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TESTING LINEARITY OF ECONOMIC TIME SERIES AGAINST CYCLICAL ASYMMETRY†

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ABSTRACT: The question of asymmetry of business cycles has not received much attention in modern theoretical work, although it was considered already in the 1920s. However, recently some researchers have studied cyclical asymmetry of economic time series empirically using modern statistical tools. In this paper we follow their example but use time series techniques not previously applied to the present problem. Our starting-point is that smooth transition autoregressive (STAR) models can generate cyclically asymmetric realizations. The asymmetry of economic time series may thus be investigated by testing linearity (symmetry) of a process against STAR. The necessary tests of Lagrange multiplier type have recently become available. The results show that the U.S. unemployment series other researchers have analyzed previously may indeed be asymmetric, but the evidence for non-linearity seems to stem largely from the pre-1960 part of the series. International evidence from 13 OECD countries based on quarterly data from 1960(1) to 1986(4) is mixed. German unemployment and industrial production series seem to offer strongest evidence for asymmetry. The industrial production in the U.S. and Japan shows signs of asymmetry, whereas unemployment does not when the period of observation begins in 1960. At the other end of the scale, business cycles in Sweden and Finland seem linear. Neither for unemployment nor for industrial production series is linearity rejected in these countries when tested against STAR. Autoregressive conditional heteroskedasticity is found to be a possible type of non-linearity for several time series investigated in our study.

KEYWORDS: Autoregressive conditional heteroskedasticity (ARCH), business cycle, economic fluctuations, non-linear time series, smooth transition autoregressive model.
1. INTRODUCTION

The idea of asymmetric business cycles is not new. Mitchell (1927, pp. 330-334 and 407-412) has discussed it and presented statistical evidence both in favour of and against it. Keynes (1936, p. 314) has argued that the contractions in an economy are more violent but also more short-lived than the expansions. Yet, modern theoretical work does not seem to be much influenced by such a possibility. For instance, there is no mention of cyclical asymmetry in the entry on business cycles describing recent work (Dotsey and King, 1987) in the New Palgrave Dictionary of Economics. On the other hand, Stock (1987) mentions a paper on nonlinearities in aggregate output supply and demand equations by Chetty and Heckman (1986) as an important foundation for his investigation of business cycle asymmetry.

Recently, some investigators armed with modern econometric tools have returned to discussing the asymmetry of business cycles. Neftçi (1984) has suggested the use of finite state Markov processes for studying the problem in the U.S. economy. The procedure can be described as follows. Consider the first differences of a quarterly, seasonally adjusted and stationary time series representing the business cycle. Dichotomize the differenced series according to the signs of the differences. Study the length of runs of positive and negative signs indicating the length of expansions and contractions. If they differ, the two conditional probabilities of remaining in the same state as before may not be equal, and that is tantamount to cyclical asymmetry. Thus the empirical conclusions may be based on estimated transition probabilities.
Neftçi (1984) uses three different unemployment series as business cycle indicators. After estimating the relevant transition probabilities using a second-order Markov process as his model and constructing appropriate confidence ellipsoids, he concludes that his evidence favours the asymmetry assumption.

Falk (1986) has used the same Markov process framework for testing the symmetry of the business cycle. A novelty lies in that he has used other U.S. series than unemployment to serve as indicators of the cycle. The evidence gathered this way favours asymmetry less than that of Neftçi (1984). Falk also investigates the issue using quarterly industrial production series from four European countries and Canada. The results support the cyclical symmetry rather than the asymmetry hypothesis. DeLong and Summers (1986) have studied the asymmetry by calculating the skewness of the distribution of growth rates for a variety of production series in several countries. Their conclusion is that there is little evidence favouring asymmetry: the only notable exception seems to be the U.S. unemployment rate 1950-1979.

Stock (1987) has approached the problem from another angle. He assumes that latent economic variables evolve according to a linear time-invariant process in economic time. Economic time is non-trivially different from calendar time if the transformation from economic to calendar time is non-linear. If this is the case, then the observable processes representing the latent variables may behave asymmetrically in calendar time. Stock (1987) is mainly interested in finding out whether there is a common cyclical indicator indicating a cyclical time scale for all variables of interest. Although the null hypothesis of a linear time scale is rejected for some of the systems he con-
siders, there is not evidence in favour of a systemwide cyclical time scale.

Contrary to Neftçi and Falk, we shall adopt a pure time series approach to the question of asymmetry. Tong and Lim (1980) and Tong (1983) have discussed a class of non-linear time series models called self-exciting threshold autoregressive (SETAR) models. As Tong and Lim (1980) explain, these models can generate time series which evolve asymmetrically over time. A generalization of the SETAR model to a model called the smooth transition autoregressive (STAR) model enables us to test linearity and thus cyclical symmetry of economic processes against non-linearity specified by the STAR model. An advantage of this approach is that no information is lost through dichotomizing the time series. We shall present the SETAR and STAR models in section 2. In section 3 we discuss tests of linearity based on the use of the STAR model as the non-linear (asymmetric) alternative. As the STAR model may not be the only conceivable non-linear alternative, we also test our linear model against autoregressive conditional heteroskedasticity and bilinearity considered possible in business cycle data. These tests are discussed in section 4. In section 5 we reanalyze the U.S. unemployment series considered by Neftçi (1984). They turn out to be non-linear: all our tests reject linearity. The strongest evidence for cyclical asymmetry seems to come from the observations before 1960. The analysis of unemployment series continues in section 6 where we consider international time series and find the evidence mixed. Section 7 reveals that the non-linearity results are sensitive to the logistic transformation. In section 8 we study international industrial output series, and section 9 concludes.
2. THE SMOOTH TRANSITION AUTOREGRESSIVE MODEL

