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Keskusteluaiheita - Discussion papers

No. 330

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PREDICTING VOLATILITY OF STOCK INDEXES FOR OPTION PRICING ON A SMALL SECURITY MARKET

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ISSN 0781-6847

01.07.1990

BERGLUND, Tom - HEDVALL, Kaj - LILJEBLOM, Eva, PREDICTING VOLATILITY OF STOCK INDEXES FOR OPTION PRICING ON A SMALL SECURITY MARKET. Helsinki : ETLA, Elinkeinoelämän Tutkimuslaitos, The Research Institute of the Finnish Economy, 1990. 20 p. (Keskusteluaiheita, Discussion Pepers, ISSN 0781-6847; no. 330).

ABSTRACT: This paper compares several alternative estimators for the volatility of a narrowly based stock market index on the Helsinki Stock Exchange using their performance in the pricing of hypothetical one-month options as evaluation criterion. Volatility estimates for daily as well as weekly returns are computed on estimation periods of half a year, one year and two years. Daily returns are shown to produce a downward biased estimate which, however, can be corrected using the procedure suggested by Cohen, Hawawini, Maier, Schwartz and Withcomb (1983). Somewhat surprisingly, the GARCH model estimated on monthly returns produces better volatility estimates than the alternative estimators on more frequent data.

KEY WORDS: Black-Scholes, GARCH, Index options, Market serial correlation

Predicting Volatility of Stock Indexes for Option Pricing on a Small Security Market

CONTENTS

1. Introduction

This paper compares the performance of several simple predictors for the volatility of a stock index or the value of a stock portfolio compiled on a small stock market. The search for a good volatility predictor for stock portfolios or indexes is justified by the fact that index options have turned out to be quite successful especially on small stock markets[1].

In evaluating index options the original Black and Scholes (1973) model or the Black (1976) version for options on futures is commonly used. This model is based on an assumption of constant volatility. However, it has been known for a long time that the volatility implicit in option prices will change substantially through time[2] . Thus, it is of considerable interest to investigate how the price history of the underlying index should be used in establishing a good volatility estimate to use in the Black and Scholes (1973) model.

There are several reasons why the solution to this problem is by no means trivial, in the sense that a mechanically calculated standard deviation computed over a sufficiently long time series of log price differences (henceforth "returns") would suffice. The most important are:

1. Portfolio returns tend to be positively serially correlated[3]. The reason for this is found in non-synchronous trading in the stocks included in the portfolio as well as in factors that create friction on the market[4]. As predicted by Cohen, Hawawini, Maier, Schwartz and Withcomb (henceforth: CHMSW) (1980) this feature is much more pronounced on a small stock market. Thus, the first order serial correlation reported by Berglund and Liljeblom (1988) for the small Helsinki Stock Exchange (HeSE) is 0.49 for returns based on a Value-Weighted Index over the 6-year period from the beginning of 1977 to the end of 1982.In contrast for the US market the first order serial correlation coefficient reported in Lo and MacKinlay (1989) for value-weighted returns based on a sample of 665 NYSE and

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[1] In Finland there are traded options exclusively based on stock indexes. In Sweden where trading in options started with stock options in 1985, index options which were launched at the end of 1987 had approximately 60 % of the trading volume in 1989.

[2] See e.g. Rubinstein (1985).

[3] Recent empirical evidence for the US market is found in Lo and MacKinlay (1989).

[4] A survey of these factors is found in CHMSW(1980).

AMEX stocks from April, 29 in 1975 through the end of 1987 is 0.146[5]. The positive serial correlation of index returns imply that the mechanically computed standard deviation will constitute a downward biased estimate of the true volatility[6].

2. The null hypothesis of conditional homoscedasticity can be rejected at least for daily returns on a stock market index as shown in several studies in which the Autoregressive Conditional Heteroscedasticity or ARCH model by Engle (1982, or its generalized form: GARCH by Bollerslev,1986), has been estimated, see e.g. Akgiray (1989)[7]. For the Finnish stock market, Booth, Hatem, Virtanen and Yli-Olli (1989) using daily data for the HeSE from 1980 to 1987 show that market returns adjusted for seasonalities exhibit a significant GARCH component in addition to first and second order serial correlation. In contrast to the extensively discussed cases of stochastic changes in the stock price volatility[8], and discrete jumps in the sample path followed by the stock price[9], the changes identified using the ARCH or GARCH model are predictable changes even on the restricted information set consisting of previous prices.

3. Finally, there are several technical problems produced by the fact that the prices on which the index is based may relate to different points in time for different days, as well as problems produced by the bid-ask spread[10]. However, these problems are more severe in the case of stock options for individual firms than in the case of index options. Since these problems are essentially idiosyncratic, they are substantially reduced by diversification in the case of index-options[11]. In the following, we will consequently neglect them in favour of those problems that fall under the first two categories.

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[5] 0.248 for the corresponding equally weighted returns, Akgiray (1989) reports a first order serial correlation coefficient of 0.201 for the value-weighted CRSP (Center for Research in Security Prices) index during 1975-80 and 0.1406 during 1981-86.

[6] See Cohen, Hawawini, Maier, Schwartz and Withcomb (1983).

[7] In the literature on options pricing Latané and Rendleman as early as 1976 observed that implied standard deviations (ISD:s) were not constant over time, which indicates that participants in the options market either did not regard the returns as constant, or the volatilities as constant, or both.

[8] See Hull and White (1987) and Johnson and Shanno (1987).

[9] See Cox and Ross (1976) and Merton (1976).

[10] See e.g. Marsh and Rosenfeld (1986).

[11] This is one of the reasons why index options are popular on markets that are plagued by thin trading.

The fact that some of the basic assumptions needed to derive the Black and Scholes (1973) formula are violated in the case of index options - i.e. the assumption of informational efficiency for the underlying instrument, and the assumption of constant volatility - seems to question the whole relevance of the Black and Scholes (1973) formula. However, as shown by Jarrow and Rudd (1982), for a large class of stochastic processes the actual value of a call can be approximated by the Black and Scholes (1973) model. They show that even substantial deviations from the Black & Scholes assumption of log-normally distributed returns can be handled by adjustments which account for the discrepancy between the actual distribution and the log-normal with respect to the first four central moments of the distribution. The crucial condition is that the assumption of risk-neutral evaluation is justified. In addition to the original Black and Scholes (1973) justification based on a risk-neutral hedge, risk neutral evaluation is justified if the risk is completely diversifiable or under certain restrictions on the combination of investors´ preferences and the return distributions of available assets.

