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A MODEL FOR THE BANKING SECTOR
IN FINLAND

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A MODEL FOR THE BANKING SECTOR IN FINLAND

1. Derivation of the equations for the supply of bank loans

The aim of this paper is to present the structure of the banking sector submodel of the financial market model and its preliminary estimation. 1) The starting point of the model is the following balance sheet of the banks

\[(1) \quad L_h + L_f = D + CBD + NFD + O\]

where
- \(L_h\) = loans to households
- \(L_f\) = loans to firms
- \(D\) = deposits
- \(CBD\) = central bank debt of the banks
- \(NFD\) = net foreign debt of the banks
- \(O\) = other liabilities, net.

The loans to the firm sector consist of loans denominated in domestic currency (\(L_f^{dom}\)) and loans denominated in foreign currency (\(L_f^{for}\)). There is a (known) relation between bank deposits and bank lending to the household sector,

\[(2) \quad D = D(L_h), \quad \frac{dD}{dL_h} > 0\]

1) See the former papers Alho (1983) and Alho (1984) on the flow-of-funds models of the households and firms.
i.e. deposits are linked to the possibility of raising loans. A part of the loans to the households is, on the other hand, linked to the presavings (deposits) of the households, and to loan promises of the banks to households, i.e. loans to the households cannot be less than some fixed "promised" amount,

\[ L_h \leq \bar{L}_h, \text{ where } \bar{L}_h = L_h(D), \frac{dL_h}{dD} \geq 0. \]

The net foreign debt of the banks consists of two components, the foreign finance raised to finance the loans denominated in foreign currency \( L_{for} \) and the net forward position of foreign exchange of the banks (FP) which is covered by an equivalent amount of foreign debt to the euromarkets. \(^1\) Thus we can write

\[ \text{NFD} = L_{for} + FP. \]

The foreign currency loans of the banks to the firms depend on three main factors: the volume of foreign transactions, the exchange regulation, and the "mutual willingness" of the firms and the banks to utilize this channel of finance. The first factor can be quite well represented by the volume of the foreign transactions, i.e. by the volume of exports \( X \) and imports \( H \). The last factor can be described by the forward premium \( fp \) of the Finnish markka against the foreign currencies. Further, it depends on the speculative element concerning the expected changes in the external value of the Finnish markka, whether there are some expectations on a devaluation or a revaluation of the markka. We take this last factor to be determined

\(^1\) Instead of foreign currency loans we should have the difference of these loans and the foreign currency deposits but here we have included the latter item in other liabilities.
outside the banking sector model and it is represented by the symbol (sp) (spot premium). The second of the above three factors we have to omit at this phase. So we can write

\[(5) \quad L_f^\text{for} = L_f^\text{for}(X + H, r_d - r_f, sp) \text{ and} \]

\[(6) \quad FP = FP(CA, fp, sp) \]

where CA is the surplus on the current account and \( r_d \) is the relevant domestic rate of interest to the firms. The cost of the foreign funds is the (weighted) euromarket interest rate \( r_f \) which is also the loan rate of interest of these loans.

The banks also get some revenues in addition to the interest incomes from loans to the firms and these are supposed to be represented by a concave revenue function \( R(L_f) \) which is defined more closely on page 16. The cost of the central bank finance is supposed to be represented (each year) well enough by a convex function \( r_{cb} \) which has the marginal properties

\[(7) \quad r_{cb} = h_0 + h_1CBD, \quad r_{cb}'' = h_1 \geq 0, \quad h_0 \text{ and } h_1 \text{ are constant.} \]

We are now ready to present the profit function of the representative bank

\[(8) \quad \pi = r(L_h + L_{f, \text{dom}}) + r_f L_f^\text{for} - r_D - r_{cb}(CBD) - r_f NFD + R(L_f). \]

We specify the forward premium of the Finnish markka to be determined by the covered interest parity,
We further suppose that \( \frac{dr_d}{dr_{cb}} = 1 \), i.e., that the domestic short-term interest rate is determined by the marginal interest rate on the central bank debt of the banks.