Stationary, cyclically symmetric differences of a time series may often be adequately represented by a sufficiently long autoregressive (AR) process. However, an AR process cannot generate asymmetric time series; a non-linear process is necessary for that. If we want to test the symmetry of an economic time series against asymmetry, we may therefore test the hypothesis of pure autoregression against an appropriate non-linear alternative. To pursue this notion and illustrate the possibilities we shall first consider the self-exciting threshold autoregressive (SETAR) model (Tong, 1983). Assume we have a stationary business cycle indicator $y_t$. It is said to follow a SETAR(2; p, p) model if

$$y_t + \pi_0 + \sum_{j=1}^{p} \pi_j y_{t-j} + (\theta_0 + \sum_{j=1}^{p} \theta_j y_{t-j}) F(z_t) = \epsilon_t$$  \hspace{1cm} (1)$$

where $\epsilon_t \sim \text{nid}(0, \sigma^2)$, $F(\cdot)$ is a Heaviside function:

$$F(z_t) = \begin{cases} 1, & z_t > 0 \\ 0, & z_t \leq 0 \end{cases}$$ \hspace{1cm} (2)$$

Furthermore $z_t = y_{t-d} - c$, where $c$ and $d$ are fixed but usually unknown parameters. If $\theta_j = 0$, $j = 0, 1, \ldots, p$, (1) collapses into an ordinary AR(p) model.

It is intuitively obvious that (1) can generate cyclically asymmetric time series. When $y_{t-d} > c$, the process follows an AR(p) regime and it switches from this to another AR(p) regime the next
period if \( y_{t-d+1} \leq c \). Because the two regimes have different dynamic structures, the conditional probability of remaining in one regime given the past is not necessarily the same as the corresponding probability of remaining in the other. For further discussion, see Tong and Lim (1980). Neftçi (1984) is directly interested in similar conditional probabilities, implicitly assuming \( c = 0 \) in (1), estimating them and testing their equality using a Markov chain approach. On the other hand, what we have just said implies that the symmetry assumption could also be tested by testing the linearity of \( y_t \) against the non-linear alternative (1). However, even if we assume \( p \) and \( d \) known in (1), deriving a feasible test is not possible if \( c \) is unknown. This is because the likelihood function of (1) is then irregular. Chan and Tong (1986a) have suggested a numerical evaluation of the likelihood function and a likelihood ratio test based on that numerical approximation. Recently, Luukkonen et al. (1988a) have explored another avenue which opens up through a generalization of (1). Assume

(i) \( F: \mathbb{R} \rightarrow \mathbb{R} \) is an odd, monotonically increasing function possessing a non-zero derivative of order \((2s+1)\) in an open interval \((-a,a)\), \(a > 0\), \(s > 0\).

(ii) \( \frac{d^kF(z_t)}{dz_t^k} \bigg|_{z_t=0} \neq 0 \) for \( k \) odd and \( 1 \leq k \leq 2s+1 \).

(iii) \( z_t = \gamma(y_{t-d}-c) \), \( \gamma > 0 \) in (1).
Assumptions (i)-(iii) generalize (1) and make the transition from one regime to the other smooth. In fact, the system is described by a mixture of the two regimes. Model (1) where (i)-(iii) define $F(\cdot)$ is the STAR model. Many cumulative distribution functions of continuous random variables with a density symmetric around the mean are suitable candidates for $F(\cdot)$. Chan and Tong (1986b) have suggested a standard normal distribution. Luukkonen et al. (1988a) recommend the logistic distribution which approximates the normal distribution quite closely and has a lot of computational advantages over using the latter.

Two things make the STAR model interesting from our point of view. First, the model is capable of generating asymmetric fluctuations just like its special case, the SETAR model. The degree of asymmetry depends on the parameters of the model. Second, there are now asymptotic tests available for testing linearity of a time series model against STAR. The tests are of Lagrange multiplier type and can as such be carried out without estimating any non-linear model. The STAR model thus provides a framework for testing linearity of economic time series against asymmetry. It has to be noted, however, that cyclical asymmetry is a rather descriptive and not a very accurately defined concept. Asymmetry is said to occur if the two main phases of the business cycle, contraction and expansion, are not equally long. There is no unique method of defining contractions and expansions and many types of data may be used to describe and represent a business cycle. As a result there is no unique way of quantifying the concept of asymmetry. The attempt to define asymmetry with STAR is only one of many possibilities. Neftči's definition based on transition probabilities of the first differences of a stationary business cycle indicator is another one. DeLong and Summers prefer to say that the business
cycles are asymmetric if the distribution of the first differences is skew. For them the time ordering of the observations is thus less important and they do not make efficient use of it. Given all this, we can hardly expect similar results from all empirical studies of cyclical asymmetry. This paper can perhaps best be seen as an attempt to test the linearity of business cycles against STAR as well as some other non-linearities.

3. LINEARITY TESTS AGAINST CYCLICAL ASYMMETRY

We shall now discuss the testing of linearity of economic time series against STAR in more detail. Luukkonen et al. (1988a) have recently considered this testing problem and come up with two tests which are easy to apply. They are Lagrange multiplier type tests and assume that both \( c \), the threshold parameter, and \( d \), the delay parameter, \( d \geq 1 \), are unknown. No estimation of the alternative non-linear model is needed. Assuming we have an observed time series \( y_{-p+1}, \ldots, y_0, y_1, \ldots, y_T \), the first test of \( H_0: \; j=0, j=0,1,\ldots, p \), in (1), is carried out as follows:

(i) Regress \( y_t \) on \( 1, y_{t-j}; \; j=1,\ldots, p \), using ordinary least squares, form the residuals \( \hat{e}_t \), \( t=1,\ldots, T \), and the residual sum of squares \( \text{SSE}_0 = \sum_{t=1}^{T} \hat{e}_t^2 \).

