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FOUR REVEALED PREFERENCE TABLES

by

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1. INTRODUCTION

Revealed preference theory is used to infer from market data which of two consumption bundles is preferred to the other. Diewert (1976) has suggested that the data from two choice situations allow us to make one of four possible inferences, depending on the particular configuration of prices and quantities observed. Subsequently, Vartia (1976) provided a nine-fold classification scheme, arguing that the borderline cases in Diewert's model could be evaluated more exactly. The purpose of this paper is to demonstrate that these results need not be viewed as contradictory. Specifically, we argue that there exists a variety of revealed preference theories rationalized by different maintained hypotheses.

The paper is divided into three parts. In the first part, we consider a consumer with a known set of preferences and discover the implications which can be drawn from alternative regularity conditions imposed on the preference ordering. This procedure has recently been called the convexity approach by Sen (1979). (See also Hammond (1979)). In particular, we will demonstrate that the strengthening of the assumption that the preference ordering is quasi-concave to the assumption that it is strictly quasi-concave changes the implications drawn from the data in a manner analogous to the differences found in Diewert's and Vartia's work.

In the second part of the paper, we start with the concept of a demand correspondence and formulate two different versions of the Weak Axiom of Revealed Preference, WARP-I and WARP-II. Variant I is the weaker of the two; it allows indeterminate consumer's choices, i.e. a set of consumption bundles given by a demand correspondence $h^*(p,Y)$. The second variant, WARP-II, restricts $h^*(p,Y)$ to be a singleton and a four-fold revealed preference table follows as in section 2.2.

The third part of the paper considers the relationship between the first two parts.

Let $X = \{x | x \in \mathbb{R}^n_+\}$ be the consumer's consumption set, where x is a consumption bundle. We use the notation p, where $p \in \mathbb{R}^n_{++}$, to denote either a price vector or a normal to a hyperplane; the particular interpretation will be clear from the context. It is convenient to recall the definitions of the Laspeyres and Paasche quantity indices. The Laspeyres quantity index is

$$Q^{La}(x^1, x^0, p^1, p^0) = \frac{p^0 \cdot x^1}{p^0 \cdot x^0} = \Sigma\, w_i^0 (x_i^1 / x_i^0) \ , \qquad (1)$$

while the Paasche quantity index is

$$Q^{Pa}(x^1,\ x^0,\ p^1,\ p^0) = \frac{p^1 \cdot x^1}{p^1 \cdot x^0} = [\ \Sigma\ w_i^1 (x_i^1/x_i^0)^{-1}\ ]^{-1}\ , \quad (2)$$

where superscripts indicate different situations or periods, $p \cdot x = p_1 x_1 + \ldots + p_n x_n$ and, e.g., $w_i^0 = p_i^0 x_i^0 / p^0 \cdot x^0$ is the 'old' value share for i^{th} commodity.

2. INFERENCES FROM PREFERENCE ORDERINGS

2.1. Quasi-Concave Preferences

The consumer is assumed to have a weak preference relation $\succsim$. We define strict preference, $\succ$, and indifference, $\sim$, in the usual fashion;

$$x \succ y \Leftrightarrow x \succsim y\ \&\ \neg\ (y \succsim x)$$

and

$$x \sim y \Leftrightarrow x \succsim y\ \&\ y \succsim x\ .$$

We also define $x \precsim y \Leftrightarrow y \succsim x$ and $x \prec y \Leftrightarrow x \succ x$. Initially, we assume that $\succsim$ satisfies Conditions I.

<u>Conditions I</u>

(i) $\succsim$ is an ordering (i.e. reflexive, total, transitive),

(ii) $\succsim$ is continuous (i.e. the sets $\succsim x = \{y | y \succsim x\}$ and $\precsim x = \{y | y \precsim x\}$ are closed for all x),

(iii) $\succsim$ is strictly increasing (i.e. $x_1 \geq y_1, \ldots, x_n \geq y_n$ and $x \neq y \Rightarrow x \succ y$), and

(iv) $\succsim$ is quasi-concave (i.e. $x \succsim y \Rightarrow \lambda x + (1-\lambda)y \succsim y$ for all $0 < \lambda < 1$).

Conditions I quarantee the existence of a utility function $u(x)$ representing $\succsim$. Strict increasingness prevents the indifference surfaces $Ix = \{y | y \sim x\}$ from having portions parallel to coordinate axes.

