ETLA Working Papers

No. 37

8 March 2016

Nelli Valmari

Estimating Production Functions of Multiproduct Firms

Suggested citation: Valmari, Nelli (8.3.2016). "Estimating Production Functions of Multiproduct Firms".
ETLA Working Papers No 37. http://pub.etla.fi/ETLA-Working-Papers-37.pdf

ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY

ISSN-L 2323–2420
ISSN 2323–2420 (print)
ISSN 2323–2439 (online)

Estimating Production Functions of Multiproduct Firms*

Nelli Valmari**

March 8, 2016

Abstract

Despite the fact that multiproduct firms constitute a considerable share of firms and account for an even greater share of production, virtually all production function estimates are based on the assumption that firms are single-product producers. The single-product assumption is made due to lack of data on input allocation across the various product lines multiproduct firms operate. I provide a method to estimate product-level production functions without observable input allocations. The empirical application and Monte Carlo simulations show that the single-product firm assumption leads to biased parameter and productivity estimates and overestimated productivity differences between firms.

Keywords: Multiproduct firm, production function, productivity
JEL codes: D24, L11, L25

Tiivistelmä

Suuri osa yrityksistä on monituoteyrityksiä eli tuottaa useita erilaisia tuotteita, ja vielä suurempi osa tuotteista on monituoteyritysten valmistamia. Silti lähes kaikki tuotantofunktioestimaatit perustuvat oletukseen, että yritykset tuottavat vain yhtä tuotetta. Oletus yksituoteyrityksistä tehdään, koska tutkijat eivät havaitse monituoteyritysten tuotetason tuotantopanosallokointeja. Tämä tutkimus tarjoaa menetelmän, jolla tuotekohtaiset tuotantofunktiot voidaan estimoida ilman havaintoja tuotetason tuotantopanosallokoinneista. Tutkimuksen empiirinen sovellus ja Monte Carlo simulaatiot osoittavat, että oletus vain yhtä tuotetta tuottavista yrityksistä johtaa harhaisiin parametri- ja tuottavuusestimaatteihin sekä yliarvioituihin tuottavuuseroihin yritysten välillä.

Asiasanat: Monituoteyritys, tuotantofunktio, tuottavuus
JEL koodit: D24, L11, L25

* I thank my PhD thesis supervisor Otto Toivanen for his constructive comments and support. I thank Dan Ackerberg, Liran Einav, Jeremy Fox, Pekka Ilmakunnas, Peter Nyberg, David Rivers, Bram De Rock, Matti Sarvimäki, Paul Scott, Valerie Smeets, Janne Tukiainen, Stijn Vanormelingen, Frédèric Warzynski and an anonymous referee for valuable comments and discussions, and seminar participants at the Helsinki Center of Economic Research, the European Economic Association annual congress 2013, and the 11th CEPR School on Applied Industrial Organization.

** Research Institute of the Finnish Economy (ETLA). Email: nelli.valmari@etla.fi

1 Introduction

A substantial share of firms is multiproduct firms, and an even greater share of goods is provided by these multiproduct producers. For example, in the US manufacturing sector in 1987 to 1997, 39% of the firms manufactured more than one product title, while these multiproduct firms accounted for 87% of the sector's output (Bernard, Redding and Schott, 2010). In a large sample of Finnish manufacturing plants (2004 - 2011), more than 60% of the plants produce at least two product titles. The product scopes range up to 82 titles, and the average product scope of multiproduct firms is 4.3 titles. In international trade multiproduct firms are even more widely present: they accounted for more than 99% of the US exports in 2000 (Bernard, Jensen, Redding and Schott, 2007). Moreover, the product assortments and their output shares vary both across firms, and across time (Bernard, Redding and Scott, 2010).

Despite the empirical fact that multiproduct firms are prevalent, and hence many firms are likely to use several production technologies, the standard practice in production function estimation is to assume that all firms are single-product firms with a single production technology. Most often the output variable is the sum of sales revenue from the various products, and hence the production functions are estimated at the firm-level. The reason for this is pragmatic: to the best of my knowledge, there is no dataset that reports input allocation at the product-firm level for a cross-section of firms.

In this paper I estimate product-level production functions of firms that are mostly multiproduct producers. I provide a simple strategy for estimating structural product-level production functions[1] when the inputs are observed only at the firm- or establishment-level, which is typical of most micro-level datasets. The challenges consist of solving for the unobservable product-level inputs and, as always in production function estimation, controlling for endogeneity problems, i.e., the endogeneity of inputs to the unobservable productivity. The first key insight underlying my estimation strategy is that by inverting the production function, the very definition of productivity can be used to control for the unobservable productivity level. The second insight that is that, once one can control for the unobservable productivity level, the demand[2] for the final good can be used to identify the unobservable input allocation as well as the parameters of the production function. One of the advantages of this estimation strategy is that I can relax the so-called monotonicity assumption made in other structural production function models to control for the endogeneity of inputs. While the monotonicity assumption implies that input demand is monotonic in productivity, in reality firms facing downward-sloping demand curves may either raise or cut their input demand due to improved productivity. The

[1] In this paper I estimate Cobb-Douglas production functions but the identification strategy accommodates also other functional forms. The requirement on the production model, as in most structural production models, is that there has to be at least one input that is chosen as a function of the unobservable productivity.

[2] The demand function estimated in this paper is isoelastic, but also other functional forms can be used.

empirical model also allows for productivity differences between a firm's product lines, which is in line with empirical findings that productivity differences within firms exist, and that they affect firms' production decisions and hence market outcomes.[3]

I demonstrate the estimation method for multiproduct firms' production functions by estimating the production functions for the goods in the wood industry. I find that the technologies used are statistically different across products. To show how the assumption of a single-product production technology, or a firm-level production function, changes the production function estimates and the implied productivity levels, I estimate also the firm-level production function typically estimated in the literature. The firm-level production function estimates differ clearly from the product-level estimates. While the firm-level productivity measures implied by the product- and firm-level estimates are positively correlated, for most firms the firm-level estimates imply either an under- or over-estimated productivity level. In addition, the productivity differences between firms are overestimated when the single-product technology assumption is imposed. This finding may explain at least partly the stylized fact that even within narrowly defined industries, estimated productivity differentials between firms are substantial and persistent (Doms and Bartelsman, 2000; Syverson, 2011).

To get a more complete picture of how the single-product firm assumption biases production function estimates, I run Monte Carlo simulations in the appendix of the paper. I generate data of an industry where firms produce two types of goods with product-specific Cobb-Douglas production functions, and then estimate the firm-level production function. I alternate the parameters of the true product-level production functions as well as the firms' product scopes. I find that the biases in the estimated firm-level parameters are substantial even when the true product-specific technologies are very similar. The simulation results are also in line with the empirical results.

In the next section I review the literature relevant to estimating multiproduct firms' production functions. The model and the estimation strategy are presented in sections 3 and 4. In section 5 I introduce the dataset and provide further details of the estimation procedure. Empirical results are presented in section 6. Section 7 provides a discussion on how the identifying assumptions of my estimation strategy relate to the current production function literature, and in particular how they compare with the identifying assumptions underlying the empirical model of multiproduct firms of De Loecker, Goldberg, Khandelwal and Pavcnik (2012), which is closest to my paper in terms of the estimation approach adopted. Section 8 concludes. In the appendix of the paper I describe the estimation biases that arise if the assumption of a firm-level technology is imposed when the true technologies are product-specific.

[3]See section 2.3 for literature view on multiproduct firms.

2 Literature

This paper relates to three bodies of literature. The first is about identification and estimation of production functions. The second body of literature is about aggregated production functions estimated and discussed in the macro literature. However, even though not previously discussed in the micro literature, aggregation takes place also when imposing the single-product firm assumption on multiproduct firms. The third literature is about characteristics of production by multiproduct firms, and how multiproduct firms have been taken into account in production function estimation.

2.1 Identification of production functions

The current literature recognizes several identification issues that challenge the estimation of production functions. Marschak and Andrews (1944) first pointed out that inputs are not independent variables because firms set them with the aim of maximizing profit. More precisely, inputs are endogeneous to the productivity level that is unobservable to the econometrician. This endogeneity bias, often referred to as the simultaneity or transmission bias, is the identification problem most carefully considered in the production function literature. Failure to correct for the simultaneity bias leads to overestimated production function parameters for the flexible inputs such as materials and possibly also labor.

Another endogeneity problem is the selection bias. As first discussed by Wedervang (1965), econometricians do not observe a random sample of firms. A firm's decision to be active in the market depends on its productivity level as well as its fixed input stocks. Firms with a large capital stock may find it profitable to stay active in the market even if they face a negative productivity shock, while the same holds for firms with a small capital stock that face a positive productivity shock. Hence the fixed input stocks and the unobservable productivity levels of the firms observed are negatively correlated. If firm selection is not accounted for, the production function parameters for the fixed inputs, such as capital, are overestimated.

Olley and Pakes (1996, henceforth OP) were the first to correct for the selection bias, while also controlling for the simultaneity of inputs with a novel structural method. To take account of selection OP estimate survival probabilities for the observed firms. The insight that allows them to correct the simultaneity problem is that a firm chooses its investment level as a function of its productivity. Hence the firm's demand for investment, which OP write as a nonparametric function, can be used to back out the unobservable productivity. The key assumptions that enable this identification strategy are (1) strict monotonicity of investment in productivity, (2) productivity as the only unobservable in investment demand, and (3) the timing of investment (labor) choices before (after) the productivity shock. To relax the rather strict assumption of a monotonic investment function, Levinsohn and Petrin (2003, henceforth LP) propose using demand for intermediate inputs, rather than investment, in inverting out productivity. Wooldridge (2009) shows how the two-step estimators of OP and LP can be implemented in one step to improve efficiency.

Ackerberg, Caves and Frazer (2006, henceforth ACF) observe that the identification strategies of OP, and especially of LP, suffer from collinearity problems. ACF point out that in both estimation strategies the static labor input is collinear with the nonparametric input demand function that is inverted for the unobservable productivity. ACF provide an alternative identification strategy that uses the insights of OP and LP but with slightly modified timing assumptions avoids the aforementioned collinearity problem. However, they also acknowledge that if a gross output production function with more than one flexible input is estimated, there is one identification problem remaining. As shown by Bond and Söderbom (2005), in the absence of inter-firm variation in the input prices, flexible inputs are collinear with each other and with any fixed inputs.

Some studies attempt to control for the collinearity problem by estimating a value added production function that has only one flexible input. However, Gandhi, Navarro and Rivers (2013) show that the value added specification is not a resolution to the collinearity problem, but induces a so-called value added bias instead. In excluding flexible inputs, which are collinear with productivity and other inputs, the degree of productivity heterogeneity is overstated and the elasticity estimates for the fixed inputs are biased. Gandhi, Navarro and Rivers show that if the value added bias is not corrected, the estimated inter-firm productivity differences are orders of magnitude larger, and even of opposite sign, than the productivity differences obtained when correcting for the bias. They provide a strategy to correct for the collinearity and simultaneity problems for both gross output and value added specifications. Gandhi, Navarro and Rivers make the same assumptions regarding timing of input choices and evolution of productivity as ACF, but identification is based on a transformation of the firm's short-run first order conditions.

Also the so-called monotonicity assumption of the aforementioned proxy estimators has been contested. Ornaghi and Van Beveren (2011) compare the performance of the proxy method proposed by OP, and modifications to it by LP, ACF, and Wooldridge. The methods differ in the proxy variables, assumptions on the timing of input decisions and when investments translate into productive capital, and moment conditions. However all the estimators are based on the so-called monotonicity assumption that the proxy variable monotonically increases in the unobservable productivity term. As noted by Ornaghi and Van Beveren, if the monotonicity assumption is violated, the estimators yield inconsistent estimates. They propose a diagnostic tool for testing whether the monotonicity assumption holds for the estimators. Ornaghi and Van Beveren find that the assumption fails to hold in the majority of cases they consider. The assumption holds in all three industries examined in at least 90% of the cases only for three estimators: OP/LP with non-linear least squares, OP/LP with GMM, and Wooldridge's one-step estimator with the assumptions of OP.

Furthermore, there is a large degree of heterogeneity in the results, which indicates that the timing assumptions and the choice of the estimator affect the estimates.

Another type of identification problem is the omitted price bias, which occurs whenever the production function is estimated using sales revenue and/or input expenditure data, and output and/or input prices are not equal across firms. Harrison (1994) discusses the bias with input prices, and Klette and Griliches (1996) with output prices. Despite the considerable biases these inter-firm price differentials can induce, they have been ignored to a large extent in the empirical literature. The explanation is again largely practical: output and input are often measured in sales revenue and expenditures only.

The most recently discussed identification problem concerns firms' endogeneous product selection. Bernard, Redding and Schott (2009) note that most firms make production decisions at a more disaggregated level than what is observed in the data and therefore studied in the productivity literature. They consider single-product firms that choose one out of two heterogeneous goods based on the productivity of the firm, as well as the production technologies and demand for the goods. Bernard, Redding and Schott derive the productivity bias that arises in revenue production function estimation when endogeneous product selection is not accounted for. The so-called product bias is determined, not surprisingly, by the same factors that influence product selection. The empirical implications of ignoring product endogeneity have not been considered.

The paper of Bernard, Redding and Schott and this study are are both based on the observation that production technologies may differ across products even within industries. However the production function estimation biases considered in these studies are different both in their causes and their implications. Bernard, Redding and Schott consider the bias in measured productivity caused by ignoring endogenous product selection. This study, in contrast, provides an estimation method for correcting the functional form misspecification problem that arises from assuming away product-level technologies.

2.2 Aggregation of production functions

A literature on aggregation of production functions has evolved within the macro literature but, despite its relevance to estimating firms' production functions, it has not gained attention among microeconomists. A key element in the neoclassical macroeconomics literature is the aggregate production function. It is constantly estimated despite numerous critical remarks that the aggregate production function does not have a sound theoretical foundation (Felipe and Fisher, 2003). There are two types of issues related to the aggregation of production functions: aggregation over various inputs and outputs, and aggregation over firms when not all inputs are efficiently allocated. Felipe and Fisher discuss the theoretical literature on the aggregation problem.

Klein (1946a, 1946b) initiated the literature on production function aggregation. His objective was to write an aggregate production function as a purely technological relationship, independent of behavioral assumptions such as profit maximization. However May (1946) pointed out that even the micro production functions assume optimization. Pu (1946) noted that if the macro variables are not derived from micro variables that satisfy equilibrium conditions, neither will the macroeconomic equilibrium conditions hold.

The first major findings were made by Leontief and Nataf. Leontief (1947a, 1947b) provides necessary and sufficient conditions for aggregation of variables into homogeneous groups within a firm. Aggregation is possible if and only if the marginal rates of substitution among variables in the aggregate are independent of the variables outside of it. This assumption may hold for some real-life producers but it is unlikely to hold for all of them. Nataf (1948) considers aggregation over different production functions. He finds that aggregation over different functions is possible if and only if the micro production functions are additively separable in capital and labor.

Fisher (1969, 1993) notes that without imposing an efficiency condition, an aggregate function almost never exists. He provides conditions for the existence of aggregates of capital, labor and output under some presumptions. Fisher assumes that, first, labor is allocated across firms efficiently, second, capital is firm-specific and hence capital markets do not exist, and third, firm-level production functions have constant returns to scale. Even under these strong assumptions the conditions for the existence of aggregate production functions are stringent. The aggregates exist only if, first, firm-level production functions are identical except for the capital efficiency coefficient, second, all firms employ different types of labor in the same proportion, i.e., specialization in labor is ruled out, and third, all firms produce all goods in the same proportions, i.e., specialization in output is ruled out. Felipe and Fisher conclude that the conditions under which a well-behaved aggregate production function can be derived are so stringent that actual economies are unlikely to satisfy them.

The firm-level aggregation problem I look at has similarities with the macroeconomic counterpart, albeit the problems are not identical. In the case of firm-level data, aggregation takes place over multiple inputs and outputs, and over various production functions, but in contrast to the macroeconomic literature, decision-makers are not aggregated over. I am not aware of a study that looks at the implications of aggregation of production functions to the firm-level.