By deriving (8) with respect to \( L_h \) we get as the first condition for the optimum of the bank

\[
\frac{d\pi}{dL_h} = r - r_D L_h - \frac{\partial r_{cb}(CB)}{\partial L_h} = 0.
\]

Here the expression \( r'_{cb} \) is quite complicated because the marginal interest rate on the central bank debt depends on the volume of the central bank debt which, further, depends on the net forward position of the banks which, further, depends on the forward premium of the markka which depends on the marginal discount rate, and so on. We get a "series expansion" of the derivative \( r'_{cb} \):

\[
\frac{\partial r_{cb}(CBD)}{\partial L_h} = r'_{cb} \left( \frac{\partial CBD}{\partial L_h} \right) = (h_0 + h_1 CB)(1-D_L h - \frac{\partial FP}{\partial L_h}) \text{ and}
\]

further

\[
\frac{\partial FP}{\partial L_h} = \frac{\partial FP}{\partial FP} \frac{\partial r_{cb}}{\partial L_h} \frac{\partial CBD}{\partial L_h} = FP_{fp} h_1 \frac{\partial CBD}{\partial L_h} = FP_{fp} h_1 (1-D_L h-h_1 FP_{fp} \frac{\partial CBD}{\partial L_h})
\]

\[
= FP_{fp} h_1 (1-D_L h-h_1 FP_{fp} (1-D_L) + h_1^2 FP_{fp} ^2 \frac{\partial CBD}{\partial L_h})
\]

\[
= \ldots = FP_{fp} h_1 (1-D_L h)(1-h_1 FP_{fp} + h_1^2 FP_{fp} ^2 - \ldots)
\]

\[
= \frac{FP_{fp} h_1 (1-D_L h)}{1 + h_1 FP_{fp}} \text{ supposing that } |h_1 FP_{fp}| < 1.
\]

Now we can insert this into (10) and solve for the optimal central bank debt

\[
fp = r_f - r_d.
\]
The second optimum condition is

\[ r_c + R'(L_f) - \frac{\partial r_{cb}(CBD)}{\partial \text{dom}_{L_f}} = 0. \]

Analogously with that above we can write the last term here as

\[ \frac{\partial r_{cb}(CBD)}{\partial \text{dom}_{L_f}} = (h_0 + h_1 CBD)(1 - D_{L_f} - \frac{FP_fh_1(1 - D_{L_f})}{1 + h_1 FP_f}). \]

We may now use the optimal central bank debt from (12) and the optimal marginal interest rate \( r_{cb} \) to solve for the optimal level of the domestic currency loans to firms from (13). From (6) we can next solve for the net forward position of the banks and then solve for the loans to the household sector from the balance sheet (1). Then we have solved all the items in the balance sheet of the banking sector.

In the 1970's Bank of Finland participated in the forward market for foreign exchange by setting the forward rates and taking the net forward position on its own risk. This phase lasted up to the end of March 1980, after which Bank of Finland participated only in the forward exchange market of the rouble.\(^1\) This phase ended in spring 1983. The phase after 1980 can be described by the above model. In the former phase we can write (11) simply as

\[ \text{---} \]

\(^1\) See Suvanto (1982), chapter 4 more closely on this.
\[(11)^\prime \quad \frac{\partial r_{CB}(CBD)}{\partial L_h} = (h_0 + h_1CBD)(1-D_{Lh})\]

and continue analogously as above.

So far we have not taken the constraint (3) into account. Let \( \mu \) be the Lagrangian multiplier related to this constraint \((\mu \geq 0)\), which we should add to the left hand side of (10). In the solution (12) this would cause an increase in the optimal central bank debt of the banks compared to the previous nonconstrained case. 1)

From the above profit maximization we can derive the optimal central bank debt to be

\[(15) \quad \hat{CBD} = CBD(h_0, h_1, r, r_D, D_{Lh}, FP_{FP}, \bar{L}_h).\]

The optimal stocks of loans to firms and households are then as follows

\[(16) \quad \hat{L}_{f}^\text{dom} = f(h_0, h_1, \hat{CBD}, FP_{FP}, D_{Lf}, \bar{L}_{f}, R'(L_f), r) \quad \text{and}\]

\[(17) \quad \hat{L}_h = D + \hat{CBD} + O + FP - \hat{L}_f^\text{dom}.\]

So we have derived the optimal values for all the different items in the balance sheet (1) of the banks.

