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A note on upward bias in the Bank of Finland currency index

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1. Introduction

The Bank of Finland currency index intends to reflect the average change in the exchange rates of the currencies most important in Finland's foreign trade; see Puro (1978). According to the Currency Act which came into force on November 1, 1977, the currency index should be an expression of the external value of the Finnish mark. The principles of calculation of the currency index and its fluctuation limits are to be confirmed by a Cabinet Decision, on the proposal of the Bank of Finland. The method of calculating the currency index was specified in a Cabinet Decision which entered into force on November 1, 1977, and hasn't been changed since. The currencies included in the index are the currencies of those countries which, in each of the last three calendar years, have accounted for at least one percent of the total value of Finnish commodity trade. (A currency not quoted by the Bank of Finland and not generally used in Finland's foreign trade with the country concerned is replaced in the

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currency index by the quoted currency most used in that trade.) The ratio between the exchange rate for the current period and the base period rate is determined for each currency included in the index. These ratios are weighted together by the corresponding countries' shares in Finland's foreign trade[1]. The weights are calculated as arithmetic averages of the shares for the base period and the last four full quarters for which official foreign trade statistics have been released; see Puro (1978).

In symbols, the currency index P^s_{1974}, comparing the base year 1974 and the observation period s (a given day), may be written:

$$(1) \qquad P^s_{1974} = \sum_{i=1}^{I} \frac{1}{2}(w_i^{1974} + w_i^h)\left(\frac{p_i^s}{p_i^{1974}}\right),$$

where p_i^s is the exchange rate of the ith currency for the observation period s and p_i^{1974} is the mean exchange rate of the same currency during the base period 1974. The weights w_i^{1974} and w_i^h are the shares of the corresponding country in Finland's foreign trade (commodity imports and exports) respectively in 1974 and in the last four full quarters for which the data is available.

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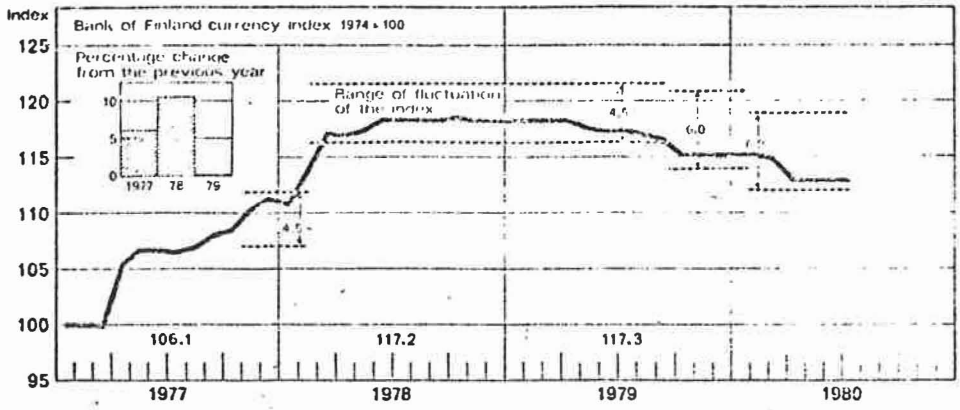
1) Our analysis is also related to the exchange rate index of the Central Bank of Sweden; see Franzén, Markowski and Rosenberg (1980). We disagree with them on some points concerning the rationalization of the index number formula of the Riksbank's currency index, particularly with their view that only exchange rate indices of the arithmetic type should be used.

Figure 1: Monthly averages of the Bank of Finland currency index and its fluctuation limits (a copy from the Bank of Finland Monthly Bulletin)



The currency index for a longer period, say a month, is calculated as an average of the daily figures. The yearly average of the currency index for, e.g., the calendar year 1977 is thus

$$(2) \qquad \bar{P}^{1977}_{1974} = \frac{1}{T} \sum_{1977} P^{s}_{1974} ,$$

where T is the number of days in 1977 for which the index was calculated and the summation is over all such days. Since the trade shares w_i^t for the last four full quarters of the most important trading countries are slowly changing characteristics, the yearly average $\bar{P}^t_{1974}$ for any calendar year t is very accurately approximated by the following yearly currency index

$$(3) \qquad P^{t}_{1974} = \sum_{i=1}^{I} \frac{1}{2}(w_i^{1974} + w_i^{t})\left(\frac{p_i^t}{p_i^{1974}}\right) ,$$

where $p_i^t = \frac{1}{T}\Sigma p_i^s$ is the average exchange rate of the ith currency in the year t and w_i^t is the trade share of the corresponding country in the same calendar year t. In (3) all the data comes from two periods, the base year 1974 and the observation year t, while, e.g., in (1) three periods are involved because of the practical difficulties related to the availability of relevant data. In the following we will analyse the yearly currency index P^t_{1974} of (3), which will very accurately approximate the yearly average of the daily currency indices