2.3 Multiproduct firms

A large share of the recent literature on multiproduct firms is written in the context of international trade, perhaps because international trade flows are dominated by multiproduct firms. In 2000, firms that exported more than one product title, as defined at the ten-digit level, accounted for more than 99% of the US export value (Bernard, Jensen, Redding and Schott, 2007). A number of studies centers on how reductions in barriers to international trade affect firms' productivity and product scope. Nearly every study finds that as reductions in trade barriers lead to increased competition, the firms that remain active become more productive. Theoretical findings on the product scope, which is a potential channel for productivity effects to take place, are mixed. As a consequence to reductions in trade barriers, product scopes are found to decrease,[4] increase,[5] or both.[6] Empirical evidence indicates that increased competition drives firms to concentrate on the goods they are most competent in and drop the least productive products from the selection of exported goods.[7] In other words, empirical evidence suggests that firms' productivity across goods vary.

Multiproduct firms are widely present also within national markets. As in the global markets, firms' production decisions are not restricted to entry and exit decisions at the extensive margin and production scale adjustments at the intensive margin. In fact, changes in product scope, i.e., in the intra-firm extensive margin, are substantially more frequent than changes in the extensive margin (Bernard, Redding and Schott, 2010; Broda and Weinstein, 2010), and they lead to productivity gains for US manufacturing firms (Bernard, Redding and Schott, 2010).

The reason why many firms produce multiple goods is economies of scope (e.g. Panzar, 1989). Economies of scope arise when joint production of multiple goods incurs lower fixed and/or variable costs of production than when the same product bundle is produced separately. Most theoretical and empirical studies assume that firms can add new goods to their product assortment without making considerable investments in production technology, albeit the good-specific marginal costs increase as the product scope grows (e.g. Eckel and Neary, 2010). This is referred to as flexible manufacturing, and implies economies of scope in the form of lower fixed costs but not in the form of lower variable costs. A key feature of flexible manufacturing is that a multiproduct firm can produce one or a few of its goods more efficiently than the rest of its goods, that is, a multiproduct firm has core competency in producing some of its goods (e.g. Bernard, Redding and Schott, 2011). Production function estimation does not typically accommodate economies of scope, flexible manufacturing or core competency, however, apart from a few exceptions discussed below.

Virtually all estimates of production functions are implicitly based on the assumption that all of the firm's output is produced with a firm-level technology.[8] The first set of papers that make an exception evaluate cost minimization with a nonparametric method-

[4] See Bernard, Redding and Schott, 2011; Eckel and Neary, 2010; Mayer, Melitz and Ottaviano, 2014; and Nocke and Yeaple, 2013.

[5] See Feenstra and Ma, 2007; and Ma, 2009.

[6] See Allanson and Montagna, 2005.

[7] See Baldwin, Caves, Gu, 2005; Bernard, Redding and Schott, 2011; and Mayer, Melitz and Ottaviano, 2013.

[8] There is an early literature on estimating cost functions of multiproduct firms. See, for example, Brown, Caves and Christensen, 1979 and Caves, Christensen and Tretheway, 1980. The early multiproduct cost functions allow for the fact that production technologies across goods vary, but they do not correct the typical endogeneity problems such as the simultaneity or selection bias.

ology. Cherchye, De Rock and Vermeulen (2008) allow for product-specific technologies as well as economies of scope that result from joint input use and input externalities. Their methodology does not require observable input allocation. Cherchye, De Rock, Dierynck, Roodhooft and Sabbe (2011) build on Cherchye, De Rock and Vermeulen (2008) using a methodology based on data envelopment analysis. In contrast to Cherchye, De Rock and Vermeulen (2008), they use information on output-specific inputs and joint inputs. As a result the discriminatory power of the efficiency measurement is higher, and the efficiency value of the decision making unit can be decomposed into output-specific efficiency values. However, the methodology is not suited for any typical firm- or plant-level dataset due to the requirement on observable input allocation. Cherchye, Demuynck, De Rock and De Witte (2011) distinguish between two assumptions: cooperative cost minimization at the firm level, and uncooperative minimization at the level of output department. The advantage of these nonparametric methodologies is that they do not require functional form assumptions. On the other hand, the typical endogeneity biases are not treated.

De Loecker, Goldberg, Khandelwal and Pavcnik (2015) estimate production functions to examine how trade liberalization affects product-specific marginal costs and price markups. They use data on single-product firms and the estimation strategy of Ackerberg, Caves and Frazer (2006) to estimate good-specific production function parameters, which are assumed to be the same for single- and multiproduct firms. The product-level input allocations are estimated given the observed data and parameter estimates, and assuming that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e., independent of the input type. De Loecker et al. show that cost efficiency as well as profitability vary across the various products firms produce. They also find a positive correlation between productivity and the size of the product scope, and suggest that firms may use reductions in marginal costs to finance the development of new products. The method adopted by De Loecker et al. is perhaps closest to the empirical strategy presented in this paper, and the assumptions underlying their estimation method are discussed in section 7.1.

Dhyne, Petrin and Warzynski (2013) study price, markup, productivity and quality dynamics of Belgian manufacturing firms. They modify the proxy approach of Wooldridge (2009) to estimate a product-level production function where the output of a given good is related to the firm-level inputs, the output quantities of the other goods the firm produces, and an unobservable firm-level productivity term. Estimating the production function does not require solving for the unobservable input allocations. However, the output elasticities of the inputs as well as the productivity levels are assumed constant across goods. Dhyne, Petrin and Warzynski also estimate a variable cost function for multiple goods, which takes into account the productivity shocks that are implied by the production function estimates.

3 The Model

The model consists of good-specific production and demand functions, and assumptions on the timing of production decisions. Production functions are typically estimated without considering demand for the goods, but in this study output demand is they key for identifying good-specific input allocations and production functions. When firms have market power in the output market, the production decisions are functions of the downward-sloping output demand curves. Functional forms and also most of the other assumptions are familiar from the empirical microeconomic literature. The only exception is that the production function is specified at the product-level instead of the firm-level. The key identifying assumptions are discussed in more detail in sections 4 and 7.

3.1 Production

Firm j produces n_{jt} goods at time t. Production technology i is a good-specific Cobb-Douglas production function with three inputs, materials M_{ijt}, labor L_{ijt}, and capital K_{ijt}:

$$Q_{ijt} = \exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right). \tag{1}$$

Parameters β_{Mi}, β_{Li}, and β_{Ki} denote the output elasticities of materials, labor, and capital for good i, and β_{0i} is a constant. All the production function parameters are good-specific. The productivity term ω_{ijt} varies across goods, firms, and time. It can be divided into expected productivity, $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$, and a mean zero productivity shock, ξ_{ijt}:

$$\omega_{ijt} = E\left[\omega_{ijt}|\omega_{ijt-1}\right] + \xi_{ijt}. \tag{2}$$

Productivity ω_{ijt} comprises all factors other than M_{ijt}, L_{ijt}, and K_{ijt} that affect the firm's production volume in a given product line and time period. Examples of such factors are management and organization of production and down-time due to, for example, maintenance work and defect rates in the manufacturing process (Ackerberg, Caves and Frazer, 2006). Productivity $\exp\left(\omega_{ijt}\right)$ is assumed to follow a first-order Markov process. The firm's decision maker forms an expectation of period t's productivity, $E\left[\omega_{ijt}\right]$, as a function of the previous period's productivity ω_{ijt-1}. The productivity shock ξ_{ijt} represents a deviation from the expected productivity that takes place or becomes observable at the beginning of period t. The shocks ξ_{ijt} may or may not be correlated across the product lines of the firm. For example, managerial changes may have a similar effect on all the product lines, but they may also have different impacts. Similarly, productivity ω_{ijt} may or may not be correlated across the product lines. The firm may have achieved heterogeneous productivity levels due to, for example, different paths of learning and experience.

Labor L and capital K are substitutable across the product lines of the firm. All the factors of production are continuously divisible and exclusive across product lines. This means that they can be flexibly allocated across the different product lines, and that any given share of a firm-level input stock is used in only one product line at a time. Furthermore, none of the production functions utilizes other inputs than M_{ijt}, L_{ijt}, and K_{ijt}. This rules out utilization of by-products as factors of production. The good-specific production functions are independent of production of other goods, which implies that there are no economies of scope in the form of lower variable costs.[9]

3.2 Demand

Firm j faces a downward sloping and isoelastic demand curve[10] for product i in period t:

$$Q_{ijt} = \exp(\alpha_{ij})\, P_{ijt}^{\eta_i} \exp(\varepsilon_{ijt}). \tag{3}$$

Price elasticity of demand, η_i, is good-specific and assumed to be lower than -1. Price elastic demand is required to rule out cases where firms produce marginally small output quantities of various goods. The level of demand, denoted by α_{ij}, depends on unobservable factors such as the quality of the good. These factors vary across goods and firms, but they are constant over time. Any shocks to the good- and firm-specific demand level are captured by ε_{ijt}. The shocks can be caused by changes in buyers' preferences or income, prices of substitutes or complementary goods, or the number of buyers in the market, for example.

3.3 Timing of production decisions

The three types of inputs, M_{ijt}, L_{ijt}, and K_{ijt}, differ in how they are determined. The product-level materials M_{ijt} is a flexible input, chosen at the time of production. It is also a static input, meaning that it doesn't have dynamic implications such as adjustment costs. The firm-level human resources[11] L_{jt} and capital stock K_{jt}, on the other hand, are fixed at the time of production, and they are formed in a dynamic process. L_{jt} is chosen in the previous period $t-1$, while the related costs are paid in the period of production. K_{jt} is determined as a function of the previous period's capital stock and investment, $K_{jt} = f(K_{jt-1}, I_{jt-1})$. However, the product-level inputs L_{ijt} and K_{ijt} are allocated across product lines in the period of production, subject to the the firm-level constraints $\sum_i L_{ijt} \leq L_{jt}$ and $\sum_i K_{ijt} \leq K_{jt}$.

The outline of the production decisions is as follows. At time $t-1$, the firm observes its current capital stock K_{jt-1}, the expected productivity in product lines i at time t,

[9] However economies of scope are allowed in the form of lower fixed costs, as noted in chapter 3.3.

[10] The identification strategy accommodates also other functional forms for demand.

[11] L_{jt} is typically a flexible input in structural production function models. I assume L_{jt} to be fixed because it is more realistic of the Finnish labor market, as discussed in section 7. However, the model can be estimated under either assumption: flexible or fixed labor input.

$E[\omega_{ijt}|\omega_{ijt-1}]$, as well as any other observable factors that affect its future profits. The firm then chooses whether to remain active in period t, and if so, what product titles i to produce.[12] Then, the firm decides on the next period's level of human resources L_{jt} and, by setting the level of capital investment I_{jt-1}, capital stock K_{jt}.

At time t the productivity shocks ξ_{ijt} and the demand shocks ε_{ijt} are realized and become observable to the firm. The firm observes also the price of materials, P_{Mjt}. P_{Mjt} is an exogenous variable, which may reflect the level of bargaining power the firm possesses in the input markets, for example. P_{Mjt} is not a function of the input quantities purchased, however, which implies that there are no cost economies of scope or scale in the form of lower input prices. The firm then chooses the quantities of product-level materials M_{ijt}. At the same time the firm decides how to allocate its human resources L_{jt} and the capital stock K_{jt} among the different product lines the firm is active in, i.e., it sets L_{ijt} and K_{ijt}.

The timing assumptions of this model are similar to the assumptions previously made in the production function literature. These assumptions are compared to those in the previous literature in section 7.

3.4 Firm's optimization problem

The firm maximizes the present discounted value of future profits by making three decisions. First, it chooses which goods i to produce in the next period $t+1$, denoted by $D_{ijt+1} = 1$ if it produces good i at $t+1$, and $D_{ijt+1} = 0$ otherwise. Second, the firm decides the human resources L_{jt+1} to be employed in the next period. Third, the firm invests I_{jt} to determine the next period's capital stock K_{jt+1}. These decisions are made given the expected demand and productivity for the goods in the next period, as well as the expected future material price.

The Bellman equation for the firm's firm-level dynamic optimization problem is:

$$V(S_{jt}) = \max_{D_{ijt+1}, L_{jt+1}, I_{jt}} \sum_i \Pi_{ijt}(S_{jt}) - C(I_{jt}) + \frac{1}{1+\rho} E[V(S_{jt+1})|S_{jt}, D_{ijt}, L_{jt+1}, I_{jt}] \quad (4)$$

where $\Pi(S_{jt})$ is the static profit earned in period t, $S_{jt} = (\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt})$ is the vector of state variables, $C(I_{jt})$ is the cost of investment, and ρ is the discount rate. The dynamic optimization problem gives rise to policy functions $D(S_{jt})$, $L(S_{jt})$ and $I(S_{jt})$.

Instead of solving for the dynamic optimization problem,[13] I follow the examples of Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg, Caves and Frazer

[12] There may be good-specific fixed costs of production that depend on the product bundle the firm is to produce, i.e., there may be economies of scope in the form of lower fixed costs. The fixed costs do not need require further consideration, however, as discussed in chapter 3.4.

[13] Because the dynamic optimization problem is not solved, further specification of the determinants of the dynamic variables is not needed.

(2006), and solve only the static profit maximization problem, which is sufficient for identifying the production function parameters. The static profit maximization problem consists of allocating the firm-level human resources L_{jt} and capital stock K_{jt} among the various product lines i, and setting the product-specific materials M_{ijt} for each product line:

$$\max_{M_{ijt},L_{ijt},K_{ijt}} \Pi_{ijt} = \sum_i P_{ijt}Q_{ijt} - P_{Mjt}M_{ijt} \text{ s.t. } \sum_i L_{ijt} \leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \quad (5)$$

Substituting in the inverse demand, $P_{ijt} = \left(Q_{ijt}\left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-1}\right)^{\frac{1}{\eta_{ijt}}}$, as well as the production functions, the static profit maximization problem becomes:

$$\begin{aligned} \max_{M_{ijt},L_{ijt},K_{ijt}} \Pi_{ijt} &= \sum_i \left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \\ -P_{Mjt}M_{ijt} \text{ s.t. } \sum_i L_{ijt} &\leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \end{aligned} \quad (6)$$

The optimization problem yields a Lagrangian equation with two constraints. The constraints account for not exceeding the firm-level human resources L_{jt} and capital stock K_{jt} when the firm makes input allocations to the product lines. More precisely, given that the firm maximizes profit, L_{jt} and K_{jt} are always fully utilized and the constraints are binding as $\sum_i L_{ijt} = L_{jt}$ and $\sum_i K_{ijt} = K_{jt}$. The Lagrangian is:

$$\begin{aligned} Lagr &= \sum_i \left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \\ &\quad -P_{Mjt}M_{ijt} + \lambda_{Ljt}\left(L_{jt} - \sum_i L_{ijt}\right) + \lambda_{Kjt}\left(K_{jt} - \sum_i K_{ijt}\right). \end{aligned} \quad (7)$$

The first-order conditions for static profit maximization are (JT is the number of firm-time

-observations):

$$\begin{aligned}
\frac{\partial Lagr}{\partial M_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Mi}}{M_{ijt}} \\
-P_{Mjt} &= 0\ \forall\ i=[1,n_{jt}] \qquad (8)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Lagr}{\partial L_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Li}}{L_{ijt}} \\
-\lambda_{Ljt} &= 0\ \forall\ i=[1,n_{jt}] \qquad (9)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Lagr}{\partial K_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)\left(\exp(\alpha_{ij}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Ki}}{K_{ijt}} \\
-\lambda_{Kjt} &= 0\ \forall\ i=[1,n_{jt}] \qquad (10)
\end{aligned}$$

$$\frac{\partial Lagr}{\partial \lambda_{Ljt}} = L_{jt}-\sum_i L_{ijt}=0\ \forall\ jt=[1,JT] \qquad (11)$$

$$\frac{\partial Lagr}{\partial \lambda_{Kjt}} = K_{jt}-\sum_i K_{ijt}=0\ \forall\ jt=[1,JT]\,. \qquad (12)$$

Although the production functions are product-specific, production of the goods is interdependent because the firm-level human resources and capital stock are fixed at the time of production, and hence the firm has to allocate these inputs across the product lines. The allocation is done as a function of the various demand conditions, production technologies, and the price of materials.