(ii) Regress \( \hat{e}_t \) on \( 1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}; \; j=1,\ldots, p \), form the residuals \( \hat{n}_t \), \( t=1,\ldots, T \), and \( \text{SSE}_1 = \sum_{t=1}^{T} \hat{n}_t^2 \).
(iii) Compute the test statistic

\[ S_1 = \frac{T(SSE_0 - SSE_1)}{SSE_0}. \]

This testing procedure is in fact the well-known linearity (or non-linearity) test of Tsay (1986). \( S_1 \) has thus an asymptotic \( \chi^2 \) distribution with \( p(p+1)/2 \) degrees of freedom. In this context the test has good power properties if \( \theta_0 = 0 \). If, on the other hand, the main source of non-linearity in (1) is \( \theta_0 \), the test lacks power.

Realizing this, Luukkonen et al. (1988a) have derived two other tests which use more degrees of freedom than \( S_1 \) but have power against the situation we have just described. We shall present the more parsimonious test procedure of the two. It consists of the following steps:

(1) Same step as before.

(ii) Regress \( \hat{e}_t \) on \( 1, y_{t-1}, y_{t-1}Y_{t-j}; i=1,\ldots,p; j=1,\ldots,p; \)
\( y_{t-1}^3, i=1,\ldots,p \), form the residuals \( \hat{\nu}_t, t=1,\ldots,T, \) and
the residual sum of squares \( SSE_3 = \sum_{t=1}^{T} \hat{\nu}_t^2 \).

(iii) Compute the test statistic \( S_3 = \frac{T(SSE_0 - SSE_3)}{SSE_0} \).

Under \( H_0 \), \( S_3 \) has an asymptotic \( \chi^2 \) distribution with \( p((p+1)/2+1) \) degrees of freedom. This test is not quite as powerful as \( S_1 \) if \( \theta_0 = 0 \) but it is generally more useful of the two if that assumption cannot be made a priori. In the applications that follow we shall make use of both \( S_1 \) and \( S_3 \).
4. OTHER LINEARITY TESTS

Rejecting linearity against a well-specified STAR model using a Lagrange multiplier type test does not entitle us to accept cyclical asymmetry. The STAR model is only one of several non-linear models considered in the time series literature and there is no economic theory available to exclude all of them from consideration here. The problem is that the STAR tests may have power against some of these other non-linearities as well; for discussion see e.g. Luukkonen et al. (1988b). It is therefore desirable to learn more about the situation before drawing very definite conclusions. One way of obtaining more information is to test linearity against more than one type of non-linearity.

A relevant form of non-linearity to which our symmetry tests may well respond is autoregressive conditional heteroskedasticity (ARCH). It is quite conceivable that the conditional variance of the error process varies according to the phase of the business cycle. Carrying out an ARCH test for the time series we consider could thus be useful. The test against ARCH we shall apply is called Q(n) and defined in the appendix. McLeod and Li (1983) who have constructed the test recommend it as a general linearity test. However, as Luukkonen et al. (1988b) have shown, it is asymptotically equivalent to the ARCH test of Engle (1982) and thus applicable here. The simulation results reported in Luukkonen et al. (1988b) indicate that in small samples Q(n) often has little power against many other types of non-linearity. Thus, if the ARCH test does not reject linearity but the STAR tests do, we may at least exclude heteroskedasticity from our set of alternatives. On the other hand, the same simulation
experiments have revealed that a few linearity tests have power against ARCH at sample sizes appearing in this paper, although they have been designed for some other non-linear alternative. Thus, if all tests reject, we cannot exclude the possibility that the STAR tests actually respond to conditional heteroskedasticity. This is particularly true if the probability value of the ARCH test is a lot smaller than that of the STAR tests. Naturally, both STAR and ARCH may appear simultaneously as well, although models accommodating this possibility have not yet appeared in the literature.

Another non-linear time series model which is of interest here is the bilinear model. It may generate stationary realizations with amplitude changes, i.e., sharp peaks and troughs, which may also be present in business cycle data. For discussion see Subba Rao and Gabr (1984, pp. 150-151). Our STAR tests may well respond to amplitude changes or conspicuous outliers in time series. With that in mind it seems useful to run linearity tests against bilinearity. If neither they nor the ARCH test reject the null of linearity whereas STAR tests do, we may argue with a little more certainty that the rejections by the latter are caused by STAR. Weiss (1986) and Saikkonen and Luukkonen (1988) have constructed tests against bilinearity: the exact form of the test we shall apply is presented in the appendix.

5. U.S. UNEMPLOYMENT SERIES

Neftci (1984) has used U.S. unemployment series in considering the possible asymmetry of business cycle fluctuations and gives two main reasons for that. First, he believes that series like unemployment
related to production give a better indication of the business cycle than, say, consumption. Second, he does not need to estimate and eliminate a trend component because the unemployment variables do not contain a trend. In fact, Neftci essentially considers and observes (2.2) with $z_t = \nabla u_t$ where $u_t$ is an unemployment rate. On the other hand, what matters in this paper is that the time series can be made approximately stationary by differencing. In testing linearity against STAR the threshold parameter $c$ is assumed unknown, and the watershed between contractions and expansions is thus not fixed in advance as in Neftci (1984). If we try to model $\nabla u_t$ assuming a STAR model and if that assumption turns out to be a plausible one, we might expect $c$ to be close to zero. The situation could be different if we were modelling industrial output using post-war data. Then it would be natural to expect $c$ to be clearly positive, if a STAR model fitted the data well.

In applying linearity tests to U.S. unemployment series there is the problem of determining $p$, the order of autoregression. It is solved by starting with a relatively high order, in this case 7, and fitting AR models for all orders not higher than that to the data. The model with the lowest value of an order selection criterion is selected. We have used both AIC (Akaike, 1974) and SBIC (Schwarz, 1978; Rissanen, 1978) as our order selection criteria. The test results are in this case rather close to each other, and the ones reported in Table 1 are based on SBIC. In this and subsequent tables we report the observed significances or p-values of the test statistics rather than the values of the test statistics themselves.
We use the same quarterly, seasonally adjusted time series from Business Conditions Digest as Neftçi (1984) does but first extend the period from 1948(i) to 1985(iv). The series are differenced once before applying the tests. The results indicate that the differenced series are non-linear. The McLeod and Li (ARCH) test with n=4 strongly rejects the null in two cases out of three, which points towards ARCH. The linearity is rejected in favour of STAR at the 0.05 level of significance in one case and at the 0.01 level in the remaining two. Even the test against bilinearity rejects linearity, if the bilinear model contains enough parameters (m=k=2). Because Q(n) is often not very powerful against non-linearities other than ARCH, we may not reject the notion that heteroskedasticity is present except for series 44. However, the linearity against STAR is strongly rejected even in that case, which lends some support to cyclical asymmetry.