With these assumed regularity conditions on $\succsim$, we can find at least one supporting hyperplane to any set $\succsim x^t = \{y | y \succsim x^t\}$ at x^t and choose a normal to this hyperplane, p^t, with strictly positive components[1]. Conditions I then allow us to conclude:

$$\forall x, x^t \in X : p^t \cdot x^t \geq p^t \cdot x \quad \Rightarrow \quad x^t \succsim x \, , \tag{3}$$

and

$$\forall x, x^t \in X : x \succsim x^t \quad \Rightarrow \quad p^t \cdot x \geq p^t \cdot x^t \, . \tag{4}$$

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1) If strict increasingness (iii) is weakened to increasingness ($\forall i : x_i \geq y_i \Rightarrow x \succsim y$) then for some x^t-points p^t may contain zero coordinates. However, these points x^t do not appear as demand points in market situations where $p_i^t > 0$ for all i.

These two conditions are also found in Afriat (1967, p. 67). The latter may be stated in an equivalent form as follows:

$$\forall x, x^t \in X : p^t \cdot x^t > p^t \cdot x \quad \Rightarrow \quad x^t \succ x \, . \tag{5}$$

No behavioural assumptions were employed in the development of (3) - (5). However, if the consumer is assumed to choose his consumption bundle by maximizing $\succsim$ subject to a budget constaint, then x^t can be interpreted as the quantities chosen at prices p^t with income $p^t \cdot x^t$. We adopt this interpretation of the theory. Condition (3) states that any bundle x which costs no more than x^t at prices p^t can be only as good as x^t (i.e. the money spent on x^t has been spent effectively, Afriat (1977, p. 51)), while (4) says that any bundle which is at least as good as x^t costs at least as much (or from (5): all cheaper bundles are worse).

Note that it is not necessarily true that more expensive bundles are at least as good, or that $p^t \cdot x > p^t \cdot x^t \Rightarrow x \succsim x^t$.

Restricting attention to two consumption bundles, x^0 and x^1, with corresponding prices p^0 and p^1, we can construct Revealed Preference Table I using (3) - (5) and recalling (1) and (2).

Revealed Preference Table I

	$Q^{Pa} < 1$ $p^1 \cdot x^1 < p^1 \cdot x^0$	$Q^{Pa} = 1$ $p^1 \cdot x^1 = p^1 \cdot x^0$	$Q^{Pa} > 1$ $p^1 \cdot x^1 > p^1 \cdot x^0$
$Q^{La} < 1$ $p^0 \cdot x^1 < p^0 \cdot x^0$	$x^1 \prec x^0$	Impossible	Impossible
$Q^{La} = 1$ $p^0 \cdot x^1 = p^0 \cdot x^0$	$x^1 \precsim x^0$	$x^1 \sim x^0$	Impossible
$Q^{La} > 1$ $p^0 \cdot x^1 > p^0 \cdot x^0$	Zone of Indeterminacy	$x^1 \succsim x^0$	$x^1 \succ x^0$

For instance if $p^1 \cdot x^1 = p^1 \cdot x^0$ and $p^0 \cdot x^1 < p^0 \cdot x^0$, (3) implies $x^1 \succsim x^0$ and (5) implies $x^0 \succ x^1$, which contradicts Condition I (i). Other cases may be derived similarly.

Essentially the same table was derived in an other way in Vartia (1976, p. 44). The four entries in the upper right-hand corner of the table are implied also by Afriat's (1967, p. 69) cyclical consistency condition.

In the case where $Q^{Pa} = Q^{La} = 1$, it is possible to have $x^1 \sim x^0$ and $x^1 \neq x^0$ because there may be flat regions on the indifference surfaces. Note that $Q^{Pa} \geq 1$ ($Q^{Pa} > 1$) quarantees $x^1 \succsim x^0$ ($x^1 \succ x^0$) regardless of the value of Q^{La}. Similarly, $Q^{La} \leq 1$ ($Q^{La} < 1$) implies $x^1 \precsim x^0$ ($x^1 \prec x^0$). If $Q^{Pa} < 1 < Q^{La}$, no inferences concerning the preference ordering between x^1 and x^0 can be drawn, as can be shown by counterexamples. Table 7 summarizes all of the valid conclusions concerning the consumer's preferences between x^1 and x^0 that can be drawn from the observations (p^0, x^0) and (p^1, x^1), if all that one knows is that $\succsim$ is a fixed preference ordering satisfying Conditions I.