In a recent article Jarrow and Wiggins (1989) advocate the use of the Black and Scholes (1973) model instead of option pricing models based on more realistic assumptions, the justification being that the Black and Scholes (1973) model is parsimonious. The implementation of more realistic models will require estimation of more parameters and the estimation of these parameters will introduce uncertainty into the pricing equation. The gain in unbiasedness may thus be offset by a loss of efficiency. Jarrow and Wiggins (1989) suggest that the Black and Scholes (1973) model should be used with the implicit volatility instead of the explicit volatility computed on observed prices. The reason for using the implicit volatility is that this measure incorporates the evaluation made by the market of those factors that can make the immediate future differ from the past as seen through the simple Black and Scholes (1973) model.

The present paper is not concerned with the markets ability to forecast the volatility of the index, however. The question in this paper is how the information set restricted to consist exclusively of past prices should be used to predict the future volatility of the index. On an efficient market this information as well as additonal e.g. macro economic information should be reflected in the volatilities implicit in the option prices. The objective of this paper is, more precisely, to compare different volatility predictors which can be computed on an information set restricted to the price history of the underlying instrument. The potential gain achieved by correcting the volatility estimate for the first order serial correlation observed in daily index returns as well as by the use of the GARCH model, generalized from Engle´s (1982) ARCH model by Bollerslev (1986), will be assessed.

The performance of alternative volatility estimates is evaluated using hypothetical options. The underlying index consists of the twelve most traded stocks on the Helsinki Stock Exchange in Finland for the period 1970 to 1987. The index is especially designed for the present project. The design is a compromise between the desire to have a representative index, and the attempt to avoid problems related to non-synchronous trading.

The paper is organized as follows: First the data to be used is surveyed. Next the estimators to be compared will be introduced. The impact of the estimation interval and the length of the estimation period will be investigated. The effect of correcting for the serial correlation in daily returns will also be analyzed. The final candidate will be the GARCH model. The following section discusses the criteria by which the performance of the candidates will be judged. This is done using positions in hypothetical options. The next section in the paper reports the results. Section five contains a summary and some suggestions for further research.

2. The data

The data used in the present study starts at the beginning of 1970 and ends at the end of 1987. During most of this period approximately 50 companies were listed on the Helsinki Stock Exchange. There were considerable differences in the trading frequency between these stocks, however. Only a few of the stocks were subject to transactions each trading day. To avoid the problems created by missing transaction prices we will focus on those stocks in which transactions were observed almost each day. For those days in which no transactions occurred (less than 1% of the total number of trading days) the bid quotation is used. The prices used in the study are the average of the lowest and highest trading price at day t, adjusted for splits, stock dividends and rights issues. The prices are multiplied by

$$\prod_{i=1}^{t}(1 + d_i),$$

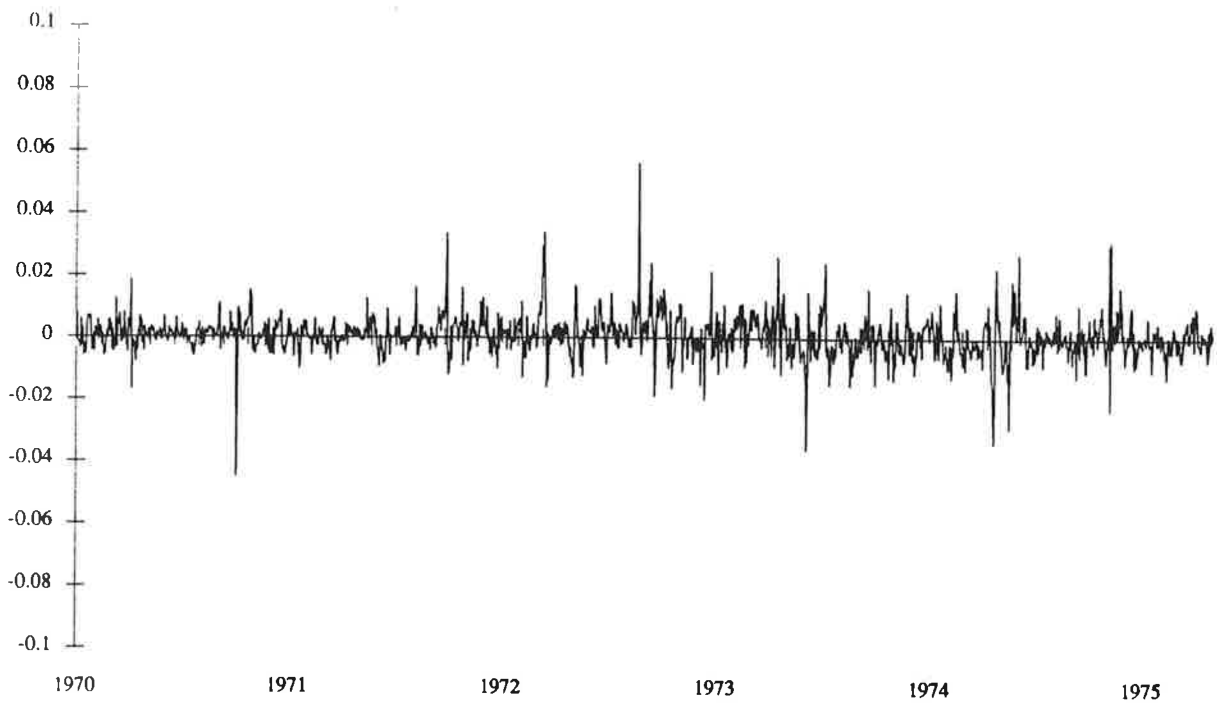
where d is the dividend yield, and the product is taken over all ex-dividend days from the beginning of 1970 up to and including t. In other words, all dividends are assumed to be reinvested in the stock on the ex-dividend day at no transaction costs.

The portfolios are equally weighted in the sense that they are rebalanced to be equally weighted[12] every 20th trading day. The reason for using an equally weighted index is to avoid the problem of having stocks with a very large weight included in the portfolio. The use of a value weighted portfolio would have led to an index with more than 50 % of its value occasionally in two of its largest stocks. The same reason applies to the use of rebalancing. In the case of no rebalancing the phenomenon with a few dominating stocks could be a potential problem toward the end of the period. Thus, this study will be based on a portfolio consisting of the 12 most frequently traded stocks[13], rebalanced to be equally weighted each 20:th trading day.

In this study we will focus on the returns of such a portfolio, where the return is measured as the change in the log of the value of the portfolio. For simplicity the value of the portfolio will be called the index. A graph of the changes in the index is given in Figure 1. The annual return of the index portfolio is approximately 19 % in the 1970-87 period.

Figure 1. Logarithmic changes in the index consisting of the the 12 most frequently traded stocks on the HeSE during 1970-1987.

