ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B 00120 Helsinki Finland Tel. 601322 Telefax 601753

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Tom Berglund* - Staffan Ringbom** - Laura Vajanne***

PRICING OPTIONS ON A CONSTRAINED CURRENCY INDEX: SOME SIMULATION RESULTS

* The Research Institute of the Finnish Economy
** The Swedish School of Economics and Business Administration
*** Union Bank of Finland

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ABSTRACT: This paper uses a simulation model to analyze the consequences for the pricing of currency options of independence between the initial exchange rate position within the currency band established by the central bank and the subsequent jump produced by re- or devaluations.

A simple form of dependence is introduced which makes the probability of a devaluation grow quadratically from zero at the upper border, while the probability of a revaluation grows quadratically from zero at the lower border.

Our results indicate that the expected change in the exchange rate produced by the independence case will lead to unrealistic outcomes. To preclude profitable speculation the exchange rate must be expected to depreciate close to the lower border, while it must be expected to appreciate close to the upper border. Furthermore the implied term structure for the domestic interest rate is marked by highly unrealistic features. It turns out that these phenomena are avoided if the occurrence of re- or devaluations depends on the position of the exchange rate within the band, given a realistic set of parameter values.

When the option values based on our simulation experiments were compared with the option prices calculated with the widely used Garman and Kohlhagen model (1983), it turned out that the Garman and Kohlhagen model gave rise to over- as well as underestimation. When the simulated standard deviation was used in the Garman and Kohlhagen model the prices generally were above the simulated values, but when the standard deviation computed without regard to the possibility of jumps was used the reverse turned out to be true.

KEY WORDS: Currency options, jump-diffusion, exchange rate risk

CONTENTS

1. Introduction

In classical arbitrage pricing of options the specification of a statistical distribution which models the price changes of the underlying security plays a fundamental role. The basic currency option valuation formulas, such as Garman and Kohlhagen (1983), and Grabbe (1983), assume that the exchange rate, like the stock price in the Black and Scholes (1973) article, follows an Ito process. The stochastic part of the exchange rate is assumed to follow a geometric Brownian motion, which implies a lognormal distribution of exchange rate changes.

However, the assumption that price changes are lognormally distributed may be less valid in the foreign exchange market than in the stock market. The reason is found in active exchange rate management policies such as market interventions, realignments, and shifts in monetary or fiscal policy, that do not have any counterparts in the stock market.

Recent studies, including those by Boothe and Glassman (1987), Hsieh (1988), and Tucker and Scott (1987), fit alternative stochastic processes to foreign exchange rate movements. Without exception, these authors report significant departures from a geometric Brownian motion. The consensus seems to be that distributions of the log price changes are more peaked and have fatter tails than the normal distribution.[1]

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[1]Distributions that violate the lognormality assumption, however, do not necessarily indicate that the Black and Scholes type of model should be discarded. If a model predicts reasonably well, violations of some of the assumptions are relatively unimportant. Unfortunately, empirical tests of the Black and Scholes model applied to currency options show systematic, significant mispricing, see e.g. studies by Bodurtha and

Moving to exchange rates for which there exists a currency agreement such as the ERM in EMS member countries or the currency index system used in Nordic countries, the assumption that underlying exchange rate movements follow a geometric Brownian motion seems even less justified. In these cases it seems clear that constraints imposed by currency agreements may shift due to the existence of realignments and the possibility of a jump must be included in a model of the exchange rate process.

In general, option pricing models that allow jumps in the price of the underlying instrument have been studied by Cox and Ross (1976), Cox, Ross and Rubinstein (1979), and Merton (1976). A generalized valuation formula for the case where the security price is a combination of an Ito process and a random point process is presented by Aase (1988).

In Merton's (1976) jump-diffusion model it is assumed that the stock price dynamics can be written as a combination of two types of changes: (i) small vibrations in price, e.g. due to a temporary imbalance between supply and demand, gradual changes in the economic outlook, or other marginally significant new information. This component has a continuous sample path, and can be modeled by a standard geometric Brownian motion with a constant variance per unit of time. (ii) "Abnormal" price changes which are due to the arrival of important new information about the stock. Typically, such information will be specific to the firm or possibly its industrial branch although occassionally general economic information could be the source. This component is modeled by a "jump"

Courtadon (1987), Goodman, Ross and Schmidt (1985), Shastri and Tandon (1986), and Tucker, Peterson and Scott (1988). However, the observed mispricing has not been large enough to create profit opportunities, when transactions costs including the bid-ask spread are taken into account.

process with an inherently noncontinuous sample path reflecting the non-marginal impact of the information. The prototype Merton uses for the jump component is a "Poisson-driven" process, which implies that the jumps are independently distributed over time.

When analyzing the jump-diffusion model in the pricing of stock options, Ball and Torous (1985) found that significant discrepancies between Black-Scholes and Merton call prices may occur if the underlying common stock return process is dominated by the presence of large jumps which occur infrequently. However, their empirical evidence suggested that no such jumps were present in the case of stocks. Finally, they suggested that the return on other financial securities, such as foreign exchange, may still be more accurately modeled as a compound Poisson jump-diffusion process characterized by infrequent large jumps.

Empirical evidence of the existence of discontinuities in the sample path of exchange rates and of the impact to currency option pricing can be found in Borensztein and Dooley (1987), and in Jorion (1988).[2]

All the mentioned references of mixed jump-diffusion models have used the assumption that the jump and the diffusion process are mutually independent. There are two reasons for this. Firstly, the model will become much more complicated, if dependence is assumed. Secondly, the exact form of the dependence is a difficult empirical question.

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[2]Jorion (1988) estimates the parameters in question and finds out that exchange rates display significant jump components, which are more manifest than in the stock market. These discontinuities seem to arise even after explicit allowance is made for possible heteroskedasticity in the usual diffusion process. He also shows that ignoring the jump component in exchange rates can lead to serious mispricing errors for currency options.

In this paper we are especially interested in the effects of allowing the exchange rate movements and the probability of a jump to be mutually dependent as compared to the assumption of the independence of these two stochastic processes. As a working hyphothesis we assume that this dependence has significant effects on the valuation of a currency option.

What we have in mind is an option on a currency with a fixed but adjustable exchange rate band. From a practical point of view the formal currency arrangement supporting the band determined by given index boundaries represents an important simplification compared to the general approach for mixed jump-diffusion processes, because exact identification of the jumps is possible.

The approach we will take in this paper is to compute prices of currency options using simulated currency data. Simulation of the underlying exchange rate movements allows us to study in detail how certain properties of the currency basket system will affect the pricing of currency options.

The simulation approach is also justified by the fact that the jump component is difficult to estimate from historical data. Neither the intensity nor the expected level of the jump may necessarily be inferable from the history of previous jumps, rather the jump is a result of complex economic and political considerations that may vary over time.

The outline of this paper is as follows. The next section briefly discusses the principle of risk-neutral valuation on pricing of options in the present context. The institutional framework of the Finnish currency basket system will be briefly described in section 3. The following sections describe the process used to generate the currency data under the alternative assumptions and the specification of the parameter values used in the simulations. In section 6, the simulation results for the prices of options on a currency index are presented. Conclusions are given in section 7.

2. The principle of risk-neutral valuation in the present context

At the heart of the derivation of the Black-Scholes type of currency option pricing formula is the arbitrage technique by which investors can follow a dynamic portfolio strategy using the underlying currency and riskless borrowing to exactly reproduce the return structure of an option. By following this strategy in combination with a short position in an option, the investor can eliminate all risk from the total position, and hence to avoid arbitrage opportunities, the option must be priced such that the return on the total position must equal the risk-free rate of interest.

However, for the Black-Scholes arbitrage technique to be carried out, investors must be able to revise their portfolios continuosly and the underlying instrument's price must follow a stochastic process that generates a continuous sample path. In effect, this requirement implies that over a short interval of time, the price of the underlying instrument cannot change by much.[3]

In our context where the prices do not follow a geometric Brownian motion because of the possibility of jumps we have to make an assumption concerning the jump risk to be able to evaluate currency options. A sufficient condition is that the jump component of an exchange rate return is diversifiable. Under this sufficient condition, investors do not require an expected compensation for the jump risk.[4]

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[3]See Merton [1976]

[4]This sufficient condition is obtained by Merton (1976) using Ross (1976) arbitrage pricing model. See also Jarrow and Rosenfeld (1984).