To facilitate comparison with Neftçi's results, we have performed the same tests using his observation period, 1948(i) to 1981(iv). The results do not change much. Linearity is generally rejected at the 0.05 level of significance. For series 44 the rejection against STAR is very strong, and there is now also a rejection against ARCH at the 0.05 level. Bilinearity is generally rejected slightly less strongly than STAR but remains a possible cause of non-linearity. The rejections against STAR are in accord with Neftçi's results which support cyclical asymmetry.

In anticipation of the next section, we drop the first observations and base our tests on data from 1960(i) to 1985(iv). The rather ambiguous results are in Table 2. Using SBIC we end up having a parsimonious AR(1) model as our linear alternative. Our conclusion then is that there is no trace of asymmetry in the unemployment series we are considering. However, if we let AIC guide us, the outcome is in
two cases a less parsimonious AR model, and the linearity hypothesis is rejected in favour of STAR at the 0.05 level of significance. Obviously, if there is STAR type non-linearity in series 44 and 45, parameters $\theta_2$ and $\theta_3$ in (2.1) are needed to characterize it. Two conclusions emerge. First, our results are sensitive to the AR specification. Second, the evidence for cyclical asymmetry is weaker if the observations before the sixties are excluded than if they are retained. The case for heteroskedasticity remains strong even if the early observations are excluded, and linearity continues to be rejected against the less parsimonious of the two bilinear alternatives indicating the possibility of amplitude changes in the series.

6. INTERNATIONAL UNEMPLOYMENT SERIES

Next we shall extend this study by looking at unemployment rates elsewhere. This is done by analyzing unemployment data from 13 OECD countries. The time series are quarterly, seasonally unadjusted unemployment rates 1960(i)-1986(iv) collected by OECD and published in their Main Economic Indicators. The seasonally unadjusted U.S. series is included in the sample for comparison. We shall make the series approximately stationary by seasonal differencing, i.e., the time series to be analyzed are of the type $\nabla_4 u_t = u_t - u_{t-4}$. No other seasonal adjustment procedure is applied.

The results of the linearity tests appear in Table 3. Taking the U.S. first, it can be seen that the results are in accord with those in
Table 2. Selecting the AR model by SBIC and testing linearity with $S_1$ and $S_3$ leads to accepting linearity as before. On the other hand, the less parsimonious (AIC) AR model using more parameters to characterize seasonality allows us to reject linearity at the 0.05 level in favour of STAR, when the test statistic is $S_1$. Note that linearity is not rejected against ARCH or bilinear alternatives. Conditional heteroskedasticity thus seems more like an artifact due to a particular seasonal adjustment procedure than a phenomenon inescapably present in cyclical quarterly U.S. economic data.

As to the other countries, Japan and the three Scandinavian countries, Finland, Norway and Sweden, distinguish themselves from the others in that linearity is accepted. In Norway, Sweden and Japan the unemployment has been very low during the whole period of observation, so that we may not expect strong cycles there, symmetric or asymmetric. Yet, the Norwegian data resemble American in that using more parameters to model seasonality does lead to rejecting the linearity in favour of STAR.

In many European countries, the unemployment has risen dramatically at the end of the seventies or early eighties and started to fluctuate around a higher country-specific level. It is conceivable that this increase in unemployment affects test results: at least the appearance of ARCH may be expected. Indeed, the ARCH tests for Belgium, West Germany, France, Italy and U.K. reject linearity in favour of ARCH. But then, for France and the Netherlands, the rejection of linearity against STAR is really overwhelming, and the possibility of cyclical asymmetry can hardly be ignored. Note, however, that these are the two countries for which the logarithm of unemployed has replaced the
quarterly unemployment rate, because the latter has not been available for the whole observation period.

The Belgian test results are apparently contradictory as $S_1$ strongly favours the null, whereas $S_3$ clearly rejects it. However, as Luukkonen et al. (1988a) point out, $S_1$ has little power if the dominating non-linearity parameter in the STAR model is $\theta_0$. The results indicate this to be the case if we model the Belgian unemployment rate by a STAR(2;2,2) model. If a less parsimonious AR model is used, $S_3$ loses power and does not any longer reject linearity.

The most clear-cut evidence in favour of possible cyclical asymmetry comes from Canada. When the AR(2) model forms the base for inference, linearity is rejected only against STAR. When the AR(5) model selected by AIC is used, linearity is also rejected against the BL(6;1,1) model, but the rejection against STAR is stronger and also a lot stronger than the corresponding rejection in the AR(2) case. For many other countries we cannot avoid the suspicion that the increase in the natural rate of unemployment occurring during the observation period influences the results. It may therefore be difficult to argue in favour of cyclical asymmetry of the series in those countries on the basis of the evidence in Table 3.

7. TRANSFORMED UNEMPLOYMENT SERIES

Since the increase in the natural unemployment rate may pose a problem to the investigator, it seems reasonable to transform the data in such
a way that the relative importance of the increase diminishes. The
unemployment rate (in per cent) \( u_t \) is a variable bounded between 0
and 100 %. For time series bounded like this, Wallis (1987) considers
the logistic transformation \( v_t = \log(u_t/(100-u_t)) \). It is a
useful transformation in modelling \( u_t \), because it ensures consist-
ency with the definition of the bounds. We shall apply it here mainly
because it enhances small absolute but large relative changes in the
unemployment rate. Thus it puts a larger weight than previously on
cyclical fluctuations before mid-seventies or early eighties in
countries where the natural rate of unemployment has soared. The
seasonal differences \( \nabla_4 v_t \) to be analyzed have the same sign as \( \nabla_4 u_t \).

