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TIMBER SUPPLY INCENTIVES

AND OPTIMAL FOREST TAXATION**

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ABSTRACT: The paper contributes to the theoretical analysis of forest taxation both under certainty and under future timber price uncertainty with risk aversion by focusing on two policy issues: First, the incentive issue of what permanent tax will elicit the highest timber supply given that government wants to collect a fixed amount of tax revenue by means of forest taxation? Second, the welfare issue of what forest taxes will be optimal subject to a government's tax revenue requirement? No sharp conclusions can be obtained concerning the timber supply incentives of a switch between forest taxes under price uncertainty. Turning to the welfare issue given the optimal lump-sum tax, it is desirable to introduce the yield tax, which at the margin serves as an insurance device by decreasing the post-tax variability of future timber price. If there is no aggregate uncertainty or if government is risk neutral, then the full insurance via the 100 % yield tax is optimal. But if there is an aggregate risk and government cares about uncertainty in its tax revenues, then the partial insurance is optimal with the yield tax being less than 100 %.

KEY WORDS: timber supply, forest taxation

1. INTRODUCTION

Theoretical analyses of rotation age and timber supply have predominantly relied on the twin assumptions of certainty and perfect capital markets in the sense that forest owners have been assumed to borrow and lend an unlimited amount at the one risk-free interest rate. Under these circumstances the Fisherian separation theorem holds: consumption decisions of individuals are made separately from their investment decisions (for more on Fisherian separation, see e.g. Copeland and Weston (1983), ch. 2). Most modern analyses start by assuming that forest owners try to maximize the present value of profits accrued from growing trees (see e.g. Chang (1982), (1983), Hyde (1980) and Jackson (1980)). There is no doubt that this framework has produced interesting results concerning how various taxes might affect rotation age and timber supply. But to the extent that there are uncertainties associated with the decision environment, these results may no longer hold.

As far as the role of uncertainty in forest management is concerned, using a generalized form of Johansson and Löfgren's (1985) two-period model[1)], Koskela (1988a), (1988b) has analyzed the relationship between various types of forest taxes and timber supply under future timber price uncertainty when forest taxes are temporary in the sense that the tax rates may be different between the current and future periods. Specifically, Koskela (1988a), (1988b) studies the *ceteris paribus* effects of various types of forest taxes on timber supply and also extends the analysis to cover *policy questions* in the following sense: What are the timber supply implications of changes in the timing of a given tax (i.e. switching the tax between the current and future periods) as well as in switching between tax types under the constraint that tax revenues to government remain unchanged?

The paper contributes to the theoretical analysis of forest taxation by focusing on two policy issues which has not received any attention in the literature: First, we are interested in the incentive issue of what permanent forest tax will elicit the highest timber supply[2)] given that the government wants to collect a fixed amount of tax revenue by means of forest taxes? Second, and more importantly, we analyze the welfare issue of what forest tax will be optimal in the sense of maximizing the (expected) utility of a representative forest owner subject to a government's tax revenue requirement? The forest taxes to be analyzed are (i) the lump-sum tax, (ii) the unit tax and (iii) the yield tax. The lump-sum tax is defined as a fixed amount of tax, which is independent of the amount of timber harvested. The unit tax is defined as a production tax, i.e. so many dollars per unit of harvested timber volume. And finally, the yield tax is defined as a tax on timber revenue from harvest.[3)] In both the incentive and welfare analyses the lump-sum tax is regarded as the benchmark case to which the unit tax and the yield tax are compared.

The paper proceeds as follows: Section 2 presents the framework to be used and the assumptions and simplifications to be adopted and characterizes the timber supply and welfare implications of structural tax policies, which will keep the tax revenues of government unchanged under certainty and perfect capital markets. In section 3 the analysis is extended to consider the same issues under future timber price uncertainty and risk aversion.

1.1. Summary of results

To anticipate results, under certainty and perfect capital markets, the permanent lump-sum and yield taxes are neutral in terms of timber supply since they do not affect the relationship between the marginal return from and the marginal cost of harvesting. Hence, the choice between them does not matter either in terms of timber supply or in terms of the welfare of forest owners. On the other hand, a permanent rise in the unit tax will affect timber supply negatively (positively) with increasing (decreasing) timber prices. This substitution effect can be explained as follows: A permanent rise in the unit tax will decrease the marginal return from current harvesting by the amount of the real rate of interest and the marginal cost of current harvesting by the amount of the growth rate of forest. Under increasing (decreasing) timber prices the real rate is higher (lower) than the growth rate of forest at the interior solution so that the timber supply will decrease (increase) as a result of a permanent rise in the unit tax. Consequently, the timber supply effects of changing the tax base between the unit tax and either the lump-sum or yield tax depend on the question of whether timber prices are increasing or decreasing. As for the welfare effects, both the lump-sum and yield taxes dominate the unit tax under the changing timber prices due to the distortionary unit tax i.e. due to the non-zero substitution effect.

Introducing the future timber price uncertainty with risk aversion changes picture in a number of ways. First, taxes may now have wealth effects. That is, when a tax goes up, the forest owner is worse-off and may be less willing to take risks, i.e. he (or she) may be more willing to harvest now rather than in the future. Under these circumstances a permanent rise in the lump-sum tax will increase timber supply. Second, in addition to these positive wealth effects, the permanent change in the unit tax will affect timber supply via the substitution effect by changing the marginal return from and the (expected) marginal cost of current harvesting. Third, the permanent change in the yield tax affects also negatively the post-tax variability of future timber price and the yield tax has an ('adjusted') risk effect, which is, however, a priori ambiguous.

