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OPTIMAL PRODUCTION OF INNOVATIONS UNDER UNCERTAINTY

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ABSTRACT: The paper reports results on a risk neutral firm's research incentives. When unrelated to the firm's own stake in the program, the risks encourage or discourage risky research spending depending on the properties of the research technology available. A non-decreasing time path of information builds into the model the idea of an asymmetric probability distribution of the state of knowledge. It follows that the required return on risky investments may actually fall short of the safe return. Since it is the upside risk that dominates, increased controllable risks will increase incentives for risky innovating activity. It is proved, but only in a more restricted framework (with differentiable processes), that the expectational effects involved will strengthen the positive relationship between controllable risks and the expected return.

KEY WORDS: Innovations, uncertainty.

I Introduction

The key feature of the research technology of a firm is the genuine risk associated with the required effort for a given progress to be made. Alternatively, given the past and current research effort, the progress to be made can only be imperfectly predictable. A research program represents, by tautology, a leap into the unknown. Relative to investment in known technology, the risks are of a different variety when a research program is concerned. First, if a research program is a failure, it has a private liquidation value of zero. But if it is a success, the reward may be quite substantial. Given this particular type of distribution of returns, the question is raised whether sufficient incentives exist for a firm to undertake research programs. This is the issue addressed in the current paper. It undertakes the task of developing a positive theory of a firm subject to uncertainty about the output of its research technology.[1] As to the interaction of inputs, a successful research program is understood to improve the productivity of current assets.[2] This provides motivation for the second objective of the paper, which concerns the spillover effects on the flow of investment in current assets.

In contrast to the previous work in the field, the paper recognizes that the distribution of returns is asymmetric. Intuitively, the maximum loss cannot exceed the firm's own stake while, in principle, there is no upper limit to the gain. It is the upside-dominated asymmetry that is characteristic of the outcome of the research technology. This provides an explanation for the major finding of the paper, i.e. that uncertainty enhances risk neutral firms' incentives to undertake risky projects.

The paper formulates a model of a firm which has two exclusive options with regard to allocation of its cash flow, after allowing for depreciation of its current capital assets. In the first alternative, the firm can retain part of its cash flow to be allocated to capital assets operating under current technology. In the second alternative, its cash flow can be channelled to a research program in order to enhance the productivity of the existing assets. The latter alternative is hoped to give rise to process innovations with a positive payoff.[3]

The key features of the model of the current paper are as follows:

(i) It will be assumed that investment in current assets has fully predictable effects on profitability while spending on an innovation process only entitles the firm to a probability distribution of returns.

(ii) The fundamental assumption to be modelled is that the future production technology depends on the output of the current research technology of the firm. The complementary inputs required in production of new knowledge are taken to be the old knowledge and current spending on research. The expected outcome, flow of innovations, depends on the firm-specific research technology, or "creativity" of the research department of the firm. However, the outcome of the research program cannot be completely predicted on the basis of this information only. There is genuine uncertainty, modelled in terms of a Wiener process.

(iii) It is an intrinsic property of the model that uncertainty of the more distant future is greater than is the uncertainty associated with tomorrow.

(iv) The model incorporates, though at some cost of increased complexity, the highly appealing idea that with a small stake the risks are also small while they are manifold with a greater stake. The model hence introduces the idea that the risks are controllable by the firm.

(v) The stochastic process which controls the output of the research technology will be restricted such that the amount of information is non-decreasing. It is this natural restriction which builds into the model the plausible idea of asymmetry of the probability distribution of the state of knowledge at any given point in time. In other words, it is the upside risk which is "unlimited". The downside risk, on the contrary, will be restricted from below.

(vi) There will be two types of intertemporal externalities (though they are internal for the firm). First, as stated above, new knowledge is always based on old knowledge. Second, old innovations tend to generate new know-how. However, the model abstracts from inter-firm externalities i.e. diffusion of know-how from one firm to another. It hence assumes that the firm is able to capture the full rent on its own innovations. It would be possible to introduce the discrepancy between the private and social returns to research, but this extension is not modelled.

(vii) The model presumes that the optimal effort depends upon the expected prize or rents associated with a success.

(viii) The firm is assumed to be risk neutral. Moreover, it is taken that the markets do not require risk premium on the required return on the innovative firm. The latter assumption may sound surprising. But it follows from the assumption that the returns on research projects cannot be predicted on the basis of the existing state of the world. The former is a highly useful simplification for an attempt to to explore the incentives for research spending, unrelated to private risk aversion. It is then only natural to argue that a positive degree of risk aversion will then curtail the detected incentives.

The model is simple enough but it is able to incorporate all the assumptions (i)-(vii) stated above.

In principle, the return on a risky project should be sufficient to provide compensation for risk. It will be shown that risks indeed alter the required return even of a risk neutral firm. Hence, the market rate of interest is a poor guide for the return on risky investment. However, our result that is not a priori anticipated is that the required return on risky investments may actually fall short of the safe return. This follows because risk taking is productive in a very important sense: given that the risks are asymmetric and that it is the upside risk that dominates increased risks will actually increase incentives for risky innovating activity. The paper proves theorems as to the sufficient conditions for this finding.