3.5 Measurement error

The observed variables are product-level Q_{ijt} and P_{ijt}, and firm-level M_{jt}, L_{jt}, K_{jt} and P_{Mjt}. The firm-level materials, M_{jt}, is measured with multiplicative measurement error:

$$\epsilon_{Mjt}=\frac{M_{jt}}{\sum_{i=1}^{n_{jt}}M_{ijt}}-1. \qquad (13)$$

The other observed variables are measured without measurement error. Also these assumptions are compared to the assumptions previously made in the literature in Section 7.

4 Identification and Estimation Strategy

Firm-level Cobb-Douglas production functions have been estimated in numerous studies. With respect to estimation, the product-specific functions of this paper differ from the firm-level functions in one important aspect: the product-specific inputs are unobservable to the econometrician. This implies that all the elements in the production function are unobservable: input quantities, the output elasticities of the inputs, and total factor productivity. In other words, not only are the the inputs endogenous to the unobservable productivity, which is a standard problem in production function estimation, but they are also unobservable. Clearly, these two problems are closely related.

My identification strategy is based on two insights: one for controlling the endogeneity of inputs to the unobservable productivity level, and another for identifying the unobservable input allocations. The first insight is that, by definition, output is a function of the firm's productivity: the more productive the firm is, the greater its output for any given level of inputs. The unobservable productivity level can be written as a function of the input allocations and the output elasticities of the three inputs, β_{Mi}, β_{Li}, and β_{Ki}. I use this definition of productivity in solving the product-level inputs.

The second insight is that firms make their production decisions as a function of supply-side factors, such as productivity, fixed inputs, and prices of the flexible inputs, but also as a function of the demand for the goods. Intuitively, the higher the demand for a given good, the more inputs the firm is willing to allocate to the product line. Shocks in output demand provide a source of variation for identifying the optimal input allocations. Furthermore, as an overidentifying assumption I can use the notion that the product-level inputs sum up to the observable firm-level inputs.

The optimal input choices are solved analytically from the firm's static profit maximization problem, as a function of the productivity term ω_{ijt} and up to the production function parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} (recall that the state variables $S_{jt} = \left(\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt}\right)$):

$$M_{ijt} = f_M\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{14}$$

$$L_{ijt} = f_L\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) \tag{15}$$

$$K_{ijt} = f_K\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right). \tag{16}$$

As explained above, the first key of the estimation strategy is using the definition of the productivity term ω_{ijt} in controlling for the endogeneity of inputs. Inverting the production function for ω_{ijt}, I get:

$$\omega_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\beta_{0i}) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}}\right). \tag{17}$$

By substituting this definition of ω_{ijt} in the analytical input functions $M_{ijt}, L_{ijt}, K_{ijt}$, I obtain:

$$
\begin{aligned}
M'_{ijt} &= g_M\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) && (18)\\
L'_{ijt} &= g_L\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) && (19)\\
K'_{ijt} &= g_K\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right), && (20)
\end{aligned}
$$

where S'_{ijt} denotes the state variables without ω_{ij}. By imposing $M'_{ijt} = M_{ijt}$, $L'_{ijt} = L_{ijt}$, and $K'_{ijt} = K_{ijt}$, and substituting $M'_{ijt}, L'_{ijt}, K'_{ijt}$ and the definition of ω_{ijt} in the production function, I take account of the unobservable productivity level. The production function for good i can then be written as:

$$
Q_{ijt} = \exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right), \tag{21}
$$

where β_{0i}, β_{Mi}, β_{Li} and β_{Ki} are the only unobservables. But when written in this form, an infinite number of parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} solve the empirical production function. This is because ω_{ijt} is inverted from the production function itself. However, the production function can be identified using the structure of the productivity process, which is a function of the expectation of productivity $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$, and the productivity shock ξ_{ijt}.

Using the productivity shock ξ_{ijt} in identification is a standard practice in structural production function models (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves and Frazer, 2006). Lagged static inputs, in this paper M_{ijt}, are correlated over time but uncorrelated with the productivity shock. Fixed inputs, in this case L_{ijt} and K_{ijt}, are chosen prior to observing ξ_{jt}. Hence, they are not correlated with the productivity shock. As the fixed inputs L_{ijt} and K_{ijt} are subject to different input costs, the two variables are not collinear.

Given the standard assumptions I make regarding the timing of input choices, and given that there are sufficiently many sources of identifying variation, the above moments can be modified to suit the production function specified in this paper. The productivity shocks only have to be specified at the product-level:

$$
\begin{aligned}
E\left[\xi_{ijt}|M_{jt-1}\right] &= 0\ \forall\ i = [1, N] && (22)\\
E\left[\xi_{ijt}|L_{jt}\right] &= 0\ \forall\ i = [1, N] && (23)\\
E\left[\xi_{ijt}|K_{jt}\right] &= 0\ \forall\ i = [1, N]. && (24)
\end{aligned}
$$

The firm-level M_{jt-1}, L_{jt}, and K_{jt} are correlated with the product-level M_{ijt}, L_{ijt}, and K_{ijt} because the firm-level variables are sums of the product-level inputs. An additional instrument is the price of the flexible input, correlated with M_{ijt} but uncorrelated with ξ_{ijt}:

$$
E\left[\xi_{ijt}|P_{Mjt}\right] = 0\ \forall\ i = [1, N]. \tag{25}
$$

P_{Mjt} is a valid instrument even if measured with error because the measurement error is not correlated with the productivity shock.

Demand for good i would also be a valid instrument. Demand for good i correlates positively with the input choices M_{ijt}, L_{ijt} and K_{ijt}, while it is uncorrelated with the productivity shock ξ_{ijt}. Unfortunately, the demand is unobservable. However, the output prices are informative about the underlying demand. Price for good i depends on the output quantity produced and the level of productivity at which it is produced, that is, P_{ijt} is correlated with the productivity shock and hence not a valid instrument. However, lagged price P_{ijt-1} is correlated with the demand for good i also at time t, and hence with the input choices M_{ijt}, L_{ijt} and K_{ijt}, because demand for good i is correlated over time as denoted by α_{ij}. At the same time, P_{ijt-1} is uncorrelated with the productivity shock:

$$E\left[\xi_{ijt}|P_{ijt-1}\right] = 0 \; \forall \; i = [1, N]\,. \tag{26}$$

I also use the fact that product-level inputs M_{ijt} add up to the firm-level input M_{jt}, which is observable but measured with measurement error. Any firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}, is expected to be zero. A valid instrument for identifying β_{Mi} is the product of output price and quantity, $P_{ijt}Q_{ijt}$, which is uncorrelated with the measurement error in materials ϵ_{Mjt}, but correlated with the use of materials M_{ijt}:

$$E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0 \; \forall \; i = [1, N]\,. \tag{27}$$

These moment conditions identify the production technologies.

Identification of the demand functions requires an instrument[14] for the endogeneous prices. The material price P_{Mjt}, human resources L_{jt}, and capital stock K_{jt} correlate with the product prices but they are uncorrelated with the product- and firm-specific demand shocks ε_{ijt}:

$$\begin{aligned} E\left[\varepsilon_{ijt}|P_{Mjt}\right] &= 0 \; \forall \; i = [1, N] \qquad (28)\\ E\left[\varepsilon_{ijt}|L_{jt}\right] &= 0 \; \forall \; i = [1, N] \qquad (29)\\ E\left[\varepsilon_{ijt}|K_{jt}\right] &= 0 \; \forall \; i = [1, N]\,. \qquad (30) \end{aligned}$$

The model is identified with these moments and estimated by GMM.

[14] For a discussion on instruments used in demand estimation, see, for example, Ackerberg, Benkard, Berry and Pakes, 2007.

4.1 Solving for ξ_{ijt}, ε_{ijt}, and ϵ_{Mjt}

The productivity shock ξ_{ijt} is:

$$\xi_{ijt} = \log\left(\frac{Q_{ijt}}{\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}}\right) - E\left[\omega_{ijt}|\omega_{ijt-1}\right] \tag{31}$$

where M_{ijt}, L_{ijt}, K_{ijt} and $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$ are unknown. M_{ijt}, L_{ijt}, and K_{ijt} are solved from the first-order conditions for static profit maximization, the definition of productivity for the estimation equation, $\omega_{ijt} = \log\left(Q_{ijt}(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}})^{-1}\right)$, and the demand function inverted for price, $P_{ijt} = \exp\left(\alpha_{ij} + \varepsilon_{ijt}\right)^{-\frac{1}{\eta_i}} Q^{\frac{1}{\eta_i}}$. By substitution:

$$M_{ijt} = \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \frac{\beta_{Mi}}{P_{Mjt}} \ \forall\ i = [1, n_{jt}] \tag{32}$$

$$L_{ijt} = \frac{\left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Li} L_{jt}}{\sum_i \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Li}} \ \forall\ i = [1, n_{jt}] \tag{33}$$

$$K_{ijt} = \frac{\left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Ki} K_{jt}}{\sum_i \left(\frac{1}{\eta_{ijt}} + 1\right) P_{ijt} Q_{ijt} \beta_{Ki}} \ \forall\ i = [1, n_{jt}]\,. \tag{34}$$

Given M_{ijt}, L_{ijt}, K_{ijt}, and the implied ω_{ijt}, the productivity process is estimated with the following estimation equation:

$$\omega_{ijt} = g\left(\omega_{ijt-1}\right) + \xi_{ijt} \tag{35}$$

where $g\left(\omega_{ijt-1}\right)$ is a second-order polynomial of the lagged productivity term $\omega_{ijt-1}\left(\beta_{Mi}, \beta_{Li}, \beta_{Ki}\right)$, and ξ_{ijt} is the productivity shock.[15]

Given the solution for M_{ijt} (32), the multiplicative input measurement error ϵ_{Mjt} is computed as:

$$\epsilon_{Mjt} = \frac{M_{jt}}{\sum_{i=1}^{n_{jt}} M_{ijt}} - 1. \tag{36}$$

The demand shock ε_{ijt} is:

$$\varepsilon_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\alpha_{ij}) P_{ijt}^{\eta_i}}\right) \tag{37}$$

where the unobservable product-firm -specific demand level, α_{ij} is (T_{ij} is the number of

[15] The parameters in the polynomial $g\left(\omega_{ijt-1}\right)$, denoted by γ_i, enter the moment conditions linearly. Hence they can be concentrated out from the estimation routine for the nonlinear parameters. The linear parameters γ_i are obtained by regressing the productivity level implied by a given set of parameter values $\omega_{ijt}\left(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki}\right)$ on the second-order polynomial terms of the implied lagged productivity $\omega_{ijt-1}\left(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki}\right)$.

time periods in which firm j has produced good i):

$$\alpha_{ij} = T_{ij}^{-1} \sum_{t=1}^{T_{ij}} \log \left(\frac{Q_{ijt}}{P_{ijt}^{\eta_i}} \right). \quad (38)$$

5 Data and Empirical Implementation

5.1 Data

I use the Longitudinal Database on Plants in Finnish Manufacturing (LDPM) and the Industrial output data of Statistics Finland on years 2004 - 2011. The two datasets include plants that belong to manufacturing firms with at least 20 employees, and a subset of plants of firms with less than 20 employees. The reporting units are mainly plants. The only exceptions are in the Industrial output data, where a few plants belonging to the same firm report jointly. For these reporting units I aggregate the observations in the LDPM accordingly.

I estimate the production functions of firms in Division 16, "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". This industry is a good example of an industry where firms manufacture several different goods and, presumably, employ multiple different technologies. The division is also one of the biggest in the Finnish manufacturing sector. The products are classified according to Eurostat's 8-digit PRODCOM (Production communautaire) codes that are supplemented by national 10-digit subclasses. Goods within the fairly narrowly defined titles are therefore comparable in physical quantities.[16] The titles are provided in Table 1. For each product title a plant produces in a given year, I observe the output measured in a physical unit as well as the sales revenue. These two yield the average price of the good in the given year. Similarly for the intermediate products and materials I observe physical quantities and expenditures by the PRODCOM titles. The "price" of materials is computed as the Elteto-Koves-Szulc (EKS) multilateral price index (see, for example, Hill, 2004, and Neary, 2004). For firm a it can be expressed as follows:

$$P_{EKS}^{a} = \prod_{j=1}^{J} \left(\frac{P_F\left(q^j, q^a, p^j, p^a\right)}{P_F\left(q^j, q^b, p^j, p^b\right)} \right)^{\frac{1}{J}}, \quad (39)$$

where q^j and p^j are the quantity and price vectors of firm j, and $P_F\left(q^j, q^a, p^j, p^a\right)$ is the bilateral Fisher price index between firm a and firm j, $j = 1, ..., J$ (J is the number of firms), which is given by

[16] Since the production function to be estimated is not a revenue production function, the outputs of a given product line need to be comparable in physical quantities. Quality differences can be accommodated by defining separate production functions for goods of different quality.

Table 1

PRODCOM	Title
16.10.10.33	Coniferous wood; sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm, end-jointed, sanded or planed
16.10.10.33.10	Spruce wood (Picea abies Karst.), sanded or planed, end-jointed, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm
16.10.10.33.20	Pine wood (Pinus sylvestris L.), sanded or planed, end-jointed, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm
16.10.10.35	Spruce wood (Picea abies Karst.), fir wood (Abies alba Mill.)
16.10.10.37	Pine wood (Pinus sylvestris L.)
16.10.10.50	Wood, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6mm (excluding coniferous and tropical woods and oak blocks, strips and friezes)
16.10.21.10	Coniferous wood continuously shaped (including strips and friezes for parquet flooring, not assembled)
16.10.23.03	Coniferous wood in chips or particles
16.10.23.05	Non-coniferous wood in chips or particles
16.10.41.00.10	Sawdust
16.10.41.00.20	Woodchips
16.10.41.00.40	Lathes, borders, etc.
16.10.41.00.60	Bark
16.10.41.00.80	Other wood waste (excluding sawdust, woodchips, bark, lathes, borders, pellets, briquettes etc.)
16.21.11.00	Plywood, veneered panels and similar laminated wood, of bamboo
16.21.12.14	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceeding 6 mm thickness, with at least one outer ply of non-coniferous wood (excluding trpical wood)
16.21.12.17	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceeding 6 mm thickness (excluding products with at least one outer ply of tropical wood or non-coniferous wood)
16.21.13.13	Particle board, of wood
16.21.21.18.30	Veneer for plywood, cross-banded plywood and other wood, of coniferous wood, sawn lengthwise, sliced or peeled, of a thickness <=6mm (excluding end-jointed, planed, sanded and board for manufacturing pencils)
16.21.21.18.80	Veneer for plywood, cross-banded plywood and other wood, of hardwood, sawn lengthwise, sliced or peeled, of a thickness <=6mm (excluding end-jointed, planed, sanded and board for manufacturing pencils)
16.21.22.00	Densified wood, in blocks, plates, strips or profile shapes

16.22.10.60	Parquet panels of wood (excluding those for mosaic floors)
16.23.11.10	Windows, French-windows and their frames, of wood
16.23.11.50	Doors and their frames and thresholds, of wood
16.23.19.00.12	Carpenter's produce for walls, of wood
16.23.19.00.16	Carpenter's produce for stairs, of wood
16.23.19.00.26	Components for sauna, of wood
16.23.19.00.32	Panel elements (also glulam and cellular panels), of wood
16.23.19.00.36	Ceiling elements, of wood
16.23.19.00.42	Glulam beams and columns
16.23.19.00.46	Vertical and horizontal beams (excluding glulam beams and columns)
16.23.19.00.52	Log frames for buildings of wood
16.23.19.00.90	Other carpenter's produce, of wood
	(excluding doors, windows, produce for floors, walls, stairs and sauna, panel and ceiling elements, beams, columns and log frames)
16.23.20.00.20	Residential buildings of wood, for permanent habitation
16.23.20.00.40	Residential buildings of wood, for recreational use
16.23.20.00.60	Saunas of wood (outdoor saunas, assembled or prefabricated)
16.23.20.00.90	Buildings of wood (assembled or prefabricated) (excluding residential buildings and saunas)
16.24.11.35	Box pallets and load boards of wood (excluding flat pallets)
16.24.13.20	Cases, boxes, crates, drums and similar packings of wood (excluding cable drums)
16.24.13.50	Cable-drums of wood
16.29.14.90	Other articles of wood (excluding pallet collars)

$$P_F\left(q^j, q^a, p^j, p^a\right) = \left(\frac{q^j * p^a}{q^j * p^j} * \frac{q^a * p^a}{q^a * p^j}\right)^{\frac{1}{2}}, \tag{40}$$

where $q^j * p^j = \sum_{n=1}^{N} q_n^j p_n^j$ (N is the number of product titles). Similarly for $P_F\left(q^j, q^b, p^j, p^b\right)$, where b stands for the base firm chosen. The EKS multilateral index satisfies the circularity (transitivity) requirement, which implies that the same index is obtained irrespective of whether firms are compared with each other directly, or through their relationships with other firms (Hill, 2004; Neary, 2004). The EKS multilateral index is thus well-suited for my purpose of comparing firms when no representative firm exists, and bundles of goods differ between firms.

