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Assessing Components of Uncertainty in Demographic Forecasts with an Application to Fiscal Sustainability

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Abstract

When the future evolution of demographic processes is described in a stochastic setting, the challenge is to communicate the meaning of forecast uncertainty in an understandable way, to decision makers and public at large. For the purpose of risk communication, a formal setting is developed, in which the roles of the demographic processes on point forecasts and predictive distributions can be elucidated. The communication problem becomes central in fiscal decision making, when eventual forecast errors have differential implications on the value of the policy options being considered. Tax rate that is required to maintain financial sustainability, until a given target year, is used for illustration.

Tiivistelmä

Väestöennusteiden epävarmuuskomponentit: arviointia ja sovellus julkisen talouden kestävyyslaskelmaan

Kun tulevaa väestökehitystä kuvaillaan stokastisilla laskelmilla, on tulosten ja niihin liittyvän epävarmuuden välittäminen sekä päätöksentekijöille että suurelle yleisölle haasteellista. Kommunikaatio on erityisen tärkeää finanssipoliittisessa päätöksenteossa, jossa ennustevirheiden seuraukset tarkasteltaville politiikkavaihtoehdoille vaihtelevat.

Tässä artikkelissa esitellään kaavamuotoinen kehikko, jolla voidaan dekomponoida tulevaan syntyvyyteen, kuolevuuteen ja siirtolaisuuteen liittyvän epävarmuuden vaikutuksia sekä piste-ennusteisiin että ennustejakaumiin. Sovellusesimerkkinä käytetään veroastetta, joka riittää ikääntyvän väestön aiheuttamien julkisten menojen kattamiseen annetun pituisella aikajaksolla. Tämä kestävyysvajetta vastaava käsite lasketaan Suomen väestölle laaditun stokastisen väestöennusteen ja suomalaiseen aineistoon perustuvien tyyliteltyjen verotuloja ja julkisia menoja kuvaavien ikäprofiilien avulla.

Eri väestötekijöiden vaikutusta koko väestön ennustejakaumaan ja siten myös väestökehityksestä johdettuihin taloutta kuvaaviin jakaumiin ei voida yksikäsitteisesti määrittää. Tulokset riippuvat siitä, missä järjestyksessä tekijät otetaan analyysiin mukaan. Kiinnostavaksi sovellusesimerkiksi osoittautui mm. vaihtoehto, jossa siirtolaisuuteen ja kuolevuuteen liittyvä epävarmuus poistettiin kokonaan. Jos syntyvyys pysyy matalalla tasolla, väestön koko luonnollisesti ajan mittaan pienenee ja ikärakenne painottuu vanhoihin ikäryhmiin. Kestävyysvaje on tällöin väistämättä suuri. Suuri syntyvyys puolestaan johtaisi todennäköisesti matalampaan kestävyysvajeeseen, mutta vajearvioon liittyvä epävarmuus kasvaisi, koska esimerkiksi suuret vaihtelut vauvabuumeista matalan syntyvyyden jaksoihin olisivat mahdollisia.

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Keywords: Aging, Demography, Predictive distribution, Risk communication, Stationary equivalent population

Asiasanat: Väestö, Ennustejakauma, Ikääntyminen, Riskitietoisuus ja rahoituksellinen kestävyys, Stationaarinen ekvivalentti väestö

JEL: J11, J18, H68

1 Introduction

We consider demographic forecasting in a stochastic setting. It has been understood, quite some time, that adopting a stochastic point of view is necessary, in order to account for the uncertain future values of the three component processes of fertility, mortality, and migration, in a consistent manner (e.g., Keilman, 1990; Alho and Spencer, 1991). Stochastic methods are an order of magnitude more complex than the formulation of alternative scenarios. This leads to problems in the communication of the results to decision makers. The fundamental reason is that from the net effect it is not possible to discover the independent roles of the component processes. When taxes, spending, and other fiscal measures are viewed as *functionals* of the population, the situation becomes even less transparent (cf., Alho and Vanne, 2005)

Yet, from the perspective of policy formulation, the roles of the component processes can be decisive. Pöysti (2014) argues that acknowledging uncertainty is a precondition for a rational public debate. Auerbach (2014) notes that ignoring uncertainty may bias decision making towards short-term optimality. But, whence comes the uncertainty?

As a starting point, we will take a stochastic version of the recent population forecast of Statistics of Finland. This gives us the point forecast. The uncertainty of the component processes is empirically calibrated to match the volatility of past demographics.

As a practical illustration we will consider *fiscal gap*, i.e., how much taxes should be raised in order that the current public sector balance could be maintained for the next H years. The *required tax multiplier* is the factor by which current taxes must multiplied to accomplish this. This measure appears in various forms in, e.g., in the U.S. (Board of Trustees, 2021), the E.U. (EU, 2020), Canada (CPP, 2019), and Finland (ETK, 2019). The *predictive distribution* that is at stake is illustrated by Figure 4, below.

The primary contribution of this paper is the development of a systematic approach to decomposing sources of uncertainty in probabilistic demographic forecasts, and measures derived with them. As regards the point forecast, a novel aspect we take up is the role of the age distribution at *jump-off*[1]. This typically differs from the stationary age distribution determined by the life table of the jump-off year. Any difference between the two reflects the *demographic inheritance* from the past population history, be it a burden or benefit.

