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Keskusteluaiheita – Discussion papers

No. 734

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INFERIOR PLAYERS IN SIMPLE GAMES†

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† We acknowledge the hospitality of the Institut für Allokation und Wettbewerb, Universität Hamburg, whose 3rd Workshop on Applied Game Theory 1999 sparked this research. We thank W. Eichhorn, D. Felsenthal, M.J. Holler, M. Machover, H. Salonen, D. Samet and S. Seifert for their comments on an earlier draft.

ISSN 0781-6847 14.09.2000

NAPEL, Stefan – **WIDGRÉN**, Mika, **INFERIOR PLAYERS IN SIMPLE GAMES**. Helsinki: ETLA, Elinkeinoelämän Tutkimuslaitos, The Research Institute of the Finnish Economy, 2000, 35 p. (Keskusteluaiheita, Discussion Papers, ISSN 0781-6847; No. 734).

ABSTRACT: Power indices like those of Shapley and Shubik (1954) or Banzhaf (1965) measure the distribution of power in simple games. This paper points at a deficiency shared by all established indices: players who are inferior in the sense of having to accept (almost) no share of the spoils in return for being part of a winning coalition are assigned substantial amounts of power. A strengthened version of the dummy axiom based on a formalized concept of inferior players in a possible remedy. The axiom is illustrated first in a deterministic and then a probabilistic setting. With three axioms from the Banzhaf index, it uniquely characterizes the Striict Power Index (SPI). The Follower-Leader Index of Power (FLIP) establishes a further refinement. SPI and FLIP are shown to be special instances of a more general family of power indices related to the Banzhaf index but obeying the inferior player axiom.

SUMMARY

Power indices measure the distribution of power in n-person simple games, such as voting games. They have been applied to evaluate political and economic institutions in numerous empirical studies. Several plausible indices have been proposed, e.g. by Shapley and Shubik, or Banzhaf. However, none of these is consistent with competitive equilibrium or the core: in a three-player simple game where the only winning coalitions are the grand coalition *ABC* and the two coalitions *AB* and *AC*, core and competitive analysis attribute *all* power to player *A*. In contrast, all established indices assign a very substantial share of total power to players *B* and *C*. This is in our view a deficiency of established power indices and motivates the paper.

We illustrate and define the concept of *inferior players* as a first step to overcome this deficiency. Player i is called *inferior* in a given game if there exists some other player j who can actively prevent all winning coalitions in which i is crucial, i.e. has a swing, and who is himself crucial in at least one winning coalition not containing i. Thus, an inferior player i can be credibly threatened to be forced into a losing coalition unless substantial concessions are made to some player j – depriving i from practically all the power commonly attributed to his swing positions. We suggest to replace the dummy axiom conventionally used in power measurement by a stricter axiom based on inferior players.

Two indices that satisfy this *inferior player axiom* are presented. The *Strict Power Index (SPI)* is based on a stricter notion of Banzhaf's swings. It is shown to be globally monotonic, and we provide an explicit axiomatic characterization. A further strengthening yields the *Follower-Leader Index of Power (FLIP)*. Inferiority is also investigated in a probabilistic context, where it translates into restrictions on players' acceptance rates in the multilinear extension of the underlying game. Finally, SPI, FLIP, and the respective probabilistic restrictions are generalised.

Future research may apply the inferior player axiom to other indices than the non-normalized Banzhaf index, e.g. of Shapley and Shubik or of Deegan and Packel. It could be worthwhile to investigate more thoroughly the mathematical properties of the respective adaptations of the Banzhaf, Shapley-Shubik or Deegan-Packel index in terms of axiomatization, monotonicity, and susceptibility to typical paradoxes in power measurement. The inferior player axiom could also be extended to the domain of general games in characteristic function form. The concept of inferior players incorporates an important aspect of non-cooperative bilateral interaction into the cooperative world of power indices. It remains a challenge for the future to provide still more comprehensive non-cooperative foundations of power measurement.

1 Introduction

Power indices are functions that map n-person simple games, such as weighted multi-party voting games, to n-dimensional real vectors. They measure the distribution of power in a game, and assign to each player a number that indicates the player's ability to shape events, i.e. to determine the outcome of the game.

Power indices have been applied to evaluate numerous political and economic institutions. Power distributions in the context of shareholders' meetings have been one focus of attention (compare e. g. Leech 1988), with the related theoretical challenge of dealing with cross-ownership whereby players exert power both directly and indirectly (see Gambarelli and Owen 1994 for one solution). In the political sphere, decision making in the U.S. Congress, U.S. presidential elections (see Owen 1975), the U.N. Security Council, and, recently, the institutions of the European Union (e. g. Laruelle and Widgrén 1998; see Nurmi 1998 for a comprehensive survey) have all been studied extensively using power indices.

Despite the wide application and more than forty years after the seminal contribution to power measurement by Shapley and Shubik (1954), there is still considerable controversy as to what constitutes an appropriate power measure. On the surface, the debate is about whether minimal winning coalitions, crucial coalitions, player permutations, or else are the best primitives of power measurement. More fundamentally, the discussion is about the realism of the distinct probability models behind alternative indices and whether properties like monotonicity are to be regarded as essential.[1]

In the wake of Shapley and Shubik's work, numerous power indices have been

[1]Compare, for example, the recent discussion about monotonicity between Holler (1997), Nurmi (1997), Turnovec (1997), and Mercik (1997), which was sparked by Freixas and Gambarelli (1997). For an indication of the ongoing research providing ever more refined indices, compare e. g. Bilbao, Jiménez, and López (1998) and the contributions in Holler and Owen (2000).

proposed and axiomatically characterized – most notably by Banzhaf (1965), Deegan and Packel (1978), and Holler and Packel (1983).[2] However, none of these indices is consistent with traditional notions of competitive equilibrium or the cooperative concept of the core: in a three-player simple game where the only winning coalitions are the grand coalition ABC and the two coalitions AB and AC, core and competitive analysis attribute *all* power to player A. In contrast, the indices of Shapley-Shubik, Banzhaf, Deegan-Packel, or Holler-Packel respectively assign $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, and $\frac{1}{2}$ of total power to players B and C.[3]

In this paper, we define the concept of *inferior players* as a first step to overcome this deficiency. Based on this definition we suggest to replace the dummy axiom conventionally used in power measurement by a stricter axiom. The proposed axiom requires indices to *not* take into account a player's supposed power (as traditionally measured by swings, pivot positions etc.) if some other player can issue the following ultimatum to him: accept (almost) no share of the spoils from a winning coalition or be prevented from taking part in one at all. Thus, power measurement is brought more in line with competitive analysis.

Section 2 starts with some preliminary definitions and then introduces the concept of inferior players. The inferior player axiom is stated, and the rest of the paper deals with possible applications of it. First, in section 3, two deterministic indices based on the Banzhaf index are presented, one of which is explicitly axiomatized, and analyzed with respect to monotonicity properties. Then, in section 4, inferiority is investigated in the realm of probabilistic power

[2]For a recent comparative investigation of power indices, their properties and applicability, see Felsenthal and Machover (1998).

[3]Note that successful attempts have been made to provide a non-cooperative foundation for the value concepts related to power indices, most notably the Shapley value (see Hart and Mas-Collel, 1996, for a recent contribution). Doubts about the realism of the highly specific bargaining procedures and respective limit considerations are, in our view, confirmed by this simple example.

measurement. Adapting probabilistic indices to the inferiority axiom amounts to imposing restrictions on players' acceptance rates in the multilinear extension of the underlying simple game. We present two rather intuitive conditions both of which imply that zero power for inferior players is indicated. They turn out to define the probabilistic counterparts of the two indices introduced in section 3. In section 5, these two conditions and the corresponding indices are generalized. A whole continuum of indices is shown to be in line with the inferior player axiom and available for selection through additional axioms. Section 6 concludes.

2 Inferior Player Axiom

2.1 Preliminary definitions

Let u and v denote n-person *simple games* and $N = \{1, 2, \ldots, n\}$ their common set of players. $\mathcal{P}(N)$ is the set of feasible coalitions. The simple game v (and u analogously) is characterized by the set $W(v) \subsetneq \mathcal{P}(N)$ of *winning coalitions.* $W(v)$ satisfies $\emptyset \notin W(v)$, $N \in W(v)$ and $S \in W(v) \wedge S \subset T \;\Rightarrow\; T \in W(v)$. v can also be described by a characteristic function $v : \mathcal{P}(N) \to \{0, 1\}$ with

$$v(S) = \begin{cases} 0; & S \notin W(v) \\ 1; & S \in W(v). \end{cases}$$

$\mathcal{G}$ denotes the set of all n-person simple games. *Voting games* are special instances of simple games that are characterized by a non-negative real vector $r_v = (q; w_1, \ldots, w_n)$, where w_i represents player i's voting weight in game v and q represents the quota of votes that establishes a winning coalition.