In the case of foreign currencies a well diversified portfolio is in principle easier to create than in the case of ordinary assets. The reason is that there is an almost perfect negative correlation in the case of jumps between the change in the value of nominally fixed domestic assets and a basket of nominally fixed foreign assets.[5]

In the following we will assume that the jump is diversifiable. This implies that we may use risk neutral evaluation to obtain the price of the options that we are analyzing, i.e. the domestic risk-free interest rate will be used to discount the expected value of the option at the expiration date.

The next step is to determine the level of the domestic interest rate. The foreign rate is assumed to be exogenously given, which means that the domestic rate will be given by the absence of expected speculative profits. With free mobility of capital, the domestic interest rate can be seen as determined by the sum of the given world interest rate and the interest rate differential. The interest rate differential is the sum of the expected depreciation of the home currency and a foreign exchange risk premium.

An important result in international asset pricing models[6] is that the risk premium of a position in domestic or

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[5] In practice, however, this strategy may be difficult to implement due to various restrictions on international capital flows.

[6] See e.g. Fama and Faber (1979), and Stulz (1981,1984). In a full general equilibrium setting in the tradition of Lucas' (1978, 1982) dynamic general equilibrium model of asset pricing the risk premium is analyzed e.g. by Hodrick and Srivastava (1984,1986), Mark (1985), and Sibert (1989).

foreign currency depends on the covariances with changes in asset prices and ultimately with changes in consumption.

The intuition behind the importance of the covariance term is as follows: The greater the covariance between the future exchange rate and the marginal consumption value of the domestic currency is, the more effective is the foreign bond relative to the domestic bond as a hedge against consumption risk. This is because the stochastic payoff on a foreign asset, in terms of the domestic currency, tends to be high when the consumption value of domestic nominal assets drop. A higher covariance leads to a lower required interest rate on foreign bonds relative to that on domestic bonds.

As in the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) the only risk that cannot be eliminated is the one which depends on the covariance with the market portfolio, in this case the world market portfolio, or ultimately with the world consumption.

In this paper we assume that the home country (Finland) has the same exposure to surprices in the world economic outlook as other countries on average. This means that foreign investors will not require a risk premium for holding the Finnish markka and neither do Finnish investors require a risk premium to hold a well diversified basket of foreign currencies.

In practice this is not necessarily true. Due to the extensive dependence on the highly cyclical forest industry, Finland may be more exposed to world market risk. Thus, if the foreign output is considered less "risky" than the home output, there is an expected return on buying foreign exchange forward, and a risk premium should be added to the relation.

The assumption that the domestic and the foreign economy are expected to react alike to exogenous stochastic shocks allows us to derive the domestic risk-free interest rate from the foreign rate by assuming that the instantaneous expected return is the same for default-free holdings irrespective of the currency it is denoted in. The risk premium is zero, and interest rates are linked by the uncovered interest rate parity condition, which relates the interest differential to expected changes in exchange rates.[7]

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[7]Even in a case of risk neutrality, the uncovered interest rate parity may seen to be violated due to Jensen's inequality. This is known as Siegel's [1972] paradox, and since it affects the computation of the domestic interest rate it will be discussed in section 6 of this paper.

3. The currency basket system of the FIM exchange rate

In this paper we are studing call options on a foreign currency index, similar to the index to which the Finnish markka is tied. This index is in effect a currency basket consisting of fourteen foreign currencies. If the currency index for the Finnish markka were pegged to a certain level, the movements of the markka with respect to any individual currency in the index would be exactly determined by the movement of that currency vis-à-vis the other currencies in the index.

However, the system is made more complicated by the fact that the central bank usually allows the markka index to move freely inside the official bounds of the index. An upward movement of the index number means a depreciation and a downward movement an appreciation of the markka. At present the fluctuation range is 6 %.

An important issue in modeling the behavior of the currency index over time is thus the behavior of the central bank. The central bank occasionally has to intervene to maintain the exchange rate within the index limits. At least in the proximity of the index barriers, these interventions will potentially be an important force affecting the exchange rate behaviour.

In keeping the exchange rate within the given band the central bank must try to avoid the application of rules which would create opportunities for systematic speculative profits. Thus, it is plausible that the actions taken by the central bank will contain a certain amount of randomness. Consequently, the actual timing and the effect of the central bank's interventions are not easily representable.

In addition to movements within the band there can be discrete changes of the band itself, i.e. the whole index range can be moved upwards or downwards (facilitating a devaluation or revaluation, respectively, of the markka).[8]

A central issue in discussing the failure of the attempts to keep the currency within the fixed band lies in macroeconomic problems of the underlying economic structure. External changes in the economic conditions affecting the country can under the commitment to the fixed band put constraints on domestic fiscal and monetary policy which may be politically impossible to implement.

The immediate reason why a defence of the currency band may fail is that when the value of the currency moves closer to the border of the band, the speculative behavior may grow so strong that it becomes too costly for the central bank to prevent the index from passing the limit.

In order to illustrate the implications of the currency index system on bilateral exchange rates, we can express a change of an individual exchange rate with the help of two components: a change of the currency index and a change of the individual currency with respect to the index.[9]

Let $s^j(t)$ denote the log of the currency j exchange rate in period t (e.g. FIM/USD) and s(t) all the other exchange rates expressed in dollars. The log of the currency index is I(t). The term **w** is the sum of all the currency basket weights except for the dollar's. The change of the markka/dollar exchange rate is thus:

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[8]In Finland, this requires a permission of the government. In the 1980's there have been e.g. two devaluations and two revaluations of the markka.

[9]This kind of expression of the currency index is used e.g. in Oksanen (1981) and Hörngren (1986).

$$(1)\ s^j(t+1) - s^j(t) = \mathbf{w}[s(t+1) - s(t)] + [I(t+1) - I(t)]$$

Hence, an observed depreciation of the markka against the dollar can be due to an overall depreciation of the markka (an increase in the currency index) and/or to an overall appreciation of the dollar against an average of the non-markka currencies.

This idea of dividing the change of the exchange rate into two parts can be used to analyze the uncertainty connected to exchange rate risk. On the one hand, there is the risk due to relative changes for a foreign currency with respect to other foreign currencies, and on the other hand there is the risk due to changes in the domestic currency with respect to all other currencies. In this paper we will be interested exclusively in the latter kind of risk.

4. Experimental design

4.1. Exchange rate movements within the index band when jumps are excluded

We start by assuming that the movements of the currency index[10] within the index band can be desrcibed as a discrete time Markov process on a compact state space.

The essential defining property of a Markov process is that the conditional distribution of its future values given all current information is the same as the conditional distribution of its future values given only its current value. This is a common assumption considering e.g. exchange rates and stock prices, where the absence of expected speculative profits imply that all the relevant information is included in the current price.

In the absence of jumps, the process of the currency index is characterized by the expression[11]:

$$(2) \quad J_t^* = \max\{\ 0,\ \min\{\ 2c, (c + b(J_{t-1}^* - c) + \sigma\Phi^{-1}(X_t^{(1)}))\ \}\ \}$$

where 2c is the width of the band, b is the strength of the assumed mean reverting tendency within the index band, and $X_t^{(1)}$ is a standarized uniformly distributed random variable, which through the inverse of the cumulative standard normal density function ϕ^{-1} produces a normally

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[10]For convience we will talk about movements of the index although movements of the specific currency (markka) with respect to the index would be more natural.

[11]In the background we have a stochastic base $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{N}}, P)$ on wich every process is measurable and adapted. $\Omega = \bigtimes_{i=0}^{\infty}\mathbb{R}^3$, $\mathcal{F} = \bigotimes_{i=0}^{\infty}\mathcal{R}^3$, $\mathcal{F}_t = \bigotimes_{i=0}^{t}\mathcal{R}^3$, is sufficiently large for our purposes.

distributed random shock. The dispersion of the shock is scaled by σ. The boundaries of the index are imposed through the min and max functions, which will keep the index value within the band.

The mean reverting property means that when the system is in a state far removed from the midpoint, it is more likely to move towards the midpoint than in the opposite direction. If b=1, the corresponding unbounded process, the second argument in the min function in (1), is a random walk. If $|b|<1$, the corresponding process is a stationary autoregressive process of order one.

