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Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

Keskusteluaiheita
Discussion papers

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THE ARCH MODEL AND THE CAPM: A NOTE

No 301 30.10.1989

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I am grateful to Tom Berglund and Robert F. Engle for helpful comments. But they cannot be held accountable for the views propagated in this note.

ISSN 0781-6847

KANNIAINEN, Vesa, THE ARCH MODEL AND THE CAPM: A NOTE, Helsinki : ETLA, Elinkeinoelämän Tutkimuslaitos, The Research Institute of the Finnish Economy, 1989. 10 p. (Keskusteluaiheita, Discussion Papers, ISSN 0781-6847 ; no. 301).

ABSTRACT: The note argues that there is mutual inconsistency between the traditional capital asset pricing model and the recent econometric ARCH (autoregressive conditional heteroscedasticity) approach if the latter is employed to model returns on single assets. If the CAPM is accepted, the current versions of the ARCH model are relevant for market returns. However, a theoretically unjustified asymmetry is found in the implicit pricing assumption used by the ARCH model.

KEY WORDS: ARCH, CAPM

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TIIVISTELMÄ: Kirjoitus väittää, että traditionaalinen riskipitoisten sijoituskohteiden hinnoitteluteoria ja uusi ekonometrinen ARCH malli ovat keskenään ristiriitaisia, jos jälkimmäistä käytetään yhden sijoituskohteen tuoton mallittamisessa. Jos CAPM hyväksytään, tämän hetken ARCH mallit sopivat markkinaportfolion tuottojen mallittamiseen. Silti osoittautuu, että ARCH-malli perustuu implisiittisesti epäsymmetriseen hinnoittelunäkemykseen, jota ei voi pitää teoreettisesti hyväksyttävänä.

ASIASANAT: ARCH, CAPM.

Contents

I Introduction

A typical data series of asset returns on equities or foreign exchange strongly suggests that the assumption of a constant conditional variance of returns is not the most fruitful one. Indeed, data seem to exhibit volatility clustering with large residuals in asset returns to be followed by large residuals. A big surprise seems to predict further big surprises while a small shock tends be be followed by another small shock. This well-documented observation (cf. Bollerslev, Engle and Wooldridge (1988)) goes back at least to Mandlebroit (1963). Two questions arise immediately. First, what kind of pricing behavior of investors is consistent with these empirical facts? Second, what are the appropriate statistical models for estimation and testing purposes?

When discussing the relationship between the financial theorizing and the ARCH approach, this note will argue that there is a contradiction between (current versions of) the ARCH model and the CAPM proposed originally by Sharpe (1964) and Lintner (1965) for valuation of risky assets. In a sense these models do not, of course, belong to the same continent. The CAPM represents a theoretical approach while the ARCH models are purely econometric. But it is legitimate to ask first, whether any finance-theoretic foundations can be developed for the ARCH specifications used and second, whether

the results based on the ARCH models violate any of the central implications of the CAPM.

I argue that a CAPM theorist would have to reject the ARCH model if the latter is to be applied to model the excess returns on single risky assets. Second, I argue that there is an implicit and theoretically unjustified asymmetry in the pricing assumption used by the ARCH model.

II The ARCH Model: A Summary

The autoregressive conditional heteroscedasticity (ARCH) model developed by Engle (1982) has been a great success in providing an econometric approach which beats the more traditional models. As documented by Bollerslew (1986), the GARCH(1,1) model generally is adequate for describing financial data.

To evaluate the ARCH model, let us summarize its major features as in Engle, Lilien and Robins (1987), who used it to resolve the empirical paradoxes in the term structure of interest rates. Let

$$(1) \qquad y_t = (q_t/p) - r$$

stand for the excess return per dollar invested in risky assets with price p and a random return q where r is a return on a riskless asset. The mean and variance of the excess return

are given

by

(2) $$E(y_t) = \mu = (\theta/p) - r \quad \text{and} \quad V(y) = \sigma^2\phi/p^2$$

where θ and ϕ are the mean and variance, respectively, of the distribution of q. Under efficient pricing,

(3) $$E_{t-1}y_t - \mu = \epsilon_t$$

where the error term ϵ_t is unforecastable. The evidence presented by Engle, Lilien and Robins, however, leads to rejection of the hypothesis of a time-invariant risk premium of 6 month T-bills over 3 month T-bills. The alternative view which is better consistent with data rests on the model

(4a) $$y_t = \beta + \delta h_t + \epsilon_t$$

(4b) $$h^2_t = \Gamma + \alpha\Sigma_\tau \, w_\tau \epsilon^2_{t-\tau}$$

where β, δ, Γ and α are parameters and w_τ are weighting parameters. In brief, the conditional variance is a determinant of the current risk premium.

III Evaluation

1. Inconsistency between the CAPM and the ARCH Model

The celebrated implication of the CAPM, based on homogenous expectations of rational investors, is that the risk premium is proportional to the undiversifiable risk of a risky asset. The latter is measured by the covariance of the asset return with the whole set of assets in the economy. Thus

$$(5) \qquad E(r_j) - r = (\theta/p) - r = \phi cov(r_j, r_m)$$

where ϕ is the market price of risk. The first point we want to make here is that the CAPM rejects the role of the variances while the ARCH formulation above ignores the role of the covariance effect. This inconsistency can be avoided if one limits the use of the ARCH approach to market returns only (like Engle, Lilien and Robins did). In the case of single assets, it is unavoidable to model the covariances.