A player who by leaving a winning coalition $S \in W(v)$ turns it into a losing coalition $S \backslash \{i\} \notin W(v)$ has a *swing* in S and is called a *crucial* or *critical member* of coalition S.

Coalitions in which at least one member is crucial are called crucial coalitions.[4]

[4]Deegan and Packel (1978) use the term 'minimal winning coalition', Felsenthal and Ma-

Coalitions where player i is critical are called *crucial coalitions with respect to i.* Let

$$C_i(v) := \{S \subset N \mid S \in W(v) \wedge S \setminus \{i\} \notin W(v)\}$$

denote the set of crucial coalitions w.r.t. i. The number of swings of player i in simple game v is thus

$$\eta_i(v) := \mid C_i(v) \mid .$$

A player i who is never crucial, i. e. $\eta_i(v) = 0$, is called *dummy player.* It is common to all established power indices that dummy players are considered powerless.

A power index is a mapping from the space of n-player simple games into the n-dimensional space of non-negative real numbers, assigning to each player $i \in N$ a number $\mu_i(v)$ that indicates i's power in the considered game $v \in \mathcal{G}$. An index $\mu : \mathcal{G} \rightarrow I\!R^n_+$ is *locally monotonic* on the domain of voting games if $w_i \geq w_j$ in r_v implies $\mu_i(v) \geq \mu_j(v)$, i. e. more weight implies more power.[5] The Shapley-Shubik and Banzhaf indices are locally monotonic, the Deegan-Packel or Holler-Packel indices are not.

Recently, monotonicity has also been defined with respect to players' position in different simple games. Following Levínský and Silárszky (2000), a game u can be considered 'better' than game v from player i's point of view if all winning coalitions of v with i also win in u (and, possibly, some other coalitions with i win in u) and if all winning coalitions of u without i also win in v (and possibly some more). Formally, we define the preference relation $\succ_i$ with

$$u \succ_i v \;:\Longleftrightarrow \begin{cases} \quad i \in S \wedge S \in W(v) \;\Rightarrow S \in W(u) \\ \wedge \;\; i \notin S \wedge S \in W(u) \;\Rightarrow S \in W(v). \end{cases}$$

An index μ is *globally monotonic* if $u \succ_i v$ implies $\mu_i(u) \geq \mu_i(v)$ for all $i \in N$. In the special case of voting games, global monotonicity requires that if player i's

chover (1998) the term 'vulnerable coalition' instead of 'crucial coalition'. We, like other authors, follow Bolger's (1980) conceptualization.

[5] Levínský and Silárszky (2000) provide a definition on the entire domain of simple games.

weight in u is greater or equal than that in v, i. e. $w_i(u) \geq w_i(v)$, and the weights of all players $j \neq i$ are not greater in u than in v, i. e. $\forall j \neq i : w_j(u) \leq w_j(v)$, then i's indicated power in u is not smaller than in v. Provided that an index – like all established indices – is symmetric, global monotonicity implies local monotonicity. The Shapley-Shubik index and the non-normalized Banzhaf index are globally monotonic, the normalized Banzhaf index is not.

When power in simple games is analysed in a probabilistic context, each player's probability of accepting a random proposal is considered, and referred to as i's *rate of acceptance* $p_i \in [0,1]$. Assuming that actual acceptance decisions are taken independently across players, the probability of forming a coalition $S \subset N$ is $\Pr(\underline{S} = S) = \Pi_{i \in S} p_i \Pi_{j \notin S}(1-p_j)$. Weighting all coalitions $S \subset N$ with their respective value $v(S) \in \{0,1\}$, we get the mathematical expectation

$$\begin{aligned} E(v) = f(p_1, \ldots, p_n) &= \sum_{S \subset N} \prod_{i \in S} p_i \prod_{j \notin S} (1-p_j)\, v(S) \\ &= \sum_{S \in W(v)} \prod_{i \in S} p_i \prod_{j \notin S} (1-p_j) \end{aligned}$$

of game v, also called its *multilinear extension (MLE)* (see Owen 1972, 1988). The MLE gives the probability of formation of a winning coalition in v.

Graphically, the set of all possible coalitions $\mathcal{P}(N)$ corresponds to the set of corner points of the n-dimensional unit cube $\{0,1\}^n$. Coordinates x_i of points $x \in \{0,1\}^n$ indicate whether player i belongs to the considered coalition or not. The characteristic function of a simple game can be formulated as the mapping $v : \{0,1\}^n \to \{0,1\}$, and MLE then simply extends the domain of v to $[0,1]^n$ and its range to $[0,1]$. Any point within the cubic gives a combination of players' rates of acceptance.

Denoting player i's marginal contribution to coalition S by

$$\Delta_i v(S) := \begin{cases} 1, & S \in C_i(v) \\ 0, & S \notin C_i(v), \end{cases}$$

we get the following first order partial derivative of the MLE of v with respect to

p_i:

$$\begin{aligned} f_i(p_1,\ldots,p_n) &= \sum_{S\in W(v)} \prod_{\substack{j\in S\\ j\neq i}} p_j \prod_{\substack{k\notin S\\ k\neq i}} (1-p_k)\,\Delta_i v\,(S) \\ &= \sum_{S\in C_i(v)} \prod_{\substack{j\in S\\ j\neq i}} p_j \prod_{k\notin S} (1-p_k)\,. \end{aligned}$$

This expression, usually referred to as player i's *power polynomial* (Straffin 1977, 1988), gives the probability of i having a swing in the random coalition to be formed in game v. Typically, players' acceptance rates for a random proposal are modeled as random variables. Let P be the distribution of random vector $(p_1,\ldots,p_n)$. Then, the expectation

$$Ef_i\,(p_1,...,p_n) = \int f_i\,(p_1,...,p_n)\,dP \tag{1}$$

is an indicator of i's power in game v. Note that v is now characterized both by the set of winning coalitions and by a specific distribution of players' acceptance rates P (cp. Owen 1972). The *probabilistic power index* defined by (1) coincides with traditional deterministic indices for several plausible probability models.

2.2 Inferior players

In the introduction, the game v with $W(v) = \{AB, AC, ABC\}$ was used to illustrate the divergence between power predictions based on conventional indices on the one hand, and competitive analysis or the concept of the core of a game on the other hand.

In the considered game v, player A can issue ultimata to both B and C.[6] Imagine that the spoils of a winning coalition are \$100 and to be split among its members. Or, alternatively, consider 100 policy units, referring to facets of a political proposal with diverging preferences. Regardless of the object of

[6] An alternative argument based on a process of mutual underbidding of B and C can be constructed. We prefer the argument based on the ultimatum game since it does not rely on the presence of *multiple* inferior players and is thus more general.

conflicting interests, whenever the situation permits negotiations before the final establishment of a winning coalition player A is in the position of the proposer in a non-cooperative *ultimatum game* with B as responder. Since A has the option to form a winning coalition without B, B cannot do better but to accept whatever A proposes in terms of B's share of spoils or political influence. A anticipates this and rationally offers B a share of (almost) nothing. The same holds for possible negotiations between A and C.

More accurately, the ultimatum game $G = (\{A, B\}, \Sigma, \Pi)$ with players A and B, strategy spaces $\Sigma_A = [0, 100]$ and $\Sigma_B = \{\sigma_B \,|\, \sigma_B : [0, 100] \to \{0, 1\}\}$, and payoffs $\Pi_A(\sigma_A, \sigma_B) = (100 - \sigma_A)\, \sigma_B(\sigma_A)$ and $\Pi_B(\sigma_A, \sigma_B) = \sigma_A\, \sigma_B(\sigma_A)$ has a unique subgame perfect equilibria (SPE): $(0, \sigma_B \equiv 1)$, i. e. A offers nothing, and B accepts regardless of A's offer.

It may look extreme that player B accepts in this equilibrium though he is in fact indifferent between accepting and rejecting. For better intuition, one may interpret the SPE as the limit of situations in which A offers an arbitrarily small $\varepsilon > 0$ to B; B then does strictly better by accepting. In any case, it is uncontroversial to summarize the ultimatum situation by saying that a rational player B will accept *practically* or *almost* nothing in return for giving his consent to A's proposal. The same applies to player C and the similarly defined game $G' = (\{A, C\}, \Sigma, \Pi)$.