Section 2 develops the notation for the forecast calculations. Section 3 details a public sector model, and defines the tax multiplier. Section 4 develops the decomposition of the systematic and stochastic aspects of the predictive

[1] This is the most recent time for which we consider the population counts and past values of the vital rates as known.

distributions. The jump-off effects on the future ratio of taxes to spending are assessed in Section 5. In Section 6 we first consider the jump-off effects on required tax, and summarize those effects together with the effects of vital rates, on point forecasts. After that a detailed analysis of the uncertainty in the required tax multiplier follows. Section 7 concludes with a discussion of the (lack of) uniqueness of the decompositions, and summarizes what was learned about the fiscal gap as a policy tool.

2 Population Evolution

2.1 Definitions

We will write the model using a one-year time unit. Age is given by $x = 0, 1, \ldots, \omega$, where ω is the highest, open-ended age-group. Gender is denoted by $s = 0, 1$, for females and males, respectively. Forecast years are $t = 1, \ldots, T$, and $t = 0$ is the jump-off year. Arrays by age are indexed starting from 0.

We define V_{xst} = number of people in age x, of gender s, in the beginning of year t. Matrices $\mathbf{V}_t = (V_{xst})_{(\omega+1)\times 2}$, have population counts for the beginning of year $t = 0, \ldots, T$. For brevity in later use, we collect these into array $\mathbf{V} = (V_{xst})_{(\omega+1)\times 2\times (T+1)}$.

Define m_{xst} = age-specific mortality rate in age x, for gender s, during year t. The matrix $\mathbf{m}_t = (m_{xst})_{(\omega+1)\times 2}$ has data for year $t = -1, 0, \ldots, T-1$. Here $t = -1$ refers to the calendar year before jump-off time. Corresponding to these, define $\mathbf{p}_t = (p_{xst})_{(\omega+1)\times 2}$ as the matrix of one-year survival probabilities from age x. (Survival from ω to $\omega + 1$ refers to survival in the highest, open-ended age-group.) Define f_{xt} = age-specific fertility rate in in age x, for females, during year t. Collect them into vector $\mathbf{f}_t = (f_{xt})_{(\omega+1)\times 1}$ for year t. The parameters $0 < \kappa_0, \kappa_1 < 1$ are the proportions of girls and boys out of the newborn, with $\kappa_0 + \kappa_1 = 1$.

Define N_{xst} = net number of migrants to age x, of gender s, during year t, who survive to $t + 1$. The matrix $\mathbf{N}_t = (N_{xst})_{(\omega+1)\times 2}$ has the numbers for year t.

The jump-off time of the analysis will be January 1, 2019. In the economic application we will have $T = 126$ corresponding to years 2020-2145. The highest (open-ended) age is $\omega = 100$. The *sex ratio at birth* is 1.05, so $\kappa_1 = 1.05/2.05$.

2.2 Jump-off Data

Assume that the jump-off population for the beginning of year $t = 0$ is known, or that an estimate exists (cf., Shryock and Siegel, 1976, 427). Similarly, mortality, fertility, and net-migration are assumed to be known for $t = -1$, i.e., the year before the jump-off. The latter are needed for the computation of the baseline point forecast, and for the computation of the stationary population.

2.3 Linear Growth Model

Simple book-keeping equation "population at t = population at $t-1$ + births during t - deaths during t + net number of migrants during t" means that we have recursively for the youngest age $x = 0$,

$$V_{0,s,t} = \kappa_s \sum_x f_{x,t-1} V_{x,0,t-1} + N_{0,s,t-1}, s = 0, 1. \tag{1}$$

For the remaining ages $x = 1, \ldots, \omega - 1$,[2] we have that

$$V_{x,s,t} = p_{x-1,s,t-1} V_{x-1,s,t-1} + N_{x-1,s,t-1}, s = 0, 1. \tag{2}$$

2.4 Stochastic Forecast of Finland 2020 - 2145

The January 1, 2019, jump-off population is as estimated from the population register. Age-specific fertility is assumed to remain at the latest observed level. Mortality is assumed to continue to decline at the rate of recent years. Net migration is assumed to remain positive, at its recent level. The assumptions, and the actual numerical values, are essentially the same as those used by Statistics Finland in their latest forecast available in 2020.

Stochastic simulation program PEP (Program for Error Propagation) has been used in this study. Formally, the stochasticity of the population $\mathbf{V}$, and the economy $\mathbf{W}$ defined below, derives from the joint predictive distributions of the age-specific fertility rates, of the underlying mortality rates that give rise the survival probabilities, and of the net migration numbers, as used in recursions (1) and (2). Details of the structure of PEP, and the empirical uncertainty estimates used, are given in Alho and Spencer (2005).[3] The

[2]For age ω terms reflecting survival and migration in this age are added.

[3]For additional details and for earlier economic applications, see Alho, Hougaard-Jensen and Lassila (2008).

uncertainty represented in these models has been empirically calibrated to such levels that produce prediction intervals of appropriate width, when they are applied to past empirical data from several European countries in the 1900s.

3 A Public Sector Model

The public sector finances are described in terms of matrix $\mathbf{W} = (W(i,t))_{3\times(T+1)}$ where $W(1,t)$ = tax revenues collected during period t; $W(2,t)$ = public sector spending during period t; and $W(3,t)$ = public sector balance in the beginning of period t. Here, the time index takes the values $t = 0, 1, \ldots, T$. Mathematically, the matrix $\mathbf{W}$ will be a function of the population $\mathbf{V}$, and the starting value $W(3,0)$.

Economic data are calibrated to Finland around year 2017. This is a small open economy with a large public sector.