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1) In the following estimations we have not so far tried to capture this effect with some kind of a proxy variable.
2. Estimation of the optimal central bank debt of the banks

2.1. The equation for the net forward exchange position of the banks

In order to derive the optimal central bank debt of the banks we need to know the parameters included in (12). First we estimate the equation for the net forward exchange position for the banks.

A simple equation which does not try to take into account the variability in the spot premium of the Finnish markka is the following

\[
\begin{align*}
FP &= 1138.8 + 0.471CA + 157.6fd + 0.335FP_{-1}, \\
R_c^2 &= 0.742, \\
(350.1) & (172) \quad (42.5) \quad (197) \\
D-W &= 1.952
\end{align*}
\]

where $FP =$ net forward position of the banks, mill.marks
$CA =$ surplus in current account, mill.marks
$fd =$ 3 month forward discount of the Finnish markka against the dollar p.a., quarterly average.

The estimation is based on quarterly data from 1980 second quarter to 1984 second quarter. Next we try to take into account the fact that there has been quite marked variations in the expected spot rate of the markka. Especially there have been two occasions when this has happened. In September 1982 there was a heavy run on the markka when expectations were spread about its devaluation which then was put into effect in October 1982. This caused the forward position of the banks to diminish rapidly. In the spring of 1984, on the other hand, the short domestic interest rates in Finland were kept much higher than what the uncovered interest parity would have predicted because the slowdown in inflation did not any more give rise to expected devaluation of the markka in a near future, and a huge inflow of foreign capital occurred.
We have tried to capture these two phases by simply adding a dummy variable to the model which we call a devaluation dummy, sd, and which gets the value 1 in the third quarter of 1982 and the value -1 in the second quarter of 1984. This model gives the following estimation result by the Cochrane-Orcutt method:

$$FP = 1540.8 + 0.444CA + 157.1fd - 804.9sd, \quad R^2 = 0.772,$$

$$\begin{align*}
(18) \quad & (173.7) \quad (.112) \quad (35.7) \quad (308.2) \quad R_1 = 0.283
\end{align*}$$

This model is somewhat better than the above model but the effect of the forward discount is very much the same in these two models. According to these estimates the value of $FP_{fp}$ would be 160 mill. marks. 1)

2.2. The dependency of deposits on loans to households

The next parameter we want to estimate is $D_{lh}$, i.e. how the deposits depend on the volume of bank lending to households. We use the following procedure. Let there be $n$ banks in the banking market. An increase in the lending of bank $i$, $dL_i$, is channelled to increased deposits $dD_i$, $i = 1, \ldots, n$ of all banks in the market, and a part of it leaks out of the banking system, which is denoted by $dV_i$. So we have

1) The models (17) and (18) do not pay any attention to the intervention of the Bank of Finland in the forward market. Empirically, however, this intervention has been active only in this year when the forward position of the Bank of Finland has risen from 2 to 13 billion marks at the end of June. In empirical estimation the intervention variable did not get a meaningful coefficient.
(19) \[ dL_1 = \sum_j dD_j + dV_1. \]

We further denote by coefficient \( a_{ij} \) the reaction of deposits of bank \( j \) with respect to loans of bank \( i \). So

(20) \[ dL_1 = a_{i1} dL_1 + \sum_j a_{ij} dL_j + dV_1 \]

and further by dividing both sides by \( dL_1 \):

(21) \[ 1 = \sum_i a_{ij} + v_1. \]

We can now analyse the change in the deposits of the bank \( i \), \( dD_i \), to be totally the following

(22) \[ dD_i = a_{i1} dL_1 + \sum_{j \neq i} a_{ij} dL_j. \]