Players B and C are not exactly in the position of dummy players, but quite close to it. Note that collusion between B and C with the objective to prevent A from exploiting her bargaining power is not stable. It suffices for A to make the credible declaration that only *one* coalition partner will be accepted, and to approach either B or C, or auction off the right to exclusive partnership.

B's position can be described as follows: there exists a player – here A – that can veto or prevent all coalitions in which B makes a positive contribution, i. e. is crucial, but who can herself form a crucial coalition without an opportunity for B to interfere. Threatened by A taking this outside option, B is in the unpleasant situation of preferring (almost) any amount of concession with respect to A's demands to being excluded from participating in a winning platform at all. B is in this sense an *inferior player* in game v (the same holds for C). Formalizing this intuitive notion of inferiority, we state:

Definition 1: Player i is *inferior* in simple game v if $\exists j \neq i$:

$$\begin{aligned} &\forall S \in C_i(v): \quad j \in S \\ \wedge \quad &\exists S' \in C_j(v): \quad i \notin S' \end{aligned}$$

Let $I(v) \subsetneq N$ denote the set of inferior players in v and $m = |I(v)|$ its cardinality. The relationship to dummy players is simple:

Corollary 1: Every dummy player in simple game v is also an inferior player. The reverse is not true.

Proof: Since $C_i(v) = \emptyset$ for a dummy player i, the first part of the definition is trivially satisfied. The second part is satisfied by any player j with $C_j(v) \neq \emptyset$ (there is at least one). That inferior players need not be dummy players is obvious from the example above. □

There is a neat equivalent definition of inferior players:

Proposition 1: Player i is inferior in simple game v $\Longleftrightarrow \exists j \neq i : C_i(v) \subsetneq C_j(v)$.

Proof: a) Let i be inferior in v, i. e. $\forall S \in C_i(v) : j \in S \wedge \exists S' \in C_j(v) : i \notin S'$. Now assume that there exists $\tilde{S} \in C_i(v)$ with $\tilde{S} \notin C_j(v)$. It follows that $\tilde{S} \in W(v)$, and $\tilde{S} \setminus \{j\} \in W(v)$ since $\tilde{S} \notin C_j(v)$. Furthermore, from $\tilde{S} \setminus \{i\} \notin W(v)$ it follows that $\tilde{S} \setminus \{j\} \setminus \{i\} \notin W(v)$. Thus, $\tilde{S} \setminus \{j\} \in C_i(v)$ – a contradiction to $\forall S \in C_i(v) : j \in S$, so we must have $C_i(v) \subseteq C_j(v)$. Together with the fact that j is crucial in at least one coalition S' without i, we have $C_i(v) \subsetneq C_j(v)$.

b) First, $C_i(v) \subsetneq C_j(v)$ implies that $\forall S \in C_i(v) : S \in C_j(v)$. By definition, $S \in C_j(v)$ implies $j \in S$. Second, assume $C_i(v) \subsetneq C_j(v)$ and $\forall S' \in C_j(v)$:

$i \in S'$. Using the argument in a), the latter implies $C_j(v) \subseteq C_i(v)$. This is a contradiction. □

Players who are not inferior are generally agreed to be *powerful players.* It is convention in power measurement to require a reasonable power index $\mu : \mathcal{G} \longrightarrow \mathbb{R}^n_+$ to indicate zero power for dummy players, i. e. $\mu_i(v) = 0$ if i is a dummy player in v. Given our stricter notion of what constitutes powerful and powerless players, we suggest to strengthen the conventional dummy axiom to:

Inferior Player Axiom: i is inferior in $v \Longrightarrow \mu_i(v) = 0$.

As illustrated by our example, none of the conventional power indices satisfies the inferior player axiom.

It is necessary to ask whether our stricter notion of powerful and powerless players applies to *all* situations in which simple games are played. This is not the case. The motivation for introducing the concept of inferior players which was given above rested on three implicit premises. First, we assumed the opportunity for *negotiation* about spoils of a winning coalition or details of a policy proposal before eventual coalition formation. This assumption is not fully appropriate when many players have anonymous votes on an exogenously given proposal without precise spoils to be split. We believe, though, that these situations are rare at the party level of political institutions, to which power indices are typically applied.

Second, we assumed that the essence of negotiations was adequately captured by a *single-shot ultimatum game*, and that, third, negotiations were with *rational* inferior players. Therefore we considered only equilibrium or close-to-equilibrium situations in which inferior players agree to accept (practically) no share of spoils or influence on policy. When the same players in a political institution interact, for example, in an indefinitely repeated manner, the single-shot ultimatum game no longer describes the relation between inferior and non-inferior players adequately. Depending on players' time preference, e. g. the discount factor they apply to future payoffs, there may be a multitude of SPE involving payoffs for inferior players which are bounded away from zero. Moreover, laboratory experiments (sparked by Güth *et al.* 1982; for an overview see Roth 1995) indicate that even a unique SPE can be a shaky predictor of actual human bargaining behaviour. We doubt that these critical experimental results allow many statements about behaviour in political institutions. Still, the ultimatum story behind inferior players loses its appeal when the simple game under investigation is played in an imperfectly rational environment.

It is also worthwhile to ask whether applicability of the concept of inferior players is restricted by a particular notion of power underlying it.[7] The context of simple games played in voting bodies is the most important. There, our understanding of "ability to shape events" refers to more than merely the chance event of being crucial with an anonymous vote when the quota is just about reached by votes of the other players. This rather narrow view of influencing outcomes is behind the concept of *I-power*, which Felsenthal and Machover (1998, pp. 35ff) define. They contrast it with *P-power* – referring to the prize of power in terms of a share of a fixed purse – and argue that power indices could and should be distinguished by what sort of power they measure.

On first view, our concept of inferior players belongs to the sphere of P-power since it assumes that a decision is somehow still a matter of negotiation – be it about associated financial spoils of contributing to a specific winning coalition or other aspects of a political deal. However, power remains a more fuzzy concept in our view than Felsenthal and Machover's precise conceptualization of I-power and P-power suggests. P-power builds on I-power since it is the potential effects of players' voting behaviour which underlies their claims when dividing the spoils. And a pure voting situation in which I-power could be analyzed is typically brought about by an externally made proposal which is, however, often

[7]We thank M. Machover for first posing this question.

influenced by players' preferences, i. e. refers to some compromise on potential spoils as captured by P-power.

We are therefore uneasy to associate inferior players too closely with P-power. Despite the appeal of I-power vs. P-power considerations, we prefer to distinguish between power indices on the basis of varying informational assumptions (cp. Straffin 1977, 1988) and the situational aspects discussed above. We regard the actual decision environment – relevance of negotiations, repeated vs. single-shot interaction, perfectly vs. boundedly rational players – as the chief determinant of which index type with or without the inferior player axiom is appropriate.

3 Application to deterministic indices

In order to show that the inferior player axiom leads to reasonable power indices with desirable other properties and plausible probability models, we will define and investigate two example indices related to the Banzhaf index (1965). This is based on the traditional deterministic formulation of power indices. Note that similar adaptations could be made to the Shapley-Shubik index, the Deegan-Packel index, or other power indices.

3.1 From Banzhaf to the Strict Power Index

The *non-normalized Banzhaf index* β is defined by

$$\beta_i(v) := \frac{\eta_i(v)}{2^{n-1}}, \qquad i \in N.$$

Since there are exactly 2^{n-1} potential coalitions in which i is a member and could theoretically have a swing, $\beta_i(v)$ can be at most 1, represents i's ratio of actual to potential number of swings, and is sometimes referred to as *swing probability* (cp. Dubey and Shapley 1979).

In order to construct an index that is based on the Banzhaf index but satisfies the inferior player axiom, it is straightforward to start with the following adaptation of the notion of swings:

Definition 2: Player i has a *strict swing* in winning coalition S if

a) i can turn S into a losing coalition by leaving it, and

b) i is not inferior in v, i.e. $i \notin I(v)$.

Formally, let

$$\tilde{\eta}_i(v) := \begin{cases} \mid C_i(v) \mid; & i \notin I(v), \\ 0; & i \in I(v), \end{cases}$$

denote the number of strict swings of player i in game v.