The values for the coefficient b and for the standard deviation σ can be estimated from historical data. Another possibility is to cover a range of different values in the simulation of the exchange rate series.

4.2 The jump characteristics

The next step is to include the possibility of a jump, which means that the whole index band is moved up or down and the index number jumps to a new location within the new band.

Since the range of the band is given, it is sufficient to model the process for the lower bound ($J_t^{(1)}$). The change of the lower bound of the index, i.e. a revaluation (A) or a devaluation (D), is defined by the following expression:

$$(3)\quad \Delta J_t^{(1)} = -\,1_{A_t}(X_t^{(2)})\cdot a_1(X_t^{(3)}) + 1_{D_t}(X_t^{(2)})\cdot d_1(X_t^{(3)})$$

Normally $\Delta J_t^{(1)}$ is zero. If the indicator function 1_A takes the value one, a revaluation occurs and the size of the jump is given by $a_1 X_t^{(3)}$, and if the indicator function 1_D takes the value one, a devaluation occurs and the size of the jump is given by $d_1 X_t^{(3)}$. $X_t^{(3)}$ is a standard uniformly distributed independent stochastic variable which is independent of $X_t^{(1)}$. The terms a_1 and d_1 denote inverse functions of lognormal cumulative distribution functions. Thus the size of the re- or devaluation is lognormally distributed.

By choosing the lognormal distribution we eliminate jumps in the wrong direction, which could occur if negative values were possible. Futhermore, very small jumps are unlikely, while large jumps are possible due to the skewness of the distribution.

$X_t^{(2)}$ is a standard uniformly distributed independent stochastic variable[12], which will determine whether a revaluation or devaluation will occur. This is determined according to Figure 1.

Figure 1. The determination of re- and devaluation

If $A: X^{(2)} \leq q(j)$, a revaluation occurs, while if $D: X^{(2)} > 1-p(j)$, a devaluation occurs.

The probability for a revaluation q(j) and p(j) for devaluation are assumed to be Bernoulli type probabilities as follows:

$$(4) \quad q(j) = k_a \left[1 - \frac{j}{2c} \right]^n ,$$

$$p(j) = k_d \left[\frac{j}{2c} \right]^n ,$$

with the natural restriction,

$$\forall j \in [0, 2c]: \; q(j), p(j) \geq 0 \;\&\; q(j)+p(j) \leq 1.$$

12 Also independent of $X^{(1)}$ and $X^{(3)}$

If n=0, the probability will not depend on the position within the band. Thus, the probability is evenly distributed over the whole fluctuation area.

The total unconditional probability of a revaluation or devaluation is given as an integral of the probabilities over the whole fluctuation area.

$$(5)\quad p = \int_{[l.b,u.b]} p(j)\,d\overline{P}(j), \qquad q = \int_{[l.b,u.b]} q(j)\,d\overline{P}(j)$$

where $\overline{P}$ is a stationary probability measure for the position within the band, $j \in [l.b.,u.b.]$.

4.3 Exchange rate movements including jumps

The process determining the position of the currency index value within the index band ($J_t^{(2)}$), including the possibility of a jump, is defined as follows:

$$(6)\quad J_t^{(2)} = 1_{A_t^c \cap D_t^c}(X_t^{(1)}) J_t^*(X_t^{(2)}) + 1_{A_t}(X^{(1)}) a_2(X^{(2)}) + 1_{D_t}(X^{(1)}) d_2(X^{(2)}),$$

where $A_t = [0, q(J_{t-1}^{(1)})]$ and $D_t = ((1 - p(J_{t-1}^{(1)})), 1]$, when $t>0$,

$A_0 = D_0 = \emptyset$.

Since the location of the index number within the band after a jump is unknown, we assume the location is Beta-distributed within the new index band. Thus, a_2 and d_2 are inverse functions of the cumulative Beta-distribution function, where the parameters α and β are determined to correspond to a priori expectations. Using the Beta-distribution the probability of the new position is centred in the middle of the band and the probability of landing outside the range is zero.

The process of the log of the currency index is now easily defined as

(7) $I = (I_t)_{t\in\mathbb{N}}$, where $\forall t\in\mathbb{N}: I_t = J_t^{(1)} + J_t^{(2)}$.

The log of the index is the sum of the lowerbound and the position in the band. Thus the currency index process can be written as

(8) $S = (S_t)_{t\in\mathbb{N}}$, where $\forall t\in\mathbb{N}: S_t = \exp\{J_t^{(1)} + J_t^{(2)}\}$

5. Specification of the parameter values used in the simulation experiments

The parameters and starting values which are used in the simualtions of the exchange rate movements are defined in Table 1.

Table 1. Parameter settings used in the simulation experiments

Symbols	Description	Value
Starting value:		
x	The relative index position within the band	$x \epsilon [0,1]$
I_{lb}	The lower bound of the index.	97
I_{ub}	The upper bound of the index.	103
I_t	The initial position within the band	97-103
Parameters:		
b	The strength of the mean reversion property.	0.9985
σ	The standard deviation of the error term in the unconstrained autoregressive process.	0.00145
Θ	The expected log size of the jump	0.05
δ	The standard deviation of log of the jump.	0.01
$p(j)+q(j)$	The probability of the jump: (j) is the relative position within the (log) band.	0.005
$E(x\|j)$	The expected relative position of the index value within the new (log) index band after the jump.	0.5
$Var(x\|j)$	The variance of the relative position of the index within the new (log) band.	0.001

The index band is set to 6 %, which corresponds to the present band used in Finland. In our experiments a sample of 11 evenly distributed initial states that start at the lower and end at the upper boundary are explored.

The strength of the mean reverting property is determined using the estimated value from the daily observations of the Finnish currency index in the period 1987-1989. The standard deviation, which is used in the simulations is estimated from the same data.

The log of the expected size of the devaluation or revaluation is set to 0.05. The standard deviation around the expected size is assumed to be 0.01. These figures correspond to our view that very small currency realigments are unnecessary, while the reagligment usually will be made before the required magnitude will grow too large, due to actual or potential speculative pressure.[13]

The case in which the probability of a jump is dependent of the position within the band is parametrized as follows: The probability of a jump as its highest is 0.005 at the boundaries of the band. For a revaluation the maximum is at the lower bound and for a devaluation at the upper bound. For the rest of the band the probability is determined as a quadratically decreasing function which reaches zero at the other boundary.

In the benchmark experiment the overall probability of jumps stays the same, the difference being that the

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[13]The trend towards smaller realignments can be observed in EMS as well as in Nordic countries. The devaluations used to correct the value of exchange rates were quite large until the middle of the 1980's but towards the end of the decade they have declined. The latest realignmets within the EMS in 1987 were between 0-3 per cent and in Finland the latest shift of the index band (revaluation) was 4 % in March 1989. Exact descriptions of foreign exchange policy measures can be found in central banks' bulletins from corresponding countries.

probability for devaluations as well revaluations will be constant throughout the band.

When dependence is assumed, the expected relative value of the index after the jump is in the middle of the new band. The variance of the expected value is set to 0.001. In the benchmark case the expected relative value of the index after the jump is the same as before the jump.

6. Simulation experiments

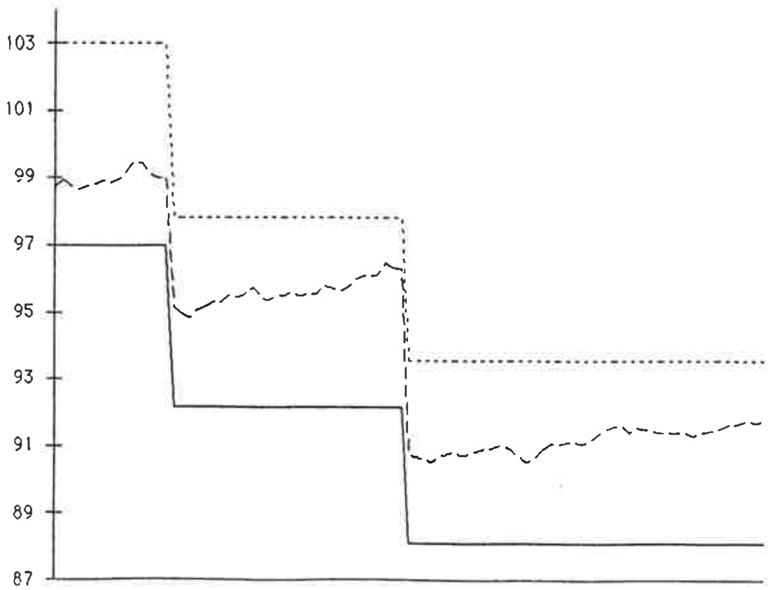
6.1. General considerations

To evaluate options on our index we start by using the above model to generate a data set. Once the data is given, the values of the currency options are computed.