The approach of the current paper can be contrasted to some earlier work in the field. First, when compared with the tradition dating back to Arrow (1962) and including subsequently Dasgupta and Stiglitz (1980), Spence (1984) and

Greenwald and Stiglitz (1990) there is a difference in modelling in that the latter allow the R&D to give rise to cost savings. Second, the model is complementary to the seminal work by Lucas (1972), extended subsequently by Grossman and Shapiro (1986) and Hagen and Kanniainen (1990b), who all focus on a given research project with a finite (known or unknown) completion time.[4] Moreover, the fundamental property of the research technology, i.e. endogenization of the technical progress is not really modelled in the works of Lucas or Grossman and Shapiro. In the current model, this feature is quite explicit as it also was in Hagen and Kanniainen (1990b). However, the major difference between all the earlier work and the current paper is in the treatment of risks. The competition between firms, surveyed thoroughly by Reinganum (1989) will not be dealt with in the current paper.

II Model of a Firm

II.1 Research Technology Let $z_t \geq 0$ stand for the stock of current technical know-how due to past research efforts. If $x_t \geq 0$ denotes the current flow of resources into the research program, the evolution of new technical knowledge is assumed to take place at the expected rate

$$(1) \qquad (1/dt)E_t(dz_t/z_t) = f(x_t) + y_t/z_t$$

where $(1/dt)E_t$ is Ito's differential operator. Function f(x) is used to describe the firm-specific, strictly concave and deterministic mechanism of the research technology with $f'(x) > 0$, $f''(x) < 0$. It is not only the assumption of concavity, i.e. diminishing returns, which is important in the model. Indeed, also the type of concavity will show up as critical. The complementarity between the inherited knowledge z_t and the new effort x_t is built into the model through the assumption $f(0) = 0$.

The research technology also has a stochastic element, y_t, assumed to follow a stochastic Ito process

$$(2) \qquad dy_t = \sigma(x_t)\sqrt{y_t}d\theta_t$$

In this equation, $d\theta_t$ is assumed to follow a Wiener process with mean zero and unit variance. In terms of the conditional variance, $var(y_s/y_t) = (s-t)\sigma^2(x_t)y_t$ while $Ey_s = y_t$, $s \geq t$. From (2), dy_t may be positive or negative. But the square root guarantees that $y_s \geq 0$ for all $s \geq t$, assuming $y_t > 0$. This, of course, captures the assumption of non-decreasing information.[5]

The function $\sigma(x_t)$, which is assumed to depend on the magnitude of the stake of the firm, x_t, is taken here as a measure of risk of the project. The risk is hence assumed to be controllable, albeit not by diversification. The assumption (iv) is built in the model as follows

(4) $\sigma(0) \geq 0$, $\sigma'(x) \geq 0$.

Below both the cases with equality and inequality signs will be considered.

The assumptions (1) through (3) indicate that the risks are asymmetric in the sense that it is only the upside risk that is unlimited while there is lower boundary for the down-side risk. Let us adopt the following definitions:

Definition 1. A project P is more risky than a project R if $\sigma_P(x) > \sigma_R(x)$ for all $x \geq 0$.

Definition 2. The riskiness of a project P has increased if $\sigma_P(x)$ has changed to $\sigma^*_P(x)$ such that $\sigma^*_P(x) > \sigma_P(x)$ for all $x \geq 0$.

Below we consider the following mechanisms to be differentiated. For any given level of research effort x_t, we consider the effects of changes in risk measured by new evaluation of the function $\sigma(x)$. This should not be confused with endogenously changed risks due to a larger stake x_t, given an unchanged function $\sigma(x_t)$. In the latter case, we show under what conditions a riskier option stochastically dominates a less risky option. This dominance will, however, not show up as the pure form of the second-order stochastic dominance.

Hagen and Kanniainen (1990b) found that the concavity of the research technology is important because it creates an incentive to smooth the innovative effort, measured by x_t, over time. It will, however, turn out below that it is the type of concavity of the research technology that becomes crucial as to the characterization of the optimal policy. Hence, it is helpful to define the elasticity of the marginal productivity of the research effort as[6]

$$g(x) = -f''(x)/f'(x) > 0. \tag{5}$$

For subsequent purposes, it is useful to note that $g'(x) = -f'''(x)/f'(x) + g^2(x)$.

