



# Borrowing constraints and housing market liquidity<sup>☆</sup>



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## ABSTRACT

We study how changes in household borrowing constraints influence housing market liquidity. To this end, we develop a housing market model with both matching and credit frictions. In the model, risk-averse households may save or borrow in order to smooth consumption over time and finance owner housing. Prospective sellers and buyers meet randomly and bargain over the price. In the model, housing market liquidity is very sensitive to changes in household credit conditions. In particular, a moderate tightening of household borrowing constraints increases the average time-on-the-market and idiosyncratic price dispersion substantially.

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## 1. Introduction

Housing market liquidity varies a lot over time, at least if liquidity is measured by the average time it takes to sell a house. Díaz and Jerez (2013) and Ngai et al. (2017) demonstrate this for the US and UK housing markets. We provide similar evidence for the Finnish housing market. While Finland has experienced very large house price fluctuations, the average time-on-the-market (TOM) has been even more volatile. Given that the primary residence typically represents a very large share of a home owner's total assets, large changes in housing market liquidity are bound to affect household welfare.

In this paper, we analyze the determinants of housing market liquidity. Specifically, we consider the possibility that the observed changes in the average TOM are driven by changes in household credit conditions. Intuitively, a tighter borrowing constraint may imply that some potential buyers are willing to buy only if the seller accepts a relatively low price. The evidence from the Finnish housing market is also consistent with this conjecture. The average TOM has increased drastically during periods when household credit conditions have tightened, and vice versa.

The relation between credit conditions and housing market liquidity is also relevant for the current debate on 'macroprudential' policies. Many countries have recently implemented loan-to-value restrictions on household borrowing. The aim of such policies is to make households and banks less vulnerable to housing market fluctuations by reducing household lever-

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age. However, by making potential home buyers more likely to be borrowing constrained, they may also reduce housing market liquidity.

Most of the existing housing market models with borrowing constraints do not allow studying how borrowing constraints might influence housing market liquidity. This is because they do not feature matching frictions. In the absence of matching frictions, all households can count on being able to instantly sell their house at the prevailing market price. On the other hand, existing housing market models with matching frictions assume risk-neutral preferences and abstract from households' savings decisions. As a result, they are not directly applicable when considering the role of credit frictions for housing market outcomes.

In order to study the interaction of credit constraints and matching frictions in the presence of precautionary savings, we develop a new modeling framework by combining two strands of literature. We introduce matching frictions following, for example, [Wheaton \(1990\)](#), [Krainer \(2001\)](#), [Piazzesi and Schneider \(2009\)](#), [Head and Lloyd-Ellis \(2012\)](#) and [Díaz and Jerez \(2013\)](#). In these models, households can become dissatisfied with their current housing and in order to move need to meet a potential trading partner and bargain over the price. We embed this setting into a Bewley–Huggett–Aiyagari framework where risk-averse households face uninsurable income shocks and make savings decisions ([Huggett, 1993](#); [Aiyagari, 1994](#)). This allows us to study how borrowing constraints and households' asset positions affect housing market outcomes in the presence of matching frictions. Similar incomplete market models with housing and housing-related borrowing constraints, but without the matching frictions, include [Ríos-Rull and Sánchez-Marcos \(2008\)](#), [Díaz and Luengo-Prado \(2010\)](#), [Iacoviello and Pavan \(2013\)](#), and [Halket and Vasudev \(2014\)](#).<sup>1</sup>

In our model, the households either rent or own their housing. Some households prefer owner housing over rental housing and the tenure preference may change over time. If a renter household becomes dissatisfied with rental housing, it wants to buy a house. Similarly, a home owner may want to move to rental housing in which case it considers selling its house. Prospective sellers and buyers meet randomly and bargain over the price. Households may save or borrow with a financial asset but can only borrow against owner housing.

In the model, the asset distributions of potential buyers and sellers are both key equilibrium objects. For instance, when bargaining over the price, the sellers need to consider the distribution of all potential buyers because it influences the value of not selling today and staying in the market. Similarly, buyers need to consider the distribution of sellers as that influences the value of not buying today. This feature distinguishes our paper from the recent very interesting work of [Hedlund \(2016a,b\)](#). He develops a general equilibrium model that is similar to ours in that it also features both matching and credit frictions in the housing market. In his model, however, trading in the housing market is intermediated by real estate brokers and does not involve bilater bargaining between individual households. The presence of these brokers gives rise to a simpler (block recursive) equilibrium where households need not keep track of the distributions to form expectations regarding housing market liquidity.

The combination of precautionary savings and matching frictions also relates our analysis to recent labor market matching models with a precautionary savings motive such as [Krusell et al. \(2010\)](#). In their model, unemployed workers and firms with vacancies are matched and bargain over the wage.<sup>2</sup> Workers are heterogeneous in their assets, but all recruiting firms are identical. In our housing market model, where current buyers are future sellers and vice versa, both parties of the bargaining process are heterogeneous in their assets.

We calibrate the model using data on Finnish households' portfolios and the Finnish housing market. In order to capture the importance of both borrowing constraints and matching frictions, we match, among other things, the share of highly leveraged recent house buyers and the average time it takes to sell a house.

We first study how the outcome of the bargaining process depends on the traders' asset positions. We find that it is sensitive to asset positions whenever either the potential buyer or seller is close to being borrowing constrained. For instance, poor sellers might be willing to sell at a relatively low price because of liquidity reasons whereas a wealthier seller would prefer to wait for a better match. Conversely, if both traders are relatively wealthy, the credit frictions are not important for them and as a result, the outcome of the bargaining process does not depend on the asset positions of the traders. Combined with asset heterogeneity, which stems endogenously in the model, this feature has two realistic implications. First, not all matches result in trade. Second, at any given point in time, identical houses sell at different prices.<sup>3</sup> In particular, the stationary equilibrium features non-trivial deviations from the average market price in cases where the seller and the buyer are both close to being borrowing constrained. Since owner houses are identical in the model, these results reflect solely the role of borrowing constraints and households' asset positions in determining the outcome of the bargaining process.

<sup>1</sup> In many of these models, houses are illiquid in the sense that buying or selling a house involves transaction costs. However, transaction costs alone do not prevent households from selling or buying instantly at the equilibrium market price.

<sup>2</sup> An earlier example of a model that combines labor market matching frictions and precautionary savings is [Costain and Reiter \(2005\)](#). In their model, bargaining takes place between worker unions and firms.

<sup>3</sup> [Merlo and Ortalo-Magne \(2004\)](#) document that in the UK one third of all matches are unsuccessful. It also seems widely recognized that there is idiosyncratic dispersion in quality adjusted house prices even though it is difficult to measure it accurately. See, for instance, [Leung et al. \(2006\)](#) and the references therein.

These two equilibrium properties arise also in some previous housing market matching models but for very different reasons. Typically, they follow from exogenous preference heterogeneity that affects the surplus from trade.<sup>4</sup> We believe that preference heterogeneity is indeed relevant for housing market outcomes. However, we find it more likely that changes in household credit conditions rather than changes in preferences are key drivers of changes in aggregate housing market outcomes, such as the average TOM. Evidence also suggests that household credit conditions indeed vary quite frequently over time (see, e.g. [Duca et al., 2011](#)). This is one reason why we focus on asset heterogeneity and credit frictions.

We then consider how changes in the borrowing constraint influence the stationary equilibrium in the presence of matching frictions. We find that credit frictions influence liquidity in the housing market by magnifying the effects of matching frictions. A tightening of the borrowing constraint decreases the share of matches that result in trade. As a result, it increases the average TOM much like an increase in matching frictions would do. Moreover, while some matching frictions are needed to generate idiosyncratic price dispersion, a tightening of the borrowing constraint increases price dispersion. Intuitively, these results reflect the fact that a tighter borrowing constraint makes the surplus from trade more sensitive to traders' asset positions. The results are quantitatively relevant in that a moderate tightening of the borrowing constraint increases both the average TOM and idiosyncratic price dispersion substantially.

We proceed as follows. In the next section, we discuss empirical observations from the Finnish housing market. In section 3, we describe the set-up, present the household problem and the matching process, define the recursive stationary equilibrium, and outline our numerical solution algorithm. In section 4, we discuss the calibration. In section 5, we present our results. Section 6 offers a conclusion.

## 2. Time-on-the-market and credit conditions in the Finnish housing market

[Díaz and Jerez \(2013\)](#) and [Ngai et al. \(2017\)](#) demonstrate that housing market liquidity, as measured by the average TOM, has varied a lot over time in the US and the UK. In this section, we present similar evidence from the Finnish housing market. In addition to the volatility of the average TOM, we are interested in its correlation with household credit conditions. Our data cover a period when there were drastic changes in household credit conditions. Until mid-1980s, loan volumes were controlled which resulted in very tight household credit rationing. Bank lending to households was liberalized in 1986.<sup>5</sup> This implied a rapid relaxation of household borrowing constraints and induced a rapid growth of household credit. The financial market liberalization is the main explanation typically put forward for the housing market boom of the late 1980s.<sup>6</sup> The subsequent bust in turn coincided with a severe depression and a banking crisis in the early 1990s, which certainly tightened household credit conditions again.

We consider quarterly price index from Statistics Finland for resales of apartment buildings for the whole country. For the TOM, we employ individual transaction data based on transactions where major real estate agencies acted as intermediary.<sup>7</sup> We focus on time period 1984–2012 and consider only dwellings in apartment buildings. With these restrictions, the transaction data include some 530,000 transactions almost 90% of which are resales. We determine for each transaction the sale time as the difference between the listing date and the date of sale. We then group the transactions according to the quarter of the sale date and calculate, for each quarter, the average TOM. [Fig. 1](#) displays the resulting demeaned log average TOM and the linearly detrended house price index.

While Finland has experienced very large house price fluctuations during this time period, the average TOM has been even more volatile. The standard deviation of the (demeaned log) average TOM is 20.4% while that of the (linearly detrended) house price index is 17.5%. [Díaz and Jerez \(2013\)](#) find similar results for the US economy. They report a standard deviation of 16.7% for the average TOM and 4.1% for the Case–Schiller house price index.<sup>8</sup>

The Finnish data also suggest that changes in the average TOM are closely related to changes in household credit conditions. As [Fig. 1](#) shows, following the credit market liberalization in the late 1980s, the average TOM was well below the average during a period of few years. During the banking crisis in the early 1990s, in contrast, the average TOM was very high. It is also likely that the 2008 financial crisis led also Finnish banks to tighten household borrowing constraints. As can be seen from the figure, the average TOM again increased at that time.<sup>9</sup> The figure also reveals that the average TOM and the price level have been strongly and positively correlated.

<sup>4</sup> The heterogeneity may be match-specific, as for instance, in [Williams \(1995\)](#) and [Díaz and Jerez \(2013\)](#). Alternatively, individuals may be inherently different as in [Carrillo \(2012\)](#), where agents differ in their intrinsic motivation to trade.

<sup>5</sup> For details on the timing of the different measures, see [Vihriälä \(2005\)](#).

<sup>6</sup> See [Koskela et al. \(1992\)](#) and [Berg \(1994\)](#).

<sup>7</sup> These data are voluntarily collected by a consortium of Finnish real estate brokers and the dataset is refined and maintained by the VTT Technical Research Centre of Finland. As not all real estate agencies participate, the dataset represents a sample (albeit rather large) of the total volume of transactions. For a more detailed description of the data, see e.g. [Eerola and Lyytikäinen \(2015\)](#).