A set of 30 000 exchange rates at the expiration date of the option are produced for each 11 initial states. The initial time to expiration in our simulations is 10 and 90 days.[14]

An example of the time path of the exchange rate produced by our simulation model is given in **Figure 2**. The exchange rate in the figure appreciates from 99 to 91.8 and two revaluations occur during 90 days.

Figure 2. Time path of the exchange rate (90 days)



[14]Initial experiments with 20, 30 and 60 days turned out to produce results that were in line with those for the extreme cases. Consequently these results are not reported.

The srike prices of the options are determined next. For our 11 starting values we define three corresponding strike prices. Thus at time 0 the options are either at-the money, in-the-money or out-of-the-money. The in-the-money and out-of-the-money options at time 0 are defined to have a strike price which is one standard deviation below or above the current rate.

The foreign interest rate is defined as the weighted average of interest rates of the currencies in the basket. We used the average of daily observations of the three months' currency basket eurorate for the year 1989, which was 9.9 %.

The domestic interest rate when determined by the model guarantees the elimination of expected speculative profits between the synthetic forward and domestic default-free money market instruments.[15]

However, even in the case of risk neutrality, the uncovered interest rate parity may be violated due to Jensen's inequality. This problem is known as Siegel's (1972) paradox.[16]

This convexity term is sometimes erroneously interpreted as a component of the risk premium, but it has nothing

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[15]See chapter 2 for a discussion of how the domestic interest rates are determined.

[16]Siegel's paradox states that, even in the absence of risk aversion, the efficiency condition in the forward foreign exchange market cannot be that expected nominal profits are zero. This is because nominal profits can be expressed in terms of either currency. It cannot be the case that the forward home-currency price of foreign currency equals the expected future spot rate of foreign currency $F=E(S)$, and the forward foreign price of home currency equals expected future spot price of home currency $1/F=E(1/S)$. Because of Jensen's inequality, expected nominal profits must exit - at least in terms of one of the two currencies.

to do with the investor's degree of risk aversion. For detailed discussion of the effects of Siegel's [1972] paradox, see Roper [1975], McCulloch [1975], Stockman [1978], Frankel [1979] and Sibert [1989].

The convexity problem of nominal interest rates arising from the Jensen's inequality is solved in our model using logarithmic values of the future exchange rates.

6.2 The results

In the following the results for two maturities are reported. These are 90 days, which represents a long time to expiration, and 10 days, which represents a short time. The results for these two maturies will be reported in parallel to highlight the effects of the time to expiration.

To make interpretation easier the main results of this study are reported in graphical form. Since the results will depend on the starting position of the exchange rate within the band, the graphs will report the outcome with the starting state on the category axis, the first state being the lower border and the eleventh or last state the upper border of the initial band.

The first results that are of crucial interest for how the options are priced are the domestic interest rates and the volatilities produced by the model. The fact that the exchange rate is not allowed to cross the border of the band unless there is a jump is clearly reflected in the domestic interest rates as seen in Figure 3a for the 90-day option, and in Figure 3b for the 10-day option.

In general, the border will push the exchange rate in the other direction. At the lower border this will produce an expected depreciation and high interest rates, and at the upper border an expected appreciation and consequently low domestic rates. This is most transparent in the case in which the jump probability is independent of the exchange rate position. In the 10-day case the presence of the border is clearly felt exclusively in the most extreme starting states.

Figure 3 a

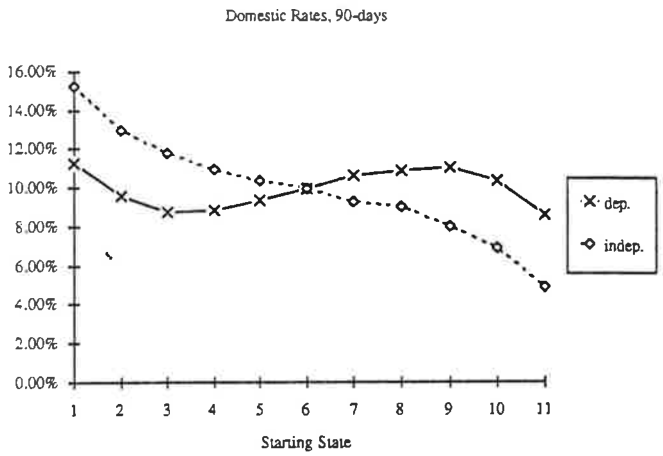
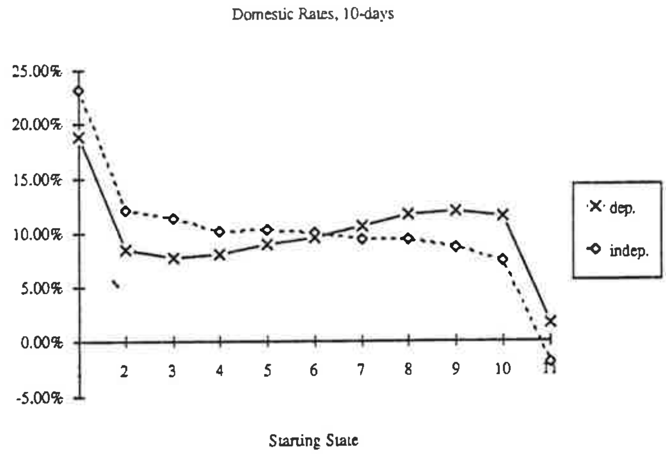


Figure 3 b



An interesting picture emerges if the level of the interest rate in the 90-day case is compared to the level of the interest rate in the 10-day case.

It is seen that a relatively steeply sloping yield curve is required to eliminate expected speculation profits close to the borders of the band. The fact that the domestic rate turns negative at the upper border for the independent case indicates that there are states in which independence between the jump and the position in the band can no longer be reconciled in the absence of speculative profits.

The main difference between our main case in which the probability of a jump depends on the state within the band, and the benchmark case, is that the effect of the closeness to the border of the band will be counteracted by an increasing jump probability in the dependent case. The outcome is an S-shaped pattern for the domestic rate, which is almost horizontal.

Figure 4 reports the observed average volatilities. The observed volatilities will to a large extent reflect jumps in the band. In the benchmark case these jumps are equally likely and of the same magnitude independently of the exact starting state within the band, and that translates into a volatility which is largely independent of the starting state. For the dependent case, however, the size of the jump will depend on the state in which it occurs.