Substituting strict swings for swings in the definition of the Banzhaf index, we get the following new index:

Definition 3: The *Strict Power Index (SPI)* $\tilde{\beta} : \mathcal{G} \longrightarrow I\!R^n_+$ is given by

$$\tilde{\beta}_i(v) := \frac{\tilde{\eta}_i(v)}{2^{n-1}}, \qquad i \in N.$$

Obviously, we have $\tilde{\beta}_i(v) \leq \beta_i(v)$. By its construction, $\tilde{\beta}$ indicates zero power for inferior players – no matter whether they are true dummy players, or whether they are crucial in some coalition(s) S but have to accept whatever the powerful members of S offer them. For our introductory example, the SPI produces the vector $\tilde{\beta}(v) = (\frac{3}{4}, 0, 0)$. Acknowledging that $\tilde{\beta}$ is not subject to any efficiency requirements – it does not indicate a distribution of spoils but of power – this result nicely complies with the notions of competitive analysis presented above. The difference between $\tilde{\beta}_A(v) = \frac{3}{4}$ and 1 indicates that though A is the only powerful player in v, A is not in the position of a proper dictator.

One may ask why the core of the game – in our example $\{(1,0,0)\}$ – is not used as an indicator of both power *and* an efficient equilibrium distribution of spoils. However, the core is a *set concept* and generally produces non-singleton or empty sets which are useless in terms of power evaluation. Normalization is a way to ensure efficiency of power vectors, but typically destroys the probability model underlying the non-normalized index, and also its monotonicity properties.

Corresponding with the non-normalized Banzhaf index β, we have the following comforting result for the SPI:

Proposition 2: The SPI $\tilde{\beta}$ is *globally monotonic*, i. e. $\forall i \in N$

$$u \succ_i v \Longrightarrow \tilde{\beta}_i(u) \geq \tilde{\beta}_i(v).$$

The proof is given in the appendix.

Before we give a full axiomatic characterization of the SPI, recall that the Banzhaf index can be characterized as the unique index to satisfy the following four axioms (cp. Dubey and Shapley 1979):

A1: (dummy players) i is a dummy player in $v \Longrightarrow \mu_i(v) = 0$.

A2: (absolute power) $\sum_{i=1}^{n} \mu_i(v) = \frac{1}{2^{n-1}} \sum_{i=1}^{n} \eta_i(v)$.

A3: (anonymity) For any permutation π of $N = \{1, \ldots, n\}$: $\mu_{\pi(i)}(\pi v) = \mu_i(v)$.

A4: (addition) $\forall u, v \in \mathcal{G} : \mu(u \vee v) = \mu(u) + \mu(v) - \mu(u \wedge v)$.

In A3, the permutation game πv is defined by $(\pi v)(S) := v(\pi^{-1}(S))$. In A4, the game $u \vee v$ is defined by the characteristic function $(u \vee v)(S) := \max\{u(S), v(S)\}$, and $u \wedge v$ by $(u \wedge v)(S) := \min\{u(S), v(S)\}$.

The conventional technique of showing that A1–A4 do, in fact, uniquely characterize the Banzhaf index is based on the possibility to decompose every game $u \in$ into $u_{S_1} \vee \ldots \vee u_{S_r}$. Here, $S_1, \ldots, S_r$ denote those coalitions in which every member is crucial in u ($r \geq 1$ is game-specific). They are also called *minimal winning coalitions (MWC)* of u, and constitute the set $M(u)$. The games $u_{S_1}, \ldots, u_{S_r}$ denote *auxiliary games* which have $S_1, \ldots, S_r$, respectively, as their only MWC. Hence, coalition S is winning in u_{S_k}, i. e. $S \in W(u_{S_k})$, if and only if it contains S_k, or $S_k \subseteq S$.

Using this fact, one first shows that A1–A3 suffice to uniquely define a power value for any single-MWC auxiliary game. Then, second, one can use A4 to show that a uniquely defined power value is defined for any simple game u by its decomposition into auxiliary games.

Our axiomatization will follow exactly the same steps. There is, however, an important difference: A4 above, and similarly the respective addition axioms of the Shapley-Shubik, Deegan-Packel, and Holler-Packel indices, is based on an essentially *linear* understanding of power. Given two simple games u and v, and player i's power value in them, i's power in the sum game $u \vee v$ – comprising exactly all winning coalitions from both u and v – is according to A4 essentially the sum of the two power values, with a correction made for double counting of swings by subtracting power from $u \wedge v$. This stands in contrast to a fundamental *non-linearity* under the inferior player axiom. A player i may be inferior, i. e. powerless, in u and in v because he faces a credible ultimatum by at least some (different) other player in both games. Yet he may have an outside option protecting him against ultimata in $u \vee v$, suddenly making i very powerful indeed. An example of this are players B and C in the two four-player games u and v with $W(u) = \{AB, AC, ABC, ABD, ACD, ABCD\}$ and $W(v) = \{AD, BCD, ABD, ACD, ABCD\}$.

Since the concept of inferior players reflects (aspects of) the entire strategic situation in a given game u, an arbitrary decomposition into $u_1 \vee u_2 \equiv u$ is not in general meaningful. An application of an index μ that obeys the inferior player axiom to u_1 and u_2 will even with knowledge about the index value for $u_1 \wedge u_2$ not say much about power in u. Exceptions are those decompositions in which all players are non-inferior in both u_1 and u_2.

Therefore, axiomatization based on the inferior player axiom has to do without an addition axiom of the same simplicity as for e. g. Banzhaf or Shapley-Shubik index. Before we present the four axioms that uniquely characterize the SPI, we state the following lemma:

Lemma 1: $\forall u, v \in : \mu_i(u \vee v) = \mu_i(u) + \mu_i(v) - \mu_i(u \wedge v)$ for a given player i

$\Longrightarrow \forall u \in : \mu_i(u) \equiv \mu_i(\bigvee_{S \in M(u)} u_S) = \sum_{T \subseteq (M(u))} (-1)^{|T|-1} \mu_i(\bigwedge_{S \in T} u_S)$

Proof: The proof is by complete induction. Let $w^r \in$ denote an arbitrary simple game with exactly $r \geq 1$ minimal winning coalitions, i. e. $M(w^r) = \{S_1, \ldots, S_r\}$. Applying the premise to $\mu_i(u_{S_1} \vee u_{S_1})$, it follows that the claim is true for $r = 1$.

We proceed to $r+1$ by adding a MWC S_{r+1} to w^r, i. e. we consider simple game w^{r+1} with $M(w^{r+1}) = \{S_1, \ldots, S_r, S_{r+1}\}$. Applying the premise and the result for r, we have

$$\begin{aligned} \mu_i(w^{r+1}) &= \mu_i(w^r \vee u_{S_{r+1}}) = \mu_i(w^r) + \mu_i(u_{S_{r+1}}) - \mu_i(w^r \wedge u_{S_{r+1}}) \\ &= \sum_{T \subseteq (\{S_1, \ldots, S_r\})} (-1)^{|T|-1} \mu_i(\bigwedge_{S \in T} u_S) + \mu_i(u_{S_{r+1}}) - \mu_i(w^r \wedge u_{S_{r+1}}). \end{aligned} \tag{2}$$

Again applying the result for r, the last term of this expression can also be written as

$$\begin{aligned} \mu_i(w^r \wedge u_{S_{r+1}}) &= \mu_i\left(\bigvee_{S \in M(w^r)} (u_S \wedge u_{S_{r+1}})\right) \\ &= \sum_{T \subseteq (\{S_1, \ldots, S_r\})} (-1)^{|T|-1} \mu_i\left(\bigwedge_{S \in T} (u_S \wedge u_{S_{r+1}})\right) \\ &= -\sum_{T \subseteq (\{S_1, \ldots, S_r\})} (-1)^{|T \cup \{S_{r+1}\}|-1} \mu_i(\bigwedge_{S \in T \cup \{S_{r+1}\}} u_S). \end{aligned}$$

Plugging this into (2) proves the result for $r+1$, and thus the lemma. □

Now we are ready to state and prove the following result:

Proposition 3: The Strict Power Index (SPI) $\tilde{\beta}$ is the unique power index satisfying the following four axioms:

A1*: (inferior players) i is inferior in $v \Longrightarrow \mu_i(v) = 0$.

A2: (absolute power) $\sum_{i=1}^{n} \mu_i(v) = \frac{1}{2^{n-1}} \sum_{i=1}^{n} \tilde{\eta}_i(v)$.