The test results appear in Table 4. It is seen that in the light of
our tests, the U.S. data now show no signs of non-linearity. If
anything, the logistic transformation seems an excellent way of
linearizing this series. The same is true for the Austrian and
Canadian unemployment rates. For the countries with a low unemployment
rate in our sample, Japan, Sweden and Norway, the linearity hypothesis
not unexpectedly continues to receive support against STAR. However,
the transformed Swedish series seems conditionally heteroskedastic,
as linearity is very strongly rejected against ARCH. As to Finland
where the seasonally unadjusted unemployment rate has not exceeded
7.8 %, the less parsimonious (AIC) model of the two hints at STAR, but
linearity is still accepted at the 0.05 level of significance.

We half expected the logistic transformation make the unemployment
series more linear in the case of European countries with increased
natural rate of unemployment. For Belgium and to some extent U.K. this
seems to have been the case. However, there are remarkable examples of the series becoming more non-linear after the transformation. Emphasizing fluctuations at low rates of unemployment more reveals important non-linearity for West Germany and Italy: linearity is rejected against both STAR and ARCH. Besides, the German data still show indications for bilinearity as in Table 3. Combining the information from Tables 3 and 4 it is evident that the test results are not invariant to data transformations. This contributes to making the symmetry or asymmetry of business cycles a far from unambiguous issue. There is not a single country for which the tests against STAR consistently reject linearity. (U.K. can be an exception if we insist that the failure of $S_i$ to reject the null in Table 4 is just an indication of that the non-linearity in the STAR model stems mainly from $\theta_0$.) Only for the three countries with the lowest unemployment rates do the results of STAR tests (supporting linearity) seem to be unaffected by the logistic transformation.

8. INTERNATIONAL INDUSTRIAL PRODUCTION SERIES

It might be interesting to compare the results obtained using unemployment series with the information gained by studying other time series as well. As our time series techniques are suitable for the purpose, we follow the example of Falk (1986) and DeLong and Summers (1986) and study the asymmetry of business cycles using output series. Because the manufacturing is likely to show more cyclical variation than GNP, we use international data on industrial production. The observations are again quarterly, seasonally unadjusted values of the logarithmic index of industrial production in 13 OECD countries,
published in the OECD Main Economic Indicators, and they cover the quarters 1960(i) to 1986(iv). The series are made approximately stationary by seasonal differencing in the same way as the unemployment series. The specification of the AR order is carried out as before using SBIC and AIC. The test results appear in Table 5.

While the U.S. unemployment rate seems linear, at least after the logistic transformation, the hypothesis of the logarithmic industrial production being non-linear and of STAR type receives support from the data. The same is also true for Japan: linearity is rejected at the 0.05 level of significance in favour of STAR. However, for the Japanese data Q(n) rejects linearity very strongly, so that ARCH cannot be excluded from consideration. There is evidence for heteroskedasticity in several other countries as well: this includes Austria, Belgium, France, Italy, The Netherlands and Norway. Thus the rejections or "near-rejections" of linearity against STAR for these countries might also arise from ARCH. An exception is West Germany: if we accept non-linearity, there is a good case for asymmetry. The bilinearity and ARCH tests do not even come close to rejecting linearity. Of the four countries with linear unemployment series, Finland and Sweden still stand out: no test rejects linearity. A reason for this may be that both countries have mostly been enjoying sustained economic growth without violent swings of any kind during the observation period. The linearity of logarithmic industrial production is also accepted for the British and Canadian economies.

These results do not fully accord with those of Falk (1986) who has analyzed the asymmetry of business cycles using Neftçi's (1984) technique and industrial production data from Canada, France, Italy,
United Kingdom and West Germany. His application differs from ours in so many respects that a reconciliation does not seem possible. Falk uses seasonally adjusted quarterly data and applies linear detrending before dichotomizing the first differences. Thus his series are much more heavily manipulated than ours. His estimation period extends from 1951(i) to 1983(iv), and he interprets the results as not making cyclical symmetry as unlikely possibility. As noticed above, we do obtain rejections against STAR at the 0.05 significance level with shorter time series already in studying the industrial output of West Germany and France.

Finally, in our application the probability values of tests against bilinearity are generally clearly higher than those of STAR tests. Many STAR probability values are around 0.1 or less, whereas that is rare for tests against bilinearity. Thus it is rather unlikely that amplitude changes or outliers have caused any low probability values in STAR tests. On the other hand, as noted above, in some cases the possibility of STAR tests responding to ARCH cannot be excluded.

9. FINAL REMARKS

If one wants to sort out countries in which the business cycles appear non-linear of STAR type in 1960-1986, our investigation leaves us with a single example, West Germany. For the U.S. and Japan, cyclical asymmetry of industrial production cannot be excluded, whereas the unemployment series appear symmetric. As to the U.S., some doubt remains for the untransformed unemployment series, but the main evidence for non-linearity in general seems to come from the pre-1960
data. When the period 1940(i)-1959(iv) is excluded from consideration, the case for asymmetry in unemployment series is weakened. At the other end of the scale, Finland and Sweden appear as examples of countries with linear business cycles. For the remaining countries the evidence is mixed but in many cases conditional heteroskedasticity at least seems a very real possibility.

