ETLA
ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

Keskusteluaiheita
Discussion papers

Yrjö O. Vartia

Comments on W.E. Diewert's paper "The economic theory of index numbers: A survey"

No. 53 14.3.1980

This paper comprises the comments, which I presented as a discussant to Professor Diewert's paper at the North American Summer Meeting of the Economic Society at Montreal in the session of Index Number and Aggregation Theory, June 28, 1979.

An important new feature in the paper is that it considers both price and quantity indices at the same time in the spirit of the weak factor reversal test. E.g. in Frisch's classical survey on index numbers only price indices were considered.

It seems to be time to recognize that there is no unique definition for the "True Cost of Living Index" but many competing definitions, which, however, coinside in the homothetic case. Diewert defines and investigates three different definitions of the "True Quantity Index", namely Konüs, Allen and Malmquist Quantity Indices and mentions on p. 57 a Quantity Index first advocated presumably by Leontief (1936), see Vartia (1976, p. 40-49). Samuelson and Swamy (1974, p. 590) remark in their note 17 that Pollak (1971) has given the same definition in an unpublished paper[1]. These all have their price counterparts (defined by the weak factor reversal test) so that at least 4 different "economic definitions" for the price and quantity indices have been proposed and applied. By combining or generalizing these definitions still others may be invented, say

$$Q_1(x^0,x^1,\bar{x}) = 1/D(F(x^1),\bar{x})D(F(\bar{x}),x^0) = \lambda^1/\lambda^0,$$

where $F(\lambda^1\bar{x}) = F(x^1)$ and $F(x^0/\lambda^0) = F(\bar{x})$. Or define

$$Q_2(x^1,x^0,\bar{x}) = m(\hat{x},x^1)/m(\hat{x},x^0),$$

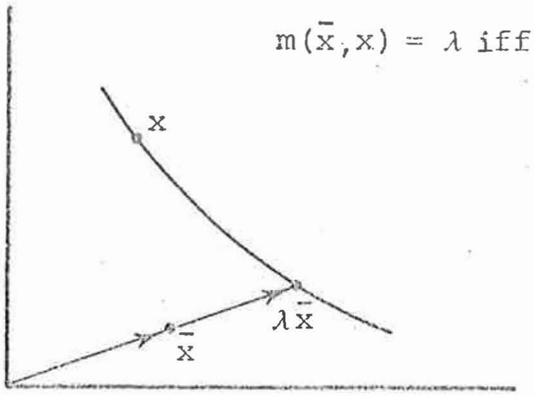
where $\hat{x} = \frac{1}{2}(m(x^0,\bar{x})x^0 + m(x^1,\bar{x})x^1)$ and $m(\bar{x},x)$ is the multiplier function defined below.

There are no major points I would like to change in the paper but perhaps some additions could be made. For instance the Leontief Quantity Index would be worthwhile presenting. Its definition is similar to that of the Malmquist index: it uses just multipliers instead of Malmquist's deflators.

1) Having not seen the paper I cannot check the remark. Leontief-Pollak's definition is often mixed with Malmquist's definition.

The *multiplier function* $m(\bar{x},x) = \min_\lambda\{\lambda : F(\lambda\bar{x}) \geq F(x),\ \lambda > 0\}$ gives the smallest multiplier λ of $\bar{x}$ such that $\lambda\bar{x}$ becomes indifferent to x, see figure 1.

Figure 1. Definition of the multiplier function $m(\bar{x},x)$.

