ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

Keskusteluaiheita
Discussion papers

Pekka Ilmakunnas

LINKING FIRM DATA TO MACROECONOMIC DATA: SOME THEORETICAL AND ECONOMETRIC CONSIDERATIONS

No 310 27.12.1989

ISSN 0781-6847

ILMAKUNNAS, Pekka: LINKING FIRM DATA TO MACROECONOMIC DATA: SOME THEORETICAL AND ECONOMETRIC CONSIDERATIONS. Helsinki: ETLA, Elinkeinoelämän Tutkimuslaitos, The Research Institute of the Finnish Economy, 1989. 38 p. (Keskusteluaiheita, Discussion Papers, ISSN 0781-6847 ; 310)

ABSTRACT: It is common to see in the forecasting literature sales forecasting models where a firm's sales are regressed on activity variables which include for example other industries' production, GNP, consumer expenditure on certain type of goods, disposable income, etc. However, there is often no discussion on what kind of economic behavior is assumed to be behind the model. This paper attempts to formulate firm-level demand models which are consistent with the relevant economic theory. Separate models are formulated for firms selling consumer goods and for those selling producer goods, or intermediate products. In each case it is shown how the assumed behavioral assumptions about the buyers affect the structure of the forecasting model and the choice of variables in the model. The theoretical framework is the differential approach to demand and production theory.

The basic goal is to formulate firm-level demand models, which could be used in practical sales forecasting. Therefore the models have to be fairly simple, but still consistent with economic theory. The models are specified as "macro to micro" models in the sense that the aggregate behavior of the demand side of the market is used for explaining the sales of individual firms. Using separability assumptions it is possible to decompose the demand decisions to several steps. In the final step, there is a decision between fairly homogenous goods produced by different firms in an industry.

The paper includes also an analysis of some possible estimation problems in this kind of models: misspecification of the relevant market too narrowly and aggregation over different submarkets in the demand side. There is also discussion about the interpretation of the price effects when a forecast is made, and on the possible effects of omitting price variables from the model.

KEY WORDS: Demand theory, production theory, sales forecasting

1. Introduction

It is common to see in the forecasting literature sales forecasting models where a firm's sales are regressed on activity variables which may include other industries' production, GNP, consumer expenditure on certain type of goods, disposable income, population, etc. (e.g. Klein (1969), Elliott (1972), Macintosh, Tsurumi and Tsurumi (1973), Browne and O'Brien (1974), Naylor (1979), Ilmakunnas (1987a, 1988)). In addition, the models may include price terms, advertising expenditure or other marketing variables. This kind of models are typically formed for forecasting corporate or division level aggregate sales, whereas individual product demand models may be more detailed. In the short run, sales may fairly accurately be forecasted using e.g. ARIMA models. The role of econometric models that contain environmental variables is in the longer run analysis. By relating the firm's sales to exogenous variables, it is possible to consider how alternative economic scenarios affect sales. However, there is often no discussion on what kind of economic behavior is assumed to be behind the model. This paper attempts to formulate firm-level demand models which are consistent with the relevant economic theory. Separate models are formulated for firms selling consumer goods and for those selling producer goods, or intermediate products. In each case it is shown how the assumed behavioral assumptions about the buyers affect the structure of the forecasting model and the choice of variables in the model. The models are specified as "macro to micro" models in the sense that the aggregate behavior of the demand side of the market is used for explaining the sales of individual firms. This approach should not be used for variables that are in the firm's control, i.e. that are endogenous to the firm, like production or employment. For these variables in the supply side of the market, the macro behavior is determined as the sum of the behavior of the micro units.

Forecasting models for a firm's sales can be derived fairly straightforwardly from modern demand and production theory. When forming the models, the following considerations are taken into account. First, the models should be simple, but still consistent with economic theory. This implies, among other things, that although demand models are typically estimated as a system of equations, in a company demand model one may want to consider a one-equation model and the ignore demand equations for competing products, although possibly taking into account the restrictions the systems approach imposes on the separate demand functions. In addition, the models should be easy to estimate, i.e. they should be linear or log-linear. Secondly, each model should have as explanatory variables such price, income or output variables for which it is easy to obtain forecasts. These forecasts could be available from different published forecasts, market research companies, firm's own information etc., but preferably they are such that the firm needs no elaborate forecasting models to forecast these exogenous, or environmental, variables.

The basic idea in the following analysis is that decision processes in the demand for different types of goods can be broken down to several steps: first there is a choice between broad aggregates of goods, in the second step between different types of goods in such an aggregate, and so on. Finally, there is choice between fairly closely related goods produced by different firms. At each stage, it can be assumed that consumer expenditure to be allocated between the alternatives has been predetermined in a previous choice. The stepwise decision process requires certain separability conditions to hold in the preferences. This kind of allocation models are widely used in demand analysis (e.g. Theil (1980a,b), Deaton and Muellbauer (1980)). Theil (1979,1980b), and Clements and Selvanathan (1988) have advocated use of allocation models also in marketing research. In the demand for producers' goods, the predetermined variable is the output in the industry buying the product. The production theory models are not allocation models in the sense that total expenditure (cost) cannot be treated as a predetermined variable. However, even in these models it is possible to use a stepwise decision process, assuming suitable separability conditions to hold in the technology (e.g. Theil (1980a,b), Fuss (1977)).

The outline of the paper is as follows. In sections 2 and 3 the consumer goods and intermediate products models are set up. In section 4 modelling of exports and imports is briefly considered. In section 5 modelling of price effects is discussed. Finally, in section 6 there is an analysis of typical estimation problems in this kind of models.

2. A model for consumer goods

2.1. The basic model

Consider first consumer goods industries. Consumers allocate their total expenditure on product groups A,B,...,Z. In group A, expenditures are allocated between firms 1,2,...,n, assuming that the products of these firms are fairly close substitutes. More steps could be included in the decision process, but here only two are used for simplicity. The first choice depends on the price aggregates of the groups, $P_A,\dots,P_Z$, and the total expenditure E. The second step choice, on the other hand, depends on the expenditure allocated to this group, E_A, and on the prices of the different firms, $p_1,\dots,p_n$. The firms are assumed to behave as price setters. For this choice to be independent of the prices in the other groups, B,...,Z, it has to be assumed that the different goods or brands in group A are weakly separable from the other groups in the representative consumer's preferences. However, conditions on the first step choice are more restrictive, if the assumption is maintained that only group price aggregates are used in the choice between groups. One possibility is to assume strong separability, or groupwise additivity of preferences. Less restrictive is to maintain weak separability, but to treat the relationships as approximations (see Deaton and Muellbauer (1980)). In particular, the group price indexes should depend on the utility, but if it is assumed that variation of the price as a function of utility is not large, available price indexes may be used for the group prices. Finally, homotheticity of the preferences could be assumed, but this would imply unitary income elasticities, which is too restrictive in most applications.

Depending on the functional form and whether a utility, an indirect utility or an expenditure function is used for deriving the demand equations, it is possible to use as dependent variables expenditure shares, expenditures or real expenditures. The quantity model is the most appropriate for the present purposes.

Consider firm 1 in industry A. When the dependent variable is quantity, the first step decision yields real expenditure equation $Q_A=Q_A(P_A,...,P_Z,E)$. Corresponding nominal expenditure is $E_A=P_AQ_A$. The second step determines real sales equation

$$q_1 = q_1(p_1,...,p_n,E_A)$$

$$= q_1(p_1,...,p_n,Q_A(P_A,...,P_Z,E)P_A). \qquad (1)$$

This can also easily be expressed in terms of nominal expenditures and sales. Economic theory imposes several constraints on the model. The demand functions have to be homogenous in prices and income, the expenditures on different goods have to add up to total expenditure, and the Slutsky matrix of compensated price effects has to be symmetric and positive semi-definite. If only one equation is estimated, as would be a practical procedure in sales forecasting for an individual firm, only homogeneity can be imposed. Therefore the explanatory variables in the sales forecasting models would in practice be relative prices and real expenditure.

These models show that a firm's sales can be related either to group real expenditure Q_A or total real consumer expenditure (or disposable income). The choice between the explanatory variables is determined partly by the availability of expenditure forecasts. There may be a tradeoff between modelling narrow market segments carefully but not having a forecast of such a narrowly defined expenditure category available, and modelling large market segments for which an expenditure forecast is available. If a higher-level expenditure variable is used, also more price variables may be needed. However, in practice some relative prices may be assumed to be unchanged in the forecast period, so that some of the price terms may be ignored. Alternatively, if it is assumed that the preferences are separable so that goods in different groups are independent (blockwise independence), or that all goods are independent (preference independence), less price terms are needed in the model.