A3: (anonymity) For any permutation π of $N = \{1, \ldots, n\}$: $\mu_{\pi(i)}(\pi v) = \mu_i(v)$.

A4*: (aggregation) i is not inferior in v
$\Longrightarrow \mu_i(u) \equiv \mu_i(\bigvee_{S \in M(u)} u_S) = \sum_{T \subseteq (M(u))} (-1)^{|T|-1} \mu_i(\bigwedge_{S \in T} u_S)$.

Proof: We first have to check that the SPI, in fact, satisfies the four axioms. A1* and A2 are satisfied by construction. A3 follows from the anonymity of swings, and therefore of strict swings. A4* refers to non-inferior players only. For those players, the SPI is constructed to coincide with the Banzhaf index. Therefore, the premise in Lemma 1 is satisfied for all non-inferior players i. Thus, A4* is satisfied.

Now, it remains to be shown that above axioms uniquely define a power index, i. e. a function $\mu : \mathcal{G} \rightarrow I\!R^n_+$. We first consider only games with a single minimal winning coalition S, i. e. the auxiliary game u_S. All players $i \notin S$ are inferior in u_S. For all inferior players i of u_S, A1* defines $\mu_i(u_S) = 0$. For all non-inferior players $j \in S$, A3 implies the same power value $\mu_j(u_S) = a$ with $a \geq 0$. Thus, we have $\sum_{i=1}^n \mu_i(u_S) = a|S|$. A2 requires $a|S| = \frac{1}{2^{n-1}} \sum_{i=1}^n \tilde{\eta}_i(u_S)$. By construction of u_S we have

$$\tilde{\eta}_i(u_S) = \begin{cases} 0, & i \not\in S \\ 2^{n-|S|}, & i \in S, \end{cases}$$

implying

$$a = \frac{1}{2^{|S|-1}}.$$

Thus, μ is uniquely defined for all auxiliary games u_S with $S \in (N) \setminus \emptyset$. By A1* and A4*, this is extended to the entire domain of simple games. □

The intuition behind A1* was given above, and that for A2 and A3 is the same as in case of the Banzhaf index. Compared to A4, A4* looks considerably more clumsy and less intuitive. However, as made clear by the proof of Lemma 1, aggregation axiom A4* is merely a specialization of the simple addition axiom A4 to the case of non-inferior players. We get a rather complicated mathematical formulation of A4* because it takes into account that a powerful player in u may be inferior in component games of decompositions $u \equiv u_1 \vee u_2$, by directly referring to the level of a game's constituting auxiliary games. Thus the general non-linearity of power is taken into account.

As should be the case for a characterization of an index by a set of axioms, we have:

Proposition 4: Axioms A1*, A2, A3, and A4* are logically independent.

Proof: In order to show independence it suffices to give an example index for each axiom which violates the considered axiom, but is consistent with the remaining three. By implication, none of the axioms is then a logical consequence of the other ones.

The Banzhaf index β obviously violates the inferior player axiom A1* but obeys A2–A4*. Hence A1* cannot be implied by A2–A4*. An index which violates the absolute power axiom A2 but obeys the remaining axioms can be obtained by normalizing the SPI to

$$\tilde{\beta}_i'(v) := \frac{\tilde{\beta}_i(v)}{\sum_{i=1}^{n} \tilde{\beta}_i(v)}.$$

An index which is consistent with A1*, A2, and A4*, but not with A3 can be constructed by allocating the entire number of strict swings in single-MWC auxiliary games to the non-inferior player $j \in N \setminus I(v)$ with lowest order number. This is in line with A1* and A2. The index is extended to general simple games by A4*. Considering a simple example, e. g. $W(v) = \{AB, AC, BC, ABC\}$, shows that the resulting index is not equivalent to SPI. Finally, a re-scaling of the Follower-Leader Index of Power defined in the next section satisfies A1*, A2, and A3, but violates A4*. □

3.2 Follower-Leader Index of Power

Another way to adapt the Banzhaf index to the inferior player axiom is to formalize the following modification of the concept of swings:

Definition 4: Player i has a *leader swing* in coalition $S \subset N$ if

a) i can turn S into a losing coalition by leaving it,

b) i is not inferior in v, i. e. $i \notin I(v)$, and

c) all inferior players $i \in I(v)$ are part of S.

Formally, let

$$\hat{\eta}_i(v) :=\mid \{S \subset N \mid S \in C_i(v) \ \wedge \ i \notin I(v) \ \wedge \ I(v) \subset S\} \mid$$

denote the number of leader swings of player i in game v.

The concept of leader swings is based, first, on the intuition that rational players anticipate that inferior players are (almost) not rewarded for participating in a winning coalition $S \in W(v)$. What is specific to leader swings is, second, the consequential assumption that inferior players are therefore ready to "follow" the powerful players or "leaders" of the game into whatever winning coalition will be established. It is straightforward to construct the following index:

Definition 5: Let $m = |I(v)| < n$ be the number of inferior players in v. The *Follower-Leader Index of Power (FLIP)* $\hat{\beta} : \mathcal{G} \rightarrow \mathbb{R}^n_+$ is given by

$$\hat{\beta}_i(v) := \frac{\hat{\eta}_i(v)}{2^{n-m-1}}, \qquad i \in N.$$

In our example, A's swing in coalition ABC is the only leader swing of the game and $m = 2$. Hence, FLIP produces the power vector $\hat{\beta}(v) = (1, 0, 0)$.

Note that in a n-person simple game with m inferior players, only the 2^{n-m-1} coalitions that include a given powerful player i *and* all m inferior players potentially deliver leader swings to i. Re-scaling with $1/2^{n-m-1}$ therefore ensures that $\hat{\beta}_i(v)$ is at most 1. Since the denominator of $\hat{\beta}$ depends on the *game-specific* number m of inferior players rather than the fixed number n of players, monotonicity investigations are more messy for the FLIP than for the SPI. We leave this for future investigation.

4 Inferiority in a probabilistic context

In this section, we show that the inferior player axiom is compatible with plausible restrictions of players' acceptance rates p_i in the MLE of a game v.

4.1 Strict Power Condition

With uniformly distributed rates of acceptance[8]

$$p_j \sim U(0,1) \qquad \forall j \in N$$

in game v's MLE, we obtain $(i \in N)$

$$Ef_i(p_1, \ldots, p_n) = \underbrace{\int_0^1 \cdots \int}_{n} \sum_{S \subset C_i(v)} \prod_{\substack{j \in S \\ j \neq i}} p_j \prod_{\substack{k \notin S \\ k \neq i}} (1 - p_k)\, dp_1 ... dp_n = \frac{\eta_i(v)}{2^{n-1}} = \beta_i(v),$$

i. e. the non-normalized Banzhaf index.[9]

To operationalize the inferior player axiom in the probability model we look for plausible restrictions on players' acceptance rates which ensure that zero power is indicated for inferior players. Remember that the original motivation for formulating the inferior player axiom was the observation that, in our example game, player A was able to play off both B and C. The equilibrium of the hypothetical bargaining process covering that notion gives inferior players a payoff of zero. This means that an inferior player is always *indifferent* between joining a winning coalition or staying outside, i. e. between voting for or against a proposal. This is formalized by:

Strict Power Condition (SPC): i is inferior in $v \Longrightarrow p_i \equiv \frac{1}{2}$.

[8]This assumption is often referred to as *independence* (Straffin 1977).

[9]Note that the Shapley-Shubik index can be derived from a slightly different and somewhat more restricting assumption where $p_i = t$ for all i and $t \sim U(0,1)$. This assumes full correlation between players' rates of acceptance.

As intended, we have:

Proposition 5: A MLE satisfying the SPC indicates zero power for inferior players.

Proof: This is obvious from the fact that p_i is no longer a variable in f, so that $\partial f(p_1, \ldots, p_n)/\partial p_i \equiv 0$. □

In the case of our example game v with $W(v) = \{AB, AC, ABC\}$ the MLE is

$$f(p_A, p_B, p_C) = p_A p_B(1-p_C) + p_A(1-p_B)p_C + p_A p_B p_C.$$

Imposing the SPC yields

$$f(p_A, p_B, p_C) = \frac{3}{4} p_A.$$

This produces the power vector $\nabla f(p_A, p_B, p_C) = (\frac{3}{4}, 0, 0)$ equal to the SPI of v (see section 3). In fact, we can show:

Proposition 6: Applying the SPC in the setting of the probabilistic Banzhaf index, i. e.

$$p_i \begin{cases} \equiv \frac{1}{2}; & i \in I(v) \\ \sim U(0,1); & i \notin I(v) \end{cases}$$

implies the probabilistic Strict Power Index (SPI) $\tilde{\beta}$.