Figure 1a: Logarithmic changes in the index consisting of the 12 most frequently traded stocks on the HeSE,1970-1975



[12] The total wealth in the portfolio split between the 12 stocks in equal proportions.

[13] These stocks are: KOP, UBF com., Kesko, Enso A, Kymmene, Nokia com., Partek, Rauma-Repola, Suomen Sokeri I (presently Cultor), Tampella, Wärtsilä I, and Yhtyneet com.

Figure 1b: Logarithmic changes in the index consisting of the 12 most frequently traded stocks on the HeSE,1976-1981

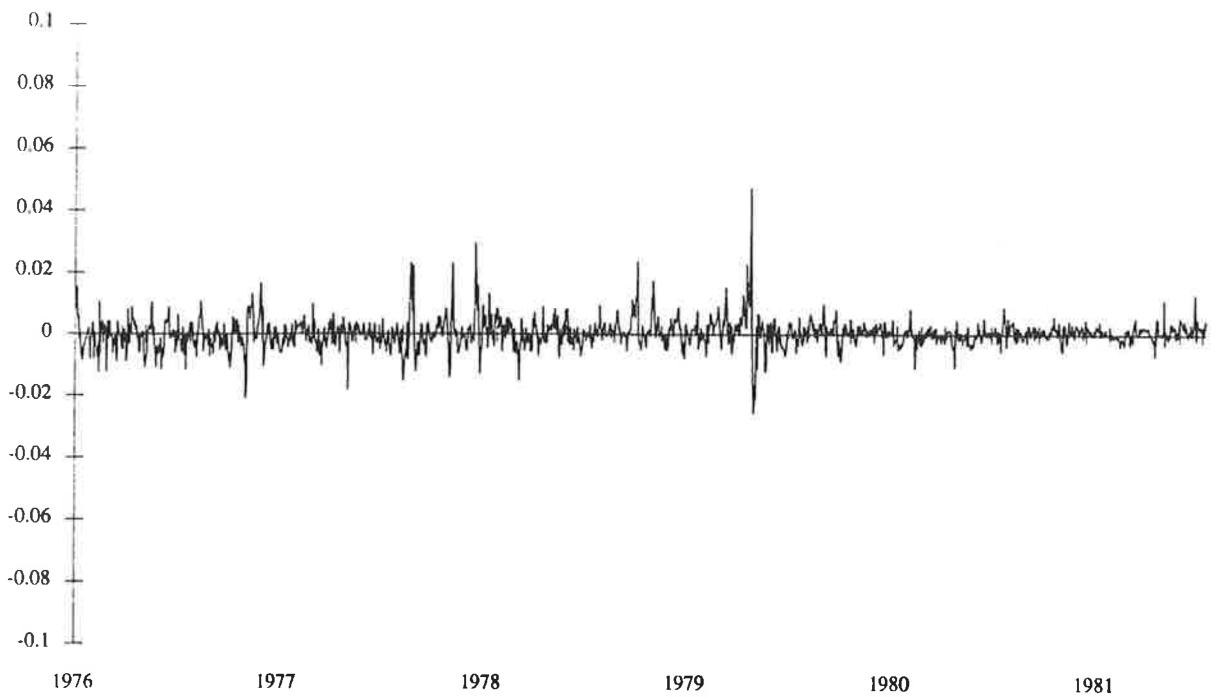
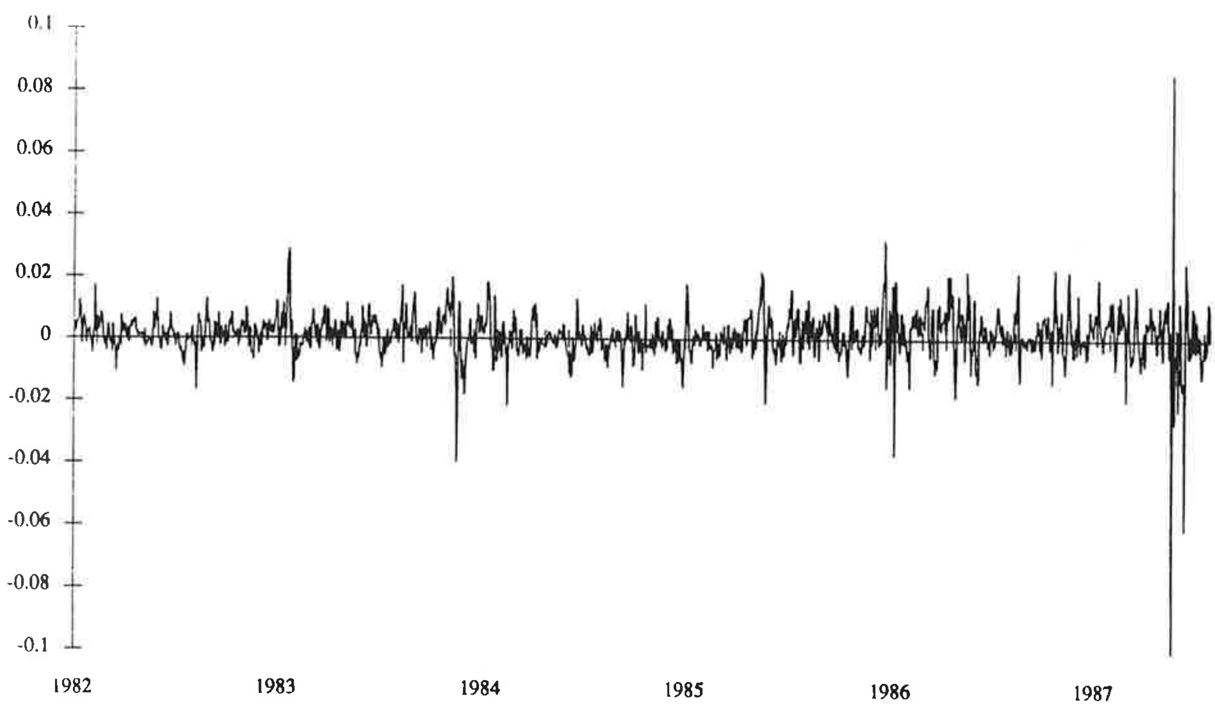


Figure 1c: Logarithmic changes in the index consisting of the 12 most frequently traded stocks on the HeSE,1982-1987



The holding period of 20 trading days, from one rebalancing of the index to the next, will be of crucial importance in this paper. The main focus will be on the prediction of the volatility for the next 20-days holding period. As opposed to the case which sequences of daily returns are assumed to be i.i.d:s, the best predictor for the volatility over a considerably shorter or longer period than our 20-day period may not necessarily be obtainable by a simple transformation of the volatility predictor for the 20-day period (henceforth called a month).

