t) / S_t$$

The supply of money,  $M^s$ , is formed of three parts. First, there is a basic exogenous supply  $\bar{M}$ . Second, there is an exponential random shock  $m_t$ . Third, there is a negative element that comes from unindexed income taxation. Increased real taxes that result from unexpected inflation draw money off from the market. We assume that all this money is returned to supply through open market operations.

$$(14) \quad M_t^s = \bar{M} \exp(m_t)$$

### 3 Short-run equilibrium

#### 3.1 The role of expectations

The four disturbance terms  $m, v, i^*$  and  $p_z^*$  are assumed to be uncorrelated and generated by white noise processes, e.g.:

$$(15) \quad m \sim N(0, \sigma_m^2)$$

and analogously for  $v, i^*$  and  $p_z^*$ . The variances are henceforth denoted by  $V_m, V_v, V_{i^*}$  and  $V_{p_z^*}$ , respectively.

It is convenient to express variables as percentage deviations around their non-stochastic equilibrium values. This non-stochastic equilibrium is achieved through solving the model under the assumption that  $m_t = v_t = i_t^* = p_{z,t}^* = 0$ . To get percentage deviations, we use log-linear approximations. We denote these transformed variables with lowercase letters.

Equating the marginal products of labor to the real product wages, we get the sectoral supplies:

$$(16) \quad n = \bar{h}(p_n - w) + hv$$

$$(17) \quad z = \bar{h}(p_z - w) + hv$$

where

$$\bar{h} = \alpha / (1 - \alpha); h = 1 / (1 - \alpha)$$

The determination of the short-run equilibrium can now be expressed with seven equations:

$$(18) \quad y = \bar{h}(p - w) + hv$$

$$(19) \quad \bar{h}(p_n - w) + hv = -a(p_n - p_z) + (1 - u)y - (1 - r)up - c(i + p)$$

$$(20) \quad m - p = (1 - gu)y - g(1 - r)up - ki$$

$$(21) \quad w = bp$$

$$(22) \quad i = i^* - s$$

$$(23) \quad p_z = p_z^* + s$$

$$(24) \quad p = \theta_n p_n + \theta_z p_z$$

Aggregate supply  $y$  in equation (18) is obtained from sectoral supplies, using also equations (2) and (3). Equation (19) is obtained by equating the supply of and demand for nontraded goods. Parameter  $g$  in the money demand equation (20) varies with the specification. The conventional specification is achieved with  $g = 0$ , and the Mankiw-Summers specification with  $g = 1$ . Equation (21) is the wage indexing rule. The coefficient  $b$  shows the amount of wage indexation, with  $b = 0$  referring to no indexation and  $b = 1$  to full indexation.

The short-run equilibrium can be thought as the first period solution of a two-period model. The second period affects the first only through expectations concerning inflation and the exchange rate. Since  $p$  is the unexpected deviation of the price level from its no-shock equilibrium value, it creates an expectation of inflation of the amount  $-p$  in the next period; thus the real interest rate is  $i + p$ , as in equation (19). Similarly, if the shocks cause the exchange rate to devalue, that creates a revaluation expectation as in (22).

Expectations are crucial for the resulting equilibrium. If there were no expectations, the model solution would be extremely simple. The price level would be determined solely in the money market. It would be affected only by domestic monetary or productivity shocks and by foreign interest shocks. The openness of the economy would not affect prices. The exchange rate would balance the supply and demand of the non-traded goods by making the relative price of non-traded to traded goods appropriate.

The solution becomes much more complicated with exchange rate expectations. A change in the exchange rate creates an expectation of a reversed

change in the future, and this immediately affects the demand for money. Prices and the exchange rate are now simultaneously determined.

Solving the model given by equations (19) - (24) yields the following expression for  $p$ :

$$(25) \quad p = D^{-1} \{m - h[1 + u(k/\phi - g)]v + (i^* + p_z^*)k(1 - c/\phi)\}$$

where

$$D = [1 + u(k/\phi - g)]\bar{h}(1 - b) + k + 1 + u(k/\phi - g)(1 - r)$$

and

$$\phi = c + [a + \theta_z \bar{h}]/\theta_n$$

We could now solve  $y$  from (18), and then other variables. As this is not, however, essential for the questions we wish to consider here, it is postponed to a later stage.

Openness can be measured in this model by the share of traded goods ( $\theta_z$ ), or by the substitutability in consumption ( $a$ ) or in production ( $\bar{h}$ ). An increase in any of these will increase  $\phi$ .