The proof is given in the appendix.

4.2 Follower-Leader Condition

After consideration of the probabilistic version of the SPI, we now turn to the FLIP. In order to operationalize the idea of leader swings in the probability model it is natural to require that an inferior player does not vote against a proposal, which is supported by a player to whom he is inferior to. Formally, we assume that the will of powerful player j determines the acceptance rate of inferior player i by requiring:

Follower-Leader Condition (FLC): i is inferior to j in $v \Longrightarrow p_i p_j = p_j$.

With this we can establish the following:

Proposition 7: A MLE satisfying the FLC indicates zero power for inferior players.
The proof is given in the appendix.

Similar to the link established between SPC and SPI, we have:

Proposition 8: Applying the FLC in the setting of the probabilistic Banzhaf index, i. e.

$$\forall i \in N : p_i \sim U(0,1) \;\text{ s. t. }\; i \text{ inferior to } j \text{ in } v \Rightarrow p_i p_j = p_j,$$

implies the probabilistic Follower-Leader Index of Power (FLIP) $\hat{\beta}$.
The proof is given in the appendix.

The FLC implies that $p_i = 1$ if $p_j > 0$. Player i thus unequivocally supports a bill if there is a positive probability that player j votes for the bill. If $p_j = 0$ there are no restrictions on p_i. The intuition behind FLC is that an inferior player i is ready to support (or follow) the superior player j if it is not sure, in probabilistic terms, that j votes against.

It is worth noting that the FLC asks for a different and stronger type of behavioural similarity than full correlation, which drives the probabilistic interpretation of the Shapley-Shubik index. In the full correlation case, there are no leaders and followers but players follow a common standard when forming their rates of acceptance and this standard is external for any coalition.

5 Generalizations

The two preceding sections illustrated the concept of inferior players by examining two examples: first, we considered swings only of non-inferior players, i. e. strict swings, to define the Strict Power Index (SPI). The SPI turned out to have a probabilistic foundation in the Strict Power Condition (SPC), which is one way of formalizing inferior players' practical indifference about joining a winning coalition. The second example strengthened strict swings to leader swings by assuming that all inferior players follow the leaders of the game into a winning coalition, possibly in an attempt to retain at least minimal influence. This produced the Follower-Leader Index of Power (FLIP) with the related probabilistic Follower-Leader Condition (FLC). Both SPI and FLIP are merely special cases from a continuum of indices that implement the inferior player axiom. To see this and also to get a clearer conception of the relation between SPI and FLIP let us start with the following generalization of leader swings:

Definition 6: Player i has a *θ-swing* in coalition $S \subset N$ if

a) i can turn S into a losing coalition by leaving it,

b) i is not inferior in v, i. e. $i \notin I(v)$, and

c) the number of inferior players $i \in I(v)$ that are part of S is θ.

Formally, let

$$\eta_i^{(\theta)}(v) := \mid \{S \subset N \mid S \in C_i(v) \ \wedge \ i \notin I(v) \ \wedge \ |S \cap I(v)| = \theta\} \mid$$

denote the number of θ-swings of player i in game v.

$\theta = m$ is the special case of leader swings. The intuition behind θ-swings is that – in contrast to the concept of leader swings – only a given number $\theta \leq m$ of inferior players might be following the leaders of the game into whatever winning coalition will be formed.

Compelling arguments in favour of a specific choice θ, perhaps with the exception of the extreme case $\theta = m$, are not evident, though. Also, for a given θ there may not exist an actual subset $\tilde{I} \subset I(v)$ of inferior players with $|\tilde{I}| = \theta$ that is joining all winning coalitions whereas $I(v) \setminus \tilde{I}$ is always abstaining. Rather, the actual set of joining inferior players – in contrast to its cardinality – may be coalition-dependent. Nevertheless, θ-swings are useful as a *primitive concept.* θ-swings for different values of θ can, for example, be weighted and combined.

This seems a natural way of incorporating especially plausible or empirically relevant assumptions about inferior players' behaviour. For example, making the default assumption that every number of inferior participants in an established winning coalition is equally likely provides a link between θ-swings and strict swings, since, quite trivially, we have

$$\tilde{\eta}_i(v) = \sum_{\theta=0}^{m} \eta_i^{(\theta)}(v).$$

With θ-swings we can define the following index:

Definition 7: Let $m = |I(v)| < n$ be the number of inferior players. The *θ-Follower-Leader Index of Partial Power (θ-FLIPP)* $\beta^{(\theta)} : \mathcal{G} \longrightarrow I\!R^n_+$ is given by

$$\beta_i^{(\theta)}(v) := \frac{\eta_i^{(\theta)}(v)}{2^{n-m-1}}, \qquad i \in N.$$

FLIP is the special case of m-FLIPP, i. e. $\hat{\beta} = \beta^{(m)}$, and SPI is merely a re-scaling of the sum (or average) of θ-FLIPP values taken over all $\theta \leq m$, i. e.

$$\tilde{\beta}_i(v) = \frac{\tilde{\eta}_i(v)}{2^{n-1}} = \frac{\sum_{\theta=0}^{m} \eta_i^{(\theta)}(v)}{2^{n-m-1}} 2^m = 2^m \sum_{\theta=0}^{m} \beta_i^{(\theta)}(v).$$

Turning to generalizations in the probabilistic realm, it is straightforward to extend the Strict Power Condition of exact indifference, i. e. $p_i \equiv \frac{1}{2}$, of inferior players to an arbitrary acceptance rate $p_i \equiv c$ with $c \in [0, 1]$. We call this

Generalized Strict Power Condition (GSPC): i is inferior in $v \Longrightarrow p_i \equiv c, \quad c \in [0, 1]$.

Note that it is implicitly assumed that c is a common fixed probability or, more generally, has a common probability distribution for all inferior players. $c = \frac{1}{2}$ is the special case of SPC. The special case $c = 1$ corresponds to the Follower Leader Condition (FLC), except that the latter makes no assumptions on p_i for $i \in I(v)$ when all powerful players $j \notin I(v)$ have an acceptance rate of $p_j = 0$ – a zero probability event under most distributions and in particular uniform distribution of the p_j's.

Characterizing a quite general family of power indices we now have:

Proposition 9: A MLE satisfying the GSPC indicates zero power for inferior players. The respective Generalized Strict Power Index (GSPI) $\tilde{\beta}^c$ based on uniformly distributed acceptance rates p_i for non-inferior players takes the form

$$\begin{aligned} \tilde{\beta}_j^c(v) &= \sum_{\theta=0}^{m} c^\theta (1-c)^{m-\theta} \frac{\eta_j^{(\theta)}(v)}{2^{n-m-1}} \\ &= \sum_{\theta=0}^{m} c^\theta (1-c)^{m-\theta} \beta_i^{(\theta)}(v) \end{aligned} \quad (3)$$

The proof is given in the appendix.

In the preceding two sections we concentrated on two special cases of $\tilde{\beta}^c$, namely $\tilde{\beta}^{\frac{1}{2}}$ (SPI – re-scaled by 2^m), and $\tilde{\beta}^1$ (FLIP). Above proposition shows that a continuum of indices that are both indicating zero power for inferior players and related to the Banzhaf index by the common assumption of independently uniformly distributed acceptance rates for non-inferior players can be constructed.

The GSPC – and thus FLC and SPC – restricts the domain of a multilinear extension to the $(n-m)$-dimensional unit cubic $[0,1]^{n-m}$ where m is the number of inferior players. Figure 1 illustrates how GSPC affects the MLE in our example game. The FLC requires $p_A = p_A p_C$ and $p_A = p_A p_B$. If $p_A > 0$ we have $p_B = p_C = 1$. The domain of the MLE is thus restricted to the line between $a = (0,1,1)$ and $b = (1,1,1)$. The SPC requires that we are moving along the line $[c,d]$ having $\left(0, \frac{1}{2}, \frac{1}{2}\right)$ and $\left(1, \frac{1}{2}, \frac{1}{2}\right)$ as its end points. In general, the plane (a,b,f,e) shows the set of uni-dimensional cases of the GSPC. The fixed rate of acceptance c determines how "high" the horizontal line is located in the cubic.[10]

[10]Note that the GSPC implies that the plane (a,b,f,e) should be interpreted as a set of parallel horizontal lines $[(0,c,c),(1,c,c)]$. If there is, however, uncertainty about the exact level

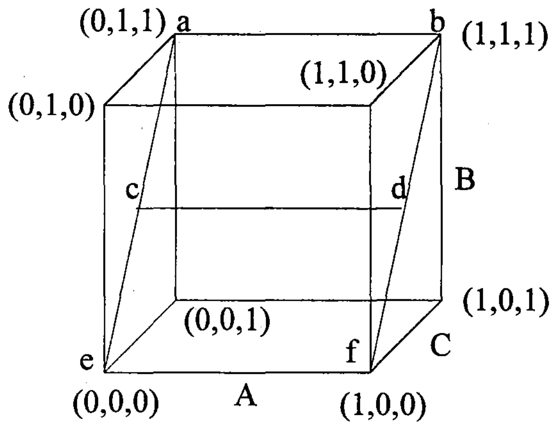

Figure 1: Graphical illustration of SPC, GSPC and FLC.