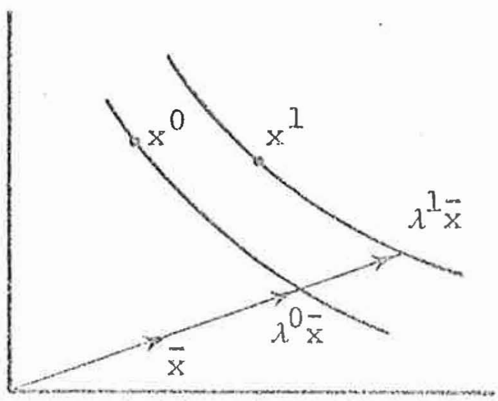


For $x^0 >> 0_N$, $x^1 >> 0_N$, $\bar{x} >> 0_N$, define now the *Leontief* (1936) *Quantity Index* as as the following ratio of multipliers

$$(1) \qquad Q_{LE}(x^0,x^1,\bar{x}) = \frac{m(\bar{x},x^1)}{m(\bar{x},x^0)} = \frac{\lambda^1}{\lambda^0} .$$

This is illustrated in figure 2.

Figure 2. Definition of the Leontief (1936) Quantity Index $Q_{LE}(x^1,x^0,\bar{x})$.



$$Q_{LE}(x^1,x^0,\bar{x}) = \frac{m(\bar{x},x^1)}{m(\bar{x},x^0)} = \frac{\lambda^1}{\lambda^0}$$

Because the multiplier and deflator are inverses of each other we have

$$(2) \qquad m(\bar{x},x) = \min_{\lambda}\{\lambda : F(\lambda\bar{x}) \geq F(x)\}$$

$$= 1/\max_{k}\{k : F(\bar{x}/k) \geq F(x)\}$$

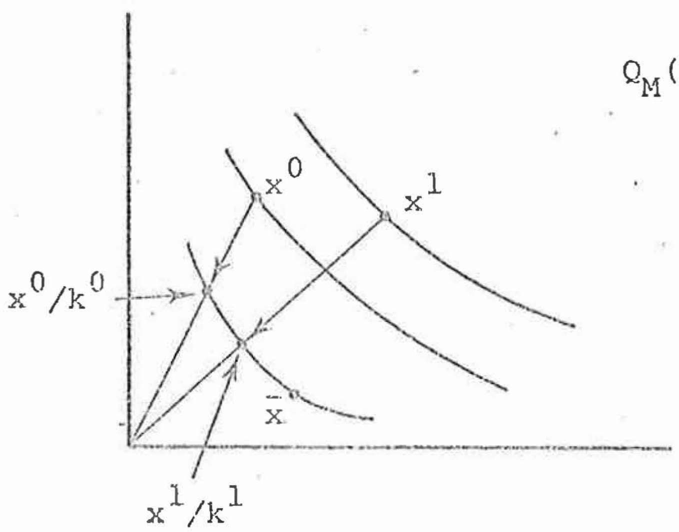
$$= 1/D(F(x), \bar{x}),$$

where the changing roles of $\bar{x}$ and x and the notational differences cause somewhat irritating difficulties. Using this relationship we get a somewhat awkward expression for the Leontief Quantity Index:

$$(3) \qquad Q_{LE}(x^0,x^1,\bar{x}) = \frac{m(\bar{x},x^1)}{m(\bar{x},x^0)} = \frac{D(F(x^0), \bar{x})}{D(F(x^1), \bar{x})} .$$

This should be compared with the Malmquist's definition illustrated in figure 3. Malmquist's and Leontief's definitions are often mixed with each other, but they are two different definitions.

Figure 3. Definition of the Malmquist (1953) Quantity Index $Q_M(x^0,x^1,\bar{x})$.



$$Q_M(x^0,x^1,\bar{x}) = \frac{D(F(\bar{x}),x^1)}{D(F(\bar{x}),x^0)} = \frac{k^1}{k^0}$$

The Leontief Quantity Index will for every choice of $\bar{x}$ correctly indicate which of the aggregates x^0 and x^1 is on the higher indifference surface or whether they are indifferent. It is also shown in Vartia (1976, p. 49) that for any given x^0 and x^1 the Allen and Leontief Quantity Indices $Q_A(x^0,x^1,p)$, $Q_{LE}(x^0,x^1,\bar{x})$ vary over the same range of values when their "nuisance parameters" p and $\bar{x}$ are varied.

Of course, they are equal and independent of their nuisance parameters in all homothetic cases.