II.2 Uncertainty in the Cash Flow and the Optimality Criterion

Let k_t denote the stock of capital assets and j_t the rate of current investment flow in these assets. Then

$$dk_t = (j_t - \phi k_t)dt \tag{6}$$

where ϕ is taken as the rate of economic depreciation. The relationship between capital in natural units and capital in efficiency units is assumed to be given by

$$(7) \qquad k^* = \alpha(z)k$$

where $\alpha(k)$ is assumed to be a strictly concave function with $\alpha'(k) > 0$, $\alpha''(k) < 0$. Moreover, the production technology is assumed to be given by a strictly concave function

$$(8) \qquad Y = F(k^*)$$

with $F_{k^*}(k^*) > 0$ and $F_{k^*k^*}(k^*) < 0$. The assumption of diminishing returns suggests the existence of positive intramarginal rents. This is quite a natural assumption in the current context if only because it helps to motivate the very existence of innovative incentives. The technical progress, which is of a disembodied type in the model, makes the output Y_t also an Ito-process.

Assume next that the firm is a price-taker and let the price of output be the numeraire. Denote p = cost of the research input, q = price of capital goods, and $\mu(j)$ = the convex cost of adjustment associated with accumulation of capital assets with $\mu'(j) > 0$, $\mu''(j) > 0$.[7] The current cash flow of the firm is given by

$$(9) \qquad \pi = F[\alpha(z)k] - px - qj - \mu(j).$$

The owners of the firm are assumed to possess a well-diversified portfolio of assets tradeable in the secondary market with the consequence that only the covariance risks are priced. However, given that the risks associated with the output of the research technology are related to "engineering" and "human" capital by their very nature, a strong justification exists for assuming that the returns cannot be predicted on the basis of the current state of the economy. Then the appropriate discount rate of future returns is the riskless rate of return, say r.[8]

The control problem of the firm will be written in three state variables as follows

$$(10) \qquad V(z_t,k_t,y_t) = \max_{x,j} E_t \int_t^{\infty} \pi_s \exp\{-r(s-t)\}ds.$$

The sufficient conditions for the differentiability of the value function V(.) derived by Benveniste and Scheinkman (1979) are assumed to be satisfied.[9]

III Derivation of the Optimality Conditions

At each point in time, the firm is assumed to face the option of evaluating its research targets. The momentary success is always stochastic but provides new information as to the future prospects. The plan formulated in the past will be revaluated at each moment such that along the control path, the value function obeys the following optimality condition

$$(11) \qquad rV(z_t,k_t,y_t)dt = \max_{x,j} \{\pi_t dt + E_t(dV)\}.$$

The left-hand side is the total mean return required by the owners of the firm over the time interval dt. The right-hand side is the total expected return, consisting of the cash return and the expected appreciation of value. For optimality it is required that the expected return is equal to the required mean return. Applying Ito's Lemma, one can solve

$$(12) \qquad (1/dt)E_t(dV) = V_k(j - \phi k) + V_z[f(x)z + y] + (1/2)V_{yy}\sigma^2(x)y$$

where $V_k = \delta V/\delta k$, $V_z = \delta V/\delta z$ etc. denote the partial derivatives. Substituting into (11) gives the Hamilton-Jacobi-Bellman equation

$$(13) \qquad rV(z_t,k_t,y_t) = \max_{x,j} \{\pi_t + V_k(j - \phi k) + V_z[f(x)z + y] + (1/2)V_{yy}\sigma^2(x)y\}.$$

At each point in time, the controls x and j have to be chosen so as to balance the current profits against changes in the sum of all discounted future profits associated with the chosen investment and research policies. The necessary conditions for this to hold are given by

(14) $\quad -p + V_z f'(x)z + V_{yy}\sigma(x)\sigma'(x)y = 0$

(15) $\quad -(q + \mu'(j)) + V_k = 0.$

Condition (15) dictates that along the control path, the ratio of the present value of expected gains from marginal investment to the cost of investment equals one, i.e. $V_k/(q + \mu'(j)) = 1$. The same holds for the risk-adjusted ratio of the present value of expected gains from marginal research spending to the current cost,

(16) $\quad V_z f'(x)z/p = 1 + \beta(x,y)$

where $\beta(x,y) = -V_{yy}\sigma(x)\sigma'(x)y$ is used to measure the risk adjustment of the valuation of the marginal research effort. This risk adjustment, of course, should not be confused with a risk premium. Due to the following lemma, proved in Appendix A, $V_{yy} < 0$ and hence $\beta(x,y) > 0$.

<u>Lemma 1.</u> V is concave in y. []

An increase in the firm's stake x raises marginally the variance of y when $\sigma'(x) > 0$. The rents resulting from the increment in the stake are then accordingly revalued. If $\sigma'(x) = 0$, this effect will disappear and $V_z f'(x)z/p = 1$. Under the assumption $\sigma(0) = 0$, $\sigma'(x) > 0$, the asymmetry of risks implies $\lim_{x\to 0} \beta(x,y) = 0$, $\lim_{x\to\infty} \beta(x,y) = \infty$. It is clear from (14) that it is the existence of controllable risk

$\sigma(x)$ that substantially increases the complexity of the problem of the firm.

