ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B 00120 Helsinki Finland Tel. 609900 Telefax 601753

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Risto Murto

TESTING CONTINUOUS TIME MODELS OF THE TERM STRUCTURE OF INTEREST RATES IN THE FINNISH MONEY MARKET*

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ABSTRACT: The purpose of this paper is to investigate empirically the continuous time models of the term structure of interest rates using data from the Finnish Money market. We estimate and test both two- and one-factor term structure models. In particular, we estimate and test a two-factor general equilibrium model by Longstaff and Schwartz (1991). In order to estimate the two-factor model we need the estimates of volatility of the short-term interest rates. These are acquired by estimating the GARCH(1,1) process for one-month interest rate changes. We conclude that the two-factor model is clearly rejected when we use data from the whole sample. The subsample results are slightly more favourable for the two-factor model, but we are still able to reject the model. The Vasicek (1977) one-factor model is rejected both for the entire period and subsamples.

KEY WORDS: GARCH, HELIBOR rates, two-factor term structure model, volatility

1. INTRODUCTION

The term structure of interest rates describes the relationship between the maturity and the interest rate of debt instrument. Term structure issues have already been of central importance in finance and financial economics for many years. Increased interest rate risk and development of new interest rate-dependent securities, such as interest rate swaps, options and futures, have increased the interest in the term structure modelling. When developing pricing models for these instruments, one needs a model of the term structure of interest rates or a model of interest rates.

The purpose of this paper is to investigate empirically the continuous time models of the term structure of interest rates using data from the Finnish money market. In particular, we are interested in testing a two-factor general equilibrium model by Longstaff and Schwartz (1991). Longstaff and Schwartz have developed a two-factor model of the term structure using the general equilibrium framework of Cox, Ingersoll and Ross (1985a). In this model, the state variables are the short-term interest rate and the volatility of the short-term interest rate. They derive closed-form expressions for discount bond prices and for discount bond option prices.

This study provides fresh evidence on the empirical validity of the term structure models for a new set of data. Most of the empirical work in this area has been done with U.S. data. In addition to the obvious size differences, the Finnish money market differs from the U.S. market in many respects. The influence of foreign interest rates on Finnish financial markets is strong, particularly since the Bank of Finland has pegged the external value of the markka against a currency basket.

The outline of this study is as follows. In the next section we discuss the Longstaff-Schwartz model. Section 3 presents the data and the markets. In section 4 we estimate the conditional volatility by using a GARCH-specification and test both one- and two-factor term structure models by GMM. Finally, conclusions are presented in section 5.

2. THE TWO-FACTOR GENERAL EQUILIBRIUM MODEL

In this section we present the Longstaff & Schwartz two-factor term structure model. The model is derived in a general equilibrium framework following Cox, Ingersoll and Ross (1985a, 1985b). An advantage of the CIR framework over partial equilibrium approaches, as Vasicek (1977) or Brennan and Schwartz (1979), is that the factor risk premiums are determined endogenously as part of the general equilibrium.

CIR (1985a) present a general rational expectations asset pricing model. In the model there is a finite number of production technologies exhibiting constant stochastic returns to scale. A single good is produced that can be allocated to either consumption or investment. There is a also fixed number of identical agents who maximise the expected utility of lifetime consumption. All investment is done by firms and the agents invest all of their unconsumed wealth in the shares of those firms. There are also random technological changes depending on the state variables. These state variables are in turn governed by a multivariate diffusion process. Equilibrium in this economy gives the market-clearing interest rate, prices for the contingent claims, and the total production and consumption plans.

In the company paper, CIR (1985b) derive a number of explicit models of the term structure of interest rates from this framework. In order to do so they make some specific assumptions about production opportunities and preferences. The empirical analysis has focused on the case in which there is one state variable and where both this and the instantaneous short term rate follow a 'square-root' process. This is the specific model to which we refer to below as the CIR model.

As in the CIR, Longstaff and Schwartz make an assumption that all physical investment is performed by a single stochastic constant-returns-to-scale technology which produces one good. This good can be either consumed or reinvested in production. The realized returns on physical investment are driven by two economic factors, X and Y, instead of by only one factor as is CIR. The realized returns are governed by the following stochastic differential equation

$$(1) \qquad \frac{dQ}{Q} = (\mu X + \theta Y)\, dt + \sigma \sqrt{Y}\, dZ_1,$$

where μ, θ, and σ are positive constants and Z_1 is a scalar Wiener process. The first factor X represents the component of expected return that is unrelated to production uncertainty, while Y represents the component common to both. Expected returns and production volatility are not required to be perfectly correlated.

The state variables X and Y are assumed to follow a square root process:

$$(2) \qquad dX = (a - bX)\, dt + c \sqrt{X}\, dZ_2,$$

$$(3) \qquad dY = (d - eY)\, dt + f \sqrt{Y}\, dZ_3,$$

where a, b, c, d, e, f > 0, and Z_2 and Z_3 are scalar Wiener processes. Changes in X are assumed to represent technological changes that are unrelated to production uncertainty. Thus we require that Z_2 be uncorrelated with Z_1 and Z_3. Following CIR (1985b), we also require that $\theta > \sigma^2$, which guarantees that the riskless rate is nonnegative.

There is a fixed number of identical individuals with time-additive preferences. We assume that the representative investor's preferences are logarithmic:

$$(4) \qquad E_t\left[\int_t^{\infty} \exp(-\rho s)\, \ln(C_s) ds\right],$$

where E[.] is the conditional expectation operator, ρ is the utility discount factor, and C_s represents consumption at time s. Finally, we assume the existence of perfectly competitive, continuous markets for riskless borrowing and lending as well as other types of contingent claims.

The investor is maximizing (4) subject to the budget constraint by selecting an optimal level of consumption. The unconsumed wealth is reinvested in the physical production. The investor's derived utility of wealth function (value function) is partially separable and has the simple form

$$(5) \qquad J(W,X,Y,t) = \frac{\exp(-\rho t)}{\rho} \ln(W) + G(X,Y,t)$$

The optimal consumption is now ρW. Substituting optimal consumption and (1) into the investor's budget constraint gives the following equilibrium dynamics for wealth:

$$(6) \qquad dW = (\mu X + \theta Y - \rho)W\, dt + \sigma W \sqrt{Y}\, Z_1.$$

A simple rescaling of the state variables gives $x = X/c^2$ and $y = Y/f^2$. Let $H(x,y,\tau)$ denote the value of a contingent claim with maturity τ and boundary conditions that do not depend on wealth. Given this framework, Theorem 3 of CIR (1985a) can be applied to obtain the fundamental partial differential equation satisfied by the contingent claim

$$(7) \qquad (x/2)H_{zz} + (y/2)H_{yy} + (\gamma - \delta x)H_z +$$

$$(\eta - \xi y - (-J_{WW}/J_W)\, \mathrm{Cov}(W,Y))H_y - rH = H_\tau,$$

where $\gamma = \alpha/c^2$, $\delta = b$, $\eta = d/f^2$, $\xi = e$, r is the instantaneous riskless rate, and Cov(W,Y) is the instantaneous covariance of changes in W with changes in Y. The utility-dependent term in the coefficient of H_y represents the market price of the risk of changes in the level of production uncertainty which is governed by Y. Because of the separable form of the derived utility of wealth function, however, (3), (5) , and (6) can be used to show that

$$(8) \qquad (-J_{ww}/J_w)\ \mathrm{Cov}(W,Y) = \lambda y,$$

where λ is a constant. Thus, the market price of risk is proportional to y.