Finally let me make some short notes

1) Frisch (1932), Rajaoja (1958), Theil, Afriat and others later have defined and used in addition to the "average" price index also the "marginal" or "incremental" price index. These coincide again only in homothetic worlds. I think that there are reasons to argue that in nonhomothetic worlds the marginal price index is the more important one of these two. I would like to see the marginal price index also included in the survey.

2) A rare but important omitted reference is a Finnish dissertation "The study in the theory of demand functions and price indexes" by Rajaoja (1958), where she defines a most interesting index, "*the price index of the competitors of a good*" and proves useful theorems. I shall be glad to give a copy of it to Prof. Diewert.

3) On pages 1 and 31 other approaches to index number theory are considered. The reference to Frisch is, however, not quite right as according to Frisch

(1936), p. 3) "There are two fundamentally different ways in which the problem of price index numbers may be approached. We term them the *atomistic* and the *functional* approach". The atomistic approach is further divided by him into the *stochastic* and the *test approach*. Apparently Allen used the term "statistical" instead of Frisch's "atomistic".

4) As my last comment to this elegant survey of the economic theory of index numbers I would like to refer to p. 52 where the concept of *exactness* of an index number formula is used for *nonhomothetic* aggregator functions. It seems to me that the concept of exactness looses much of its importance in nonhomothetic cases. By making the nonhomotheticity strong enough any price index getting values between Laspeyres and Paasche indices may be made "exact".

I think that even in generalizing the concept of exactness to nonhomothetic cases there is still work to be done.

As a summary I would like to say that Professor Diewert's survey of the economic theory of index numbers is an important paper which will certainly awake new interest in index numbers and will be a firm basis for further important work on the area.

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Comments on W.E. Diewert's paper "The economic theory of index numbers: A survey"

First I would like to make some general remarks on the scope of the paper. Diewert's article deals mainly with the economic theory of index numbers where various index number concepts are defined and investigated either in demand or production theory. The results derived in this approach are results of some economic theory and are *known to be valid* (strictly taken) only as far as the underlying economic theory is valid. In empirical work we should remember that whenever we are using results of the economic theory of index numbers, we should in fact believe that the underlying economic theory is a good description of the empirical phenomenon investigated. The beautiful and powerful results of this approach to index numbers are restricted to the world of economic theory and become results of the empirical world only when the economic theory is a good description of it. It is a fortunate fact that many results of the economic theory of index numbers are not contradictory to more elementary considerations. For instance Fisher's ideal index finds its place among superlative index numbers in the economic as well as in the statistical, test theoretic or, as I have called it, in the descriptive theory of index numbers. Therefore Fisher's ideal index does not need the economic theory (e.g. the quadratic utility function) for its support (although Afriat (1972, p. 45) seems to disagree with me). Or more clearly stated: Fisher's ideal index does not fall together with the utility hypothesis because it possesses many attractive properties even when prices and quantities are freely changing variables. Therefore I would like to characterize Fisher's ideal index (as well as other superlative index numbers) as theory/independent compared to many other indices which need some particular demand system for their support and work badly outside that particular demand world.

Thus there are genuine results in the economic theory of index numbers the validity of which depends heavily on the economic assumptions, e.g. that utility is a function of quantities only, or that there are no taste changes, or that the utility function has a special functional form.

It is a deep question in the methodology of science how to act in a situation where the validity of the standard theory is doubtful. The depth of this question lies perhaps in the fact that a theory need not be true to be useful (Even lies are useful sometimes). Here lies the demarcation line between realism and instrumentalism in science.

I think that we economists too often believe or act as if we believed in our simple and therefore beautiful theories. A more theory-independent or robust course of action would be more recommendable.

Turning now to the contents of the paper I should first like to congratulate the author on the clearness and explicitness of presentation which is so characteristic of Prof. Diewert. This is seen especially in accurate notation and in successful terminology. Much confusion has arisen because e.g. the utility level in the Konüs Cost of Living Index has been omitted or when the Konüs Implicit Quantity Index and the Allen Quantity Index are not distinguished from each other.