In principle, all information concerning the optimal path is imbedded in equations (14) and (15). Indeed, the model is fully determined in the sense that there are two conditions to determine, though implicitly, the paths of the two controls. But note that there are three unknown forward-looking shadow prices in these optimality conditions, V_k, V_z and V_{yy}. The first two of them can be eliminated . While no analytic way exists for the elimination of the third one, an economic interpretation of its role can be produced.

Since $V(.)$, V_k, V_z and V_{yy} are all Ito processes, they are not differentiable with respect to time. But equation (13) is an identity and it can be differentiated with respect to the state variables z,k and y to obtain

(16a) $0 = F_z + V_{zz}[f(x)z + y] + V_z f(x) + (1/2)V_{yyz}\sigma^2(x)y - rV_z + V_{zk}(j - \phi k)$

(16b) $0 = F_k - (r + \phi)V_k + V_{kk}(j - \phi k) + V_{zk}[f(x)z + y] + (1/2)V_{yyk}\sigma^2(x)y$

(16c) $0 = V_{ky}(j - \phi k) + V_{zy}[f(x)z + y] + V_z - rV_y + (1/2)V_{yy}\sigma^2(x) + (1/2)V_{yyy}\sigma^2(x)y.$

The notation $F_k = (\delta F/\delta k^*)\alpha(z)$, $F_z = (\delta F/\delta k^*)\alpha'(z)k$ has been used. Utilize then the information that the shadow prices

V_z, V_k and V_y are all Ito processes written as $V_z = V_z(z,k,y)$, $V_k = V_k(z,k,y)$ and $V_y = V_y(z,k,y)$. Applying Ito's Lemma again and the differential operator, one can derive

(17a) $(1/dt)E_t(dV_z) = V_{zk}(j - \phi k) + V_{zz}[f(x)z + y] + (1/2)V_{zyy}\sigma^2(x)y$

(17b) $(1/dt)E_t(dV_k) = V_{kk}(j - \phi k) + V_{kz}[f(x)z + y] + (1/2)V_{kyy}\sigma^2(x)y$

(17c) $(1/dt)E_t(dV_y) = V_{ky}(j - \phi k) + V_{yz}[f(x)z + y] + (1/2)V_{yy}\sigma^2(x)y.$

Substituting into (16a)-(16c) gives the stochastic versions of the Euler-equations

(18a) $-[f(x) - r]V_z = F_z + (1/dt)E_t(dV_z)$

(18b) $(r + \phi)V_k = F_k + (1/dt)E_t(dV_k)$

(18c) $rV_y = V_z + (1/2)V_{yy}\sigma^2(x)(1-y) + (1/2)V_{yyy}\sigma^2(x)y$
$(1/dt)E_t(dV_y).$

It is clear that, perhaps except in some very special case, the solution of a stochastic model like the current one cannot be of an open loop type since optimality requires reoptimization after observing each successive shock. The feedback mechanisms can be introduced by noting that along the control path, certain relations have to hold between the state variables and the control variables. Let these be denoted by $x = x(z,k,y)$ and $j = j(z,k,y)$. The proposal is that at least some non-zero partial derivatives exist. This is indeed the case. Taking total differentials, applying the expectations operator, and noting that $(dy)^2 = dt$, we can prove:

Lemma 2. The controls satisfy the following conditions along the optimal path

(19a) $$(1/dt)E_t(dx_t)^2 = x^2_y\sigma^2(x)y_t$$

(19b) $$(1/dt)E_t(dj_t)^2 = j^2_y\sigma^2(x)y_t. \quad []$$

The expression x^2_y denotes $(\delta x/\delta y)^2$ etc. It is now possible to evaluate the expectational terms on the right-hand side of (18a) and (18b). This is done in Appendix B. Then, using (B.1) and (B.3), (18a) can be re-written as

(20a) $$f'(x)z[F_z/p] + g(1/dt)E_t(dx_t) = r + \theta_z$$

with

(20b) $$\theta_z = y/z + (1/2)g^2(g^2 - g')x^2_y\sigma^2y - [f(x) - r]\beta/p - [f'(x)z/p](1/dt)E_td[\beta/f'(x)z].$$

Using (B.4), (18b) can be re-written as

(21a) $F_k + \mu''(j)(1/dt)E_t(dj_t) =$

$$(r + \phi)[q + \mu'(j)] + \theta_k$$

with

(21b) $\theta_k = - (1/2)\mu'''(j)j^2{}_y\sigma^2$

To summarize the derivation of equations (20) and (21), recall that use was made both of the optimality conditions (14) and (15) and the background condition (13) to eliminate the two unknown shadow prices V_z and V_k. It is not possible to eliminate analytically the expectational term $(1/dt)E_t[V_{yy}\sigma\sigma'y/f'z]$. But luckily enough, this does not prevent describing the optimal policy.