### 3.2 Taxation and the demand for money

Taxation brings three different ingredients into the analysis. One is progressivity, which means that also real income surprises affect real disposable income even though taxation may be indexed. The second is tax indexation or its absence, which determines whether price surprises have direct effects on real disposable income. The third effect is the option of modelling money demand to depend on real disposable income, instead of conventional total real income. Our primary interest lies with the second of these three points, but its inclusion requires non-proportional taxation and the results turn out to be sensitive with respect to the third option.

We notice that if taxation were proportional, that is  $u = 0$ , all tax parameters would vanish from (25). With  $u \neq 0$ , the degree of tax indexation ( $r$ ) and the specification of money demand ( $g$ ) have both separate and combined effects.

To show that the demand for money specification really makes a difference here, let us have a closer look at (25). Let us denote

$$(26) \quad \mu = k/\phi - g$$

The term  $\mu$  is clearly greater than  $-1$ , and thus  $1 + u\mu$  is positive. But  $\mu$  is not necessarily positive. This term is in the coefficients of all the tax terms in (25). Clearly,  $\mu$  can only be negative if we assume that money demand depends on real disposable income, so that the parameter  $g$  is set to 1. Then the sign of  $\mu$  depends on the magnitudes of  $k$  and  $\phi$ .

Progressive taxation is often thought to be an 'automatic stabilizer'. But this depends on the specification of money demand, as Holmes and Smyth (1972) and Mankiw and Summers (1986) have noted. In our model this is straightforwardly seen from (25), assuming taxes to be fully indexed. If  $\mu$  is negative, progressive taxation decreases the denominator and thus increases the effects of demand shocks (monetary and foreign shocks) and thus becomes an 'automatic destabilizer'. It may, however, stabilize supply shock effects. With positive  $\mu$  these stabilitory properties are reversed. We may thus define the automatic stabilization condition as follows.

AUTOMATIC STABILIZATION CONDITION: Income taxation is an automatic stabilizer if and only if

$$(27) \quad k > g\phi$$

To interpret this condition, let us consider a shock that creates excess demand in the market for non-traded goods. The exchange rate probably appreciates to lower the relative price of traded goods, although this would also create depreciation expectation and increase the real interest rate. Whatever the change in exchange rate, an increase in prices balances the market more with the tax effects included than without them, because the tax effects reduce demand. The tax effects thereby dampen price variability in the non-traded goods market. The situation is nevertheless different in the money market. A rise in prices reduces the real money stock, but this balancing effect is reduced by the tax effects, because they reduce the demand for money. Any disequilibrium in the money market requires larger price movements to balance, and thus taxation enlargen the variability of prices. Condition (27) tells when the money market effects of taxation are stronger than the opposing effects in the goods market.

Mankiw and Summers argued that for the U.S. economy the interest elasticity of money demand was sufficiently low compared to the interest elasticity of goods demand so that the analogy of condition (27) was probably not met. In our model the interest rate effects come from movements in the exchange rate, which create expectations about future reversed movements.

In addition the exchange rate changes the relative price of goods. Both these channels are captured in the term  $\phi$ . The more open the economy the more likely it is that condition (27) does not hold. Whether or not it holds, the term  $D$  in (25) is positive.

It is noticeable that taxation has effects also on the relative price adjustment. The equation of the non-traded goods market can be expressed as

$$(28) \quad p_n - p_z = -c(i^* + p_z^*)\theta_n/\phi - u[1 - r + \bar{h}(1 - b)]p - uhv$$

A transitory foreign price or interest rate increase leads to a higher real interest rate, which reduces the demand for both goods. The relative price of non-traded goods must fall to balance the supply and demand of non-traded goods. On the other hand, higher inflation that follows the shocks, according to (25), reduces disposable income through taxation and thus further reduces demand. Therefore the needed adjustment of the relative price is larger the more progressive taxation is and the less it is indexed.

With progressive taxation, also domestic shocks change the relative price. This is evident from (28), where productivity shocks are explicitly present and monetary shocks have effects through changes in the price level.