3.1 Revenues, Spending, and Balance

The public sector consists of the state and municipalities. State collects progressive income tax, added value tax, mandatory pension contributions, and various fees. Municipalities collect income tax at flat rates. These result in average *age-specific taxes* per person $z_{xst} \geq 0, x = 0, \ldots, \omega; s = 0, 1; t = 0, \ldots, T$ (cf., Alho and Vanne, 2005). The *total revenue* collected in year t is $W(1,t) = \sum_{x,s} z_{xst} v_{xst}$.

The public sector pays for the daycare and education of the young, health care for all ages, long-term care, old-age pensions, and administration. We define $c_{xst} \geq 0$ as the average *spending* per person in age $x = 0, \ldots, \omega$, for $s = 0, 1$, during year $t = 0, \ldots, T$. The *total spending* during year t equals $W(2,t) = \sum_{x,s} c_{xst} v_{xst}$.

The public sector balance is assumed to satisfy the book-keeping equations

$$W(3,t) = W(3,t-1)(1+r) + W(1,t) - W(2,t), \quad t = 0, \ldots, T-1, \quad (3)$$

where $r = 0.01$ is the interest rate used.

In numerical calculations the genders are combined and the schedules are kept fixed over time (so $z_{xst} \equiv z_x$ and $c_{xst} \equiv c_x$). The euro values of the profiles have been set equal to the actual government spending in 2018 (or $t = -1$). These profiles are displayed in Figure 1.[4]

[4]We thank R. Vanne, for help with these data; cf., Vaittinen and Vanne (2011) and

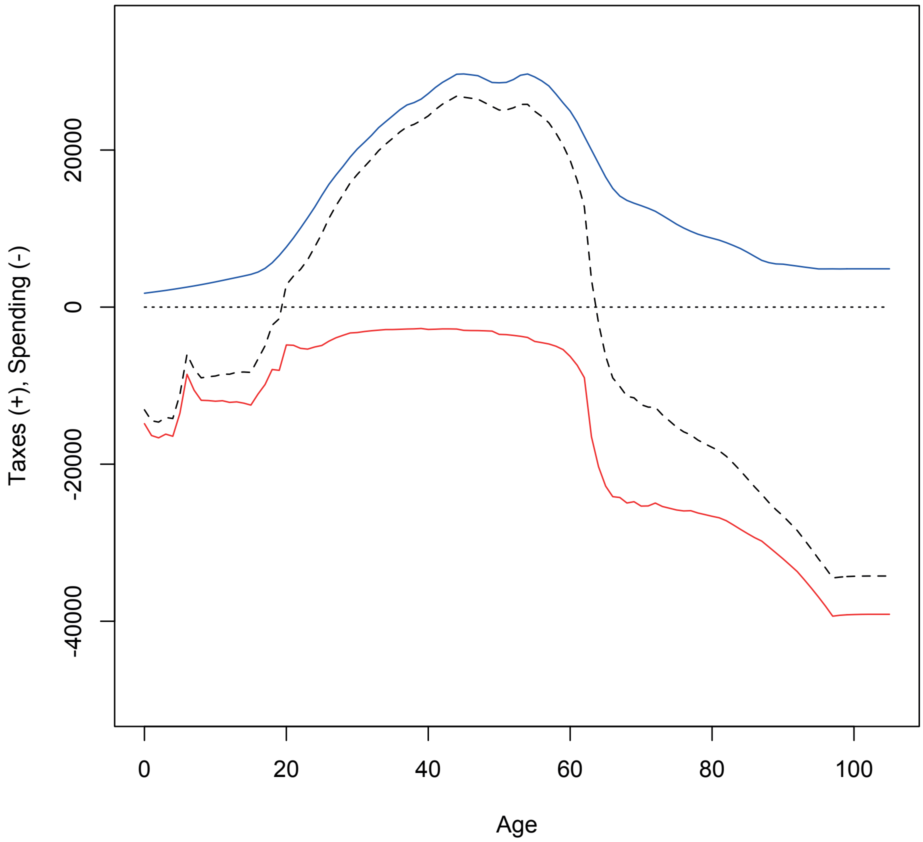

Figure 1: Average Tax (Blue), (the Negative of) Spending (Red), and their Balance (Dashed), by Age (in Euros).

3.2 Fiscal Policies with Balance Targets

Write $u_{xst} = z_{xst}q_t$, where $q_t = \sum_{x,s} z_{xst}$ is the *sum of age-specific taxes per person* across age and gender groups, so $z_{xst} = q_t u_{xst}$.

Suppose we set, in the beginning of year t, the target that exactly $H > 0$ years ahead the balance equals $W_3^*(H)$, in current value. Discounting the balance, future taxes, and spending, to time t, we look for the value of $q(H)$ that would result in the desired balance, or

$$W(3,t) + q(H) \times Y_t(H) - W_{2t}(H) = (1+r)^{-H} W_3^*(H), \tag{4}$$

Lee and Mason (2011).

where

$$Y_t(H) = \sum_{h=1}^{H} (1+r)^{-h} \sum_{x,s} u_{x,s,t+h} v_{x,s,t+h} \tag{5}$$

is the *discounted age-standardized tax base* in the following H years, and

$$W_{2t}(H) = \sum_{h=1}^{H} (1+r)^{-h} \sum_{x,s} c_{x,s,t+h} v_{x,s,t+h} \tag{6}$$

is the *discounted spending* in the coming H years. For lack of established terminology, we call $q(H)$ as the *required tax multiplier*.