2.2. Strictly Quasi-concave Preferences

If preferences are restricted to be strictly quasi-concave, somewhat more can be inferred.

<u>Conditions II</u>

(i), (ii) and (iii) as before.

(iv) $\succsim$ is strictly quasi-concave. (i.e. $x \succsim y \Rightarrow \lambda x + (1-\lambda)y \succ y$ for all $0 < \lambda < 1$).

Let p^t be the normal of some supporting hyperplane of $\succsim x^t$ at x^t. From Conditions II we conclude,

$$\forall x, x^t \in X,\ x \neq x^t :\ p^t \cdot x^t \geq p^t \cdot x \quad \Rightarrow \quad x^t \succ x\ , \qquad (6)$$

and

$$\forall x, x^t \in X,\ x \neq x^t : x \succsim x^t \quad \Rightarrow \quad p^t \cdot x > p^t \cdot x^t \ . \qquad (7)$$

However, as (6) and (7) are equivalent here (because $\neg (x \succsim x^t) \Longleftrightarrow x^t \succ x$), we have obtained only one independent condition.

The differences in the two models can be seen by comparing (3) and (4) with (6) and (7). Assuming $x \neq x^t$, the premises in (3) and (6) are identical, but their conclusions differ. The sharper conclusion in (6) follows from the assumed absence of flat segments on indifference surfaces. Similarly, the difference in the conclusions of (4) and (7) are accounted for by the possibility of flat segments in the former.

We again adopt the behavioural interpretation introduced in Section 2.1. Analogous to that discussion, we may construct the following Revealed Preference Table II:

Revealed Preference Table II

	$Q^{Pa} < 1$ $p^1 \cdot x^1 < p^1 \cdot x^0$	$Q^{Pa} \geq 1$ $p^1 \cdot x^1 \geq p^1 \cdot x^0$
$Q^{La} \leq 1$ $p^0 \cdot x^1 \leq p^0 \cdot x^0$	$x^1 \prec x^0$	Impossible (unless $x^1 = x^0$)
$Q^{La} > 1$ $p^0 \cdot x^1 > p^0 \cdot x^0$	Zone of Indeterminacy	$x^1 \succ x^0$

Only strict preferences can be inferred here (unless $x^1 = x^0$ and thus $Q^{Pa} = Q^{La} = 1$). If $Q^{Pa} \geq 1$, then $x^1 \succ x^0$ or $x^1 = x^0$; if $Q^{La} \leq 1$, then $x^1 \prec x^0$ or $x^1 = x^0$; and if $Q^{Pa} < 1 < Q^{La}$, then any of cases $x^1 \succ x^0$, $x^1 \sim x^0$, $x^1 \prec x^0$ may be realized by the choice of a suitable preference ordering. Note also that if $x^1 \neq x^0$, $Q^{Pa} \geq 1$ implies $Q^{La} > 1$ although $Q^{Pa} > Q^{La} > 1$ is possible. For homothetic preferences we have $Q^{La} \geq Q^{Pa}$ always. Similarly $Q^{La} \leq 1$ implies $Q^{Pa} < 1$ unless $x^1 = x^0$.

Diewert (1976, p. 144) gave a similar table, however, the cases $Q^{Pa} = 1$ and $Q^{La} = 1$ were misplaced in the first column and second row respectively.

Thus, strengthening quasi-concavity to strict quasi-concavity has made it possible to collapse the nine-entry table to a four-entry table. The two cases in Table I where weak preference had been indicated have been sharpend to strict preference, and it is no longer possible to discover two distinct indifferent bundles from these comparisons.

3. INFERENCES FROM REVEALED PREFERENCE AXIOMS

3.1. General Remarks

Here we take a choice correspondence, not a preference relation, as our starting point. Let the set of <u>alternatives</u> X be all non-negative n-vectors, its elements x are interpreted to be consumption bundles. $\mathcal{B}$ is a nonempty collection of nonempty subsets of X, each element B of $\mathcal{B}$ is called <u>a budget</u>. A mapping h defined on $\mathcal{B}$ and assigning a nonempty subset $h(B) \subset B$ for every budget $B \in \mathcal{B}$ is called a <u>choice correspondence</u>. The set h(B) of alternatives in B is called the <u>choice set</u> for budget B. If $x \in h(B)$ and $y \in B$ we say x <u>is chosen</u> and y <u>could have been chosen</u>. This general framework is from Richter (1971).