Turning to policy questions, a change in the tax base, which will keep the (expected) tax revenues of government unchanged, will cancel wealth effects out, while the substitution and risk effects remain. Hence, the timber supply effect of a switch between the lump-sum and unit tax depends on the sign of the unit tax substitution effect, while the effect of a switch between the lump-sum and yield tax depends on the sign of the ('adjusted') risk effect. Proposition 3 gives sufficient conditions under which the signs can be determined unambiguously. Finally, under timber price uncertainty the unit tax is generally distortionary and is thus dominated by the lump-sum tax on welfare grounds. As for the comparison between the lump-sum and the yield tax, it is generally desirable given the optimal (expected utility maximizing) lump-sum tax to introduce the yield tax, which at the margin serves as an insurance device by decreasing the post-tax variability of the future timber price. If there is no aggregate uncertainty or if government is risk-neutral, then the full insurance via the 100 % yield tax is optimal. On the other hand, if there is aggregate risk and government cares about uncertainty in its tax revenues, then the partial insurance is optimal with the yield tax being less than 100 %.

2. TIMBER SUPPLY AND OPTIMAL FOREST TAXATION UNDER CERTAINTY

2.1. A Theoretical Framework

The forest owner is assumed to have a preference ordering over the present and future consumption, c_1 and c_2 respectively, which is represented by a intertemporally additive, thrice differentiable utility function $U = u(c_1) + \beta u(c_2)$, where $\beta = (1+\rho)^{-1}$ denotes the rate of time preference factor and $u' = \partial u(.)/\partial c > 0$ and $u'' = \partial^2 u(.)/\partial c^2 < 0$, i.e. u is strictly concave. Thus U describes the discounted utility from consumption in both periods. In what follows the partial derivatives are denoted by primes for functions with one argument and by subscripts for functions with many arguments. E.g. $A_x(x,y) = \partial A/\partial x$, $A_{xy}(x,y) = \partial^2 A/\partial x \partial y$ etc.

The forest owner chooses between harvesting today (current timber supply) x and harvesting to-morrow (future timber supply) z according to the 'production possibility' frontier

(1) $$z = (Q-x) + F(Q-x), \quad F'(.) > 0, \; F''(.) < 0$$

where Q refers to the initial stand yield (i.e. volume/unit of land) and F(Q) defines stand growth as a function of the stand yield Q. Therefore, $F' = F'(Q-x)$ gives the growth rate for the stand after the current period harvest x. The more one cuts to day, the less is available for to-morrow because $\partial z/\partial x = -(1+F')$. Moreover, the greater is the volume per unit land, the lower is the growth rate, i.e. $F''(.) < 0$. Under certainty this assumption about growth as a positive, but decreasing function of the volume per unit land is important. As shown later, assuming the linear 'production possibility' frontier with

$F''(.) = 0$ implies that there is no unique solution interior; either the forest owner cuts everything now ($z = 0$) or everything in the future ($z = Q+F(Q)$) or is indifferent between cutting now or cutting in the future.

Denote the lump-sum, unit and yield taxes by T, t and τ respectively. Thus T = tax dollars lost due to fixed taxes, t = tax dollars lost per unit of harvest and τ = tax dollars lost per unit of harvest revenue. If we denote the current and future before-tax timber prices by p_1 and p_2 respectively and assume that all taxes are operative, then the respective post-tax prices can be written as $p_i^* = p_i(1-\tau)-t$ so that $p_i^* = p_i-t$ if the yield tax is not operative and $p_i^* = p_i(1-\tau)$ if the unit tax is not operative, where $i = 1,2$. In the first period the forest owner can allocate the net revenues from timber harvested between current consumption c_1, lump-sum tax T and saving ($S > 0$), or, in the case where current harvest revenues do not meet current consumption and tax needs, finance the shortfall by borrowing ($S < 0$) so that we have $p_1^*x = c_1 + T + S$ for the current period flow-of-funds equation. The forest owner's consumption in the future, c_2, depends on the net revenues from harvested timber, p_2^*z, lump-sum tax, T, and on capital income from saving RS (if $S > 0$), or on the capital expense from borrowing RS (if $S < 0$) so that $c_2 = p_2^*z - T + RS$, where $R = 1+r$ = the interest rate factor in the capital market. With the perfect capital market the forest owner can borrow and lend freely at the risk-free interest rate r so that we can combine the flow-of-funds equations to get the the intertemporal budget constraint with taxes

(2) $$c_2 = p_2^*z - T + R(p_1^*x - T - c_1).$$

2.2. Comparative statics of forest taxation

The decision problem of the forest owner is now to choose current consumption c_1 and current timber supply x - which here means current timber harvested - so as to maximize the intertemporally additive utility function $U = u(c_1) + \beta u(c_2)$ subject to the 'production possibility' frontier (1) and the intertemporal budget constraint (2). In what follows we are interested only in the qualitative properties of the current timber supply x. The first-order condition for the utility maximization in terms of x is

$$(3) \qquad U_x = 0 \iff e = [p_1(1-\tau)-t]R - [p_2(1-\tau)-t](1+F') = 0$$

According to (3) the harvesting decision is separable from the consumption preferences of the forest owner and depends only on the post-tax marginal returns of harvesting more today, $[p_1(1-\tau)-t]R$, and on the post-tax marginal costs of harvesting more today, $[p_2(1-\tau)-t](1+F')$. Thus at the optimum, x is determined so as to make the post-tax marginal return equal to the post-tax marginal cost of harvesting. The expression (3) can be rewritten as

$$(3') \qquad r - F' = \frac{[p_2 - p_1]}{[p_1 - t(1-\tau)^{-1}]}(1+F')$$

so that $r \gtreqless F'$ as $p_2 \gtreqless p_1$. At the interior solution for x the real rate of interest, r, exceeds, is equal to, or falls short of the growth rate of forest, F', as timber prices are increasing, unchanged or decreasing respectively.