As a proxy for the risk-free interest rate the average change in the consumer price index will be used. In the 1970-87 period this change was approximately 9.13 % p.a. The justification for this proxy lies in absence of meaningful alternatives in Finland. There are no short-term money market instruments that have been traded throughout the period, and the market for bonds with a short remaining maturity is too thin to produce reliable estimates, while the banking sector has been heavily regulated throughout the seventies.

3. Predictors to be evaluated

The purpose of this paper is to compare some simple alternative estimators computed on presently avaliable data for the index volatility during the upcoming month. In practice it is possible to construct an overwhelming number of such slightly different estimators simply by varying the return interval, the estimation period and the weights given to the included observations. To keep the problem manageable we picked a small number of commonly used alternatives. In the group of crude, unadjusted measures we include the standard deviation of logarithmic returns computed on daily and weekly data. The alternatives regarding the length of the estimation period are 6 months, 12 months, and 24 months.

The return computed from an index consisting of several stocks is known to be subject to serial correlation, due to nonsynchronous trading, as shown by Fisher (1966), and for various other reasons[14] as discussed in Berglund and Liljeblom (1988) specifically in connection with the Helsinki Stock Exchange. This serial correlation will make mechanically computed standard deviations biased estimates of the true volatility. As shown by Cohen Hawawini, Maier, Schwartz and Withcomb (1983) positive serial correlation implies that the computed standard deviation will be a downward biased estimate of the true variance.

They prove that an unbiased estimate of the true variance is provided by:

$$(1) \qquad s^2_{adj.} = s^2(1+2\sum_{i=1}^{n} \rho_{-i}),$$

where s^2 is the sample variance, ρ_{-i} is the serial correlation coefficient of order i, and n is the number of relevant coefficients. Based on the results on beta estimation reported in Berglund, Liljeblom, and Löflund (1989), we decided to include the alternatives n =1,2, and 3 in our comparison.

The final alternative to be compared in the present paper will differ from the previous ones in the sense that this alternative will produce an explicit prediction for next month´s volatility, while the predictions in the previous cases are based on static expectations in the sense that next month is assumed to be like the average of the months included in the estimation period. The last alternative will in fact produce a weighting scheme in which the

[14] See e.g. Atchinson, Butler and Simmonds (1987) and Lo and MacKinlay (1989).

most recent observations are given the largest weights. This alternative is based on the Auto Regressive Conditional Heteroscedasticity or ARCH approach introduced by Engle (1982) and generalized by Bollerslev (1986) to what is known as the Generalized ARCH or GARCH[15] model. The GARCH(m,n) model is :

$$r_t = \mu_0 + \varepsilon_t$$

$$(2) \quad \varepsilon_t \sim N(0,\sigma_t)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{n} \beta_j \sigma_{t-j}^2 ,$$

where r_t is the log-return for period t, ε_t is normally distributed around zero with variance σ^2 which depends on m lagged squared residuals and on n lagged variances. On the basis of a number of preliminary regressions the most parsimonious model on monthly data that was not subject to apparent specification errors turned out to be the GARCH(1,1) model[16].

Since the GARCH model gives higher weights to recent observations than to older ones the problem of selecting an appropriate estimation period will not be present in the GARCH-case. The estimation of the GARCH model was carried out on all data from the beginning of 1970 to the month preceding the one for which the volatility is predicted. The volatility estimates produced by the different estimators are given in Figure 2.

The set-up for the present project in which only one-month options are evaluted while monthly returns are assumed to follow a GARCH-process, avoids the problem produced by a changing volatility during the life of the option. If e.g. daily returns were assumed to follow a GARCH-process the changing volatility would affect the prices of a one month option. As shown by Ng (1989) simulated option prices based on the GARCH(1,1) process for daily retruns may differ substantially from the corresponding Black and Scholes (1973) values in the case of approximately one and two month options[17].

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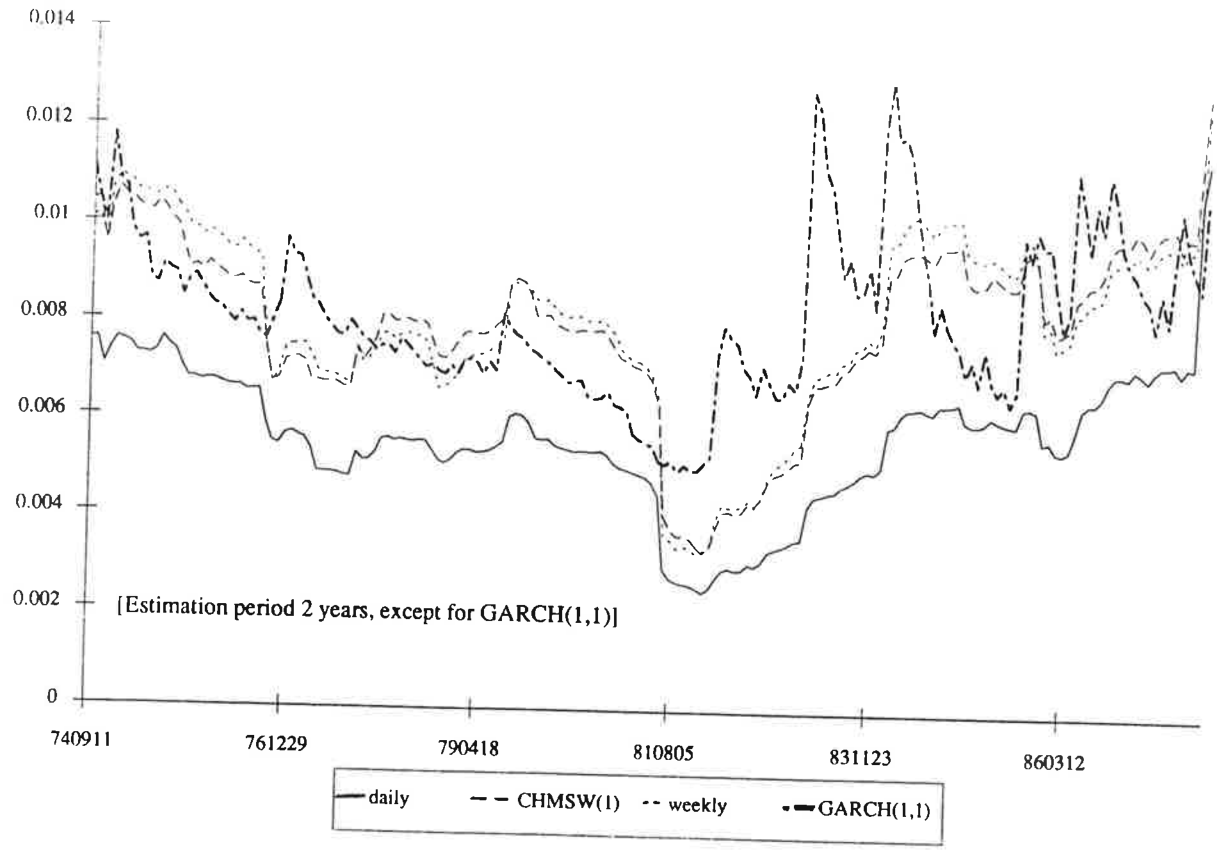
[15] Ng (1989) shows that the GARCH-model for stock returns can be supported in a general equilibrium model with time-separable constant relative risk aversion utility.