6 Conclusion

In this paper we argue in favour of strengthening the commonly used dummy player axiom of power measurement to an axiom based on the concept of inferior players. Motivation for this is the discrepancy between power indications given by, on the one hand, established indices that are based on the dummy axiom and, on the other hand, an important aspect of power related to competitive equilibrium and core analysis. The latter was illustrated by the example of one player who is on the short side of the market and can play off two potential coalition partners.

In order to demonstrate that meaningful indices which comply with the infe-

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of c we are in fact violating the inferior player axiom and approaching the more traditional power indices: c also becomes a variable in the MLE. For instance $c \sim U(0,1)$ gives us $p_A \sim U(0,1)$ and $p_B = p_C = c \wedge c \sim U(0,1)$. Hence in Straffin's (1977) terminology we have one independent player (A) and two homogeneous players (B and C). This set-up would give us the power distribution $\left(\frac{2}{3}, \frac{1}{4}, \frac{1}{4}\right)$.

rior player axiom can be constructed, we proposed two example indices, referred to as Strict Power Index (SPI) and Follower-Leader Index of Power (FLIP). Both were first analysed in a traditional deterministic setting. For a comprehensive understanding of the concept of inferior players we then investigated the probabilistic version of our example indices. We derived – and later generalized – probabilistic conditions that imply SPI and FLIP respectively. We think that the two alternative restrictions which were imposed in order to adapt established indices both make sense. Whether SPI or FLIP – or possibly yet a different member of the index family derived in section 5 – is most appropriate will typically depend on the context in which the distribution of power is to be gauged.

Future research may apply the inferior player axiom to other indices than the non-normalized Banzhaf index, e. g. those of Shapley and Shubik or of Deegan and Packel. It could be worthwhile to investigate more thoroughly the mathematical properties of the respective adaptations of the Banzhaf, Shapley-Shubik or Deegan-Packel index in terms of axiomatization, monotonicity, and susceptibility to typical paradoxes in power measurement. The inferior player axiom could also be extended to the domain of general games in characteristic function form. The concept of inferior players incorporates an important aspect of non-cooperative bilateral interaction into the cooperative world of power indices. It remains a challenge for the future to provide still more comprehensive non-cooperative foundations of power measurement.

Appendix

Proof of proposition 2

Proposition 2 claims that the Strict Power Index $\tilde{\beta}$ is globally monotonic, i. e. $\forall i \in N$

$$u \succ_i v \Longrightarrow \tilde{\beta}_i(u) \geq \tilde{\beta}_i(v).$$

We show that this is true by observing, first, that the the swing numbers $\eta_i(\cdot)$ are globally monotonic, and, second, that when player i is inferior in u, $u \succ_i v$ implies that i is also inferior in v.

Lemma 2: $u \succ_i v \Longrightarrow \eta_i(u) \geq \eta_i(v)$.

Proof: For every swing that i has in v, there is a coalition $S \in W(v)$ with $i \in S$ and $S \setminus \{i\} \notin W(v)$. $u \succ_i v$ implies that S also wins in the 'better' game u, i. e. $S \in W(u)$. Suppose that i is no longer crucial in S in game u, i. e. $S\setminus\{i\} \in W(u)$. $u \succ_i v$ then implies that $S\setminus\{i\}$ also wins in the 'worse' game v, i. e. $S\setminus\{i\} \in W(v)$ – a contradiction. □

First, it follows from the proof that not only is i's number of swings non-decreasing when moving from game v to u, but i keeps his crucial positions in *every* single coalition S, i.e. $C_i(v) \subset C_i(u)$. Second, note that i is not necessarily the *only* player that keeps or increases his number of swings when moving from v to u – $u \succ_j v$ may be true for more than one player in N (though not for all, unless $u = v$). Finally, note that the reverse of Lemma 1 is not true since $\succ_i$ is not necessarily complete: u and v with $W(u) = \{A, AB, AC, ABC\}$ and $W(v) = \{B, AB, BC, ABC\}$ produce $\eta_C(u) = \eta_C(v) = 0$, but neither $u \succ_C v$ nor $v \succ_C u$ (AC wins in u but not v, BC in v but not u).

Lemma 3: $u \succ_i v \;\wedge\; i$ not inferior in $v \Longrightarrow i$ not inferior in u.

Proof: When player i is not inferior in v, we can distinguish two cases:

Case 1: $\forall j \neq i: \; \exists S_j \in C_i(v): \; j \notin S_j$.

Player i is protected from being played off in game v by having an outside option S_j with respect to any player $j \neq i$. As it followed from the proof of Lemma 1, $C_i(v) \subset C_i(u)$, so $S_j \in C_i(u)$ with $j \neq S_j$.

Case 2: $\forall S \in C_i(v): \; j \in S$.

There is a player j that is member of every crucial coalition w.r.t. i in v. However, since i is not inferior in v, i must also be member of every crucial coalition w.r.t.

j, implying $C_i(v) = C_j(v)$. Player i keeps his swings in all coalitions $S \in C_i(v)$ in game u, and possibly gains some more. If either j gains no new swings in u, except when in coalition with i, or if there is a new coalition $S \in C_i(u)$ with $j \notin S$, we are finished.

Otherwise, for i to become inferior in u, it must be true that a) j is part of all crucial coalitions w.r.t. i in u and that b) there is a new coalition $\hat{S} \in C_j(u)$ with $i \notin \hat{S}$. $u \succ_i v$ implies $\hat{S} \in W(v)$. Now, we either have $\hat{S} \in C_j(v)$, which contradicts $C_i(v) = C_j(v)$. Or $\hat{S} \notin C_j(v)$, i.e. j is not crucial in $\hat{S}$ and we have $\hat{S} \setminus \{j\} \in W(v)$. Since $\hat{S} \setminus \{j\} \cup \{i\}$ wins in v, it is also winning in the 'better' game u. Player i cannot be crucial in $\hat{S} \setminus \{j\} \cup \{i\}$ because that would contradict Case 2. So, $\hat{S} \setminus \{j\} \in W(u)$, contradicting $\hat{S} \in C_j(u)$. □

Note that the reverse of Lemma 3 is obviously not true: i may well be inferior in the 'worse' game v without being so in u.

Now it is clear that Proposition 2 is true: if $u \succ_i v$ and i is neither inferior in u nor in v, then the number of strict swings $\tilde{\eta}_i$ in games u and v is equal to the number of swings η_i in u and v, for which global monotonicity was established in Lemma 2. If i is inferior in v only, then $\tilde{\eta}_i(v) = 0$ and $\tilde{\eta}_i(u)$ cannot be smaller. Finally, if $u \succ_i v$ and i is inferior in u, then by Lemma 3 i is also inferior in v and $\tilde{\eta}_i(u) = \tilde{\eta}_i(v) = 0$. Hence,

$$u \succ_i v \Longrightarrow \frac{\tilde{\eta}_i(u)}{2^{n-1}} \geq \frac{\tilde{\eta}_i(v)}{2^{n-1}} \Leftrightarrow \tilde{\beta}_i(u) \geq \tilde{\beta}_i(v).$$