In the case of share equations, in the first step the expenditure shares of the groups are determined. They are $W_i=P_iQ_i/E=E_i/E$, $i=A,\dots,Z$. Specifically, the share W_i is a function of the prices $P_A,\dots,P_Z$, and expenditure E. In the second step, the market shares of different brands in each group are defined conditionally on expenditures on that group, e.g. w_1' is a function of the prices $p_1,\dots,p_n$, and group expenditure E_A. The conditional share is defined as $w_1'=p_1q_1/E_A$. Alternatively, the unconditional share is defined as $w_1=p_1q_1/E=w_1'W_A$. If one combines the steps, the share equation gives real sales for firm 1 in the form

$$q_1 = w_1'E_A/p_1 = w_1'W_AE/p_1=w_1E/p_1. \qquad (2)$$

This does not easily yield a model where q_1 is directly explained by prices and expenditure. Instead, w_1 and E would have to be forecasted separately and then a forecast for q_1 is obtained.

The share equations can take different forms. For example, the almost ideal demand system has expenditure shares as functions of the logarithms of prices and income, and the translog indirect utility function yields nonlinear share equations. On the other hand, if some of the shares can be assumed constant, the model is considerably simplified. The corresponding price terms can be ignored and, assuming the shares known, no estimation is needed. An application of the constant shares assumption can be found in Goudie and Meeks (1984), where firm-level variables are linked to a macro model.

In this paper the quantity model will be used. There are several alternative forms, e.g. the linear expenditure system and log-linear demand equations. The latter is economically less justified, as discussed e.g. by Deaton and Muellbauer (1980) and Lau (1986), but many sales forecasting models are in this form: log of real sales is regressed on the log of an activity variable. The firm-level demand function is

$$\log q_1 = \alpha_1 + \Sigma_j \beta_{1j} \log p_j + \theta_1 \log E_A + u_1 \qquad (3)$$

However, the firm-level demand functions should also fulfill the adding-up constraint that the expenditures should add up to total expenditure. It is well known that imposing this constraint implies that for all goods, the demand equation (3) must have the form

$$\log q_1 = \alpha_1 - \log p_1 + \log E_A + u_1 \qquad (4)$$

with constraint $\Sigma_j e^{\alpha_j}=1$. This implies that own price elasticity is minus one, cross price elasticities are zero and income elasticity is one. Hence, although the model is convenient for estimation, it is difficult to justify from the theoretical point of view. It is still often used in Engel curve estimation, especially if it is justifiable to argue that the goods are so different from each other that it may be unreasonable to specify a functional form that is applicable to all goods.

When one starts from taking an approximation to the true preferences or the true demand function, the resulting functional form of the demands may, however, be log-differenced. Examples of this approach are Sato's (1972) approximation to a demand functions derived from a CES utility function, and the Rotterdam model, which is discussed below. An essential feature of the approximations is that the parameters of the resulting demand functions are, strictly speaking, not constant.

Another aspect in the choice of the form of the model is that many macro forecasts are given in the form of percentage change of quantities. Therefore a convenient form for a forecasting model would be the approximations mentioned above, which have a log-differenced form with prices and real expenditure as explanatory variables. The Rotterdam model is based on an approximation of the demand functions (1). The absolute price version of the second-step model in the present case is

$$w_1 d(\log q_1) = \theta_1' W_A d(\log Q_A) + \Sigma_{j=1}^{n} \pi_{1j} d(\log p_j) + u_1 \qquad (5)$$

In applied work, where finite changes have to be used, the variables are log differences, i.e. $Dq_{1t}=\log q_{1t}-\log q_{1,t-1}$ etc. In this case the shares are averages of current and past shares, i.e. $\bar{w}_{1t} =.5(w_{1t}+w_{1,t-1})$. $d(\log Q_A)$ is a Divisia volume index: $d(\log Q_A)=\Sigma_{j=1}^{n} w_j' d(\log q_j)$. Its coefficient

θ'_i is the conditional marginal budget share of good 1 of expenditures on group A. The adding-up constraint that when summed over all firms, both sides of (5) yield $d(\log Q_A)$, implies that $\Sigma_j\theta'_j=1$ and $\Sigma_i\pi_{ij}=0$. The parameters π_{ij} are the Slutsky coefficients. They are constrained by the homogeneity condition $\Sigma_j\pi_{ij}=0$, symmetry $\pi_{ij}=\pi_{ji}$, and negative semi-definity of the matrix $\pi=[\pi_{ij}]$ of the coefficients. The conditional income elasticity of demand for q_1 is θ'_1W_A/w_1, the own price elasticity π_{11}/w_1 and the cross price elasticity π_{1j}/w_1. Taking into account the homogeneity constraint equation (5) can be written

$$w_1d(\log q_1) = \theta'_1W_Ad(\log Q_A) + \Sigma^n_{j=2}\pi_{1j}(d(\log p_j) - d(\log p_1)) + u_1. \qquad (6)$$

In this model one can use the forecast of percentage change in expenditure Q_A directly in the equation to forecast q_1. If it is assumed that the preferences are blockwise independent, i.e. the utility function is additive in subfunctions, each of which are functions of goods in one group only, demand for q_1 can be directly explained by real total expenditure and the prices of the goods in group A. In this case $\theta'_1W_Ad(\log Q_A)$ in (6) is replaced by $\theta_1d(\log Q)$, since the choice between groups does not depend on the group price aggregates.

If the assumption of weak separability, rather than blockwise independence, is adopted, a higher level model is

$$W_Ad(\log Q_A) = \theta_Ad(\log Q) + \Sigma^Z_{i=A}\pi_{Ai}d(\log P'_i) + u_A \qquad (7)$$

where $d(\log Q)$ is the Divisia volume index of aggregate expenditure, $d(\log Q)=\Sigma^Z_{i=A}W_id(\log Q_i)$, and $d(\log P'_i)$ are Frisch price indexes of the groups, e.g. $d(\log P'_A)=\Sigma^n_{j=1}\theta'_j d(\log p_j)$. The constraints on the parameters are similar to those in the lower-level model. The homogeneity constraint $\Sigma_i \pi_{Ai}=0$ imposed, the model is

$$W_A d(\log Q_A) = \theta_A d(\log Q) + \Sigma^Z_{i=B}\pi_{Ai}(d(\log P'_i) - d(\log P'_A)) + u_A. \tag{8}$$

The two levels, (6) and (8), can be combined to the model

$$w_1 d(\log q_1) = \theta'_1\theta_A d(\log Q) + \Sigma^n_{j=2}\pi_{1j}(d(\log p_j) - d(\log p_1))$$
$$+ \Sigma^Z_{i=B}\theta'_1\pi_{Ai}(d(\log P'_i) - d(\log P'_A)) + e_1. \tag{9}$$

where $e_1=u_1+\theta'_1 u_A$. Both sides of the model can be divided by W_A if it is reasonable to assume that the parameters are proportional to the group share W_A (see Theil (1980a), p. 167). In this case $d(\log q_1)$ is multiplied by the conditional share of firm 1 in group A, $w'_1=w_1/W_A$, and not by its share in total expenditure, w_1. In addition, $\theta'_1=\theta_1/\theta_A$, so that $\theta'_1\theta_A=\theta_1$ is the unconditional marginal share. Use of the conditional share is justified especially when the unconditional share would be very small. This is the case for example when the firm under consideration is small or when the product group is defined so narrowly that its share of the consumer expenditure is small.

In the Rotterdam model the condition under which the stepwise modelling is possible is that $Cov(u_1,u_A)=0$, which holds under the assumption of "rational random behavior" (See Theil (1979a)). It implies here that $d(\log Q_A)$ and u_1 are uncorrelated in (5) and $d(\log Q)$ and e_1 are uncorrelated in (9).

