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INTANGIBLE INVESTMENTS IN A DYNAMIC THEORY OF A FIRM

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ABSTRACT: The paper introduces intangible capital to recast the dynamic theory of a firm. Sato-Beckmann neutral technical progress is assumed and a firm's R&D incentives in terms of the innovation technology are derived. An optimal non-lump-sum tax subsidy is derived to eliminate the disincentives due to the anticipated diffusion. The existence of a unique rest point is established. The strict conditions for the innovation technology and the anticipated diffusion, necessary for a unique stable manifold to exist, are derived.

KEYWORDS: Intangible capital, Innovations, Technical Progress

I Introduction

The standard model of a firm is based on an idea that a firm's assets can be aggregated into a sigle "stock of capital". This can be a highly restrictive approach. R&D effort is an example of resource use creating assets which are intangible in the sense that they subsequently show up as new technology and better know-how. Technical progress is most often taken as exogenous. Here it is, instead, regarded as an outcome of innovation technology entering the production function in the Sato-Beckmann neutral manner. One of the targets of the paper is to contribute to the modelling of the R&D incentives and intangible capital in a dynamic theory of a firm.

The suspision that socially suboptimal amount of resources is devoted to the innovative activity at the level of the firm goes back at least to Arrow's (1962) classic work on production of information. Two major potential failures in market allocation have since been identified. Both of these will be explicitely introduced in the model of a firm of the current paper. The firm is assumed to be endowned with production and innovation technologies both with diminishing returns. Innovative incentives are reduced due to imperfect appropriability. This argument is of an externality type and reflects the public good property of information.[1] It is the anticipated diffusion that operates as the social externality.

The paper shows in which way this externality can be integrated into the model of a firm.

Second, due to high riskiness of investments in new know-how, private risk aversion is viewed as creating strict financial constraints for firms carrying R&D programs. These constraints result from the well-identified informational asymmetries between the investing firms and the outside credit and equity markets. Not only is the adverse selection an obvious problem for outside suppliers of finance. No obvious mechanism exists for elimination of the potential for moral hazard either. By implication, inefficient sharing of economic risks arises in private markets. Management's risk aversion adds to this problem. As a consequence, mispricing of risks in terms of stock valuation is a logical eventuality (cf. footnote 8).

The allocational inefficiencies above seem to make a strong case for public intervention.[2] The paper introduces an initially neutral profit taxation scheme and derives an optimal R&D subsidy so as to equalize the private and social discount rates. Thereby it claims that it is suboptimal to tax away the economic rents if they are due to innovative activity. In contrast, distortionary taxation appears second best optimal.

Even in the absence of changes in relative prices between labor and capital, these inputs will adjust over time due to technical progress. The paper derives conditions for the existence of a unique long-term equilibrium, as well as for a unique stable manifold. The firm's optimal reaction to accelareted diffusion is characterized using phase diagram techniques.

II R&D Incentives at the Level of a Firm

II1. Production Technology and Technical Progress. Assume that the firm's existing production technology is summarized by a triple $\{f,\alpha,\beta\}$ where

(1) $y = \max Y = f(K^*,L^*)$

(2) $K^* = \alpha(Z)K, \quad L^* = \beta(Z)L.$

The following assumptions are made. For non-negative capital and labor inputs K^* and L^*, measured in efficiency units, $y > 0$. Second, f is continuous, twice differentiable with $f_{K^*} > 0$, $f_{L^*} > 0$, $f_{K^*K^*} < 0$, $f_{L^*L^*} < 0$, and the marginal rate of substitution $mrs(K^*,L^*) = f_{L^*}/f_{K^*}$ is diminishing. Third, for a single firm, the returns to scale are diminishing.[3]

In (2), it is assumed that there exist twice and continuously differentiable concave functions α and β which are cardinal and which map the set of pairs of non-negative productive inputs, measured in natural units, to the sets of inputs, meaasured in efficiency units. Variable Z is used to describe the state of current accumulated know-how. It is assumed that $\alpha(Z_o) = \beta(Z_o) = 1$, $\alpha(Z) \geq 1$, $\beta(Z) \geq 1$ if $Z > Z_o$, $\alpha' \geq 0$, $\beta' \geq 0$, $\alpha'' \leq 0$, $\beta'' \leq 0$.[4]

Equations (1) and (2) model disembodied technological process innovations. In the case of labor, this involves job training, for example. The firm is assumed to know the functions α and β with certainty.[5] Though it has been widely employed in growth theory (cf. Drandakis and Phelps (1966)), the assumption of disembodied technical progress is of course quite restrictive. For the stability analysis of the current paper, this assumption appears much less restrictive.

