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### **UNDERVALUATION OF CORPORATE SHARES REVISITED: A NOTE**

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**ABSTRACT:** The paper shows that the well-known result on undervaluation of corporate shares is inapplicable for the tax systems prevailing in a majority of the OECD-countries. It is proved that the relative undervaluation is determined by the tax rate on profits and the extent to which dividends receive preferential tax treatment with respect to interest income.

**KEY WORDS:** undervaluation of shares, taxation



## I Introduction

It has been one of the most celebrated results in the theory of capital income taxation ever since the late 1970's that the equilibrium stock market valuation of shares of incorporated firms should fall short of the replacement value of their assets to the extent that the overall taxation of distributed and undistributed profits is non-harmonized. The mechanism behind this result, which was independently reached in a number of studies (Bergström and Södersten (1977), Auerbach (1979), Bradford (1979), King (1977), Sinn (1987)), is as follows. One unit, say \$1, of profits retained in the firm is worth  $1-\tau_d$  for a shareholder when her personal (marginal) tax rate on dividends is  $\tau_d$ . Hence, if  $\tau_c$  is the tax rate at which the appreciation of the share value is taxed, appreciation may be limited to  $(1-\tau_d)/(1-\tau_c) < 1$  to provide a return of  $1-\tau_d$  on an after-tax basis.<sup>1</sup> Appreciation is less than unity if  $\tau_c < \tau_d$ .

This note proves that this result is inappropriate for most of the corporate tax systems prevailing in the OECD countries. In contrast, it is shown that the relative undervaluation is determined by the tax rate on corporate profits and the extent to which dividends receive preferential tax treatment with respect to interest income.

## II Model

A distinctive feature of the corporate taxation in most OECD-countries is the close connection between taxable income and book profits. Due to the legally required use of uniform reporting<sup>2</sup>, corporations actually face a dividend constraint limiting the maximum amount of dividends payable as taxable income (net of taxes).<sup>3</sup> The reason why this constraint has not obtained sufficient attention in the earlier literature seems to be that much of the relevant theory in this area has been developed for the Anglo-Saxian case where uniform accounting is not enforced.

The dividend constraint is especially important when the rate of tax depreciation (say  $g$ ) can exceed the economic depreciation ( $\delta$ ) as is often the case in the OECD area. Let  $K$  = capital,  $C$  = book capital and  $B$  = corporate debt stand for the state variables in an optimal control problem of the firm. Normalize all prices equal to one. Let  $I$  = the rate of investment and  $dB/dt$  = net issue of debt stand for the controls. Then the flow of dividends satisfies

$$(1) \quad D = (1-\tau)[F(K) - iB] + \tau gC + dB/dt - I \\ \leq (1-\tau)[F(K) - iB - gC]$$

where  $i$  = debt interest,  $\tau$  = tax rate on corporate profits, both distributed and undistributed, and  $gC$  = the depreciation allowance

claimed by the firm.<sup>4</sup> The standard non-arbitrage condition  $(1-\tau_d)D + (1-\tau_c)dV/dt = i(1-\tau_p)V$  with  $V$  = the market value of shares and  $\tau_p$  = the tax rate on interest income provides the valuation of shares as

$$(2) \quad V(K_t, C_t, B_t) = (1-\tau_d)/(1-\tau_c) \int_t^{\infty} D(s) \exp[-r(s-t)] ds$$

with  $r = i(1-\tau_p)/(1-\tau_c)$  as the cost of retained profits. Define the associated costate variables as  $\mu_K$ ,  $\mu_C$  and  $\mu_B$  and  $n \geq 0$  as the Kuhn-Tucker price associated with the constraint (1). The Hamiltonian then reads as

$$(3) \quad H = D(1-\tau_d)/(1-\tau_c) + \mu_K(dK/dt) + \mu_C(dC/dt) + \mu_B(dB/dt) + n[(1-\tau)(F(K) - iB - gC) - D]$$

where  $K$  and  $C$  evolve as  $dK/dt = I - \delta K$ ,  $dC/dt = I - gC$ . Assuming that the unique optimal control solution exists, it has to satisfy the following first-order conditions

$$(4a) \quad \theta - \mu_K - \mu_C - n = 0$$

$$(4b) \quad (\mu_K + \mu_C + n)(1-\tau)F_K - (r + \delta)\mu_K = -d\mu_K/ds$$

$$(4c) \quad \mu_K + \mu_C + \mu_B = 0$$

$$(4d) \quad -(\mu_K + \mu_C + n)(1-\tau)i - \mu_B r = -d\mu_B/ds$$

$$(4e) \quad (\mu_K + \mu_C)\tau g - \mu_C(r + g) - n(1-\tau)g = -d\mu_C/ds.$$

We have set  $\theta = (1-\tau_d)/(1-\tau_c)$ . The marginal valuation of equity

$q$  from (4c) is  $q = -\mu_B = \mu_K + \mu_C$ , i.e. the sum of the marginal valuation of capital and the present value of the tax shields. Then (4a) gives  $q = \theta - n$ , or  $d\mu_B/ds = dn/ds$ . From (4d) one can solve  $n = \theta[r - (1-\tau)i]/r$ . The value of  $n$  thus depends on the relative costs of financing. The tax systems in the vast majority of the OECD-countries are structured in such a way that debt is preferred to equity as the marginal source of funds, cf. Sinn (1987) fig. 3.1 or OECD (1991), Ch.2. According to the latter source, this holds for 21 out of the 24 countries included in the study. In other words, the total tax rate on retained profits exceeds the tax rate on interest income,  $(1-\tau)(1-\tau_C) < 1-\tau_P$ . This implies  $r > (1-\tau)i$  and suggests that the constraint (1) is actually binding,  $n > 0$ . Eliminating  $r$ , the marginal valuation of equity reads as  $q = (1-\tau)(1-\tau_d)/(1-\tau_p)$ .