As noted above the model could be simplified by assuming that relative prices do not change, either between groups or within a group, so that some of the price effects can be ignored, or alternatively the preference structure could be assumed to be simpler. Another simplification in the case where there are many firms in group A is to make some assumptions about how the price elasticities relate to each other. A useful form would be such that the prices of the other firms do not appear in the equation, since this kind of information is difficult to obtain. One alternative is to assume preferences to be separable in the goods produced by the different firms. In this case the price term in (6) would be replaced by $d(\log p_1)-d(\log P'_A)$ with coefficient $\phi\theta_1$ and Q_A would be replaced by Q (see Theil (1979a), p. 12). This implies zero cross price elasticities and own price elasticity proportional to marginal share. This is a fairly restrictive assumption and more appropriate in the choice between broader product groups. As an alternative, which in practice leads to a similar form, Keller (1984) has suggested the following decomposition: $\pi_{ij}=-\sigma(\phi_i\delta_{ij}-\phi_i\phi_j)$, where δ_{ij} is the Kronecker delta, i.e. δ_{ij} equals 1 if $i=j$ and 0 if $i\neq j$. Adding-up and homogeneity imply that $\Sigma_i\phi_i=1$ and the concavity condition is $\sigma>0$ and $0\leq\phi_i\leq 1$. Using this assumption in equation (5) yields

$$w_1 d(\log q_1) = \theta'_1 W_A d(\log Q_A) - \sigma\phi_1(d(\log p_1) - d(\log P_{A\phi})) + u_1. \qquad (10)$$

where $P_{A\phi}=\Sigma^n_{j=1}\phi_j d(\log p_j)$ is a new price index for group A. This model has the advantage that in the price term only the firm's own price and the industry price index appear. In practice $P_{A\phi}$ may be approximated by some easily available price index. A similar simplification can be made also in the choice between groups of goods.[1)]

When a final form of the demand model has been chosen and the model has been estimated in the finite change form, forecasts of the explanatory variables for the period t+h can be inserted in it. This yields the forecast $(\bar{w}_1 Dq_1)^f_{t+h}$. To obtain a forecast of the change in real sales, $Dq^f_{1,t+h}$, $(\bar{w}_1 Dq_1)^f_{t+h}$ has to be divided by a share variable. A reasonable choice seems to be to use w_{1t} as the divisor so that $w^f_{1,t+h}$ need not be separately forecasted.

As noted, the Rotterdam model is an approximation, and as such is particularly suitable for the two-step model where in the first step only price aggregates are used (see Deaton and Muellbauer (1980)). Sometimes it is argued that if the model has constant parameters it implies a level form equation which is log-linear, and hence suffers from the problems discussed above. The use of the model can nevertheless be justified by aggregation considerations. Aggregation over individuals with different Rotterdam demand functions, where the coefficients are fuctions of prices and income, leads to an aggregate Rotterdam equation. The aggregate parameters are weighted averages of the micro parameters and the homogeneity and symmetry conditions on parameters hold also at the aggregate level (see e.g. Barnett (1979), Clements and Selvanathan (1988)). On the other hand, Mountain (1988) has argued that treating the parameters of the Rotterdam model as constant approximations, and the elasticities dependent on the data, does not necessarily differ from approximations used in various flexible functional forms.

2.2. Aggregation over goods

Several aggregation issues arise in sales forecasting models. On one hand, there is the issue of aggregation of individual consumers to obtain aggregate demand for consumer goods. On the other hand, if the firm under consideration produces several types of products, forecasting models for different types may be aggregated to obtain a sales forecasting model, where total sales are explained by e.g. total consumer expenditure. The first aggregation issue will not be discussed here. It is assumed that not too much error is made by using a representative consumer.[2)] In fact, as shown e.g. by Barnett (1979) and Clements and Selvanathan (1988), if the number of consumers is large, it is justifiable to treat the Rotterdam demand equations as aggregates of the micro equations. The homogeneity and symmetry restrictions carry over to the aggregate demand equations.

Consider the second aggregation issue. To simplify the model, assume that the preferences are independent. The model that relates the firm's sales in product group i to total expenditure is

$$w_{1i}d(\log q_{1i}) = \theta_{1i}d(\log Q) + \phi\theta_{1i}(d(\log p_{1i})-d(\log P')) + u_{1i}, \quad i=A,\dots,Z \tag{11}$$

where now q, w, θ and p have been indexed both by firm and by product group. To obtain a model where summing over i, $d(\log q_1)$ is obtained as the dependent variable, the weight to be used for $d(\log q_{1i})$ has to be w^s_{1i}, the share of sales to group i in firm 1's total sales. The share w_{1i} can be written as $w_{1i}=q_{1i}p_{1i}/E=w^s_{1i}s_1/E=w^s_{1i}w_1$, where $s_1=\Sigma_i p_{1i}q_{1i}$ is the nominal sales of firm 1, and $w_1=s_1/E$. The demand model can be rewritten as

$$w_1 w^s_{1i}d(\log q_{1i}) = \theta_{1i}d(\log Q) + \phi\theta_{1i}(d(\log p_{1i})-d(\log P')) + u_{1i}. \tag{12}$$

Sum over i and note that $d(\log q_1)=\Sigma_{i=A}^{Z} w_{1i}^{s} d(\log q_{1i})$. This yields

$$w_1 d(\log q_1) = (\Sigma_i \theta_{1i}) d(\log Q) + \phi \Sigma_i \theta_{1i} (d(\log p_{1i}) - d(\log P')) + \Sigma_i u_{1i}$$

$$= (\Sigma_i \theta_{1i}) d(\log Q) + \phi(\Sigma_i \theta_{1i})(d(\log p'_{1i}) - d(\log P')) + e_1^{*}. \qquad (13)$$

where $d(\log p'_{1i})=\Sigma_i(\theta_{1i}/\Sigma_i \theta_{1i}) d(\log p_{1i})$ is the firm's own average price. The equation shows that total sales of firm 1 can be related to total consumer real expenditure, with a coefficient that is the sum of the marginal shares of the firm in different product groups, and to the difference of the firm's average price and an aggregate price index.

Relaxing the assumption of preference independence would result in the same kind of term for d(logQ), but also in some additional price terms. One would have to take into account all the firms' prices in all the product groups.

The fairly easy formulation above is essentially a feature of the the Rotterdam model. If one had, for example, double-logarithmic, differenced demand functions, the aggregation would have to be done by weighting the equations by some share variables. In this case there would be a weighted average of group demand terms multiplied by corresponding elasticities. To aggregate the group demands to an aggregate demand term with a coefficient wich is an average of the group elasticities would involve an approximation error. The approximation error is discussed in more detailed in the model for intermediate goods.

3. A model for intermediate goods

3.1. The basic model

For producer goods industries the analysis is different from the consumer goods case. Since the firms purchasing the production are assumed to minimize costs given a level of output, expenditures on inputs cannot be treated as predetermined. Another difference is the input-output structure of production. Above it was assumed that all the production of the consumer goods industries goes to final consumption. Here it is assumed for simplicity that all the production goes to other industries as intermediate inputs and hence there are no sales for final demand directly. Finally, aggregation of the demand side is much less justified than in the case of consumer goods, since the industries purchasing the intermediate good can be fairly different.

The starting point is the cost function for industry j

$$C_j = C_j(p_1,\ldots,p_n,\ldots,p_z,P_{Kj},P_{Lj},Y_j) \qquad (14)$$

where p_i, $i=1,2,\ldots,z$, are prices of the different firms from which the intermediate goods are purchased. P_{Kj} and P_{Lj} are the prices of capital and labor inputs for the industry, and Y_j is industry gross production. Assuming that the intermediate inputs purchased from different industries A,B,...,Z are homothetically weakly separable from capital and labor, and that inputs purchased from firms in different industries A,...,Z are homothetically weakly separable from each other, the cost function can be written as (see e.g. Fuss (1977), Chambers (1988))

$$C_j = C_j(P_{Mj}(P_{Aj}(p_1,\ldots,p_n),\ldots,P_{Zj}(p_s,\ldots,p_z)),P_{Vj}(P_{Kj},P_{Lj}),Y_j) \quad (15)$$

where P_{Vj} is the price aggregate of value added. Differentiating the cost function with respect to the intermediate goods price aggregate P_{Mj} yields the total demand for intermediate goods: $M_j=\delta C_j/\delta P_{Mj}=F_j(P_{Mj},P_{Vj},Y_j)$. Alternatively, the functional form may be such that it is easier to obtain the cost share $W_{Mj}=P_{Mj}M_j/C_j=W_{Mj}(P_{Mj},P_{Vj},Y_j)$. Use of price aggregates in these choices is more easily justified than in the consumer goods model, since homotheticity of the input aggregates is a more realistic assumption than would have been homotheticity of preferences. To ensure that the product of the price and volume indexes of the aggregates is equal to the sum of the costs of their components, it has to be assumed further that the aggregates are linearly homogenous in their components.