So, the total increase in the volume of deposits is

(23) \[ dD = \sum_i dD_i = \sum_i a_{i1} dL_1 + \sum_{ij} a_{ij} dL_j \]

\[ = \sum_i a_{i1} dL_1 + \sum_{j \neq i} a_{ij} dL_j \]

\[ = \sum_i a_{i1} dL_1 + \sum_{j \neq i} (1-v_j) dL_j. \]

We next utilize the following aggregation result

(24) \[ \sum_i x_i y_i = \bar{x} \sum_i y_i + n \text{cov}(x,y) \]
With the aid of this we get the following result

\[(25) \quad dD = (1 + a_{11} - v_1) dL + n \text{cov}(a_{11}, dL) + n \text{cov}(v_1, dL).\]

The covariance terms are here naturally unobservable but this does not very much bother us if we can suppose them to have been constant over time, something we are also bound to do here.

By estimating a linear relationship between real changes in deposits and bank loans to households we get the following result by the Cochrane-Orcutt method

\[(26) \quad dD = 254.357 + 1.0048 dL, \quad R^2 = 0.724, \quad R_1 = 0.410.\]

\[ (241.338) \quad (.154) \]

In order to be able to derive the coefficient \(\frac{dD}{dL} = \bar{a}_{11}\) we have to get an idea of the average leakage coefficient \(\bar{v}_1\). For this purpose we have estimated a regression equation between total capital exports of the firm sector and flow of bank credits to households.\(^1\) This equation is, however, very poor and the coefficient of the bank loan variable does not differ significantly from zero. So on the basis of this evidence and equations (25) and (26) we should just set the parameter \(\frac{dD}{dL_h}\) to be zero.

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1) The direct foreign capital transactions of the households are not allowed by the exchange regulation and because of this we have estimated an indirect relationship.
2.3. Estimates of the optimal central bank debt

Next, the parameters of the linear approximation \( r_m = h_0 + h_1 \text{CBD} \) have been estimated as yearly averages of the various interest rate penalty schemes adopted by the Bank of Finland. Of invaluable help in this task has been the document written by Saarinen (1983) without which the task would have been much more difficult.\(^1\) Figures 1 and 2 present estimates of the parameters \( h_0 \) and \( h_1 \). As can be seen, the schedules have been varied a lot during the period covered in our study. One central feature is the remarkable shift towards a more fixed marginal interest rate on central bank debt, i.e. a marked decrease in the value of the parameter \( h_1 \). As can be seen from the equation (12) if \( h_1 \) is zero profit maximization becomes impossible and we have to formulate the optimization problem of the bank in a new way.

On the basis of these estimates we can derive the estimates for the optimal central bank debt of the banks. These are presented together with the actual debt in figure 3. The ratio between these has on the average been 53 per cent in years 1966-82. So, the actual debt has clearly been above the profit maximizing level of the central bank debt. There are two possible explanations to this.\(^2\) First, the banks' decision making may be influenced by other factors than profit maximization, most notably the market share goal may drive them to

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1) Oksanen (1977) has done a similar effort on an earlier period.

2) There is a third possibility in addition to the two considered below, namely that the parameter \( \frac{dD}{DL_h} \) is (clearly) positive which would raise the optimal central bank debt of the banks.
FIGURE 2: VALUES OF THE PARAMETER H1, 1960-82
take more central bank debt than is optimal on profit maximization goals solely. This would give us the following situation where the objective function of the bank would be

\[(27) \quad U = U(\pi, L).\]

We can illustrate this situation by the following diagram.

\[U(\pi, L) = U(\pi, a + bCBD) = \bar{U}\]

The second explanation is the possibility of a (very) slow adjustment towards the optimum. This could be rationalized by the fact that it is generally quite difficult for the banking sector as a whole to change the central bank debt rapidly in large amounts.\(^1\) By estimating a simple partial adjustment model for the central bank debt we cannot even get an estimate for the speed of adjustment that would differ from zero.