Figure 4 a

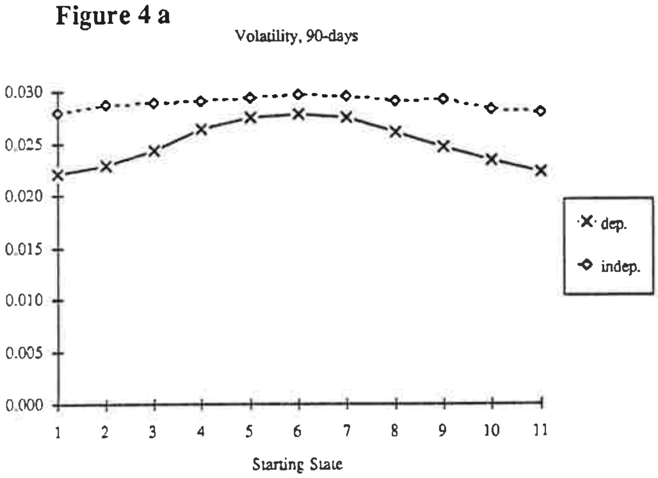
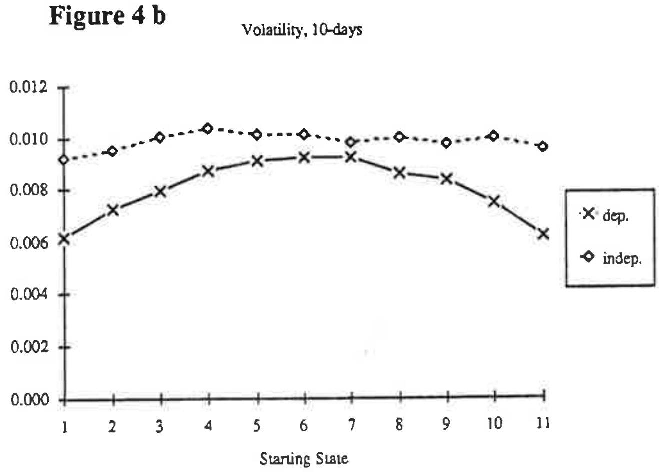


Figure 4 b



Close to the borders the volatility is affected by two counteracting tendencies:

1. The jump probability is higher in the dependent case than in the independent case, which implies a higher volatility for the dependent case.
2. The expected size of the jump is smaller in the dependent case [17], which implies a lower volatility for the dependent case.

---

[17]The expected size of the jump in the exchange rate at the border is in the independent case jump in border, whereas in the dependent case a c should be substracted.

It turns out that 2 will dominate 1 and the volatility will be markedly lower close to the borders in the dependent case.

Figure 5 reports the call option prices based on our simulation data. These prices are obtained using the formula:

$$(9) \quad C_t(s) = \left. \frac{e^{-r\tau} \sum_{i=1}^{30000} \max(S_{t+\tau}(\omega^i) - X,\ 0)}{30000} \right|_{S_t = s} ,$$

where τ is time to expiration , i indexes the simulation run, S is the exchange rate, and X the exercise price. Three sets of expiration prices where used: out-of-the-money, at-the-money, and in-the-money. The at-the-money options have an exercise price equal to the present exchange rate, the out-of-the-money an exercise price one standard deviation (computed ex post) above the present rate, and the in-the-money an exercise price one standard deviation below the present rate.

The results exhibited in Figure 5 clearly mimic the results for the interest rates reported in Figure 3. As expected this is most apparent when the relative impact of volatility is smallest, which is the case for the prices of the in-the-money options in 5e and 5f.

The relatively large difference for the out-of-the-money options close to the lower border of the band is due to the fact that the independent case makes the probability of an upward jump in the band (i.e. essentially in the "wrong" direction) much more probable than the dependent case does.

Figure 5 a

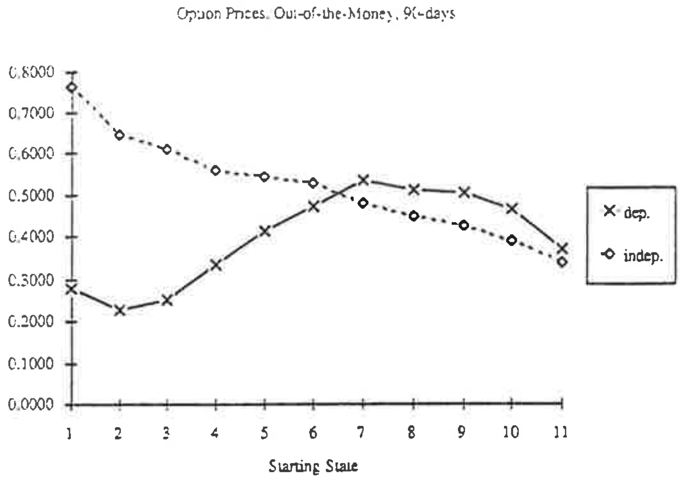


Figure 5 b

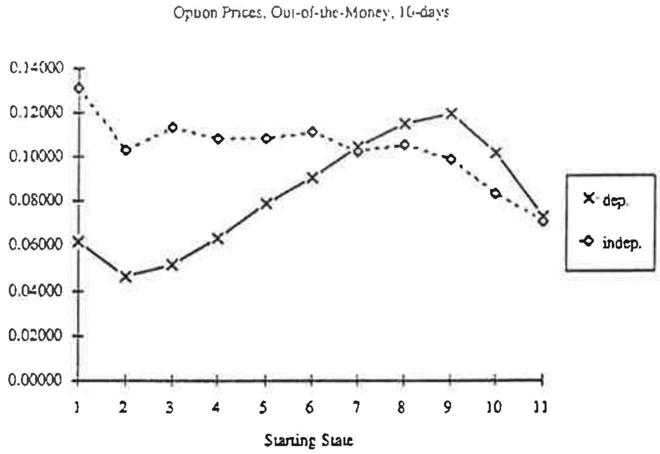


Figure 5 c

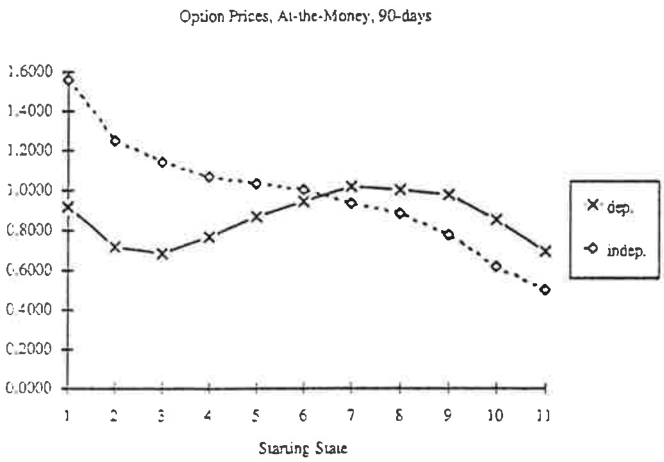


Figure 5 d

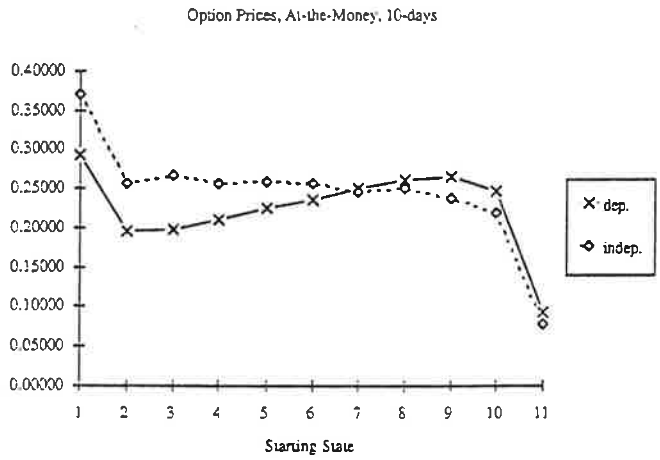


Figure 5 e

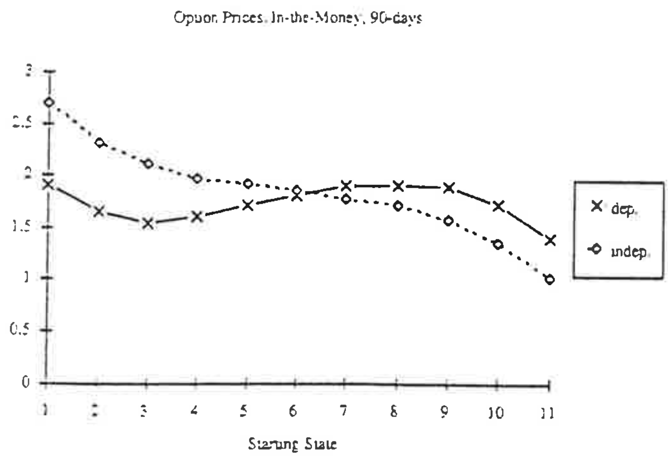
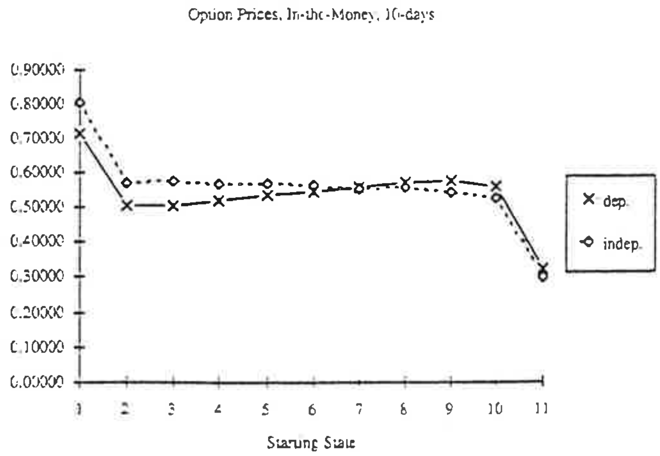


Figure 5 f



In the dependent case the probability of a jump in the wrong direction approaches zero as the exchange rate approaches the border, whereas for the independent case this probability stays on the same level as for all other states within the band.