[16] The GARCH-model was also estimated on weekly and daily data. However, on daily data severe problems with convergence of the routine used to compute the maximum likelihood estimates were encountered. To some extent this problem was still present in weekly returns due to the higher degree of fat-tailedness in weekly than in monthly returns.

[17] Evidence from the HeSE contradicts this specification, however. The residual return distributions for indvidual stocks, as well as stock indexes on the HeSE tend to be much more fat-tailed for daily returns than for weekly and monthly returns.

Figure 2: The volatility estimates



4. Evaluation Criteria

The evaluation of the performance of the volatility predictor is more complicated than the evaluation of the performance of a predictor for a single number, e.g. the level of the index 20 trading days from now. The volatility measure is supposed to measure uncertainty about the future development of the index, but as time passes this uncertainty is dissolved. At the end of the 20-day period the outcome is known with certainty.

To handle this problem we will use hypothetical options. To avoid the problem caused by the direction in which the index will happen to change, we will use hypothetical straddles, i.e. a call and a put with the same exercise price and the same expiration date, which in our analysis is the last day of each 20-trading-day period. A bought straddle will yield a profit if the index has decreased or increased considerably when the expiration date closes. This will be more likely if the volatility is high than if the volatility is low. Buying or writing a straddle is likely to yield a profit or a loss over a single month even if the price at the beginning of the month is based on an unbiased volatility estimate. However, over a large number of months the buying, or writing, of straddles at the price given by an unbiased volatility estimator ought to yield approximately zero average profits.

By choosing the exercise price to equal the expected value of the index at the expiration date the straddle will be approximately neutral with respect to the direction of the change in the index. The sensitivity of the value of the straddle V with respect to a change in the underlying index (S) will be:

$$(3) \qquad \frac{\partial V}{\partial S} = \frac{\partial C}{\partial S} + \frac{\partial P}{\partial S} = N(d_1) + [\, N(d_1) - 1 \,] ,$$

where:

C - price of the call,

P - price of the put,

N - cumulative standard normal distribution function and,

$$d_1 = \frac{\ln\left(\frac{S e^{(r + \frac{\sigma^2}{2})\tau}}{X} \right)}{\sigma \sqrt{\tau}}$$

where:

ln - the natural logarithm,

r - the risk-free interest rate,

σ - the return standard deviation,

τ - the time to expiration, and

X - the exercise price .

It is easily seen that by setting X equal to the expected value of the index on the expiration date in d_1, the argument in the ln-function will be approximately 1. Thus d_1 will be approximately zero, and $N(d_1)$ approximately 1/2. Inserting this value into expression (1) proves that V is not affected by small changes in S and consequently not by the sign of this change either.

The evaluation of the alternative volatility estimators will be based on two simulated trading strategies:

1. A simple strategy consisting of systematic buying of straddles evaluated at the Black and Scholes (1973) price computed with the volatility estimate produced by each of the estimators to be compared. Straddles are bought for 100 FIM each month[18]. If the estimator produces unbiased estimates, we would expect the average profit of this strategy to be insignificantly different from zero.

2. A pairwise comparison of the alternatives. The options are assumed to be priced at the Black and Scholes (1973) price computed using the estimate of one volatility estimator, and trading is simulated on the assumption that one of the alternatives, the contender, is correct. Thus, if the contender gives a higher volatility estimate than the volatility on which the option prices are based, straddles are bought for 100 FIM, and if the contender gives a lower volatility estimate, straddles are written for the same amount.

If a straddle is bought the money is borrowed at the risk-free rate and if the straddle is written the money is invested at the risk-free rate, which is set equal to 9.125 % continuously compounded per annum. The use of the risk-free rate can be justified using the CAPM by Sharpe (1964) and Lintner (1965) since the ß-coefficient of the straddle equals zero. As shown by Cox and Rubinstein (1986, p.190) the ß-coefficient of a position in options (in our case the straddle V) equals:

(4) $$\beta_V = \frac{\partial V}{\partial S}\frac{S}{V} \times \beta_S ,$$

where β_S is the beta-coefficient of the underlying index. As shown in connection to expression (3) above the partial derivative on the right-hand-side is approximately zero for a straddle in the present case. This proves that the ß-coefficient of V is approximately zero, which justifies the use of the risk-free rate.

18 Undivisibilities which would make this strategy difficult to apply in practice are disregarded.

5. Results

Table 1 reports the results obtained when the first strategy described in the previous section was simulated. Since this strategy consisted of buying straddles for 100 FIM each month borrowing the proceeds at the risk free rate (9.125 %) we would expect this strategy to produce neither systematic losses nor gains if option prices are based on unbiased volatility estimates.

Table 1. Results from the first trading strategy (Buying straddles)

Estimation period: 0.5 YEARS

	DAILY DATA	WEEKLY DATA	DAILY DATA with CHMSW-correction for autocorrelation (1)	(1)-(2)	(1)-(3)	GARCH
SUM (FIM)	10121.97	2616.47	2032.93	-336.44	-541.53	-202.33
AVG	44.59	11.53	8.96	-1.49	-2.39	-1.17
STD of average	7.89	6.10	5.98	5.53	5.67	5.85
Skewness	1.59	0.20	1.49	1.58	1.78	1.43
Kurtosis (excess)	4.10	0.94	3.27	3.58	4.46	3.25
MIN	-100.57	-100.59	-100.61	-100.61	-100.61	-100.45
MAX	676.88	372.65	445.36	403.87	396.40	363.09

Estimation period: 1 YEAR

	DAILY DATA	WEEKLY DATA	(1)	(1)-(2)	(1)-(3)	GARCH
SUM (FIM)	9524.04	1573.77	1214.15	-1116.51	-1835.85	-202.33
AVG	43.10	7.12	5.49	-5.05	-8.31	-1.17
STD of average	8.02	6.02	5.93	5.36	5.17	5.85
Skewness	1.78	1.70	1.75	1.77	1.78	1.43
Kurtosis (excess)	5.31	4.59	4.99	5.16	5.21	3.25
MIN	-100.62	-100.64	-100.64	-100.64	-100.65	-100.45
MAX	705.45	451.33	460.11	397.00	388.40	363.09