(4) $$\bar{P}^t_{1974} = \frac{1}{T}\sum_t P^s_{1974} .$$

We intend to demonstrate that the Bank of Finland currency index has the unfortunate feature of a permanent upward bias compared to, e.g., Fisher's ideal index and to other unbiased indices. This is most easily done by analysing the implied yearly currency indices (3).

2. Some topics in index number theory

The price relatives $p^t_i/p^0_i = \pi^t_0(i)$ of the exchange rates of various currencies from the base year t^0 to some observation year t may be aggregated into a currency index P^t_0 in various ways. In the simplest methods some well-known weighted average (e.g., arithmetic, geometric or harmonic) is used, while the weights w_i are value shares of some kind of the currencies either for the base year or for the observation year. We do not discuss here whether, instead of the trade shares of the currencies, some other shares (e.g., import or export shares or contract shares) should be used[1]. The choice of the type of shares on which the weights of the currencies are based is an important but difficult question, which affects the meaning and scope of the currency index, but this problem is put aside in the present paper. We consider here only trade-weighted currency indices and show how the results are affected by the choice of the index number formula. We will demonstrate, in particular, that the choice of the index number formula for the Bank of Finland currency index is an unfortunate one, since the formula will produce, in any data, upwards biased results compared to index number formulas which may be classified as superlative in index theory. Our analysis is based on the results of Vartia (1978). Our results will not depend on the length of the observation period or on the type of shares on which the weights are based. For the sake of simplicity we will consider only consecutive years t^0, t^1, t^2,... as observation periods, and the shares w_i^0, w_i^1, w_i^2,... of the various currencies a_i will here be trade shares calculated from commodity imports from and commodity exports to the country having the currency a_i. The exchange rate p_i^t is the mean "price" of the currency a_i in Finnish marks during the year t and the "price relative" $\pi_0^t(i) = p_i^t/p_i^0$ indicates the relative change in the price of the currency a_i from the base year t^0 to year t. The statistical units are the n currencies $a_1,\ldots,a_n$ and we have the vectors

$$(5) \qquad \pi_0^t = (\pi_0^t(1),\ldots,\pi_0^t(n)) = (p_1^t/p_1^0,\ldots,p_n^t/p_n^0)$$

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1) For instance, the Laspeyres-type currency index (1970=100) published by ETLA since 1974 uses export shares; see Suhdanne D7-D18.

$$(6) \qquad w^t = (w_1^t, \ldots, w_n^t)$$

of price relatives and value shares for any year t. Specifically, for the base year t^0, $\pi_0^0 = (1,\ldots,1)$ and

$$(7) \qquad w^0 = (w_1^0, \ldots, w_n^0) .$$

Weighted arithmetic, geometric and harmonic means of positive numbers $x_1,\ldots,x_n$ with positive weights $c_1,\ldots,c_n$ are defined and denoted respectively

$$(8) \qquad A(x,c) = \sum_{i=1}^{n} c_i x_i / \sum_{i=1}^{n} c_i$$

$$(9) \qquad G(x,c) = \left(\prod_{i=1}^{n} x_i^{c_i}\right)^{\frac{1}{\Sigma c_i}} = \exp\left(\left(\sum_{i=1}^{n} c_i \log x_i\right) / \sum_{i=1}^{n} c_i\right)$$

$$(10) \qquad H(x,c) = \sum_{i=1}^{n} c_i / \sum_{i=1}^{n} (c_i/x_i)$$

$$= \left(\sum_{i=1}^{n} c_i x_i^{-1} / \sum_{i=1}^{n} c_i\right)^{-1}$$

It is well known, that for all x's and c's,

$$(11) \qquad A(x,c) \geq G(x,c) \geq H(x,c)$$

with equality if and only if all x's are equal. Applying (11) to $x_i = \pi_0^t(i)$ and $c_i = w_i^0$ we get for the <u>base-year weighted</u> arithmetic, geometric and harmonic indices

$$(12) \qquad L = A(\pi_0^t, w^0) = \Sigma\, w_i^0 \pi_0^t(i) \qquad \text{"Laspeyres"}$$