As indicated earlier, the incentive and welfare analyses are conducted by keeping the lump-sum tax as the benchmark case to which other taxes are compared. On the basis of the expression (3') the ceteris paribus effects of forest taxes on timber supply are as follows. Both the lump-sum tax and the yield tax with $t = 0$ are neutral since they affect neither the post-tax marginal return from nor the post-tax marginal cost of current harvesting. In the case of the unit tax with $\tau = 0$ we have $\text{sgn}(x_t) = -\text{sgn}(r-F')$ so that by utilizing the expression (3') we get $x_t \lesseqgtr 0$ as $p_2 \gtreqless p_1$. This can be interpreted as follows: a permanent rise in the unit tax will decrease the post-tax marginal return of harvesting by the amount of the real rate of interest r and the post-tax marginal cost of harvesting by the amount of the growth rate of forest F'. Under increasing (decreasing) timber prices the real rate of interest exceeds (falls short of) the growth rate of forest so that a permanent rise in the unit tax will decrease (increase) timber supply when timber prices are rising (falling).[4)]

2.3. Timber supply incentives, welfare and forest taxes

The earlier section dealt with the ceteris paribus effects of the permanent changes in forest taxes. Government may not, however, always be in a position - or it may not want - to change only one tax parameter at a time. Instead, it may ask: Given the tax revenue requirement from forest taxation, what structure of forest taxes is most effective in the sense of eliciting the highest current timber supply?[5)] This necessitates the study of the timber supply effects of forest tax switches.

We noticed that both the lump-sum and yield taxes had no effect on timber supply so that the tax switch between them does not affect timber supply. In order to compare the lump-sum and unit taxes we first define the present value of forest tax revenue

(4) $T^{o} = T(1+R^{-1}) + t(x+R^{-1}z)$

Before going on it is useful to make one further assumption. If taxes are not neutral, then they will affect the present value of the tax revenues (4) both directly via T and t and indirectly via behavioural responses, when x and z change as a response to changes in tax parameters. All subsequent analyses will be carried out under the assumption that the direct effect of taxes dominate the indirect ones. This means that the government tax revenues and tax rates are assumed to be positively related. This relationship is sometimes called in the literature the Laffer curve, which is thus assumed to be upward-sloping.[6)] This is both plausible and simplifies the analysis. Taking the partial derivative of (4) with respect to T and t and accounting for the behavioural responses gives $\partial T^{o}/\partial T = R^{-1}[1+R+t(r-F')x_T]$ and $\partial T^{o}/\partial t = R^{-1}[(Rx+z)+t(r-F')x_t]$, which we assume to be positive.

The tax switch between T and t, which will keep T^{o} unchanged is defined by $dT = -(Rx+z)(1+R)^{-1}dt - t(r-F')(1+R)^{-1}dx$. The total effect of the shift on timber supply is given by the following total differential $dx = x_t dt + x_T dT$. Substituting the above expression for dT into dx gives the timber supply response only in terms of t, where the offsetting change in T has been taken into account. This yields with $x_T = 0$

$$(5) \qquad \left.\frac{dx}{dt}\right|_{\substack{(t,T)\\ dT^0=0}} = x_t \lesseqgtr 0 \quad \text{as } p_2 \gtreqless p_1$$

Thus we have

Proposition 1: In the presence of certainty, upward-sloping Laffer curve and perfect capital markets the choice between the lump-sum and yield tax does not matter in terms of timber supply. On the other hand, changing the tax base towards the unit tax, in the tax switch between the lump-sum and unit tax so as to keep the present value of government tax revenues unchanged, will decrease, leave unchanged or increase timber supply as timber prices will increase, remain constant or decrease respectively.[7)]

Instead of asking "What forest tax policy is most effective in the sense of eliciting the highest timber supply?" one may ask: "What forest tax policy is optimal in the sense of maximizing the utility of a (representative) forest owner?". The problem is then to choose the tax rates so as to maximize a (representative) forest owner's utility subject to his (or her) behavioural responses and the government tax revenue requirement.[8)]