In applications below, we will take $W_3^*(H) = W(3,0)$. This means that the current balance is maintained for H years.

4 Systematic and Stochastic Components of Error

4.1 Effect of Baseline Age-Distribution

Actual population $\mathbf{V}$ is taken to be stochastic, around the point forecast $\hat{\mathbf{V}}$. This gives rise to a stochastic component of error. The point forecast $\hat{\mathbf{V}}$ deviates from the *baseline forecast*, in which one assumes that mortality rates $\mathbf{m}_{-1}$, and fertility rates $\mathbf{f}_{-1}$ persist indefinitely, and there is no migration.

Suppose the total population size in the beginning of the year $t = 0$ is V_0. The *stationary equivalent population* has size V_0, but an age distribution that is determined by the (female and male) survival probabilities, of year $t = -1$. Denote this hypothetical population as $\bar{\mathbf{V}}_0$, as distinct from the estimated population $\hat{\mathbf{V}}_0$. All decompositions presented below can be further decomposed in terms of the difference between the actual jump-off population and the stationary equivalent population.

4.2 Baseline Forecast and its Total Error

For the youngest age $x = 0$ the baseline forecast is calculated recursively as

$$\tilde{V}_{0,s,t} = \kappa_s f_{x,-1} \tilde{V}_{x,0,t-1}, s = 0, 1, \tag{7}$$

and for the remaining ages $x = 1, \ldots, \omega - 1$, as

$$\tilde{V}_{x,s,t} = p_{x-1,s,-1}\tilde{V}_{x-1,s,t-1}, s = 0, 1. \tag{8}$$

The (negative of the)*total error of the baseline forecast* can now be decomposed as $\mathbf{V} - \tilde{\mathbf{V}} = \mathbf{S} + \mathbf{D}$, where

$$\mathbf{S} = \mathbf{V} - \hat{\mathbf{V}}, \quad \text{and} \quad \mathbf{D} = \hat{\mathbf{V}} - \tilde{\mathbf{V}}. \tag{9}$$

Here the first difference is the stochastic error, and the second difference is the (presumed) gain from using a better point forecast than the baseline.

4.3 Decomposing Deterministic Error

We use three superscripts, each either "+" or "-", to distinguish forecast variants that include a forecast of one or more of fertility, mortality, and migration. For example, if mortality would be forecasted, but fertility would be kept at the baseline value and net migration would be set to zero, the resulting forecast would be denoted as $\tilde{\mathbf{V}}^{-+-}$. Then, the limiting cases are the actual point forecast $\hat{\mathbf{V}} = \tilde{\mathbf{V}}^{+++}$ and the baseline forecast $\tilde{\mathbf{V}} = \tilde{\mathbf{V}}^{---}$.

A decomposition of $\mathbf{D}$ that can be relevant in sustainability analyses is

$$\mathbf{D} = (\tilde{\mathbf{V}}^{+--} - \tilde{\mathbf{V}}) + (\tilde{\mathbf{V}}^{++-} - \tilde{\mathbf{V}}^{+--}) + (\hat{\mathbf{V}} - \tilde{\mathbf{V}}^{++-}). \tag{10}$$

Roughly speaking, the first term on the right hand side tells us about the effect of fertility,[5] the second about the effect of longevity, and the third about the effect of net migration.

4.4 Decomposing Stochastic Error

We will use notation similar to that of Section 4.3, but this time superscripts are attached to $\hat{\mathbf{V}}$. For example, if *uncertainty* of mortality is present, but that of fertility and migration is not, the resulting stochastic forecast is denoted as $\hat{\mathbf{V}}^{-+-}$. Using this convention, the actual future population is represented by $\mathbf{V} = \hat{\mathbf{V}}^{+++}$, and the deterministic point forecast is $\hat{\mathbf{V}} = \hat{\mathbf{V}}^{---}$.[6]

In analogy with (10) we have the decomposition of the stochastic error as

$$\mathbf{S} = (\hat{\mathbf{V}}^{+--} - \hat{\mathbf{V}}) + (\hat{\mathbf{V}}^{++-} - \hat{\mathbf{V}}^{+--}) + (\mathbf{V} - \hat{\mathbf{V}}^{++-}). \tag{11}$$

The three differences need not be statistically independent, but one would expect them to be approximately so, for short lead times.

[5]But, recall that in our empirical application the point forecast of fertility is the current value, so this component vanishes.

[6]So we also have $\hat{\mathbf{V}}^{---} = \tilde{\mathbf{V}}^{+++}$.