<sup>8</sup> Their TOM measure relates to newly built houses. They compute the standard deviations from quarterly data HP detrended with  $\lambda = 1600$ . Using the same detrending gives us a standard deviation of 13.7% for the average TOM and a standard deviation of 7.5% for the price index.

<sup>9</sup> [Fig. 3](#) in [Ngai et al. \(2017\)](#) shows that a similar pattern was observed in the UK. The average TOM ("time to sell") increased drastically from 2007 to 2008.



**Fig. 1.** Demeaned log average TOM and linearly detrended house price index between 1984/1 and 2012/4. Note: Quarterly average TOM calculated from individual transaction data (dwellings in apartment buildings). For each quarter and for each transaction during that quarter, the sale time is determined as the difference between the listing date and the date of sale. The sale times are used to calculate average TOM for each quarter between 1984/1 and 2012/4. The price index from Statistics Finland is based on resales in apartment buildings.

### 3. Model

#### 3.1. Set-up

Time is discrete and there is a continuum of households of mass one. Households live forever. In each period, households work, consume nondurables, and occupy a house. The economy is small and open to the international capital markets in the sense that the interest rate and the wage rate are exogenously determined.

There are two types of dwellings: owner houses and rental houses. Owner houses are in fixed supply.<sup>10</sup> The mass of owner houses is equal to  $m^o \in (0, 1)$ . Each household either owns or rents one house. The ownership rate is then  $m^o$ , and the share of households living in rental houses is  $1 - m^o$ . In state  $d = r$ , the household is renting. In state  $d = o$ , the household owns the house it lives in.

We assume that some households derive a higher utility flow from owner houses than rental houses, whereas other households derive the same utility flow from the two house types. We capture this tenure preference by assuming that some households incur a utility cost when living in rental housing.<sup>11</sup> As in Wheaton (1990) and the related literature, in each period, a household may be hit by a mismatch shock which affects the utility it derives from its current house. In our setting, the mismatch shock changes the tenure preference of the household.<sup>12</sup>

The story we have in mind is that rental houses are smaller than owner houses reflecting the fact that the rental market for larger apartments and single-family houses tends to be very thin. Households without children are happy with a relatively small and inexpensive rental flat. Later, they may have children, which leads them to prefer larger, more expensive houses. They then consider becoming home owners because large rental houses are hard to find.

For simplicity, we assume that there are no trading frictions in the rental market: a household can always find a rental house at a fixed exogenous rental rate. The market for owner housing, in contrast, is characterized by matching frictions. In order to sell or buy a house, a household must first meet a potential trading partner.

The tenure preference is denoted by  $z = 1, 2$ . In state  $z = 1$ , the household derives the same utility from owner housing and rental housing. In state  $z = 2$ , the household incurs a utility cost if it lives in rental housing. Given current state  $z$ , the probability of next period state  $z'$  is  $P(z', z)$ .

Each household will therefore be in one of the following four situations: i) Those with  $d = r$  and  $z = 1$  are renting without incurring a utility cost related to rental housing. We refer to them as 'happy renters'. ii) Those with  $d = r$  and  $z = 2$  are renting, but suffer a utility cost relative to owning. We refer to them as 'unhappy renters'. iii) Those with  $d = o$  and

<sup>10</sup> Assuming that the stock of owner houses is fixed simplifies the analysis substantially. Head and Lloyd-Ellis (2012) assume that owner housing can be converted to rental housing and vice versa but assume that the costs from doing so are high enough for no conversion to take place in their benchmark analysis.

<sup>11</sup> We could also assume that all households derive higher utility from owner housing than from rental housing, as in Head and Lloyd-Ellis (2012). What matters for our analysis is that some households value the (larger) owner houses more than others.

<sup>12</sup> In this respect, our model is similar to that in Head and Lloyd-Ellis (2012) where a household that wishes to downsize needs to rent.

$z = 1$  are owning but would not suffer a utility cost if they were renters. We refer to them as ‘unhappy owners’. iv) Those with  $d = o$  and  $z = 2$  are owning and would suffer a utility cost if they were renters. We refer to them as ‘happy owners’.

Unhappy renters would like to buy so as to avoid the utility cost of rental housing. Unhappy owners in turn would like to sell because of the cost of housing. This is because the equilibrium user cost of owner housing (capital and maintenance costs) will be higher than the rent since all households value owner housing at least as much as rental housing (given non-housing consumption) and some households strictly prefer owner housing to rental housing. Therefore, the unhappy owners pay the higher cost of owner housing but would receive the same utility flow from rental housing. Happy renters and happy owners on the other hand have little reason to trade.

In the baseline model, we assume that all unhappy renters and unhappy owners participate in the housing market, that is, search for a house to buy or put their house for sale, while happy renters and happy owners do not enter the market. However, we also consider an extension with endogenous market entry where unhappy renters first decide whether to participate in the market (and incur a market participation cost) or not.

Each period, households receive wage income  $\varepsilon w_z$ , where  $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_{n_\varepsilon}\}$  is an iid income shock and  $w_z$  the average wage rate of those with tenure preference  $z$ . Probability of income shock  $\varepsilon_i$  is  $\varphi_i$ . By allowing the wage rate to depend on the tenure preference, we can capture the fact that owners have on average higher income than renters. This helps in replicating the empirical fact that home buyers tend to have little savings. In addition, by assuming that the persistent income shock is perfectly correlated with the tenure preference, we save a state variable. This is particularly important in our setting, because the number of possible buyer and seller matches grows exponentially with the number of individual state variables. Note also that since the wage depends on the tenure preference, instead of the actual tenure, households do not attain a higher wage by buying a house.

In each period, timing is the following. First, potential buyers (unhappy renters) and potential sellers (unhappy owners) are randomly matched and can meet at most one trading partner. Upon having met, they bargain over the price. If there exists a price that makes trade mutually beneficial, trade takes place and the price is determined by Nash bargaining. The price will depend on the seller's and buyer's continuation values, which in turn depend on their asset positions. Next, unsuccessful matches break down and the transactions of successful matches take place. Buyers move to owner housing and sellers move to rental housing.<sup>13</sup> The renters pay the rent and the owners the maintenance cost. Finally, all households decide on non-housing consumption and financial saving or borrowing.

The periodic utility of the household is given by  $u(c, z, d)$  where  $c$  denotes non-housing consumption. Each household's financial asset position,  $a$ , evolves as

$$a' = Rs + \varepsilon' w',$$

where  $s$  denotes financial saving or borrowing, interest rate is  $R - 1$ , and primes indicate next period values. In what follows, we refer to  $a$  as ‘financial wealth’.

Borrowing is limited by a borrowing constraint.<sup>14</sup> We require that  $s \geq \underline{s}^d$  for  $d = r, o$ . We will later assume that  $\underline{s}^r = 0$  and  $\underline{s}^o < 0$ , that is, only owners can borrow. These assumptions mean that home owners can costlessly refinance their mortgage.<sup>15</sup>

If a household does not buy or sell a house, its current non-housing consumption is

$$c = a - s - g, \tag{1}$$

where  $g$  is the direct cost of housing services. This cost equals the rent,  $g = \nu$ , for the renters and the maintenance cost,  $g = \kappa$ , for the owners.

If a renter buys a house with price  $p$ , its current non-housing consumption is

$$c = a - s - \kappa - (1 + \tau)p, \tag{2}$$

where  $\tau \geq 0$  denotes a transaction tax.

Finally, if an owner sells a house, its current non-housing consumption is

$$c = a - s - \nu + p. \tag{3}$$

All households must be able to afford strictly positive non-housing consumption. Denoting this minimum consumption level by  $c_{\min} > 0$  (close to zero), it directly follows from (2) that the maximum price a buyer can pay is

$$p = \frac{a - \underline{s}^o - \kappa - c_{\min}}{1 + \tau}. \tag{4}$$

<sup>13</sup> We abstract from own-to-own trades. According to Wheaton and Lee (2009), in the US housing market, there are generally more purchases of homes by renters or new households than there are by existing owners. Having both own-to-own trades and rent-to-own trades would substantially complicate the model. See Anenberg and Bayer (2013) for a model where the choice of whether to first buy or sell is endogenous.

<sup>14</sup> In our model, the borrowing constraint is entirely exogenous. See e.g. Hedlund (2016b) and Mitman (2016) for a model with endogenous mortgage supply.

<sup>15</sup> Most mortgage contracts in Finland specify an amortization schedule and mortgage refinancing is relatively rare. However, mortgage contracts typically provide an option to pay interest only for several years and also reverse mortgages are available.

Similarly, from (3) it follows that the minimum price a seller needs to receive is

$$p = \underline{z}^r + v - a + c_{\min}. \tag{5}$$

### 3.2. Household problem and bargaining

We now define the household optimization problem recursively. Let  $V^d(a, z)$  denote the value function before current period matches and  $v^d(a, z)$  the value function conditional on not trading. The latter is determined as:

$$v^d(a, z) = \max_{s \geq \underline{z}^d} \left\{ u(c, z, d) + \beta \sum_{j=1}^2 P(j, z) \sum_{i=1}^{n_\varepsilon} \varphi_i V^d(Rs + \varepsilon_i W_j, j) \right\}$$

subject to (1)

where  $\beta < 1$  is the subjective discount factor,  $z = 1, 2$  and  $d = r, o$ . We use  $s^d(a, z)$  to denote the associated savings policy. Because happy renters and happy owners do not participate in the housing market

$$V^r(a, 1) = v^r(a, 1) \tag{7}$$

$$V^o(a, 2) = v^o(a, 2). \tag{8}$$

For unhappy renters and unhappy owners, we have to take into account the value of being matched with a potential trading partner. Let  $W^b(a^b, a^s)$  denote the value of a potential buyer (unhappy renter) with financial wealth  $a^b$  matched with a potential seller (unhappy owner) with financial wealth  $a^s$ . Similarly,  $W^s(a^b, a^s)$  denotes the value of the potential seller.

Finally, let us denote the population of households with financial wealth  $a$ , occupancy state  $d$ , and tenure preference state  $z$  by  $\mu^d(a, z)$ . The mass of potential sellers is  $m^s = \int \mu^o(a, 1) da$  and the mass of potential buyers is  $m^b = \int \mu^r(a, 2) da$ .

We can now define  $V^r(a, 2)$  as

$$V^r(a, 2) = \phi^s \int W^b(a, \tilde{a}) \frac{\mu^o(\tilde{a}, 1)}{m^s} d\tilde{a} + (1 - \phi^s) v^r(a, 2), \tag{9}$$

where  $\phi^s$  denotes the probability of meeting a potential seller. The first term is the expected value of a match weighted by the probability of being matched with a potential seller. The second term is the value of a renter not trading weighted by the probability of not being matched.

Similarly, we define  $V^o(a, 1)$  as

$$V^o(a, 1) = \phi^b \int W^s(\tilde{a}, a) \frac{\mu^r(\tilde{a}, 2)}{m^b} d\tilde{a} + (1 - \phi^b) v^o(a, 1), \tag{10}$$

where  $\phi^b$  denotes the probability of meeting a potential buyer.