IV Description of the Optimal Research and Investment Policies

The results (20a,b) and (21a,b) can now be used to evaluate the optimal policy of the firm under uncertainty. These equations, which are first-order differential equations in the two control variables, together with (1), (2) and (6) constitute a dynamic system with three state variables (z_t, k_t, y_t). Some care should be taken in the interpretation of these equations. Clearly, they do not provide explicit solutions for the time paths of the controls. Rather, they state the equality between the expected marginal revenues (the left-hand sides) and marginal costs along the control path. They can hence be used to evaluate the incentives for the firm for adjusting the state variables, the desired progress z_t and the stock of capital k_t.

There is a highly useful anchor for sorting out the incentive effects. Note that an increase in the real rate of interest unambiguously lessens incentives for expansion of both state variables z_t and k_t, as confirmed by equations (20a) and (21a). Well-known as it is, this is the very standard opportunity cost effect. Then, it is easy to uncover the interaction of other variables with the incentives simply by comparing whether these variables tend to counteract or strengthen the dampening effect caused by, say a boost in the real rate of interest.

The interpretation of F_z and F_k on the left-hand sides of (20a) and (21a) is rather straightforward. They are the momentary marginal productivities associated with a slight change in the state variables z and k, ceteris paribus. The firm knows them with certainty. The second terms on the left-hand side represent the saving in the marginal decline in the productivity of a research effort and saving in the marginal cost of adjustment, respectively due to current values of z and k. μ'' is the decline in the marginal cost of adjustment tomorrow, in (20), given more investment today. The saving in the reduction of productivity of research effort tomorrow, measured by the elasticity g, operates in (21) principally in the same way. The right-hand sides of (20a) and (20b) represent the current marginal costs. It is rather useful to evaluate these costs in steps, not least of all because they show up in the first-order conditions in a rather complicated manner, though additively.

The first result to be noted is that even in the absence of uncertainty, the model does not possess any steady state. Having $\sigma(x) = 0$ implies $dy = 0$. But even when $x = 0$, $dz/dt = y = \text{constant} > 0$. $\alpha' > 0$ implies that the marginal productivity F_k has to grow over time. The percentage rate of change dz/z, however, is lessened in time and $\lim_{t\to\infty} y/z = 0$. In the absence of future uncertainty, $r + y/z$ and $r + \phi$ represent the required rates of return on marginal research and capital investments, respectively. The discovered feature of the solution, the endless growth and technical know-how, though at the diminishing rate, captures one of the intertemporal externalities in the model.

Future uncertainty introduces some additional important mechanisms. Whether uncertainties dampen or enhance the research and investment incentives depends upon whether the adjustments in the rate of return requirements, θ_z and θ_k in (29a) and (21a) are positive or negative. Non-zero values of θ_z and θ_k suggest that the risks interact with the firm's return requirement making the market rate of interest a poor guide as to these incentives. The firm faces no uncertainty as to the gains due to higher current z or k, measured by F_z and F_k. But knowing only the momentary probability distributions of all future values of $\{z_s\}$ makes all the future F_z and F_k stochastic. The impact of this future uncertainty on the required rates of return is measured by θ_z and θ_k.

It is very clear that the momentary distribution of z_s for each future $s \geq t$ is asymmetric and that it has a positive tail. Since $y \geq 0$ always, z_s cannot be reduced as it can only grow. The question of major interest is whether the presence of risks increases or decreases the incentives for the firm to invest in risky projects.

Consider first the case where σ^2 is positive and constant and hence totally unrelated to the stake of the firm. Then $\sigma'(x) = 0$. This represents uncontrollable risk, which is asymmetric in the sense that it makes the growth of z uncertain for a firm. What the firm knows for sure is that z will not fall even in the worst state of the world. This asymmetry is not, however, suggestive per se as to whether the firm would be encouraged to engage more in risky research programs. It is the third term on the right-hand side of (20a) that reveals whether the existence of fully exogenous risk enhances or dampens the firm's incentives to be engaged in a research program. The answer solely depends upon whether $g^2(x) - g'(x) < 0$ or > 0. This condition, however, is equivalent to $f'''(x)$

> 0 or < 0. With this conclusion, we have proved:

Theorem 1. Whether the risks associated with the research activity, when unrelated to the firm's own stake in the programs, encourage or discourage risky research spending depends on the properties of the research technology available for the firm. If the firm has access to research technology with strictly convex marginal productivity, i.e. $f'''(x) > 0$, risk-taking is discouraged. In the case of strictly concave marginal productivity with $f'''(x) < 0$, the risks curtail the incentives for risky spending. []

Our Theorem 1 can be compared to the well-known results of Diamond and Stiglitz (1974). They show that the optimal response of a risk averse agent to an increase in the mean preserving risk involves increased risk taking if the marginal utility is a convex function in the state of the world. There is, however, an important difference here. The fact that according to our Theorem 1, this result extends to the case of a risk neutral firm suggests that an increase in risks in our case is of the non-mean-preserving type. $f'''(x) > 0$ then means that the expected return on risky spending must be larger when compared to the no-risk case, while it is lower in the opposite case which holds when $f'''(x) < 0$.