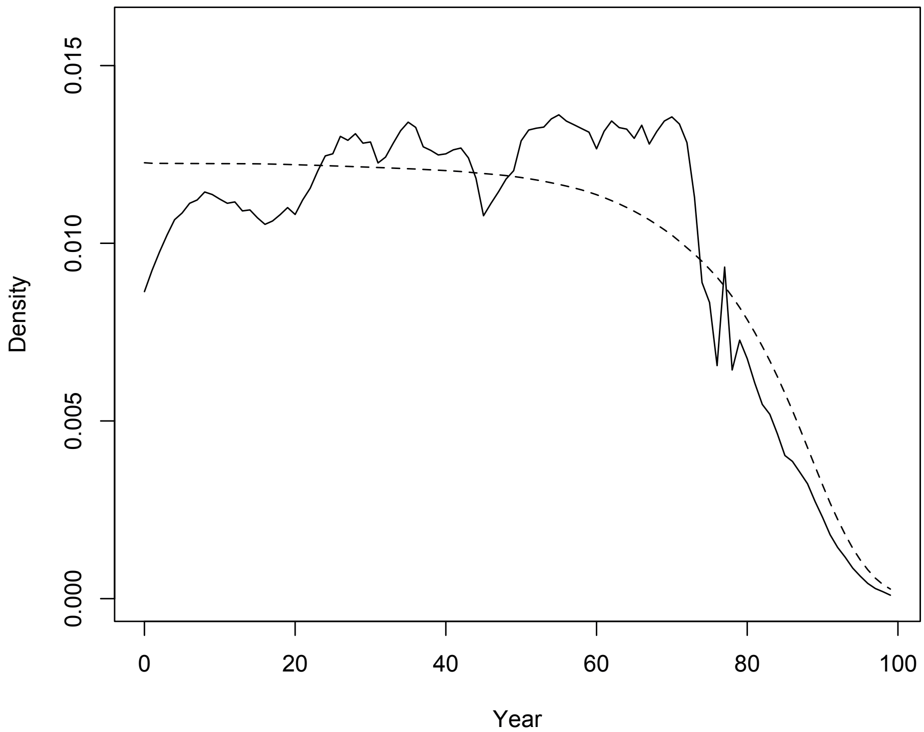

Figure 2: Density of Population, Jan. 1, 2019 (Solid), and Stationary Population for 2018 (Dashed).

5 Effects on Taxes and Spending

5.1 Jump-off Effects

Figure 2 displays the density of the actual jump-off population as of January 1, 2019, and the density of the stationary life table population of the previous year (both genders combined). The latter exceeds the former in the highest ages. This is due to mortality decline during recent decades. A comparison to Figure 1 shows that the actual age distribution tends to make the health care and pension costs *lower* than under the stationary equivalent population.

In ages 50-75 the actual population density is higher. A part of this excess is in working ages, and *boosts public revenues* as compared to the stationary alternative. But, an increasing part is in retirement, and tends to *increase expenditures* later on.

A third difference is in the youngest ages. This is due to the low fertility of the past 25 years. The actual age distribution tends to *lower public expenditures* related to rearing children (child care, education), as compared to the stationary alternative. However, this also means that the future workforce will be smaller, which *reduces expected future revenues*.

5.2 Component Effects

Applying the schedules of Figure 1 to population counts we obtain ratios of Taxes to Spending, in 2018 - 2117, corresponding to different decompositions of point forecasts. These are given in the four panels of Figure 3.

Panel (a) of Figure 3 shows that both under the actual jump-off population, and the stationary equivalent model, the taxes are not enough to support the expected expenditures (i.e., the ratio is below 1). The fact that the dashed curve is higher, after about a decade, says that the current relatively favorable situation is going to turn into an unfavorable one.

The bottom, dashed curve of panel (b) assumes that net migration is zero (case $\tilde{\mathbf{V}}^{++-}$). The dot-dashed curve assumes further that mortality remains at its jump-off value (case $\tilde{\mathbf{V}}^{+--}$). Thus, the difference between the two reflects the decline in mortality. The difference between the solid curve and the dashed curve reflects the effect of migration, or $\tilde{\mathbf{V}}^{+++} - \tilde{\mathbf{V}}^{++-}$.

Panels (c) and (d) of Figure 3 add 67 % prediction intervals to the top panel. From panel (c) we find that the legacy of the past demographic development that is present in the age distribution of the jump-off population