As a consistency check the value of call options with exercise prices given by the lower border and the upper border were also computed. The results are reported in Figure 6.

Figure 6 a

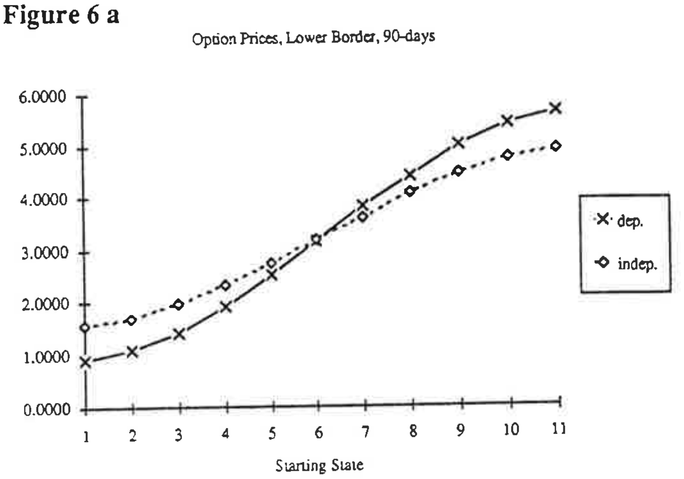


Figure 6 b

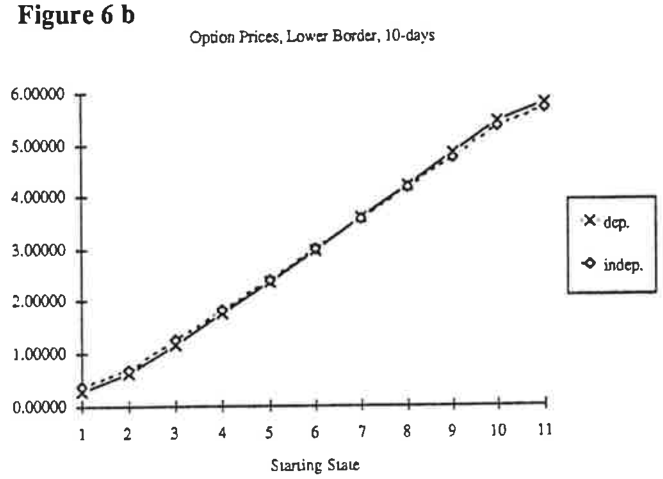


Figure 6 c

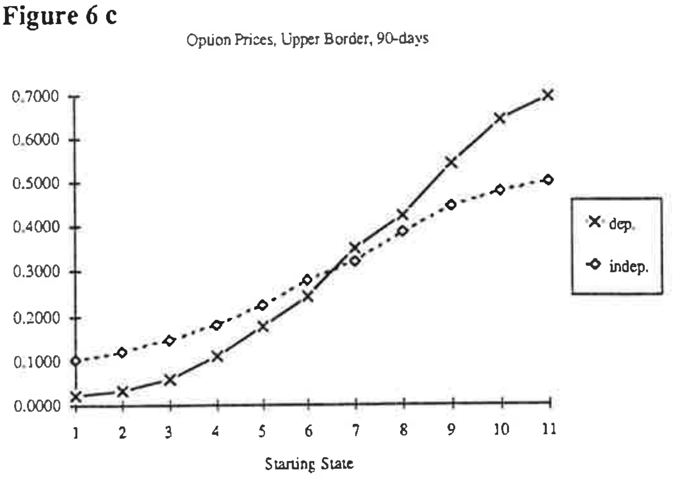
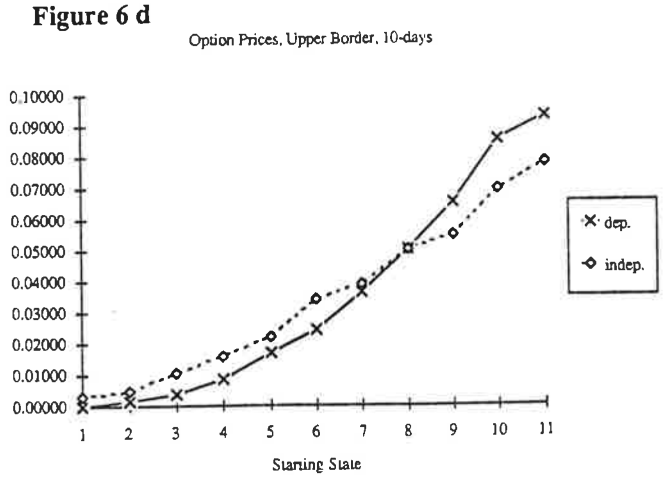


Figure 6 d



The prices for the independent and the dependent case follow each other quite closely, which shows that the degree of out-of or in-the-moniness is a more important explanatory factor for the observed option prices than is the difference between our main case and the benchmark. The observed main case versus benchmark difference is mainly due to the lower probability for jumps in the "wrong" direction and a higher probability for jumps in the "right" direction when the jumps depend on the location of the exchange rate within the band as compared to the independent case. This difference is less pronounced for the 90-day than for the 10-day option because of the tendency in the dependent case for a jump to switch the process to the middle of the band.

To be able to focus more directly on the impact of the differences in second and higher moments of the exchange rate distribution on the expiration date, we computed a new set of option prices with different striking prices. The striking prices in this new case were determined on the basis of the average log value of the exchange rate at the expiration date. This implies that the expiration prices will differ considerably between our main case and the benchmark case. However, the deviation produced by the differences in the expected exchange rate[18] on the expiration date is removed. The results based on the adjusted strike are reported in Figure 7.

Figure 7 reveals that adjusting the strike price to account for the expected change in the exchange rate will make the differences between the option prices correspond quite closely to the differences in volatilities as displayed in Figure 4 for the at-the-money as well as for the in-the-money case.

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[18] The stochastic component in the average as compared to the actual mean or the simulation error is disregarded.