Let Ω be a cone of strictly positive vectors in $\mathbb{R}^{n+1}$ with vertex at the origin. We write $(p,M) \in \Omega$ and interpret p as a vector of prices and M as income (or rather, expenditure). The <u>competitive budget</u> corresponding to $(p,M) \in \Omega$ is $B(p,M) = \{x \in X | p \cdot x \leq M\}$, a subset of X, and the collection of all competitive budgets is $\bar{\mathcal{B}}$. <u>A demand correspondence</u> h* is defined as follows: $h^*(p,M) = h(B(p,M))$; h* assigns to every (p,M) in Ω a nonempty subset of the competive budget B(p,M). Because $B(\lambda p,\lambda M) = B(p,M)$ for all $\lambda > 0$, h* is homogenous of degree zero.

In addition we assume that all income is spent:

$$\forall (p,M) \in \Omega : p \cdot h^*(p,M) = M.$$

3.2. WARP-I

Using Richter's (1971, p. 32) terminology, we will present a theory of "direct revealed preference". We shall say that a bundle x is "directly revealed at least as good as" y, $x \boxed{\succsim} y$, if for some $(p,M) \in \Omega$, x is chosen when y could have been chosen. More explicitly,

$$x \boxed{\succsim} y \Leftrightarrow \exists\ (p,M) \in \Omega : x \in h^*(p,M)\ \&\ M \geq p \cdot y .$$

Similarly, x is "directly revealed better than" y, $x \boxed{\succ} y$, iff

$$\exists\ (p,M) \in \Omega : x \in h^*(p,M)\ \&\ M > p \cdot y ,$$

and x is "directly revealed indifferent to" y, $x \boxed{\sim} y$, if $x \boxed{\succsim} y$ and $y \boxed{\succsim} x$. In what follows we assume that the demand correspondence h* satisfies the following regularity condition:

Weak Axiom of Revealed Preference - Variant I (WARP-I):

$$\forall x,y \in X : [\, x \boxed{\succsim} y \Rightarrow \neg\, (y \boxed{\succ} x)\,]$$

$$\&\ [\, x \boxed{\succ} y \Rightarrow \neg\, (y \boxed{\succsim} x)\,]\ .$$

We now consider two choices $x^0 \in h^*(p^0,M^0)$ and $x^1 \in h^*(p^1,M^1)$. Using WARP-I we obtain[1]:

$$p^t \cdot x^t \geq p^t \cdot x^{t'} \Rightarrow p^{t'} \cdot x^{t'} \leq p^{t'} \cdot x^t,\ t,t'=0,1,\ t \neq t', \tag{13}$$

and

$$p^t \cdot x^t > p^t \cdot x^{t'} \Rightarrow p^{t'} \cdot x^{t'} < p^{t'} \cdot x^t,\ t,t'=0,1,\ t \neq t'. \tag{14}$$

We can express this information in the following Revealed Preference Table III:

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1) It is possible to adopt a rigid empirical interpretation of the theory. With (p^0,x^0) and (p^1,x^1) as the <u>only</u> observations, one would suppose $M^0 = p^0 \cdot x^0$ and $M^1 = p^1 \cdot x^1$, and set $\Omega = \{(p,M) \mid (p,M) = (\lambda p^0, \lambda M^0)$ or $(p,M) = (\lambda p^1, \lambda M^1),\ \lambda > 0\}$. With this domain for the demand correspondence, (13) and (14) are not merely implied by WARP-I but are, in fact, equivalent to WARP-I. Similarly, in Section 3.3 (17) would be equivalent to WARP-II.

Revealed Preference Table III

	$Q^{Pa} < 1$ $p^1 \cdot x^1 < p^1 \cdot x^0$	$Q^{Pa} = 1$ $p^1 \cdot x^1 = p^1 \cdot x^0$	$Q^{Pa} > 1$ $p^1 \cdot x^1 > p^1 \cdot x^0$
$Q^{La} < 1$ $p^0 \cdot x^1 < p^0 \cdot x^0$	$x^1 \boxed{\prec} x^0$	Impos- sible	Impossible
$Q^{La} = 1$ $p^0 \cdot x^1 = p^0 \cdot x^0$	$x^1 \boxed{\precsim} x^0$	$x^1 \boxed{\sim} x^0$	Impossible
$Q^{La} > 1$ $p^0 \cdot x^1 > p^0 \cdot x^0$	Zone of Indeterminacy	$x^1 \boxed{\succsim} x^0$	$x^1 \boxed{\succ} x^0$

Comparing this with Revealed Preference Table I, we have the same nine cases. As now there is no presumption that underlying preference ordering exists, the interpretation of the symbols has changed[1]. In Table I actual preferences $\succsim$ are revealed, while in Table II $\boxed{\succsim}$ is defined entirely from choices.