$$(13) \qquad \ell = G(\pi_0^t, w^0) = \exp(\Sigma\, w_i^0 \log \pi_0^t(i)) \qquad \text{"Log-Laspeyres"}$$

$$(14) \qquad Lh = H(\pi_0^t, w^0) = (\Sigma\, w_i^0 \pi_0^t(i)^{-1})^{-1} \qquad \text{"Harmonic Laspeyres"}$$

the inequality

$$(15) \qquad A(\pi_0^t, w^0) \geq G(\pi_0^t, w^0) \geq H(\pi_0^t, w^0),$$

or shortly $L \geq \ell \geq Lh$ in the notation of Vartia (1978). Similarly, the <u>observation-year weighted</u> indices

$$(16) \qquad P\ell = A(\pi_0^t, w^t) = \Sigma\, w_i^r \pi_0^t(i) \qquad \text{"Palgrave"}$$

$$(17) \qquad p = G(\pi_0^t, w^t) = \exp(\Sigma\, w_i^t \log \pi_0^t(i)) \qquad \text{"Log-Paasche"}$$

$$(18) \qquad P = H(\pi_0^t, w^t) = (\Sigma\, w_i^t \pi_0^t(i)^{-1})^{-1} \qquad \text{"Paasche"}$$

satisfy

$$(19) \qquad A(\pi_0^t, w^t) \geq G(\pi_0^t, w^t) \geq H(\pi_0^t, w^t),$$

or shortly $P\ell \geq p \geq Lh$. In the "three tined forks" (L, ℓ, Lh) and $(P\ell, p, P)$ of the base- and observation-year weighted indices only L and P have a basket interpretation[1]. Very few generally used index number formulas (except Laspeyres and Paasche) have a basket interpretation, and thus it is often not quite legimitate to speak of a "currency basket" in connection with currency indices. For instance, neither the Bank of Finland index nor the Riksbank's currency index has a currency basket interpretation. It may be that the Bank of Finland currency index (4) (or rather (3)) was meant to be an approximation to and Edgeworth type of currency index $P^t_{1974} = p^t \cdot (q^{1974} + q^t)/p^{1974} \cdot (q^{1974} + q^t)$, which has a basket interpretation, but that the currency baskets q^t were confounded with the value shares w^t. In currency index literature, the vector of value shares is not infrequently treated as if it were a currency basket.

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1) Let v_i be the value, in Finnish marks, of foreign trade (exports and imports) with a country having the currency a_i and let p_i be the price of currency a_i in terms of Finnish marks. Then, $q_i = v_i/p_i$, or the value of foreign trade with the country having the currency a_i in this country's own currency. Now, $v_i = p_i q_i$ and $w_i = v_i / \sum_{j=1}^{n} v_j = v_i/V$. For Laspeyres' index we have $L = \Sigma\, w_i^0 (p_i^t/p_i^0) = \Sigma\, v_i^0 (p_i^t/p_i^0)/\Sigma\, v_i^0 = \Sigma\, p_i^t q_i^0/\Sigma\, p_i^0 q_i^0 = p^t \cdot q^0/p^0 \cdot q^0$. Here $q^0 = (q_1^0, \ldots, q_n^0)$ is a base year basket of currencies, the amount of every currency q_i^0 representing the base year foreign trade with the corresponding country. Thus, e.g., $p^t \cdot q^0$ is the value of the base-year basket of currencies q^0 at the observation-period exchange rates p^t. Similarly we have for the Paasche's index:

$P = (\Sigma\, w_i^1 (p_i^1/p_i^0)^{-1})^{-1} = p^1 \cdot q^1/p^0 \cdot q^1$.

The relative differences of the base year weighted indices depend on the variance

$$(20) \qquad s^2_{0p} = \Sigma\, w^0_i(\log \pi^t_0(i) - \log G(\pi^t_0, w^0))^2$$

$$= \Sigma\, w^0_i(\log (p^t_i/p^0_i) - \log \ell\,)^2$$

of the price log-changes $\log\pi^t_0(i) = \log(p^t_i/p^0_i)$. If all price relatives p^t_i/p^0_i happen to be the same, the log-changes $\log (p^t_i/p^0_i)$ of the prices coincide with the log-change of the price index $\log\ell$, which now also equals $\log L$ and $\log Lh$. As shown in Vartia (1978), the relative differences of the base-year weighted indices satisfy

$$(21) \qquad \log \frac{L}{\ell} \approx \log \frac{\ell}{Lh} \approx \frac{1}{2} s^2_{0p} \,.$$