The labor input is measured in labor costs that comprise salary and social payments. The monetary value of the capital stock is estimated using the perpetual inventory method, $K_{jt} = \delta K_{jt-1} + I_{jt-1}$, where $\delta = 0.9$ and I_{jt} is investment.

The estimation methodology poses certain requirements on the observations. First, all product titles need to be observed in at least four pairs of observations, each pair being from two consecutive years in a given firm. This is because for each product title there are four non-linear parameters to be estimated, and because estimating the 1st order Markov process of productivity evolution requires sequences of at least two observations. Second, observations with missing variables cannot be used in estimation. Observations that do not fulfill the aforementioned criteria are dropped from the sample.

Note that measurement error in output is assumed to be zero. Unfortunately, there is no other output variable that could be used to verify the accuracy of the product-specific sales revenue variables. The only other output variable available is the plant-level gross output reported in the LDPM. Gross output is defined as the sum of sales revenue, deliveries to other plants of the firm, changes in inventories, production for own use, and other business revenue, deducting capital gains and acquisition of merchandise. Not suprisingly, gross output is not equal to the sum of product-specific sales revenues from production in all of the plants. As the definition of gross output goes, there are several potential explanations for this. Plants may produce output that is not included in the sales revenue from production (deliveries to other plants of the firm, positive changes in inventories, production for own use), or the sales revenue data may include output produced in some previous year (negative changes in inventories). Moreover, because capital gains and acquisition of merchandise are deducted from gross output, it is not possible to make strong inferences about potential measurement error in output. Unfortunately, the various components of gross output are not reported in the LDPM, and hence I cannot identify why gross output may differ from sales revenue. However, to reduce the likelihood of using observations with major measurement error in output, I use only those observations for which the ratio of sum of sales revenue to gross output is at least 0.6 but not more than 1.4.

In the final sample there are 2053 good-plant-year -level observations and 904 plant-year -level observations, collected from 190 plants during 8 years. In total, 42 different product titles are produced. Plants' product assortments range from 1 up to 17 product titles, 659 out of 904 firms producing at least two product titles. A plant produces on average 3.25 product titles. Products assortments vary across firms, i.e., there are no typical product combinations. The correlation between producing two goods is low for most of the product pairs: the absolute value of the correlation coeffiecient is lower than 0.05 (0.1) [0.2] for 63% (81%) [91%] of all the product pairs.

5.2 Product line specification

Every product title i is related to four nonlinear parameters that need to be estimated: price elasticity η_i, and output elasticities β_{Mi}, β_{Li} and β_{Ki}. If I defined the parameters at the 8- or 10-digit level, I would need to estimate $42 \times 4 = 168$ nonlinear parameters. At least in my setting this is too many. Instead, I define the parameters at the 3-digit level, which yields two product categories: "Sawmilling and planing of wood" (PRODCOM code 161), and "Manufacture of products of wood, cork, straw and plaiting materials" (162). This specification implies estimating $2 \times 4 = 8$ nonlinear parameters. The parameters governing the productivity process $g(\omega_{ijt-1})$ are also specified at the 3-digit level. The constants β_{0i} are specific to the goods as defined at the 8- or 10-digit level. Also the productivity levels ω_{ijt} and the productivity shocks ξ_{ijt} are specific to the 8- or 10-digit titles.

There are 15 product titles in category 161, and 27 titles in category 162. A plant produces on average 2.17 titles in category 161, and 1.08 titles in category 162. 56% of the plants in the sample produce at least one good in category 161, and 61% of the plants produce at least one good in category 162. 86% of the plants that produce any good in category 161 produce at least two titles in that category. Similarly, 43% of the plants that produce any good in category 162 produce more than one title in that category.

5.3 Optimal instruments

To improve the estimator's efficiency, I replace some of the moment conditions discussed above by moments with optimal instruments. Amemiya (1974) derives optimal instruments for non-linear models, and Arellano (2003) provides an overview of optimal instruments in linear and nonlinear models. Reynaert and Verboven (2012) show that adopting Chamberlain's (1987) optimal instruments in estimating the randon coefficients logit demand model of Berry, Levinsohn, Pakes' (1995) reduces the small sample bias and increases the estimator's efficiency and stability.

The optimal instrument is the expected value of the derivative of the structural error term with respect to the parameter, computed at an initial estimate of the parameters:

$$z_{ijt} = E\left[\frac{\partial \xi_{ijt}(\theta)}{\partial \theta'} \mid X_{ijt}\right] \tag{41}$$

where θ contains the parameters to be estimated, $\theta = (\eta, \beta, \gamma)$, and X_{ijt} comprises the observables, $X_{ijt} = (Q_{ijt}, P_{ijt}, P_{Mjt}, L_{jt}, K_{jt})$. Because the optimal instruments are non-linear functions of the parameters to be estimated, they cannot be computed directly from the data. Instead the optimal instruments are updated after each stage of GMM. In the first stage I use starting values that are an educated guess of the parameters. For the subsequent rounds, the optimal instruments are recomputed using the parameter estimates from the previous stage of GMM.

I replace all the supply-side moments with productivity shocks ξ_{ijt} and standard instruments by moments with optimal instruments. As compared to the empirical model with standard instruments, the objective function appears smoother, and the estimates less responsive to the starting values. This is because the functional forms imposed are exploited to a fuller extent.

I do not adopt optimal instruments for the other moments, i.e., the moments that contain the measurement error ϵ_{Mjt} or demand shock ε_{ijt}. The reason is that writing optimal instruments when the structural error term is a function of endogenous observations is complicated (Arellano 2003). In summary, the moment conditions I use are:

$$\begin{array}{cc}
\text{Moment} & \text{Parameter identified} \\
E\left[\xi_{ijt}|z_{Mijt}\right] = 0 \;\forall\; i = [1,N] & \beta_{Mi} \\
E\left[\xi_{ijt}|z_{Lijt}\right] = 0 \;\forall\; i = [1,N] & \beta_{Li} \\
E\left[\xi_{ijt}|z_{Kijt}\right] = 0 \;\forall\; i = [1,N] & \beta_{Ki} \\
E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0 \;\forall\; i = [1,N] & \beta_{Mi} \\
E\left[\varepsilon_{ijt}|P_{Mjt}\right] = 0 \;\forall\; i = [1,N] & \eta_i \\
E\left[\varepsilon_{ijt}|L_{jt}\right] = 0 \;\forall\; i = [1,N] & \eta_i \\
E\left[\varepsilon_{ijt}|K_{jt}\right] = 0 \;\forall\; i = [1,N] & \eta_i
\end{array} \tag{42}$$

As four moment conditions are sufficient for exact identification of the model, there are three overidentifying restrictions in the above set of moments. Some of the 8- or 10-digit product titles have at least four but less than seven observation pairs. In these cases I cannot use all the seven moment conditions. Instead of dropping observations of the product title entirely, I drop some of the overidentifying moments for these products. For product i with only four observations pairs, I adopt moments $E\left[\xi_{ijt}|z_{Mijt}\right] = 0$, $E\left[\xi_{ijt}|z_{Lijt}\right] = 0$, $E\left[\xi_{ijt}|z_{Kijt}\right] = 0$, and $E\left[\varepsilon_{ijt}|P_{Mjt}\right] = 0$. Moment $E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0$ ($E\left[\varepsilon_{ijt}|L_{jt}\right] = 0$) [$E\left[\varepsilon_{ijt}|K_{jt}\right] = 0$] is used when there is at least five (six) [seven] observation pairs.

5.4 Estimation algorithm for the moments

The parameters are estimated by iterated GMM. The estimation algorithm for computing the moments is as follows. In the beginning of each outer GMM iteration, I compute the optimal instruments given the starting values for η_i, β_{Mi}, β_{Li}, and β_{Ki}. First, I compute the productivity level implied by the starting values for η_i, β_{Mi}, β_{Li}, and β_{Ki}, call it h_{ijt} (instead of ω_{ijt}). Second, I estimate the parameters in the implied productivity process $g(h_{ijt-1})$ by OLS where h_{ijt} is the dependent variable, and the explanatory variables are polynomial terms of h_{ijt-1} as in the Markov process for productivity. Finally, I compute the optimal instruments as functions of the data, the starting values for η_i, β_{Mi}, β_{Li}, and β_{Ki}, and the implied estimates of the parameters in $g(h_{ijt-1})$. On the second and any subsequent outer GMM iteration, the starting values for η_i, β_{Mi}, β_{Li}, and β_{Ki} are the parameter estimates obtained on the previous outer GMM iteration.

On each inner GMM iteration, I compute the residuals ξ_{ijt}, ϵ_{Mjt}, and ε_{ijt} given some values for η_i, β_{Mi}, β_{Li}, and β_{Ki}, and then compute the moments given the optimal instruments computed in the beginning of the outer GMM iteration. First, I compute the productivity level implied by the values for η_i, β_{Mi}, β_{Li}, and β_{Ki} of the iteration, again call it h_{ijt}. Second, I estimate the parameters in the implied productivity process $g(h_{ijt-1})$ by OLS where h_{ijt} is the dependent variable, and the explanatory variables are polynomial terms of h_{ijt-1}. Third, I compute the productivity shock ξ_{ijt} as well as the measurement error in the static input ϵ_{Mjt} as described in equations (31) and (36), respectively. The demand shocks ε_{ijt} are computed given the output and price data, and some value for η_i, as described in equations (37) and (38). Finally, I compute the moments.

6 Results

As there are multiple parameters to be estimated that enter the GMM objective function non-linearly, finding the global minimum can be challenging. To make sure that the estimation routine reaches the global minimum of the GMM objective function, I experiment with various minimization algorithms, of which the Gauss-Newton algorithm turns out to perform best in finding the global minimum among the local minima. I also run the estimation routine with a large set of alternative starting values.[17]

The estimation results are presented in Table 2. The two production functions and demand functions estimated are for two groups: "Sawmilling and planing of wood" (PRODCOM titles 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM titles 162). All the non-linear parameter estimates are statistically significant.[18] Also, the estimates of the output elasticities are statistically different for the

[17] The starting values for β_{Mi}, β_{Li} and β_{Ki} range between 0.15 and 0.5, and the starting values for η_i between -8 and -1.5.

[18] The product-firm specific demand levels α_{ij}, the 42 constants β_{0i}, and the parameters governing the productivity process $g(\omega_{ijt-1})$ are not reported.

technologies of the two product groups. The output elasticity of materials is considerably higher in the technology for titles 162 than in the technology for 161 (β_{Mi} for 162 is 0.74 and β_{Mi} for 161 is 0.38). The output elasticity of labor, again, is considerably lower in the technology for titles 162 (β_{Li} for 162 is 0.12 and β_{Li} for 161 is 0.35). Both technologies have output elasticity of capital of the same magnitude (β_{Ki} for 161 is 0.19 and β_{Ki} for 162 is 0.18). Returns to scale are different for the two technologies: the technology for product titles 161 is subject to decreasing returns to scale ($\beta_{Mi} + \beta_{Li} + \beta_{Ki} = 0.93 < 1$) , while the technology for titles 162 has increasing returns to scale ($\beta_{Mi} + \beta_{Li} + \beta_{Ki} = 1.04 > 1$). In short, the various goods in the product groups "Sawmilling and planing of wood" and "Manufacture of products of wood, cork, straw and plaiting materials", which many multiproduct firms simultaneously produce, are not manufactured with a single firm-level production technology.

Table 2. Parameter estimates
PRODCOM 161 Sawmilling and planing of wood
PRODCOM 162 Manufacture of products of wood, cork, straw and plaiting materials

	Parameter estimate (standard error)	
	PRODCOM 161	PRODCOM 162
Materials	0.37 (0.008)	0.73 (0.002)
Labor	0.36 (0.011)	0.13 (0.003)
Capital	0.20 (0.008)	0.18 (0.003)
Price elasticity of demand	-1.29 (0.020)	-1.13 (0.004)
Prob[Chi-sq.(264)>J]	0.4632	
Number of obs.	2053	

The demand for titles 161 is more price elastic than the demand for titles 162, as η_i for titles 161 is -1.30 and η_i for titles 162 is -1.12. This is intuitive because products of wood, cork, straw and plaiting materials are likely to be more differentiated than the output of sawmilling and planing of wood. Hansen's J-test does not reject the null hypothesis of valid overidentification restrictions (Prob[Chi-sq.(264)>J] is 0.4632).

6.1 Comparison with firm-level estimates

To show how the assumption of a firm-level, or single-product, production technology changes production function estimates and the implied productivity and inter-firm productivity differences, I estimate the production function also with the single-product assumption. Estimating the single-product production function differs from the multiproduct specification in three respects. First, while the product-level output variables can be easily measured in at least one physical measure, there is no obvious physical output variable at the firm-level. Typically the firm-level output variable used in the literature is the sum of sales revenue from all the goods. However the sales revenue is a problematic measure of output because it involves also price-cost markups and is hence dependent on the firm's market power. The output measure I use is the EKS multilateral quantity index, which I compute similarly as for the material price as explained in section 5.1.

The second difference that imposing the single-product assumption entails is that the input variables are now observable at the level the production function is specified at, i.e., at the firm-level. Hence there is no need to estimate them. This relates to the third difference: because there is no firm-level demand function, demand cannot be used in identification of the production function parameters and solving for the inputs. Therefore also measurement error in the static input M has to be assumed away. As a consequence the set of moment conditions reduces to:

$$\begin{array}{cc} \text{Moment} & \text{Parameter identified} \\ E\left[\xi_{jt}|z_{Mjt}\right]=0 & \beta_M \\ E\left[\xi_{jt}|z_{Ljt}\right]=0 & \beta_L \\ E\left[\xi_{jt}|z_{Kjt}\right]=0 & \beta_K \end{array} \tag{43}$$

As opposed to the multiproduct production function model, the single-product specification is exactly identified. In addition, finding the global minimum of the objective function turns out to be less dependent on the starting values.

The firm-level estimation results are presented in Table 3. The estimated output elasticity of materials β_M is 0.56 which is in between the estimated product-level elasticities β_{Mi} (0.37 and 0.73 for titles 161 and 162, respectively.) The firm-level estimate of output elasticity of labor β_L is 1.04 which is considerably higher than any of the two product-level estimates β_{Li} (0.36 for titles 161 and 0.13 for titles 162). The estimated firm-level output elasticity of capital β_K is -0.21 which is not only considerably lower than the product-level estimates β_{Li} (0.20 and 0.18 for titles 161 and 162, respectively) but also negative. However none of the output elasticity estimates is statistically significant. In short, the firm-level estimates give a rather different description of how materials, labor and capital augment output, as compared to the multiproduct production functions estimated above.

Table 3. Firm-level parameter estimates

	Parameter estimate (standard error)
Materials	0.56 (0.814)
Labor	1.04 (1.737)
Capital	−0.21 (0.627)
Number of obs.	904

In the Monte Carlo simulations reported in the appendix of this paper, I characterize the directions and magnitudes of the parameter estimate biases caused by firm-level production function estimation. The biases can go in either direction, towards or away from zero, depending on how the true product-level technologies compare with each other. The magnitudes of the biases grow in the difference in the returns to scale between the true product-level technologies. As the returns to scale of the estimated technologies for titles 161 and 162 are as different as 0.93 and 1.04, the large biases in the firm-level estimates are not surprising.