Though the current model is based on the assumption of diminishing returns, it is convenient to use the established terminology by calling the technical progress Sato-Beckmann neutral whenever the production function can be presented in form (1) and (2). The impact of technical progress on the marginal rate of substitution is then given by $mrs(K^*,L^*) = (f_L/f_K)(\alpha(Z)/\beta(Z))$. The Hicksian neutral progress is obtained as the special case where mrs(.) is unchanged (cf. Eichhorn (1978)). Moreover, the labor-augmenting Harrod-neutral ($\alpha(Z) = 1$) and the capital-augmenting Solow-neutral ($\beta(Z) = 1$) cases are also obtained as special cases. No non-malleability conditions are imposed for the capital. Hence, the elasticity of substitution ex post is assumed to equal to that of ex ante.

The adjustment of capital and labor crucially depends upon the way the technical progress changes their marginal productivities. These are given below:

$$
\begin{aligned}
f_K &= f_{K^*}(.)\alpha(Z) > 0, \; f_L = f_{L^*}(.)\beta(Z) > 0 \\
f_{KK} &= \alpha^2(Z) f_{K^*K^*}(.) < 0, \; f_{LL} = \beta^2(Z) f_{L^*L^*}(.) < 0 \\
f_Z &= K f_{K^*}(.)\alpha'(Z) + L f_{K^*}\beta'(Z) > 0 \qquad (3) \\
f_{ZZ} &= K[\alpha''(Z) f_{K^*} + K(\alpha'(Z))^2 f_{K^*K^*}] \\
&\quad + L[\beta''(Z) f_{L^*} + L(\beta'(Z))^2 f_{L^*L^*}] < 0.
\end{aligned}
$$

In the Sato-Beckmann case, new technology is thus assumed to improve the marginal productivity of both productive inputs, though in a biased fashion.

II2. Innovation Technology. The key concept in the current model is the innovation technology. The fundamental idea is as follows. Innovations are produced by a combination of existing knowledge (Z) and the flow of current R&D effort (measured by x) using a well-defined technology. The latter, to be denoted by h, maps the pairs of inputs (x_t, Z_t) to the set of non-negative real numbers, $dZ/dt \geq 0$, describing the accumulation of new know-how per unit of time,

$$(4) \qquad dZ/dt = h^f(x,Z).$$

where t is the time index. The output $h^f(.)$ depends critically upon the capability of the innovation unit, in a word "creativity". It is firm-specific and it distinguishes the innovation production possibility functions among firms. Its properties are also crucial for the stability conditions to be studied below. Since the focus here is on a single firm at the microeconomic level, the subscript f will be deleted.

An assumption is made that there are positive but diminishing returns to the R&D effort. Thus, $h_x > 0$ and $h_{xx} < 0$. It is important to note that this assumption rules out the most rapid solution. Due to diminishing returns, it is hence optimal for the firm to smooth out its R&D effort over time. Moreover, it is assumed $h_Z > 0$, $h_{ZZ} < 0$. It will be also assumed that $h(0,Z) = 0$ i.e. that with no additional effort, output is zero. In this sense, the inputs are complementary. The type of equilibrium obtained, hinges upon this assumption. The interaction between the two inputs is highly relevant for the dynamic properties. An assumption is made that $h_{xZ} < 0$. Intuitively, this amounts to saying that the marginal impact of an R&D effort is greater when very little know-how has been accumulated as compated to the case where more has been accumulated. Below, a strict upper limit will be derived for h_{xZ} for stability.

Finally, it is important to note that the condition $dZ/dt \geq 0$ builds into the model a strong irreversibility condition.

II3. Markets and Diffusion. Ruling product innovations out, output will be assumed to be priced competitively in the market place. However, higher efficiency brings along technological advantages for the firm in comparison with its competitors. This means that the innovative firm is assumed to capture some rents which are, however, subject to economic depreciation over time both due to imitations and competing innovative activities.[6]

From the society's point of view, no depreciation takes place. But a single innovative firm is assumed to face an anticipated diffusion process, depicted (cumulatively) as $\phi(\theta)$ with $\theta = Z-Z^P$ in figure 1. (For microfoundations, cf. Jensen (1982)). Initially, $\theta = 0$ since there is no gap between the private information Z and the public information Z^P. A successful innovation creates a positive wedge $\theta = Z-Z^P$ that, however, tends to be eliminated over time. The rate of anticipated diffusion ϕ_θ is fully exogenous for a single firm. For any particular diffusion process it is assumed to hold