Similarly, the observation year weighted indices satisfy

$$(22) \qquad \log \frac{P\ell}{p} \approx \log \frac{p}{P} \approx \frac{1}{2} s^2_{tp} \,,$$

where

$$(23) \qquad s^2_{tp} = \Sigma\, w^t_i(\log \pi^t_0(i) - \log G(\pi^t_0, w^t))^2$$

$$= \Sigma\, w^t_i(\log (p^t_i/p^0_i) - \log p)^2 \,.$$

Usually the variances s^2_{0p} and s^2_{tp} of the price log-changes are almost equal. The relative difference between Log-Laspeyres ℓ and Log-Paasche p

$$(24) \qquad \log (p/\ell) = \Sigma\ (w_i^t - w_i^0) \log (p_i^t/p_i^0)$$

$$\approx \Sigma\ \tfrac{1}{2}(w_i^t + w_i^0) \log (w_i^t/w_i^0) \log (p_i^t/p_i^0)$$

$$= \Sigma\ \tfrac{1}{2}(w_i^t + w_i^0) \left[\log\left(\frac{v_i^t}{v_i^0}\right) - \log\left(\frac{V^1}{V^0}\right)\right] \log \left(\frac{p_i^t}{p_i^0}\right)$$

depends on changes in the prices of the currencies and in the value shares $w_i = v_i/\Sigma v_j = v_i/V$ (or values v_i). Actually, $\log(p/\ell)$ is approximately equal to the covariance $\mathrm{cov}(\Delta \log v_i, \Delta\log p_i)$ between the log-changes in values and those in prices as calculated with the mean weights $\frac{1}{2}(w_i^t + w_i^0)$. Regardless of the value of this covariance,

$$(25) \qquad F = \sqrt{LP} = \sqrt{(\Sigma\ w_i^0(p_i^t/p_i^0))(\Sigma\ w_i^t(p_i^t/p_i^0)^{-1})^{-1}} \qquad \text{"Fisher"}$$

$$(26) \qquad t = \sqrt{\ell p} = \exp(\Sigma\ \tfrac{1}{2}\ (w_i^0 + w_i^t) \log (p_i^t/p_i^0)) \qquad \text{"Törnqvist"}$$

and a third member in the family of Diewert's (1976, 1978) superlative indices,

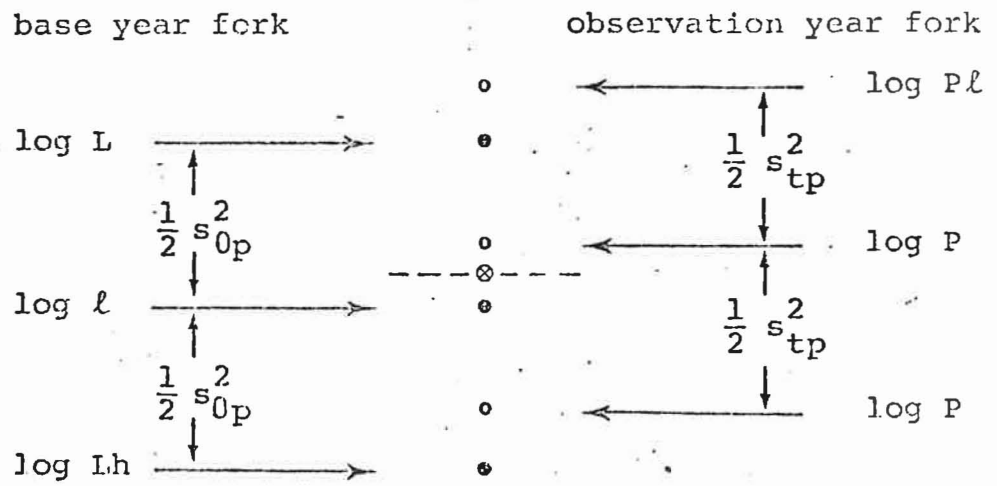
$$(27) \qquad \sqrt{Lh \cdot P\ell} = \sqrt{(\Sigma\ w_i^0(p_i^t/p_i^0)^{-1})^{-1}(\Sigma\ w_i^t(p_i^t/p_i^0))},$$

are all very nearly equal:

$$(28) \qquad F \approx t \approx \sqrt{Lh \cdot P\ell}.$$

This follows from the relative positions of the relevant indices in the "base-year fork" (L, ℓ, Lh) and in the "observation-year fork" ($P\ell, p, P$) and from the fact that $s^2_{0p} \approx s^2_{tp}$. An index number formula is <u>unbiased</u> if it lies (like the indices of (28)) in the middle of the following figure constituted by the base-year and observation-year forks.