Figure 7 a

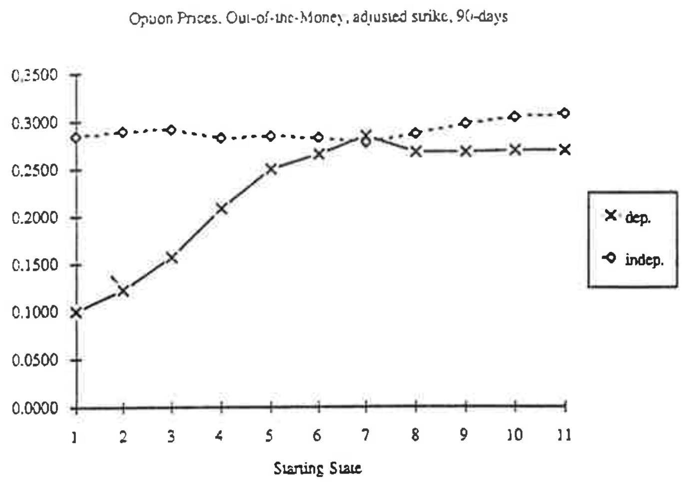

Figure 7 b

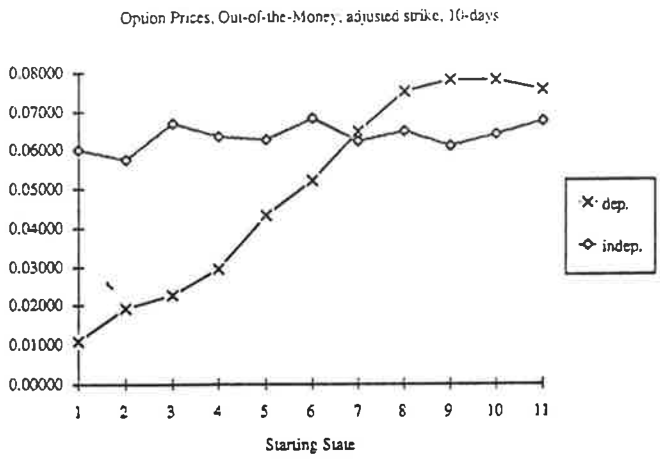

Figure 7 c

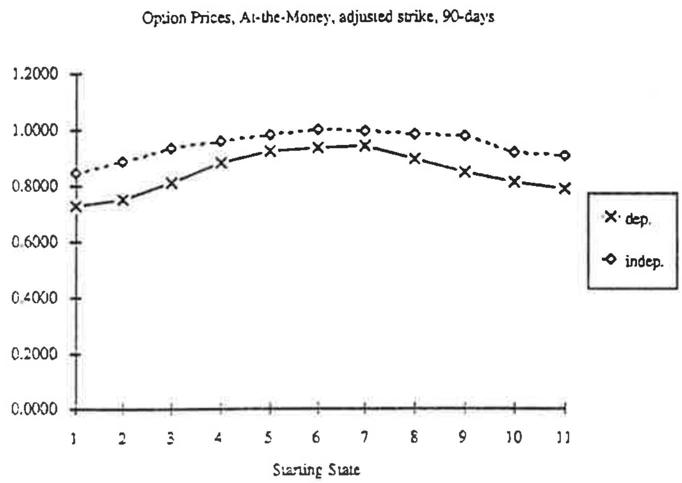

Figure 7 d

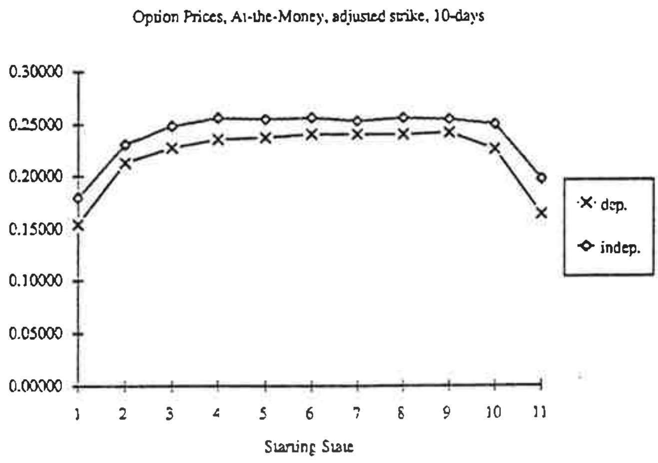

Figure 7 e

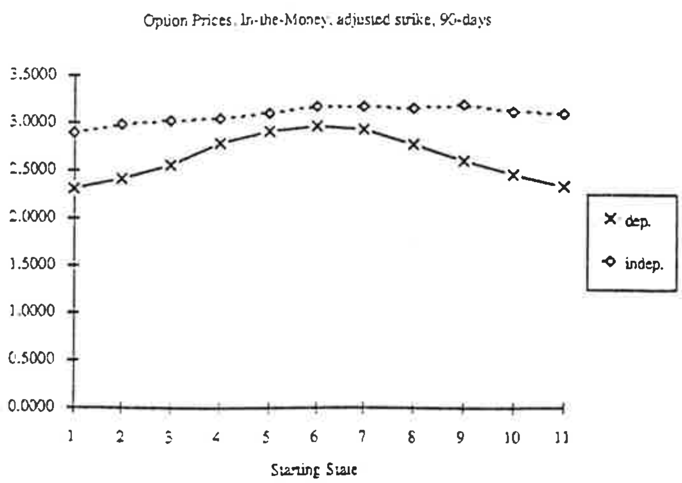

Figure 7 f

However, this is not the case for the out-of-the-money option. For that option the differences are most notable, as they were in Figure 5, for the states close to the lower border reflecting the higher probability for an upward jump in the independent case.

When moving from the first starting state towards the eleventh in the 90-day case, the option value computed for the dependent process approaches the value for the independent process up to the state 7, and then falls off again, whereas for the 10-day case, the value for the dependent process intersects the value for the bench mark, and stay above it up to and including the upper border. This difference between the two maturities is explained by the higher probability in the 10-day dependent case of an upward jump when the exchange rate is close to the upper border. This effect will have a smaller impact for the 90-day option. For that option the fact that after a jump the exchange rate is expected to land in the middle of the band, where the probability of further jumps is at its minimum, will make the overall effect of jumps smaller than for the 10-day option.

As a summary of the comparison of the option values produced under our specification of the dependence between the location within the band and the jumps produced by re- or devaluations and the option values produced under the assumption of independence, we conclude that the most notable difference will arise because of differences in the expected exchange rate change.

The independence assumption will produce the somewhat pathological result that the exchange rate must be expected to depreciate close to the lower border while it has to be expected to appreciate close to the upper border. A constant jump probability, furthermore, requires a steeply sloping yield curve for domestic interest rates close to the borders of the band, not to give rise to expected speculative profits. In general our results indicate that the shape of the yield curve is quite sensitive to the specific assumptions made about the exact exchange rate dynamics.

The largest differencies in option prices produced by the two models will arise for out-of-the-money options close to the lower border. Given the way the dependence between the location within the band and the jumps is modeled, the probability of a revaluation will be highest at the lower border. This will make the call option on the foreign currency far less valuable than in the independent case where the probability of a devaluation at the lower border is as high as of a revaluation.

6.3. Comparisons to the Garman-Kohlhagen model

Since the most widely used model for pricing exchange rate options is the Garman and Kohlhagen (1983) version of the Black and Scholes (1973) model, it is natural to ask how the prices produced by the present model will differ from those produced by the Garman and Kohlhagen model. The Garman and Kohlhagen model gives the following expression for the value of a call option:

(10) $$C = Se^{-r_f\tau}\cdot\Phi(d_1) - Xe^{-r_d\tau}\cdot\Phi(d_2),$$

where $$d_1 = \frac{\ln(S/X) + (r_d - r_f + \sigma^2/2)\cdot\tau}{\sigma\sqrt{\tau}}$$ and

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

When applying the above formula to the valuation of the options produced by our simulation experiment, proper values for the input variables have to be selected. Since all other input variables than the volatility normally will be observed, we will focus on the choice of the volatility measure. The foreign interest rate is taken to be the 9.9 % used in our simulations, and the domestic rate is the one produced as a result of our simulations. An analogous choice for the volatility parameter would be the volatility produced by our 30 000 simulation runs.

The other two candidates used in the present paper are: the correct volatility in the case of no restrictions for the autoregressive process, which is called the unconstrained standard deviation[19], abbreviated unc.std., and a simulated proxy for the standard deviation of the truncated process conditional on no jumps, which will be called the in-the-band standard deviation. The Garman-Kohlhagen option prices produced by these standard deviations are given along with our simulated option values in Figure 8.

Figure 8 reveals that our simulated option values in most cases will remain below the Garman and Kohlhagen option value based on our simulated standard deviation, whereas it will lie above the option values produced by the other candidates. The reason for the tendency of the simulated volatility to overestimate the true value of the option lies in the fact that the standard deviation is more sensitive to extreme values than is the option price itself.

---

[19]With no constrains the index will follow an AR(1) process. It is easily shown using the formula for the sum of geometric series that the variance of the index k-periods from now will be: $\sigma^2 \dfrac{1 - b^{2k}}{1 - b^2}$,
where σ^2 is the variance of the error term, k is the number of periods ahead, and b is the autoregressive coefficient in the model $|b|<1$.