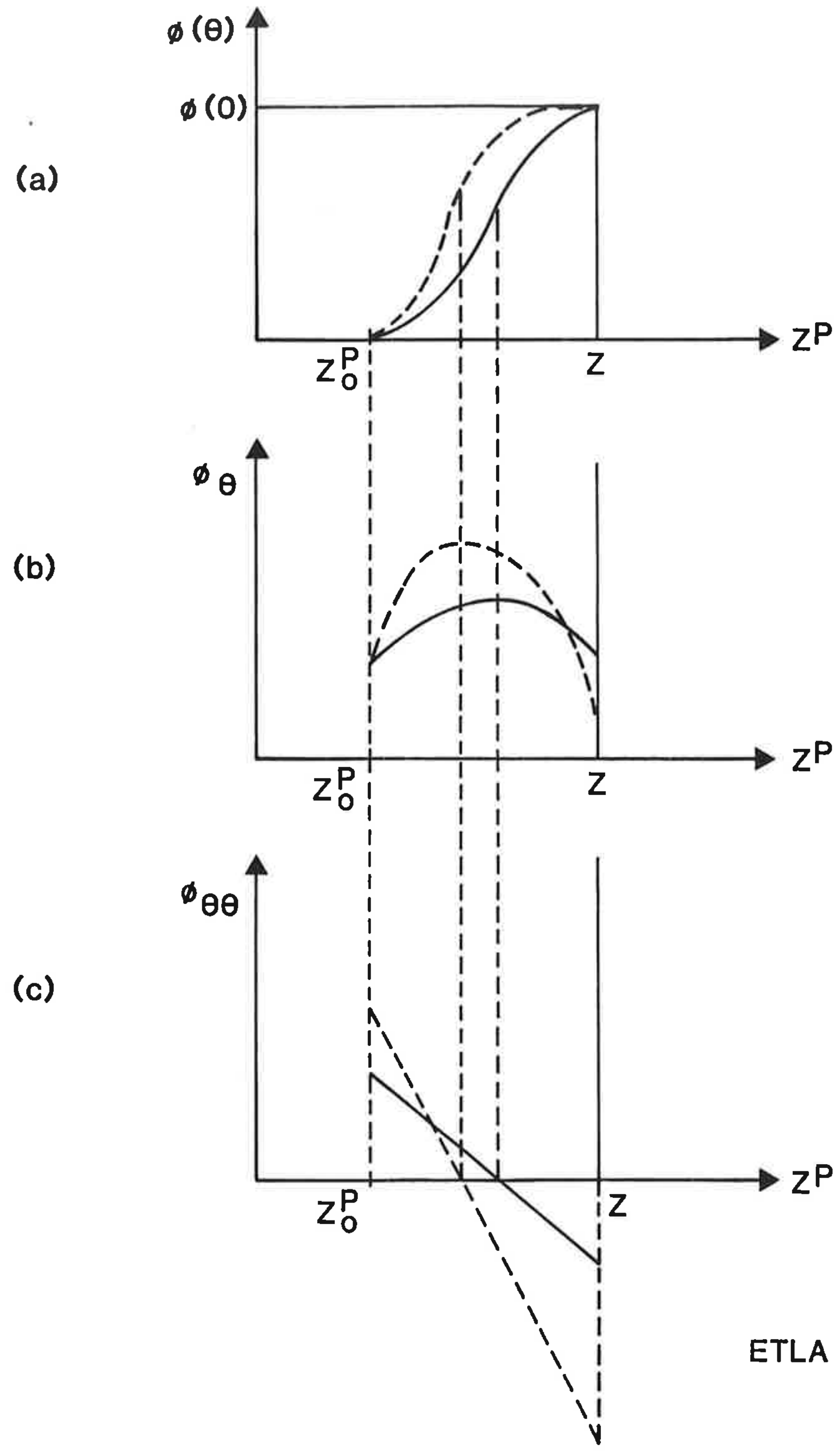
(5a) $\phi_\theta(0) = \lim_{\theta \to 0} \phi_\theta(\theta) > 0$

(5b) $\phi_{\theta\theta}(0) = \lim_{\theta \to 0} \phi_{\theta\theta}(\theta) < 0.$

Then the earnings flow of the firm allocated to profit distributions will be modelled as

(6) $\pi = [f(K^*,L^*) - \phi(\theta)] - mx - qj - wL.$

Figure 1. Expected Cumulative Diffusion.



Variables m and q stand for the prices of R&D input and capital goods (j), respectively and w for the price of labor.[7] The output price is used as the numeraire and set equal to one.

II4. Optimality Conditions. Suppose that the firm chooses its R&D, investment, and production program so as to maximize the functional (the share price)

$$(7) \qquad p(Z(t),K(t)) = \int_t^{\infty} \pi(s)\exp\{-r(s-t)\}ds$$

subject to (1)-(6), the initial conditions K_o, Z_o, and a motion $dK/dt = j - uK$, where $0 < u < 1$ stands for the economic depreciation of tangible capital assets. An assumption will be made that all risks are priced in the discount rate r.