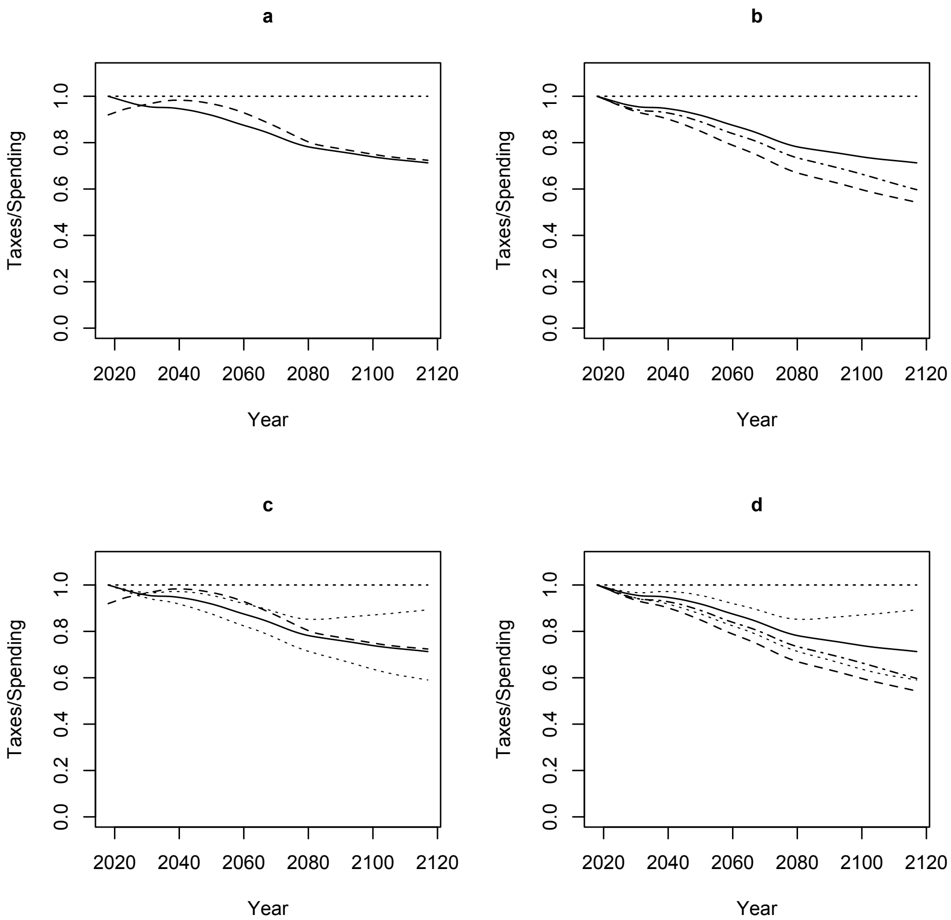

Figure 3: The Ratio of Total Taxes to Total Spending, in 2018-2117: (a) Using the Actual Jump-off Population (case $\tilde{V}^{+++}$; Solid) and Stationary Equivalent Population (Dashed), (b) Using the Actual Point Forecasts (Solid), Setting Net Migration to Zero (case $\tilde{V}^{++-}$; Dashed), and Setting Both Net Migration to Zero and Assuming the Mortality to Remain at Jump-off Value (case $\tilde{V}^{+--}$; Dot-Dashed), (c) Same as (a) but with 67 % Prediction Intervals (Dotted), (d) Same as (b) but with 67 % Prediction Intervals (Dotted).

Table 1: Alternative Point Forecasts of the Required Tax Multiplier.

POP	NET	MORT	$H = 50$	$H = 75$	$H = 100$
+	+	+	1.074	1.120	1.154
+	+	−	1.040	1.064	1.083
+	−	+	1.130	1.200	1.255
+	−	−	1.094	1.143	1.184
−	+	+	1.049	1.094	1.131
−	+	−	1.018	1.041	1.060
−	−	+	1.102	1.169	1.224
−	−	−	1.070	1.115	1.154

is comparable, 10-50 years into the future, to that of all other sources of uncertainty population.[7] Panel (d) indicates that persistent effect of positive net migration is large indeed, because without it the ratio would be below the bottom 67 % prediction interval for all years.

6 Decomposing Forecasts of the Required Tax Multiplier

6.1 Point Forecasts of the Required Tax Multiplier

As in Figure 3, we consider alternative point forecasts of the required tax multiplier. We will introduce abbreviated symbols, and adapt the earlier +/- notation, as follows: POP – we use either the actual jump-off population (+) or the stationary equivalent population (-); NET – net migration is either forecasted (+) or set to zero (-); MORT – mortality is either forecasted (+), or kept at jump-off value (-). Since the point forecast for fertility equals the jump-off value, it is left out from this discussion. Three target years $H = 50, 75, 100$ are considered. The results are collected in Table 1.

To summarize, we fit a four-way Analysis of Variance model "$POP + NET + MORT + H$" to the numbers in the table. The findings are clear. As expected, the actual jump-off population, instead of its stationary equivalent, adds 0.026 to the tax multiplier. Net migration is expected to lower the required tax multiplier by 0.076. This is as expected, as migrants typically increase

[7] This large effect might not come as a surprise as it has long been known that the jump-off error is a major source of error in short term population forecasting (cf., Alho and Spencer, 1985).

the share of the working age population. Increase in longevity increases the required tax multiplier by 0.053, as compared to what would happen if mortality declines would stop. All findings are in line with what one would expect from Figure 2.

The new information we learn from the decomposition of the point forecast is that extending the target period H from 50 to 75, increases the required tax multiplier by 0.046, and extending it from 50 to 100 increases the tax even more, by 0.084. This calls for a more detailed assessment, and some qualification.

First, the large positive effect of net migration is partly due to the assumption that immigrants will have the same tax/spending characteristics as the receiving population. Kaihovaara and Larja (2019) find that the net effect is neutral as the employment rate of the immigrants is approximately 60 %. Sarvimäki (2017) agrees, but stresses the mediating importance of work-related migration policies.

Second, the negative effect of mortality is exaggerated, because tax and spending schedules have been kept unchanged over time. But, when deaths are postponed, death related costs are also postponed (Häkkinen et al., 2007). On the other hand, Määttänen (2014) suggests that longer lifetimes are likely to lead to longer work careers. Taking both points of view into account, Lassila and Valkonen (2018) concluded that the effect of mortality decline is likely to be small. – Clearly, there is a need for further empirical study.

6.2 Components of Uncertainty in the Required Tax Multiplier and the Role of H

Boxplots in Figure 4 show how the location and scale of the predictive distribution of the tax multiplier behaves as a function of the target year H. Consonant with Table 1, the location, which is best described by the median depicted inside the boxes, increases with H. At the same time the width of the box (that contains the central 50 % of the data) widens. In other words, both the location and spread of the predictive distribution increase.