The equilibrium riskless interest rate can be obtained by applying Theorem 1 of CIR (1985a) which relate the riskless rate to the expected rate of change in marginal utility. Because of the logarithmic form of the derived utility of the wealth function, the short-term interest rate is simply the expected return on production minus the variance of production returns

$$(9) \qquad r = \alpha\, x + \beta\, y,$$

where $\alpha = \mu c^2$ and $\beta = (\theta - \sigma^2)f^2$, which is nonnegative for all feasible values of the state variables. The instantaneous variance of changes in the riskless rate V can be obtained by applying Ito's Lemma to the expression for r in (9) and deriving the appropriate expectation. The resulting expression is

$$(10) \qquad V = \alpha^2 x + \beta^2 y,$$

which is also negative for all feasible values of the state variables. Together, (9) and (10) form a simple system of two linear equations in x and y. Provided that α is not equal to β, this system is globally invertible and we can solve for x and y in terms of r and V, obtaining

$$(11) \qquad x = \frac{\beta r - V}{\alpha(\beta - \alpha)},$$

$$(12) \qquad y = \frac{V - \alpha r}{\beta(\beta - \alpha)}.$$

Equations (11) and (12) allows us to make the change of variables from x and y to r and V. These two variables, the instantaneous spot rate and variance of the spot rate, serve as observable instruments on which the pricing will be based.

The dynamics of r and V are presented in equations (13) and (14). These can be obtained by applying Ito's Lemma to (9) and (10) and making the corresponding change of variables.

$$(13) \qquad dr = \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r - \frac{\xi - \delta}{\beta - \alpha} V\right) dt + \alpha\left(\frac{\beta r - V}{\alpha(\beta - \alpha)}\right)^{1/2} dZ_2 + \beta\left(\frac{V - \alpha r}{\beta(\beta - \alpha)}\right)^{1/2} dZ_3$$

$$(14) \qquad dV = \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V\right)dt + \alpha^2\left(\frac{\beta r - V}{\alpha(\beta - \alpha)}\right)^{1/2} dZ_2 + \beta^2\left(\frac{V - \alpha r}{\beta(\beta - \alpha)}\right)^{1/2} dZ_3 .$$

Let $F(r,V,\tau)$ denote the value of a riskless unit discount bond with τ periods until maturity. The equilibrium value of $F(r,V,\tau)$ can be determined by solving (7), subject to the maturity condition that the bond value equals on, when $\tau = 0$, and then making a change of variables to r and V. A separation of variables results in the following solution for $F(r,V,\tau)$:

(15) $$F(r,V,\tau) = A^{2\gamma}(\tau)B^{2\eta}(\tau)\exp(\kappa\tau + C(\tau)r + D(\tau)V),$$

where $$A(\tau) = \frac{2\,\phi}{(\delta + \phi)(\exp(\phi\tau)-1) + 2\phi},$$

$$B(\tau) = \frac{2\,\psi}{(\nu + \psi)(\exp(\psi\tau)-1) + 2\psi},$$

$$C(\tau) = \frac{\alpha\phi(\exp(\psi\tau)-1)B(\tau) - \beta\psi(\exp(\phi\tau)-1)A(\tau)}{\phi\psi(\beta - \alpha)},$$

$$D(\tau) = \frac{\psi(\exp(\phi\tau)-1)A(\tau) - \phi(\exp(\psi\tau)-1)B(\tau)}{\phi\psi(\beta - \alpha)},$$

$$\nu = \xi + \lambda,$$

$$\phi = (2\alpha + \delta^2)^{1/2},$$

$$\psi = (2\beta + \nu^2)^{1/2},$$

$$\kappa = \gamma(\delta + \phi) + \eta(\nu + \psi).$$

From (15), we obtain the following expression for the yield on a τ-maturity bond $R(\tau)$:

(16) $$R(\tau) = -(\kappa\tau + 2\gamma \ln A(\tau) + 2\,\eta \ln B(\tau) + C(\tau)r + D(\tau)V)/\tau.$$

The yield to maturity and likewise the discount bond price is a function of two state variables r and V, and depends on the six parameters α, β, γ, δ, η, and ν.

The yield to maturity converges to a constant as $\tau \to \infty$. The yield on this bond is $\gamma(\phi-\delta) + \eta(\psi-\upsilon)$.

The Longstaff-Schwartz model can produce a rich variety of different shaped yield curves. In this respect the two-factor model is clearly superior to the one-factor models, which produce only much simpler shaped yield curves.

Furthermore, in the Lonstaff-Schwartz model the term structure can change even without any change in the level of the instantaneous interest rate. A change in the volatility can cause the term structure to change even if the front-end of the yield curve remains the same.

More detailed explanations about the behaviour of interest rates with respect to the different parameters can be found in Longstaff and Schwartz (1991). Longstaff and Schwartz derive also a two-factor model of discount bond options prices.

3. THE DATA

The data used in this study is from the Finnish money market. Most of the testing of the continuous time term structure model have been based on U.S. data. The Finnish money market provides an interesting alternative data source, which differs in many respects from that of the U.S. The Finnish money markets did not emerge until the beginning of 1987. The most liquid securities in the money market are certificates of deposits (CDs) issued by banks and the Bank of Finland. The empirical work in this study is based on HELIBOR rates. The Bank of Finland calculates daily HELIBOR rates (Helsinki Interbank Offer Rates) for 1, 2, 3, 6, 9 and 12 month maturities as the average bid rate for the bank's CDs quoted by the five largest banks. The time-series of the 1-, 3- and 12-month HELIBOR rates are presented in figure 1.

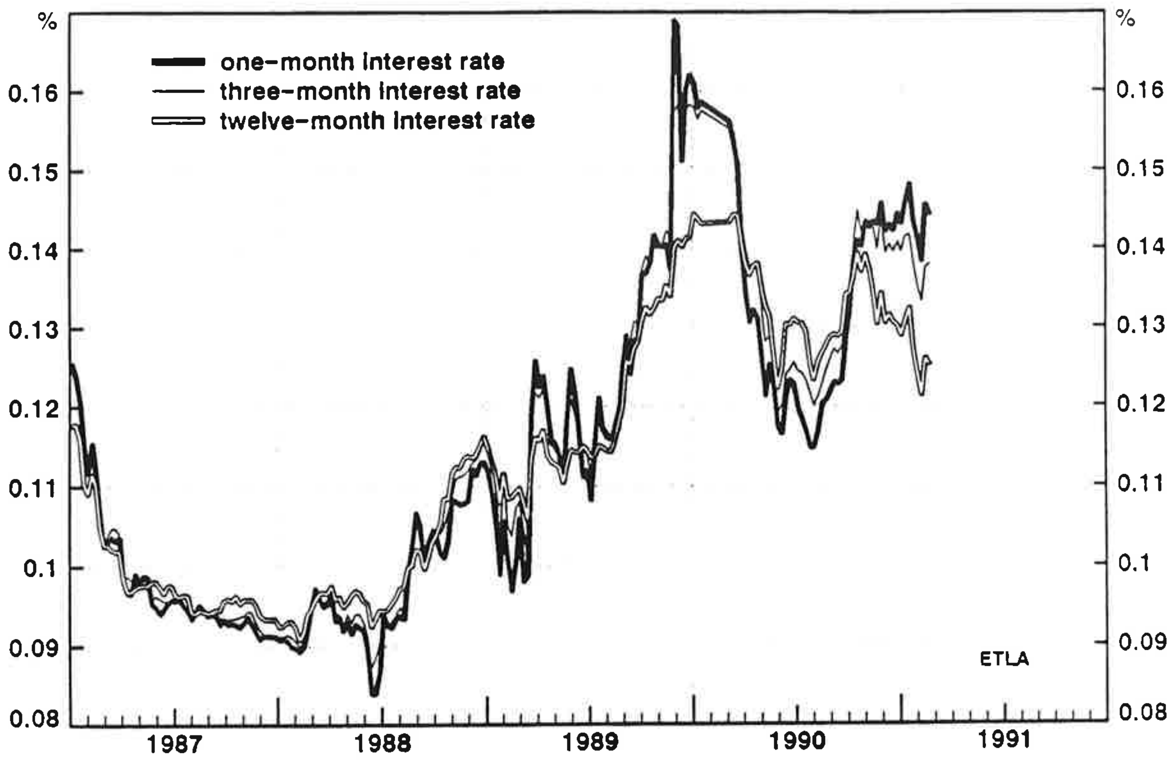
The banks' CDs are not homogeneous instruments per se issued only by a sole party as are Treasury notes. Since the number of issuers has been limited, because of the small number of banks in the Finnish money markets, the CDs have nevertheless been quite homogeneous. The market participants have also treated these banks as very creditworthy so the difference between rates for the treasury bills and the bank's CDs has been negligible. The situation changed, however, in autumn 1990 when the market participants started requiring an extra premium for CDs issued by one of the five banks (Skopbank). In order to mitigate the effect of this premium on our results we have recalculated the average bid rate from autumn 1990 without Skopbank's bid rate. So that from the beginning of 1987 to autumn 1990 the basic data is the HELIBOR-rates and from autumn 1990 to spring 1991 it is the average bid rate for the bank's CDs quoted by four of the five largest banks.