## 4 Optimal income tax indexation

In the foregoing analysis employment was determined by labor demand. The labor market did not clear, but rather the wage level was set by the wage indexing rule (21). The resulting welfare loss from the labor market disequilibrium is proportional to the expected squared discrepancy of output from its equilibrium level, obtained with full market clearing, see e.g. Aizenman and Frenkel (1985) and Aizenman (1985). Thus we use the loss function adopted by e.g. Gray (1976) and Flood and Marion (1982).

$$(29) \quad H = E_0(y - \tilde{y})^2$$

where a tilde  $\tilde{\cdot}$  above a variable refers to the value of a variable in a fully flexible economy. In a flexible economy it follows from the labor market

clearing<sup>2</sup> that

$$(30) \quad \tilde{w} = \tilde{p} + \frac{h}{\delta + h}v$$

$$(31) \quad \tilde{y} = hv - \frac{\bar{h}h}{\delta + h}v$$

From equations (18) and (31) we get that

$$(32) \quad H = (\bar{h})^2 E_0[(1 - b)p + \frac{h}{\delta + h}v]^2$$

Replacing (25) for  $p$  in (32), squaring, taking the expectation and differentiating with respect to  $r$ , we get the optimal value for tax indexation:

$$(33) \quad r^* = b + \delta(b - 1) + \frac{k + 1 - (1 - b)[\Phi/(1 + u\mu) + 1 + \delta]}{u\mu}$$

where

$$\Phi = \frac{\delta + h}{h^2} \left[ \frac{V_m}{V_v} + k^2(1 - c/\phi)^2 \frac{V_{i^*} + V_{p_z^*}}{V_v} \right]$$

and

$$b \neq 1.$$

It is evident from (33) that the automatic stabilization condition is indeed critical here. We proceed by assuming conventionally that  $\mu$  is positive and condition (27) is not met. If  $\mu$  is negative, all the results below change signs.

The less flexible the real wage is, that is the more wages are indexed, the higher is the optimal degree of income tax indexation<sup>3</sup>.

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<sup>2</sup>If the supply of labor depends on the net real wage the following results may change. Assuming that wage taxation is similarly progressive as total income taxation, all depends on how we allow progression to affect the share of taxes relative to income. Assuming that the share of taxes in the full equilibrium is always  $1 - e^{-\tau}$  would keep qualitative results similar. If instead the share of taxes is  $1 - e^{-\tau - u\tilde{y}}$  in the new equilibrium, optimal income tax indexation (and also wage indexation) would depend on progressivity in a much more complex way. Complexity would further increase if the share of taxes would be  $1 - e^{-\tau - u\tilde{y} - u(1-r)\tilde{p}}$ .

<sup>3</sup>With fully indexed wages, optimal tax indexation would, however, be indeterminate. This is straightforwardly seen from equation (32): with  $b$  equal to one, the loss function would depend only on the variance of productivity shocks, and the tax indexation parameter is not included in that relation.

The relative importance of shocks plays a role here. The higher the ratio of productivity shock variance to the variances of other shocks, the higher is the optimal tax indexation.

The optimal rate of income tax indexation depends negatively on the real wage elasticity of the supply of labor.

The interest elasticity of the demand for money and the progressivity of taxation have ambiguous effects on the optimal degree of tax indexation. This is hardly surprising. And whatever the signs are, they change if the automatic stabilization condition is not met.

The openness of the economy has an ambiguous effect on the optimal tax indexation, except that the more open the economy the more likely it is that (27) does not hold and the results above change sign.

Nothing guarantees here that optimal tax indexation yields a value for  $r$  that is between zero and one or in the limit. The necessary inequalities can be obtained from (33), but the results are too messy to be fruitful.

## 5 Temporary shocks vs. permanent shifts

The role of expectations is crucial in the determination of the short-run equilibrium. The shocks are assumed to be temporary, and the shocks of today are assumed to contain no information about the shocks of tomorrow. Thus it is rational to expect that whatever deviations from the no-shock equilibrium occur this period, these deviations are reversed in the next period because the best forecast of the next period is simply the no-shock equilibrium:

$$(34) \quad (E_t P_{t+1} - P_t)/P_t \approx -p, \quad (E_t S_{t+1} - S_t)/S_t \approx -s$$

These reversed-deviations expectations affect the present period through inflation expectations via the real interest rate and through exchange rate expectations via the interest parity condition.

Since the expectations are so crucial, it is useful to check how sensitive the results are to the assumption that shocks are strictly temporary. We forsake the assumption of temporariness in this section, and assume that all shocks are permanent shifts. All agents perceive that the shifts are permanent, and make rational forecasts of the next period.

Indexation here reflects rigidities both in wage formation and in taxation that are due to contract length. New contracts are made before the next



period. Wages are set so that the labor market is in equilibrium, with the new shifted parameters. Taxes are assumed to be set so that the ratio of gross to net real income is  $e^r$  (see equation 1). This is consistent with the idea that progression is not allowed to increase the share of taxes in time in a growing economy.