Note that there are a number of parameters, including g^2, $x^2{}_y$, σ^2 and y, which determine the magnitude of this risk effect. The larger is the variance σ^2, for example, the greater is the impact on the expected return.

When the outcome of the research program is uncertain, the properties of the optimal path over a longer horizon are even less predictable than they were above when the case $\sigma = 0$ was discussed. Though y_s stays non-negative, it varies along the path somewhat erratically. Hence, the path of z_s is volatile, too, though it is constrained to be non-decreasing. The firm can control the movement of y_s by adjusting its own stake in the program x_t, but only imperfectly. The minimum values of future k_s and y_s, $s \geq t$ can be stated with no difficulty: they are zero. Moreover, the current z_t is the lower boundary for all future values of z_s. But the upper boundaries are clearly infinite for all the state variables. There is no doubt that these boundary values are accessible in the sense of Malliaris and Brock (1982) ch. 9. There is clearly no appealing way to limit the joint density function for the triple (z_s,k_s,y_s) when $s \to \infty$ such that the boundaries would be inaccessible and that some stationary distribution would exist.[10] Our model seems to fit better with the complex reality than a standard model of a firm with a well-defined steady state. Though it hence suggests that $x^2{}_y > 0$ in (20b), it does not, of course, provide a clue as to the size of this parameter. Yet it is interesting to observe that it is only

x_y and j_y that matter for the optimal policy in (20b) and in (21b) but that the other partial derivatives x_k, x_z, j_k and j_z are fully irrelevant.

When the risks involved are related to the firm's own stake with $\sigma'(x) > 0$, there will be two important additional mechanisms involved. The firm now has to recognize the dependence both of the current risk and the expected change in the future risk on the particular value of x_t that the firm chooses today. Recall from the definitions 1 and 2 (p.7) that one now has to distinguish between an exogenous change and endogenous change in risks. It was the former which we had in mind when we discussed the impetus of changed risks. But this time the firm also recognizes that the magnitude of the risk effects depends on its own policy through the function $\sigma(x)\sigma(x)' > 0$. The other parameters which determine the magnitude of this risk effect include V_{yy} and y.

Whether the incentives to undertake risky research projects are enhanced or dampened now depend on the discrepancy between the productivity of the research effort and the real rate of interest, $f(x) - r$. If $f(x) > r$, the fourth term in (20b), $-[f(x)-r]\beta/p$, is negative. This clearly tends to strengthen the incentives for risk-taking of a risk-neutral firm. The converse case is obtained when $f(x) < r$. We have thus given the proof for the following theorem:

Theorem 2. If the risks are positively related to the firm's own stake in the research program, the incentives for undertaking risky research programs are strengthened for a risk-neutral firm when risks get greater, provided that the productivity of the research effort exceeds the real rate of interest. []

If anything, the theorem 2 is a very strong one. One observes that as long as $f'(x) > 0$, the firm can always, if it chooses so, obtain $f(x) > r$ by expanding its research spending sufficiently. This leads to the most important finding:

Theorem 3. If the stochastic process generating the output of the research technology is of type (1) and (2) with $d\theta_t$ following a Wiener process, the expected return on research spending is positively related to the risks of the type $\sigma = \sigma(x)$.[]

The proof is an indirect one. A risk neutral firm's research and investment policies are governed by the expected return. We have proved in Theorem 2 that the risk neutral firm prefers risky spending. This will not be case unless the expected return is increased.

It should be noted immediately that this mechanism is counteracted by the mechanism introduced in Theorem 1.

There is a final term in (20) which relates the required return to the currently held expectation concerning the change in the ratio of risk adjustment and the marginal productivity of research effort, as measured by $\beta/f'(x)z$. This change, if positive, unambiguously reduces the required return on risky spending and hence further enhances the incentives for innovating effort. The third Euler-equation (18c) shows that the valuation of marginal y, V_y and the valuation of marginal z, V_z are related. This is rather expected since the time path of the state z is partly dictated by the time path of y. However, as (18c) shows, this relationship, though it can be algebraically derived, does not lend itself to any useful analytic simplification.

For evaluation of the expectation term in (20b) denote

$$(22) \qquad H(x) = \beta(x)/f'(x)z.$$

To gain intuition, consider for a moment an analogous case where all the variables can be assumed differentiable with respect to time. Then

$$(23) \qquad dH/dt = (d/dx)(dx/dt)[\beta(x)/f'(x)z]$$
$$= (dx/dt)[(d/dx)\beta(x)/f'(x)z].$$

It is clear that $(d/dx)\beta(x)/f'(x)z > 0$ since the risk adjustment $\beta(x)$ is increasing in x while the marginal productivity $f'(x)$ is decreasing. Note that β depends on V_{yy} but that a marginal change in the current x_s leaves V_{yy} unchanged. The latter can be verified by introducing $-px$ on the RHS of (A.1) in Appendix 1 and going through the derivatives up to V_{yy}. Thus we have

Lemma 3. $dH/dt > 0$ (< 0) iff $dx/dt > 0$ (< 0). []