Figure 2: The geometry of the index number problem



⊗ denotes the point of gravity around which the unbiased index numbers gather

The index number formulas F, t and $\sqrt{Lh \cdot P\ell}$ are examples of superlative and unbiased index number formulas[1], which (excluding exceptional cases of very unequal price changes)

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1) Most of the index numbers which Fisher (1922, p. 255) empirically assigned to his groups of excellent or superlative index numbers may be shown to be <u>unbiased</u> in our sense. These include, in addition to Fisher's ideal index F = 123, at least 124, 125, 126, 323, 325, 1123, 1124, 1153, 1154, 1323, 1353, 2153, 2154, 2353, 5323, 8053, 8054 in Fisher's numbering system; cf. also Fisher (1922, p. 265). Unbiased indices invented after Fisher include, e.g., Stuvel's (1957) index, see van Yzeren (1958), Diewert's (1976, 1978) indices and Sato-Vartia index, see Sato (1976) and Vartia (1974, 1976, 1976b).

almost coincide with each other. In this connection it is worth while to point out that Fisher's (1922) contention that Laspeyres formula (12) is approximately equal to these unbiased indices does not usually hold. None of the indices in (L,ℓ,Lh) or (Pℓ,p,P) can be generally recommended. The biases of L,ℓ,p and P are usually only moderate, whereas Pℓ and Lh are usually clearly biased upwards and downwards respectively. It is possible, however, that even Palgrave's ordinarily upwards biased formula (16) is more accurate than, e.g., Laspeyres' or Paasche's formulas; see Vartia (1978) and Sihtola and Vartia (1978).

But if Paasce's index $P = (\Sigma\, w_i^t(p_i^t/p_i^0)^{-1})^{-1}$ in Fisher's formula $F = \sqrt{PL}$ is replaced by Palgrave's index $P\ell = \Sigma\, w_i^t(p_i^t/p_i^0)$, a new index number formula

$$(29) \qquad \sqrt{P\ell \cdot L} = \sqrt{(\Sigma\, w_i^t(p_i^t/p_i^0))(\Sigma\, w_i^0(p_i^t/p_i^0))}$$

is defined[1]. Because $P\ell \geq P$ always and because

$$(30) \qquad \log(P\ell/P) \approx s_{tp}^2$$

1) This formula was used by accident instead of Fisher's formula in calculating the final unit value indices (1954=100) of Finnish foreign trade statistics, see Kajander (1965, 1957) and Vartia (1979).

we have

$$(31) \qquad \sqrt{P\ell \cdot L} \geq \sqrt{P \cdot L} = F$$

always and

$$(32) \qquad \log \frac{\sqrt{P\ell \cdot L}}{F} \approx \frac{1}{2} s_{tp}^2 .$$

This shows that $\sqrt{P\ell \cdot L}$ is always biased upwards compared to Fisher's ideal index, the relative difference between $\sqrt{P\ell \cdot L}$ and F being half of the new price variance s_{tp}^2. The bias, compared to any unbiased index number formula is of the same order of magnitude. The upward bias is not a constant but increases with increasing dispersion of the price relatives.

Because the arithmetic mean is always greater than or equal to the geometric mean, we have

$$(33) \qquad \frac{1}{2}(P\ell + L) \geq \sqrt{P\ell \cdot L} \geq F .$$

Thus the bias of the arithmetic mean of Palgrave $P\ell$ and Laspeyres L

$$(34) \qquad \frac{1}{2}(P\ell + L) = \frac{1}{2} \Sigma w_i^t (p_i^t/p_i^0) + \frac{1}{2} \Sigma w_i^0 (p_i^t/p_i^0)$$

$$= \Sigma \frac{1}{2} (w_i^t + w_i^0)(p_i^t/p_i^0)$$

is of the same order of magnitude as the bias of $\sqrt{P\ell \cdot L}$:

$$\text{(35)} \qquad \log \frac{\frac{1}{2}(P\ell + L)}{F} \approx \frac{1}{2} s_{tp}^2 \; .$$