The data is a weekly sample consisting of HELIBOR rates for every Tuesday. The sample period starts on 6.1.1987 and ends at 12.02.1991. There are 209 observations on each maturity. The data set do not include the observations from the period 18.1 - 2.3.1990, when the interest rate level was frozen due to the bank strike. All interest rates are continuously compounded and expressed in percent per annum.

We estimate term structure models as well as GARCH models both for the whole period and for subperiods. The first subperiod starts on 1.1.1987 and ends on 21.02.1989. In this period changes in the interest rates were quite moderate. The interest rates were in a declining trend in the beginning of the period and start to rise towards to the end of the first subperiod.

The second subperiod is from 28.02.1989 to 12.02.1991. In the beginning of the period, on March 16, 1989, the fluctuation range of the currency index for the Finnish markka was shifted by about 4 per cent facilitating a revaluation. This was a period of turbulence in the foreign exchange market and money market. Unrest in currency markets and money market increased the volatility of the interest rate changes, as can be seen from figure 1. For example, devaluation speculation against the markka caused a dramatic increase in interest rates in the end of the year 1989, which can be seen as a high spike in the data set.

FIGURE 1. HELIBOR rates



4. TESTING THE TWO-FACTOR GENERAL EQUILIBRIUM MODEL

4.1 Estimating the Volatility

In order to test the Longstaff and Schwartz two-factor model we need observations of the instantaneous variance of chances in the instantaneous spot rate. We use conditional variance estimates acquired by estimating a GARCH-process for the one-month interest rate. In addition we use a number of different specifications estimated both for the whole period and for the two subperiods in order to have robust results.

We model discrete changes in the interest rate by the following GARCH(1,1) specification:

$$(17) \qquad \Delta\, r(t) = \alpha_0 + \alpha_1 \text{spread}(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0,\, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\, V(t-1) + \beta_3\, \epsilon^2(t-1).$$

The mean of the change in the one-month interest rate was modelled by the spread of the two- and one-month interest rates at time t. This is motivated by the simple expectations hypothesis which states that the difference between the long- and short-term interest rates forecasts the future movements in the short-term interest rates. See Murto (1990) for evidence in favour of the expectations hypothesis with respect to the Finnish money market.

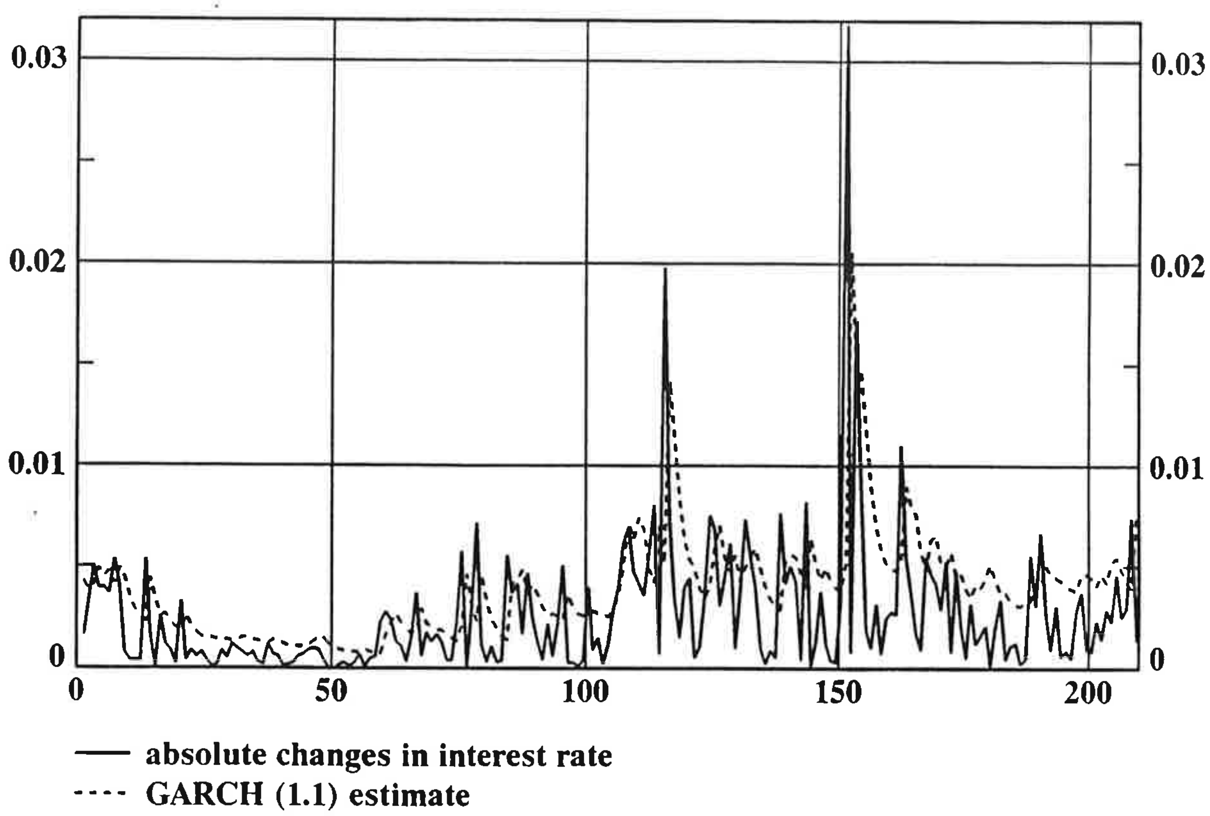
The conditional variance is modelled by the GARCH(1,1) model. The use of the GARCH model is motivated by the extensive evidence on conditional heteroscedasticity of different financial time series. In addition to the lagged conditional volatility and the lagged error term in the variance equation, we also incorporate a term for the level of the interest rate. This implies that the variance of changes in the interest rate increases with the interest rate level.

Table 1 presents the results of estimating model (17) on weekly data. Table 1 incorporates parameter estimates, the corresponding t-statistics and the pattern of diagnostic tests.

The estimated GARCH model appears to provide a reasonably good representation of the data. All estimated parameters, except the constant term in the mean equation, are significantly different from zero at the usual significance levels. GARCH effects, in particular are apparent.

Figure 2 gives a plot of the estimated weekly conditional variance and ex post volatility measure series. The ex post volatility is measured by the absolute weekly changes in the one-month interest rates.