Figure 8 a

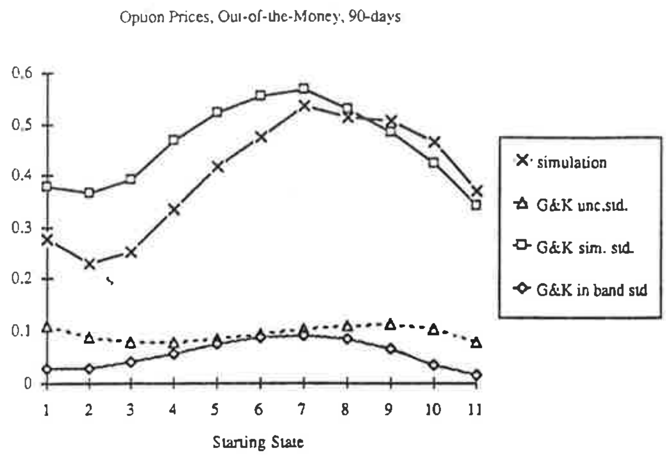

Figure 8 b

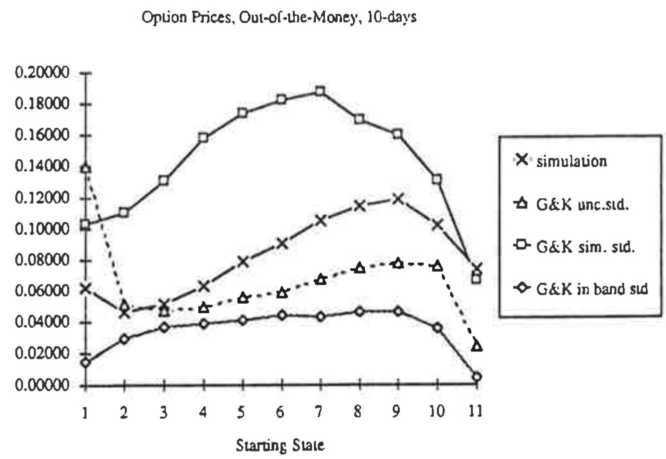

Figure 8 c

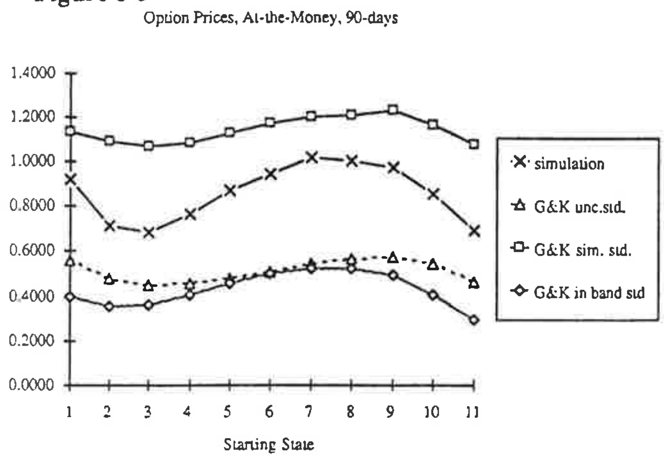

Figure 8 d

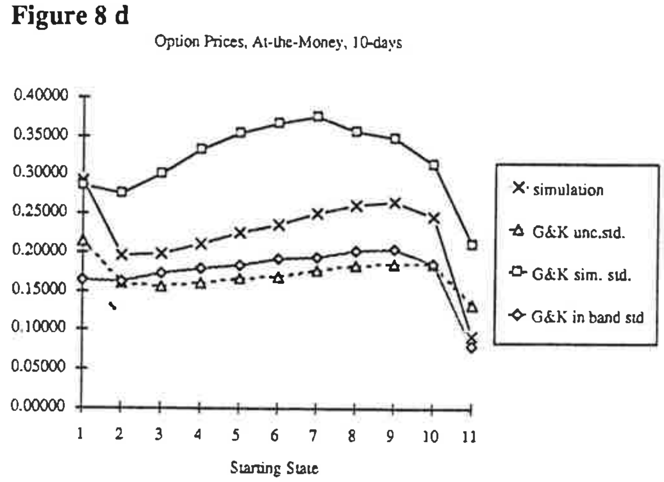

Figure 8 e

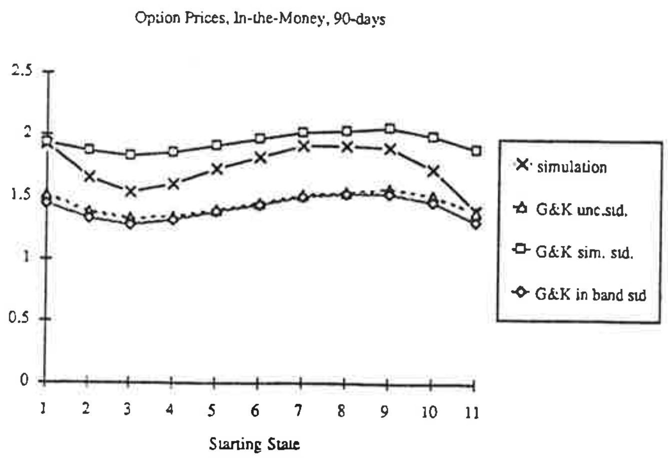

Figure 8 f

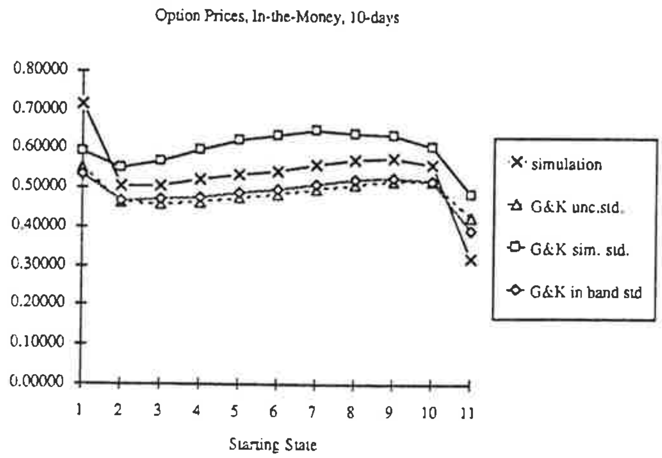

Figure 8 also shows that jumps may contribute significantly to the value of the option. This is especially true in the out-of-the-money case, where the use of a standard deviation estimated on observations that do not include a jump would produce grossly misleading results.

7. Conclusions

This paper uses a simulation model to analyze the consequences of removing the assumption of independence between the jump produced by re- or devaluations and the exchange rate position within the band. A simple form of dependence is introduced which makes the probability of a devaluation grow quadratically from zero at the upper border, while the probability of a revaluation grows quadratically from zero at the lower border.

Our results indicate that the expected change in the exchange rate in the independence case will produce unrealistic outcomes. To preclude profitable speculation the exchange rate must be expected to depreciate close to the lower border, while it must be expected to appreciate close to the upper border. It turns out that these phenomena are avoided if the occurrence of re- or devaluations depends on the location within the band, given a realistic set of parameter values.

The largest difference in the call option values produced by the two models are for out-of-the-money options close to the lower border. The constant probability of an upward jump in the independent case will drive up the option price as opposed to our dependent case in which it will be driven down by a relatively high probability of a downward jump. Even for in-the-money options a difference of 25 % at its maximum was observed for the 90-day option. Thus, whether independence between the jump and the location within the band is assumed or not seems to be of considerable importance for the pricing of an option on the index.

Finally, when the option values based on our simulation experiments were compared with the option prices calculated with the widely used Garman and Kohlhagen model, it turned out that the Garman and Kohlhagen model gave rise to over- as well underestimation. When the simulated standard deviation was used in the Garman and Kohlhagen model, the prices generally were above the simulated values, but when jumps were disregarded in the standard deviation the reverse turned out to be true. Once again the out-of-the-money options exhibited the most notable differences.

The present simulation model is still subject to several apparently unrealistic features. Perhaps the most striking is the fact that de- and revaluations are made dependent on the exchange rate position within the band in a rather simplistic fashion. A more realistic model would derive probability estimates for re- and devaluations from simulated interest rate differentials. That would guarantee realistic interest rate dynamics, which is not the case in the present model.

Another interesting extension of the present model would be to replace the given band in the model with an assumed target exchange rate supported by the central bank. The soft boundaries around this target level would produce a different stochastic process and consequently different option prices than the present model does.

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ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
LÖNNROTINKATU 4 B, SF-00120 HELSINKI

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