□

Proof of Proposition 6

Proposition 6 states that applying the Strict Power Condition in the setting of the probabilistic Banzhaf index implies the probabilistic Strict Power Index. Let $\theta(S)$ denote the number of inferior players belonging to coalition S. $p_i \equiv \frac{1}{2}$

for all $i \in I(v)$ implies the MLE

$$\begin{aligned} f(p_1,\ldots,p_n) &= \sum_{S\in W(v)} \prod_{\substack{i\in I(v)\\ i\in S}} \left(\frac{1}{2}\right) \prod_{\substack{j\in I(v)\\ j\notin S}} \left(1-\frac{1}{2}\right) \prod_{k\in S\setminus I(v)} p_k \prod_{l\notin S\cup I(v)} (1-p_l) \\ &= \sum_{S\in W(v)} \left(\frac{1}{2}\right)^{\theta(S)} \left(1-\frac{1}{2}\right)^{m-\theta(S)} \prod_{k\in S\setminus I(v)} p_k \prod_{l\notin S\cup I(v)} (1-p_l) \\ &= \left(\frac{1}{2}\right)^m \sum_{S\subset W(v)} \prod_{k\in S\setminus I(v)} p_k \prod_{l\notin S\cup I(v)} (1-p_l) \end{aligned}$$

Obviously, $f_i(p_1,\ldots,p_n) \equiv 0 = \tilde{\beta}_i(v)$ if $i \in I(v)$. For powerful players $j \notin I(v)$ – suppose these are $m+1,\ldots,n$ – acceptance rates are uniformly distributed, i. e. $\forall j \in N \setminus I(v) : p_j \sim U(0,1)$. This yields

$$\begin{aligned} Ef_j(p_1,\ldots,p_n) &= \left(\tfrac{1}{2}\right)^m \underbrace{\int_0^1 \cdots \int}_{n-m} \sum_{S\in C_j(v)} \prod_{\substack{k\in S\setminus I(v)\\ k\neq j}} p_k \prod_{l\notin S\cup I(v)} (1-p_l) dp_{m+1} ... dp_n \\ &= \left(\tfrac{1}{2}\right)^m |C_j(v)| \left(\tfrac{1}{2}\right)^{n-m-1} \\ &= \left(\tfrac{1}{2}\right)^{n-1} \tilde{\eta}_j(v) = \tilde{\beta}_j(v) \end{aligned}$$

□

Proof of proposition 7

Proposition 7 claims that the Follower-Leader Condition ensures that zero power is indicated for inferior players.

Let i be inferior to j. Then $p_i p_j = p_j$ allows for two cases:

Case 1: $p_j > 0 \ \wedge \ p_i = 1$.

In this case, multilinear extension

$$f(p_1,\ldots,p_n) = \sum_{S\subset N} \prod_{k\in S} p_k \prod_{l\notin S} (1-p_l)\, v(S)$$

becomes

$$f(p_1,\ldots,p_{i-1},p_{i+1},\ldots,p_n) = \sum_{S\subset N} \prod_{k\in S\setminus\{i\}} p_k \prod_{l\notin S\setminus\{i\}} (1-p_l)\, v(S)\,,$$

i.e. it ceases to be a (non-degenerate) function of p_i. Therefore, under $p_j > 0 \wedge p_i = 1$ we have

$$\frac{\partial f(p_1, \ldots, p_n)}{\partial p_i} = 0.$$

Case 2: $p_j = 0$.

In this case, $f(p_1, \ldots, p_n)$ *is* a (non-degenerate) function of p_i. Nevertheless, its partial derivative with respect to p_i is zero since the first term in

$$\begin{aligned} f_i(p_1, \ldots, p_n) &= \sum_{\substack{S \in C_i(v) \\ j \in S}} \prod_{\substack{k \in S \\ k \neq i}} p_k \prod_{\substack{l \notin S \\ l \neq i}} (1 - p_l) \\ &+ \sum_{\substack{S \in C_i(v) \\ j \notin S}} \prod_{\substack{k \in S \\ k \neq i}} p_k \prod_{\substack{l \notin S \\ l \neq i}} (1 - p_l) \end{aligned}$$

is zero because of $p_j = 0$. The second term is zero because i is never crucial in coalitions without j, i.e. $\{S \subset N \mid S \in C_i(v) \wedge j \notin S\} = \emptyset$. □

Proof of proposition 8

Proposition 8 claims that the Follower-Leader Condition implies the Follower-Leader Index of Power when acceptance rates of non-inferior players are independently uniformly distributed.

Restricting analysis to the probability-one case of $p_i \equiv 1$ for all $i \in I(v)$, we have the MLE

$$f(p_1, \ldots, p_n) = \sum_{S \in W(v)} \prod_{\substack{i \in I(v) \\ i \in S}} 1 \prod_{\substack{j \in I(v) \\ j \notin S}} 0 \prod_{k \in S \setminus I(v)} p_k \prod_{l \notin S \cup I(v)} (1 - p_l).$$

Here, the index set $\{j \mid j \in I(v) \wedge j \notin S \wedge S \in W(v)\}$ is empty whenever all inferior players are part of a winning coalition S. The second product is 1 in these cases. Hence,

$$f(p_1, \ldots, p_n) = \sum_{\substack{S \in W(v) \\ I(v) \subset S}} \prod_{k \in S \setminus I(v)} p_k \prod_{l \notin S \cup I(v)} (1 - p_l).$$

Obviously, $f_i(p_1,\ldots,p_n) \equiv 0 = \hat{\beta}_i(v)$ if $i \in I(v)$. For powerful players $j \notin I(v)$ – suppose these are $m+1,\ldots,n$ – acceptance rates are uniformly distributed, i. e. $\forall j \in N \setminus I(v) : p_j \sim U(0,1)$. This yields

$$\begin{aligned} Ef_j(p_1,\ldots,p_n) &= \underbrace{\int_0^1 \cdots \int}_{n-m} \sum_{\substack{S\in C_j(v)\\ I(v)\in S}} \prod_{\substack{k\in S\setminus I(v)\\ k\neq j}} p_k \prod_{l\notin S\cup I(v)} (1-p_l)dp_{m+1}...dp_n \\ &= \hat{\eta}_j(v)\left(\tfrac{1}{2}\right)^{n-m-1} = \hat{\beta}_j(v). \end{aligned}$$

□

Proof of proposition 9

Proposition 9 states first that GSPC indicates zero power for inferior players. Following the proof of proposal 2 this is trivial since c is presumed to be fixed. Second, it was claimed that the respective GSPI with uniformly distributed rates of non-inferior players' acceptance takes the form $\tilde{\beta}_i^c(v) = \sum_{\theta=0}^{m} c^\theta (1-c)^{m-\theta} \hat{\beta}_i^\theta(v)$.

Let m denote the number of inferior players in v. Using GSPC in the MLE gives

$$\begin{aligned} f(p_1,\ldots,p_n) &= \sum_{S\in W(v)} \prod_{\substack{i\in I(v)\\ i\in S}} c \prod_{\substack{j\in I(v)\\ j\notin S}} (1-c) \prod_{k\in S\setminus I(v)} p_k \prod_{l\notin S\cup I(v)} (1-p_l) \\ &= \sum_{S\in W(v)} c^{\theta(S)}(1-c)^{m-\theta(S)} \prod_{k\in S\setminus I(v)} p_k \prod_{l\notin S\cup I(v)} (1-p_l). \end{aligned}$$

Taking the first-order partial derivatives yields

$$f_i(p_1,\ldots,p_n) = \sum_{S\in C_i(v)} c^{\theta(S)}(1-c)^{m-\theta(S)} \prod_{k\in S\setminus(I(v)\cup\{i\})} p_k \prod_{l\notin S\cup I(v)} (1-p_l)$$

whose expectation under uniformly distributed acceptance rates for powerful players is

$$\begin{aligned} Ef_i(p_1,\ldots,p_n) &= \sum_{S\in C_i(v)} c^{\theta(S)}(1-c)^{m-\theta(S)} \left(\frac{1}{2}\right)^{\sigma(S)-1} \left(1-\frac{1}{2}\right)^{n-m-\sigma(S)} \\ &= \sum_{S\in C_i(v)} c^{\theta(S)}(1-c)^{m-\theta(S)} \left(\frac{1}{2}\right)^{n-m-1} \end{aligned}$$

where $\sigma(S)$ denotes the number of powerful players in coalition S. This sum can be simplified and re-ordered to

$$\begin{aligned}\tilde{\beta}_i^c(v) = Ef_i(p_1,\ldots,p_n) &= \sum_{\theta=0}^{m} c^\theta (1-c)^{m-\theta} \eta_i^{(\theta)}(v) \left(\frac{1}{2}\right)^{n-m-1} \\ &= \sum_{\theta=0}^{m} c^\theta (1-c)^{m-\theta} \beta_i^{(\theta)}(v).\end{aligned}$$

□

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