Estimation period: 2 YEARS

	DAILY DATA	WEEKLY DATA	(1)	(1)-(2)	(1)-(3)	GARCH
SUM	9458.15	1494.64	1364.31	-990.18	-1781.97	-202.33
AVG	45.25	7.15	6.53	-4.74	-8.53	-1.17
STD of average	8.34	6.23	6.20	5.56	5.32	5.85
Skewness	1.95	1.83	1.94	1.96	1.93	1.43
Kurtosis (excess)	6.96	5.40	6.55	6.65	6.52	3.25
MIN	-100.41	-100.50	-100.50	-100.52	-100.52	-100.45
MAX	778.77	497.43	534.21	467.72	441.00	363.09

Table 1 contains some rather striking results. First of all it is clear that the standard deviation computed on daily returns produces a downward biased estimate of the true volatility. The average return on the straddles is clearly significantly positive for all three estimation periods. Table 1 indicates that the easiest way to avoid most of this problem is to use weekly returns instead of daily returns to compute the standard deviation. Especially for the one- and two-year estimation periods the average return on the straddles bought at the price based on the weekly standard deviation is no longer significant indicating that the volatility estimate is approximately unbiased[19].

Almost the same result as by using weekly returns is obtained by using the CHMSW(1) correction, i.e. expression (1) restricted to one lead and one lag. The inclusion of more leads and lags will make the average return on the straddles negative, and this tendency is stronger for the longer estimation periods. Whether the slight improvement in absolute terms motivates the use of two leads and lags in favour of one is questionable. Finally, the best result measured by how close the average return is to zero is obtained using the GARCH(1,1) volatility prediction on monthly data.

The plot of the cumulative profits[20] produced by the prices based on the different volatility estimators in Figure 3 indicates a certain lack of stability in the estimation errors over time. All estimators overestimate the true volatility in the second half of the seventies and the beginning of the eighties. Around 1982 this trend was reversed. As expected on the basis of the results in Table 1, GARCH(1,1) seems to produce the smallest deviations from the horizontal axis.

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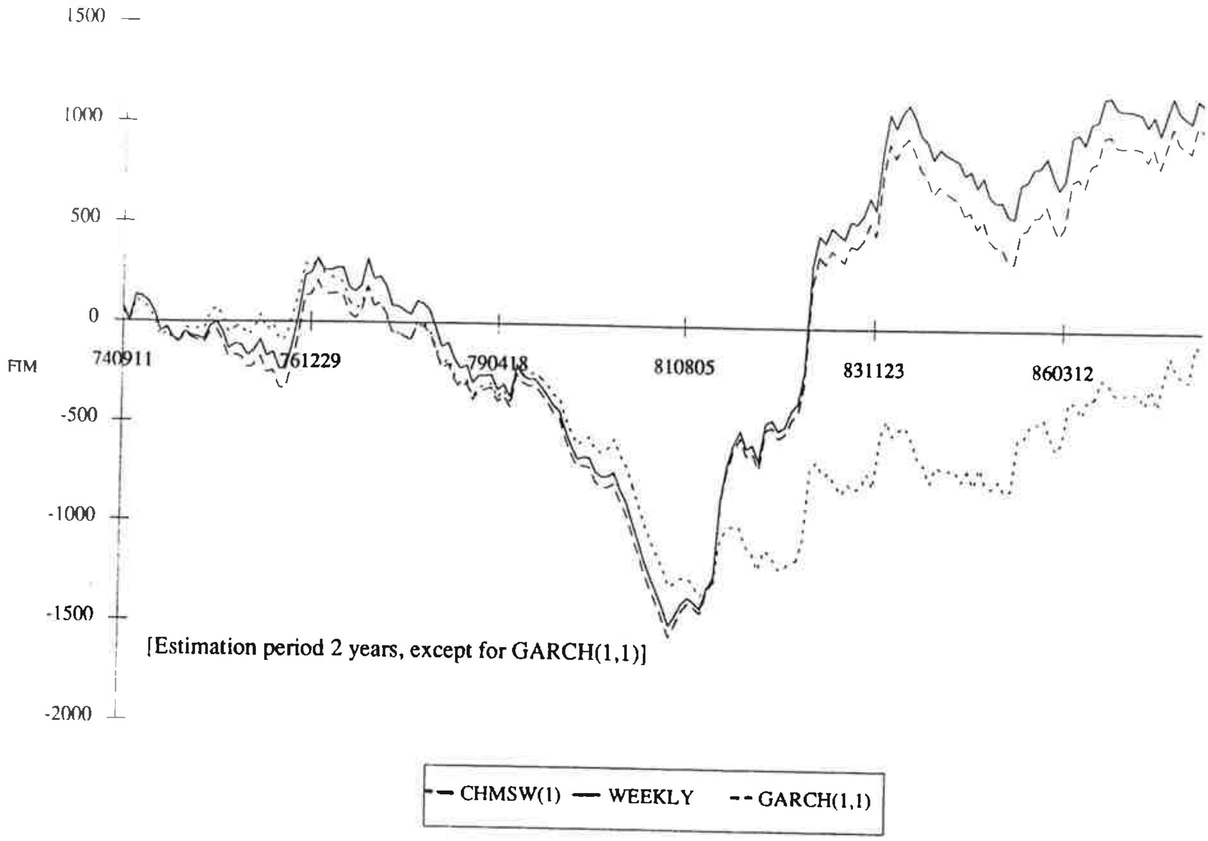
[19] The apparent skewness makes the use of critical values based on a normally distributed population questionable.

[20] No interest rate is taken into account in the cumulation not to exaggerate the importance of the profits made in the beginning of the period as opposed to the profits made at the end of the period.

Figure 3. The results from the first trading strategy. (Buying straddles)



The second step in our empirical analysis was to evaluate the performance of the estimators relative to each other. Thus the pricing was assumed to be made using estimates by one estimator, and a straddle position was taken on the basis of the value produced by another estimator. A hypothetical long position was taken if the value provided by the contender was above the price, and a short position was taken, if the value was below the price. When the straddle was bought(sold) the proceeds were borrowed(invested) at the risk-free rate. The results are reported in Table 2. From this step the CHMSW estimates with 2 and 3 lags were excluded since these alternatives did not clearly outperform the more parsimonious CHMSW(1) alternative in our first step.