We assume that the agents are able to calculate the second period full equilibrium values of the relevant variables. This means either that the shocks are directly observable or that their magnitudes can be inferred from observable variables, following the line of reasoning of Karni (1983). Here this latter route can be followed assuming that output, domestic prices, the domestic interest rate and the exchange rate are observable.

In the full equilibrium the labor market also clears, so that we replace the wage indexation rule by the following:

$$(35) \quad \tilde{w} = \tilde{p} + \frac{h}{\delta + h}v$$

As there are no expected changes in the full equilibrium, the model solution is easily obtained. The price level comes from the money market equation:

$$(36) \quad \tilde{p} = m - \frac{1 + \delta}{h + \delta}v + ki^*$$

The exchange rate is determined through the goods market:

$$(37) \quad \tilde{s} = \tilde{p} + \frac{\theta_n c(i^*)}{h\theta_z + a} - p_z^*$$

These would be the required changes in the prices and exchange rate to achieve the full equilibrium. The actual changes in the first period are different, however, and that creates the rational expectation that there are further changes to come. These expectations are:

$$(38) \quad (E_t P_{t+1} - P_t)/P_t \approx \tilde{p} - p, \quad (E_t S_{t+1} - S_t)/S_t \approx \tilde{s} - s$$

Comparing (34) and (38) we notice that in both cases inflation this period lowers the expected inflation next period, although in the permanent shift case from a higher initial expectation level, and analogously for the exchange rate expectations.

Solving the model with these modified expectations yields the following expression for the price level adjustment.

$$(39) \quad p = D^{-1} \left\{ (k+1)m - \left[ 1 + u\mu - \frac{1+\delta}{h+\delta} \right] hv + k^2 i^* \right\}$$

where  $D$  is the same as in equation (25).

Although these expectations make the short-run equilibrium somewhat different, the qualitative change is not very significant. The biggest difference is that, with permanent shifts, the foreign price of traded goods has no effects on the aggregate price level. The reason is that the immediate reaction in  $s$  is  $-p_z^*$ , which is also the full equilibrium change, so no further adjustment is expected. We also notice that the effects of monetary disturbances have grown while productivity effects have diminished.

## 6 Taxation and optimal wage indexation

Replacing (25) for  $p$  in (32), squaring, taking the expectation and differentiating (32) with respect to  $b$ , we get the optimal value for wage indexation. It is interesting to note that  $b$  and  $r$  are determined only in relation to each other. Namely, differentiating with respect to either  $b$  or  $r$  yields the same conditions for the optimal values. For a given  $r$  we get

$$(40) \quad b^* = 1 - [k+1 + (1-r)u\mu] / [\Phi(1+u\mu)^{-1} + (1+u\mu)(1+\delta)]$$

Equations (40) and (33) are the same. This means that in this model, forgetting for the moment that  $b$  and  $r$  should be either zero or unity or in between, wage indexation and income tax indexation are equally efficient methods for minimizing the loss function. The mathematics are looked upon more closely in the Appendix. The economic explanation might be that both instruments affect the transmission of shock effects only from price movements onwards to other variables. Notice that this redundancy of one of these two instruments does not follow from the fact that we have two instruments for one target. We could get a better social optimum by optimizing the rate of progressivity  $u$  also. This progressivity parameter affects the transmission of shock effects also from output movements onwards to other variables.

We could, of course, set an additional target, e.g. the stability of the exchange rate, and optimize either  $b$  or  $r$  with respect to that target, without sacrificing anything from the original target.

Aizenman's (1985) two main results were that an increase in openness will increase optimal wage indexation, and that optimal wage indexation decreases in accordance with the ratio of productivity variance to the variance of the other shocks. These results can be seen in a rather straightforward manner from (40), assuming that  $u = 0$ . Productivity variance case is obvious, and for openness, all three measures used by Aizenman lead to a similar conclusion:

$$b_a^* > 0, b_{\theta_z}^* > 0, b_{\bar{h}}^* > 0$$

where the partial derivatives are taken with respect to the share of traded goods ( $\theta_z$ ), the substitutability in consumption ( $a$ ) and in production ( $\bar{h}$ ).

Do these results hold when taxes are not indexed? For openness, not necessarily, because progressive taxation has made the case more complex. In Aizenman's article openness only showed in the term  $\phi$  and thus in  $\Phi$ . Here  $\phi$  shows up also in  $\mu$ , and it has effects in both directions. The results are ambiguous, and they may turn around for some parameter combinations, especially with high progressivity. Consider e.g. the case where the variance of productivity shocks is huge compared to the variances of other shocks. As  $V_v \rightarrow \infty$ ,  $\Phi \rightarrow 0$ . Now, as openness increases,  $\mu$  gets smaller in (40) and the optimal  $b$  also gets smaller.