Figure 2. Ex post- and conditional volatility



To assess the general descriptive validity of the model, a series of misspecification tests was performed. We test for $\epsilon^2(t-2)$ as an additional variable in the variance equation. The result is negative. The 1-degree-of-freedom LM test statistics is 0.33. The 5-degrees-of-freedom likelihood ratio test statistic for the null hypothesis that $\Delta\ r(t)$ is generated by a normal model with constant mean and variance is 110.44, which is significant at the usual significance levels. Ljung-Box statistics for 12-order serial correlations are compute for the levels and squares of the normalized residuals. The statistics are 16.23 and 4.66 respectively. They are insignificant at the 5 % level suggesting that there are little unexplained time dependence in the data. However, the coefficient of skewness and the coefficinet of kurtosis of normalized residuals, 1.32 and

8.93 respectively, indicate slight evidence of misspecification of the conditional distribution.

In addition to using GARCH(1,1) specifications, presented by equation (17), when testing the LS model, we also use several other conditional volatility specifications. Longstaff and Schwartz model discrete changes in the riskless rate by the GARCH(1,1)-M process with interest rate levels both in the mean and variance equation. This specification is implied by equation (13). The results from the corresponding specification is presented in appendix. The results are not favorable for this specification. The conditional variance term in the mean equation is insignificant. The sum of the β_2 and β_3 is greater than one, possibly implying an integrated GARCH process.[1]

Also other GARCH- and ARCH-specifications were estimated and used when testing the Longstaff and Schwartz term structure model. The robustness of the GMM test results with respect to changes in the volatility measure is examined in section 4.3.

The results from the subperiods are presented in tables 2 and 3. These results are notably weaker in the sense that many parameters are now insignificant.

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[1] We also estimated the same specification without the conditional variance term in the mean equation. In this case the sum of β_2 and β_3 was 0.964, but the interest rate term in the mean equation was insignificant.

Table 1. GARCH(1,1)-specification

$$\Delta\ r(t) = \alpha_0 + \alpha_1 \text{spread}(t-1) + \epsilon(t),$$

$$\epsilon(t) \sim N(0,\ V(t)),$$

$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\ V(t-1) + \beta_3\ \epsilon^2(t-1)$$

Parameter estimates		Diagnostics	
α_0	-0.0001 (-0.71)	loglik.	900.26
α_1	0.7884 (4.49)	LM1	0.33
β_0	-0.0000 (-3.98)	LR5	110.44
β_1	0.0001 (4.16)	Skew	1.32
β_2	0.4836 (9.28)	Kurto	8.93
β_3	0.4867 (4.79)	Q12	16.23
		QS12	4.66

LM1 = a 1 degree of freedom LM test statistic (Chi-square-1) for adding a one order higher ARCH term in the variance equation,
LR5 = a 5 degrees of freedom likelihood ratio test statistic for the null hypothesis that the endogenous variable follows a normal model with constant mean and variance,
Skew = coefficient of skewness for normalized residuals,
Kurto = coefficient of kurtosis for normalized residuals,
Q12 = point for Ljung-Box(12) for normalized residuals,
QS12 = point for Ljung-Box(12) for squares of normalized residuals.
t-values in parentheses

Table 2. GARCH(1,1)-specification. The first subperiod: 06.01.1987 - 21.02.1989.

$$\Delta\ r(t) = \alpha_0 + \alpha_1 \text{spread}(t-1) + \epsilon(t),$$

$$\epsilon(t) \sim N(0,\ V(t)),$$

$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\ V(t-1) + \beta_3\ \epsilon^2(t-1)$$

Parameter estimates		Diagnostics	
α_0	-0.0001 (-0.60)	loglik.	532.40
α_1	0.3212 (1.23)	LM1	0.95
β_0	-0.0000 (-2.12)	LR5	38.05
β_1	0.0002 (2.34)	Skew	0.70
β_2	0.0918 (0.72)	Kurto	6.39
β_3	0.7766 (3.34)	Q12	14.76
		QS12	16.86

LM1 = a 1 degree of freedom LM test statistic (Chi-square-1) for adding a one order higher ARCH term in the variance equation,
LR5 = a 5 degrees of freedom likelihood ratio test statistic for the null hypothesis that the endogenous variable follows a normal model with constant mean and variance,
Skew = coefficient of skewness for normalized residuals,
Kurto = coefficient of kurtosis for normalized residuals,
Q12 = point for Ljung-Box(12) for normalized residuals,
QS12 = point for Ljung-Box(12) for squares of normalized residuals.
t-values in parentheses

Table 3. GARCH(1,1)-specification. The second subperiod: 28.02.1989 - 12.02.1991.

$$\Delta\ r(t) = \alpha_0 + \alpha_1 spread(t-1) + \epsilon(t),$$

$$\epsilon(t) \sim N(0,\ V(t)),$$

$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\ V(t-1) + \beta_3\ \epsilon^2(t-1)$$

Parameter estimates		Diagnostics	
α_0	0.0005 (1.05)	loglik.	375.30
α_1	0.8307 (3.03)	LM1	8.97
β_0	0.0000 (1.61)	LR4	30.77
β_1	-0.0003 (-1.32)	Skew	1.59
β_2	0.1110 (0.96)	Kurto	11.98
β_3	0.6485 (2.22)	Q12	18.88
		QS12	18.64

LM1 = a 1 degree of freedom LM test statistic (Chi-square-1) for adding a one order higher ARCH term in the variance equation,
LR5 = a 5 degrees of freedom likelihood ratio test statistic for the null hypothesis that the endogenous variable follows a normal model with constant mean and variance,
Skew = coefficient of skewness for normalized residuals,
Kurto = coefficient of kurtosis for normalized residuals,
Q12 = point for Ljung-Box(12) for normalized residuals,
QS12 = point for Ljung-Box(12) for squares of normalized residuals.
t-values in parentheses

4.2 The Generalized Method of Moments Estimator

In comparison to the vast number of works in which the traditional expectation hypothesis has been investigated empirically, the number of empirical applications of the continuous time models of the term structure is still manageable. Some of the earlier research has used two-state methods in estimation. They have started by estimating parameters of the process followed by the short-term interest rate using the time series of a short-term rate, and estimating the utility dependent parameter on the basis of term structure equation. Part of the research concentrates on identifying the stochastic processes of interest rates (e.g. Marsh and Rosenfeld (1983)).

We will start directly from the resulting term structure equations. The estimation procedure, the generalized method of moments (GMM) instrumental variables procedure, was set out by Hansen (1982). The procedure has been recently used in many financial market applications. For example, Hansen and Singleton (1982) used the GMM to test a consumption based CAPM. The basic idea was to apply the GMM estimator directly to orthogonality conditions implied by the first-order conditions of agents' intertemporal optimization problems.

We define the following pricing errors:

(18) $e_{it}(\Phi) = R_{it}^{O} - R_{it}^{M}$,

where R_{it}^{O} = the observed interest rate of the security i at time t,
R_{it}^{M} = model interest rate of the security i at time t and
Φ = parameters to be estimated.

If the theoretical model is correct and markets are efficient,

$E(e_{it}(\Phi^*))$ (i.e. the expected pricing errors evaluated at the true parameter value) should be zero. Then equation (18) implies that the errors must be uncorrelated with variables in the information set I(t). Thus by the law of iterated expectations, we can write:

(19) $E(e_{it}(\Phi^*)z_{jt}) = 0,$

where z_{jt} = instrumental variable j at time t.

A more compact way to write (19) is

(20) $E(e_t(\Phi^*) \otimes z_t) = 0$ or

(21) $E(g_t(\Phi^*)) = 0,$

where $e_t(\Phi^*)$ is M × 1 column vector comprised of the $e_{it}(\Phi^*)$ and $g_t(\Phi) = e_t(\Phi) \otimes z_t$.

Condition (21) is the basis for the GMM technique. When the above equation is correct, the sample moment

(22) $$g_T(\Phi) = T^{-1} \sum_{t=1}^{T} g_t(\Phi),$$

should be close to zero when evaluated at $\Phi = \Phi^*$. It is therefore reasonable to estimate Φ^* by choosing Φ_T so that $g_T(\Phi)$ is close to zero, i.e., by solving

(23) $\min_b g_T(\Phi)' W_T g_T(\Phi),$

where W_T is the inverse of a consistent estimate of the covariance matrix.