Assume then that there exist continuous and differentiable functions of time, $\mu_Z(s)$ and $\mu_K(s)$ for the current marginal valuations of Z and K. Then the optimal program can be found from the extremal values of the following Hamiltonian function, defined in current values for all $t \leq s \leq \infty$

$$(8) \qquad H(x,j,L;Z,K;\mu_Z,\mu_K) = \pi + \mu_Z h(x,Z) + \mu_K(j - uK).$$

Any candidate for the optimal control has to satisfy the following necessary conditions

(9a) $\mu_Z = m/h_x, \quad \mu_K = q$

(9b) $f_Z = Kf_K{*}\alpha' + Lf_L{*}\beta' = \mu_Z(r - h_Z) + \phi_\theta - d\mu_Z/dt$

(9c) $f_K = f_K{*}\alpha(Z) = \mu_K(r + u) - d\mu_K/dt$

(9d) $f_L = f_L{*}\beta(Z) = w$

Later, strict conditions will be derived as to when the transversality conditions $\lim_t \mu_Z \exp(-r^*t) = \lim_t \mu_K \exp(-r^*t) = 0$ are satisfied.

Among the conditions (9a)-(9d), there are two major novelties. Conditions (9a) and (9b) suggest the particular way in which the economic incentives for R&D effort are related to the nature of the innovation technology and the expected diffusion. (9a) suggests that the equilibrium marginal valuation of new technical know-how (μ_Z) is related to the ratio of the cost of the R&D relative to the productivity of the marginal R&D effort. When substituted in (9b), the latter provides an analytic expression for the gross private discount rate (equilibrium required return) associated with the R&D program. It is given by $f_Z = (r - h_Z)m/h_x + \phi_\theta$. This return is inversely related to the marginal contribution of both current knowledge and the productivity of R&D to the accumulation of new knowledge. The marginal social externality (ϕ_θ) associated with the new information created privately appears as an additional private cost in this expression. This externality is precisely the difference between the private and social marginal returns and seems to provide a case for public intervention.

Secondly, the equations (9c) and (9d) implicitely provide conditions for steady state factor demands in efficiency units under conditions of endogenously evolving Sato-Beckmann neutral technical knowledge.

It should be noted that the equilibrium valuation of the firm is given by $p(Z,K) = qK + (m/h_x)Z$. Hence, the standard empirical approaches used to interpret Tobin's q-ratio can be quite misleading in as they neglect the intangible assets.

Unanticipated changes in relative factor prices have strong implications for the optimal policy (even if the firm has to take the α- and β-functions as given). For example, an increase in the wage-rental ratio pivots the isocost lines along the isoquants, conditional on output and accumulated technical know-how. However, the resulting adjustment (increase) in the capital- labor ratio changes the required equilibrium return on R&D capital f_Z, cf.(3). Then the nature of the technical progress determines whether the firm adjusts by investing more in the R&D. The condition for the latter case is $df_Z < 0$. This amounts to $(dK/dL) < (f_{L*}/f_{K*})(\beta'/\alpha')$ or

(10) $$\beta'/\alpha' > 1.$$

which requires that the technical progress is biased towards the labor-augmenting type. If this condition does not hold, the firm regrets ex post having made excessive R&D investments earlier. There is, however, no way out by adjustment because these investments are strictly irreversible. Subsequent reversal of relative factor prices or otherwise improved profitability provide channels out of the resulting disequilibrium. A similar reasoning applies if the wage-rental ratio changes due to a change in the discount rate.

III Aspects of Corporate Finance

Rather than claiming to present a rigorous analysis, this section more serves to demonstrate that the financial aspects may be crucial for the theory of the R&D capital. The model of this section thus is complementary to the basic model.

So far the optimality conditions have been derived for the case where information is perfect and equally distributed. There are three interest groups whose risks will now be briefly considered, i.e. current shareholders, the management, and potential new outside suppliers of funds.

No attempt will be made to model explicitely the game between the stock market and the management. Instead, it will simply be assumed that as a result of the prevalent principal-agent relationship, an implicit contract has emerged. It delegates the firm's operations to the management. The owners abstract from continuous monitoring effort due to high opportunity costs. Instead, it is thought that they have reserved the right to fire and replace the management team if it deviates "too much" from the owners' interest.

There is a simple way of making an allowance for the view that no first-best contracts actually can emerge because of informational asymmetries. In the investment program, the diverging interests of the management and the owners can be argued to be culminated in the return requirement, i.e. the discount rate. It will be assumed here that it is the discount rate of the management team, say r^*, not that of the owners, which determines the investment behavior. It is not implausible that due to the management's risk aversion, r^* exceeds the owners' rate of return requirement. To the extent some investment opportunities are therefore passed, there will be a residual agency cost reflected in share values and born by the owners.[8]