Our analysis also reveals that one has to be careful in speaking about the asymmetry of business cycles. The results are sensitive not only to the choice of series to represent the cycle but to data transformations as well. They are also dependent on any particular definition of asymmetry. Furthermore, some conclusions reached with U.S. data are reversed when the observation period is changed. Our study of the international series shows at any rate that non-linearity is often rejected already with rather short series and against more than one type of non-linear alternative. This may cast some doubt on economic theories which completely preclude non-linear business cycles.
FOOTNOTE

1) Luukkonen et al. (1988b) also show that, asymptotically, the power of Q(n) against bilinearity or exponential autoregression is not higher than the size of the test.
REFERENCES


Table 1. Observed significances or p-values of linearity tests against STAR and other non-linearities using quarterly, seasonally adjusted U.S. unemployment data 1948(1) – 1985(iv) (1948(1) – 1981(iv))

<table>
<thead>
<tr>
<th>BCD Series</th>
<th>AR order p</th>
<th>Test statistics</th>
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</tr>
</thead>
<tbody>
<tr>
<td>37 ÷ 441</td>
<td>2 (2)</td>
<td>$S_1$</td>
<td>0.003</td>
<td>(0.002)</td>
<td>0.133</td>
<td>(0.037)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$S_3$</td>
<td>0.008</td>
<td>(0.002)</td>
<td>0.082</td>
<td>(0.003)</td>
</tr>
<tr>
<td>&quot;44</td>
<td>3 (2)</td>
<td>$BLT(p;1,1)$</td>
<td>0.019</td>
<td>(&lt;0.0001)</td>
<td>0.028</td>
<td>(0.030)</td>
</tr>
<tr>
<td>45</td>
<td>2 (2)</td>
<td>$BLT(p;2,2)$</td>
<td>0.031</td>
<td>(0.050)</td>
<td>0.026</td>
<td>(0.038)</td>
</tr>
</tbody>
</table>

Note: (a) The series are from Business Conditions Digest and cover the years 1948-1985. Series 37 (in the coding system of BCD) is "number of persons unemployed", 441 is "total civilian labour force", 44 is "unemployment 15 weeks and over" (unemployment rate), and 45 is "average weekly insured employment". (b) The figures in parentheses are based on data from 1948(1) – 1981(iv), the observation period used by Neftçi (1984).
Table 2. Observed significances or p-values of linearity tests against STAR and other non-linearities using quarterly, seasonally adjusted U.S. unemployment data 1960(i) - 1985(iv)

<table>
<thead>
<tr>
<th>BCD Series</th>
<th>AR order</th>
<th>Test statistics</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p</td>
<td></td>
<td>$S_1$</td>
<td>$S_3$</td>
<td>BLT($\hat{\rho};1,1$)</td>
<td>BLT($\hat{\rho};2,2$)</td>
</tr>
<tr>
<td>37 441</td>
<td>1</td>
<td>0.329</td>
<td>0.598</td>
<td>0.639</td>
<td>0.008</td>
<td>0.002</td>
</tr>
<tr>
<td>44</td>
<td>1</td>
<td>0.776</td>
<td>0.957</td>
<td>0.134</td>
<td>0.020</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(0.021)</td>
<td>(0.018)</td>
<td>(0.509)</td>
<td>(0.191)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>45</td>
<td>1</td>
<td>0.795</td>
<td>0.768</td>
<td>0.272</td>
<td>0.002</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(0.020)</td>
<td>(0.030)</td>
<td>(0.037)</td>
<td>(0.011)</td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

Note: (a) The BCD series are the same as in Table 1. (b) The figures in parentheses are related to tests in which the AR model has been selected by using AIC. If there are no values in parentheses, SBIC and AIC yield the same AR model.
Table 3. Observed significances or p-values of linearity tests against STAR and other non-linearities using quarterly, seasonally unadjusted unemployment rates of 13 OECD countries, 1960(i) - 1986(iv)

<table>
<thead>
<tr>
<th>Country</th>
<th>AR order p</th>
<th>Test statistic</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_1$</td>
<td>$S_3$</td>
<td>BLT(p;1,1)</td>
<td>BLT(p;2,2)</td>
<td>Q(4)</td>
</tr>
<tr>
<td>Austria</td>
<td>1</td>
<td>0.078</td>
<td>0.006</td>
<td>0.102</td>
<td>0.382</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.938</td>
<td>0.024</td>
<td>0.529</td>
<td>0.077</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>(0.176)</td>
<td>(0.172)</td>
<td>(0.290)</td>
<td>(0.011)</td>
<td>(0.100)</td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>0.011</td>
<td>0.027</td>
<td>0.156</td>
<td>0.342</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.019)</td>
<td>(0.092)</td>
<td>(0.306)</td>
</tr>
<tr>
<td>FR Germany</td>
<td>5</td>
<td>0.118</td>
<td>0.120</td>
<td>0.415</td>
<td>0.024</td>
<td>0.027</td>
</tr>
<tr>
<td>Finland</td>
<td>4</td>
<td>0.908</td>
<td>0.287</td>
<td>0.053</td>
<td>0.440</td>
<td>0.265</td>
</tr>
<tr>
<td>France*</td>
<td>5</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
<td>0.477</td>
<td>0.0001</td>
<td>0.009</td>
</tr>
<tr>
<td>Italy</td>
<td>5</td>
<td>0.256</td>
<td>0.056</td>
<td>0.452</td>
<td>0.390</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>(0.082)</td>
<td>(0.069)</td>
<td>(0.287)</td>
<td>(0.566)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>Japan</td>
<td>1</td>
<td>0.206</td>
<td>0.393</td>
<td>0.508</td>
<td>0.208</td>
<td>0.919</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(0.612)</td>
<td>(0.413)</td>
<td>(0.613)</td>
<td>(0.184)</td>
<td>(0.400)</td>
</tr>
<tr>
<td>The Netherlands*</td>
<td>6</td>
<td>0.0002</td>
<td>&lt; 0.0001</td>
<td>0.860</td>
<td>0.019</td>
<td>0.099</td>
</tr>
<tr>
<td>Norway</td>
<td>1</td>
<td>0.692</td>
<td>0.748</td>
<td>0.216</td>
<td>0.150</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(0.042)</td>
<td>(0.044)</td>
<td>(0.078)</td>
<td>(0.274)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Sweden†</td>
<td>1</td>
<td>0.709</td>
<td>0.181</td>
<td>0.892</td>
<td>0.885</td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(0.552)</td>
<td>(0.487)</td>
<td>(0.697)</td>
<td>(0.823)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>6</td>
<td>0.007</td>
<td>0.024</td>
<td>0.975</td>
<td>0.568</td>
<td>0.035</td>
</tr>
<tr>
<td>United States</td>
<td>2</td>
<td>0.381</td>
<td>0.390</td>
<td>0.777</td>
<td>0.690</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>(0.032)</td>
<td>(0.124)</td>
<td>(0.591)</td>
<td>(0.962)</td>
<td>(0.358)</td>
</tr>
</tbody>
</table>

Note: The figures in parentheses are related to tests in which the AR model has been selected by using AIC. If there are no values in parentheses, SBIC and AIC yield the same AR model.