Now the choice between purchases from industries A,...,Z can be studied without having to consider the price of value added. Given the unit price function for the intermediate inputs, $P_{Mj}=P_{Mj}(P_{Aj},\ldots,P_{Zj})$, cost minimizing purchases from different industries can be determined by differentiating P_{Mj} with respect to the industry price aggregates. Note that these are indexed by j, since different industries buy different combinations of goods from each industry and hence the unit prices differ. This yields demands for inputs from different industries. For industry A, the demand is $M_{Aj}=M_j\delta P_{Mj}/\delta P_{Aj}=F_{Aj}(P_{Aj},\ldots,P_{Zj})M_j$ and the corresponding conditional cost share equation is $W'_{Aj}=P_{Aj}M_{Aj}/P_{Mj}M_j=W'_{Aj}(P_{Aj},\ldots,P_{Zj})$.

The next step is to consider the unit cost of purchases from industry A: $P_{Aj}=P_{Aj}(p_1,\ldots,p_n)$. From this, cost minimizing purchases from different firms 1,...,n can be determined without having to consider the prices in other industries. It is obtained that $q_{1j}=M_{Aj}\delta P_{Aj}/\delta p_1=F_{1j}(p_1,\dots,p_n)M_{Aj}$. Alternatively, the conditional cost share is $w'_{1j}=p_i q_{1j}/P_{Aj}M_{Aj}=w'_{1j}(p_1,\dots,p_n)$.

From the assumption of cost minimization follow some constraints on the system of this kind of demand equations for firms 1,...n, or industries A,...,Z. The equations are homogenous in the prices and the cross price effects are symmetric. In addition, the cost and price functions are concave. The share equations are constrained by adding-up, i.e. the sum of the shares is one. If one equation is considered separately, only the homogeneity constraint can be imposed.

The real sales equation for firm 1 is obtained as

$$
\begin{aligned}
q_{1j} &= F_{1j}(p_1,\dots,p_n)M_{Aj} \\
&= F_{1j}(p_1,\dots,p_n)F_{Aj}(P_{Aj},\dots,P_{Zj})M_j \\
&= F_{1j}(p_1,\dots,p_n)F_{Aj}(P_{Aj},\dots,P_{Zj})F_j(P_{Vj}(P_{Kj},P_{Lj}),P_{Mj},Y_j). \qquad (16)
\end{aligned}
$$

There is a choice between different aggregation levels in the forecasting model, depending on whether a forecast of Y_j, M_j or M_{Aj} is available. The model can also be formulated in terms of cost shares:

$$
\begin{aligned}
q_{1j} &= w'_{1j}P_{Aj}M_{Aj}/p_1 = w'_{1j}W'_{Aj}P_{Mj}M_j/p_1 \\
&= w'_{1j}W'_{Aj}W_{Mj}C_j/p_1 = w_{1j}C_j/p_1. \qquad (17)
\end{aligned}
$$

The exact functional form that one uses in a forecasting model derived above, depends on the functional form of the cost and price functions. To obtain log linear equations for the firm-level sales model, logs of cost shares should be used in the share approach. The most popular cost function model, translog, however, yields cost shares as functions of the logarithms of prices. Therefore the translog approach is best suited to either stepwise modelling where each share equation w'_{1j}, W'_{Aj} and W_{Mj} is modelled separately and and the real sales forecast is obtained from the share and total cost forecasts. The Cobb-Douglas cost and price functions yield constant cost shares, which may be a useful approximation. For example, Nakamura (1986) uses constant cost shares in some stages of a multisectoral model of factor demand.

In many cases it is easier to use the quantity model. In this case simple linear forms are obtained using a Cobb-Douglas technology. Again, this is fairly restrictive, but may suffice for a forecasting model that is not concerned of the price substitution effects; in the Cobb-Douglas case the elasticities of substitution for all input pairs are constrained to be unity. More flexible is the differential approach (see Theil (1980a)), which gives log difference of real sales as the dependent variable. It has also the advantage that under weak separability and "rational random behavior" the stepwise modelling is possible even without assuming homotheticity.

In the models discussed the concept of output has been gross output, which is produced using primary inputs and intermediate inputs. However, most forecasts of industry or aggregate production refer to net output, or real value added. One instance where net output can be used is when real value added and intermediate goods produce gross output in fixed proportions. If real value added- output coefficient in industry j is β_j and intermediate inputs - output coefficient is α_j, the intermediate inputs - value added ratio is α_j/β_j, or $M_j=\alpha_j V_j/\beta_j$. Therefore the first step of the model is left out. This result is obtained also from the cost function, which in the fixed coefficient case is specified as $C_j=(\alpha_j P_{Mj}+\beta_j P_{Vj})Y_j$. Demands for intermediate inputs and value added are $M_j=\delta C_j/\delta P_{Mj}=\alpha_j Y_j$ and $V_j=\delta C_j/\delta P_{Vj}=\beta_j Y_j$. This implies that the coefficient of $\log V_j$ in a model for $\log M_j$ should be 1. Also in the firm-level sales equation the same coefficient for $\log V_j$ appears. A possible justification for including value added with a non-unity coefficient, although fixed coefficients are assumed, is that the gross output - value added ratio is constant e.g. during any given year, but can change over longer time periods (see Adams et al. (1976)).

It is possible to assume a strongly separable form for the cost function, which allows a non-unity coefficient for $\log M_j$. One example is cost function $C_j=a_{0j}P_{Vj}Y_j^{a_{1j}}+b_{0j}P_{Mj}Y_j^{b_{1j}}$. Now no substitution is allowed between value added and materials, but their ratio can change when gross output changes if $a_{1j}\neq b_{1j}$. Solving for optimal demands for M_j and V_j and eliminating Y_j yields M_j as a function of V_j with coefficient which is different from one: $M_j=\delta C_j/\delta P_{Mj}=b_{0j}Y_j^{b_{1j}}$, $V_j=\delta C_j/\delta P_{Vj}=a_{0j}Y_j^{a_{1j}}$ and $\log M_j=\log b_{0j}-(b_{1j}/a_{1j})\log a_{0j}+(b_{1j}/a_{1j})\log V_j=\alpha_{0j}+\alpha_{1j}\log V_j$.

The fixed proportions assumption can be extended also to other stages of the model. This gives the input-output model as a special case. If the price function P_{Mj} has the form $P_{Mj}=a_{Aj}P_{Aj}+...+a_{Zj}P_Z$, then $a_{Aj}=M_{Aj}/M_j$ is the ratio of intermediate inputs from industry A to total intermediate inputs purchased by industry j. Hence $M_{Aj}=a_{Aj}M_j$. Given $M_j=\alpha_j Y_j$, it is obtained that $M_{Aj}=a_{Aj}\alpha_j Y_j$ where $a_{Aj}\alpha_j$ is an input-output coefficient. Similarly at the industry level the price function may be $P_{Aj}=a_{1j}p_1+\ldots+a_{nj}p_n$ where $a_{1j}=q_{1j}/M_{Aj}$ is the real market share of firm 1 in the purchases of industry j from industry A. Hence $q_{1j}=a_{1j}M_{Aj}=a_{1j}a_{Aj}\alpha_j Y_j=a_{1j}a_{Aj}\alpha_j V_j/\beta_j$. As is well known, one can work backwards and express output Y in terms of final demands for the products. This makes it possible to link firm sales either to demand by different industries, or production in these industries, or the final demands.

Here the differential approach to input demand is adopted. In the first step, demand for q_1 by industry j is determined conditionally on the industry's purchases from industry A. The absolute price model with homogeneity constraint is

$$w_{1j}d(\log q_{1j}) = \theta'_{1j}W_{Aj}d(\log M_{Aj}) + \Sigma_{i=2}^{n}\pi^{j}_{1i}(d(\log p_i) - d(\log p_1)) + u_{1j} \tag{18}$$

where $d(\log M_{Aj})=\Sigma_{i=1}^{n}w'_{ij}d(\log q_{ij})$. Interpretation of the coefficients and variables is the same as in the consumer goods model. Note that (18) is not an approximation of the first line in (16), since in the latter case homotheticity is assumed.