Next, a taxonomical assumption will be made that only tangible assets qualify as collateral. The outside suppliers of debt know that the R&D programs are highly risky. For the same very reason, new share issues are not regarded feasible. Then, due to adverse selection and moral hazard reasons, no external funds are available for these programs. Thus, if $\{B, r_B\}$ is the debt contract with B denoting the stock of (one-period, renewable) debt and r_B the interest, the following two financial constraints will be introduced

$$(11a) \qquad B = bqK \qquad\qquad 0 < b \le 1$$

$$(11b) \qquad \pi \ge 0$$

It will be shown that these constraints result in different required returns for the R&D and the capital investment, respectively. The redefined π is given by

$$(12) \qquad \pi = [f(K^*,L^*) - \phi(\theta)] - mx - qj - wL + dB/dt - r_B B.$$

From (11b), $\pi \ge 0$ requires $x \le x^* = (1/m)[f(K^*,L^*) - \phi(\theta) - qj - wL + bq(j-uK) - r_B bqK]$ along the path of adjustment. Let $p_x(t)$ denote the shadow price of the cash flow constraint (11b). Then define a new Hamiltonian function

$$(13) \qquad H' = \pi + \mu_Z h(x,Z) + \mu_K(j - uK) + + p_x[x^* - x].$$

For any given financial policy $0 \le b \le 1$, the optimality conditions for the investment program are

$$(14a) \qquad \mu_K = q(1-b)[1+p_x/m]$$

$$(14b) \qquad f_K[1+p_x/m] = \mu_K(r^*+u) + bq[(1+p_x/m)(r_B+u)] - d\mu_K/dt$$

$$(14c) \qquad \mu_Z = m(1+p_x/m)/h_x$$

$$(14d) \qquad (f_Z-\phi_\theta)[1+p_x/m] = \mu_Z(r^*-h_Z) - d\mu_Z/dt.$$

In the absence of full debt financing, the cash flow constraint significantly complcicates the description of the adjustment (though not necessarily the nature of the equilibrium). The mechanisms of major interest in the current paper can fortunately be looked into in the case with full debt financing of capital investments. Since this concerns also the analysis of public policy below, set b = 1. Then (14a) and (14b) simplify to $\mu_K = 0$ and $f_K * \alpha(Z) = q[r_B + u]$. Note, however, that this is the only case where the management's risk aversion does not lead to underinvestment of capital.

Consider now the adjustment of a firm to a reduction in the relative price of R&D inputs m/q with given factor prices. Since Z is predetermined, the firm does not tend to alter momentarily its capital or employment policies even though the constraints do not bind these decisions. Rather, it adjusts to the created R&D incentive by expanding its R&D budget as is described by (14c) and (14d). The speed of adjustment is, however, affected by the potential cash flow constraint. At any rate, it is optimal to cut back profit distributions and start adjusting capital and employment so as to enhance internal finance for the R&D budget. Accumulation of subsequent internal finance could be accelerated by current overinvestment in capital and labor. However, this will not be part of the optimal stretagy. The conclusion is that the latter option cannot release the firm from being constrained by its current cash flow.

IV Public Policy

The above analysis has traced two major allocational problems in the production of new knowledge. The first is the positive externality of new information. The second is private risk aversion, both of the management and of the financial markets. There is thius quite a strong case for public intervention.

Here the question is raised as to the optimal tax subsidy in the form of an optimal tax base adjustment of a firm. Since the innovations considered here do not give rise to monopolistic pricing, the consumer surplus can be taken to be zero in the welfare evaluation.

It would be unconventional to claim that a tax policy should be used for improving the allocation due to financial distortions. Hence, let us first focus on policy towards the externality using as the yardstick a full information economy with $r^* = r$. Assume that the R&D expenditures qualify for free write-offs, as they generally do. Consider an R&D related tax base adjustment $A(x)$. The tax base is then given by

(15) $$TB = [f(.) - \phi(\theta)] - mx - A(x) - r_B bqK - uqK - wL.$$

With $b = 1$, the firm's equilibrium again gives $\mu_K = 0$ (hence, $d\mu_K/dt = 0$ along the optimal path) and $f_K = q(r_B + u)$. If anything, this only states the expected result: under deductibility of the cost of borrowing and economic depreciation, the corporation tax does not distort the demand for capital in efficiency units.

As to the optimal policy towards Z, one obtains

(16) $\mu_Z = [(1-\tau)m - \tau A_x + p_x]/h_x$

(17) $(1-\tau)(1+p_x/m)(f_Z-\phi_Z) = \mu_Z(r-h_Z) - d\mu_Z/dt.$

The tax effect then depends on the initial state of the firm. Starting with the equilibrium, there is only the valuation effect on μ_Z (with $A(x) = 0$). Dividends are cut just to finance the tax surcharge. But a tax hike in the situation with a binding liquidity constraint $x = x^*$ raises the shadow price $\delta p_x/\delta\tau > 0$. This can be proved as follows. $\delta p_x/\delta\tau = (\delta p_x/\delta x^*)(\delta x^*/\delta\tau) = (\delta/\delta x^*)(\delta p(Z,K)/\delta x^*)(\delta x^*/\delta\tau) = (\delta^2 p(Z,K)/\delta x*^2)(\delta x^*/\delta\tau)$. This expression is strictly positive because both terms are greater than zero. Generally there are thus tax effects under the liquidity constraint, though they are operative during the phase of adjustment rather than in equilibrium.