* Results are based on the logarithm of the number of unemployed, as the unemployment rate has not been available for the whole period of observation.

† Results are based on data from 1962(i) to 1986(iv).
Table 4. Observed significances or p-values of linearity tests against STAR and other non-linearities using the logistic transformation of quarterly, seasonally unadjusted unemployment rates of 13 OECD countries, 1960(i) - 1986(iv)

<table>
<thead>
<tr>
<th>Country</th>
<th>AR order p</th>
<th>Test statistic</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_1$</td>
<td>$S_3$</td>
<td>BLT($\hat{p};1,1$)</td>
<td>BLT($\hat{p};2,2$)</td>
<td>Q(4)</td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>2</td>
<td>0.220</td>
<td>0.178</td>
<td>0.066</td>
<td>0.370</td>
<td>0.896</td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>5</td>
<td>(0.486)</td>
<td>(0.015)</td>
<td>(0.518)</td>
<td>(0.106)</td>
<td>(0.246)</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>2</td>
<td>0.477</td>
<td>0.263</td>
<td>0.930</td>
<td>0.554</td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td>FR Germany</td>
<td>2</td>
<td>0.166</td>
<td>0.085</td>
<td>0.696</td>
<td>0.049</td>
<td>0.817</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>4</td>
<td>0.004</td>
<td>0.008</td>
<td>0.019</td>
<td>0.128</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>France*</td>
<td>6</td>
<td>&lt; 0.0001</td>
<td>&lt; 0.0001</td>
<td>0.477</td>
<td>0.0001</td>
<td>0.113</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>6</td>
<td>0.004</td>
<td>0.005</td>
<td>0.704</td>
<td>0.116</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>4</td>
<td>0.165</td>
<td>0.317</td>
<td>0.476</td>
<td>0.044</td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td>The Netherlands*</td>
<td>4</td>
<td>0.0002</td>
<td>&lt; 0.0001</td>
<td>0.860</td>
<td>0.019</td>
<td>0.099</td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>4</td>
<td>0.240</td>
<td>0.083</td>
<td>0.315</td>
<td>0.136</td>
<td>0.370</td>
<td></td>
</tr>
<tr>
<td>Sweden†</td>
<td>1</td>
<td>0.833</td>
<td>0.313</td>
<td>0.418</td>
<td>0.700</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>(4)</td>
<td>(0.682)</td>
<td>(0.159)</td>
<td>(0.992)</td>
<td>(0.678)</td>
<td>(&lt; 0.0001)</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>2</td>
<td>0.406</td>
<td>0.390</td>
<td>0.876</td>
<td>0.691</td>
<td>0.756</td>
<td></td>
</tr>
</tbody>
</table>

Note: The figures in parentheses are related to tests in which the AR model has been selected by using AIC. If there are no values in parentheses, SBIC and AIC yield the same result.

* Results are based on the logarithm of the number of unemployed; they are the same as in Table 3.

† Results are based on data from 1962(i) to 1986(iv).
Table 5. Observed significances or p-values of linearity tests against STAR and other non-linearities using quarterly, seasonally unadjusted logarithmic indices of industrial production from 13 OECD countries, 1960(1) - 1986(iv)

<table>
<thead>
<tr>
<th>Country</th>
<th>AR order</th>
<th>Test statistic</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\rho}$</td>
<td></td>
<td>$S_1$</td>
<td>$S_3$</td>
<td>BLT($\hat{\rho};1,1$)</td>
<td>BLT($\hat{\rho};2,2$)</td>
</tr>
<tr>
<td>Austria</td>
<td>5</td>
<td>0.048</td>
<td>0.072</td>
<td>0.020</td>
<td>0.096</td>
<td>0.046</td>
</tr>
<tr>
<td>Belgium</td>
<td>5</td>
<td>0.054</td>
<td>0.069</td>
<td>0.024</td>
<td>0.044</td>
<td>0.0002</td>
</tr>
<tr>
<td>Canada</td>
<td>5</td>
<td>0.138</td>
<td>0.107</td>
<td>0.266</td>
<td>0.969</td>
<td>0.889</td>
</tr>
<tr>
<td>FR Germany</td>
<td>4</td>
<td>0.027</td>
<td>0.096</td>
<td>0.514</td>
<td>0.544</td>
<td>0.998</td>
</tr>
<tr>
<td>Finland</td>
<td>1</td>
<td>0.891</td>
<td>0.793</td>
<td>0.459</td>
<td>0.927</td>
<td>0.720</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(0.717)</td>
<td>(0.866)</td>
<td>(0.880)</td>
<td>(0.653)</td>
<td>(0.873)</td>
</tr>
<tr>
<td>France</td>
<td>5</td>
<td>0.109</td>
<td>0.039</td>
<td>0.381</td>
<td>0.553</td>
<td>0.019</td>
</tr>
<tr>
<td>Italy</td>
<td>5</td>
<td>0.124</td>
<td>0.056</td>
<td>0.986</td>
<td>0.987</td>
<td>0.013</td>
</tr>
<tr>
<td>Japan</td>
<td>5</td>
<td>0.033</td>
<td>0.083</td>
<td>0.889</td>
<td>0.227</td>
<td>0.001</td>
</tr>
<tr>
<td>The Netherlands</td>
<td>1</td>
<td>0.197</td>
<td>0.102</td>
<td>0.255</td>
<td>0.436</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(0.676)</td>
<td>(0.307)</td>
<td>(0.650)</td>
<td>(0.632)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Norway</td>
<td>5</td>
<td>0.046</td>
<td>0.090</td>
<td>0.463</td>
<td>0.724</td>
<td>0.022</td>
</tr>
<tr>
<td>Sweden</td>
<td>4</td>
<td>0.998</td>
<td>0.890</td>
<td>0.820</td>
<td>0.934</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(0.584)</td>
<td>(0.384)</td>
<td>(0.932)</td>
<td>(0.974)</td>
<td>(0.945)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>5</td>
<td>0.140</td>
<td>0.181</td>
<td>0.332</td>
<td>0.818</td>
<td>0.521</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>(0.278)</td>
<td>(0.280)</td>
<td>(0.372)</td>
<td>(0.808)</td>
<td>(0.286)</td>
</tr>
<tr>
<td>United States</td>
<td>2</td>
<td>0.033</td>
<td>0.036</td>
<td>0.201</td>
<td>0.110</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td>(0.017)</td>
<td>(0.062)</td>
<td>(0.170)</td>
<td>(0.092)</td>
<td>(0.061)</td>
</tr>
</tbody>
</table>