The CAPM rejects the role of variances in pricing of single risky assets and many empirical studies, starting with Fama and MacBeth (1973), seem to support this implication. If it appears that the ARCH approach is successful in bringing the own variance effect back to life in the case of a single asset, the CAPM will be in great trouble /1/.

2. Asymmetric Pricing Implied by the ARCH Model

There is an asymmetry in the pricing assumption implicit in the ARCH model but unrecognized earlier. Any large current disturbance, regardless of whether it is an unexpected capital gain or loss, will increase the predicted conditional risk premium $E_t y_{t+1} - \mu$, cf.(4a)-(4b). But then, given the equilibrium pricing of rational investors, the market value has to be reduced to compensate for the increased risk premium. It is not only that a relatively large reduction in current price would reduce the equilibrium price through the adjustment of the risk premium. Also a relatively large increase in the current price has precisely the same effect if the investors revise their beliefs of riskiness according to the ARCH model. Thus, there is a strong built-in downward bias in price formation of risky securities. But this should then be reflected in negative unconditional skewness. This is an implication that the evidence clearly contradicts (cf. Fama (1976) and Beedles (1979)).

3. Informed and Uninformed Investors

Was the October 1987 crash a surprise? To whom? Is it rational to predict the future risk premia using the observations of the most recent shocks?

If the recent shocks do contain information, it is irrational

not to utilize this information. But expected shocks do not always necessitate the revision of the view concerning the underlying stochastic processes generating the observations. Assume that Shiller (1984) is right about the informed "smart-money" and uninformed "ordinary" investors. Which group then more likely behaves like the ARCH model predicts? For informed investors, the crash could not have been a surprise (though its timing certainly was). It is not a priori clear why these investors would have any reason to revise their view of how the stock markets function and what the risks are. It is taken for granted that the market behavior of uninformed investors may be a source of uncertainty to informed investors. But is its pattern predictable enough?

If uninformed investors behave something like the ARCH model predicts, they revise their subjective assessment of the relative risk premia each time a new observation is made. For the actual determination of prices, it is relevant what the informed investors think about the trading behavior of the uninformed investors. That information can be used for speculative trading. Yet no theory is at hand of the trading policies of the two groups. Real progress in the explanation of changing volatility of stock returns necessitates understanding the strategic interaction between the differently informed traders.

IV Conclusions

This note has discussed the potential mutual inconsistency between the traditional capital asset pricing model and the very recent econometric approach called the ARCH model. The prime contribution of the ARCH approach is the incorporation of the dynamics of changing information into asset pricing models which traditionally have assumed unchanging parameters. Support for this view can be found from Bachrach and Galai (1979), who argue that shifts in the debt-to-equity ratios also change the systematic measure of risk of equity. Inconsistency with the CAPM, nevertheless, arises if the current versions of the ARCH models are applied to excess returns on single assets without incorporating the covariance effects. The CAPM will be in trouble if the variance effects show up as significant in this case. The bad news for the ARCH model here is, however, the theoretically unjustified asymmetry in the pricing view inherent in that approach.

Footnotes.

/1/ An early warning regarding the CAPM was provided by Stiglitz (1972), who long ago pointed out that the economy operates on its efficiency frontier only if firms are independent. If this is not the case, externalities destroy the mean variance efficiency by placing too much weight on own variance relative to the covariance. If anything, this view is bad news for the CAPM in a very fundamental way.

References:

Bachrach B. and Galai D.: "The Risk-Return Relationship and Stock Prices", Journal of Financial and Quantitative Analysis, XIV, 1979, 421-441.

Beedles W.L.: "On the Asymmetry of Market Returns", Journal of Financial and Quantitative Analysis, XIV, 1979, 653-660.

Bollerslev T.: "Generalized Autoregressive Conditional Heteroscedasticity", Journal of Econometrics, 31,1986, 307-327.

Bolleslev T., Engle R.F. and Wooldridge J.M.: "A Capital Asset Pricing Model with Time-varying Covariances", Journal of Political Economy, 96, 1988, 116-131.

Engle R.F.: "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", Econometrica, 50, 1982, 987-1007.

Engle R.F., Lilien D.M. and Robins R.P.:"Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model", Econometrica, 55,1987, 391-407.

Fama E.F.: Foundations of Finance, 1976, Basil Blackwell.

Fama E.F. and MacBeth J.D.: "Risk, Return and Equilibrium:

Empirical Tests", Journal of Political Economy, 71, 1973, 607-636.

Lintner J.: "The Valuation of Risk Assets and Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, 47, 1965, 13-37.

Mandlebroit B.: "The Variation of Certain Speculative Prices", Journal of Business, 36, 1963, 394-419.

Sharpe W.F.:"Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", Journal of Finance, 19, 1964, 452-42.

Shiller R.J.: "Stock Prices and Social Dynamics", Brookings Papers on Economic Activity", 1984:2, 457-510.

Stiglitz J.E.: "On the Optimality of the Stock Market Allocation of Investments", Quarterly Journal of Economics, 1972, 25-60.

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0033A/30.10.1989