The dimension of the right-hand side of (22) is MR, where R is the number of components of the instrument vector. If MR exceeds p, which is the number of parameters to be estimated, then $g_T(\Phi)$

cannot in general equal zero for any Φ, and only asymptotically will the minimized value of the objective function vanish. Hansen (1982) showed that under regular conditions the estimator Φ that minimized (23) is a consistent estimator Φ^*.

Furthermore, the statistic T $J_T(\Phi)$, where $J_T(\Phi)$ is the objective function in equation (23), is asymptotically distributed as a chi-squared random variable with degrees of freedom equal to the dimension of $g_T(\Phi)$ less the number of estimated parameters. This statistic provides a test of the overidentifying restrictions.

When testing the models the pricing errors are hypothesized to have a mean zero and to be uncorrelated with the instruments. The next question is to choose the instruments. In general, using theoretically optimal instruments is difficult. In practice, the most common approach has been to use the independent variables as instrumental variables. This is basically the approach we follow below. Furthermore, we use a number of other instrumental variables as suggested by Bossaerts and Hillion (1989).

There are no simulation studies on small sample properties of the J-statistics concerning testing of the continuous-time term structure models. Tauchen (1986) and Korcherlakota (1990) provide simulation evidence for testing the Euler restrictions. Tauchen (1986) finds that the test statistics perform well with relative few annual observations, even though there is a slight tendency to reject the model too infrequently. Using a different set of parameter values, however, Korcherlakota (1990) finds that J-statistics tend to overreject the model when it is true small samples. Furthermore, the estimators with multiple instruments perform poorly in small samples.

4.3 Tests of a Two-Factor Model

In this section we test the two-factor term structure model and estimate the model parameters using Hansen's generalized method of moments. We use weekly data from 06.01.1987 to 12.02.1991.

The one-month interest rate is used as the short rate. As a variance we use forecasts for next period's variance acquired from GARCH specifications.

We test a two-factor model with a difference form as follows:

(24) $\varepsilon(t,\tau) = \Delta R(t,\tau) - a(\tau) + b(\tau)\ \Delta\ r(t) + c(\tau)\ \Delta\ V(t),$

where Δ =the difference operator,
$a(\tau) = 0,$
$b(\tau) = -\ C(\tau)/\tau,$
$c(\tau) = -\ D(\tau)/\tau$ and
$R(t,\tau)$ = observation of the τ-months interest rate at time t.

The instrumental variables set consists both of model variables and of other predetermined variables. The instruments are the domestic short-term interest r(t), the conditional volatility V(t), the average international interest rates $r^f(t)$, the difference between domestic and international interest rates $r^d(t)$, and bank's net call money deposits dep(t). All variables are in first differences. In all cases the instrumental variable set includes a constant.

Domestic short-term interest rate and conditional volatility are obvious choices for the instruments. These are used in the same form as when used as state variables in the two-factor model. In addition to the model variables we use some other predetermined variables. The first is the average international one-month interest rate weighted in accordance with the Bank of Finland currency index. The influence of foreign interest rates are strong, particularly since the Bank of Finland has pegged the external value of the markka against a currency basket. The second variable which is not one of the model variables is the interest rate differential between the domestic one-month interest rates and average international one-month interest rates. It captures among others expected devaluations. The last variable is the change in the banks' aggregate net call money deposits. We use the previous friday's observations to ensure that the call money position is known by the market participant's. Murto (1990) investigates the relationship between the term structure and banks' call money position. The summary statistics for the instrumental variables are presented in table 4.

We will present results both including and excluding the current value of the instrument. Ferson and Constantinides (1991) notice that the measurement errors and the interpolation when calculating monthly consumption expenditures can cause spurious correlation between instruments and error terms when testing Euler equations. To avoid erroneous conclusions they also use instrumental variables which are lagged twice instead of once in relation to the asset return in the Euler equations.

The spurious correlations due to measurement errors are less likely to happen with interest rate series than with the macroeconomic data. One possible source for the measurement errors is the updating procedure for the bid rates. We assume that the respective bid rates are updated to correspond to true transaction prices. Lags in updating can produce non-syncronities in the data series. The problem is, however, expected to be small especially with domestic interest rates series. The average international interest rates is a weighted rate over ten foreign currencies, which means that the problem can be more serious here. Same applies to the interest rate differential. The likelihood for the measurement errors in the variance is high, because it is a generated variable.

When estimating the two-factor model we do not always achieve converged results. The problem is most severe with subsamples and with current values of instruments included in instrumental variable set. Furthermore the variance as instrument do not produced converged results. In the next section we estimate and test a one-factor term structure model. When estimating the one-factor model convergence problems do not appear.

Table 4. Summary statistics for instrumental variables

Variable	Mean	Std. dev.	Autocorrelations ρ_1	ρ_2	ρ_3	ρ_4
Δ r(t)	-9.7×10^{-5}	0.0043	0.04	-0.23	0.09	0.07
Δ V(t)	-1.9×10^{-7}	3.5×10^{-5}	-0.28	0.02	-0.09	-0.10
Δ r^f(t)	8.6×10^{-5}	0.0014	-0.11	0.05	-0.02	0.06
Δ r^d(t)	1.0×10^{-5}	0.0042	0.00	-0.23	0.08	0.05
Δ dep(t)	0.2896	8.7201	-0.11	-0.12	-0.08	-0.07

Correlations of the instrumental variables

	Δ r(t)	Δ V(t)	Δ r^f(t)	Δ r^d(t)	Δ dep(t)
Δ r(t)	1.00				
Δ V(t)	0.49	1.00			
Δ r^f(t)	0.23	0.04	1.00		
Δ r^d(t)	0.94	0.48	-0.11	1.00	
Δ dep(t)	-0.38	-0.22	0.04	-0.40	1.00

The results obtained from the orthogonality conditions are presented in table 5. The table displays parameter estimates, their t-statistics, the GMM minimized criterion (χ^2) value, its associated degrees of freedom and the p-value. The variance estimates were acquired from the GARCH(1,1) specification presented in equation (16) in section 4.1.

The test of overindentifying restrictions, the χ^2 statistic, clearly rejects the two-factor term structure model with domestic interest rates or interest rate differentials as instruments. P-values for the goodness-of-fit tests are very close to zero with these instruments. The values of the goodness-of-fit test are bigger when we include the current observations in our instrumental variable set than when we exclude them.

In contrast the model is not rejected by the goodness-of-fit test using the international interest rates or banks' call money debt as instruments. The p-value of the goodness-of-fit test varies from 0.531 to 0.987 for these instruments.

In most cases the parameter estimates are not significantly different from zero. The estimated t-values for parameters are biggest when we use current and lagged values of domestic short-term interest rate as instruments. The parameter estimates vary with the different instruments.