The corresponding model of the demand for inputs from industry A, conditionally on the total demand for intermediate inputs, is

$$W_{Aj}d(\log M_{Aj}) = \theta'_{Aj}W_{Mj}d(\log M_j) + \Sigma_{i=B}^{z}\pi^{j}_{Ai}(d(\log P'_{ij}) - d(\log P'_{Aj})) + u_{Aj} \tag{19}$$

where $d(\log P'_{Aj})=\Sigma_{i=1}^{n}\theta'_{Aj}d(\log p_i)$, etc. are the Frisch price indexes for the input groups. The total demand for intermediate inputs is given by

$$W_{Mj}d(\log M_j) = \theta_{Mj}d(\log G_j) + \pi^j_{MV}(d(\log P'_{Mj}) - d(\log P'_{Vj})) + u_{Mj} \qquad (20)$$

where $d(\log G_j)=W_{Mj}d(\log M_j)+W_{Vj}d(\log V_j)$ is the total quantity of inputs. The price indexes are similar to those defined earlier. Finally, the total input decision is given by

$$d(\log G_j) = \mu_j d(\log Y_j) + u_{Gj}. \qquad (21)$$

where μ_j is the output elasticity of total cost. These elements, (18)-(21), could be combined to a sales forecasting model. If, as above, value added rather than gross output is used as the activity variable, it is assumed that intermediate inputs and value added are strongly separable. Therefore $W_{Mj}d(\log M_j)=\theta_{Mj}d(\log G_j)+u_{Mj}$ and $W_{Vj}d(\log V_j)=\theta_{Vj}d(\log G_j)+u_{Vj}$. Solving these for M as a function of V yields $W_{Mj}d(\log M_j)=$ $\theta_{Mj}(W_{Vj}/\theta_{Vj})d(\log V_j)+u_j$, where $u_j=u_{Mj}-\theta_{Mj}u_{Vj}/\theta_{Vj}$. Combining this with (18) and (19) gives

$$w_{1j}d(\log q_{1j}) = \theta'_{1j}\theta'_{Aj}\theta_{Mj}(W_{Vj}/\theta_{Vj})d(\log V_j) + \Sigma^n_{i=2}\pi^j_{1i}(d(\log p_i) - d(\log p_1))$$

$$+ \theta'_{1j}\Sigma^Z_{i=B}\pi^j_{Ai}(d(\log P'_{ij}) - d(\log P'_{Aj})) + e_{1j} \qquad (22)$$

where $\theta'_{1j}\theta'_{Aj}\theta_{Mj}=\theta_{1j}$ is the unconditional marginal share of firm 1 in the total cost of industry j, and $e_{1j}=u_{1j} + \theta'_{1j}u_{Aj} + \theta'_{1j}\theta'_{Aj}u_j$. Depending on the forecasts available, q_{1j} can be modelled also as a function of M_{Aj}, M_j or Y_j. The price terms can be simplified using Keller's (1984) model in the same way as in the consumer goods case. Also, w_{1j} could be replaced by the conditional share $w'_{1j}=w_{1j}/W_{Aj}$ if it is assumed that the parameters are proportional to the group share. Again, this is helpful when the unconditional share is very small.

A potential difficulty in the above approach of replacing gross output by value added is that the latter is now correlated with the error term of the model, i.e. $d(\log V_j)$ and u_{Vj} are correlated. Only in the fixed coefficients case is this avoided, but then it is difficult to justify a non-unity coefficient for value added in the logarithmic equation.

3.2. Aggregation over industries

To obtain total sales for firm 1, the above models are summed over j. Combining them to a simple model, which has total industry output as an explanatory variable, requires fairly strong assumptions about the similarity of the industries. Again, it is assumed that firms that purchase the production are sufficiently similar within each industry, so that a representative firm can be used for each industry.

Consider again the Rotterdam model for the demand for intermediate inputs. Ignoring the price effects, the model (22) that relates firm sales to industry j to the value added of that industry is

$$w_{1j}d(\log q_{1j}) = (\theta_{1j}W_{Vj}/\theta_{Vj})d(\log V_j) + e_{1j}. \qquad (23)$$

To obtain a model where summing over j $(j=1,\dots,k)$, $d(\log q_1)$ is obtained as the dependent variable, the weight to be used for $d(\log q_{1j})$ has to be w^s_{1j}, the share of sales to industry j of firm 1's total sales. The share w_{1j} can be written as $w_{1j}=q_{1j}p_1/C_j=w^s_{1j}s_1/C_j$, where $s_1=p_1q_1$ is the nominal sales of firm 1. It is assumed that the production is sold at the same price to all industries. The demand model can be rewritten as

$$s_1 w^s_{1j} d(\log q_{1j}) = (C_j \theta_{1j} W_{Vj}/\theta_{Vj}) d(\log V_j) + C_j e_{1j}. \qquad (24)$$

Sum over j and divide by $C=\Sigma_i C_j$, noting that $d(\log q_1)=\Sigma^k_{j=1} w^s_{1j} d(\log q_{1j})$ and $d(\log V)=\Sigma^k_{j=1} v_j d(\log V_j)$, where $v_j=P_{Vj}V_j/\Sigma_j P_{Vj}V_j=P_{Vj}V_j/P_V V$ is the share of industry j of total industrial value added; further, denote $f_1=s_1/C$ and $c_j=C_j/C$. This yields

$$f_1 d(\log q_1) = \Sigma^k_{j=1}(C_j \theta_{1j} W_{Vj}/\theta_{Vj} C) d(\log V_j) + \Sigma^k_{j=1} c_j e_{1j}$$

$$= (P_V V/C)\Sigma^k_{j=1} v_j (\theta_{1j}/\theta_{Vj}) d(\log V_j) + e^*_1$$

$$= (P_V V/C)\bar{\theta}_{1V} d(\log V) + (P_V V/C)\mathrm{Cov}(\theta_{1j}/\theta_{Vj}, d(\log V_j)) + e^*_1. \qquad (25)$$

where $\bar{\theta}_{1V}=\Sigma^k_{j=1} v_j \theta_{1j}/\theta_{Vj}$, and $\mathrm{Cov}=\Sigma^k_{j=1} v_j (\theta_{1j}/\theta_{Vj}-\bar{\theta}_{1/V})(d(\log V_j)-d(\log V))$. This kind of decompositions are often used in aggregation analysis; see e.g. van Daal and Merkies (1984), and Pylkkänen and Vartia (1986).

The equation shows that total sales of firm 1 can be related to total industry value added, with a coefficient that is a weighted average of the industry coefficients. Regressing change in sales on change in total value added ignores the covariance of the industry coefficients and changes in industry value added. This causes an omitted variable bias in the estimates. In some cases it is possible to formulate tests of the aggregation bias, as Stoker (1986) has shown. If this covariance is constant, it can be dealt with by adding a constant term in the aggregate equation. Alternatively, one could estimate separate industry demand models and get an estimate of the covariance from past data. This could then be used in forecasting aggregate sales. However, the main idea of using the aggregate model, rather than summing the separate disaggregate models, is that often it is difficult to obtain the necessary information for estimating the disaggregate models. One can, however, make adjustments to the constant term when forecasting with the model, for example if it is felt that output is going to grow much slower in industries where the firm has a large marginal share.[3)] It is also possible to model total sales so that the value added in different industries appear separately as explanatory variables. In this case the change in each value added term appears in the equation weighted by the value added share of the industry in question, as can be seen from the second line of (25).

In the above analysis the price terms were omitted. If the price elasticities and prices vary across industries, the same kind of decomposition as was done for the output term can be applied to the price term of the model. However, if the good is sold at the same price to different industries, the covariance term related to the firm-level prices and their coefficients is zero and the coefficient of the price term is a weighted average of the industry price coefficients. For the industry-level price terms there will be similar aggregation problems as for the output term. An additional aggregation issue would be summing of the firm's sales of different intermediate inputs, i.e. equation (23)-(24) would be indexed also by the type of good, m, and in (25) there would be also summing over m.

4. Exports and imports

In the case of an open economy and an industry producing a good with foreign substitutes, the models have to be extended in various ways. If foreign goods are imported, the demand equations should include the import price as an additional variable, and the market shares should be redefined to include imports. If it can be assumed that the imported goods are weakly separable from the domestically produced goods in the preferences or the technology in the demand side of the market, the model is simplified. In this case the imports need to be considered only if a higher-level model is specified, where e.g. total consumer expenditure is an explanatory variable.

If the industry is exporting part of its production, a foreign demand model has to be specified. It is possible to specify demand functions for different countries in a manner similar to that used above for consumers (see Armington (1969a,b)), although one may have to form demand functions for several different countries unless some kind of aggregation of the demand side can be made. Also the specification of the price variables is more difficult than in the domestic demand case, since now also the exchange rates have to be taken into account. One simplification is to use a forecast for total exports of the industry and to model only the firm's export share. This would allow the separate analysis of the effects of a general increase in exports, with a constant export share for the firm, and the price effects on the firm's export share. This is similar to the constant market share analysis often used in analyzing the export performance of different countries. Its theoretical rationale can be derived from an Armington-type demand model (see Merkies and van der Meer (1988)).