In order to determine  $W^b(\cdot)$  and  $W^s(\cdot)$ , we need to find out, for all possible matches, whether the match leads to trade and if so, at what price. This can be computationally very costly when we discretize the state space with a reasonably fine grid for financial wealth. However, the computational costs can be reduced drastically by exploiting the fact that the surplus from trade can be expressed using the no-trade value functions defined above. The benefit of this approach is that one does not have to solve the household optimization problems when determining  $W^b(\cdot)$  and  $W^s(\cdot)$ .

Consider a potential buyer with financial wealth  $a$  who has met a potential seller and contemplates buying the house with price  $p$ . If it buys the house, it becomes an owner and faces the same problem as a happy owner with financial wealth equal to  $a - (1 + \tau)p$ . If it does not buy, its value is the same as that of an unmatched renter. Hence, its surplus from trade can be written as

$$S^b(a, p) = v^o(a - (1 + \tau)p, 2) - v^r(a, 2). \tag{11}$$

In the same way, if a potential seller with financial wealth  $a$  sells its house with price  $p$ , it becomes a happy renter with financial wealth equal to  $a + p$ . If it does not sell, its value is the same as that of an unmatched owner. Therefore, its surplus from trade is

$$S^s(a, p) = v^r(a + p, 1) - v^o(a, 1). \tag{12}$$

If there exists a price  $p$  such that  $S^b(a^b, p) \geq 0$  and  $S^s(a^s, p) \geq 0$ , the match leads to trade and the equilibrium price is

$$\arg \max_p \left\{ S^s(a^s, p)^\omega S^b(a^b, p)^{(1-\omega)} \right\} \tag{13}$$

where  $\omega$  denotes the bargaining power of the seller.

We denote the equilibrium price by  $p(a^b, a^s)$ . Appendix A.1 shows that if trade takes place, the Nash bargaining price is uniquely determined.

Finally, let  $Tr(a^b, a^s)$  be an indicator function that equals one if trade takes place and zero otherwise. Hence,  $Tr$  is defined as

$$Tr(a^b, a^s) = \begin{cases} 1 & \text{if } \exists p \text{ s.t. } S^b(a^b, p) \geq 0 \text{ and } S^s(a^s, p) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

We can now define  $W^b(a^b, a^s)$  and  $W^s(a^b, a^s)$  as

$$W^b(a^b, a^s) = \begin{cases} v^o(a^b - (1 + \tau)p(a^b, a^s), 2) & \text{if } Tr(a^b, a^s) = 1 \\ v^r(a^b, 2) & \text{if } Tr(a^b, a^s) = 0 \end{cases} \quad (15)$$

$$W^s(a^b, a^s) = \begin{cases} v^r(a^s + p(a^b, a^s), 1) & \text{if } Tr(a^b, a^s) = 1 \\ v^o(a^s, 1) & \text{if } Tr(a^b, a^s) = 0 \end{cases} \quad (16)$$

### 3.3. Matching

Trading frictions can be modeled in various ways. Existing housing market matching models make different assumptions concerning the frictions related to meeting a potential trading partner and price determination after potential traders have met. However, most papers assume either directed search with posted prices or random matching with some type of bargaining.<sup>16</sup>

It seems clear that both competitive forces and bargaining are present in the housing market. Actual transaction prices often differ from listing prices and the listing price does not seem to constitute a ceiling nor a floor. However, it also seems that TOM depends on the listing price.<sup>17</sup>

As we wish to focus on the bargaining after the match has been formed, we assume that trading frictions can be represented by a matching function, which specifies the number of trading opportunities in a given period.<sup>18</sup> More specifically, we follow [Piazzesi and Schneider \(2009\)](#) in assuming that matching is governed by Cobb–Douglas matching technology and that the potential buyers and sellers are matched at rate

$$M(m^s, m^b) = \chi (m^s)^\alpha (m^b)^{1-\alpha}$$

where the masses of potential buyers and sellers are  $m^b$  and  $m^s$ ,  $\chi \in (0, 1]$  is a matching parameter and  $0 < \alpha < 1$ . The probability of being matched with a potential seller,  $\phi^s$ , and the probability of being matched with a potential buyer,  $\phi^b$ , are then given by

$$\phi^s = \chi \left( \frac{m^s}{m^b} \right)^\alpha \quad \text{and} \quad \phi^b = \chi \left( \frac{m^s}{m^b} \right)^{\alpha-1} \quad (17)$$

In addition,  $M(m^s, m^b) \leq m^b$  and  $M(m^s, m^b) \leq m^s$  have to hold.

In our benchmark calibration, the ownership rate is 50% and there are equal numbers of owners and renters. We also assume that the tenure preference transitions are symmetric, so that  $P(1, 2) = P(2, 1)$ . Therefore, the stationary tenure preference distribution is such that half of the households strictly prefers owner housing to rental housing. Under these assumptions, the mass of potential buyers equals the mass of potential sellers in a stationary equilibrium.<sup>19</sup> From (17) it then follows that  $\phi^b = \phi^s = \chi$ .

In the sensitivity analysis, we allow the ownership rate to differ from 50%, but retain the assumption of symmetric preference shocks. This means that the ownership rate does not match the (stationary) tenure preference distribution and there is shortage or oversupply of owner housing relative to what the households prefer. Therefore, the mass of potential buyers differs from the mass of potential sellers and, as a result, the traders on the short side of the market are more likely to meet a potential trading partner.

<sup>16</sup> Exceptions include [Albrecht et al. \(2016\)](#) who consider directed search with limited commitment to the asking price, [Díaz and Jerez \(2013\)](#) who consider a combination of directed search and random matching with posted prices and [Carrillo \(2012\)](#) who considers directed search with bargaining.

<sup>17</sup> See e.g. [Díaz and Jerez \(2013\)](#) and [Albrecht et al. \(2016\)](#) for more discussion on this issue.

<sup>18</sup> Other studies assuming random matching include [Albrecht et al. \(2007\)](#), [Caplin and Leahy \(2011\)](#), [Krainer \(2001\)](#), [Piazzesi and Schneider \(2009\)](#), and [Wheaton \(1990\)](#).

<sup>19</sup> This is true even if the mass of potential buyers initially differs from the mass of potential sellers.

### 3.4. Stationary equilibrium

We consider a stationary equilibrium where the distribution of households over their asset, tenure preference, and occupancy states is constant over time. The interest, wage and rental rates as well as the ownership rate are exogenously given.

**Definition 1.** The stationary equilibrium consists of value functions  $\{V^d(a, z), v^d(a, z), W^b(a^b, a^s), W^s(a^b, a^s)\}$ , household savings function  $s^d(a, z)$ , prices  $p(a^b, a^s)$ , indicator function  $Tr(a^b, a^s)$ , matching probabilities  $\phi^b$  and  $\phi^s$ , and distribution  $\mu^d(a, z)$  (containing the information of  $m^b$  and  $m^s$ ) which satisfy

**Matching:**

Given  $\mu^d(a, z)$ ,  $\phi^b$  and  $\phi^s$  are determined by (17).

**Household optimization and bargaining:**

- a) Given  $V^d(a, z)$ ,  $s^d(a, z)$  solves (6) with  $v^d(a, z)$  as the resulting value function.
- b) Given  $v^d(a, z)$ , surpluses  $S^b(a, p)$  and  $S^s(a, p)$  are determined by (11) and (12). Given the surpluses,  $Tr(a^b, a^s)$  is determined from (14). For pairs  $\{a^b, a^s\}$  such that  $Tr(a^b, a^s) = 1$ ,  $p(a^b, a^s)$  is determined by (13). Given  $Tr(a^b, a^s)$  and  $p(a^b, a^s)$ ,  $W^b(a^b, a^s)$  and  $W^s(a^b, a^s)$  are determined by (15) and (16).
- c) Given  $v^d(a, z)$ ,  $V^r(a, 1)$  and  $V^o(a, 2)$  are determined by (7) and (8). Given  $v^d(a, z)$ ,  $W^b(a^b, a^s)$ ,  $W^s(a^b, a^s)$ , and  $\mu^d(a, z)$ ,  $V^r(a, 2)$  and  $V^o(a, 1)$  are determined by (9) and (10).

**Consistency:**

$\mu^d(a, z)$  is the time invariant distribution that follows from the household savings policy, the outcome of the Nash bargaining, the probabilities  $P(z', z)$  and  $\varphi_i$  for all  $i = 1, 2, \dots, n_\varepsilon$ , and the exogenously determined ownership rate.

### 3.5. Solving the model

When making decisions, households need to take into account the distribution of potential trading partners. This is the key computational challenge in solving the model. For instance, a potential seller wants to consider the distribution of asset holdings of all potential buyers. This is because its surplus from a match depends on the asset position of the potential buyer. Therefore, the value of not selling today depends on the whole distribution of traders.

When solving the model, we thus need to find a distribution which is consistent with households' information about the distribution and the resulting household behavior. In practice, we iterate over the distribution. We first make an initial guess for the distribution. We then use that distribution to determine the matching probabilities and the value functions of the households that are not in the market (equations (9) and (10)). After that we solve recursively for all the value and policy functions. Finally, we simulate the model to find the associated stationary distribution. The resulting distribution provides us with a new guess.

In practice, this iteration converges nicely. We have also experimented with very different initial guesses for the distribution and the equilibrium was always independent of it. We discuss computational issues in more detail in Appendix A.2.

## 4. Calibration

When calibrating the model, we try to capture the importance of both borrowing constraints and matching frictions. We match the average TOM in the data. In addition, we impose a reasonable loan-to-value restriction and try to match the share of households that are close to it.

The model does not allow us to replicate the entire wealth distribution in the data. Replicating the wealth distribution would at least require introducing very large and highly persistent income shocks. The reason we focus on the left tail of the wealth distribution is twofold. First, changes in the borrowing constraint do not directly influence the trade-offs faced by wealthier households. Second, as we show below, the outcome of the bargaining process is sensitive to small changes in household asset positions only when the trading partners are relatively poor. In other words, housing market outcomes are mainly driven by the left tail, rather than the entire asset distribution.

We base our calibration on the Wealth Survey that was conducted by Statistics Finland in 2004. The survey contains register data about the asset holdings and incomes of a representative sample of Finnish households. The register data are supplemented by survey information. In the survey, households were asked, among other things, to give an estimate of the current market value of their house and to report the length of stay in their current dwelling.

We consider only households where the age of the household head is between 25 and 60, in order to focus on the working age population. In addition, we only use data from Helsinki Metropolitan Area (HMA) because we wish to focus on



a single housing and labor market.<sup>20</sup> We also exclude households living in service housing. These restrictions imply that we have a data set of about 600 households.

We set the model period to 3 months. We set the interest rate parameter at  $R = 1.01$  implying an annual interest rate of about 4%. We also consider a shorter model period in the sensitivity analysis.

We construct two variables for the analysis: ‘house value’ and ‘financial wealth’. House value is the value of primary residence as estimated by the household. Financial wealth is the sum of all financial assets, quarterly after-tax return to financial assets, quarterly after-tax non-capital income, less mortgage debt and quarterly interest payments on it.

We consider the following utility function

$$u(c, z, d) = \frac{c^{1-\sigma}}{1-\sigma} - I(z, d) f,$$

where

$$I(z, d) = \begin{cases} 1 & \text{if } z = 2 \text{ and } d = r \\ 0 & \text{otherwise} \end{cases}.$$

Parameter  $\sigma > 0$  measures risk-aversion ( $\sigma = 1$  corresponds to log-utility) and  $f$  is the utility cost of living in rental housing when having a preference for owner housing. We set  $\sigma = 2$ , which is a relatively conventional value.