Since the lump-sum and yield tax are non-distortionary with $t = 0$, the choice between them does not matter from the welfare point of view. This leaves us the task to compare the lump-sum and unit tax. The first-order conditions for the maximization of U subject to the 'production possibility' frontier (1) and the intertemporal budget constraint (2) implicitly determine current consumption and current timber supply as functions of exogenous parameters. For convenience, we are here interested only in the role of the lump-sum and unit taxes so that $c_1 = c_1(T,t,..)$ and $x = x(T,t,..)$. Substituting these for c_1 and x in the direct utility function $U = u(c_1) + u(c_2)$ gives the indirect utility function $V(T,t,..) = U^o$, which indicates the maximum utility that can be achieved given the lump-sum tax T and the unit tax t. Applying the envelope theorem[9)] gives

$$(6) \quad \begin{cases} \text{(a)} \quad V_T = -\beta(1+R)\, u'(c_2) < 0 \\ \text{(b)} \quad V_t = (Rx+z)(1+R)^{-1} V_T < 0 \end{cases}$$

so that the maximum utility is negatively related to the tax rates. The welfare-maximizing tax structure can now be obtained by choosing T and t, which maximize $V(T,t,..)$ subject to the government tax revenue requirement (4) so that the Lagrangian to be maximized is

$$(7) \qquad L = V(T,t,..) - \lambda(T^o - T(1+R^{-1}) - t(x+R^{-1}z))$$

where λ = the Lagrangian multiplier associated with the government tax revenue requirement.

Differentiating (7) with respect to T and t, accounting for (1) and for endogeneity of x and using (6a-6b) gives the first-order conditions for the welfare optimum

$$(8) \quad \begin{cases} \text{(a)} \quad L_T = 0 = V_T + \lambda(1+R^{-1}) \\ \text{(b)} \quad L_t = 0 = (Rx+z)(1+R)^{-1} L_T + \lambda R^{-1} t(r-F')x_t \end{cases}$$

Using (6a) and the fact that $u'(c_1) = Ru'(c_2)$ at the optimum for c_1 (8a) can be written as $\lambda = -V_T(1+R)^{-1} = u'(c_1) = \beta Ru'(c_2)$. The permanent lump-sum tax should be set so as to make the marginal utility of consumption in both periods equal to the marginal cost of the lump-sum tax λ. Given the optimal lump-sum tax T* and using $x_t = ((p_2-t)F")^{-1}(r-F')$ the equation (8b) can be rewritten as

$$(9) \qquad L_{t|T*} = 0 \iff R^{-1}(r-F')^2((p_2-t)F")^{-1}t = 0$$

Under changing timber prices the unit tax is distortionary and its optimal value $t^* = 0$ given that the lump-sum tax can be used. Thus we have

Proposition 2: In the presence of certainty and perfect capital markets the choice between the lump-sum and yield tax does not matter in terms of welfare. On the other hand, under changing timber prices the unit is distortionary and the lump-sum tax dominates it in the sense that given the optimal lump-sum tax the welfare-maximizing unit tax is zero.[10)]

3. TIMBER SUPPLY AND OPTIMAL FOREST TAXATION UNDER FUTURE PRICE UNCERTAINTY

3.1. Comparative statics of timber supply under price uncertainty

This section extends the earlier analysis by allowing for future timber price uncertainty and developing its implications for comparative statics of timber supply as well as for policy questions analyzed in section 2.3. The forest owner decides how much to harvest today given today's known price and a (subjective) probability distribution of to-morrow's stochastic price. The decision problem for the forest owner is thus to choose c_1 and x so as to maximize the discounted expected utility from consumption $U^* = u(c_1) + \beta E(u(c_2))$, where E denotes the expectations operator and c_2 is now stochastic because of future price uncertainty. The expected utility of future consumption can be expressed as $E(u(c_2)) = \int_{\underline{p}_2}^{\bar{p}_2} u(c_2)f(p_2)dp_2$, where $f(p_2)$ = the probability density function of p_2 defined over the range $[\underline{p}_2, \bar{p}_2]$.

We assume that forest owners are risk-averse ($u''(c_2) < 0$) and that the so-called Arrow-Pratt measure of absolute risk aversion $A(c_2) = -u''(c_2)/u'(c_2)$ is non-increasing in c_2. This means that willingness to take a given risky choice will not decrease as c_2 increases. This can be regarded as widely plausible. It is not dimensionless and depends on the units in which c_2 is measured (for a seminal presentation and further discussion, see Arrow (1974) pp. 90-111).

The first-order conditions for the maximization of U* in terms of the decision variables c_1 and x are now expressed as

(10) $$U_c^* = u'(c_1) - \beta R E(u'(c_2)) = 0$$

(11) $$U_x^* = \beta E(u'(c_2)e) = 0$$

where $U_c^* = \partial U^*/\partial c_1$, $U_x^* = \partial U^*/\partial x$ and $e = p_1^*R - p_2^*(1+F')$ with $p_i^* = p_i(1-\tau)-t$ (i = 1,2). By using the rule $E(ab) = E(a)E(b)+cov(a,b)$ for stochastic variables a and b, the expression (11) can be written as

(12) $$E(u'(c_2))\bar{e} = cov(u'(c_2),p_2)(1+F')(1-\tau)$$

where $\bar{e} = p_1^*R - \bar{p}_2^*(1+F')$ and where $cov(u'(c_2),p_2) < 0$ because of risk aversion. The expression (12) has two important implications: First, $\bar{e} < 0$ at the interior solution with $F'' < 0$ so that the forest owner harvests to the point where the post-tax marginal return from harvesting more today falls short of the expected post-tax marginal cost of harvesting more today so that allowing for price uncertainty will increase timber supply.[11)] Second, the harvesting decision is no longer separable from consumption preferences of risk averse forest owners and has thus to be analyzed simultaneously with the consumption-saving decision. The Fisherian separability condition no longer holds.