Table 2. Results from the second trading strategy (Buying and selling straddles)

ESTIMATION PERIOD 0.5 year [price estimator / contender]

	CHMSW/WEEKLY	CHMSW/GARCH	WEEKLY/GARCH
SUM (FIM)	1224.65	2585.57	4395.49
AVG	**7.08**	**14.95**	**25.41**
STD of average	6.60	6.52	6.75
Skewness	0.14	1.08	1.36
Kurtosis (excess)	4.66	4.18	4.63
MIN	-331.83	-232.44	-176.77
MAX	393.61	393.61	364.44
# pos.profits	103/173	102/173	106/173
z-value	2.51	2.36	2.97

1.0 year

	CHMSW/WEEKLY	CHMSW/GARCH	WEEKLY/GARCH
SUM (FIM)	1358.92	1846.44	3611.38
AVG	**7.86**	**10.67**	**20.88**
STD of average	5.90	5.87	6.68
Skewness	-1.75	0.65	0.75
Kurtosis (excess)	5.39	4.40	4.50
MIN	-375.46	-221.14	-192.87
MAX	144.08	375.46	434.09
# pos.profits	108/173	101/173	105/173
z-value	3.27	2.20	2.81

2.0 years

	CHMSW/WEEKLY	CHMSW/GARCH	WEEKLY/GARCH
SUM (FIM)	-375.96	5001.58	4666.39
AVG	**-2.17**	**28.91**	**26.97**
STD of average	6.87	6.51	6.46
Skewness	1.05	1.67	2.81
Kurtosis (excess)	7.97	7.54	10.78
MIN	-381.10	-170.22	-167.92
MAX	534.21	534.21	497.43
# pos.profits	79/173	112/173	112/173
z-value	-1.14	3.88	3.88

(price estimator = the volatility estimator used by "the market", contender = the volatility estimator used by "the trader")

The first thing to note in Table 2 is that the use of daily data does not yield an advantage relative to the use of weekly data. In fact the weekly standard deviations outperform the CHMSW(1) alternative except for the 2-year estimation period. None of the differences are statistically significant though. The second, and more interesting fact revealed in Table 2 is that the GARCH(1,1) estimator outperforms both alternatives for all estimation periods.

Assuming that the profits are normally distributed the return is significantly different from zero even on a 5 % significance level, the only exception being the CHMSW(1) with a one year estimation period. This finding is supported by the additional non-parametric test employed.

Table 2 reveals that the maximum profit obtained for one month is quite large compared to the average. Thus the results may still reflect a small number of large outcomes produce by chance, rather than a systematic pattern[21]. To detect possible outliers which may affect the result, the cumulative profits produced by the simulated strategies were plotted. A sample of these plots is shown in Figure 4.

Figure 4. The results from the second trading strategy. (Buying and selling straddles)

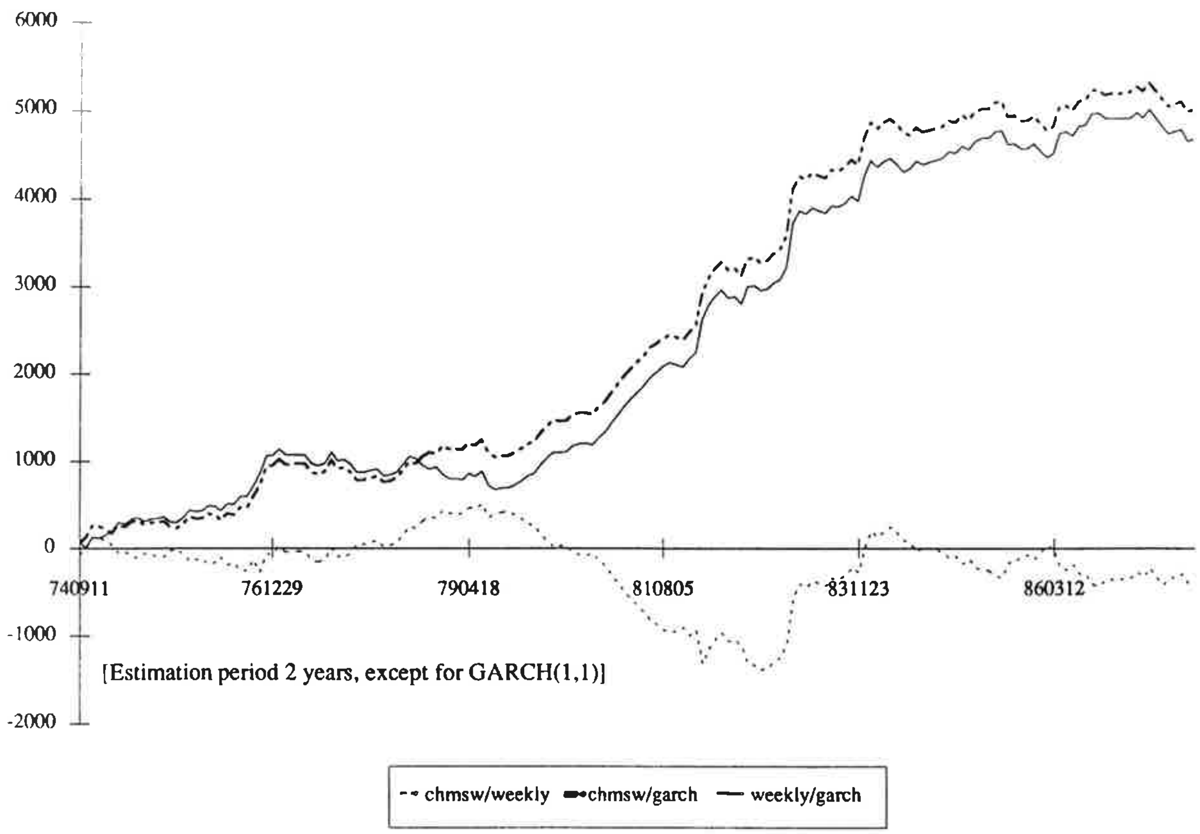


Figure 4, which is a representative sample for all estimation periods, shows that the cumulative profits obtained when using the GARCH(1,1) estimates against the alternatives

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[21] The proportions of straddles bought and sold were approximately equal.

tend to increase almost linearly through time. This pattern corroborates the conclusion drawn on the basis of the results in Table 2, namely that GARCH(1,1) tends to produce superior estimates for the volatility of the index during the coming month. This is all the more remarkable when we recall that the GARCH(1,1) model is estimated on monthly data whereas the alternatives use more frequent observations.