In our data set, the ownership rate is 45%.<sup>21</sup> However, in the benchmark calibration, we focus on the case where the ownership rate equals 50% and the tenure preference shocks are symmetric. As discussed in section 3.3, we can directly control the matching probabilities  $\phi^b$  and  $\phi^s$  with the matching parameter  $\chi$ . We set the weight in the Cobb–Douglas technology determining the number of matches at  $\alpha = 0.5$ . We assume symmetric bargaining power, i.e.  $\omega = 0.5$ .

In our data set, the average length of stay in current house is roughly 7 years. Based on this and the assumption of symmetric tenure preference shocks, we set the tenure preference transition probabilities at

$$P(z', z) = \begin{bmatrix} 0.967 & 0.033 \\ 0.033 & 0.967 \end{bmatrix}.$$

The income shock can take two values, that is  $\varepsilon \in \{\varepsilon_1, \varepsilon_2\}$ . We interpret the first shock as unemployment. The unemployed households receive an unemployment compensation. In Finland, most workers are covered by an earnings-related unemployment insurance scheme. During the first two years of unemployment, the replacement rate is typically about 50%. We therefore set

$$\varepsilon_1 = 0.5 \text{ and } \varepsilon_2 = 1.$$

We choose the probabilities of the income shocks so that the unemployment rate is 8%, which implies

$$\varphi_1 = 0.08 \text{ and } \varphi_2 = 0.92.$$

We future reference, we also define

$$\bar{\varepsilon} = \varphi_1 \varepsilon_1 + \varphi_2 \varepsilon_2 = 0.96.$$

In the data, the mean after-tax non-capital income of renters is 58% of the mean income of owners. Let  $y_r$  and  $y_o$  denote the average non-capital income of renters and owners in the model, respectively. We will normalize the wage rates so that the average non-capital income equals 1 and  $\frac{y_r}{y_o} = 0.58$ . This results in  $y_r = 0.73$  and  $y_o = 1.27$ . In what follows, we refer to the after-tax non-capital income as simply ‘income’. In the data, the median rent-to-income ratio is 0.27. Hence, we set the rent at  $v = 0.20$  ( $0.2/0.73 \approx 0.27$ ).

We set the transaction tax at  $\tau = 0.016$ .<sup>22</sup> We assume that households can only borrow against owner housing. Therefore, the borrowing constraint for renters is  $\underline{g}^r = 0$ .

We are then left with seven parameters: owners’ borrowing limit,  $\underline{g}^o$ , maintenance cost,  $\kappa$ , matching parameter,  $\chi$ , discount factor,  $\beta$ , utility cost,  $f$ , and wage rates  $w_1$  and  $w_2$ . We set these parameters so that the model matches certain empirical targets.

First, we want the model to feature a realistic average house price-to-average income ratio. In the data, the median ratio of house value to quarterly income among owners is 17.2. Given that the average income of owners is 1.27 in the model, the average house price in the model should be 21.8.

Second, we want to capture the role of borrowing constraints as realistically as possible. As we show below, the housing market outcomes in the model are sensitive to changes in traders’ asset positions only when the traders are close to being

<sup>20</sup> The Helsinki Metropolitan area consists of four municipalities (Helsinki, Espoo, Vantaa and Kauniainen) and roughly one fifth of the overall population of Finland.

<sup>21</sup> Based on our calculations using the Wealth Survey, the ownership rate is 67% in the whole of Finland. In HMA without the age restriction we use in calibrating the model, the ownership rate is 51%.

<sup>22</sup> This was the tax rate for dwellings in apartment buildings in Finland up until 2013 when it was set to 2% of the transaction price.

borrowing constrained. We therefore wish to target the share of buyers that are likely to be borrowing constrained. Those buyers have little savings relative to the value of the house they are contemplating buying. After buying a house, they are highly leveraged.

The asset positions of renters in our data offer little guidance in this respect because the data do not reveal which renters are considering buying a house or the value of the house they would like to buy. Fortunately, we know the length of stay in current house for each household. Therefore, instead of looking at the asset positions of all renters, we focus on the asset positions of those who recently bought a house. These recent buyers have had very little time to repay their mortgage and should therefore have a very similar financial position as right after having bought the house. We define recent buyers as home owners that have lived in their current house for up to two years. We then match the share of recent buyers with a financial wealth-to-house-value ratio less than  $-0.8$ . In the data, that share is approximately 25%.

Third, we want the model to feature a realistic average TOM, so that households in the model economy face a realistic trade-off between trading now and waiting for a better match. According to Eerola and Lyttikäinen (2015), the average TOM for repeat sales in Finnish cities between 2003 and 2011 has been 48 days. This corresponds to 0.53 model periods.<sup>23</sup>

Fourth, we choose the borrowing limit for owners,  $\underline{s}^o$ , so that it reflects a realistic down payment requirement for mortgages. In 2010, according to a survey conducted by the Financial Supervisory Authority about half of the housing loans for first time buyers exceeded 90% of the house value, but loans exceeding 100% of the value of the house were rare and involved special arrangements (Financial Supervisory Authority, 2011). We therefore assume that owners can borrow up to 95% of the average house price.<sup>24</sup> Fifth, in the data, the average annual maintenance cost of owner-occupiers is 1.6% of the average house value. We use this information to pin down the maintenance cost  $\kappa$ .

Finally, we match the average income of renters relative to owners and normalize the average income in the model to one. As explained above, this means that we should have  $y_r = 0.73$  and  $y_o = 1.27$ . Since the wage rate is determined by the tenure preference, the average incomes of renters and owners in the model are given by

$$y_r = \frac{[(1 - m^o - m^b) w_1 + m^b w_2] \bar{\epsilon}}{1 - m^o} \text{ and } y_o = \frac{[(m^o - m^s) w_2 + m^s w_1] \bar{\epsilon}}{m^o},$$

where  $m^o$  is the ownership rate and  $w_1$  is the wage rate of those with  $z = 1$  and  $w_2$  is the wage rate of those with  $z = 2$ .

To summarize, we choose  $\beta$ ,  $f$ ,  $\chi$ ,  $\underline{s}^o$ ,  $\kappa$ ,  $w_1$  and  $w_2$  so as to match the following targets: i) Average house price equal to 21.8 (median house value-to-income ratio in the data), ii) share of recent buyers with financial wealth-to-house value ratio less than  $-0.8$  equal to 0.25, iii) average TOM equal to 0.53 model periods, iv) owners can borrow up to 95% of the average house price, v) average annual maintenance cost is 1.6% of the average house price, vi) average income of renters equal to 0.73, and vii) average income of owners equal to 1.27.

Given the targeted average house price, targets iv) and v) directly imply  $\underline{s}^o = -0.95 * 21.8 = -20.71$  and  $\kappa = (0.016 * 21.8)/4 = 0.0872$ . The other targets depend on all remaining parameters. With parameter values  $\beta = 0.986$ ,  $f = 0.33$ , and  $\chi = 0.76$ ,  $w_1 = 0.73$  and  $w_2 = 1.353$  the model closely matches all the remaining targets. (The resulting stationary distribution features  $m^b = m^s = 0.0248$ .)

Table 1 shows selected percentiles of the distribution of the financial wealth-to-house value ratio in the data and the model. For the table, we calculated, for each owner-occupier in the data and in the model, the financial wealth-to-house value ratio. This ratio is close to the usual loan-to-value ratio. In the data, 'recent buyers' refers to owners who have lived in their current house up to two years. In the model, it refers to households who have become owners within the last 8 model periods.

The model roughly replicates the left tail of the two empirical distributions. In particular, the model is consistent with two important features of the data. First, a non-trivial share of all owner households are very highly leveraged.<sup>25</sup> Second, recent buyers are typically even more leveraged than other home owners suggesting that borrowing constraints are particularly relevant for the potential buyers. As we discuss below, the reason why recent buyers are more leveraged than other owners relates to the correlation between tenure preference and income.

The model also roughly matches the empirical distribution of TOM. In the model, 65% of the houses are sold within the same period they are put on the market, 23% in the second period and 8% in the third period, while 4% of the houses (that

<sup>23</sup> In the model, we compute the average TOM by following households that have just become unhappy owners. In the model, transactions occur in the beginning of a period. Therefore, an unhappy renter that buys a house in a given period avoids paying the utility cost associated with rental housing for the whole period. Accordingly, if a household sells its house in the same period it enters the housing market, the TOM is recorded as zero. If it sells in the next period, TOM is recorded as 1 period, or 90 days, and so on. TOM is not recorded if a household is hit by a new tenure preference shock before selling the house and withdraws the house from the market.

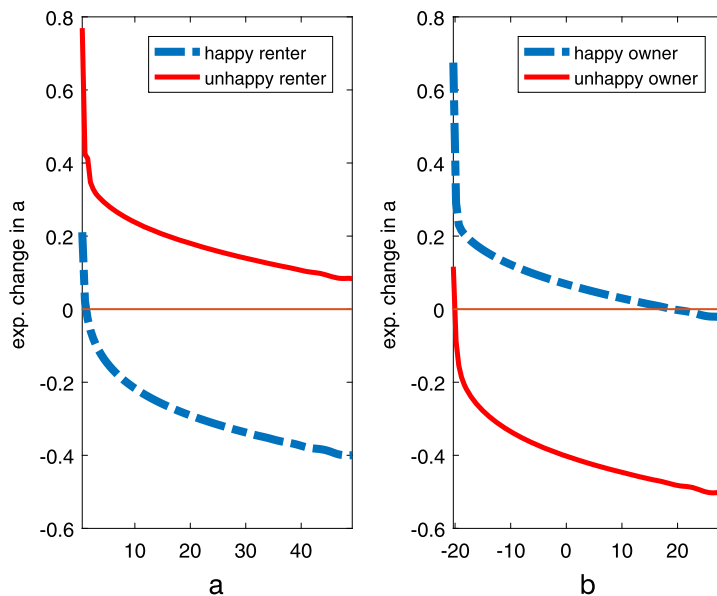
<sup>24</sup> The fact that the borrowing constraint does not depend on the trading price simplifies the computation. In our view, this formulation of the borrowing constraint is also reasonable as it is the average price, rather than trading price, that reflects the expected collateral value of the house for the bank.

<sup>25</sup> Because of higher house prices, home owners living in the HMA are typically more leveraged than households living in other parts of Finland. Moreover, Finnish households in general have little private pension savings because the mandatory pension system is quite generous. However, in an international comparison, households in our data are unlikely to be exceptionally highly leveraged. For instance, according to the OECD, the aggregate gross household debt-to-income ratio in 2004 was 82% in Finland, 104% in Germany, and 123% in the US. Since then, this ratio has increased in most OECD countries, including Finland. Cowell et al. (2017) compare household net worth distribution in 5 countries. Fig. 1 and Table 1 in their paper suggest that the left tail of this distribution is quite similar in Finland, US and UK.

**Table 1**  
Distribution of financial wealth-to-house value ratio in the data and in the model.

Percentile	Owners, data		Owners, model	
	All	Recent buyers	All	Recent buyers
5th	−0.85	−0.93	−0.84	−0.87
10th	−0.69	−0.87	−0.81	−0.84
25th	−0.40	−0.78	−0.70	−0.81
50th	−0.01	−0.50	−0.52	−0.73
95th	0.69	0.29	0.02	−0.21

Note: The table shows the financial wealth-to-house value ratio for all households in the model and our data. In the data, ‘recent buyers’ refers to owners who have lived in their current house up to two years. In the model, it refers to households who have become owners within the last 8 model periods.