The second-order conditions for the expected utility maximization are

$$(13) \qquad U^*_{cc} = u''(c_1) + \beta R^2 E(u''(c_2)) < 0$$

$$(14) \qquad U^*_{xx} = \beta[E(u''(c_2)e^2) + E(u'(c_2)p_2^*)F''] < 0$$

and

$$(15) \qquad \Delta = U^*_{cc}U^*_{xx} - (U^*_{cx})^2 > 0$$

where $U^*_{cc} = \partial^2U^*/\partial c_1^2$, $U^*_{xx} = \partial^2U^*/\partial x^2$ and $U^*_{cx} = \partial^2U^*/\partial c_1\partial x = \partial^2U^*/\partial x\partial c_1 = U^*_{xc} = -\beta RE(u''(c_2)e)$. Under the stated assumptions the second-order conditions hold. It is easy to show that decreasing (constant) absolute risk aversion implies that $E(u''(c_2)e) < 0$ (=0) so that $U^*_{cx} > 0$ (=0).

Given the second-order conditions, the first-order conditions define implicitly c_1 and x in terms of exogenous parameters. But now, due to the non-separability, the first-order conditions (10) and (11)) must be evaluated simultaneously. In what follows the results are presented only for timber supply, which we are ultimately interested in.[12)]

Before developing comparative statics of forest taxes we look at the relationship between changes in pure risk associated with future timber price and the harvesting decision. This is both interesting per se and turns out convenient in understanding an impact of forest taxation.

Let us define a small increase in risk as a "streching" of the probability distribution of future price around a constant mean. This requires the introduction of two parameters, one multiplicative and one additive. Let us write the before-tax price as $\tilde{p}_2 = \varepsilon + \eta p_2$, where ε is the additive shift parameter and η is the multiplicative one. An increase in η alone will blow up all the values of p_2 by increasing the expected value as well as the variance. To restore the mean for $\tilde{p}_2$, i.e. to compensate for a change in η, we have to reduce ε so that $dE(\varepsilon + \eta p_2) = 0$ or $\bar{p}_2 d\eta + d\varepsilon = 0$ giving $d\varepsilon/d\eta = -\bar{p}_2$, where $\bar{p}_2 = E(p_2)$ (for a use of this technique, see e.g. Sandmo (1971)). Hence, the ceteris paribus change in x due to an increase in pure risk can be obtained by evaluating the effects of a change in η from the point $\varepsilon = 0$, $\eta = 1$ with $d\varepsilon/d\eta = -\bar{p}_2$. This gives the timber supply response of a compensated change in η as

$$(16) \qquad (x_{\eta}\big|_{\substack{\varepsilon=0\\ \eta=1}}) = \Delta^{-1}[-U^*_{x\eta}U^*_{cc} + U^*_{c\eta}U^*_{cx}]$$

where $U^*_{cc} < 0$, $U^*_{cx} > 0$ (=0) if the absolute risk aversion is decreasing (constant) and where $\Delta > 0$. The terms $U^*_{x\eta} = \partial^2 U^*/\partial x \partial \eta$ and $U^*_{c\eta} = 2U^*$ refer to compensated changes in η. It is shown in appendix 1 that $U^*_{x\eta} > 0$ and $U^*_{c\eta} < 0$ under non-increasing absolute risk aversion. Thus $(x_{\eta}\big|_{\substack{\varepsilon=0\\ \eta=1}}) > 0$ is positive under constant absolute risk aversion, while a priori ambiguous under decreasing absolute risk aversion. On the one hand, a rise in future price risk means that the forest owner is worse off. Current timber supply will tend to increase via the "precautionary effect" (the term $\Delta^{-1}(-U^*_{x\eta}U^*_{cc}) > 0$). On the other hand, riskier future price may induce the forest owner to take a "hedging" position. Current timber supply will tend to decrease via the "hedging effect" to the extent that the forest owner is worried about the worst possible realizations of future price and wants to avoid the negative future consumption effects resulting from low prices (the term $\Delta^{-1}(U^*_{c\eta}U^*_{cx}) < 0$ if the absolute risk aversion is decreasing). If the "hedging effect" dominates the "precautionary effect", then an increase in pure timber price risk will decrease current harvesting!

Let us now turn to the comparative statics of forest taxation on timber supply. For the lump-sum tax T we have

$$(17) \qquad x_T = \Delta^{-1}\beta(1+R)u''(c_1)E(u''(c_2)e) \gtreqless 0 \quad \text{as } A'(c_2) \lesseqgtr 0$$

Under decreasing absolute risk aversion the lump-sum tax and current harvesting are positively related; a rise in T will make forest owners worse-off and they become less willing to take risks. Hence, timber supply will increase due to the wealth effect.

Let us next consider the unit tax. It is shown in appendix 2 that the ceteris paribus effect of the unit tax can be decomposed as

$$(18) \qquad x_t = x_t^c + (Rx+z)(1+R)^{-1}x_T$$

where $x_t^c = \Delta^{-1}U^*_{cc}\beta E(u'(c_2))(r-F')$ so that $\text{sgn}(x_t^c) = -\text{sgn}(r-F')$.