1) Instead of using WARP-I as an axiom, one could treat it as an hypothesis concerning choices. With this interpretation, the entries in the table labelled "Impossible" would be replaced by "Inconsistent with WARP-I".

Should a preference ordering $\succsim$ satisfying Conditions I exist - which is completely compatible with the assumptions made above - then, e.g., in the case where $Q^{Pa} < 1$ and $Q^{La} < 1$ we can infer not only $x^1 \boxed{\prec} x^0$ but also $x^1 \prec x^0$. Indeed, $x \boxed{\succsim} y$, $x \boxed{\succ} y$, and $x \boxed{\sim} y$ will imply $x \succsim y$, $x \succ y$, and $x \sim y$, respectively. However, the reverse implications are not valid since, e.g., it may be true that $x \succsim y$, yet if $Q^{Pa} < 1 < Q^{La}$ then $x \succsim y$ can not be revealed by direct comparisons of market choices. Stating this formally, $\boxed{\succsim} \subset \succsim$, $\boxed{\succ} \subset \succ$, and $\boxed{\sim} \subset \sim$, where the relations are interpreted as being sets in X^2. In fact, Richter (1971, p. 33) regards this observation as "the kernel of Samuelson's initial insight ... and the justification of his revealed preference terminology".

3.3. WARP-II

Variant I above seems to us to be a natural way to model a revealed preference relation. However, it is not the theory of revealed preference found in Samuelson (1947). To model Samuelson's theory, we define a new revealed preference relation, x is "directly revealed preferred to" y, $x \circledS y$, iff $x \boxed{\succsim} y$ and $x \neq y$. In this variant of the theory, there is no need for a definition of either weak revealed preference or of revealed indifference. As a revealed preference axiom we take, as in Samuelson (1947), the asymmetry of $\circledS$:

Weak Axiom of Revealed Preference - Variant II (WARP-II):

$$\forall x,y \in X : x \textcircled{>} y \Rightarrow \neg\,(y \textcircled{>} x) \;.$$

It is clear from this axiom that $h^*(p,Y)$ must be a singleton in $B(p,Y)$; we denote its only value by $\bar{h}(p,Y)$.

Now restricting attention to $x^0 = \bar{h}(p^0,M^0)$ and $x^1 = \bar{h}(p^1,M^1)$, WARP-II implies:

$$p^t \cdot x^t \geq p^t \cdot x^{t'} \quad \Rightarrow \quad p^{t'} \cdot x^{t'} < p^{t'} \cdot x^t \,,\; t,t'=0,1,\; t \neq t',\; x^t \neq x^{t'} \,. \tag{17}$$

Constructing the relevant table, there are only four entries.

Revealed Preference Table IV

	$Q^{Pa} < 1$ $p^1 \cdot x^1 < p^1 \cdot x^0$	$Q^{Pa} \geq 1$ $p^1 \cdot x^1 \geq p^1 \cdot x^0$
$Q^{La} \leq 1$ $p^0 \cdot x^1 \leq p^0 \cdot x^0$	$x^1 \textcircled{<} x^0$	Impossible (unless $x^1 = x^0$)
$Q^{La} > 1$ $p^0 \cdot x^1 > p^0 \cdot x^0$	Zone of Indeterminacy	$x^1 \textcircled{>} x^0$

Although Tables II and IV have different foundations and different interpretations, they present the same four-fold classification. Should a preference ordering $\succsim$ satisfying Conditions II exist, it is easy to see that $x \textcircled{$\succ$} y$ will imply $x \succ y$ as well.

In Section 2, Conditions II were a strengthening of Conditions I; analogously WARP-II is stronger than WARP-I.

Theorem: WARP-II implies WARP-I.