**Fig. 2.** Renters’ (left) and owners’ (right) savings policy. Note: The figure plots the difference between the expected next period financial wealth and current financial wealth as a function of current financial wealth (a).

are eventually sold) are sold in the fourth period or later. In the data, the comparable figures are about 79% (sold within the first 3 months), 16%, 3%, and 2% (not sold within the first 9 months).

## 5. Results

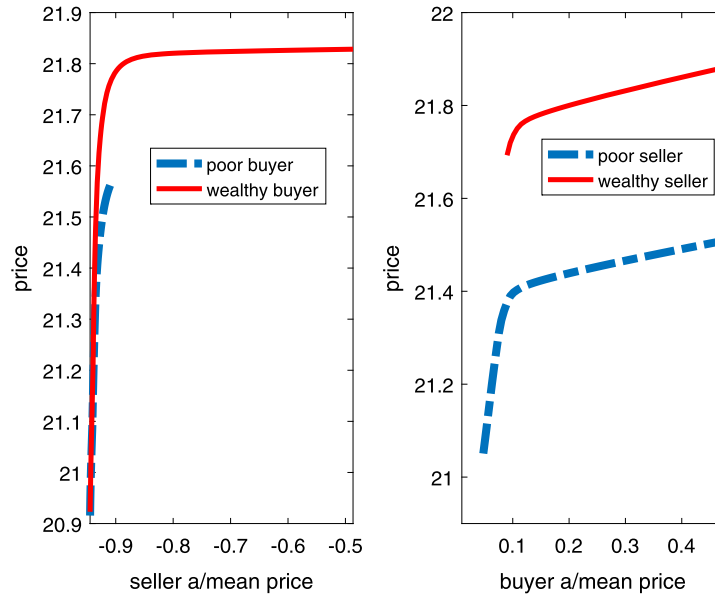
### 5.1. Household policies and price determination

Let us first briefly discuss the household savings policy, illustrated in Fig. 2. In the figure, current financial wealth is on the horizontal axis.<sup>26</sup> The vertical axis shows the difference between the expected next period financial wealth (that is,  $Rs^d(a, z) + E\varepsilon w$ ) and current financial wealth. If this difference is positive (negative), the household is expected to become wealthier (poorer). The left hand panel shows the savings policy for renters and the right hand panel that for owners separately for those on the market and those not on the market.

For very low asset holdings, any increase in the current financial wealth is spent on non-housing consumption in the current period. Therefore, an increase in current financial wealth is associated with a one-to-one reduction in the difference between expected future financial wealth and current financial wealth. This happens as long as households are borrowing constrained. Borrowing constrained owners borrow up to  $-\underline{s}^o$  and borrowing constrained renters choose to save nothing ( $\underline{s}^r = 0$ ). For those close enough to the borrowing constraint, the expected next period financial wealth is nevertheless higher than current financial wealth because financial wealth includes the wage income and the unemployment benefit.<sup>27</sup>

<sup>26</sup> The lowest financial wealth levels in the figure correspond to the maximum and minimum prices defined in equations (4) and (5). See Appendix A.2 for details.

<sup>27</sup> For instance, the expected next period financial wealth for a happy renter with no savings is approximately 0.7. This is much higher than the lowest possible realization of the financial wealth level (i.e., no savings and the worst income shock).



**Fig. 3.** Price as a function of seller's (left) and buyer's (right) asset position. Note: "a/mean price" denotes the financial wealth-to-average house price ratio.

For a given level of financial wealth, unhappy renters and happy owners save more than happy renters or unhappy owners. This reflects the fact that unhappy renters and happy owners expect their income to decrease sometime in the future (as  $w_2 > w_1$ ). Unhappy renters also expect to spend more on housing in the future since the user cost of owner housing is higher than that of rental housing.

Fig. 3 illustrates how the Nash bargaining outcome depends on potential buyer's and seller's asset positions. The horizontal axis shows the ratio of buyer's or seller's financial wealth to the average house price. For sellers, this ratio is close to the usual loan-to-value ratio. The left hand panel plots the Nash bargaining price as a function of seller's asset position and the right hand panel as a function of buyer's asset position. Both panels show two different cases: one where the potential trading partner is relatively poor in terms of financial wealth and another where the potential trading partner is relatively wealthy. For some combinations of the seller's and buyer's financial wealth, a match does not result in trade.

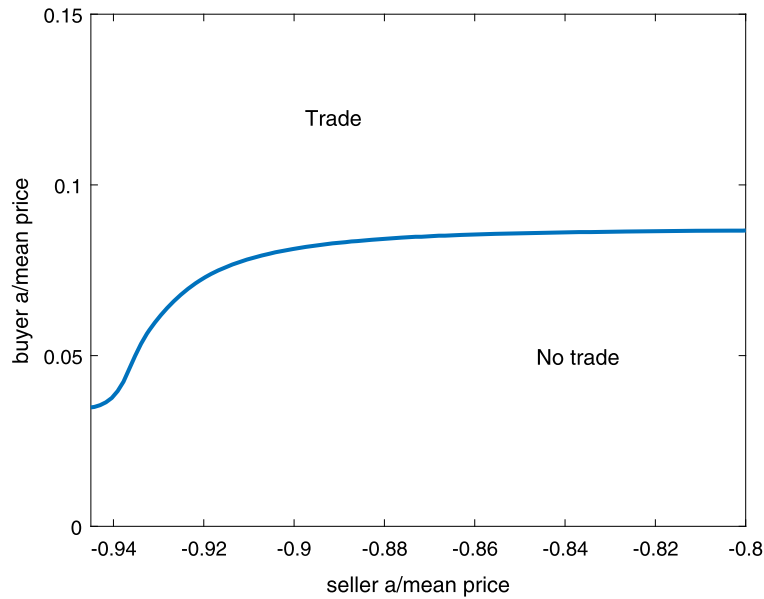
Consider first the left hand side of the figure and the case of a wealthy buyer. When the seller is close to the borrowing constraint, the Nash bargaining price is relatively low. Selling the house allows a highly leveraged owner to smooth consumption over time. For a given price, these households benefit more from trade than wealthier sellers. Therefore, the Nash product is maximized at a relatively low price. However, the need to sell for liquidity reasons diminishes quickly as we increase seller's financial wealth. This means that the outside option of the seller increases rapidly. Hence, for there to be trade, the price must also increase rapidly. Further away from the borrowing constraint, the price curve becomes flat. As wealthier sellers do not need to sell in order to smooth non-housing consumption, they all face the same trade-off between selling today and waiting for a better match. Hence, the price does not depend on seller's asset position.

When looking at the case of a relatively poor buyer, one observes that trade only occurs if the seller is also poor. Because of the borrowing constraint, a poor buyer is only able to trade if the price is relatively low. When faced with such a buyer, a wealthier seller, who does not have to sell for liquidity reasons, prefers to wait for a better match.

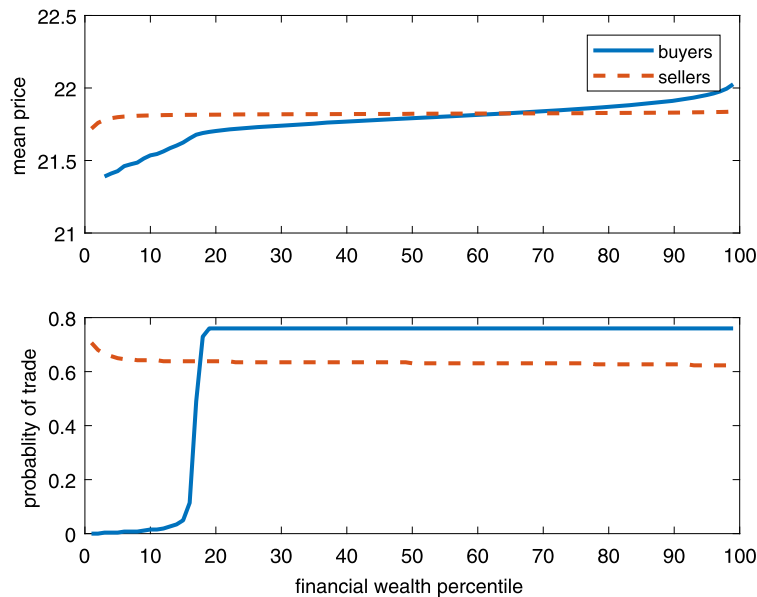
Similarly, the right hand side panel shows that when the seller is wealthy, trade occurs only when the buyer is not very poor. In contrast, a poor seller is willing to trade also with a relatively poor buyer. In that case, the price first increases rapidly with buyer's financial wealth. At higher wealth levels, when the borrowing constraint no longer limits consumption smoothing, the price curve becomes much less steep. However, unlike in the left hand panel, it does not become completely flat. This is because a wealthier buyer is always willing to pay more than a poorer one in order to avoid the utility cost associated with rental housing today rather than later.

Fig. 4 shows the combinations of buyer's and seller's asset positions (relative to the average house price) that result in trade. The figure only covers matches where traders are relatively poor. Even very poor buyers end up buying if matched with a very poor seller trading for liquidity reasons. As the seller's financial wealth increases, the potential buyer needs to be wealthier for the match to result in trade.

In the absence of matching frictions, all trades (within the same period or in the stationary equilibrium) would take place at the same market price and all households could count on being able to buy or sell at the prevailing market price. One important implication of matching frictions is that poor sellers that would like to sell quickly for liquidity reasons may not be able to sell or may have to sell at a relatively low price. In this sense, matching frictions make borrowing constraints more relevant for household welfare.



**Fig. 4.** Matches that result in trade. Note: “a/mean price” denotes the financial wealth-to-average house price ratio. Given seller’s asset position, the figure shows the asset position of the poorest buyer with which the seller would trade.



**Fig. 5.** Average price (top) and probability of trade (bottom) by buyers’ and sellers’ financial wealth percentile.

Fig. 5 shows the average price and the probability of trade for buyers and sellers in different asset positions. The x-axis shows the financial wealth percentile in the stationary distribution associated with the benchmark calibration. (The percentiles correspond to very different asset positions for buyers and sellers.) Both the average price and the probability of trade are much more sensitive to the buyer’s asset position than that of the seller’s. The very poorest potential buyers are very unlikely to trade, as they only trade when matched with a very poor seller. Over a certain range (between percentiles 10 and 20), the probability of trade increases steeply with the buyer’s financial wealth as the borrowing constraint becomes less relevant. For buyers above percentile 20, the probability of trade is the same as the probability of being matched with a potential seller. In other words, these buyers end up buying whenever they have the opportunity to do so. The sellers, in contrast, may always face a poor buyer with whom they would not trade.