\(^1\) This can be best understood in the context of a closed economy, where the central bank debt is identically the same as the stock of cash in the hands of (the banks and) the private sector. In an open economy we should naturally allow the possibility of rapid adjustment through foreign capital flows but there are also essential limits to this channel in the case of Finland.
This implies that there is a need to reformulate the optimization problem of the bank and allow also for the market share goal. Next, we estimate an equation that is based on the idea that the profit maximizing level of the central bank debt is exceeded each year in a fixed proportion because of the market share goal. Excluding the early 1960's from the estimation because of the fairly unreliable estimates of the profit maximizing level of the central bank debt CBD in those years we get the following result

(28) \[ CBD = 0.595CBD + 0.786CBD_{-1}, \quad R^2_C = 0.758, \quad D-W = 1.160. \]

\[ (.344) \quad (.192) \]

According to this result the speed of adjustment towards optimum is anyway very slow. Because of the market share goal the banks would have a debt which is 40 per cent higher than the profit maximizing level.

3. The loan supply equations of the banks

Let us now turn to the condition for the optimal volume of bank loans to the firms, the equation (13). In order to solve this equation we need to specify the function \( R'(L_f) \). We assume that there is simply a fixed revenue factor denoted by \( R \) related to each loan of the bank to the firms, a factor which depends i.a. on bank guarantees on loans, on revenues from the limits of the cheque account credits and the provisions on loan promises. In addition to this we suppose that there is a diminishing marginal revenue related to the quality of the clients. So we simply specify
(29) \( R'(L_f) = a + b \frac{L_f}{L_{f-1}} \), where \( a > 0 \) and \( b < 0 \).

Replacing the optimal central bank debt from (12) into (13) and taking into account (29) we can derive as the optimal loan supply to the firm sector

\[
\hat{L}_{\text{dom}} = \frac{r_D \text{dom} + a}{-b} \frac{L_f}{L_{f-1}} \tag{30}
\]

In empirical estimation of this equation we allow partial adjustment with respect to two variables. First, as discussed above, the actual central bank debt adjusts slowly towards the optimum,

\[
\text{CBD} = \beta \text{CBD} + \gamma \text{CBD}_{-1} \tag{31}
\]

and, secondly, we allow partial adjustment in the loan supply equation itself, i.e. we have

\[
\frac{\Delta L_{\text{dom}}}{L_{f-1}} = \alpha \left( \frac{\hat{L}_{\text{dom}}}{L_{f-1}} - 1 \right) \tag{32}
\]

Combining these two partial adjustment schemes we get as the the final equation to be estimated

\[
\frac{\Delta L_f}{L_{f-1}} = \varnothing_0 + \varnothing_1 r + \varnothing_2 h_0 + \varnothing_3 h_1 \text{CBD}_{-1} \tag{33}
\]

where
\[ \begin{align*}
\Theta_0 &= -\frac{a}{b} + \alpha \\
\Theta_1 &= \frac{1+\beta(1-D_L)}{b} \\
\Theta_2 &= \frac{(1-\beta)(1-D_L)}{b} \\
\Theta_3 &= \frac{\gamma(1-D_L)}{b}
\end{align*} \]

We should also pay attention to other factors which certainly in practice influence the bank’s loan supply policy. One such factor is variation in the risks related to the projects which the bank is financing. In an earlier paper (Alho (1984)) we showed how a risk neutral bank is willing to extend credit to a firm strictly complementary with the own capital of the firm. We add this factor to the above equation by including in it a variable which presents the profitability of the firms.

We have estimated the following equation

\[ \Delta L_{dom}^{dom} = -39.414 + 4.649r - 0.131h_{CBD-1} + 0.247 \frac{\Pi}{L_{dom}^{dom}} + \frac{\Pi}{L_{f-1}^{dom}} \]

\[ (13.795) (1.133) (0.088) (0.099) \]

\[ R_c^2 = 0.645, \text{ D-W } = 1.983 \]

where \( \Pi \) is the gross profit (net profit and the depreciation allowances) of the nonfinancial firm sector. The variable \( h_0 \) does not get a coefficient of the right sign so we have deleted it from equation (34).