Proof: The cases (a) $x = y$ and (b) $x \neq y$ are considered separately. In case (a), WARP-II, i.e., $(x \boxed{\succsim} y \ \& \ x \neq y) \Rightarrow \neg (y \boxed{\succsim} x \ \& \ x \neq y)$, is trivially true because the antecedent is false. Using the contrapositive of the first statement defining WARP-I, WARP-I is $[y \boxed{\succ} x \Rightarrow \neg (x \boxed{\succsim} y)]$ & $[x \boxed{\succ} y \Rightarrow \neg (y \boxed{\succsim} x)]$. For convenience, call these two statements A and B respectively. Since both antecedents, $y \boxed{\succ} x$ and $x \boxed{\succ} y$, are necessarily false, WARP-I is trivially true as well.

In case (b), by hypothesis WARP-II is valid and $x \neq y$. From predicate calculus we know (i) $(r \Rightarrow s) \Leftrightarrow (\neg r \vee s)$ and (ii) $\neg (r \ \& \ s) \Leftrightarrow \neg r \vee \neg s$. Consequently, WARP-II may also be stated as

$$\neg (x \boxed{\succsim} y \ \& \ x \neq y) \vee \neg (y \boxed{\succsim} x \ \& \ x \neq y)$$

or $$\neg (x \boxed{\succsim} y \ \& \ y \boxed{\succsim} x \ \& \ x \neq y)$$

or $$x \neq y \Rightarrow \neg (x \boxed{\succsim} y) \vee \neg (y \boxed{\succsim} x) .$$

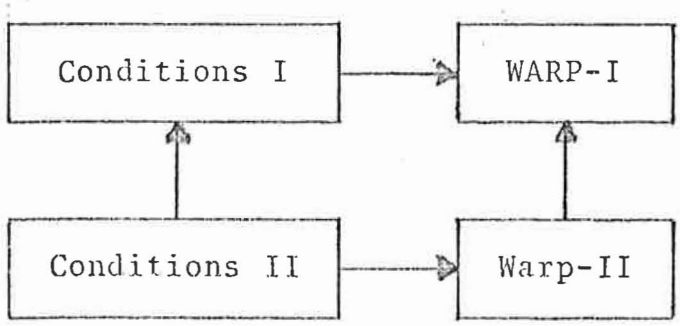
If $\neg(x \boxed{\succsim} y)$ is true, then the first part of WARP-I (A) is trivially true. As a consequence of the definitions of $\boxed{\succ}$ and $\boxed{\succsim}$, $(x \boxed{\succ} y \Rightarrow x \boxed{\succsim} y)$ or, equivalently, $[\neg (x \boxed{\succsim} y) \Rightarrow \neg (x \boxed{\succ} y)]$. Thus, if $\neg (x \boxed{\succsim} y)$ is true, the second part of WARP-I (B) is trivially true as well, and hence WARP-I = A & B is true.

The proof that $\neg (y \boxed{\succsim} x)$ is true implies WARP-I is true can be demonstrated in a similar manner.

$\square$

4. A COMPARISON OF RESTRICTIONS ON DEMAND CORRESPONDENCES

Since in the preference ordering approach a demand correspondence can be obtained by maximizing $\succsim$ subject to a budget constraint, it is possible to view Conditions I and II as well as WARP-I and II as restrictions on demand correspondences. We have shown that WARP-II implies WARP-I. If $\succsim$ satisfies Conditions I (Conditions II), then the associated demand correspondence will satisfy WARP-I (WARP-II). As Conditions II are stronger than Conditions I, we have a partial ordering of these four restrictions on demand correspondences. Schematically,



Thus for demand correspondences, Conditions II are the most restrictive of the conditions we have considered while WARP-I is the weakest. Conditions I and WARP-II are non-comparable, e.g., WARP-II is consistent with a single-valued demand function generated by a nontransitive consumer while Conditions I do not require single-valuedness.

5. CONCLUSIONS

In this paper we have considered four revealed preference tables. The first two tables were constructed with the assumption that a preference ordering exists while the second two tables are associated with the demand correspondence approach. If preferences satisfy Conditions I, we are able to summarize the comparisons of interest in a 3 x 3 table (Table I); a similar table (Table III) arises in Variant I of our demand correspondence approach. Eliminating the possibility of flat regions on indifference surfaces by strengthening the quasi-concavity of Conditions I to strict quasi-concavity results in a 2 x 2 summary table (Table II); an analogous table (Table IV) is obtained with Variant II of our demand correspondence approach[1).]

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1) Subsequent to deriving the results for this paper, we have discover that Hicks (1956) considered two versions of revealed preference theory and noted that they would result in different implications from the same data. However, it is difficult to discern with certainty all of Hicks' maintained assumptions.

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