In the stationary equilibrium of the benchmark calibration, the very lowest realized house price is about 4.5% below the average price. Hence, the model can account for non-trivial deviations from the average price. The lowest house prices correspond to matches where *both* trading partners are close to the borrowing constraint. However, given the stationary

**Table 2**  
Percentage changes in selected statistics relative to the benchmark calibration.

	$\bar{a}^o$	$\bar{a}^r$	$\bar{p}$	<i>tom</i>	<i>cv(p)</i>	<i>tr</i>
Matching probability						
$\chi = 1$	1	1	0	−64	−19	−1
$\chi = 0.6$	0	−1	0	66	21	2
Borrowing constraint						
$\underline{s}^o = -18.53$	20	6	−6	84	28	−24
$\underline{s}^o = -16.53$	40	13	−11	185	60	−41

Note: In the table,  $\bar{a}^o$  and  $\bar{a}^r$  denote the average asset holdings of owners and renters, respectively, and  $\bar{p}$  denotes the average price. The measures of market inefficiency are average time-on-the-market (*tom*), coefficient of variation of house prices (*cv(p)*), and share of matches that lead to trade (*tr*). All figures in the table refer to percentage changes relative to the benchmark calibration where  $\chi = 0.76$  and  $\underline{s}^o = -20.71$ . Owners' average financial wealth is negative. The table shows the change relative to the absolute value of average financial wealth.

asset distribution, such matches are very rare. Most transactions take place at a price close to the average. As a result, the coefficient of variation for the transaction price is just 0.38%. By comparison, the coefficient of variation of total wealth is about 104%, where total wealth is equal to financial wealth in the case of renters and financial wealth (*a*) plus the average house price in the case of owners. On the other hand, because of the borrowing constraint and wealth dispersion, about 16% of the matches do not result in trade in the benchmark calibration.

## 5.2. Frictions and housing market outcomes

In this section, we analyze how the housing market outcomes in the stationary equilibrium depend on the matching friction and the borrowing constraint. We vary one friction at a time, keeping all other parameters fixed, and recompute the stationary equilibrium. We first vary the matching parameter  $\chi$ . We consider values  $\chi = 1.0$  and  $\chi = 0.4$  (benchmark:  $\chi = 0.76$ ). We then tighten the borrowing constraint for owners by setting  $\underline{s}^o = -18.53$  and  $\underline{s}^o = -16.35$  (benchmark:  $-20.71$ ). These numbers correspond to a borrowing limit that is 85% and 75% of the average house price in the benchmark calibration, respectively.

We report changes in the average house price and the average asset positions for owners and renters. We also consider the average TOM, the share of matches that result in trade, and the coefficient of variation of house prices.

The average TOM is a commonly used measure of housing market conditions. In the model, it also indirectly measures the welfare cost related to the misallocation of housing units because it reflects the share of households that pay the utility cost associated with rental housing. This share changes almost one-to-one with the average TOM. There are two reasons why unhappy renters do not trade immediately. First, since  $\chi < 1$ , some potential buyers are not matched with a potential seller. Second, because of credit frictions and asset heterogeneity, some matches do not result in trade.

Also price dispersion is closely linked to the frictions we are interested in. Since all owner houses are identical in the model, absent matching frictions, they would sell at the same price. If better matches are instantaneously available, all differences in prices stemming from the characteristics of the current trading partners must vanish. In other words, some matching frictions are needed to create any price dispersion. However, in this set-up, matching frictions alone would not be able to generate price dispersion, if there was no wealth heterogeneity. Moreover, if all potential future trading partners were alike and identical to the current trading partner, waiting for a new match would never be profitable. Therefore, in the model, both unsuccessful matches and house price dispersion stem from matching frictions together with wealth heterogeneity.

Table 2 displays the results as percentage changes relative to the benchmark calibration. The first three columns report the relative changes in the average financial wealth of owners ( $\bar{a}^o$ ) and renters ( $\bar{a}^r$ ) and the average house price ( $\bar{p}$ ). The last three columns report the relative changes in the average TOM (*tom*), the coefficient of variation of house prices (*cv(p)*), and the share of matches that result in trade (*tr*).

Consider first the matching frictions. Changes in the matching parameter  $\chi$  have virtually no effect on households' average financial asset holdings or the average house price. Also the coefficient of variation of total wealth (not shown in the table) is almost unaffected: Increasing  $\chi$  from 0.6 to 1 decreases the coefficient of variation of the total wealth distribution from 104.1% to 102.7%.

Naturally, changes in the matching efficiency do affect the average TOM. For instance, reducing matching frictions by increasing  $\chi$  from 0.76 to 1.0 decreases the average TOM by 64%. This directly follows from buyers and sellers being more likely to be matched. There is a small countervailing effect, however, because now the share of matches that result in trade is smaller. This is because a reduction in matching frictions makes deferring trade less costly. Interestingly, this effect also shows up in reduced price dispersion. The coefficient of variation decreases by 19%. Because deferring trade is less costly, the bargaining outcome becomes less sensitive to the asset positions of the traders.

Increasing matching frictions has the opposite effects: The average TOM goes up as trading opportunities are less frequent. However, at the same time, a larger share of matches leads to trade because waiting for a better match is more costly. This also implies higher price dispersion.

Consider then the borrowing constraint. Not surprisingly, tightening the borrowing constraint increases households' average financial wealth. This is because more savings are required in order to be able to buy a house. The effects are relatively large, reflecting the fact that many home owners are close to the borrowing constraint in the benchmark calibration. For instance, lowering the maximum mortgage by about 10% (from 20.71 to 18.53) increases owners' average financial wealth by 20%. In absolute terms, this increase corresponds to about 1.5 times the owners' average periodic income. It also lowers the average house price by 6%. The distribution of total wealth becomes only slightly more equal with the coefficient of variation of total wealth decreasing from 103.5% to 101.8%.

A tighter borrowing constraint also leads to a substantially longer average TOM. Lowering the maximum mortgage by about 10% increases the average TOM by 84%, while the more drastic reduction increases it by 185%. The increase in the average TOM reflects the fact that a smaller share of matches result in trade. Tightening the borrowing constraint reduces the surplus from trade for matches where the buyer is relatively poor. As a result, even though households are on average wealthier, there are more matches that do not lead to trade.<sup>28</sup>

Tightening the borrowing constraint also increases the price dispersion. Lowering the maximum mortgage by about 10% or 20% increases the coefficient of variation of house prices by 28% and 60%, respectively. In the latter case, we also find that the very lowest realized house prices are 11% below the average price (compared to about 4.5% in the benchmark case). As the borrowing constraint is tightened, it becomes relevant to a larger share of potential buyers, even though renters' average financial wealth also increases. As a result, buyers end up trading only if they are matched with a seller that needs to sell for liquidity reasons. In those cases, the realized price is relatively low.

More generally, these results illustrate how credit frictions interact with the effects of matching frictions. Tightening the borrowing constraint increases the average TOM, much like increasing matching frictions would do. While borrowing constraints do not alter the frequency of finding a trading opportunity, they influence the surplus from trade and hence the share of successful matches. Moreover, while some matching frictions are needed to create any price dispersion between identical houses, tightening the borrowing constraint increases price dispersion substantially.

The intuition behind both the smaller share of successful matches and the increased price dispersion relates to the fact that the bargaining outcome depends on traders' asset positions mainly through the borrowing constraint. When the borrowing constraint is very lax, all matched sellers and buyers face a similar trade-off between trading now and deferring trade. As a result, there is no reason to wait for a better match and trade takes place at (approximately) the same price in all matches. A tighter borrowing constraint creates heterogeneity in the match surplus because of liquidity concerns. As a result, fewer matches result in trade and the price dispersion (of identical houses) is larger.

These results are broadly in line with the empirical observations discussed in Section 2. In particular, a tighter borrowing constraint lowers house prices and increases the average TOM, with the relative increase in the average TOM being much larger than the relative decrease in prices. Ngai et al. (2017) show that in the UK and US, the average TOM is positively correlated with the number of viewings per transaction. Our results are also consistent with this observation if we interpret a match in the model as a viewing in the data. Following a tighter borrowing constraint, the share of matches leading to trade decreases.

Since these results are based on comparing stationary equilibria, they do not directly inform us about the impact effects of the changes considered here. The impact effects of an unanticipated tightening of the borrowing constraint on the average TOM, the average house price and price dispersion may be even larger than those presented above. This is because it takes some time for potential buyers to increase their savings as a response to the tighter borrowing constraint. However, the impact effects would also depend on how the new borrowing constraint is applied to those home owners who are already highly indebted when the tightening takes place.

### 5.3. Sensitivity analysis

In this section, we consider alternative parametrizations. Our main interest is in studying whether the above conclusions regarding the effects of the borrowing constraint are sensitive to changes in some key parameters. In what follows, we vary the ownership rate, consider a lower risk aversion (logarithmic utility) and a lower discount factor than in the benchmark calibration. We also analyze the effects of asymmetric bargaining power. When varying these parameters, we recalibrate the utility cost  $f$  so as to get the same average house price as in the benchmark calibration. We do not recalibrate other parameters. We first report how these changes affect housing market outcomes relative to the benchmark case. We then consider the effects of tightening the borrowing constraint in these alternative parametrizations.

In the benchmark calibration, the supply and demand for owner and rental houses is balanced in the sense that the (fixed) ownership rate equals the share of households that prefer owner to rental housing. Here we consider ownership rates of 45% and 55%. We retain the symmetric tenure preference shocks of the benchmark calibration. As a result, the supply of owner houses may now be smaller or larger than the mass of households that strictly prefer owner housing to rental housing and the matching probabilities are no longer directly given by  $\chi$ . In order to target the same average house price, the utility cost  $f$  must be higher than in the benchmark case when the ownership rate is higher, and vice versa.

<sup>28</sup> Despite the increase in the average TOM, the transaction volume hardly changes. This is because almost all unhappy owners eventually sell their house. As explained in Ngai et al. (2017), sustained changes in the transaction volume must be related to changes in the moving rate rather than the average TOM.

**Table 3**  
Percentage changes in selected statistics relative to the benchmark calibration.

	$\bar{a}^o$	$\bar{a}^r$	<i>tom</i>	<i>cv(p)</i>	<i>tr</i>
Ownership rate 45%, $f = 0.260$	1	0	-59	-12	0
Ownership rate 55%, $f = 0.440$	0	0	65	19	3
Lower risk-aversion ( $\sigma = 1$ ), $f = 0.310$	-42	-52	-1	-20	0
Lower discount factor ( $\beta = 0.983$ ), $f = 0.370$	-30	-40	15	9	-5
Higher seller barg. power ( $\omega = 0.6$ ), $f = 0.275$	-1	-1	6	12	-2
Lower seller barg. power ( $\omega = 0.4$ ), $f = 0.405$	1	2	-11	-7	4

Note: In the table,  $\bar{a}^o$  and  $\bar{a}^r$  denote the average asset holdings of owners and renters, respectively. The measures of market inefficiency are average time-on-the-market (*tom*), coefficient of variation of house prices (*cv(p)*), and share of matches that lead to trade (*tr*). All figures in the table show the percentage changes relative to the benchmark calibration where the ownership rate is 50%,  $\sigma = 2$ ,  $\omega = 0.5$ , and  $f = 0.33$ . Owners' average financial wealth is negative. The table shows the change relative to the absolute value of average financial wealth. In all cases, the utility cost parameter  $f$  has been recalibrated so that the average house price is the same as in the benchmark calibration.