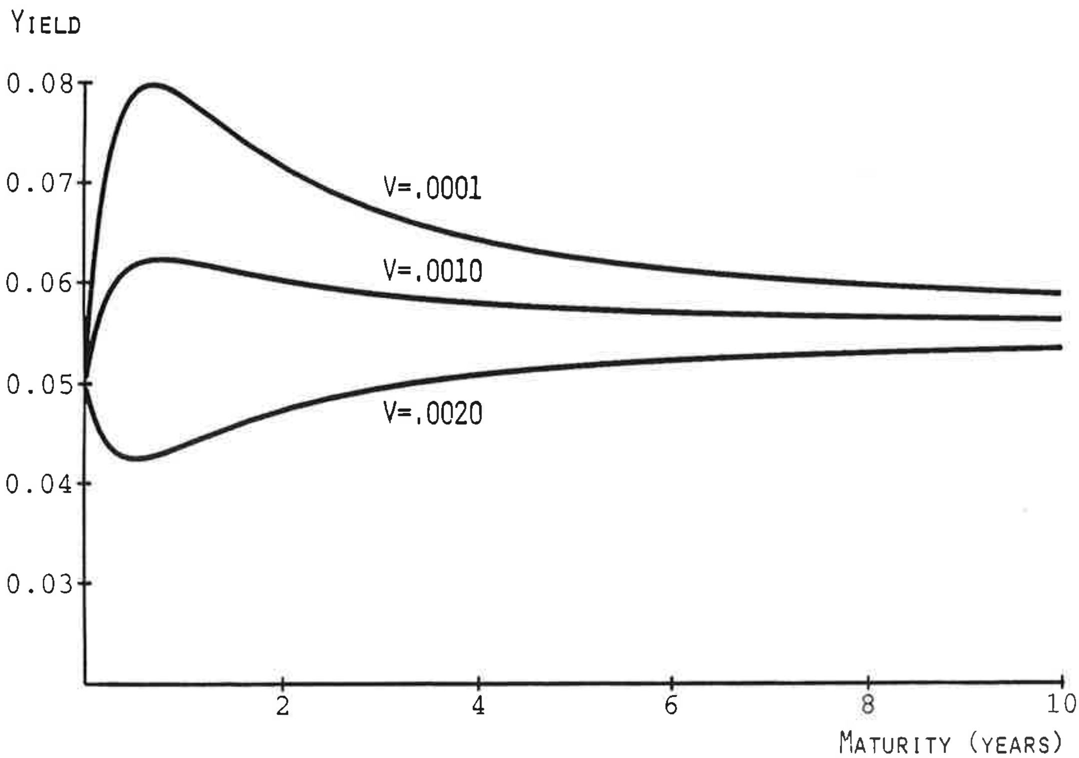
Table 5. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. 209 weekly observations from 06.01.1987 to 12.02.1991. Volatility estimates are produced by the following GARCH(1,1) specification:

$$\Delta r(t) = \alpha_0 + \alpha_1 \mathrm{spread}(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2 V(t-1) + \beta_3 \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
-0.0080	0.0124	1.3209	5.4722	80.97	21	0.000
(-1.06)	(2.35)	(2.17)	(3.86)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
-0.0042	0.0008	4.2024	6.6463	44.47	21	0.002
(-0.36)	(0.06)	(0.85)	(0.94)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3						
-0.0483	0.0161	-1.0601	3.2167	19.85	21	0.531
(-0.46)	(0.67)	(-0.66)	(1.13)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4						
-0.1034	0.0003	-2.2935	3.0822	9.07	21	0.987
(-0.38)	(0.30)	(-1.01)	(1.19)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3						
-0.0201	0.0121	0.9156	4.8356	73.31	21	0.000
(-0.95)	(1.97)	(1.02)	(4.51)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4,5						
-0.0162	-0.0007	1.7073	8.4249	52.03	26	0.002
(-1.51)	(-0.13)	(1.37)	(2.79)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4						
-0.0107	0.0085	1.1246	14.2333	19.40	21	0.559
(-0.82)	(0.52)	(0.98)	(0.80)			

In figure 3 we display the term structure of interest rates given different volatility levels. We use the following parameter values: α = -0.008, β = 0.0124, δ = 1.3209, ν = 5.4722, γ = 6.56113 and η = 41.8827. The first four parameter values correspond to the parameter estimates obtained by using current and lagged domestic short-term interest rates as instruments. The last two parameter were acquired by using the unconditional distributions for r and V. The instantanous interest rate is 0.10. Given that we use the point estimate of α which is negative, we must interpret the following results with caution.[2]

<u>Figure 3.</u> The Term Structure of Interest Rates Implied by the Two-Factor Model



[2] The value of α should be nonnegative. In table 5 the point estimates of α are negative, but they are not significantly different from zero according to the t-ratios. In the subperiods we have estimates of α, which are significantly different from zero. Then α is usually positive.

Figure 3 demonstrate that the change in the level of interest rate volatilty can have significant impacts on the yield curve. The yield curves can be downward or upward humped. The long-term interest rates approach the same asymptotic value, which is 5.5 %. This appears to be low given the sample. The asymptotic value is quite sensitive to the value of α.

In order to check the robustness of our results with respect to the variance estimates we estimated and tested the two-factor model with several other volatility estimates. The results appear to be insensitive to the volatility estimates produced by the different GARCH specifications. In appendix we report results using a GARCH(1,1)-M specification with interest rate level both in the mean and variance equations. This is the specification consistent with the LS-model. Both the parameter estimates and values of the goodness-of-fit statistics are very close to the results presented above.

The LS model was also estimated and tested with respect to the same two subsamples as in section 4.1. The first subperiod starts at the beginning of 1987 and ends on 21.02.1989. The second subperiod is from 28.02.1989 to 12.02.1991.

As in the previous whole period estimations, the model is estimated with several different volatility estimates. We report results using conditional volatility estimates acquired by GARCH(1,1) specification estimated for both the entire period and the subperiods.

Tables 6 and 7 present the results for the subperiods when GARCH(1,1) is estimated from the whole period. Now the results are slightly more favourable to the LS model than in the entire period.

Table 6. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. 112 weekly observations from 06.01.1987 to 21.02.1989. The variance estimates are acquired by the following GARCH(1,1) specification estimated from the whole period:

$$\Delta\, r(t) = \alpha_0 + \alpha_1 \mathrm{spread}(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0,\, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\, V(t-1) + \beta_3\, \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
141.7737	-3.7470	-70.9718	2.8991	35.19	21	0.027
(0.02)	(-0.12)	(-0.01)	(0.37)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
3.8821	0.0543	-26.0263	3.3504	23.56	21	0.315
(0.59)	(0.23)	(-1.84)	(0.89)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4						
-0.0004	0.2179	2.9056	-4.4703	14.77	21	0.834
(-0.10)	(0.08)	(1.02)	(-0.17)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4						
0.5002	0.0028	-14.2372	3.9323	33.14	21	0.045
(3.72)	(1.34)	(-10.52)	(1.24)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4						
-19.0817	6.5165	6.4384	-1.0157	17.56	21	0.677
(-0.01)	(0.08)	(0.02)	(-0.02)			

Table 7. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. 97 weekly observations from 28.02.1989 to 12.02.1991. The variance estimates are acquired by the following GARCH(1,1) specification estimated in the whole period:

$$\Delta r(t) = \alpha_0 + \alpha_1 spread(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2 V(t-1) + \beta_3 \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
-0.0045	0.0130	1.5074	7.1596	57.08	21	0.000
(-1.18)	(2.85)	(3.41)	(3.97)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
-0.0029	0.0098	3.3134	11.1706	32.14	21	0.057
(-0.49)	(0.89)	(1.69)	(1.34)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3						
-0.0449	0.0158	-0.9736	4.3202	17.07	21	0.707
(-0.59)	(0.78)	(-0.88)	(1.68)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4						
-0.0396	0.0036	-0.9717	2.9336	11.62	21	0.960
(-0.48)	(0.28)	(-0.44)	(1.16)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3						
0.0138	-0.0088	6.5892	1.4423	53.86	21	0.000
(2.30)	(-1.29)	(4.02)	(2.62)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4						
-0.0078	0.0085	2.8006	13.7778	30.79	21	0.077
(-1.20)	(0.85)	(1.73)	(1.44)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4						
-0.0148	-0.0002	2.1129	9.0500	19.79	21	0.535
(-0.77)	(-0.02)	(0.53)	(1.67)			