Succesful R&D activity creates rents in the short run. Since they serve as an incentive for the R&D effort, the traditional argument for taxing away pure profits (Diamond and Mirrlees (1970) and Munk (1978)) does not apply here. Over the long run, the firm looses its rents and hence there will be no tax revenue, either.

During the transition phase to the long-run equilibrium, the anticipated diffusion greatly limits the firm's R&D incentives. A tax base adjustment of the type A(x) introduced above can be thought to be used to eliminate this disincentive. The first-order condition (17) is valid for a general $\phi(.)$-function and holds along the adjustment path. To obtain a closed-form solution for A(x), make for a moment a simplifying assumption that ϕ_θ is constant, i.e. $\phi(.) = \phi_\theta Z$. In equilibrium, the first-best allocation requires $f_Z = m(r - h_Z)/h_x$. Substituting in (17) and setting $p_x = 0$ it is possible to solve A_x and integrate to obtain A(x):

$$(18) \qquad A(x) = [(1-\tau)/\tau][h_x/(r-h_Z)]\phi_\theta x$$

where h_x and h_Z are evaluated at the equilibrium. The constant of integration has been set equal to zero to eliminate any lump-sum subsidies.

Under zero consumer surplus, A(x) given in (18) is the optimal non-lump-sum subsidy if aim is to eliminate the disincentives resulting from the social externality. The important property is that this subsidy is proportional to x, the R&D investments of the firm with a proportionality factor which depends on the present value of the diffusion losses. The relevant discount factor is given by $(r-h_Z)/h_x$, the first best required return on R&D. The negative dependence of the optimal subsidy on the tax rate has a clear-cut explanation. The base adjustment turns the corporation tax distortive, though it originally was neutral. Then, respectively, the desired distortion is obtained with a smaller base adjustment if the tax rate is correspondingly higher.

Constancy of ϕ_θ may not be a very good estimate if $\phi(\theta)$ is S shaped as in figure 1. The S shape of the cumulative diffusion process does not, however, necessarily imply that the above-proposed present value is not be a reasonable estimate for the present value of the actual diffusion losses.

The results reported in the previous section show the various impacts of the informational asymmetries on the firm's decisions concerning R&D. A question can be raised whether the tax system could or should be used to improve the allocation. At the very general level the answer is affirmative. No doubt, financial matters have been important in the public policy towards the R&D subsidies. It is feasible to derive some precise results of appropriate subsidies within the scope of the current model. This is, however, quite tedious and probably does not add to the insight. A more comprehensive subsidy is also of the proportional type. But the proportionality factor is more complicated.

V Existence of Unique Equilibrium and Adjustment to Accelerated Diffusion

One of the novelties of the above model with intangible capital is its true intrinsic dynamics contrary to models that do not allow for endogenous technical innovations. This is because of diminishing returns to the innovative activity, though the financial constraints may slow down the adjustment. In technical terms, since the Hamiltonian function is concave in x, the optimal path will not in general be of the bang-bang type. Consequently, productive inputs will also have smooth adjustment patterns even in the absence of costs of adjustment. Thus, along the optimal path, capital and labor will adjust even if there have been no changes in the past or anticipated future relative prices of these inputs. This is something which is lacking from the existing models. Moreover, depending on the nature of the technical progress, the capital investments and employment decisions of the firm are affected by the tax treatment of intangible capital. This is of some interest from the policy point of view.

An accelerated diffusion will obviously cut down the firm's incentives for R&D investments. To gain more insight into this problem and to derive the precise stability conditions one needs a closer look into the equilibrium and the dynamics. Let us abstract from the flow constraints and taxes and return to the basic model of section II. It pays to simplify the notation as follows, $\mu_Z = \mu$.

Then, the dynamics are described by the following motions (K is a jump variable)

(19a) $dZ/dt = h(x,Z)$

(19b) $d\mu/dt = \mu(r-h_Z) - (f_Z-\phi_\theta)$.

It should be noted that this system is non-autonomous because of the exogenous time-dependent drift term $Z^P{}_t$ in $\phi_\theta(Z-Z^P)$. However, the behavior of the system can be stated in the neighbourhood of the rest point, (Z^o,μ^o), provided the latter exists.

From $\mu = m/h_x$, the optimal control reads as

(20) $x = x(\mu,Z)$

with the partial derivatives

(21) $\delta x/\delta\mu = -m/h_{xx}\mu^2 > 0$, $\delta x/\delta Z = -h_{xZ}/h_{xx} < 0$.