Table 3 displays changes in selected statistics relative to the benchmark calibration. The first row of the table relates to the case where the ownership rate is 45%. In this case, there are more buyers than sellers in the market and the probability of meeting a buyer is higher than in the benchmark case. This results in shorter average TOM. On the other hand, since sellers are more likely to meet a buyer, they are less likely to accept a low price when matched with a relatively poor buyer. As a result, there is less price dispersion.<sup>29</sup>

When the ownership rate is 55%, the probability of meeting a buyer is lower than in the benchmark case. This increases the average TOM. However, a lower probability of meeting a buyer also means that deferring trade is more costly for the seller. Therefore, the seller is more willing to sell at a relatively low price when she is matched with a borrowing constrained buyer. This increases price dispersion.

The logarithmic utility function results in much lower average financial wealth than the benchmark calibration. Because households are less risk-averse, being borrowing constrained is not as serious a concern. The average TOM is virtually unchanged relative to the benchmark calibration. This reflects two opposite effects. Since households are less keen on smoothing consumption over time, the borrowing constraint is less relevant for the bargaining outcome. On the other hand, as households are on average poorer, it is more likely that the potential buyer is close to being borrowing constrained.

Assuming a lower discount factor ( $\beta = 0.983$ ) decreases the average financial wealth, thereby increasing the importance of the borrowing constraint. In this case, however, households are still equally concerned about consumption smoothing as in the benchmark calibration. As a result, the average TOM and price dispersion increase relative to the benchmark calibration.

Assuming that the seller has a higher bargaining power than the buyer ( $\omega = 0.6$ ) increases the average TOM and price dispersion somewhat relative to the benchmark calibration. These effects are reversed when the bargaining power of the buyer is increased ( $\omega = 0.4$ ).

Table 4 shows the effect of a tightening of the borrowing constraint in the different cases discussed above. In each case, we lower the maximum mortgage by about 10%. For comparison, the table also shows the effects of the same experiment in the benchmark calibration. Qualitatively, the effects are the same across all the cases. Renters' and owners' average financial wealth, average TOM and price dispersion increase while the average house price and the share of matches that result in trade decrease. Quantitatively, when looking at the average TOM, the effects are large except when the ownership rate is 55%. In this case, however, the average TOM is very high to start with. The absolute increase is in fact quite similar in all cases. The effect on price dispersion does seem to depend on the ownership rate, but is very similar across all other cases. The effect on the share of matches that result in trade is remarkably similar in all cases.

Finally, we analyze an alternative calibration where the model period is interpreted to be six weeks instead of three months. We set the interest rate at  $R = 1.005$  in order to have approximately the same annual interest rate as in the baseline calibration. We still normalize the average income to one. This means that the average house price should now be twice as high as in the baseline calibration, i.e. 43.6. Also the target for the average TOM doubles to 1.06. The other targets remain the same. By setting  $\beta = 0.994$ ,  $f = 0.330$ ,  $\chi = 0.690$ ,  $w_1 = 0.750$ , and  $w_2 = 1.333$ , while keeping other parameters the same as in the benchmark calibration, the model economy closely matches all the calibration targets.<sup>30</sup> The results in Table 4 related to the six week calibration show that the effects of tightening the borrowing constraint are very similar to those in the benchmark calibration.

<sup>29</sup> It should be noted, however, that even though the average TOM for the sellers is shorter than in the benchmark case, potential buyers may have to stay longer in the market than in the benchmark case, because they are less likely to meet a seller.

<sup>30</sup> It is worth noting that the value of the matching parameter  $\chi$  is almost the same as in the benchmark calibration. In other words, the per period probability of meeting a potential trading partner is only slightly lower than in the benchmark calibration even though the model period is now much shorter. The explanation relates to the fact that a shorter model period lowers the cost of waiting for a better match. This makes matches less likely to result in trade, which works to increase the average TOM. The share of matches that lead to trade is now 64% versus 84% in the benchmark calibration.



**Table 4**

Percentage changes in selected statistics following a tightening of the borrowing constraint in different cases.

	$\bar{a}^o$	$\bar{a}^r$	$\bar{p}$	$tom$	$cv(p)$	$tr$
Benchmark calibration	20	6	-6	84	28	-24
Ownership rate 45%	20	5	-6	183	22	-22
Ownership rate 55%	20	7	-5	58	32	-27
Lower risk-aversion ( $\sigma = 1$ )	15	12	-7	81	25	-23
Lower discount ( $\beta = 0.983$ )	16	7	-7	78	33	-24
Higher seller barg. power ( $\omega = 0.6$ )	19	5	-6	91	25	-26
Lower seller barg. power ( $\omega = 0.4$ )	21	7	-6	80	32	-21
Model period six weeks	41	5	-6	63	15	-25

Note: In the table,  $\bar{a}^o$  and  $\bar{a}^r$  denote the average asset holdings of owners and renters, respectively, and  $\bar{p}$  denotes the average price. The measures of market inefficiency are average time-on-the-market ( $tom$ ), coefficient of variation of house prices ( $cv(p)$ ), and share of matches that lead to trade ( $tr$ ). All numbers in the table show the effect (percentage changes) of lowering the maximum mortgage by about 10% relative to the case where  $\bar{g}^o = -20.71$ . Owners' average financial wealth is negative. The table shows the change relative to the absolute value of average financial wealth. The utility cost parameter  $f$  has been recalibrated so that the average house price is the same as in the benchmark calibration. In the case of a shorter model period, also other parameters have been recalibrated.

#### 5.4. Endogenous market participation

So far we have assumed that all unhappy renters and unhappy owners participate in the housing market, i.e. search for a potential seller or buyer, whereas happy renters and happy owners stay out of the market. In this section we relax this assumption and consider endogenous market participation.

In many other housing market matching models, it is possible to show that there exists a participation cost such that all “mismatched” households enter the market whereas all “matched” households stay out. This is not the case in our model because of wealth heterogeneity and the fact that the value of entering the market increases with wealth. For instance, a very wealthy happy renter might want to buy a house in order to insure against a possible change in her tenure preference.

In principle, we could endogenize all market participation decisions by assuming that all households need to incur some fixed cost in order to have a chance of meeting a potential seller or buyer. However, this would complicate the analysis substantially. The reason is that the number of possible buyer and seller matches grows exponentially with new types of buyers or sellers.

For this reason, we only endogenize the market participation of unhappy renters. Given our focus on borrowing constraints, endogenous participation might be especially relevant for this group. Very poor unhappy renters, who are unlikely to be able to finance a house, may not be willing to incur even a relatively small cost to participate in the market. By the same token, changes in the borrowing constraint may influence their participation decision.

Let  $\eta > 0$  denote a periodic utility cost of participating in the market for unhappy renters. In the beginning of the model period, unhappy renters decide whether to incur it or not. Formally, the value function  $V^r(a, 2)$  becomes

$$V^r(a, 2) = \max \left( -\eta + \phi^s \int W^b(a, \tilde{a}) \frac{\mu^o(\tilde{a}, 1)}{m^s} d\tilde{a} + (1 - \phi^s) v^r(a, 2), v^r(a, 2) \right).$$

Otherwise, the household problem remains the same.

It is hard to pin down a realistic value for  $\eta$ . We therefore consider several values, namely 0.005, 0.02 and 0.04. These market participation costs are equivalent to a fall in average non-housing consumption of about 0.4%, 1.7% and 3.3%, respectively.<sup>31</sup> When introducing these costs, we again recalibrate the utility cost  $f$  so as to get approximately the same average house price as in the benchmark calibration while keeping all other parameters fixed. The resulting utility costs are somewhat lower than in the benchmark case.

Introducing the participation cost changes the stationary equilibrium, compared to the benchmark calibration. The poorest unhappy renters, who are most likely to be borrowing constrained, choose not to enter the market. This affects the average TOM in two ways. First, a match is now more likely to result in trade, which works to decrease the average TOM. In fact, with  $\eta$  equal to 0.02 or higher, virtually all matches result in trade. On the other hand, there are also less buyers relative to sellers. This lowers the probability of meeting a buyer, which works to increase the average TOM. The overall effect is a shorter average TOM. With  $\eta$  equal to 0.02, for instance, the average TOM is 0.29 periods compared to 0.53 in the benchmark calibration. Also price dispersion decreases somewhat as the buyers are less heterogeneous in their assets.

Table 5 shows the effects of tightening the borrowing constraint with different participation costs. We again lower the maximum mortgage by about 10%. Again, price dispersion increases but less than in the benchmark calibration. As in the benchmark calibration, the share of matches that result in trade decreases for a small participation cost. However, with  $\eta$

<sup>31</sup> The average non-housing consumption of unhappy renters is approximately 0.83.

**Table 5**  
Percentage changes in selected statistics following a tightening of the borrowing constraint with different participation costs.

	$\bar{a}^o$	$\bar{a}^r$	$\bar{p}$	<i>tom</i>	<i>cv(p)</i>	<i>tr</i>
Benchmark calibration	20	6	−6	84	28	−24
$\eta = 0.005$	16	6	−4	50	27	−5
$\eta = 0.02$	17	6	−4	45	20	0
$\eta = 0.04$	19	5	−6	71	14	0

Note: In the table,  $\bar{a}^o$  and  $\bar{a}^r$  denote the average asset holdings of owners and renters, respectively, and  $\bar{p}$  denotes the average price. The measures of market inefficiency are average time-on-the-market (*tom*), coefficient of variation of house prices (*cv(p)*), and share of matches that lead to trade (*tr*). All numbers in the table show the effect (percentage changes) of lowering the maximum mortgage by about 10% relative to the case where  $\bar{s}^o = -20.71$ . Owners' average financial wealth is negative. The table shows the change relative to the absolute value of average financial wealth. The utility cost parameter *f* has been recalibrated so that the average house price is the same as in the benchmark calibration.

equal to 0.02 or 0.04, virtually all matches result in trade even with the tighter borrowing constraint, and hence there is no effect on the share of successful matches.

Interestingly, the effect of a tighter borrowing constraint on the average TOM is very similar to that in the benchmark calibration. However, the mechanism is somewhat different. With a participation cost, a tightening of the borrowing constraint does not decrease the share of successful matches as much as in the benchmark calibration. However, it now reduces the number of potential buyers in the market, thereby lowering the probability that a seller meets a potential trading partner.

## 6. Discussion

We have analyzed how changes in credit frictions influence liquidity in the housing market. To this end, we developed a model of the owner-occupied housing market that features both credit and matching frictions. Our main result is that housing market liquidity is very sensitive to changes in the household borrowing constraints. Even a moderate tightening increases both the average TOM and idiosyncratic price dispersion substantially. Intuitively, this is because borrowing constraints make the outcome of the bargaining process between a house buyer and a seller more sensitive to traders' asset positions. A tightening of the borrowing constraint may also induce some potential buyers not enter the market at all. Given matching frictions, this would also make it more difficult to sell a house. The results show that the observed large fluctuations in the average TOM can be explained by moderate changes in household credit conditions.

Our results also suggest some caution when imposing stricter loan-to-value restrictions for housing lending. The aim of such 'macroprudential' policies is to make households and banks less vulnerable to housing market fluctuations by reducing household leverage. In the model economy, reducing the maximum loan-to-value ratio indeed limits household leverage. However, by making potential home buyers more likely to be borrowing constrained, it effectively makes the housing market less liquid. This may complicate the situation for those home owners that need to sell quickly for liquidity reasons.

We see our model as a first step towards a more general housing market theory that takes into account both matching and credit frictions. The basic structure could be extended and modified in several ways. It might be particularly interesting to consider different assumptions regarding how house buyers and sellers meet and how prices are determined.