Table 8. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. 112 weekly observations from 06.01.1987 to 21.02.1989. The variance estimates are acquired by the following GARCH(1,1) specification estimated in the subperiod:

$$\Delta r(t) = \alpha_0 + \alpha_1 spread(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2 V(t-1) + \beta_3 \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
141.7576	-3.7452	-70.9731	2.8988	35.19	21	0.027
(0.02)	(-0.12)	(-0.01)	(0.37)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
4.1564	0.0672	-24.7843	3.3891	22.97	21	0.346
(0.60)	(0.23)	(-1.82)	(0.86)			
Instruments: a constant and Δ $r^f(t-i)$; i = 1,2,3,4						
-0.0008	0.2881	2.9447	-4.2263	14.94	21	0.826
(-0.10)	(0.08)	(1.02)	(-0.17)			
Instruments: a constant and Δ $r^d(t-i)$; i = 0,1,2,3,4						
0.6006	0.0000	-17.9637	2.1112	43.80	26	0.016
(3.91)	(0.15)	(-5.07)	(6.06)			
Instruments: a constant and Δ $r^d(t-i)$; i = 1,2,3,4						
0.5518	0.0013	-17.7445	4.2567	26.56	21	0.186
(4.07)	(0.86)	(-6.43)	(1.26)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4						
-19.5451	6.4954	6.5125	-0.9952	17.55	21	0.678
(-0.01)	(0.08)	(0.02)	(-0.02)			

Table 9. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. 97 weekly observations from 28.02.1989 to 12.02.1991. The variance estimate is acquired by the following GARCH(1,1) specification estimated in the subperiod:

$$\Delta r(t) = \alpha_0 + \alpha_1 spread(t-1) + \epsilon(t),$$
$$\epsilon(t) \sim N(0, V(t)),$$
$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2 V(t-1) + \beta_3 \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
-0.0139	0.0199	1.4825	7.1269	57.72	21	0.000
(-1.83)	(2.26)	(3.34)	(4.01)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
-0.0114	0.0156	3.3095	11.6231	32.86	21	0.048
(-0.99)	(0.67)	(1.66)	(1.27)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3						
-0.0808	0.0274	-0.9484	4.4175	17.15	21	0.702
(-0.56)	(0.62)	(-0.98)	(1.66)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4						
-0.0768	0.0051	-1.1120	3.0739	11.60	21	0.950
(-0.47)	(0.22)	(-0.57)	(1.21)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3						
-0.0086	1.1741	4.4677	-4.9453	52.97	21	0.000
(-1.05)	(5.24)	(10.63)	(-4.97)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4						
-0.0203	0.0130	2.8118	13.8930	31.38	21	0.066
(-1.59)	(0.64)	(1.66)	(1.38)			

In the first subperiod we reject the model by the goodness-of-fit test only when we use current and lagged values of domestic short-term interest rates or lagged values of interest rate as instruments. The p-values of the goodness-of-fit tests are 0.027 and 0.045 respectively. Otherwise the model is not rejected. In this period, converged results were difficult to achieve with many instruments.

The parameter estimates vary in the wide range. In most cases parameter estimates are not however significantly different from zero.

In the second subperiod we reject the model at the 5 percent significance level only when we use domestic short-term interest rates or interest rate differentials including the current values as instruments. However with same instruments excluding the current values the p-values are very close to 0.005. With other instruments and lags we do not reject the two-factor term structure model.

The parameter estimates differ between different subperiods. For example, in the first subperiod δ is estimated to be -14.74 ; in the second subperiod it is estimated to be 2.80 with interest rate differentials as instruments. This confirms our assumption that there have been some structural changes in the interest rate formation that can not be captured solely by different volatility estimates. If there are some changes in the process governing the short-term interest rate movements, this would also affect the parameter values in equation (13) and (14) and thus in equation (16).

Tables 8 and 9 present estimation results from the two subperiods when the GARCH specification was estimated independently for both subperiods. In general the results do not differ appreciably from the results presented in tables 6 and 7. The values of goodness-of-fit statistics, χ^2, and the parameter estimates do not vary with the different variance estimates. The most notable exception is when we use changes in the interest rates differentials as instruments. Using only the lags of interest rate differentials the p-value of goodness-of-fit statistics is 0.045 with GARCH volatility estimates acquired from the whole period estimation. The respective p value is 0.186 with GARCH volatility estimates acquired by the model estimated in the subperiod. In other cases, the results are robust with respect to the variance estimates.

Also other GARCH and ARCH models were used, and the results appear to be quite robust as regards the volatility estimates generated different specifications. This confirms our conclusion that the results are not particularly sensitive to the specification of the conditional volatility.

We conclude that the two-factor model is clearly rejected when we use data from the whole sample. When we use data from the subperiods the rejection relies in most cases on the use of the most current values of instruments. Given our small sample in the subperiods we must however interpret these results with caution. Furthermore there are also cases when we reject the model in the subperiods using only lagged values of instruments.

4.4 Tests of a One-Factor Model

Above we estimated and tested a two-factor term structure model. In this section we estimate a one-factor model and ask whether the second factor does make a difference. Can we reject a one-factor model for the same estimation periods ?

One of the most commonly cited one-factor term structure model is the Vasicek (1977) partial equilibrium model.[3] The factor is

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[3] Another often cited one-factor term structure model is the CIR model, which is nested to the LS two factor model. However, the estimation of the CIR model in the difference form does not produce any satisfactory results, so we report only results for the Vasicek model. The results from the level data were similar in both the Vasicek and CIR model cases.

the instantaneous spot rate, which is assumed to follow mean-reverting diffusion process. The market price of risk is assumed to be a constant (see also Murto (1991)).

According to the Vasicek model the term structure equation can be written as:

$$(25)\qquad R(\tau) = R(\infty) + (r-R(\infty))\frac{1}{\alpha\tau}(1-e^{-\alpha\tau}) + \frac{\sigma^2}{4\alpha^3\tau}(1-e^{-\alpha\tau})^2,$$

where $R(\infty)$ is the yield on a very long-term bond, as $\tau \to \infty$, is the following:

$$(26)\qquad R(\infty) = r_0 + \sigma q/\alpha - \tfrac{1}{2}\,\sigma^2/\alpha^2.$$

This can be estimated in the same manner as the LS model. We estimate the Vasicek model with the same data for the same periods.

The results from the whole period and two subperiods are reported in tables 10 - 12. As above the tables display parameter estimates, their t-statistics, the GMM minimized criterion (χ^2) value, its associated degrees of freedom and the p-value. We report results using five different instrumental variables with both including and excluding the current variables. In addition to the variables used in estimating and testing the two-factor term structure model we are now also able to present results using conditional variance as an instrumental variable.

The Vasicek model is clearly rejected with respect to the whole period, as was the LS model. The one-factor term structure is rejected by the goodness-of-fit test when using domestic short-term interest rates, interest rate differentials or variance as instrumental variables. The model is rejected independently whether or not we include the current values in our instrumental variable set. The point estimate of α varies from 1.15 to 9.74.

Table 11 presents results from the first subperiod. The one-factor model is rejected by the goodness-of-fit test with the same instrumental variables as above when including the current values of the instrumental variables. In addition the model is rejected excluding the current values with interest rate differentials. This corresponds to the results obtained in estimating and testing the two-factor term structure model.

The results from the second subsample are presented in table 12. The Vasicek term structure model is rejected for domestic short-term interest rates, interest rate differentials and variance including and excluding the current values as instrumental variables. The corresponding p values are very close to zero. With other instruments the model is not rejected.

The Vasicek one-factor term structure model is therefore clearly rejected. Our conclusion can be based on more systematic results than in the case of the two-factor model. The estimation of the one-factor factor model was more realiable; we achieve converged results with all instruments.

Table 10. GMM tests of the cross-sectional restriction on weekly yield changes implied by the one-factor model.