An interesting finding implied by production function estimates is how producers compare with each other in terms of productivity. The production function estimates of the single- and multiproduct specifications are likely to imply different productivity levels. I compare the productivity level implied by the firm-level estimates to a firm-level weighted average of the product-specific productivity levels. The weights are the output shares generated by the estimated product-level inputs with productivity level $\exp(\omega_{ijt}) = 1$ for each good, and the firm-level weighted average is denoted by $\varkappa_{jt}$:

$$\varkappa_{jt} = \sum_{i=1}^{N} \frac{M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}}{\sum_{i=1}^{N} M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}}} \exp(\omega_{ijt}) \tag{44}$$

Figure 4. Productivity levels implied by single- vs. multiproduct production function estimates

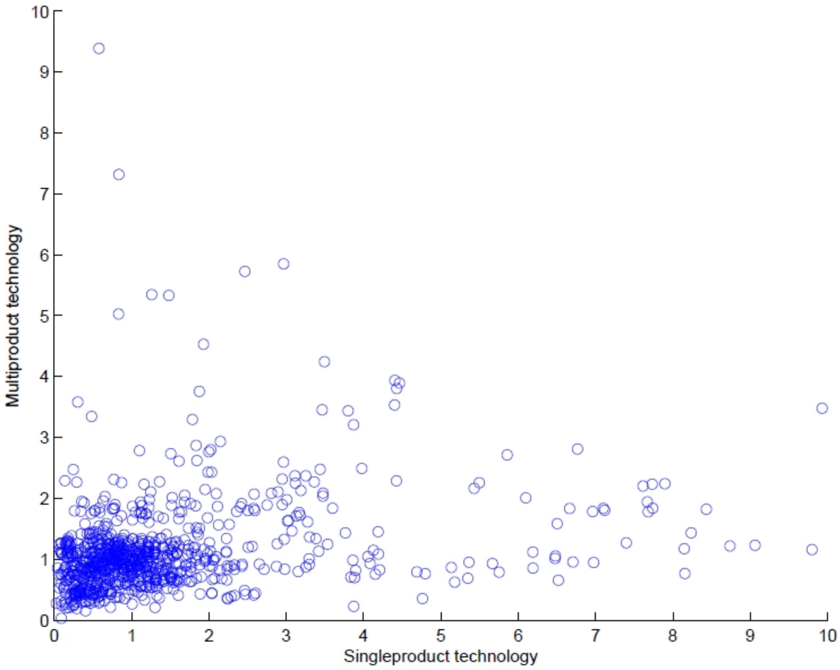

Figure 4 shows a scatter plot of the firm-level productivity measures obtained with the single- and the multiproduct production function specifications. If the difference in the single- vs. multiproduct assumption had no effect on the implied productivity levels, all the points would lie on a 45 degree line. Clearly this is not the case: the productivity level of some of the firms is underestimated, while for others the productivity level is overestimated. Table 5 gives an alternative description of this finding by showing how the firm-level productivity measures obtained under the multiproduct and the single-product firm assumptions are jointly distributed within the respective productivity percentiles. The large share of the off-diagonal entries shows that the single-product technology assumption affects the productivity estimates of a large share of observations. The correlation coefficient between the firm-level productivity implied by the product-specific estimates, $\varkappa_{jt}$, and the firm-level production function estimates is 0.44. The correlation coefficient is positive, but not close to 1. This finding is line with the Monte Carlo simulations which show that the productivity level implied by the firm-level estimates and the true productivity level have a low correlation when the firms' product scopes are heterogenous, as in the dataset used in this paper.

Table 5. Joint distribution of firm-level productivity percentiles estimated under multiproduct (MP) and single-product (SP) assumptions, measured in percentages (due to approximating the percentages do not add up to 100)

	Percentile, SP					
Percentile, MP	P ≤ 10	10 > P ≤ 25	25 > P ≤ 50	50 > P ≤ 75	75 > P ≤ 90	P > 90
P ≤ 10	2	3	2	1	1	0
10 > P ≤ 25	1	3	3	5	1	1
25 > P ≤ 50	2	4	7	7	3	2
50 > P ≤ 75	3	2	9	8	3	1
75 > P ≤ 90	1	2	3	4	4	1
P > 90	0	0	1	1	3	4

The estimated productivity differences between producers are also affected by the single- vs. multiproduct production function assumption. Figure 6 shows the disributions of the firm-level productivity measures implied by the product- and the firm-level production function estimations (solid and dashed lines, respectively). The firm-level weighted averages of the product-specific productivity levels, $\varkappa_{jt}$, have a narrower distribution than the productivity levels implied by firm-level estimates. In other words, the productivity differences implied by the multiproduct specification are smaller than the productivity differences implied by the single-product production function estimates. Table 7 reports percentiles of productivity implied by the single- and the multiproduct specifications. According to the firm-level production function estimates, the firm at the 90th percentile is approximately 12.7 times as productive as the firm at the 10th percentile. The equivalent ratio obtained using the product-level estimates is 4.2. Imposing the single-product assumption on multiproduct firms may therefore explain some of the large inter-firm productivity differences estimated in the productivity literature (Doms and Bartelsman, 2000;

Syverson, 2011).

Figure 6. Distributions of productivity levels implied by single-vs. multiproduct production function estimates

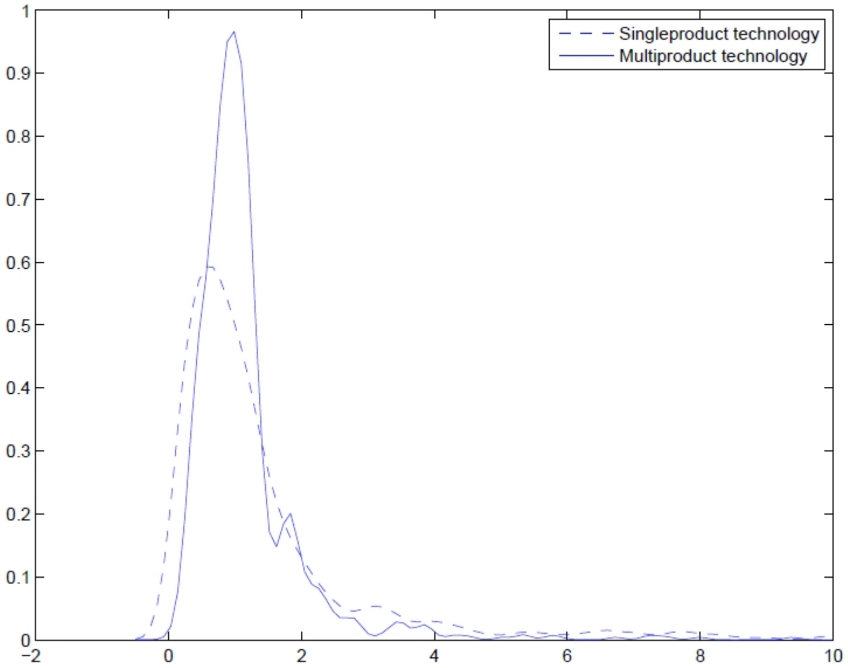

Table 7. Percentiles of productivity distributions implied by single -vs. multiproduct production function estimates

	Percentile				
	10	25	50	75	90
Productivity, SP	0.27	0.53	9.95	1.61	3.42
Productivity, MP	0.46	0.72	0.98	1.26	1.94

7 Discussion on Identification

The structural production function literature focuses on correcting for endogeneity biases. Several papers build on the insight of Olley and Pakes (1996) that because inputs are set as a function of the firm's productivity, input demand can be inverted for the unobservable productivity term. Subsequently this idea, referred to as the proxy method, has been used by Levinsohn and Petrin (2003), Ackerberg, Caves and Frazer (2006), Wooldridge (2009), and Doraszelski and Jaumandreu (2013). Gandhi, Navarro and Rivers (2013) use firms' short run first order conditions to control for the collinearity of inputs. Most of the assumptions underlying my identification strategy are familiar from this literature. I make also some novel assumptions, and relax some of the assumptions previously made.

All the moment conditions, in my and other structural production function estimation strategies, are based on assumptions about the timing of input choices with respect to productivity shocks. In addition, I specify the role of demand shocks in production choices. Materials M_{ijt} are chosen only after the demand and productivity shocks ε_{ijt} and ξ_{ijt} have been observed, while the firm-level labor L_{jt} and capital stock K_{jt} are determined before the shocks. These assumptions are standard in the literature, apart from taking account of the demand shocks in production decisions, and assuming L_{jt} to be a fixed variable. The reason for treating L_{jt} as a fixed input is not technical, but this assumption is made to account for the environment in which the data has been generated: employment protection legislation plays a significant role in Finland. The OECD indicators of employment protection (OECD, 2013) measure the strictness of legislation on individual and collective dismissals and the strictness of hiring employees on temporary contracts. The measures are based on information about statutory and case laws, collective bargaining agreements, and advice by officials from OECD member countries and country experts. According to these indicators, the Finnish labor market was of the OECD average in the strictness of employment protection during the period of 2004 to 2011. Based on this measure, fixed labor input is a realistic assumption. In case the method of this paper is to be used for estimating production functions in an economy where flexible labor input is a more appropriate assumption, the empirical model can be adjusted accordingly. As in other structural production function models, one flexible input is required for inverting out the unobservable productivity ω_{ijt}. I also further specify that the product-level labor and capital allocations L_{jt} and K_{ijt} are set as endogeneous to ε_{ijt} and ξ_{ijt}. This assumption not only facilitates the estimation of L_{jt} and K_{ijt}, but also allows firms to reallocate human resources and capital as response to demand and productivity shocks.

One more difference in the timing assumptions of this and other structural estimation strategies is that I assume away productivity shocks once the flexible inputs have been set, and measurement error in output Q_{ijt}. I make these assumptions in order to solve for the unobservable input allocations, while controlling for the unobservable productivity ω_{ijt}. At the same time, and in contrast to the rest of the literature, I allow for measurement error in the flexible inputs M_{jt} observed at the firm-level. This provides me an additional moment condition for identifying β_{Mi}, as compared to the other production models: sales revenue from a given product correlates positively with the flexible input M_{ijt} allocated to the product line, but is uncorrelated with the firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}.

In addition to the timing assumptions, the proxy methods require two more key assumptions. First, input demand is assumed monotonic in productivity. In other words, cases where input demand may decrease due to improved efficiency are assumed away. However, this assumption may be unrealistic in settings where firms face downward sloping demand curves. In fact, Ornaghi and Beveren (2011) find that the monotonicity assumption fails to hold for the majority of structural production function estimators. I relax the monotonicity assumption by using the definition of productivity itself in controlling for endogeneity.

Second, the proxy methods require the assumption that productivity ω_{ijt} is the only scalar unobservable that affects the input choices. Unobservable inter-firm variation in, say, input prices or output demand, as well as optimization and measurement error in the flexible inputs, are assumed away. I also need to make the scalar unobservability assumption for estimating product-level inputs. However, I do allow for measurement error in the flexible inputs. I also allow for inter-firm variation in input prices and output demand. In fact, I need input prices and estimates of output demand for estimating the input allocations. At the same time, variation in the input prices resolves the collinearity problem between the flexible input M_{ijt} and the other inputs. What the scalar unobservability assumption in my application implies is that the price a firm pays for its flexible input, P_{Mjt}, does not depend on the quantity purchased M_{ijt}. By modelling supply in the input market this assumption could be relaxed, however. As in other empirical strategies, I also assume that the input demand function is continuous. In other words, firms can purchase precisely the input quantity that maximizes their profit. This seems justified after eyeballing the firm-level input data.

The last set of supply-side assumptions that I make concerns the inputs. Units of the firm-level input stocks L_{jt} and K_{jt} are substitutable between product lines, and there are no adjustment costs in (re)allocating labor or capital to other product lines. Also, a firm does not use production of a given good as an input for another good. These assumptions are not specific to this product-specific model, but they are made implicitly in all firm-level estimations when firms produce more than one type of good.

In contrast to the other structural methods, the one of this paper requires demand estimates for identifying the unobservable input allocations. Identification of the demand function is based on two assumptions. First, any unobservables that affect the demand for a given good of a given firm, e.g. product quality, are constant over time. This assumption may be realistic for some industries, and unrealistic for others. If unrealistic, the demand model can be replaced with a more flexible one. Second, changes in input prices and fixed input stocks shift the supply curve, while the demand curve, including the demand shock ε_{ijt}, is not affected. Using material prices and fixed input stocks as instruments is a standard practice. Also note that the estimated product-level inputs M_{ijt}, L_{ijt}, and K_{ijt} enter the production function as generated regressors. In order for the production function estimates to be consistent, all the instruments, generated and observed, need to be uncorrelated with the residuals (Wooldridge, 2002). In other words, if the moment conditions are valid, the parameter estimates are consistent.

To sum up, recall that the estimation biases acknowledged in the literature are: selection, simultaneity, collinearity, omitted price, and product bias, as discussed in section 2. The estimation strategy of this paper does not consider the selection bias, but it could be taken into account. The model could be extended to control for market entry and selection into various product lines by computing propensity scores for entry, as in Olley and Pakes (1996).[19] Furthermore, the selection bias may be less of a problem when product-level capital and labor are quasi-flexible variables, i.e., capital and labor allocations to product lines are made in the period of production given fixed firm-level capital and labor. Recall that the selection bias arises due to a negative correlation between firms' capital stock and productivity level in the sample. But when capital allocations to product lines are set as a function of productivity and demand, as in the multiproduct case, it is not obvious whether the correlation between capital and productivity is positive or negative. Hence identifying β_{Ki} and β_{Li} is now potentially subject to two opposing biases: selection bias (towards zero), and simultaneity bias (away from zero). The simultaneity biases of β_{Ki} and β_{Li} are corrected as is the bias of β_{Mi}.

The other four of the five biases are accounted for. The simultaneity bias is corrected by writing input functions explicitly as a function of the unobservable productivity. Identifying variation in material prices and fixed inputs stocks resolves the collinearity problem. The omitted price bias doesn't occur because input and output prices are observed, and physical quantity measures are used instead of sales revenues and input expenditures. The so-called product bias is corrected by allowing for good-specific production technology, and by taking account of the role of output demand in production decisions.

The identification strategy accommodates also other functional forms than the Cobb-Douglas production function and the isoelastic demand function used in this paper. The requirement on the production model, as in most structural production models, is that there has to be at least one input that is chosen as a function of the unobservable productivity. The data is required to include observations of at least two consecutive periods, and report physical output and sales revenue by product title. Such data, fortunately, is provided by many national statistical offices in Europe, for example.

7.1 Comparison with De Loecker et al.

There are a few recent papers that also accommocate for multiproduct firms and product-specific production technologies, as mentioned in the literature review. The method of De Loecker, Goldberg, Khandelwal and Pavcnik (2015, henceforth DLGKP) is perhaps closest to the method presented in this paper. DLGKP and I have rather similar datasets where input allocations within firms are unobservable. We also use many similar identifying assumptions that are standard in the structural production function literature, as DLGKP use the empirical model and estimation strategy of Ackerberg, Caves and Frazer (2006). Nevertheless, our key assumptions and empirical strategies that address the unobservable input allocations are quite different. We also use somewhat different assumptions regarding

[19] In fact, the method of Olley and Pakes (1996) is the only one that corrects for the selection problem, while the other structural methods focus on accounting for the simultaneity problem.

firms' productivity levels.

Both DLGKP and I assume that single- and multiproduct firms use similar product-specific technologies. DLGKP are able to utilize this assumption to a fuller extent, however, because they observe sufficiently many single-product firms to estimate the technology parameters using data on those firms only. This enables DLGKP to estimate the parameters without simultaneously solving for the unobservable input allocations.