## Appendix A

### A.1. Nash bargaining price

In this appendix, we show that the Nash bargaining price is unique. The value function of the household in occupancy state *d* and tenure preference state *z* with financial wealth *a* is

$$v^d(a, z) = \max_{s \geq \bar{s}^d} \left\{ u(c, z, d) + \beta \sum_{j=1}^2 P(j, z) \sum_{i=1}^{n_\varepsilon} \varphi_i V^d(Rs + \varepsilon_i w_j, j) \right\}.$$

Denote the savings policy that solves the household problem by  $s^d(a, z)$ . The savings policy is determined by the first-order condition

$$-\frac{\partial u(c, z, d)}{\partial c} + \beta R \sum_{j=1}^2 \sum_{i=1}^{n_\varepsilon} P(j, z) \varphi_i \frac{\partial V^d(Rs + \varepsilon_i w_j, j)}{\partial a} + \mu^d = 0, \tag{A.1}$$

where  $\mu^d$  is the Kuhn–Tucker multiplier on the borrowing constraint  $s \geq \bar{s}^d$ .

Taking into account that households optimally choose savings after trade, we can write the surplus from trade for the potential buyer and the potential seller as

$$S^b(a, p) = u(c^b, 2, o) + \beta \sum_{j=1}^2 \sum_{i=1}^{n_\varepsilon} P(j, 2) \varphi_i V^o(Rs^o(a - (1 + \tau)p, 2) + \varepsilon_i w_j, j) - v^r(a, 2)$$

$$S^s(a, p) = u(c^s, 1, r) + \beta \sum_{j=1}^2 \sum_{i=1}^{n_\varepsilon} P(j, 1) \varphi_i V^r(Rs^r(a + p, 1) + \varepsilon_i w_j, j) - v^o(a, 1)$$

where

$$c^b = a - \kappa - (1 + \tau)p - s^o(a - (1 + \tau)p, 2)$$

and

$$c^s = a - v + p - s^r(a + p, 1)$$

Using the above expressions for the surpluses and taking into account condition (A.1), the effect of price changes on the surplus of the buyer and the seller can be written as

$$\frac{\partial S^b(a, p)}{\partial p} = -(1 + \tau) \frac{\partial u(c^b, 2, o)}{\partial c}$$

and

$$\frac{\partial S^s(a, p)}{\partial p} = \frac{\partial u(c^s, 1, r)}{\partial c} \tag{A.2}$$

In addition,

$$\frac{\partial^2 S^b(a, p)}{\partial p \partial p} = (1 + \tau)^2 \frac{\partial^2 u(c^b, 2, o)}{\partial c \partial c} \left( 1 - \frac{\partial s^o(a^b - (1 + \tau)p, 2)}{\partial a} \right)$$

and

$$\frac{\partial^2 S^s(a, p)}{\partial p \partial p} = \frac{\partial^2 u(c^s, 1, r)}{\partial c \partial c} \left( 1 - \frac{\partial s^r(a^s + p, 1)}{\partial a} \right) \tag{A.3}$$

The surplus from trade only depends on the price through its effect on current non-housing consumption. (The standard Kuhn–Tucker optimality conditions imply that  $\mu^d > 0$  if the borrowing constraint is binding. In this case, however,  $\frac{\partial s^d(a, z)}{\partial a} = 0$ . If, in turn, the borrowing constraint is not binding,  $\mu^d = 0$ .)

Assume that  $S^b(a^b, p) > 0$  and  $S^s(a^s, p) > 0$ . Then the Nash bargaining price  $p$  maximizes

$$S(a^b, a^s, p) = S^s(a^s, p)^\omega S^b(a^b, p)^{(1-\omega)}.$$

The first order condition for the optimal price is given by

$$(1 - \omega) S^b(a^b, p)^{-\omega} \frac{\partial S^b(a^b, p)}{\partial p} S^s(a^s, p)^\omega + \omega S^s(a^s, p)^{\omega-1} \frac{\partial S^s(a^s, p)}{\partial p} S^b(a^b, p)^{(1-\omega)} = 0$$

By using (A.2) this can be written as

$$\omega \frac{\partial u(c^s, 1, r)}{\partial c} S^b(a^b, p) - (1 - \omega)(1 + \tau) \frac{\partial u(c^b, 2, o)}{\partial c} S^s(a^s, p) = 0. \tag{A.4}$$

The second order condition is

$$\begin{aligned} \frac{\partial^2 S(a^b, a^s, p)}{\partial p \partial p} &= -\omega(1 - \omega) S^b(a^b, p)^{-\omega-1} \left( \frac{\partial S^b(a^b, p)}{\partial p} \right)^2 S^s(a^s, p)^\omega \\ &\quad + (1 - \omega) S^b(a^b, p)^{-\omega} \frac{\partial^2 S^b(a^b, p)}{\partial p \partial p} S^s(a^s, p)^\omega \\ &\quad + 2\omega(1 - \omega) S^b(a^b, p)^{-\omega} \frac{\partial S^b(a^b, p)}{\partial p} \frac{\partial S^s(a^s, p)}{\partial p} S^s(a^s, p)^{\omega-1} \end{aligned}$$

$$\begin{aligned}
 & -\omega(1-\omega)S^s(a^s, p)^{-\omega-2} \left( \frac{\partial S^s(a^s, p)}{\partial p} \right)^2 S^b(a^b, p)^{(1-\omega)} \\
 & + \omega S^s(a^s, p)^{\omega-1} \frac{\partial^2 S^s(a^s, p)}{\partial p \partial p} S^b(a^b, p)^{(1-\omega)}.
 \end{aligned}$$

Together with  $1 - \frac{\partial s^r(a^s+p, 1)}{\partial a} > 0$  and  $1 - \frac{\partial s^o(a^b-(1+\tau)p, 2)}{\partial a} > 0$  and (A.3), this implies that  $\frac{\partial^2 S(a^b, a^s, p)}{\partial p \partial p} < 0$ . Therefore, whenever trade is mutually beneficial, (A.4) determines a unique equilibrium price.

### A.2. Computational issues

We use the following algorithm to solve the model: i) Guess distribution  $\mu^d(a, z)$  and determine the matching probabilities  $\phi^s$  and  $\phi^b$ . ii) Solve for the value and policy functions using value function iteration. iii) Simulate to find the resulting stationary distribution. iv) Update the guess for distribution. v) Repeat i)–iv) until the distribution has converged.

In step ii), given a guess for  $V^d(a, z)$ , we first solve for  $v^d(a, z)$  from (6). We then determine  $Tr(a^b, a^s)$  and  $p(a^b, a^s)$ . It is clear that  $S^b(a, p)$  is decreasing and  $S^s(a, p)$  is increasing in price: other things equal, the buyer’s surplus from trade is always smaller and the seller’s larger the higher the price. We therefore begin by calculating prices  $\bar{p}^b$  and  $\underline{p}^s$  such that  $S^b(a, \bar{p}^b) = 0$  and  $S^s(a, \underline{p}^s) = 0$ . If  $\bar{p}^b < \underline{p}^s$ , there is no price that would render trade mutually beneficial. If instead  $\bar{p}^b \geq \underline{p}^s$ , we know that trade takes place. In this case, we find  $p(a^b, a^s) \in [\underline{p}^s, \bar{p}^b]$  by solving (13), which is a one dimensional maximization problem. Given  $Tr(a^b, a^s)$  and  $p(a^b, a^s)$ , we first determine  $W^b(a^b, a^s)$  and  $W^s(a^b, a^s)$  from (15) and (16). We then solve  $V^d(a, z)$  for  $d = r, o$  and  $z = 1, 2$  from (7), (8), (9), and (10).

Of course, we need to use discrete grids of possible financial wealth levels for both owners and renters. We use cubic splines to interpolate value functions  $V^d(a, z)$  and  $v^d(a, z)$  between grid points and apply linear interpolation to the match value functions  $W^b(a^b, a^s)$  and  $W^s(a^b, a^s)$ . Even though the match value functions feature kinks around asset levels where trade becomes mutually beneficial, value function  $V^d(a, z)$  turns out to be concave. This is because the location of the kink (in terms of agent’s own asset position) depends on the asset position of the potential trading partner and the value function  $V^d(a, z)$  reflects the expected value of  $W^b(a^b, a^s)$  or  $W^s(a^b, a^s)$  over all possible asset positions of the trading partner.

For a given match and a given price, we compute the surplus from trade using value functions  $v^d(a, z)$  according to (11) and (12). The minimum financial wealth levels that we may need to consider correspond to the maximum and minimum prices defined in (4) and (5). The minimum financial wealth is  $\underline{s}^r + v + c_{\min}$  for renters and  $\underline{s}^o + \kappa + c_{\min}$  for owners. We assume  $c_{\min} = 0.01$ . In the benchmark calibration, these limits are approximately 0.21 and  $-20.61$ , respectively. These limits provide the lower bounds for the financial wealth grids. We set the maximum financial wealth levels in the benchmark at 51.8 for renters and at 30 for owners. There is no mass close to these limits in the stationary distribution. The difference between these limits corresponds to the average house price.

We approximate the distribution by a discrete density function. The financial wealth of a household in occupancy state  $d$  is forced to belong to a set  $A^d = \{a_1^d, a_2^d, \dots, a_m^d\}$ . As usual, we use a lottery to force next period financial wealth to be on the grid  $A^d$  (see algorithm 7.2.3 in Heer and Maussner (2010)).

In step iii), we first determine savings policy for all unmatched households as well as the outcome of the bargaining process and the associated savings policy for all possible matches. That is, we determine, among other things,  $Tr(a_j^r, a_k^o)$  and  $p(a_j^r, a_k^o)$  for all  $j = 1, 2, \dots, m$  and  $k = 1, 2, \dots, m$ . Since  $Tr(a^b, a^s)$  is a discrete function and the price is not defined everywhere, we do not interpolate these functions, but solve for the outcome of the bargaining process in the same way as in step ii).

We then determine three transition probability matrices. The first one determines transition probabilities from a given current state  $(a, d, z)$  to different next period states for unmatched households. The next period financial wealth is determined by the savings policy and the income shock. With two income shocks and two tenure preference states (and the lottery), an unmatched household in a given state may generally move to 8 different states. The second matrix determines probabilities with which a potential buyer (unhappy renter) with a given financial wealth  $a^b \in A^r$  that is matched with a potential seller with a given financial wealth  $a^s \in A^o$  moves to different next period states. Given the match, there are again generally 8 different states to which the household can move. Similarly, the third matrix determines the probabilities at which a potential seller that is matched with a given potential buyer moves to different next period states. Given these transition probability matrices and an initial density function, we iterate over the density function to find the stationary distribution. At this stage, we need to take into account the probabilities of different matches which are in turn determined by the density function. For instance, of unhappy renters in state  $(a_j^r, r, 2)$  that are matched with a potential seller, fraction

$$\mu^o(a_k^o, 1) / \sum_{l=1}^m \mu^o(a_l^o, 1)$$

are matched with a potential seller with financial wealth equal to  $a_k^o$ .

The results reported here have been computed using 150 non-linearly spaced gridpoints for financial wealth in the value function and 200 non-linearly spaced gridpoints for financial wealth in the density function. Before we simulate to find the stationary distribution in step iii), we need to determine the outcome of the bargaining process for  $200^2$  combinations of seller's and buyer's financial wealth. Further increasing the number of gridpoints had virtually no impact on the reported statistics of the benchmark calibration.

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