The equation (18) is a Slutsky decomposition for timber supply under uncertainty and it separates the total effect into the substitution effect, x_t^c, on the one hand and into the wealth effect, $(Rx+z)(1+R)^1 x_T$, on the other hand. The substitution effect describes the timber supply response to a change in the unit tax, which is compensated by changing the lump-sum tax so as to keep the expected utility of the forest owner unchanged. The wealth effect is positive under decreasing absolute risk aversion. As for the sign of x_t^c notice that $\bar{e} < 0$ at the interior solution so that with $\tau = 0$ we have $r-F' < (\bar{p}_2-p_1)(1+F')(p_1-t)^{-1}$. Hence, $\bar{p}_2 \leqq p_1 \Rightarrow r-F' < 0$, while sgn$(r-F')$ is a priori indeterminate with $\bar{p}_2 > p_1$. Thus we have

$$(19) \qquad x_t^c = \begin{cases} 0 & \text{as } \bar{p}_2 \leqq p_1 \\ ? & \text{otherwise} \end{cases}$$

Under price uncertainty with risk aversion current harvesting is at the point, where the post-tax marginal return falls short of the expected post-tax marginal cost of harvesting. If the timber prices are not expected to increase, then the real rate of interest is less than the growth rate of forest. Under these circumstances a rise in the unit tax will decrease the marginal return less than the marginal cost and timber supply will increase via the substitution effect. Thus if timber prices are not expected to increase, the substitution and wealth effects reinforce each other and the unit tax will have a positive effect on current harvesting.

Finally, we consider the yield tax with t = 0. It is shown in appendix 2 that its ceteris paribus effect can be decomposed as

(20) $$x_\tau = S - (1-\tau)^{-1}(x_{\eta|\varepsilon=0,\eta=1}) + (Rp_1x + \bar{p}_2z)(1+R)^{-1}x_T$$

where $S = -(1-\tau)^{-1}(p_1x^c_{p_1} + \bar{p}_2x^c_{\varepsilon|\varepsilon}) > 0$. On the one hand, a rise in the yield tax will make the forest owner worse-off. He (or she) will become less willing to take risks and increases timber supply due to the wealth effect, $(Rp_1x+\bar{p}_2z)(1+R)^{-1}x_T$. One the other hand, a rise in the yield tax will decrease the post-tax risk of timber revenues, which risk effect, $-(1-\tau)^{-1}(x_{\eta|\varepsilon=0,\eta=1})$, is, however, a priori indeterminate. Finally, because the yield tax will affect directly neither the marginal return from nor the marginal cost of harvesting at the interior solution, the risk effect overestimates the timber supply effect, which is due to the negative relationship between the yield tax and the variability of the post-tax timber price. In order to correct this we have to add the positive 'weighted" substitution effect, S, where the terms $x^c_{p_1}$ and $x^c_{\varepsilon|\varepsilon=0}$ describe the compensated timber supply with respect to the current and expected future timber price respectively, when compensation is done in terms of the lump-sum tax. The term $S - (1-\tau)^{-1}(x_{\eta|\varepsilon=0,\eta=1})$ can be called the 'adjusted' risk effect. The total effect of the yield tax is thus a priori ambiguous.

3.2. Timber supply incentives and forest taxes

After the ceteris paribus analysis of forest taxes let us turn to consider the question of what structure of forest taxes is most effective in the sense of eliciting the highest timber supply given the government tax revenue requirement. In the presence of future timber price uncertainty government tax revenues are stochastic so that it is not immediately evident, what is meant by a change in the forest tax switch. In what follows we consider the changes in taxes, which will keep the expected present value of tax revenues unchanged. This is defined by

$$(21) \qquad T^{o} = T(1+R^{-1}) + t(x+R^{-1}z) + \tau(p_1x+R^{-1}\bar{p}_2z)$$

and assuming either $\tau = 0$ or $t = 0$ gives two alternative tax revenue requirements.

There are two justifications for the neglect of risk in tax revenues on the part of government. To the extent that risks are independent across agents and their number is large, the law of large numbers will guarantee government a constant total tax revenue despite uncertainty at the individual level. Government is simply a more efficient risk-pooler (see, e.g. Vickrey (1964)). To the extent that risks are aggregative risks, however, (21) only means that government is risk-neutral, i.e. indifferent to tax revenue risk.

Like in the certainty case we use the lump-sum tax as the benchmark case, to which other taxes are compared. No sharp conclusions are possible at this level of generality as the following proposition indicates.

Proposition 3: In the presence of future timber price uncertainty, upward-sloping Laffer curve and perfect capital markets the timber supply effects of the expected present value preserving forest tax switches can be characterized as follows: (a) the non-increased expected timber prices imply that changing the tax base towards the unit tax will increase timber supply, while (b) the non-positive pure risk effect implies that changing the tax base towards the yield tax will increase timber supply.

Proof: (a) The tax switch, which keeps T^o constant with $\tau=0$, is obtained by differentiating (21) with respect to t, T and x so that $dT^o = 0$. This gives $dT = -(Rx+z)(1+R)^{-1}dt - t(r-F')(1+R)^{-1}dx$. Substituting this for dT in the total differential $dx=x_t dt + x_T dT$ and accounting for the equation (18) yields

$$(22) \qquad \left(\frac{dx}{dt}\Big|_{dT^o}^{(t,T)}\right) = m^{-1}(x_t^c)$$

where $m = 1 + t(r-F')(1+R)^{-1}x_T$. In order to get the relationship between T^o and T we take the partial derivative of (21) with respect to T which gives $\partial T^o/\partial T = R^{-1}[1+R+t(r-F')x_T] = R^{-1}(1+R)m > 0$ due to the upward-sloping Laffer curve. Thus sgn $\left(\frac{dx}{dt}\Big|_{dT^o}^{(t,T)}\right) = \text{sgn}(x_t^c)$ so that $x_t^c > 0$ as $\bar{p}_2 \lesseqgtr p_1$ according to (19). This proves (a).