DLGKP, on the other hand, do not observe input quantities and prices but input expenditures only. For this reason they estimate input prices, which together with the observed input expenditures yield input quantities. The input allocations are then computed using the parameter estimates and the observable variables. DLGKP assume that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e., independent of the input type. This implies that a firm produces all of its goods with the same materials-labor-capital -ratio. However, a profit maximizing or cost minimizing firm would not allocate inputs to product lines with such constant ratios. Even when the technology parameters are correctly estimated, estimates of the unobservable productivity levels are affected by this assumption. In order to solve for the unobservable input allocation consistently with profit maximization I estimate the output demand.

Both DLGKP and I allow for economies of scope in the form of lower fixed costs. DLGKP allow for economies of scope also in the form of lower variable costs, by letting firm-level productivity to depend on the number of goods produced. DLGKP, on the other hand, assume away productivity differences between product lines. I allow for productivity differences between product lines because there is empirical evidence for them, as discussed in section chapter 2.3, and because firms' core competencies may be important in explaining market outcomes.

8 Conclusion

This paper contributes to a large empirical literature on production function estimation which underlies an even larger body of applied economic research. The standard assumption made in production function estimation is that firms produce all of their output with a single technology. However, an empirical fact is that a remarkable share of firms is multiproduct firms. Empirical literature has, apart from a few exceptions, disregarded this fact in production function estimation because datasets do not report how firms allocate their inputs to the various product lines. In this paper I provide a method for estimating product-specific production functions. The empirical model does not require data on input allocations to various product lines, making it applicable to data available in, for example, many European countries. Instead, output demand is estimated to identify the input allocations to the product lines and the production functions. Endogeneity of the input allocations to the unobservable productivity levels is controlled by using inverses of the production functions in solving for the input allocations. Using the inverses also allows relaxing the so-called monotonicity assumption which is used for controlling the endogeneity of inputs in other structural production function strategies. While the monotonicity assumption implies that input demand is monotonic in productivity, in reality firms facing downward-sloping demand curves may either raise or cut their input demand due to improved productivity. The empirical model also accommodates productivity differences between a firm's product lines, which empirical literature suggests to be important in determining firms' production decisions and market outcomes.

I demonstrate the estimation method for multiproduct firms' production functions by estimating the production functions for the goods in the industry "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". I find that the technologies used in "Sawmilling and planing of wood" (PRODCOM 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM 162) are statistically different from each other. To show how the assumption of a single-product production technology, or a firm-level production function, changes the production function estimates and the implied productivity levels, I estimate also the firm-level production function typically estimated in the literature. The firm-level production function estimates differ clearly from the product-level estimates. While the firm-level productivity measures implied by the product- and firm-level estimates are positively correlated, for most firms the firm-level estimates imply either an under- or overestimated productivity level. In addition, the productivity differences between firms are overestimated when the singleproduct technology assumption is imposed. These findings are supported by simulations that I run to characterize the implications of misspecifying multiproduct firms' as singleproduct producers, reported in the appendix of this paper. For example, the finding on the false singleproduct technology assumption and overestimated productivity differences between firms may explain some of the surprisingly large productivity differences reported in the empirical literature. In short, the findings of this paper suggest that the singleproduct technology assumption should not be imposed on multiproduct firms.

References

Ackerberg, D., L. C. Benkard, S. Berry, and A. Pakes (2007): *Econometric Tools for Analyzing Market Outcomes*vol. 6 of *Handbook of Econometrics*, chap. 63. Elsevier, 1 edn.

Ackerberg, D., K. Caves, and G. Frazer (2006): "Structural identification of production functions," MPRA Paper 38349, University Library of Munich, Germany.

Allanson, P., and C. Montagna (2005): "Multiproduct firms and market structure: An explorative application to the product life cycle," *International Journal of Industrial Organization*, 23(7-8), 587–597.

Amemiya, T. (1977): "The Maximum Likelihood and the Nonlinear Three-Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model," *Econometrica*, 45(4), 955–968.

Arellano, M. (2003): *Panel Data Econometrics*. Oxfor University Press.

Baldwin, J. R., R. Caves, and W. Gu (2005): "Responses to Trade Liberalization: Changes in Product Diversification in Foreign- and Domestic-controlled Plants," Economic Analysis (EA) Research Paper Series 31, Statistics Canada, Analytical Studies Branch.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105–130.

Bernard, A. B., S. J. Redding, and P. K. Schott (2009): "Products and Productivity," *Scandinavian Journal of Economics*, 111(4), 681–709.

——— (2010): "Multiple-Product Firms and Product Switching," *American Economic Review*, 100(1), 70–97.

——— (2011): "Multiproduct Firms and Trade Liberalization," *Quarterly Journal of Economics*, 126(3), 1271–1318.

Bond, S., and M. Söderbom (2005): "Adjustment costs and the identification of Cobb Douglas production functions," IFS Working Papers W05/04, Institute for Fiscal Studies.

Broda, C., and D. E. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review*, 100(3), 691–723.

Brown, R. S., D. W. Caves, and L. R. Christensen (1979): "Modelling the Structure of Cost and Production for Multiproduct Firms," *Southern Economic Journal*, 46(1), 256–273.

Caves, D. W., L. R. Christensen, and M. W. Tretheway (1989): "Flexible Cost Functions for Multiproduct Firms," *Review of Economics and Statistics*, 62(3), 477–481.

Chamberlain, G. (1987): "Asymptotic efficiency in estimation with conditional moment restrictions," *Journal of Econometrics*, 34(3), 305–334.

Cherchye, L., T. Demuynck, B. D. Rock, and K. D. Witte (2011): "Nonparametric analysis of multi-output production with joint inputs," Center for Economic Studies - Discussion Papers ces11.35, Katholieke Universiteit Leuven, Centrum voor Economische Studiën.

Cherchye, L., B. D. Rock, B. Dierynck, F. Roodhoft, and J. Sabbe (2011): "Opening the 'black box' of efficiency measurement: input allocation in multi-output settings," Center for Economic Studies - Discussion papers ces11.10, Katholieke Universiteit Leuven, Centrum voor Economische Studiën.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2012): "Prices, Markups, and Trade Reform," NBER Working Papers 17925, National Bureau of Economic Research, Inc.

De Rock, B., L. Cherchye, and F. Vermeulen (2008): "Analyzing cost efficient production behavior under economies of scope: a nonparametric methodology," ULB Institutional Repositary 2013/7536, ULB – Universite Libre de Bruxelles.

Dhyne, E., A. Petrin, and F. Warzynski (2013): "Prices, Markups and Quality at the Firm-Product Level," .

Doms, M., and E. J. Bartelsman (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569–594.

Doraszelski, U., and J. Jaumandreu (2013): "RD and productivity: Estimating endogenous productivity," *Review of Economic Studies*, (80), 1338–1383.

Eckel, C., and J. P. Neary (2010): "Multi-Product Firms and Flexible Manufacturing in the Global Economy," *Review of Economic Studies*, 77(1), 188–217.

Feenstra, R., and H. Ma (2007): "Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition," NBER Working Papers 13703, National Bureau of Economic Research.

Felipe, J., and F. M. Fisher (2003): "Aggregation in Production Functions: What Applied Economists Should Know," *Metroeconomica*, 54, 208–262.

Fisher, F. M. (1969): "The Existence of Aggregate Production Functions," *Econometrica*, 37(4), 553–577.

Fisher, M. F. (1993): *Aggregation: Aggregate Production Functions and Related Topics. Collected Papers of Franklin M. Fisher*. Cambridge London: MIT Press.

Gandhi, A., S. Navarro, and D. Rivers (2013): "On the Identification of Production Functions: How Heterogeneous is Productivity?," .

Harrison, A. E. (1994): "Productivity, imperfect competition and trade reform: Theory and evidence," *Journal of International Economics*, 36(1-2), 53–73.

Hill, R. J. (2004): "Constructing Price Indexes across Space and Time: The Case of the European Union," *American Economic Review*, 94(5), 1379–1410.

Klein, L. R. (1946a): "Macroeconomics and the Theory of Rational Behavior," *Econometrica*, 14(2), 93–108.

——— (1946b): "Remarks on the Theory of Aggregation," *Econometrica*, 14(4), 303–312.

Klette, T. J., and Z. Griliches (1996): "The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous," *Journal of Applied Econometrics*, 11(4), 343–361.

Leontief, W. (1947a): "Introduction to a Theory of the Internal Structure of Functional Relationships," *Econometrica*, 15(4), 361–373.

——— (1947b): "A Note on the Interrelationship of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives," *Bulletin of the American Mathematical Society*, 53(4), 343–350.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

Ma, H. (2009): "Firm Sorting, Ownership and Product Scope: Evidence from Chinese Enterprises," .

Marschak, J., and J. William H. Andrews (1944): "Random Simultaneous Equations and the Theory of Production," *Econometrica*, 12(3-4), 143–205.

May, K. (1946): "The Aggregation Problem for a One-Industry Model," *Econometrica*, 14(4), 285–298.

Mayer, T., M. Melitz, and G. I. P. Ottaviano (2014): "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review*, 104(2), 495–536.

Nataf, A. (1948): "Sur la possibilité de construction de certains macromodèles," *Econometrica*, 16(3), 232–244.

Neary, P. J. (2004): "Rationalizing the Penn World Table: True Multilateral Indices for International Comparisons of Real Income," *American Economic Review*, 94(5), 1411–1428.

Nocke, V., and S. R. Yeaple (2013): "Globalization and Multiproduct Firms," NBER Working Papers 9037, National Bureau of Economic Research, Inc.

OECD (2013): "OECD Indicators of Employment Protection," http://www.oecd.org.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Ornaghi, C., and I. V. Beveren (2011): "Using proxy variables to control for unobservable when estimating productivity: A sensitivity analysis," Discussion Papers (IRES - Institut de Recherches Economiques et Sociales) 2011029, Institut de Recherches Économiques et Sociales de l'Université Catholique de Louvain.

Panzar, J. (1989): *Technological determinants of firm and industry structure*vol. 1 of *Handbook of Industrial Organization*, chap. 1, pp. 3–59. Elsevier, 1 edn.

Pu, S. S. (1946): "A Note on Macroeconomics," *Econometrica*, 14(4), 299–302.

Reynaert, M., and F. Verboven (2014): "Improving the performance of random coefficients demand models: The role of optimal instruments," *Journal of Econometrics*, 179(1), 83–98.

Syverson, C. (2011): "What Determines Productivity?," *Journal of Economic Literature*, 49(2), 326–365.

Wedervang, F. (1965): *Development of a population of industrial firms: the structure of manufacturing industries in Norway, 1930-1948.*, Scandinavian university books. Universitetsforlaget.

Wooldridge, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

Wooldridge, J. M. (2002): *Econometric Analysis for Cross Section and Panel Data*. MIT Press.

Appendix to Estimating Production Functions of Multiproduct Firms

1 Simulations

If production technologies are assumed to be firm-level functions while they actually are product-specific, the estimation equations are misspecified. First, the output elasticities of the inputs may not be equal across product lines. Second, if multiproduct firms are present, productivity levels are not necessarily constant across product lines within firms. Third, even if all product-level production functions were identical, they would not add up to a firm-level function without changing the functional form, unless the returns to scale were constant for all the technologies.

To find how production function estimates are determined under the above functional form misspecfication, I run simulations. I first generate a dataset where the product-level technologies are known. I then estimate the production functions at the firm-level, as is the practice in the empirical literature, and compare the firm-level estimates to the true product-level technologies.

I consider functional form misspecification for the Cobb-Douglas technology. Back in 1955, Houthakker characterized the Cobb-Douglas function as sufficiently consistent with notions of economic theory to be an useful approximative device, even though the function is not firmly established as an empirical regularity. Many microeconomists still agree with Houthakker, as even today the Cobb-Douglas function dominates the literature on firms' production. For example, most structural estimation strategies assume the Cobb-Douglas technology (e.g. Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves and Frazer, 2006; and De Loecker, Goldberg, Khandelwal and Pavcnik, 2012). The only exception is the translog approximation used by Gandhi, Navarro and Rivers (2013). Hence the findings of this study cater to interpreting a large number of empirical papers.

1.1 Data generation

The data generating process for the simulations is such that the implications of the functional form misspecification for production function estimates are transparent. The data generating process is also the simplest one that allows for substitution between inputs and output types within firms. Any identification issues, such as simultaneity, selection, and collinearity problems, unobservable price and quality differences, technological change, and small sample size, are assumed away.

I generate datasets where firms produce one or two goods, each with the respective production technology. Choices on what product types to produce are exogenous.[1] In reality the number of goods produced and technologies used in an industry may of course be more than two. However the qualitative effects of the misspecification are likely to be the same when the number of true technologies is greater.

The data generated is an outcome of firms maximizing static profits. Firms are typically assumed to have at least one dynamic factor of production, capital and perhaps also knowledge investments. Decisions on dynamic factors affect production and profit also after the current period. Whether an input is a static variable or has dynamic implications does not change the effects of the functional form misspecification studied in this paper, however. Therefore, to simplify the data generating process, the firms have two static inputs, labor L_{ij} and capital K_{ij}:

$$Q_{ij} = L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp\left(\omega_{ij}\right). \tag{1}$$

The output elasticities of the two inputs, β_{Li} and β_{Ki}, are product-specific. Total factor productivity, which is product- and firm-specific, is denoted by $\exp\left(\omega_{ij}\right)$. I treat L_{ij} and K_{ij} as exogenous to the unobservable productivity level ω_{ij} both in the data generation as well as in the estimation process. Allowing L_{ij} and K_{ij} to be endogenous to ω_{ij} would require the consequent endogeneity bias to be treated by using an appropriate estimator. However, I don't know how the present estimators, such as Olley and Pakes (1996), Levinsohn and Petrin (2004), Ackerberg, Caves and Frazer (2006), or Wooldridge (2009), perform if the assumption of firm-level production functions does not hold. Allowing for endogeneity would make the analysis less straightforward because the effects of the functional form misspecification would have to be distinguished from the misperformance of the estimator in the presence of the functional form misspecification. Hence I assume L_{ij} and K_{ij} to be exogenous to ω_{ij}.[2]

[1] As discussed in the literature review, Bernard, Redding and Schott (2009) discuss the implications of ignoring endogenous product choices in production function estimation.

[2] Because both inputs, L_{ij} and K_{ij}, are static decision variables and exogenous to the productivity

The firm chooses its input and hence also output levels as a function input and output prices. Input prices, W_j for L_{ij}, and R_j for K_{ij}, vary across firms. In the output market firms face downward-sloping demand curves with a product-firm -specific demand level:

$$Q_{ij} = \exp(\alpha_{ij}) \, P_{ij}^{\eta_i}, \tag{2}$$

where P_{ij} is price of good i produced by firm j, η_i is price elasticity of demand for good i, and α_{ij} captures the good-firm -specific demand level. Variation in output demand induces firms to substitute between goods, while variation in W_j and R_j induce substitution between the two inputs.

The firm sets inputs L_{ij} and K_{ij} to maximize the static profits in all the product lines i it is active in:

$$\max_{L_{ij},K_{ij}} \Pi_{ij} = \sum_i P_{ij} Q_{ij} - W_j L_{ij} - R_j K_{ij} \tag{3}$$

Substituting in the inverse demand functions, $P_{ij} = \left(Q_{ij} \left(\exp(\alpha_{ij})\right)^{-1}\right)^{\frac{1}{\eta_{ij}}}$, and the production functions, the static profit maximization problem becomes:

$$\max_{L_{ij},K_{ij}} \Pi_{ij} = \sum_i \left(\exp(\alpha_{ij} + \varepsilon_{ij})\right)^{-\frac{1}{\eta_i}} \left(L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp(\omega_{ij})\right)^{\frac{1}{\eta_i}+1} - W_j L_{ij} - R_j K_{ij} \tag{4}$$

The first-order conditions for static profit maximization for firm j producing product i are:

$$\frac{\partial Lagr}{\partial L_{ij}} = \left(\frac{1}{\eta_i} + 1\right) \left(\exp(\alpha_{ij})\right)^{-\frac{1}{\eta_i}} \left(L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp(\omega_{ij})\right)^{\frac{1}{\eta_i}+1} \frac{\beta_{Li}}{L_{ij}} - W_j = 0 \tag{5}$$

$$\frac{\partial Lagr}{\partial K_{ij}} = \left(\frac{1}{\eta_i} + 1\right) \left(\exp(\alpha_{ij})\right)^{-\frac{1}{\eta_i}} \left(L_{ij}^{\beta_{Li}} K_{ij}^{\beta_{Ki}} \exp(\omega_{ij})\right)^{\frac{1}{\eta_i}+1} \frac{\beta_{Ki}}{K_{ij}} - R_j = 0 \tag{6}$$

$$\forall\, i = [1, n_j]$$

$$\forall\, j = [1, J]$$

These first-order conditions give the profit-maximizing inputs L_{ij}^* and K_{ij}^*.

level ω_{ij}, a cross-sectional dataset is sufficient in this study. In contrast, if at least one of the inputs were dynamic, generation of these inputs would produce a longitudinal dataset. If at least one of the inputs were endogenous to ω_{ij}, the estimation methods of Olley and Pakes (1996), Levinsohn and Petrin (2004), and Ackerberg, Caves and Frazer (2006), for example, could be used for identifying the production function, but that would require a longitudinal dataset of at least two consecutive time periods.