Parameter estimate α	Goodness-of-fit χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3			
3.5059	94.79	24	0.000
(13.24)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4			
6.0983	54.45	24	0.000
(4.06)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3			
1.1528	28.63	24	0.234
(3.11)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4			
2.0689	19.16	24	0.743
(1.39)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3			
3.6877	81.59	24	0.000
(12.67)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4			
7.0003	53.43	24	0.001
(4.32)			
Instruments: a constant and Δ dep(t-i); i = 0,1,2,3			
5.7679	27.37	24	0.288
(6.37)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4			
1.9418	25.81	24	0.363
(2.77)			
Instruments: a constant and Δ V(t-i); i = 0,1,2,3			
6.3674	91.60	24	0.000
(9.66)			
Instruments: a constant and Δ V(t-i); i = 1,2,3,4			
9.7434	83.70	24	0.000
(4.08)			

Table 11. GMM tests of the cross-sectional restriction on weekly yield changes implied by the one-factor model. 112 weekly observations from 06.01.1987 to 21.02.1989.

Parameter estimate α	Goodness-of-fit χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3			
1.9563	56.23	24	0.000
(8.04)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4			
7.2072	31.76	24	0.133
(1.18)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3			
1.1321	20.01	24	0.696
(1.34)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4			
2.8049	15.61	24	0.902
(1.16)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3			
2.1592	45.40	24	0.005
(6.97)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4			
4.8878	39.96	24	0.022
(1.17)			
Instruments: a constant and Δ dep(t-i); i = 0,1,2,3			
3.5398	31.40	24	0.143
(3.15)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4			
1.2906	17.62	24	0.821
(1.39)			
Instruments: a constant and Δ V(t-i); i = 0,1,2,3			
0.6179	46.91	24	0.003
(0.79)			
Instruments: a constant and Δ V(t-i); i = 1,2,3,4			
7.9879	33.23	24	0.099
(0.97)			

Table 12. GMM tests of the cross-sectional restriction on weekly yield changes implied by the one-factor model. 97 weekly observations from 28.02.1989 to 12.02.1991.

Parameter estimate	Goodness-of-fit		
α	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3			
4.4849	72.70	24	0.000
(12.12)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4			
5.1570	44.26	24	0.007
(3.86)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3			
1.2930	26.91	24	0.309
(3.24)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4			
2.4240	19.93	24	0.701
(1.57)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3			
4.5232	64.17	24	0.000
(11.75)			
Instruments: a constant and Δ r^d(t-i); i = 1,2,3,4			
6.2057	43.52	24	0.009
(4.28)			
Instruments: a constant and Δ dep(t-i); i = 0,1,2,3			
7.3667	21.15	24	0.630
(5.98)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4			
8.4870	22.14	24	0.571
(3.25)			
Instruments: a constant and Δ V(t-i); i = 0,1,2,3			
6.2071	63.76	24	0.000
(9.56)			
Instruments: a constant and Δ V(t-i); i = 1,2,3,4			
9.9387	56.90	24	0.000
(3.76)			

5. CONCLUSIONS

The aim of this paper was to estimate and test continuous-time term structure models in the Finnish money market data. The main focus was on testing the two-factor term structure model developed by Longstaff and Schwartz (1991), where the state variables are short-term interest rates and the volatility of the short-term interest rates. In addition to the two-factor model we estimated and tested the Vasick (1977) one-factor term structure model.

In order to estimate the two-factor model we needed the estimates of volatility of the short-term interest rates. These were acquired by estimating the GARCH(1,1) process for one-month interest rate changes. We used the specification implied by the expectations hypothesis. We checked the robustness of our results also by estimating and testing the term structure model with different volatility estimates. For example, results were essentially the same when we used volatility estimates acquired by the GARCH(1,1)-M specification implied by the Longstaff-Schwartz term structure model instead of the GARCH(1,1) specification implied by the expectations hypothesis.

The two-factor model was rejected when estimated and tested over the entire sample. The sample was also divided into two subperiods, before and after the revaluation of the markka. The subsample results were slightly more favourable for the two-factor model, but we still reject the model when the current values of instrumental variables are included in the instrumental variable set. The one-factor model was rejected for both the entire period and subsamples.

Even thought the evidence in favour of the two-factor model was not overwhelming with respect to the Finnish money market, the Longstaff and Schwartz term structure model can have important practical implications. The rejection of the model does not necessary imply that it is not useful for hedging purposes.

Under the Longstaff and Schwartz term structure model we can assess interest rate risks arising from two factors: instantaneous interest rate changes and volatility changes. This feature can be utilized in immunization and calculation of stochastic duration measures, see more in Murto (1991).

APPENDIX

Table A1. GARCH(1,1)-M specification

$$\Delta r(t) = \alpha_0 + \alpha_1 r(t-1) + \alpha_2 V(t) + \epsilon(t),$$

$$\epsilon(t) \sim N(0, V(t)),$$

$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2 V(t-1) + \beta_3 \epsilon^2(t-1).$$

Parameter estimates		Diagnostics	
α_0	-0.004 (-2.52)	loglik.	892.05
α_1	0.043 (2.58)	LM1	0.60
α_2	-18.761 (-0.77)	LR5	94.02
β_0	-0.000 (-3.19)	Skew	1.51
β_1	0.000 (3.29)	Kurto	9.40
β_2	0.347 (5.46)	Q12	12.44
β_3	0.767 (4.95)	QS12	3.48

LM1 = a 1 degree of freedom LM test statistic (Chi-square-1) for adding a one order higher ARCH term in the variance equation,
LR5 = a 5 degrees of freedom likelihood ratio test statistic for the null hypothesis that the endogous variable follows a normal model with constant mean and variance,
Skew = coefficient of skewness for normalized residuals,
Kurto = coefficient of kurtosis for normalized residuals,
Q12 = point for Ljung-Box(12) for normalized residuals,
QS12 = point for Ljung-Box(12) for squares of normalized residuals.
t-values in parentheses

Table A2. GMM tests of the cross-sectional restriction on weekly yield changes implied by the two-factor model. Volatility estimates are acquired by the following GARCH(1,1)-M specification:

$$\Delta\, r(t) = \alpha_0 + \alpha_1 r(t-1) + \alpha_2 V(t) + \epsilon(t),$$

$$\epsilon(t) \sim N(0,\ V(t)),$$

$$V(t) = \beta_0 + \beta_1 r(t-1) + \beta_2\, V(t-1) + \beta_3\, \epsilon^2(t-1)$$

Parameter estimates				Goodness-of-fit		
α	β	δ	ν	χ^2	d.o.f.	p-value
Instruments: a constant and Δ r(t-i); i = 0,1,2,3						
-0.0109	0.0227	1.4514	5.6521	80.12	21	0.000
(-1.03)	(2.52)	(2.58)	(3.79)			
Instruments: a constant and Δ r(t-i); i = 1,2,3,4						
-0.0060	0.0021	4.4594	6.5116	44.44	21	0.002
(-0.22)	(0.07)	(0.73)	(0.78)			
Instruments: a constant and Δ r^f(t-i); i = 0,1,2,3						
-0.1010	0.0245	-1.2724	2.9142	19.68	21	0.542
(-0.50)	(0.93)	(-0.80)	(1.25)			
Instruments: a constant and Δ r^f(t-i); i = 1,2,3,4						
-0.1346	-0.0029	-2.0824	2.9687	8.76	21	0.991
(-0.50)	(-0.18)	(-1.05)	(1.20)			
Instruments: a constant and Δ r^d(t-i); i = 0,1,2,3						
-0.0168	0.0221	1.4356	5.3094	72.24	21	0.000
(-0.98)	(2.11)	(1.92)	(3.83)			
Instruments: a constant and Δ dep(t-i); i = 1,2,3,4,5						
-0.0188	0.0215	1.1400	13.5630	19.98	21	0.523
(-0.85)	(0.73)	(1.04)	(0.88)			

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E L I N K E I N O E L Ä M Ä N T U T K I M U S L A I T O S (ETLA)
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
LÖNNROTINKATU 4 B, SF-00120 HELSINKI

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