5. On price reactions

A point often emphasized in the forecasting literature is that sales forecasting models cannot forecast competitors' price or marketing reactions. The models specified above assume that the prices of the other firms are known and the task is to estimate the impact of these prices on own demand. In real life applications two problems emerge. First, it may be very difficult to obtain information on the competitors' past or present prices or marketing policies. This makes estimation of the sales forecasting models difficult and in practice the first step price terms are often left out, or the competitors' prices are ignored and only own price is used. An example of simplification of the model is the constant market share assumption, which eliminates the need to make estimations if a forecast of total market demand is available. Especially if the cost conditions of the firms in the industry are similar and price is not a very important marketing variable, it is reasonable to assume that, at least in the forecasting period, relative prices do not change.

Secondly, even if the past prices were known and the full model could be estimated, in the forecast period one needs to forecast the price reactions. If the competitors' reactions can be assumed to follow a consistent pattern, it is possible to use some results of oligopoly theory so that the assumptions about the reactions become more apparent. This is discussed in the present section.

In the estimation period prices are observed ex post and already include all price reactions made. In the forecast period, however, the competitors' prices may react to the price decision of firm 1. Assume that in a model for $\log q_1$, an explanatory variable is $\log p_1/p_2$, with coefficient β_{11}; all other firms are treated as an aggregate, firm 2. Firm 1 chooses a price policy for the forecast period. Given this, a forecast of p_2 is needed.

To consider effects of changes in firm 1's price, differentiate the firm-level demand function with respect to $\log p_1$. This yields

$$\delta \log q_1/\delta \log p_1 = \beta_{11}(1 - \delta \log p_2/\delta \log p_1) = \beta_{11}(1 - \epsilon) \qquad (26)$$

where ϵ is the assumed elasticity of the other firms' price reaction to firm 1's price change. It is a conjectural price variations elasticity, i.e. it shows how firm 1 expects prices to change. Several cases can be considered. $\epsilon=0$ corresponds to a Bertrand-Nash assumption about the price reactions, i.e. it is assumed that competitors do not change their prices in response to changes in p_1. If $\epsilon=1$, firm 1's price change has no effect on its quantity sold, or its real market share, since the other firms have correspondingly changed their prices. This behavioral assumption implies maintenance of constant market shares and is often interpreted as perfectly collusive behavior.[4)]

The implication of the analysis is that if the effects of alternative price policies in the forecast period are studied, a simple way to take into account the other firms is to use a modified own-price elasticity $\beta_{11}(1-\epsilon)$. This is used for forecasting the effect of price change on change in demand, and the other firms' prices need not be explicitly forecasted. The value of ϵ is based on the firm's subjective beliefs about the competitors' reactions. These reactions need not be constant, but can depend on the general economic situation.

6.Econometric issues

In this section, some issues in the estimation of firm-level demand models are discussed. In the discussion the finite change form of the Rotterdam model is used. Deaton (1986) considers several other estimation issues that are relevant when a system of demand equations is estimated. Here, as in the previous sections, the emphasis is on the estimation of only a single firm-level demand equation.

Consider first the effect of ignoring the price terms in a demand model. Often it is difficult to obtain data or forecasts of the prices of the competitors. On the other hand, it may be assumed in some stages that the price effects are negligible. It would be important to get an idea how an incorrect assumption would affect the results.

Consider Keller's (1984) version of the Rotterdam model in finite change form, when the price term has been left out:

$$\bar{w}_{1t} Dq_{1t} = \theta_1' W_{At} DQ_{At} + e_{1t} . \tag{27}$$

The error term now includes the left-out variable: $e_1 = u_1 - \sigma\phi_1 (Dp_{1t} - DP_{A\phi t})$.
The estimate of θ_1' is

$$\begin{aligned}\theta_1^{*\prime} &= \Sigma_t \bar{W}_{At} DQ_A \bar{w}_{1t} Dq_{1t} / \Sigma_t (\bar{W}_{At} DQ_{At})^2 \\ &= \theta_1' - \sigma\phi_1 \Sigma_t \bar{W}_{At} DQ_{At} (Dp_{1t} - DP_{A\phi t}) / \Sigma_t (\bar{W}_{At} DQ_{At})^2 \\ &\quad + \Sigma_t u_{1t} \bar{W}_{At} DQ_{At} / \Sigma_t (\bar{W}_{At} DQ_{At})^2 \end{aligned} \tag{28}$$

which has expectation

$$E(\theta_1^{*\prime}) = \theta_1' - \sigma\phi_1 b_{PQ} \qquad (29)$$

where b_{PQ} is the slope from a regression of the relative price term on $\bar{W}_{At}DQ_{At}$. Hence ignoring the price terms gives a biased estimate of the marginal share parameter. However, the bias tends to be small if $\sigma\phi_1$ or b_{PQ} is small. Consider the latter term. DQ_{At} varies inversely with $DP_{A\phi t}$, the industry price index, and with Dp_{1t}. Therefore, the sign of b_{PQ} depends on whether changes in p_1 or $P_{A\phi}$ have a stronger influence on demand for the whole group of goods. If firm 1's market share is small, group demand is likely to be more strongly negatively correlated with the industry price index. Hence b_{PQ} would be positive, and the estimate (28) would have a negative bias. On the other hand, if firm 1 has a dominating position in the market, group demand is likely to have stronger negative correlation with Dp_1. In this case, b_{PQ} is negative, and the bias in (28) positive. In any case, if it is justifiable to argue that price elasticities are small, also the bias is small.

As the second estimation issue, simultaneity is discussed. It is a feature of many allocation models that an explanatory variable is the sum of the dependent variables of the system of equations. Consider the demand system

$$\bar{w}_{it}Dq_{it} = \theta_i DX_{At} + \Sigma_{j=1}^{n} \pi_{ij} Dp_j + v_{it}, \qquad i=1,\ldots,n \qquad (30)$$

where DX_{At} measures the change in the expenditure on group A. To find out, what this variable should be in a consistent system, sum (30) over i:

$$\Sigma_{i=1}^{n} \bar{w}_{it}Dq_{it} = DQ_{At} = DX_{At} + \Sigma_i v_{it}, \qquad (31)$$

where the parameter constraints $\Sigma_i\theta_i=1$ and $\Sigma_i\pi_{ij}=0$ have been used. Solving

(31) for DX_{At} and inserting it in (30) yields finally

$$\bar{w}_{it}Dq_{it} = \theta_i DQ_{At} + \Sigma^n_{j=1}\pi_{ij}Dp_j + v_{it} - \theta_i\Sigma_i v_{it}, \quad i=1,\ldots,n. \qquad (32)$$

Denote the combined error term in (32) by u_{it}. It is easily shown that $Cov(DQ_{At},u_{it})=\Sigma_j\sigma_{ij}-\theta_i\Sigma_j\Sigma_k\sigma_{jk}$. Therefore the estimation of the demand equations (32) with ordinary least squares results in biased estimates. Theil (1979a) has, however, shown that under "rational random behavior" DQ_{At} is not correlated with u_{it}. This means that the consumers trade off the costs of exact maximization against the cost of not doing so. Deaton (1986) discusses other cases where the errors are likely to be uncorrelated with DQ_{At}.

It should be noted that simultaneity does not arise in the intermediate goods model when gross output is the activity variable in the model. However, replacing it by value added leads to a situation where this explanatory variable is correlated with the composite error term in (22). The source of this simultaneity is, however, different from that in the example in this section.

Next consider the case of too narrowly defined activity variable. This kind of cases may arise when some relevant firms are omitted from the market definition or when the assumed separability conditions do not hold. In the case of producer goods, also omission of some industries where part of the output is sold is possible. For simplicity, the price terms are ignored in this analysis. Assume that group A, which consists of firms 1 and 2, is treated as the relevant group, but in reality also firm 3 should be included. The true model is

$$\bar{w}_{it}Dq_{it} = \theta_i DQ_t + u_{it}, \quad i=1,2,3 \tag{33}$$

where $DQ_t = \Sigma_{i=1}^{3} \bar{w}_{it} Dq_{it}$. However, in the estimation of the model, firm 3 is omitted and the estimated model is

$$\bar{w}_{1t}Dq_{1t} = \theta'_1 \bar{W}_{At} DQ_{At} + e_{1t}. \tag{34}$$

Given the true model, $\bar{W}_{At}DQ_{At} = \bar{w}_{1t}Dq_{1t} + \bar{w}_{2t}Dq_{2t} = (\theta_1+\theta_2)DQ_t + u_{1t} + u_{2t}$. The estimate of θ'_1 is

$$\theta_1'^{*} = \Sigma_t \bar{w}_{1t}Dq_{1t}\bar{W}_{At}DQ_{At} / \Sigma_t (\bar{W}_{At}DQ_{At})^2 \tag{35}$$

which can be written as

$$\theta_1'^{*} = \Sigma_t(\theta_1 DQ_t + u_{1t})((\theta_1+\theta_2)DQ_t + u_{1t} + u_{2t}) / \Sigma_t((\theta_1+\theta_2)DQ_t + u_{1t} + u_{2t})^2. \tag{36}$$