1.1.1 The number of goods produced

I generate datasets for four scenarios: (1) 1/2 of the firms produce good 1, and the other 1/2 of the firms produce good 2, (2) 1/3 of the firms produce good 1, another 1/3 of the firms produce good 2, and the remaining 1/3 of the firms produce both goods, (3) 1/10 of the firms produce good 1, another 1/10 of the firms produce good 2, and the remaining 8/10 of the firms produce both goods, and (4) all firms produce both goods. Demand for the goods is not correlated within firms, nor are the product-specific productivity levels. The only difference between the four datasets is the exogeneous variation in product selection.

In the first scenario all firms are single-product firms. According to datasets on firms in the manufacturing sector, such a scenario is very unlikely (Bernard, Redding and Schott, 2010), but because most studies implicitly assume single-product firms, results for the scenario may also be of interest. The other three cases are empirically more relevant. In the US manufacturing sector 40% of the firms produce at least two goods (Bernard, Redding and Schott, 2010), while more than 60% of Finnish manufacturing plants produce multiple goods.[3] The two scenarios with 1/3 and 8/10 of the firms producing two goods may therefore be considered as illustrations of a national manufacturing industry, for example. The fourth case where all firms are multiproduct producers corresponds to a dataset on exporting firms, where virtually all firms are multiproduct firms (Bernard, Jensen, Redding and Schott, 2007).

1.1.2 Production function parameters

To cover different production technology combinations that may prevail within industries, I consider altogether 18 different sets of product-level technologies, displayed in Table 1. The 18 cases differ in the technology parameters: in the technologies' output elasticities and returns to scale. Apart from the technology parameters, the data generating process for the 18 cases is identical.

In cases 1 to 9 (10 to 18), the technologies have equal (unequal) returns to scale. In cases 1 to 3 (4 to 6) [7 to 9], both technologies have constant returns to scale, $\beta_{Li}+\beta_{Ki}=1$, (increasing returns to scale, $\beta_{Li}+\beta_{Ki}>1$) [decreasing returns to scale, $\beta_{Li}+\beta_{Ki}<1$]. In cases 10 to 15, technology for good 1 has constant returns to scale, while technology

[3] According to the Industrial output data of Statistics Finland on years 2004 - 2011.

for good 2 has increasing (cases 10 to 12) or decreasing (cases 13 to 15) returns to scale. In cases 16 to 18, technology for good 1 has increasing returns to scale, and technology for good 2 has decreasing returns.

In all cases, technology for good 1 has higher output elasticity for L than for K, $\beta_{L1} > \beta_{K1}$. Depending on the case, β_{L1} ranges between 0.71 and 0.69, while β_{K1} is 0.3 across all the cases. The output elasticities of the technology for good 2 can be divided into three groups. In the first group, the elasticities are identical (cases 1, 4, 7 with equal returns to scale), or very close to the parameters of the technology for good 1 (cases 10, 13, 16 with unequal returns to scale). In the second group, the two elasticities of the technology for good 2 are exactly or approximately 0.5 each (cases 2, 5, 8, 11, 14 and 17), and hence the parameters differ from those of technology for good 1 by about 0.2 in absolute value. In the third group, technology for good 2 has lower output elasticity for L than for K, such that β_{L1} is close to β_{K2}, and β_{K1} is close to β_{L2}.

1.1.3 Other exogeneous parameters and variables

There are four exogenous parameters or variables that induce firms to substitute between the inputs and, in the case of two-good producers, between the output and the respective technology types. These exogenous parameters and variables yield identifying variation in the input choices for the two goods. Factor prices W_j and R_j induce substitution between the inputs. They are normally distributed with mean 10 and standard deviation 1. To avoid the problem of collinear inputs, the input prices are not correlated. Demand for the goods, i.e., the price elasticity of demand η_i and the level of demand α_{ij}, bring about variation in the two output types. Demand for both types of goods is elastic with price elasticity 1.05,[4] while α_{ij} is normally distributed with mean 23 and standard deviation 0.1.

1.2 Estimation

Due to how the input and output data has been generated, the product-level production functions may be estimated by OLS to obtain unbiased and efficient estimates. To examine the estimates obtained when imposing the assumption of a firm-level technology, I estimate

[4] If demand was inelastic, the model would imply negative input choices.

the following equation:

$$Q_j = L_j^{\beta_L} K_j^{\beta_K} \exp(\omega_j). \tag{7}$$

where the dependent variable is $Q_j = \sum_{i=1}^{N_j} Q_i$, and the explanatory variables are $L_j = \sum_{i=1}^{N_j} L_{ij}$ and $K_j = \sum_{i=1}^{N_j} K_{ij}$. After taking logarithms the equation can be estimated by OLS.

Before turning to the estimation results, I consider how the above estimation equation compares with the true firm-level production function aggregates when all the producers are one-product firms (scenario 1), and when at least one of the firms is a multiproduct firm (scenarios 2 - 4).

1.2.1 One-product firms (scenario 1)

Consider product-level production functions for N goods, denoted by subscript i. All the production technologies use H types of inputs. The output elasticities of the inputs are captured in a product-specific parameter vector β_i, which may vary across goods i. Taking logs, the produduction functions of the firms are written in matrix form as follows:

$$\mathbf{q}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\omega}_i \tag{8}$$

where q_i is the log of output, X_i is the log of inputs, and residual ω_i is the log of productivity level for goods of type i.

The standard estimation strategy of the literature, in this scenario where all firms are one-product firms, implies assuming that all production functions i are identical:

$$\mathbf{q} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}. \tag{9}$$

Imposing the assumption of a single technology can be considered as an example restricted least squares estimation with the following parameter restriction:

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_g \ \forall \ i = [1, N], \ \forall \ g = [1, N]. \tag{10}$$

The restricted least squares estimator is (for example, Greene, 2002):

$$\widehat{\boldsymbol{\beta}}_R = \widehat{\boldsymbol{\beta}} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{R}'\left[\mathbf{R}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{R}'\right]^{-1}\left(\mathbf{r}-\mathbf{R}\widehat{\boldsymbol{\beta}}\right) \tag{11}$$

where the parameter restriction is

$$\mathbf{R}\boldsymbol{\beta} = \mathbf{r}. \tag{12}$$

In the case of two-input technologies and two goods,[5] (10) translates into (12) such that:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \tag{13}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix}, \text{ and} \tag{14}$$

$$\mathbf{r} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{15}$$

If the constraints hold in reality, the restricted estimator $\widehat{\boldsymbol{\beta}}_R$ equals the unrestricted $\widehat{\boldsymbol{\beta}}$. If the constraints are not correct, the estimators are different. In that case the restriced estimator $\widehat{\boldsymbol{\beta}}_R$ consists of the unrestricted estimator and a correction term that accounts for the failure of the unrestricted estimator to satisfy the constraints. In short, whenever the constraint is not true, the restricted estimator $\widehat{\boldsymbol{\beta}}_R$ is biased. The directions of such biases, as well as the implications on the residuals, are considered by the simulations.

1.2.2 N-product firms (scenarios 2, 3, and 4)

Like the majority of production functions, Cobb-Douglas is non-linear in inputs. As a consequence, even if the product-level production functions were identical, they would not necessarily add up to a firm-level function without changing the functional form and the parameters. Hence finding the correct level of specification is important whenever using a

[5] In a general case of H inputs and N goods, R is a matrix of size $(H(N-1), HN)$ where each odd (even) row has a 1 in the first (second) column, a -1 in each cell $(f, f+H)$ $\forall$ $f = [1, H(N-1)]$, and zeros elsewhere. β is a vector of length HN where the product-specific β_i's are stacked, and r is a vector of 0's of length HN.

production function that is non-linear in inputs.

Assume now that all firms j produce N_j types of goods i using N_j separate product-level Cobb-Douglas production functions, again with H types of inputs for each good. The product-level production function is (now writing in scalar form):

$$Q_{ij} = \prod_{h=1}^{H} X_{hij}^{\beta_{hi}} \exp(\omega_{ij}) \tag{16}$$

and hence the total output of the firm is given by:

$$\sum_{i=1}^{N_j} Q_{ij} = \sum_{i=1}^{N_j} \prod_{h=1}^{H} X_{hij}^{\beta_{hi}} \exp(\omega_{ij}) . \tag{17}$$

Instead of the true aggregate function above, the following equation is typically estimated in the literature:

$$\sum_{i=1}^{N_j} Q_{ij} = \prod_{h=1}^{H} \left(\sum_{i=1}^{N_j} X_{hij} \right)^{\widehat{\beta}_h} \exp(\widehat{\omega}_j) . \tag{18}$$

To see how (17) and (18) differ, I rewrite the true aggregate using the following auxiliary terms:

$$\overline{X}_{hj} = \frac{\sum_i^{N_j} X_{hij}}{N_j} \tag{19}$$

$$x_{hij} = 1 + \frac{X_{hij} - \overline{X}_{hj}}{\overline{X}_{hj}} \tag{20}$$

$$\Delta\beta_{hi} = \beta_{hi} - \widehat{\beta}_h \tag{21}$$

where $\widehat{\beta}_h$ is the input elasticity of output estimated for input type h. The true firm-level output aggregate can then be rewritten as:

$$\sum_{i}^{N_j} Q_{ij} = \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\widehat{\beta}_h + \Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp(\omega_{ij}) \right) \tag{22}$$

$$\sum_{i}^{N_j} Q_{ij} = \prod_{h=1}^{H} \overline{X}_{hj}^{\widehat{\beta}_h} \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp(\omega_{ij}) \right) \tag{23}$$

$$\sum_{i}^{N_j} Q_{ij} = \prod_{h=1}^{H} \left(N_j \overline{X}_{hj} \right)^{\widehat{\beta}_h} \sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(\overline{X}_{hj}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h + \Delta\beta_{hi}} \right) \exp(\omega_{ij}) \right) \prod_{h=1}^{H} N_j^{-\widehat{\beta}_h} \tag{24}$$

$$\sum_{i}^{N_j} Q_{ij} = \underbrace{\prod_{h=1}^{H} \left(\sum_{i}^{N_j} X_{hij} \right)^{\widehat{\beta}_h}} \underbrace{\sum_{i}^{N_j} \left(\prod_{h=1}^{H} \left(X_{hij}^{\Delta\beta_{hi}} x_{hij}^{\widehat{\beta}_h} \right) \exp(\omega_{ij}) \right) N_j^{-\sum_{h=1}^{H} \widehat{\beta}_h}} . \tag{25}$$

Note that in the true aggregate rewritten in (25), the first term, $\prod_{h=1}^{H} \left(\sum_{i}^{N_j} X_{hij} \right)^{\widehat{\beta}_h}$, is equal to the deterministic part of the typical estimation equation in the literature (18). This implies that if the typical estimation equation (18) is adopted when the data generating process is product-specific (17), the estimated residual $\exp(\widehat{\omega}_j)$ is in fact the second part of (25). Clearly, the second part of (25) is not a term of unobservable productivity or output measurement error only. Instead, the firm-level productivity term estimated in the literature is a function of the true product-specific technology parameters β_{hi} and the product-level input allocations X_{hij}, as well as the true product-specific productivity levels ω_{ij}. In other words, the residual $\widehat{\omega}_j$ of the typical estimation equation (18) captures any output that remains unexplained by the deterministic part of the estimation equation (18). As a consequence, the distribution of $\widehat{\omega}_j$ may provide an unrealistic description of true productivity ω_{ij}.

As the logarithm of the estimation equation involves a logarithm of a sum, there is no analytical solution to how the parameter estimate $\widehat{\beta}_h$ is determined. The parameter estimates are therefore considered using simulations.

2 Results

The firm-level estimation equation (7) is misspecified when the true techologies are product-specific. Hence, a one-to-one comparison between the firm-level estimates and the true parameters cannot be made. Instead, I contrast the estimates with the two product-level technologies. I also compare the estimated and true returns to scale, as well as the estimated and the true firm-level productivity[6] terms. The estimation results are displayed in Tables 2, 3, 4 and 5 for the four different scenarios.

I start by characterizing the biases in the estimated firm-level parameters $\widehat{\beta}_L$ and $\widehat{\beta}_K$. Unbiased estimates are obtained only in exceptional cases. Only if the product-specific technologies are identical, and all firms produce the same number of goods (i.e., cases 1, 4 and 7 in scenarios 1 and 4, Tables 2 and 5), or the true technologies are not only identical but also subject to constant returns to scale (i.e., case 1 in scenarios 2 and 3, Tables 3 and 4), the firm-level estimates are unbiased. These circumstances are hardly realistic.

Consider first scenario 1 where all firms are single-product firms, one half of the firms producing good 1 and the other half producing good 2. Of the cases where both product-level technologies have constant returns to scale, cases 2 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.5$, $\beta_{K2} = 0.5$), 5 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.51$, $\beta_{K2} = 0.5$) and 8 ($\beta_{L1} = 0.69$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$) stand out. The product-level technology parameters for a given input differ by 0.2 in absolute value, and the industry output shares of the two goods are no more different than 52% and 48%. Yet the estimated firm-level parameter estimates are identical (cases 2 and 5) or very close (case 8) to the parameters of the technology for good 1, and hence clearly biased from the parameters of the technology for good 2.

Estimates in cases 10 to 18 of scenario 1, where the returns to scale of the two product-level technologies differ, are subject to considerably higher parameter biases. In case 10 the true technologies are almost identical ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.71$, $\beta_{K2} = 0.3$) and the difference in the returns to scale is small (only 0.01), but the firm-level estimates have a substantial upward bias for labor ($\widehat{\beta}_L = 1.07$), and a downward bias for capital ($\widehat{\beta}_K = 0.10$). In case 11 the biases go in the opposite direction: $\widehat{\beta}_L$ is biased downwards ($\beta_{L1} = 0.7$, $\beta_{L2} = 0.51$ and $\widehat{\beta}_L = 0.34$), and $\widehat{\beta}_K$ is biased upwards ($\beta_{K1} = 0.3$, $\beta_{K2} = 0.5$ and $\widehat{\beta}_K = 0.68$). The firm-level estimates are not even between the true parameters in either of the cases. In cases 14 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$), 16 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$) and 17 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and

[6] The true firm-level productivity is computed by taking a weighted average of the product-specific productivity levels, where the weights are the output shares generated with the product-level inputs and productivity level $\exp(\omega_{ij}) = 1$ for each good: $\exp(\omega_j) = \frac{L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}}{\sum_i L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}} \exp(\omega_{1j}) + \frac{L_{2j}^{\beta_{L1}} K_{2j}^{\beta_{K1}}}{\sum_i L_{1j}^{\beta_{L1}} K_{1j}^{\beta_{K1}}} \exp(\omega_{2j})$

$\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$), where the technology for product 2 is subject to decreasing returns to scale, $\widehat{\beta}_K$ is actually negative. Perhaps the most surprising estimates are obtained for case 16, where the two technologies are rather similar in the magnitudes of the output elasticities, with a difference in returns to scale of 0.02: $\widehat{\beta}_L$ is 2.06, a multiple of either of the true output elasticities of labor, and $\widehat{\beta}_K$ is -0.46, substantially below zero.