Note: The figures in parentheses are related to tests in which the AR model has been selected by using AIC. If there are no values in parentheses, SBIC and AIC yield the same result.
APPENDIX

Preliminaries

Consider the autoregressive model of order $p$,

$$y_t + a_1 y_{t-1} + \ldots + a_p y_{t-p} = \mu + \varepsilon_t$$  \hspace{1cm} (A.1)

where the zeroes of the polynomial $a(z) = 1 + a_1 z + \ldots + a_p z^p$ lie outside the unit circle and

$$\varepsilon_t \sim \text{nid}(0, \sigma^2).$$  \hspace{1cm} (A.2)

Suppose that we observe a time series $y_{-p+1}, \ldots, y_0, y_1, \ldots, y_T$, where the initial values $y_{-p+1}, \ldots, y_0$ are treated as fixed. Let $\hat{\mu}, \hat{a}_1, \ldots, \hat{a}_p$ be the least squares estimators of $\mu, a_1, \ldots, a_p$ in (A.1) and set $z_{0t} = [-1, y_{t-1}, \ldots, y_{t-p}]'$. Define

$$\hat{\varepsilon}_t = y_t + \hat{a}_1 y_{t-1} + \ldots + \hat{a}_p y_{t-p} - \hat{\mu}$$
$$\text{and } \hat{\sigma}^2 = \frac{1}{T-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2$$

as the estimators of $\varepsilon_t$ and $\sigma^2$, respectively.

ARCH model

If (A.2) is replaced by

$$\varepsilon_t | F_{t-1} \sim \text{nid}(0, h_t), \quad h_t = \sigma^2 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_n \varepsilon_{t-n}^2$$  \hspace{1cm} (A.3)

where $F_t$ is the $\sigma$-algebra (information set) determined by $(\varepsilon_u, u \leq t)$, then (A.1) combined with (A.3) forms an ARCH($p,n$) model. The null hypothesis of linearity is $H_0: \alpha_1 = \ldots = \alpha_n = 0$. Set
\[ \hat{c}_{EE}(j) = T^{-1} \sum_{t=j+1}^{T} \left( \hat{\epsilon}_t^2 - \hat{\sigma}^2 \right) \left( \hat{\epsilon}_{t-j}^2 - \hat{\sigma}^2 \right), \quad 0 \leq j < T. \]

Then we may define

\[ Q(n) = T(T+2) \sum_{j=1}^{n} \hat{r}_{EE}^2(j)/(T-j) \quad (A.4) \]

where \( \hat{r}_{EE}(j) = \hat{c}_{EE}(j)/\hat{c}_{EE}(0) \) is the sample autocorrelation coefficient between \( \hat{\epsilon}_t^2 \) and \( \hat{\epsilon}_{t-j}^2 \). This is the test statistic of McLeod and Li (1983).

Under \( H_0 \), (A.4) has an asymptotic \( \chi^2 \) distribution with \( n \) degrees of freedom.

**Bilinear model**

The BL(p,m,k) model is defined as

\[ y_t + a_1 y_{t-1} + \ldots + a_p y_{t-p} = \mu + \varepsilon_t + \sum_{i=1}^{m} \sum_{j=1}^{k} c_{ij} \varepsilon_{t-i} y_{t-j} \quad (A.5) \]

where \( \varepsilon_t \) satisfies (A.2). The null hypothesis of linearity is \( H_0 : c_{ij} = 0 \), \( i=1, \ldots, m; j=1, \ldots, k \), in (A.5). Our test statistic is

\[ \text{BLT}(p;k,m) = \hat{\sigma}^{-2} \left( \sum_{t=1}^{T} \varepsilon_t^2 \right) \left( \sum_{t=1}^{T} \varepsilon_t \right)^{-1} \left( \sum_{t=1}^{T} \varepsilon_t \right) \left( \sum_{t=1}^{T} y_t \right) \left( \sum_{t=1}^{T} y_t \right)^{-1} \left( \sum_{t=1}^{T} \varepsilon_t \right) \quad (A.6) \]

where

\[ \hat{M}_{10} = \hat{M}_{10} = \sum_{t=1}^{T} z_{1t} z_{2t}, \quad \hat{M}_{11} = \sum_{t=1}^{T} z_{1t} z_{1t}, \quad i=0,1 \]

with \( z_{1t} = [-\hat{\epsilon}_{t-1} y_{t-1}, \hat{\epsilon}_{t-1} y_{t-1}, \ldots, \hat{\epsilon}_{t-m} y_{t-k}]' \). Under \( H_0 \), (A.6) has an asymptotic \( \chi^2 \) distribution with \( mk \) degrees of freedom.

For further details, see Weiss (1986) and Saikkonen and Luukkonen (1988).
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