Taking probability limit one obtains

$$\text{plim}\theta_1'^{*} = (\theta_1(\theta_1+\theta_2)\sigma_{DQ}^2 + \sigma_1^2 + \sigma_{12}) / ((\theta_1+\theta_2)^2\sigma_{DQ}^2 + \sigma_1^2 + \sigma_2^2 + 2\sigma_{12}) \tag{37}$$

where σ_{DQ}^2 is the variance of DQ, σ_i^2 is the variance of u_i $(i=1,2)$, and σ_{12} is the covariance of u_1 and u_2. Under "rational random behavior" DQ is uncorrelated with the error terms. Further, note that because of the adding-up condition, $u_{3t} = -u_{1t} - u_{2t}$, so that $\sigma_1^2 + \sigma_{12} = -\sigma_{13}$ and $\sigma_1^2 + \sigma_2^2 + 2\sigma_{12} = -\sigma_{13} - \sigma_{23} = \sigma_3^2$, and expression (37) can be written as

$$\text{plim}\theta_1'^{*} = (\theta_1(\theta_1+\theta_2)\sigma_{DQ}^2 - \sigma_{13}) / ((\theta_1+\theta_2)^2\sigma_{DQ}^2 + \sigma_3^2). \tag{38}$$

It could be said that the estimate is consistent if $\text{plim}\theta_1'^{*} = \theta_1/(\theta_1+\theta_2)$, i.e. the probability limit of the estimate is equal to the conditional marginal share of firm 1 when only firms 1 and 2 are taken into account. It is clear from the above expression that by this criterion the estimate is consistent if $\sigma_{13}=\sigma_3^2=0$. The estimate is biased downwards if the errors for firms 1 and 3 are positively correlated. If the correlation is negative, the bias can be in either direction. It is instructive to see how this conclusion is affected by the sizes of the marginal shares of the different firms. In the limit, when the marginal share of the left-out firm approaches zero, i.e. θ_3->0, and hence $\theta_1+\theta_2$->1, $\text{plim}\theta_1'^{*}=(\theta_1\sigma_{DQ}^2-\sigma_{13})/(\sigma_{DQ}^2+\sigma_3^2)$. On the other hand, when θ_3->1, $\text{plim}\theta_1'^{*}=-\sigma_{13}/\sigma_3^2$. Therefore, the estimate is inconsistent irrespective of the marginal share of the left-out firm.

Often it is difficult to obtain forecasts of the explanatory variables. It may be tempting to formulate forecasting models where the sales of a firm are related to total production in the industry. If a forecast of the industry production is available, this would provide a fairly simple forecasting model for sales. Especially in the case of producer goods this would make it possible to avoid considering the disaggregation of the demand side to different industries. A problem in using production to forecast sales is the existence of inventories. Consider a simple model, where the true relation between the real sales of firm 1 and total quantity demanded from the industry is $Dq_{1t}=\beta_1 DQ_{At}+u_{1t}$. Let total industry production be denoted by Y_A and firm 1's production by y_1. It holds that $Dq_{1t}=Dy_{1t}-i_{1t}$ where i is the contribution of inventory change to sales. If inventories are increased, there is a smaller change in sales for a given change in output. Aggregating over firms gives $DQ_{At}=DY_{At}-I_{At}$, where DY is a sales share weighted index of production and I a weighted average contribution of inventory change. The relationship of Dq_1 and DQ_A can be written $Dq_{1t}=\beta_1(DY_{At}-I_{At})+u_{1t}$.

Now consider the estimation of a model where Dq_1 is explained by industry production. The estimate of β_1 is

$$\beta_1^* = \Sigma_t Dq_{1t} DY_{At} / \Sigma_t DY_{At}^2$$

$$= \beta_1 + \Sigma_t u_{1t} DY_{At} / \Sigma_t DY_{At}^2 - \theta_1' \Sigma_t I_{At} DY_{At} / \Sigma_t DY_{At}^2. \quad (39)$$

This has expectation $E\beta_1^* = \beta_1(1-b_{IY})$ where b_{IY} is the slope from a regression of inventory change on change in industry production. If it is assumed that the firms increase production and decrease inventories when demand is high, b_{IY} would be negative. It could also be argued that when the cost of holding inventories increases it is profitable to run down inventories and increase production. Again, output and inventory decisions tend to be inversely related. Therefore there is a positive bias from omitting inventories and using only production as an explanatory variable instead of industry demand. The above example was based on gross production. Assuming that value added is in a fixed relationship with gross production, essentially the same conclusion emerges.

8. Final remarks

Some theoretical models for linking firms to macroeconomic or industry-level variables have been presented in this paper. In Ilmakunnas (1990) some simple models will be used for forming sales forecasting models for some Finnish firms. After such a model has been estimated, forecasts of the firm's sales are obtained by inserting forecasts of the activity and price variables in the model. It is important to note that these forecasts of the environmental variables are themselves subject to error. Therefore it is important to evaluate the past record of the forecasts to see whether they are accurate enough to be useful in the sales forecating model. Some tests for such a forecast evaluation are discussed in Ashley (1983) and Ilmakunnas (1987b, 1989a,b).

Footnotes

1) Theil (1980b) has suggested an "equicorrelated substitutes" approach to goods that are close substitutes. For this kind of goods also advertising or other marketing expenditures may be important determinants of market shares and they could be incorporated into the Rotterdam model (see Theil (1979, 1980b), Clements and Selvanathan (1988)).

2) It may be noted that aggregation considerations sometimes support the inclusion of income distribution or demographic variables in demand functions; see e.g. Deaton and Muellbauer (1980) and Stoker (1986). This may be a useful practice also in sales forecasting models.

3) Pylkkänen and Vartia (1986) discuss this kind of adjustments in the context of a macro model.

4) These values can be compared to conjectural variations in quantity-setting oligopoly models. It can be shown that $\epsilon=1$ corresponds to $\delta \log q_2/\delta \log q_1=1$ in a quantity-setting case, whereas zero price conjectural variations correspond to positive quantity conjectural variations. Zero quantity conjectures, i.e. the Cournot-Nash model, corresponds to negative price conjectures (see Kamien and Schwarz (1983)).

References

Adams, F.G., Duggal, V.G., and Thanwala, S.: Industrial linking functions for the macromodels, Business Economics 11, 1976, No. 4, 87-90

Armigton, P.S.: A theory of demand for products distinguished by place of production, IMF Staff Papers 16, 1969, 159-178 (a)

Armington, P.S.: The geographical pattern of trade and the effects of price changes, IMF Staff Papers 16, 1969, 179-199 (b)

Ashley, R.: On the usefulness of macroeconomic forecasts as inputs to forecasting models, Journal of Forecasting 2, 1983, 211-223

Barnett, W.A.: Theoretical foundations for the Rotterdam model, Review of Economic Studies 46, 1979, 109-130

Browne, S.J., and O'Brien, D.: Product line forecasting and planning, Business Economics 9, 1974, No. 4, 63-71

Chambers, R.G.: Applied Production Analysis: A Dual Approach, Cambridge: Cambridge University Press, 1988

Clements, K.W., and Selvanathan, E.A.: The Rotterdam demand model and its application in marketing, Marketing Science 7, 1988, 60-75

Daal, J. van, and Merkies, A.H.Q.M.: Aggregation in Economic Research, Dordrecht: Reidel, 1984

Deaton, A.: Demand analysis, in Z. Griliches and M.J. Intriligator, eds.: Handbook of Econometrics, vol. III, Amsterdam: North-Holland, 1986, 1767-1839

Deaton, A., and Muellbauer, J.: Economics and Consumer Behaviour, Cambridge: Cambridge University Press, 1980

Elliott, J.W.: Forecasting and analysis of corporate financial performance with an econometric model of the firm, Journal of Financial And Quantitative Analysis 7, 1972, 1499-1526

Fuss, M.: The demand for energy in Canadian manufacturing: an example of the estimation of production structure with many inputs, Journal of Econometrics 5, 1977, 89-116

Goudie, A.W., and Meeks, G.: Individual companies in a macroeconomic model: UK companies in Cambridge MDM, Journal of Policy Modelling 6, 1984, 289-309

Ilmakunnas, P.: On the use of macroeconomic forecasts in some British companies, ETLA Discussion Papers, No. 244, 1987 (a)