Figure 4 illustrates the question raised in the Introduction. Although the dependency of the predictive distribution has clear overall features, one can ask, what are the roles of the demographic components in those features? Table 1 shows clearly the negative effect of mortality and and the positive effect of migration. The effect of the jump-off age distribution (as opposed to the hypothetical stationary equivalent), especially in the youngest ages

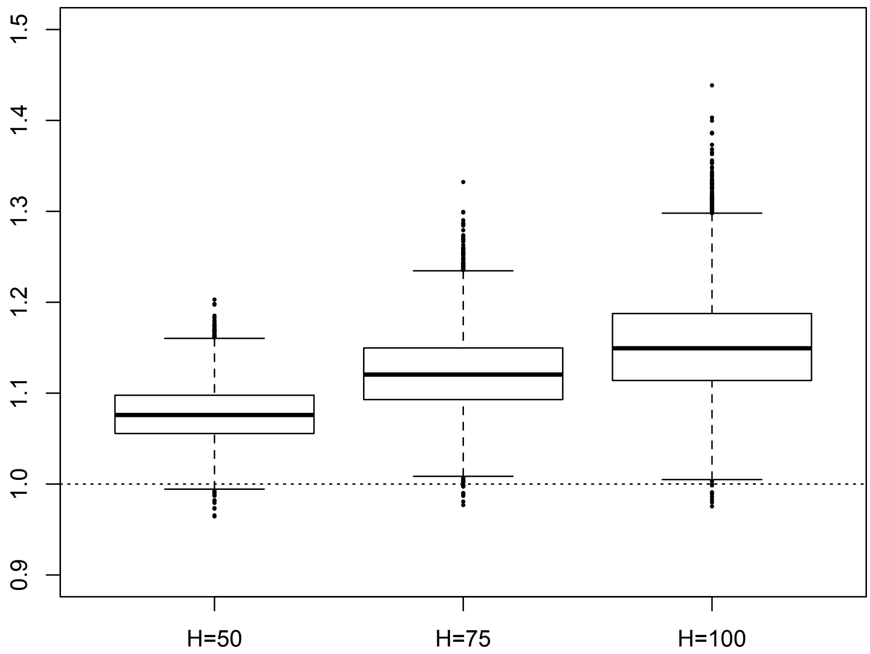

Figure 4: Boxplot of the Predictive Distribution of the Required Tax Multiplier for $H = 50, 75, 100$.

provides a hint as to the effect of fertility, which is otherwise absent from Table 1.

The graphical displays in Figure 5 are helpful in many ways. When the uncertainty related to migration is taken out (case $\hat{\mathbf{V}}^{++-}$), the uncertainty shrinks in a similar manner across the values of H. When the uncertainty related fertility is taken out, (case $\hat{\mathbf{V}}^{-+-}$) the remaining uncertainty relating to mortality increases as a function of H. Gradual increase in longevity tends to make the age distribution older. In contrast, when fertility is the only source of uncertainty (case $\hat{\mathbf{V}}^{+--}$) we see a marked difference. After $H = 75$ the uncertainty related to fertility increases clearly. This appears to be due to the (economically positive) outcomes, in which fertility persists at a higher than expected level, and leads to a younger population age structure.

The latter issue can further be elucidated. Figure 6 displays the required tax as a function total discounted taxes, when fertility is the only source of random variation in total discounted taxes over the H years (case $\hat{\mathbf{V}}^{+--}$),. The finding is striking. Total taxes are highly correlated with population size. In large populations fertility has been exceptionally high, so the age distribution is younger than one would anticipate, on average. This leads to lower required taxes. Small populations are a result of exceptionally low fertility, with ensuing old age distribution and high required tax multiplier.

There are some outliers in the upper right hand corner of Figure 6 that have both high discounted taxes and high required tax multiplier. To be sure, the have very low probability. In case $\hat{\mathbf{V}}^{+--}$ this can come about if initially high fertility (“baby boom”) is followed by a sustained drop in fertility (“baby bust”), a phenomenon we have witnessed after World War II in many countries of Europe and the United States.

7 Discussion

A well-known result from analysis of variance (and more general linear models) is that the contributions of different factors to the outcome variable cannot be uniquely determined, unless the factors or other explanatory variables are orthogonal. Some order must be imposed. (cf., Searle, 1987) A similar indeterminacy occurs in the measurement of changes in consumer prices, for which numerous price indices have been proposed (Laspeyres, Paasche etc.; for an application to changes in mortality rates, see Kitagawa, 1955). It is, therefore, not surprising that a unique decomposition does not exist when it comes to predictive distributions of demography. Also here, the order in which component effects are considered matters.