In most of the OECD-countries interest income and dividend income are taxed at the very same rate,  $\tau_d = \tau_p$ . Hence,

$$(5) \quad q = 1-\tau.$$

In contrast, the conventional undervaluation result  $q = \theta$  is obtained only when the tax schedules allow for the existence of a Miller equilibrium, with no equilibrium tax advantage to issuing debt. In our notation, this requires that  $\tau + \tau_C(1-\tau) = \tau_P$ , which implies that  $n = 0$  and hence  $q = \theta$ . If this holds, the constraint (1) is not binding. Thus,

Theorem 1. If the financial decisions of the firms are effectively constrained by a rule which prohibits firms from paying dividends in excess of their taxable profits, the equilibrium undervaluation of equity relative to the replacement cost of assets is solely determined by the corporate tax rate  $\tau$ , and the extent to which the tax on dividends differs from that on interest income. []

Theorem 2. For the standard undervaluation result (dictated by the personal tax rates on dividends and capital gains) to hold, it is necessary that the tax rates on interest and dividends satisfy the conditions for a Miller equilibrium in capital markets . []

The conditions (4a)-(4e) can be utilized to derive the following expressions for the cost of capital:<sup>5</sup>

$$(6a) \quad F_K = (r+\delta)(i+g)/(r+g) \quad (n > 0)$$

$$(6b) \quad F_K = (r+\delta)[1 - \tau g/(r+g)]/(1-\tau) \quad (n = 0).$$

Expression (6b) is the conventional result. Equation (6a) points to neutrality of the corporation tax under debt financing at the margin as claimed by Stiglitz (1973) a long time ago. Indeed, if  $g = \delta$ , one obtains in this case  $F_K = i + \delta$ .

### III Final Remarks

Intuitively, the role of the corporation tax rate in share



valuation with a binding constraint (1) follows from the need to pay more corporate taxes if the dividend payout ratio is raised. The celebrated "trapped equity" view loses its relevance in this case: It only re-emerges if the mechanisms behind the Miller equilibrium are taken seriously. The latter is far from being warranted given the tax reforms in the 1980's which led to both substantially reduced marginal tax rates and preserved incentives for issuing debt.

## Footnotes:

1. Homogeneity of the production technology is sufficient for this result to hold for the whole share capital.
2. Separate reporting is allowed in the United Kingdom, the Netherlands, Denmark, Ireland, the United States, Canada, Australia and New Zealand. On the other hand, uniform reporting is required in Germany, France, Italy, Greece, Sweden, Norway, Finland, Spain, Switzerland, Belgium, Portugal, Luxembourg, Japan and Turkey. Cf. the OECD Report (1986).
3. Current dividends may be paid out of past accumulated profits. This can nonetheless be at most a temporary phenomenon. They cannot be "financed", for example, by borrowing.
4. In many countries (including Finland and Sweden) the tax code only states the upper limit for the rate of tax depreciation, which hence is non-predetermined. In other countries, firms have no choice as to the rate of tax depreciation. In the present model, this distinction is unessential, however, since the firm will always have an incentive to maximize its use of depreciation allowances. As proved by Sinn (1987) p.381, this is the case also when there is a rivalry between deduction of debt interest and accelerated depreciation, provided that  $\tau_p > \tau_c$ .

5. Use first (4e) to show that  $\mu_C = \theta g(1-\tau)(i-r)/r(r+g)$  when  $n > 0$  and  $\mu_C = \tau g\theta/(r+g)$  when  $n = 0$ . Use then (4a) to solve for  $\mu_K$  and substitute into (4b).

## References:

Accounting for taxes on income. Report of the subgroup on income tax accounting. OECD, Paris, 1986.

Auerbach A.J.: "Wealth maximization and the cost of capital", Quarterly Journal of Economics 94, 1979, 433-436.

Bergström V. and J.Södersten, "Double Taxation and Corporate Capital Costs", A paper presented at the Econometric Society Meeting, Vienna, September 6-9, 1977.

Bradford D.F.: "The incidence and allocation effects of a tax on corporate distributions", Journal of Public Economics 15, 1981, 1-22.

King M.A.: Public Policy and the Corporation (Chapman and Hall, London), 1977.

Sinn H.-W.: Capital Income Taxation and Resource Allocation (North-Holland, Amsterdam), 1987.

Stiglitz J.E.: "Taxation, corporate financial policy, and the cost of capital", Journal of Public Economics 2, 1973, 1-34.

Taxing profits. The experience of OECD countries. Forthcoming, OECD, Paris, 1991.



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