In this model, Z is the predetermined and μ the forward-looking state variable, respectively. Then one has the following necessary conditions for a (unique) stable path to exist:

(22) $d(dZ/dt)/dZ = h_x x_Z + h_Z < 0$

(23) $d(d\mu/dt)/d\mu = r - h_Z - \mu^o h_{Zx} x_\mu > 0.$

Thus, a high enough discount rate is necessary for stability. Moreover, to have any hope of stability, inequalities (21) and (22) dictate quite a stringent condition for the innovation technology:

(24) $h_{xZ} < h_Z h_{xx}/h_x < 0.$

In words, h_x has to decline sufficiently rapidly in Z so as to keep the adjustment on a stable manifold. The existence of such a manifold, will be proved below. The existence of a unique rest point can be proved as follows. Ask, whether there exists a triple $\{x \geq 0, Z > 0, \mu \geq 0\}$ that satisfies both of the motions when $dx/dt = dZ/dt = d\mu/dt = 0$. Note that $h(0,Z) = 0$ for all $Z \geq 0$. Hence, it does not determine Z uniquely. However, from (19b), $d\mu/dt = 0$ gives a condition $\Omega(Z,x,\mu,Z^P) = 0$. Set $Z^P = Z$. Then, $h(x,Z) = 0$ requires $x = 0$ and μ can be eliminated from $\mu = \mu(x,Z) = m/h_x(x,Z)$. Thus $\Omega = 0$ can be stated as a function of Z only, say $\Omega^*(Z) = 0$ to solve for Z. A sufficient condition for this particular Z to be positive is provided by the equality

(25) $f_Z(0) + \mu(0,0)h_Z(0,0) > \mu(0,0)r + \phi_\theta(0).$

We have no difficulty whatsoever in accepting this requirement. Monotonicity of Ω^* guarantees uniqueness of the stationary Z-value. Then the proof has been given to

Proposition 1. The system (19a)-(19b) has a unique rest point (Z^o, μ^o) with $x = 0$ and $Z > 0$.

The conditions for the existence of the unique stable manifold can be stated as follows. Close to the rest point, the system obeys

$$(26) \qquad \begin{bmatrix} dZ/dt \\ d\mu/dt \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} Z - Z^o \\ \mu - \mu^o \end{bmatrix}$$

where $J_{11} = h_x x_Z + h_Z, \quad J_{12} = h_x x_\mu,$

$J_{21} = -\mu^o[h_{ZZ} + h_{Zx}x_Z] - f_{ZZ} + \phi_{\theta\theta}$

$J_{22} = r - h_Z - \mu^o h_{Zx} x_\mu$

By (22) $J_{11} < 0$, by (21) $J_{12} > 0$ and by (23) $J_{22} > 0$. One more condition is needed to guarantee that $Det(J) < 0$. It is sufficient that $J_{21} > 0$. For this to hold it is required that the rate of diffusion satisfies the following "smoothness" condition

$$(27) \qquad 0 > \phi_{\theta\theta} > f_{ZZ} + \mu^o[h_{ZZ} + h_{Zx}x_Z]$$

The smoothness condition states that the rate of diffusion is not expected to decline too rapidly towards the end of the diffusion stage (i.e. increase too fast after the new innovation). Then one has:

<u>Proposition 2.</u> Under the assumptions (21), the conditions (22) and (23), and the smoothness condition (27), there exists a unique stable manifold.

Given that the eigenvalues of the matrix $J = (J_{ij})$ are both real and of opposite sign, it remains to be asked whether it is optimal for the firm to follow the stable path. Regardless of the values obtained by the controls, the Hamiltonian function given in (8) is concave with respect to the state Z. Thus it is concave also along the path suggested by the first-order conditions. No doubt hence remains as to the optimality of the stable manifold. That path is governed by the eigenvalue

$$(28) \qquad \sigma = (J_{11}+J_{22}) - \sqrt{(J_{11}+J_{22})^2 - 4(J_{11}J_{22}-J_{12}J_{21})} < 0.$$

Then the slope of the stable path is given as the ratio of a_2/a_1 where $a = [a_1 \quad a_2]'$ is the right-hand eigenvector corresponding to the stable root σ. Solving from $Ja = \sigma a$, one obtains

$$(29) \qquad a_2/a_1 = J_{21}/(\sigma - J_{22}) < 0.$$

Accelerated expected diffusion now means a path II in figure 1a. This means rotation of the $\phi_{\theta\theta}$ function in figure 1c with a reduction in $\phi_{\theta\theta}(0)$. Then J_{21} gets smaller and the eigenvalue σ larger. This rotates the path of the stable manifold downwards from ss to s's', as depicted in figure 2. Adjustment to an accelerated diffusion hence involves both a "jump" to a less steep saddle-path and the subsequent adjustment to the new equilibrium characterized by a lower value of the state variable Z. Note that the adjustment process is governed more by the innovation technology and the diffusion process than the type of technical progress emerging. The latter is more relevant for the time path of the capital-labor ratio.