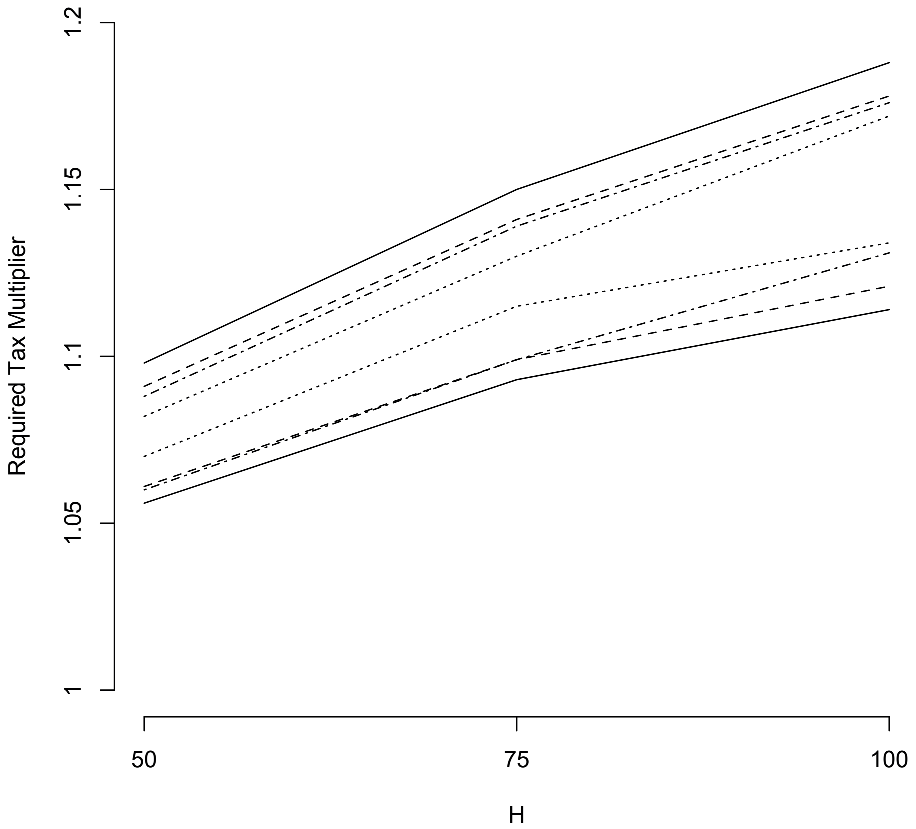

Figure 5: Graphs of the First and Third Quartile of the Predictive Distributions of the Required Tax Multiplier, When All Sources of Uncertainty Are Present (case $\mathbf{V} = \hat{\mathbf{V}}^{+++}$; Solid); When Uncertainty in Migration Has Been Omitted (case $\hat{\mathbf{V}}^{++-}$; Dashed); When Uncertainty in Migration and Mortality Has Been Omitted (case $\hat{\mathbf{V}}^{+--}$; Dotted); and When Uncertainty in Migration and Fertility Has Been Omitted (case $\hat{\mathbf{V}}^{-+-}$; Dash-Dotted).

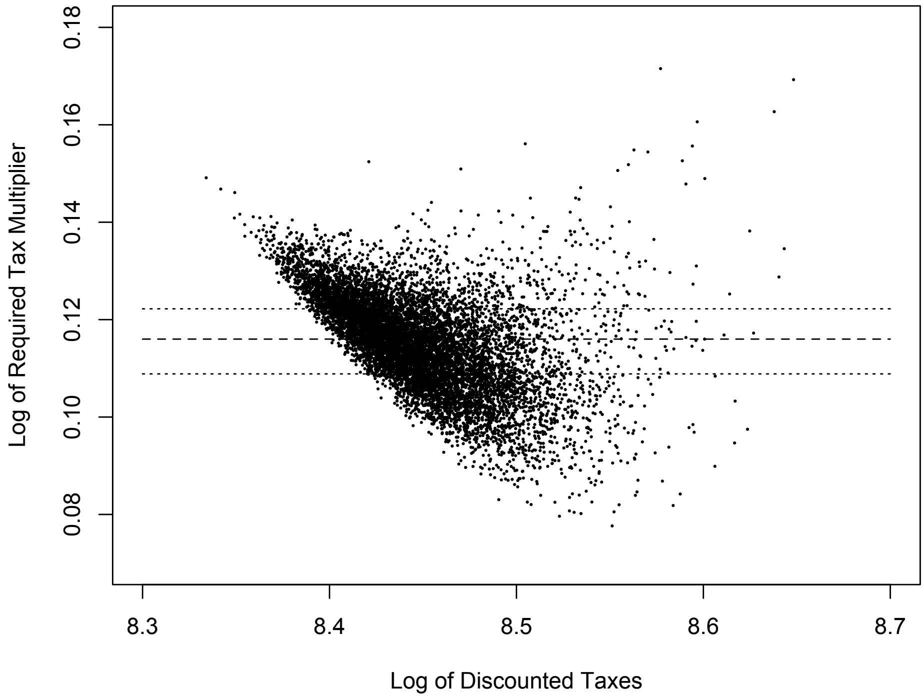

Figure 6: Required Tax as a Function of Discounted Taxes ($H = 75$), Both in Log-Scale, for a Sample of Simulated Values from the Predictive Distribution When Only Fertility Uncertainty Is Included (case $\hat{\mathbf{V}}^{+--}$), with Median (Dashed), First and Third Quartiles (Dotted).

Demographic teaching emphasizes that migration differs from fertility and mortality as it does not have a well-defined “population at risk”, so it is often the first component to be taken out. The roles of fertility and mortality differ by age, so this may provide a guide for the order in which their effects might be meaningfully quantified.

Births, deaths, and migration are more understandable to the public at large than such factors of fiscal gap as productivity, and rates of return of financial instruments. But, even if births, deaths, and migration only vary, the fiscal functionals vary in an opaque fashion.

In the case of the fiscal gap/required tax multiplier, the general finding is that the level of the predictive distribution of the required tax multiplier increases with target year H. This is related to the age distribution that is becoming older, with high probability. The fact that there is a clear increase in the spread of the predictive distribution depends essentially on the uncertainty of future fertility, a finding that is not obvious, for non-demographers, at least.[8] This means that estimates of the required tax will have to be updated frequently. But, updates are problematic, e.g., for pension systems that rely on intergenerational transfers, and depend politically on intergenerational trust (for examples of mechanisms that react to expected imbalances from Canada, Germany, and Sweden, see, e.g, CPP, 2019; BMAS, 2020; Settergren and Mikula, 2006).

[8]The qualifications of Section 6.1 further accentuate the role of fertility.

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