In an economy with progressive taxation, openness affects optimal wage indexation even if there are no foreign shocks.

The productivity variance case is stronger. The results are the same qualitatively with both the conventional money demand specification and the Mankiw - Summers specification.

## 7 The shock sensitivity of prices and output

Bruce (1981) claimed that both wage and tax indexation make prices more sensitive to shocks. This holds also here for wage indexation but not necessarily for tax indexation. This can be seen from (25). Both indexation parameters are in the denominator of  $p$ 's sentence. But  $(1 - r)$ 's coefficient  $u(k/\phi - g)$  is not necessarily positive. It is positive if we use the conventional

money demand specification but may well be negative if we use disposable income in the money demand equation. Then tax indexation would reduce the sensitivity of the price level to all kinds of shocks considered here.

Tax indexation affects also output variance. Using equations (18) and (25) we get

$$(41)y = D^{-1}\{\bar{h}(1-b)[m+k(1-c/\phi)(i^*+p_z^*)] + [k+1+u\mu(1-r)]hv\}$$

Income tax indexation makes output less sensitive to productivity shocks and more sensitive to all other shocks, at least in the conventional money demand case. The automatic stabilization condition is crucial here, too, for it changes the signs of the effects on output sensitivity with respect to all shocks.

## 8 Concluding remarks

Income tax indexation may provide an alternative or a complement to wage indexation for stabilizing an economy subject to various kinds of shocks. It is an alternative, because it turned out to be equally efficient in minimizing the loss function studied here as wage indexation. It can nevertheless also be a complement, because the optimal values for wage and tax indexation are not necessarily in the range between zero and one, but there may be a combination of optimal values that fulfills this condition.

It seems, however, that there is much more theoretical ambiguity concerning income tax indexation and its effects than there is concerning wage indexation. Especially, the demand for money specification is crucial for tax indexation results but not for wage indexation results. If we accept the specification where disposable income rather than total income affects the demand for money, then the more open the economy is the more likely it is that we get results that are exactly the opposite to those of Bruce (1981).

Moreover, the presence of progressive taxation makes qualitative results concerning the effects of wage indexation more complex. If the variance of productivity shocks is large, compared to other shock variances, it is possible that the optimal degree of wage indexation is a decreasing function of the openness of the economy, not increasing as in Aizenman (1985).

## A Appendix: Optimizing the tax parameters

The loss function (32) can be expressed as follows, after squaring and taking expectations:

$$(42) \quad H = C_1 z^2 + 2C_2 z + C_3$$

where

$$z = (1 - b) / [(1 + u\mu)\bar{h}(1 - b) + k + 1 + (1 - r)u\mu]$$

$$C_1 = V_m + (1 + u\mu)^2 h^2 V_v + (V_{i^*} + V_{p_z^*}) k^2 (1 - c/\phi)$$

$$C_2 = -\frac{h^2}{h + \delta} (1 + u\mu) V_v$$

$$C_3 = \frac{h^2}{(h + \delta)^2} V_v$$

The  $C_i$  terms are constants that do not depend on  $b$  or  $r$ . So we can minimize (42) with respect to  $z$ , which yields equations (33) and (40).

## References

Aizenman, Joshua (1985): Wage Flexibility and Openness. *Quarterly Journal of Economics*, 539- 550, May 1985.

Blinder, Alan S. (1973): Can Income Tax Increases Be Inflationary? An Expository Note. *National Tax Journal*, Vol. XXVI, 295 - 301.

Bruce, Neil (1981): Some Macroeconomic Effects of Income Tax Indexation. *Journal of Monetary Economics* 8, 271- 275.

Gray, J. (1976): Wage Indexation: A Macroeconomic Approach. *Journal of Monetary Economics*, n:o 2, 1976, 221 - 235.

Holmes, James M. and David J. Smyth (1972): The Specification of the Demand for Money and the Tax Multiplier. *Journal of Political Economy*, vol. 80, no 1, 179 -185.

Karni, Edi (1983): On Optimal Wage Indexation. *Journal of Political Economy*, vol. 91 no. 2, 282 - 292.

Mankiw, N. Gregory, and Lawrence Summers (1986): Money Demand and the Effects of Fiscal Policies. *Journal of Money, Credit and Banking*, Vol. 18, No. 4, 415 - 429, November 1986.

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