(b) Analogously, with t = 0 the tax switch, with keeps T^o constant, is defined by $dT = -(Rp_1x+\bar{p}_2z)(1+R)^{-1}\,d\tau - \tau\bar{e}^o(1+R)^{-1}\,dx$, where $\bar{e}^o = p_1R - \bar{p}_2(1+F') < 0$. Substituting this for dT in $dx = x_\tau d\tau + x_T dT$ and accounting for the equation (20) yields

$$(23) \qquad \left(\frac{dx}{d\tau}\Big|_{dT^o=0}^{(\tau,T\ )}\right) = n^{-1}[S - (1-\tau)^{-1}(x_\eta\big|_{\eta=1}^{\varepsilon=0})]$$

where $n = 1 + \tau\bar{e}^o(1+R)^{-1}x_T$ and $S = -(1-\tau)^{-1}(p_1x_{p_1}^c + \bar{p}_2x_\varepsilon^c|_{\varepsilon=0}) > 0$. It can be shown, as in part (a), that the upward-sloping Laffer curve implies $n > 0$. If the pure risk effect is non-positive, i.e. $(x_\eta\big|_{\eta=1}^{\varepsilon=0}) \leq 0$, then $(23) > 0$. □

3.3. Optimal forest taxation under price uncertainty

The section 3.2. developed the timber supply implications of various forest tax switches with given government tax revenue requirement for any arbitrary combination of forest taxes so that the expected utility of a (representative) forest owner was allowed to change as a residual. But what tax policy is optimal in the sense of maximizing the expected utility under price uncertainty? In our framework this question can be answered as follows.

Proposition 4: In the presence of timber price uncertainty with risk risk aversion and perfect capital markets optimal forest taxation can be characterized as follows: (a) the lump-sum tax dominates the unit tax in the sense that given the optimal lump-sum tax, the welfare maximizing unit tax is zero, but (b) given the optimal lump-sum tax it is desirable to introduce the yield tax, which at the margin serves as an insurance device by decreasing the post-tax variability of the future net timber price. Moreover, in the absence of aggregate risk with the non-distortionary yield tax the full insurance is in fact optimal. Finally (c) if in the presence of aggregate risk government cares about uncertainty in its tax revenues, the partial insurance with the yield tax rate being less than 100 % is optimal.

Proof: The first-order conditions (10)-(11) implicitly define c_1 and x as a function of forest taxes and we get the expected indirect utility function $V^*(T,t,\tau,..) = u^0$. According to the envelope theorem $V^*_T = -\beta(1+R)E(u'(c_2)) < 0$, $V^*_t = (Rx+z)(1+R)^{-1}V^*_T < 0$ and $V^*_\tau = (Rp_1x + \bar{p}_2z)(1+R)^{-1} -(1-\tau)^{-1}V^*_\eta$, where $V^*_\eta = \beta\,\mathrm{cov}\,(u'(c_2)p_2)z(1-\tau) < 0$ describes the effect of a pure change in future price risk on the expected indirect utility.

(a) Let us first compare the lump-sum and unit taxes with $\tau = 0$. Maximizing the Lagrangian $L = V^*(T,t,..) - \lambda[T^o - T(1+R^{-1}) - t(x+R^{-1}z)]$ with respect to T defines the optimal lump-sum tax T*. It is characterized by

$$(24) \qquad L_T = 0 = V_T^* R(1+R)^{-1} + \lambda[1+t(r-F')(1+R)^{-1}x_T]$$

where λ is the Lagrangian multiplier associated with the government tax revenue requirement. Maximizing now the Lagrangian L with respect to t and utilizing the relationship between V_T^* and V_t^* gives

$$(25) \qquad L_t = 0 = (Rx+z)L_T + \lambda t(r-F')x_t^c$$

where we have also used the expression (18) for x_t. Given that the lump-sum tax has been chosen optimally, the expression (25) is reduced to

$$(26) \qquad L_t|_{T^*} = \Delta^{-1}\lambda t(r-F')^2 E(u'(c_2))U_{cc}^* = 0$$

For the distortionary unit tax ($r \neq F'$) this implies that $t^* = 0$. Thus (a) is proved.

(b) As for the relationship between the lump-sum and yield taxes with $t = 0$ the Lagrangian $K = V^*(T,\tau,..) - \lambda[T^o - T(1+R^{-1}) - \tau(p_1x + R^{-1}\bar{p}_2z)]$ has to be maximized with respect to T and τ. The optimal T* is now characterized by

$$(27) \qquad K_T = 0 = V_T^* R(1+R)^{-1} + \lambda[1+\lambda\tau\bar{e}^o(1+R)^{-1}x_T]$$

where $\bar{e}^o = p_1R - \bar{p}_2(1+F') < 0$ according to the first-order condition (11) and where λ is the Lagrangian multiplier associated with government tax revenue requirement. Maximizing K with respect to τ gives, after using the relationship between V^*_T and V^*_τ and the equation (20) for x_τ,