The above dynamic analysis obviously serves as an argument for public intervention. Even temporary positive rents are necessary for sufficient R&D incentives. Two additional mechanisms studied in the precing chapters strengthen this argument. First, due to informational asymmetries, credit constraints can make the adjustment process suboptimally slow. Second, given that the corporation tax falls on the rents, the tax system itself may spur a suboptimal adjustment path.

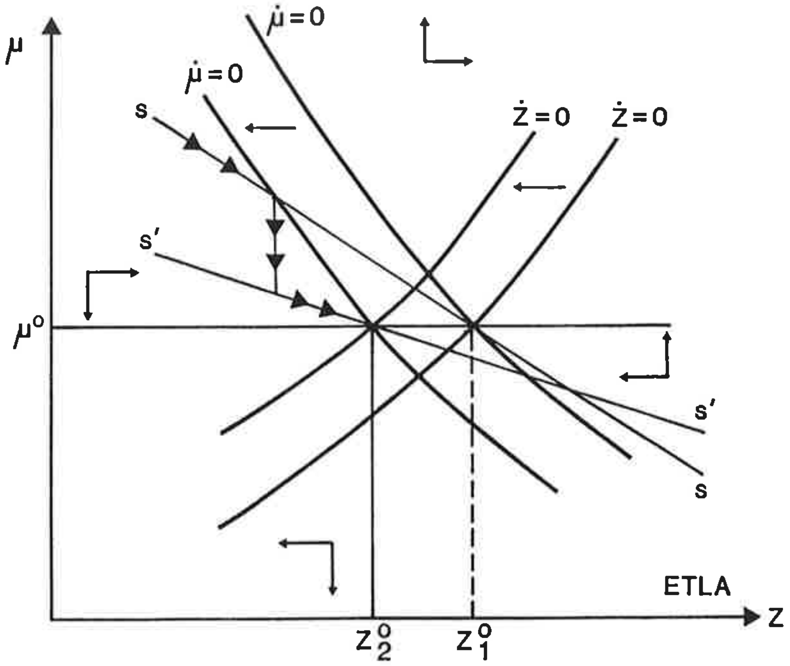


Figure 2. Adjustment to Accelerated Diffusion.

VI Final Remarks

This paper has presented a dynamic model of a firm's R&D incentives in terms of its innovation technology and the anticipated diffusion. The innovations have been assumed to interact with the productivity of factors in the Sato-Beckmann neutral manner. However, the invention possibility hypothesis of Kennedy (1964) and Drandakis and Phelps (1968) has not been introduced because factor prices have been assumed given and because the focus has been on the allocation between current assets and new technical know-how. In this sense, the current model is complementary to those two.

Footnotes:

1. This view has been challenged by Dasgupta and Stiglitz (1980) who emphasize competition in the innovation race.

2. In practise, a variety of various types of R&D subsidies have been experimented. For a summary, see Justman and Teubal (1986) or Fölster (1989).

3. In the growth models of macroeconomics, it is usually assumed that the aggregate production function is of the constant returns to scale type. Here the assumption of diminishing returns is more appropriate because then each firm has an optimal size. On the other hand, more than one firm is needed due to the diffusion phenomenon.

4. A justification for the formulation (1)-(2) is given by Sato and Beckmann (1968). They show that if the technical progress is such that the elasticity of factor substitution is unaffaected as income shares are constant, (1)-(2) can be derived from a simple assumption of the relationship between the elasticity of substitution and the share of capital.

5. More generally, if changes in future relative prices of productive inputs are anticipated, the firm will target some input-related innovations among a set of potential innovation processes. Just for the purposes of the current analysis,

static expectations are assumed with respect to input prices. This justifies the focus on the given α- and β-functions.

6. Thus, no monopoly situation will be created and no overpricing relative to the competitors can occur in spite of existence of positive rents. This conclusion is important for the welfare evaluation below.

7. If x is acquired from markets, m is the market price. Otherwise it is the non-observable shadow value of R&D resources tied to the innovation process. When an advanced technology is concerned, the markets for R&D inputs, like qualified human capital, may be highly imperfect. It would be possible to incorporate this extention to the analysis by assuming $m = m(Z)$ with $m' > 0$, $m'' > 0$.

8. Securities are priced in the market place according to public information by the individual shareholders. The management does not participate in portfolio investment and is unable to diversify its own risks, i.e. return on its human capital. Assuming an exponential utility of management, say $U(A) = -\exp(-cA)$, $c > 0$, and normal distribution of its income A (it could be included explicitly in the analysis above, though this would unnecessarily complicate the model), the risk premium is $cVAR(A)/2$. The inequality $r^* > r$ can be defended on the grounds that the variance is priced by the management in the required rate of return while the

shareholders only price the covariance with the market, as the CAPM suggests.

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