3. Conclusions

Equation (35) shows that the currency index P_{1974}^{t} of (3) and, therefore, the Bank of Finland currency index (4) are upwards biased compared to Fisher's ideal index (or any other unbiased index number formula) calculated from the same data. It can also be inferred that, if the Bank of Finland currency index P_{1974}^{s} is kept constant, the corresponding currency index calculated from Fisher's (or any unbiased) formula will slowly drift downwards as the price relatives have a tendency to disperse more from each other. Thus the policy of keeping the Bank of Finland currency index constant means, in fact, a gradual revaluation of the Finnish Mark. When writing this in July 1980, the upward bias is about 1.5 %. This means that, according to the official currency index, the Finnish mark is shown to be devalued since 1974 by 1.5 percentage points more than is actually the case[1]. Thus a devaluation of 1.5 % would now be necessary in order for the properly measured external value

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1) This bias is exclusively the result of the unfortunate choice of the index number formula. If, instead of the trade weights, contract weights (giving the shares of different currencies in foreign trade contracts) and an unbiased index number formula were used, the discrepancy would increase to 3 percentage points.

of the Finnish mark to agree with the recently obtained value of the official but biased index. The ceteris paribus increase in export earnings corresponding to such a devaluation would be about 750 million marks a year.

A properly calculated (unbiased) index has temporarily even been below the confirmed official lower fluctuation limit for the external value of the Finnish mark, see figure 1. That errors of this order of magnitude are not irrelevant is clear from the heated political discussion concerning the 2 percent revaluation in the spring of 1980.

REFERENCES:

DIEWERT, W.E. (1976): "Exact and superlative index numbers", Journal of Econometrics, 4, 115-145.

DIEWERT, W.E. (1978): "Superlative index numbers and consistency in aggregation", Econometrica, 46, 883-900.

ETLA (1974,...,1980): "Suhdanne-Economic prospects in Finland", The Research Institute of the Finnish Economy (ETLA), D7-D18, Helsinki.

FISHER, I. (1922): "The making of index numbers", Houghton Mifflin Company, Boston.

FRANZEN, T., MARKOWSKI, A. and ROSENBERG, I. (1980): "Effective exchange rate index", Occasional paper I, Central Bank of Sweden, Stockholm.

KAJANDER, S. (1957): "Ulkomaankaupan uudet yksikköarvo- ja volyymi-indeksit, De nya enhetsvärde- och volymindexarna för utrikeshandeln", Tilastokatsaus 9/57.

KAJANDER, S. (1965): "Ulkomaankaupan uudet yksikköarvo- ja volyymi-indeksit, De nya enhetsvärde- och volymindexarna för utrikeshandeln", Tilastokatsaus 12/65.

PURO, I. (1978): "The amendment of the currency act", Bank of Finland Monthly Bulletin, February 1978, 20-24.

SATO, K. (1976): "The ideal log-change index number", The Review of Economics and Statistics, 58, 223-8.

SIHTOLA, K. (1978): "Yksikkötyökustannusten kansainvälinen vertailu" (An international comparison of unit labour costs, with an English summary), ETLA B19, Helsinki.

SIHTOLA, K. and VARTIA, Y. (1978): "Indeksikaavan valinnan vaikutus kilpailijamaiden yksikkötyökustannusindekseihin" (Effect of choice of an index number formula on the index of unit labour cost of competing countries), Appendix 5 in Sihtola (1978).

STUVEL, G. (1957): "A new index number formula", Econometrica, 25, 123-131.

VARTIA, Y.O.(1974): "Relative changes and economic indices", (an unpublished licentiate's thesis in Statistics), University of Helsinki, Department of Statistics.

VARTIA, Y.O. (1976): "Relative changes and index numbers", The Research Institute of the Finnish Economy, Serie A4, Helsinki.

VARTIA, Y.O. (1976b): "Ideal log-change index numbers", Scandinavian Journal of Statistics, 3, 121-126.

VARTIA, Y.O. (1978): "Fisher's five-tined fork and other quantum theories of index numbers", in Theory and applications of economic indices, ed. by W. Eichhorn, R. Henn, O. Opitz, R.W. Shephard, Physica-Verlag, Würzburg.

VARTIA, Y.O. (1979): "Tuonnin yksikköarvoindeksin (1954=100) vuoden 1954 puuttuvien neljännesvuosiarvojen arvioiminen" (Estimation of the missing quaterly figures for 1954 of the unit value index of imports (1954=100)), Discussion paper No 29, The Research Institute of the Finnish Economy, Helsinki.

vanYZEREN, J. (1958): "A note on the useful properties of Stuvel's index numbers", Econometrica, 26, 429-439.