Ilmakunnas, P.: On the profitability of using forecasts, Economics Letters 25, 1987, 345-349 (b)

Ilmakunnas, P.: Suhdanne-ennusteiden käyttömahdollisuudet yrityksissä: esimerkkejä Iso-Britanniasta, Kansantaloudellinen Aikakauskirja 84, 1988, 396-403

Ilmakunnas, P.: Forecast pretesting, presented at Econometric Society European Meetings, Munich, Sept. 4-8, 1989 (a)

Ilmakunnas, P.: A note on forecast evaluation and correction, ETLA Discussion Papers, No. 309, 1989 (b)

Ilmakunnas, P.: Suhdanne-ennusteiden käyttömahdollisuudet yrityksissä, ETLA, 1990 (forthcoming)

Kamien, M. and Schwartz, N.: Conjectural variations, Canadian Journal of Economics 16, 1983, 191-211

Keller, W.J.: Some simple but flexible differential consumer demand systems, Economics Letters 16, 1984, 77-82

Klein, L.R., ed.: Essays in Industrial econometrics, vol. I-III, Philadelphia: University of Pennsylvania, 1969

Lau, L.J.: Functional forms in econometric model building, in Z. Griliches and M.D. Intriligator, eds.: Handbook of Econometrics, vol. III, Amsterdam: North-Holland, 1986, 1515-1566

Macintosh, N.B., Tsurumi, H., and Tsurumi, Y,: Econometrics for strategic planning, Journal of Business Policy 3, 1973, 49-60

Merkies, A.H.Q.M., and Meer, T. van der: A theoretical foundation for constant market share analysis, Empirical Economics 13, 1988, 65-80

Mountain, D.C.: The Rotterdam model: an approximation in variable space, Econometrica 56, 1988, 477-484

Nakamura, S.: A dynamic multisectoral model of production, investment and prices based on flexible cost functions, DIW Vierteljahrsheft zur Wirtschaftsforschung, Heft 3, 1986, 110-122

Naylor, T.H.: Corporate Planning Models, Reading, Mass.: Addison-Wesley, 1979

Pylkkänen, E., and Vartia, P.: Some comments on fine-tuning macro-model forecasts, Finnish Journal of Business Economics 35, 1986, 58-70

Sato, K.: Additive utility functions with double-log consumer demand functions, Journal of Political Economy 80, 1972, 102-124

Stoker, T.S.: Simple tests of distributional effects on macroeconomic equations, Journal of Political Economy 94, 1986, 763-795

Theil, H.: Can economists contribute to marketing research?, Sloan Management Review 20, No. 4, 1979, 19-29

Theil, H.: The System-Wide Approach to Microeconomics, Chicago: University of Chicago Press, 1980 (a)

Theil, H.: System-Wide Explorations in International Economics, Input-Output Analysis and Marketing Research, Amsterdam: North-Holland, 1980 (b)

ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)
The Research Institute of the Finnish Economy
Lönnrotinkatu 4 B, SF-00120 HELSINKI Puh./Tel. (90) 601 322
Telefax (90) 601 753

KESKUSTELUAIHEITA - DISCUSSION PAPERS ISSN 0781-6847

No 280 CHRISTIAN EDGREN, Tulorakenteen hyväksikäytöstä veronalaisen tulon kasvua arvioitaessa. 22.12.1988. 32 s.

No 281 PEKKA ILMAKUNNAS - HANNU TÖRMÄ, Structural Change of Factor Substitution in Finnish Manufacturing. 09.01.1989. 22 pp.

No 282 MARKKU RAHIALA - TIMO TERÄSVIRTA, Labour Hoarding Over the Business Cycle: Testing the Quadratic Adjustment Cost Hypothesis. 18.01.1989. 22 pp.

No 283 ILKKA SUSILUOTO, Helsingin seudun aluetalous panos-tuotostutkimuksen valossa. 08.02.1989. 27 s.

No 284 JAMEL BOUCELHAM - TIMO TERÄSVIRTA, How to Use Preliminary Values in Forecasting the Monthly Index of Industrial Production? 08.03.1989. 14 pp.

No 285 OLLE KRANTZ, Svensk ekonomisk förändring i ett långtidsperspektiv. 28.02.1989. 29 p.

No 286 TOR ERIKSSON - ANTTI SUVANTO - PENTTI VARTIA, Wage Setting in Finland. 20.03.1989. 77 p.

No 287 PEKKA ILMAKUNNAS, Tests of the Efficiency of Some Finnish Macroeconomic Forecasts: An Analysis of Forecast Revisions. 30.03.1989. 19 p.

No 288 PAAVO OKKO, Tuotantomuodon muutos ja sen merkitys yritys- ja aluerakenteelle. 08.05.1989. 14 s.

No 289 ESKO TORSTI, The Forecasting System in ETLA. 10.05.1989. 36 p.

No 290 ESKO TORSTI, MAT-ohjelmointitulkin käyttö ja rakenne. 11.05.1989. 67 s.

No 291 GUJA BACCHILEGA - ROBERTO GOLINELLI, Medium Term Prospects for the European Economies. 17.05.1989. 27 p.

No 292 KARI ALHO, Deregulation of Financial Markets: A General Equilibrium Analysis of Finland. 31.05.1989. 43 p.

No 293 PAAVO OKKO - EERO KASANEN, A Model of Banking Competition. 15.06.1989. 20 p.

No 294 HILKKA TAIMIO, Naisten kotityö ja taloudellinen kasvu Suomessa vuosina 1860-1985. 28.06.1989. 38 s.

No 295 PETTERI HIRVONEN, Kysyntä - tarjonta -kehikon mukainen siirtofunktiomalli bruttokansantuotteelle. 23.08.1989. 38 s.

No 296 PAAVO OKKO, Suomen aluekehityksen ja aluepolitiikan nykyvaihe. 01.09.1989. 20 s.

No 297 ANTTI RIPATTI - PENTTI VARTIA - PEKKA YLÄ-ANTTILA, Suomen talouden ja yritysrakenteen muutokset 1938-1988. 11.09.1989. 95 s.

No 298 ROBERT HAGFORS, On Economic Welfare Equality as a Policy Goal and Social Transfers as Instruments. 11.09.1989. 20 p.

No 299 SYNNÖVE VUORI - PEKKA YLÄ-ANTTILA, Joustava tuotantostrategia puu- ja huonekaluteollisuudessa. 27.09.1989. 60 s.

No 300 SEVERI KEINÄLÄ, Finnish High-Tech Industries and European Integration; Sectoral Study 1: The Telecommunications Equipment Industry. 12.10.1989. 85 p.

No 301 VESA KANNIAINEN, The Arch Model and the Capm: A Note. 30.10.1989. 10 p.

No 302 VESA KANNIAINEN, Research Issues in Corporate Taxation. 30.10.1989. 10 p.

No 303 TOM BERGLUND, Perceived and Measured Risk; An Empirical Analysis. 30.10.1989. 29 p.

No 304 SEVERI KEINÄLÄ, Finnish High-Tech Industries and European Integration; Sectoral Study 2: The Data Processing Equipment Industry. 01.11.1989. 44 p.

No 305 MASSIMO TAZZARI, Numeeriset yleisen tasapainon ulkomaankaupan mallit, teoria ja sovellutuksia. 02.11.1989. 64 s.

No 306 JUKKA LASSILA, Preliminary Data in Economic Databases. 10.11.1989. 10 p.

No 307 SEVERI KEINÄLÄ, Finnish High-Tech Industries and European Integration; Sectoral Study 3: The Pharmaceutical Industry. 15.11.1989. 78 p.

No 308 T.R.G. BINGHAM, Recent Changes in Financial Markets: The Implications for Systemic Liquidity. 12.12.1989. 39 p.

No 309 PEKKA ILMAKUNNAS, A Note on Forecast Evaluation and Correction. 27.12.1989. 13 p.

No 310 PEKKA ILMAKUNNAS, Linking Firm Data to Macroeconomic Data: Some Theoretical and Econometric Considerations. 27.12.1989.38 p.

Elinkeinoelämän Tutkimuslaitoksen julkaisemat "Keskusteluaiheet" ovat raportteja alustavista tutkimustuloksista ja väliraportteja tekeillä olevista tutkimuksista. Tässä sarjassa julkaistuja monisteita on rajoitetusti saatavissa ETLAn kirjastosta tai ao. tutkijalta.

Papers in this series are reports on preliminary research results and on studies in progress; they can be obtained, on request, by the author's permission.

0033A/27.12.1989