The estimates for the other three scenarios, where some or all firms produce multiple goods, are displayed in Tables 3 to 5. The parameter biases are similar in direction as in scenario 1 with single-product firms, but the magnitudes of the biases are somewhat lower. The greater the share of two-product firms, the smaller the biases. However even in scenario 3, where 80% of the firms produce two goods, the parameter biases are substantial. In cases 10 to 18, where the true technologies have slightly different returns to scale, none of the estimated firm-level technologies have parameters, $\widehat{\beta}_L$ and $\widehat{\beta}_K$, that both fall in between the true product-specific parameters, β_{L1} and β_{L2}, and β_{K1} and β_{K2}. Again even negative parameter estimates are obtained. The biases are lowest, albeit clearly different from zero, in scenario 4 where all firms produce two goods. For example, in case 17 the firm-level parameter estimates, $\widehat{\beta}_L = 0.80$ and $\widehat{\beta}_K = 0.21$, are clearly outside the ranges of the product-specific parameters, $\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.49$, $\beta_{K2} = 0.5$.

Two characteristics of the two true production technologies determine the directions of the parameter biases. First, when the true production technologies of the two goods are asymmetric in the sense that $|\beta_{Li} - \beta_{Ki}| > |\beta_{Lh} - \beta_{Kh}|$, where i stands for good 1 and h for good 2, or vice versa, then the parameter estimates are biased away from the true parameters of the technology for good h, the directions of the biases being towards the parameters of the technology for good i. Second, when the two true technologies have different returns to scale, i.e., $\beta_{Li} + \beta_{Ki} > \beta_{Lh} + \beta_{Kh}$, the parameter estimates are biased away from the true parameters of the technology for good h, the directions of the biases being towards the parameters of the technology for good i. When technology i is more asymmetric, i.e., $|\beta_{Li} - \beta_{Ki}| > |\beta_{Lh} - \beta_{Kh}|$, and technology h has higher returns to scale, $\beta_{Li}+\beta_{Ki} < \beta_{Lh}+\beta_{Kh}$, the biases in the parameter estimates are a mix of the two opposite effects. Depending on the parameters of the two true production technologies, and hence on the magnitudes of the estimation biases, the estimates may or may not be in between the parameters of the two true technologies. The sizes of the parameter bias grow in two characteristics. First, the greater the difference in the returns to scale between the two true technologies, the greater the bias. Second, the more there are one-good producers, the greater the bias.

When all firms are one-good firms (scenario 1) or all firms are two-good firms (scenario 4), and the true technologies have equal returns to scale, the estimated returns to scale are correctly estimated. If the true technologies have unequal returns to scale, the estimated returns are over- or underestimated in the case of one-good firms, and over-estimated in the case of two-good firms. When both one- and two-good firms are present (scenarios 2 and 3), the returns to scale are over- or underestimated depending on how similar the true technologies are. The more similar (different) the technologies, the more the returns to scale are overestimated (underestimated).

Often the most interesting results in production function estimation are, paradoxically, the residuals that are considered as the producers' unobservable productivity levels. In most of the cases of scenarios 1 and 4 the standard deviation of the productivity distribution is correctly estimated. Also the correlation between the estimated and the true firm-level productivity levels is very high, in some cases even equal to one. The exceptions are in fact cases 10 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.71$, $\beta_{K2} = 0.3$), 13 ($\beta_{L1} = 0.7$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$) and 16 ($\beta_{L1} = 0.71$, $\beta_{K1} = 0.3$ and $\beta_{L2} = 0.69$, $\beta_{K2} = 0.3$), where the product-level technologies are very similar but with small differences in the returns to scale. In these cases the correlations between the estimated and true firm-level productivity terms, $\exp(\widehat{\omega}_j)$ and $\exp(\omega_j)$, are 0.53, 0.53 and 0.31, respectively. The standard deviations of the productivity distributions are overestimated: they are estimated to be 0.19, 0.19 and 0.31, respectively, while the standard deviation of the true firm-level productivity aggregate is only 0.10.

Scenarios 2 and 3 differ from scenarios 1 and 4 in how the estimated and true firm-level productivity terms compare. First, with the exception of case 16, the estimated productivity distributions are narrower ($\exp(\widehat{\omega}_j)$ is between 0.08 and 0.16) than the true firm-level distributions ($\exp(\omega_j)$ is between 0.21 and 0.29). Second, the correlation between $\exp(\widehat{\omega}_j)$ and $\exp(\omega_j)$ ranges not higher than 0.19 to 0.41 (scenario 2) and 0.26 to 0.32 (scenario 3).

To sum up, estimations on the simulated datasets show that the biases in the estimated firm-level parameters are substantial even when the true product-level technologies are very similar. The directions and the magnitudes of the parameter biases are determined as intricate functions of the true product-level technologies and the product scopes of the firms in the industry. Also the residuals, which are often considered as the unobservable productivity levels, are affected: the more heterogeneous the product scopes of the firms, the lower the correlation between the estimated and true firm-level productivity levels.

References

Ackerberg, D., K. Caves, and G. Frazer (2006): "Structural identification of production functions," MPRA Paper 38349, University Library of Munich, Germany.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2007): "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105–130.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2012): "Prices, Markups, and Trade Reform," NBER Working Papers 17925, National Bureau of Economic Research, Inc.

Gandhi, A., S. Navarro, and D. Rivers (2013): "On the Identification of Production Functions: How Heterogeneous is Productivity?," .

Greene, W. H. (2002): *Econometric Analysis*. Prentice Hall, 5 edn.

Houthakker, H. S. (1955-1956): "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," *Review of Economic Studies*, 23(1), 27–31.

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 70(2), 317–341.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), 1263–1297.

Wooldridge, J. (2009): "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 104(3), 112–114.

3 Tables

Table 1: The true production function parameters

	Technology for good 1			Technology for good 2				
Case	β_{L1}	β_{K1}	$\sum \beta_1$	β_{L2}	β_{K2}	$\sum \beta_2$	$\sum \beta_1 = \sum \beta_2$	$\beta_{L1} - \beta_{L2}, \beta_{K1} - \beta_{K2}$
1	0.7	0.3	1	0.7	0.3	1	yes	0,0
2	0.7	0.3	1	0.5	0.5	1	yes	0.2,-0.2
3	0.7	0.3	1	0.3	0.7	1	yes	0.4,-0.4
4	0.71	0.3	1.01	0.71	0.3	1.01	yes	0,0
5	0.71	0.3	1.01	0.51	0.5	1.01	yes	0.2,-0.2
6	0.71	0.3	1.01	0.31	0.7	1.01	yes	0.4,-0.4
7	0.69	0.3	0.99	0.69	0.3	0.99	yes	0,0
8	0.69	0.3	0.99	0.49	0.5	0.99	yes	0.2,-0.2
9	0.69	0.3	0.99	0.29	0.7	0.99	yes	0.4,-0.4
10	0.7	0.3	1	0.71	0.3	1.01	no	0.01, 0
11	0.7	0.3	1	0.51	0.5	1.01	no	0.19,-0.2
12	0.7	0.3	1	0.31	0.7	1.01	no	0.39,-0.4
13	0.7	0.3	1	0.69	0.3	0.99	no	0.01,0
14	0.7	0.3	1	0.49	0.5	0.99	no	0.21,-0.2
15	0.7	0.3	1	0.29	0.7	0.99	no	0.41,-0.4
16	0.71	0.3	1.01	0.69	0.3	0.99	no	0.02,0
17	0.71	0.3	1.01	0.49	0.5	0.99	no	0.22,-0.2
18	0.71	0.3	1.01	0.29	0.7	0.99	no	0.42,-0.4

Table 2: Scenario 1; 1/2 of the firms produce good 1, and 1/2 of the firms produce good 2

	Estimated industry-specific technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum\beta_1 = \sum\beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j, \omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1	0.10	1	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.10	yes
2	0.70	0.30	1	0.10	1	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.10	yes
3	0.50	0.50	1	0.10	0.99	0.7	0.3	1	0.5	0.3	0.7	1	0.5	0.10	yes
4	0.71	0.30	1.01	0.10	1	0.71	0.3	1.01	0.5	0.71	0.3	1.01	0.5	0.10	yes
5	0.71	0.30	1.01	0.10	1	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.10	yes
6	0.52	0.49	1.01	0.10	0.99	0.71	0.3	1.01	0.5	0.31	0.7	1.01	0.5	0.10	yes
7	0.69	0.30	0.99	0.10	1	0.69	0.3	0.99	0.5	0.69	0.3	0.99	0.5	0.10	yes
8	0.68	0.31	0.99	0.10	1	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.10	yes
9	0.48	0.51	0.99	0.10	0.99	0.69	0.3	0.99	0.5	0.29	0.7	0.99	0.5	0.10	yes
10	1.07	0.10	1.17	0.19	0.53	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.10	no
11	0.34	0.68	1.02	0.10	0.98	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.10	no
12	0.32	0.69	1.01	0.10	0.99	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.10	no
13	1.06	0.10	1.16	0.19	0.53	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.10	no
14	1.05	-0.06	0.98	0.10	0.96	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.10	no
15	0.68	0.32	1	0.10	0.99	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.10	no
16	2.06	-0.46	1.6	0.34	0.31	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.10	no
17	1.40	-0.41	0.99	0.11	0.89	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.10	no
18	0.86	0.14	1.00	0.10	0.97	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.10	no

Table 3: Scenario 2; 1/3 of the firms produce good 1, 1/3 of the firms produce good 2, and 1/3 of the firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j,\omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		$\sum\beta_1=\sum\beta_2$
1	0.70	0.30	1	0.09	0.21	0.7	0.3	1	0.50	0.7	0.3	1	0.50	0.24	yes
2	0.68	0.29	0.98	0.09	0.21	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.25	yes
3	0.45	0.45	0.9	0.09	0.19	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.24	yes
4	0.71	0.30	1.00	0.09	0.21	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.24	yes
5	0.69	0.29	0.98	0.09	0.21	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.25	yes
6	0.46	0.44	0.9	0.09	0.19	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.24	yes
7	0.70	0.31	1	0.09	0.21	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.24	yes
8	0.68	0.30	0.98	0.09	0.21	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.25	yes
9	0.44	0.46	0.9	0.09	0.19	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.24	yes
10	0.91	0.12	1.03	0.16	0.29	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.25	no
11	0.32	0.67	0.98	0.09	0.2	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.25	no
12	0.27	0.65	0.92	0.09	0.19	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.25	no
13	0.91	0.12	1.03	0.16	0.29	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.25	no
14	1.06	-0.05	1.01	0.1	0.22	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.26	no
15	0.63	0.28	0.91	0.09	0.19	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.25	no
16	1.51	-0.39	1.12	0.29	0.41	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.28	no
17	1.44	-0.37	1.07	0.11	0.22	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.29	no
18	0.84	0.11	0.95	0.10	0.19	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.28	no

Table 4: Scenario 3; 1/10 of the firms produce good 1, 1/10 of the firms produce good 2, and 8/10 of the firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum \beta_1 = \sum \beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum \widehat{\beta}$	s.d. $\exp \widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j, \omega_j)$	β_{L1}	β_{K1}	$\sum \beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum \beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1	0.08	0.29	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.21	yes
2	0.68	0.30	0.98	0.08	0.29	0.7	0.3	1	0.52	0.5	0.5	1	0.48	0.21	yes
3	0.45	0.45	0.90	0.08	0.27	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.21	yes
4	0.71	0.30	1	0.08	0.29	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.21	yes
5	0.69	0.29	0.98	0.08	0.29	0.71	0.3	1.01	0.52	0.51	0.5	1.01	0.48	0.21	yes
6	0.46	0.44	0.90	0.08	0.26	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.21	yes
7	0.70	0.31	1	0.08	0.30	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.21	yes
8	0.67	0.31	0.98	0.08	0.29	0.69	0.3	0.99	0.52	0.49	0.5	0.99	0.48	0.21	yes
9	0.44	0.46	0.90	0.08	0.27	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.21	yes
10	0.78	0.25	1.03	0.11	0.28	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.22	no
11	0.33	0.65	0.98	0.08	0.28	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.21	no
12	0.28	0.64	0.92	0.08	0.27	0.7	0.3	1	0.42	0.31	0.7	1.01	0.58	0.22	no
13	0.77	0.26	1.03	0.11	0.28	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.22	no
14	1.03	-0.02	1.01	0.08	0.30	0.7	0.3	1	0.61	0.49	0.5	0.99	0.39	0.22	no
15	0.63	0.28	0.91	0.08	0.27	0.7	0.3	1	0.58	0.29	0.7	0.99	0.42	0.22	no
16	0.99	0.11	1.10	0.17	0.30	0.71	0.3	1.01	0.67	0.69	0.3	0.99	0.33	0.23	no
17	1.39	-0.32	1.07	0.09	0.32	0.71	0.3	1.01	0.69	0.49	0.5	0.99	0.31	0.23	no
18	0.84	0.12	0.95	0.08	0.28	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.23	no

Table 5: Scenario 4; All firms produce both goods

	Estimated firm-level technology					Technology for good 1				Technology for good 2				s.d. ω_j	$\sum\beta_1 = \sum\beta_2$
Case	$\widehat{\beta}_L$	$\widehat{\beta}_K$	$\sum\widehat{\beta}$	s.d. $\exp\widehat{\omega}_j$	$Corr\,(\widehat{\omega}_j, \omega_j)$	β_{L1}	β_{K1}	$\sum\beta_1$	$\frac{\sum Q_1}{\sum Q_1+Q_2}$	β_{L2}	β_{K2}	$\sum\beta_2$	$\frac{\sum Q_2}{\sum Q_1+Q_2}$		
1	0.70	0.30	1.00	0.07	1	0.7	0.3	1	0.5	0.7	0.3	1	0.5	0.07	yes
2	0.62	0.38	1.00	0.07	1	0.7	0.3	1	0.53	0.5	0.5	1	0.47	0.07	yes
3	0.50	0.50	1.00	0.07	1	0.7	0.3	1	0.50	0.3	0.7	1	0.50	0.07	yes
4	0.71	0.30	1.01	0.07	1	0.71	0.3	1.01	0.50	0.71	0.3	1.01	0.50	0.07	yes
5	0.63	0.38	1.01	0.07	1	0.71	0.3	1.01	0.53	0.51	0.5	1.01	0.47	0.07	yes
6	0.51	0.50	1.01	0.07	1	0.71	0.3	1.01	0.50	0.31	0.7	1.01	0.50	0.07	yes
7	0.69	0.30	0.99	0.07	1	0.69	0.3	0.99	0.50	0.69	0.3	0.99	0.50	0.07	yes
8	0.61	0.38	0.99	0.07	1	0.69	0.3	0.99	0.53	0.49	0.5	0.99	0.47	0.07	yes
9	0.49	0.50	0.99	0.07	1	0.69	0.3	0.99	0.50	0.29	0.7	0.99	0.50	0.07	yes
10	0.71	0.30	1.01	0.07	0.99	0.7	0.3	1	0.41	0.71	0.3	1.01	0.59	0.07	no
11	0.54	0.47	1.01	0.07	0.99	0.7	0.3	1	0.44	0.51	0.5	1.01	0.56	0.07	no
12	0.39	0.61	1.01	0.07	0.99	0.7	0.3	1	0.41	0.31	0.7	1.01	0.59	0.07	no
13	0.70	0.30	1.00	0.07	0.99	0.7	0.3	1	0.59	0.69	0.3	0.99	0.41	0.07	no
14	0.70	0.29	1.00	0.07	0.99	0.7	0.3	1	0.62	0.49	0.5	0.99	0.38	0.07	no
15	0.61	0.39	1.00	0.07	1	0.7	0.3	1	0.59	0.29	0.7	0.99	0.41	0.07	no
16	0.72	0.30	1.02	0.08	0.96	0.71	0.3	1.01	0.68	0.69	0.3	0.99	0.32	0.08	no
17	0.80	0.21	1.01	0.08	0.96	0.71	0.3	1.01	0.70	0.49	0.5	0.99	0.30	0.08	no
18	0.72	0.28	1.01	0.08	0.98	0.71	0.3	1.01	0.67	0.29	0.7	0.99	0.33	0.08	no