Hence, greater research effort currently increases the actual (and thus its expected) value of the ratio of the risk adjustment and the marginal productivity. Given that there is a minus sign in front of the last term of (20b), we have established

Lemma 4. For a process which is differentiable with respect to time, the expectational mechanism enhances the incentives for risky spending. []

Mathematically, this result cannot be extended to the case where the process is not differentiable with respect to time. Hence, we have to be satisfied with a weaker result in our current model. Since we are only interested with the sign of the incentive effect, this result is stated as

Theorem 4. If the expectational incentive effects only have the same sign in the differentiable and non-differentiable processes, the expectational mechanism tends to enhance the incentives for risky research spending. []

It is customary to compare the optimal policies under certainty and under uncertainty. In our model, there is really no interesting case when the policies would be equivalent, i.e. when a closed-loop policy would not differ from an open-loop one. Static expectations about the risk to the marginal productivity ratio together with $f'''(x) = 0$ and the equality $f(x) = r$ would no doubt reduce the last three terms in (20a) and (20b) to zero. But this case can hardly be of any special interest. It does hence seem that avoiding the explicit treatment of the genuine upside risk in the research effort would indeed be quite a radical simplification.

All the results above relate to the research incentives. In the current model, there are no spillover effects from the capital investments to the research spending on the cost side though there clearly are positive spillovers on the revenue side. From the research spending, however, there are spillower effects on capital expenditures both on the revenue side and the cost side. On the revenue side, the spillover effect is imbedded in the term $F_k(\alpha(z)k)$. This captures the idea that a successful research effort enhances the productivity of existing inputs, here capital.

On the cost side, a rather interesting result is obtained. It is not the case that increased risk associated with risky project would unambiguously lead to channelling of more resources into the safe project. It is the nature of the costs of adjustment that is informative as to the direction of this impact. More precisely, whether the riskiness of the research spending will encourage or discourage capital spending depends solely the nature of the costs of adjustment of capital. In the case where $\mu''' = 0$, there is no cost side impact on capital investments from risky projects. However, if the marginal costs of adjustment are concave, i.e. $\mu'''(j) < 0$, the spillover effects (on the cost side) from risky spending tend to reduce capital expenditures. The opposite holds if the marginal costs are convex, i.e. if $\mu''' > 0$:

> Theorem 5. The cost side effects of greater risks associated with the research program will encourage less risky capital investments if $\mu''' > 0$ while the opposite holds if $\mu''' < 0$. []

Due to lack of empirical findings, it is not advisable to speculate as to which case is the dominating one.

IV Final Remarks

The issue whether the innovating incentives of a firm are sufficient in market economies has been addressed in a number of papers earlier. Our work has produced results which suggest that the desired progress is positively related to the risks of the firm. This incentive can be regarded as complementary to the prize of winning an R&D race (cf. Reinganum (1989)). Since the magnitude of the risks can be controlled by the firm and as long as the valuation of the marginal progress is positive, the paper suggests that the innovative incentives may be quite substantial. Fundamentally, this result follows from the central idea of the model that the risks involved are asymmetric in the sense that it is the upside risk that is relevant while there is a natural floor to the downside risk.

It is not easy to judge whether the assumption that the shocks follow a Wiener process really biases our results. Obviously, there may be waves of optimism and pessimism as far as the expectations of an innovative firm are concerned. Clearly, this feature is in no way excluded by adoption of the assumption of a Wiener process. What the Wiener process excludes is the chance of a notable break-through (since the Wiener-process is continuous). Introduction of a Poisson process to capture the break-through phenomenon (i.e. abnormal success) would only reinforce our results of the existence of significant research incentives.

Our results do not, of course, wipe out completely the concern of socially optimal research incentives. First, an assumption was made that the firm is risk neutral. However, if risk aversion is the dominating mode of behavior, as suggested by Greenwald and Stiglitz (1990), a positive premium may be created for risky investments by private risk aversion. As we have suggested, there may be no covariance to be priced. Rather, it would be the own variance of returns of the firm that might be priced. Second, and equally important, the paper has assumed that the firm is fully able to capture the rents associated with the progress made. To the extent information is a public good due to positive externalities via diffusion, this assumption may be overly optimistic.

Due to the very nature of the problem, we have chosen to work with a non-separable production technology and a concave research technology. As is understandable in the type of stochastic control problem studied, it is not possible to obtain explicit solutions for the control variables.[11] We were nevertheless able to characterize the research and investment incentives of the firm along the optimal path. We were able to derive explicit results as to the effects of increased uncertainty on the research policy of a firm. Interestingly enough, increased uncertainty tends in a sense to convexify the present value of returns giving rise to increased incentives because it is the upside risk that is dominating.

Appendix A. Concavity of the Value Function in y.