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1) All variables are here multiplied by one hundred except the constant and CBD-1.
So far we have only considered the allocation of the desired stock quantities. In empirical analysis we should also take into account the flow aspect of the allocation process. In the case of banks the flow to be allocated is, first, the increase in the deposits $D$ during the year considered and, second, the change in the residual item $O$, other liabilities, net. Next we add these variables to the right hand side of equations (28) and (34).

In the case of the central bank debt we, however, were not able to get effects from the flow variables which would have been significant and of the right sign. In the case of the supply of loans to firms the result is the following equation

$$(35) \quad \frac{\Delta L_{f}^{\text{dom}}}{L_{f-1}^{\text{dom}}} = -13.882 + 1.641r - 0.187h^{\text{CBD}} + 0.158 \frac{\Pi}{L_{f-1}^{\text{dom}}} + 0.435 \frac{\Delta \text{Dep}}{L_{f-1}^{\text{dom}}} + 0.456 \frac{\Delta O}{L_{f-1}^{\text{dom}}} , \quad R_{c}^{2} = 0.732, \quad \text{D-W} = 2.560$$

where Dep refers to time deposits at banks. Change in the other component of the deposits, the demand deposits, did not get a coefficient significantly different from zero. According to this result almost half of the increase in time deposits and other liabilities is channelled to loans to the firm sector. This equation differs from equation (34) in that the coefficient of the interest rate variable and the profitability variable are now markedly smaller than in (34). The fit of the equation is presented in figure 4.
The last behavioural equation of the banking sector model is the equation for the foreign net debt (NFD) of the banks less the foreign currency loans (L_f^for) of the banks to the firms. The variables explaining this item have already been discussed above. We got the following equation in estimation: \(^1\)

\[
NFD - L_f^{for} = 16.884 \cdot h_0 - 30.786 \cdot \frac{\Delta D}{L_f^{for} \cdot L_f^{for}} + 0.823 \cdot (NFD - L_f^{for})_{-1}
\]

\[
R^2 = 0.781, \text{ D-W = 1.480}
\]

According to this result there is a significant shift, although quite small, from the central bank debt of the banks to use foreign sources of finance when the cost of the central bank debt, i.e. \(h_0\), rises. There is also some substitution between other liabilities, net, and foreign finance. The last term again shows the slow adjustment towards bank optimum which has existed in the period which the estimation covers (1962-80).

We can now derive the last item in the balance sheet of the banks, the loans to the household sector. This has been calculated from the balance sheet by replacing the loans to the firm sector, the central bank debt and the net foreign debt less foreign currency loans by their fit from the corresponding equations. The deposits and other liabilities, net, of the banks have been set to their historical values. The fit of this "equation" transformed in rate of growth form is presented in figure 5. The fit is fairly accurate except the last year, 1980, when the prediction is quite much lower than the realization.

\(^1\) The foreign interest rate variable did not work properly in the equation.
FIGURE 5: FIT FOR THE SUPPLY OF LOANS TO HOUSEHOLDS

% CHANGE

*—* = FIT
4. Some concluding remarks

Above we have derived and estimated a fairly simple model for the banking sector in Finland based on profit maximization and, because of empirical evidence, modified to take into account also the market share goal. The model resembles in some respects that of Oksanen (1977) but also covers the foreign items in the balance sheet of the banks as endogeneous and presents the partition of the loan supply to the firms and households which we need in our sectoral flow-of-funds model.

The equations of the bank loans were taken to be supply determined with no attention paid to the demand for loans. The supply equations seemed, however, to work quite well, even though there seems to years when the demand for loans is likely to determine the solution of the loan market. We intend to study this more closely in a later phase with the aid of the recently developed tests for this purpose, see e.g. Davidson and MacKinnon (1981). We should also consider more closely the links between the allocations of the various items in the balance sheet of the banks because here we considered them more on an item by item basis.

The next phase of our work is to combine the flow-of-funds models for the households, firms and banks and the budget constraint of the government sector to work together as a full model for the financial markets of Finland.
References