6. Summary

The purpose of the present paper is to compare several different estimators to be used in the pricing of index options based on the Black and Scholes (1973) formula. Since the underlying instrument is an index it is known that the returns as such as well as their volatility are subject to serial correlation. This makes the decision concerning what return-interval and what estimation period to use as well as whether to use a weighting scheme when computing the return standard deviation, a difficult problem.

In this paper several alternatives are analyzed. These include: daily and weekly returns estimated over half a year, one year, and two years, daily returns corrected for first second and third order serial correlation and finally volatility estimates produced by a GARCH(1,1) model applied to monthly data.

Our results clearly indicate that the GARCH(1,1) model produces estimates superior to the alternatives. This is in spite of the fact that the alternatives use more frequent observations than the GARCH(1,1) model estimated in this paper.

The general evaluation of the pricing errors produced by the alternative estimators revealed that the pricing errors were smallest in the GARCH(1,1) case for the whole period as well as cumulatively most of the time from the beginning of 1974 to the end of 1987. A hypothetical speculation strategy in which options were assumed to be traded at prices produced by the alternative estimators, revealed that going long or short in an at-the-money straddle on the basis of whether the GARCH(1,1) estimate indicated that the option was under- or over valued would have produced significant positive profits. The superiority of the GARCH(1,1) estimate, furthermore, turned out to be rather stable over time.

The results reported in this paper should be regarded as basic results, in the sense that the comparison includes only rather simple estimators. More sophisticated estimators can easily be obtained by using e.g. other conditional distributions[22] than the normal

[22] This would require a modification of the Black and Scholes (1973) model to preserve consistency.

distribution in the GARCH case[23], and by using exogenous variables, e.g. relative changes in trading volume[24], or the volatility on e.g. the NYSE, in the volatility equation in the GARCH model.

The fact that the time to expiration is kept at 20 trading days throughout the study is another apparent limitation in the present paper requiring further research. The application of our results to a one-week time to expiration for instance is far from trivial. On most stock markets the deviation of logarithmic returns from the normal distribution will become more striking the shorter the return interval is. The search for a practical way to handle this problem as the option matures should be an interesting subject for further research.

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[23] See Booth, Hatem, Virtanen and Yli-Olli (1989).

[24] See Lamoreux and Lastrapes (1990).

REFERENCES

Akgiray, V. (1989): Conditional Heteroscedasticity in Time Series of Stock Returns: Evidence and Forecasts, *Journal of Business*, Vol.62, No.1, pp. 55-80.

Atchison M., K.Butler & R.Simonds (1987): "Non-Synchronous Security Trading and Market Index Autocorrelation", *Journal of Finance*, Vol.42, No.1, March, pp. 111-118.

Berglund T. and E. Liljeblom (1988): "Market Serial Correlation on a Thin Security Market: A Note", *Journal of Finance*, Vol.43, No.4, September, pp. 1265-1274.

Berglund,T., E.Liljeblom and A. Löflund (1989): Estimating Betas on Daily Data for a Small Stock Exchange, Journal of Banking And Finance, Vol.13, No.1, pp. 41-64.

Black (1976): "The Pricing of Commodity Contracts", *Journal of Financial Economics*, Vol.3, January-February, pp. 167-179.

Black,F. & M. Scholes (1973): "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, Vol. 81, May-June, pp. 637-659.

Bollerslev T. (1986): Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics*, Vol. 31, pp. 307-327.

Booth G.G., J.Hatem, I. Virtanen and P. Yli-Olli (1989): Stochastic Modelling of Security Returns: Evidence from the Helsinki Stock Exchange, unpublished manuscript.

Cohen,K., G.Hawawini, S.Maier, R.Schwartz and D.Withcomb (1980): Implications of Microstructure Theory for Empirical Research on Stock Price Behaviour, *Journal of Finance*, Vol.35, pp. 249-257.

Cohen,K., G.Hawawini, S.Maier, R.Schwartz and D.Withcomb (1983): Friction in the Trading Process and the Estimation of Systematic Risk, *Journal of Financial Economics*, Vol.12, August, pp.263-278.

Cox J.and S. Ross (1976): The Valuation of Options for Alternative Stochastic Processes, *Journal of Financial Economics*, Vol. 3, pp. 145-166.

Cox J. & M. Rubinstein (1985): Options Markets, Prentice Hall Inc., Engelwood Cliffs, New Jersey.

Engle R. (1982): Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, Vol. 50, No.4, July, pp. 987-1007.

Fisher, L. (1966): "Some New Stock Market Indexes", *Journal of Business*, Vol. 38, January, pp. 191-225.

Hull J. and A. White (1987): The Pricing of Options on Assets with Stochastic Volatilities, *Journal of Finance*, Vol.42, June, pp. 271-301.

Jarrow R. and A. Rudd (1982): Approximate Option Valuation for Arbitrary Stochastic Processes, Journal of Financial Economics, Vol.10, No.2., pp. 347-369.

Jarrow R. and J. Wiggins (1989): Option Pricing and Implicit Volatilities, Journal of Economic Surveys, Vol.3, No.1., pp. 59-81.

Johnson H. and D. Shanno (1987): Option Pricing when the Variance is Changing, *Journal of Financial and Quantitative Analysis*, Vol. 22, No.2, June, pp. 143-153.

Lamoreux C. and W. Lastrapes (1990): Heteroscedasticity in Stock Return Data; Volume versus GARCH Effects, *Journal of Finance*, Vol.45, March, pp. 221-229.

Latané H. and R. Jr. Rendleman (1976): Standard Deviation of Stock Price Ratios Implied in Option Prices, *Journal of Finance*, Vol.31, May, pp. 369-381.

Lintner, John (1965): "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", *Review of Economics and Statistics*, Vol.47, No.1, February, pp. 13-37.

Lo and MacKinlay (1989): When are Contrarian Profits due to Stock Market Overreaction, Proceedings of the 16th annual meeting of the European Finance Association, Stockholm.

Marsh T. and E. Rosenfeld (1986): Non-Trading, Market Making, and Estimates of Stock Price Volatility, *Journal of Financial Economics*, Vol. 15, pp. 359-372.

Ng V. (1989): Asset Pricing with Time Varying Volatility: Theory and Empirical Studies, Dissertation at the University of California, San Diego.

Merton R. (1976): Option Pricing when Underlying Stock Returns are Discontinuous, *Journal of Financial Economics*, Vol.3, March, pp. 125-144.

Rubinstein, M. (1985): Nonparametric Tests of Alternative Option Pricing Models Using all Reported Trades and Quotes on Thirty most Active CBOE Option Classes from August 23, through August 31, 1978, *Journal of Finance*, Vol. 40, June, pp. 455-480.

Sharpe, W. F. (1964): "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", *Journal of Finance*, Vol. 19, No.3, September, pp. 425-442.

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