Instead of the value function V, defined over continuous time, it is illuminating to start with its discrete counterpart, say V^D, where D = length of the time period. It is assumed $\lim_{D\to 0} V^D = V$, $\lim_{D\to 0} V^D_y = V_y$ and $\lim_{D\to 0} V^D_{yy} = V_{yy}$. Set $x_s = j_s = 0$ for all $s \geq t$. Then

$$\text{(A.1)} \qquad V^D = E_t \sum_{s=t}^{\infty} \{1/(1+r)^{s-t}\} F[\alpha(z_s)k_s]$$

where $z_s = \sum_{\tau=t}^{s} y_\tau$. Then evaluating the derivative with respect to the state variable y

$$\text{(A.2)} \qquad V^D_y = E_t \sum_{s=t}^{\infty} 1/(1+r)^{s-t} F_{k*}\alpha'(\sum_{\tau=t}^{s} y_\tau)$$

and

$$\text{(A.3)} \qquad V^D_{yy} = E_t \sum_{s=t}^{\infty} 1/(1+r)^{s-t} \{F_{k*k*}(\alpha'(\sum_{\tau=t}^{s} y_\tau))^2 + F_{k*}\alpha''(\sum_{\tau=t}^{s} y_\tau)\}.$$

Then $F_{k*} > 0$, $\alpha' > 0$, $F_{k*k*} < 0$ and $\alpha'' < 0$ imply that $V^D_y > 0$ and $V^D_{yy} < 0$. By definition, $V = \lim_{D\to 0} V^D$. By the above assumption, $\lim^D{}_{\to 0} V^D_{yy} = V_{yy}$. This implies $V_{yy} < 0$.

Appendix B. Derivation of the Expected Adjustment of the Costate Variables along the Optimal Path

From (14) one can write

$$(B.1)\quad (1/dt)E_t(dV_z) = (1/dt)E_t d[p/f'(x)z] + (1/dt)E_t d[\beta(x)/f'(x)z].$$

Using the Taylor approximation and applying (19a)

$$(B.2)\quad d[f'(x)z] \simeq f''(x)z(dx) + f'(x)(dz) + (1/2)f'''(x)zx^2{}_y\sigma^2 y dt.$$

Then

$$(B.3)\quad (1/dt)E_t d[p/f'(x)z] = \{f''(x)z(1/dt)E_t(dx_t) + f'(x)[f(x)z + y] + (1/2)f'''(x)zx^2{}_y\sigma^2(x)y\}(-p)/[f'(x)z]^2.$$

Substituting in (B.1) and then in (18a), one can obtain equation (20) in the main text.

Applying the Taylor approximation in (15) and using (19b), one obtains

$$(B.4)\quad (1/dt)E_t(dV_k) = \mu''(j)(1/dt)E_t(dj) + (1/2)\mu'''(j)j^2{}_y\sigma^2(x)y.$$

Substituting in (18b), one can obtain equation (21) in the main text.

Footnotes:

1. Focusing on this fundamental uncertainty does not eliminate the need for understanding the effects of some other sources of uncertainty. For example, the rents associated with the achieved level of success may depend on uncertain future market conditions. Similarly, the response of the competitors in the research game is relevant (cf.Reinganum (1989)). Moreover, the rate of diffusion of innovations to outsiders may be unpredictable and interacts with the innovation incentives.

2. Cf. Hagen and Kanniainen (1990a) and Hagen and Kanniainen (1990b) for modelling this interaction.

3. Instead of utilizing the output in its own research technology, the firm could alternative sell its information in the market place. This alternative will not be modelled because, in equilibrium, the returns on both policies would have to be equal.

4. What the elegant analysis of Lucas suggests is that the optimal effort and its timing depend on the reward from success. As shown by Hagen and Kanniainen (1990b), there are conflicting mechanisms involved as to the optimal timing. The discount effect tends to lead to postponement of spending while the concavity of the research technology tends to create

an incentive for an earlier effort. These mechanisms work in the current model, too, though the difference is there that the current paper builds on the idea of continuous research effort with continuous (but uncertain) accumulation of new knowledge.

5. Cf. Cox, Ingersoll and Ross (1985) from whom this technical trick has been borrowed.

6. The reader realizes that g(x) is analogous to the celebrated Arrow-Pratt measure of local risk aversion, commonly referred as the "absolute" risk aversion.

7. Above, the assumption of diminishing returns to research inputs was adopted. Concavity of the f(x) function could, however, be given an alternative justification in terms increasing marginal costs of introducing new technology.

8. This is analogous to the Arrow-Lind (1970) criterion on public investments where non-correlation with the rest of the economy is assumed. More recently, Dixit (1989) has shown how a positive covariance could be introduced.

9. Below it is required that the first, second and third derivatives of the V(.) function exist.

10. There are not many cases when it has been possible to

establish the existence of a stochastic equilibrium. Lucas and Prescott (1971) provide such a proof in the case of a competitive firm with constant returns and convex costs of adjustment. Merton (1975) on the other hand had to assume the existence without a proof.

11. Abel (1983) was able to obtain an explicit solution in an investment problem but at the cost of simple parametrization.

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