$$(28) \qquad K_\tau = 0 = (Rp_1x+\bar{p}_2z)K_T-(1-\tau)^{-1}V^*_\eta R + \lambda\tau\bar{e}^o[S-(1-\tau)^{-1}(x_\eta\big|_{\substack{\varepsilon=0\\ \eta=1}})]$$

where $S = -(1-\tau)^{-1}(p_1x^c_{p_1} + \bar{p}_2x^c_{\varepsilon|\varepsilon=0}) > 0$. Given that the lump-sum tax has been chosen optimally, (28) is reduced to

$$(29) \qquad K_{\tau|T^*} = -(1-\tau)^{-1}V^*_\eta R + \lambda\tau\bar{e}^o[S-(1-\tau)^{-1}(x_\eta\big|_{\substack{\varepsilon=0\\ \eta=1}})] = 0$$

Evaluating this expression at the endpoints $\tau = 0$ and $\tau = 1$ gives

$$(30) \qquad \begin{cases} \text{(a)} \quad K_{\tau|T^*,\tau=0} = -V^*_\eta R > 0 \\ \text{(b)} \quad K_{\tau|T^*,\tau=1} = 0 \end{cases}$$

(30b) results from the fact that with $\tau = 1$ there is no uncertainty and $\bar{e}^o = 0$ at the interior solution. This proves (b).

(c) Finally, we consider the case, where government cares about the stochasticity of tax revenues. The Lagrangian to be maximized can be written as $M = V^*(T,\tau,..) - \psi[W(T^o)-E(W(y))]$, where $y = T(1+R^{-1}) + \tau(p_1x + R^{-1}p_2z)$ and ψ is the Lagrangian multiplier associated with the constraint $W(T^o) = E(W(y))$ and where $W' > 0$, $W'' < 0$. The optimal T* is now characterized by

$$(31) \qquad M_T = 0 = V^*_T R(1+R)^{-1} + \psi E(W'(y))[1 + \tau\bar{e}^o(1+R)^{-1}x_T] - \psi\tau(1+F')(1+R)^{-1}\,\mathrm{cov}\,(W'(y),p_2)x_T$$

where we have used the rule $E(ab) = E(a)E(b)+\text{cov}(a,b)$ for stochastic variables. Maximizing M with respect to τ gives, after using the relationship between V^*_T and V^*_τ, the equation (20) for x_τ and manipulating a bit,

$$(32)\qquad M_\tau = 0 = \bar{y}RM_T - (1-\tau)^{-1}\, V^*_\eta R + \psi\tau z\,\text{cov}\,(W'(y),p_2) + \psi\tau\,[\bar{e}^o E(W')-(1+F')\,\text{cov}\,(W'(y),p_2)]\,Q$$

where $Q = S-(1-\tau)^{-1}(x_\eta|_{\varepsilon=0,\,\eta=1})$. Now given that the lump-sum tax has been chosen optimally, (32) is reduced to

$$(33)\qquad M_{\tau|T^*} = -(1-\tau)^{-1}V^*_\eta R + \psi\tau z\text{cov}(W'(y),p_2) + \psi\tau[\bar{e}^o E(W')-(1+F')\text{cov}(W'(y),p_2)]Q$$

Evaluating this expression at the endpoints $\tau = 0$ and $\tau = 1$ gives

$$(34)\qquad \begin{cases} \text{(a)}\quad M_{\tau|T^*,\tau=0} = -V^*_\eta R > 0 \\ \text{(b)}\quad M_{\tau|T^*,\tau=1} = z\,\text{cov}\,(W'(y),p_2) < 0 \end{cases}$$

where (34b) results from the fact that with $\tau = 1$ there is no uncertainty and $S = 0$ at the interior solution. This proves (c). □

Under price uncertainty with risk aversion the lump-sum tax dominates the unit tax to the extent if it is distortionary. On the other hand, the lump-sum tax alone is not optimal when the yield tax is a possible instrument of taxation. Introducing the yield tax at the margin serves as an insurance device by decreasing the post-tax variability of future net timber price.[13)] Moreover, the yield tax is non-distortionary at the margin. This implies that if risks are individual risks or if government is risk neutral, the full insurance via the 100 % yield tax is optimal, while if risks are aggregate risks and government cares about the stochasticity of tax revenues, then the partial insurance with the yield tax being less than 100 % is welfare-maximizing.

4. CONCLUDING REMARKS

The paper has contributed to the theoretical analysis of forest taxation both under certainty and under future timber price uncertainty with risk aversion by focusing on two policy issues: First, the incentive issue of what permanent forest tax will elicit the highest timber supply given that government wants to collect a fixed amount of tax revenues by means of forest taxes? Second, the welfare issue of what forest taxes will be optimal in the sense of maximizing the (expected) utility of a (re-presentative) forest owner subject to a government's tax revenue requirement? The timber supply incentive effects are presented in propositions 1 and 3, while the welfare effects of forest taxes are summarized in propositions 2 and 4.

These findings are only a beginning. In this paper it has been assumed implicitly that the forest owners pay the taxes, which may not be the case, however. Studying the incidence of forest taxes is thus an area of research and necessitates modelling the determination of pre-tax timber prices. Moreover, the two-period model ignores future rotations in the sense that all remaining timber is assumed to be harvested in the second period. Accounting for future rotations and building land valuations into the model provides an agenda for research.

FOOTNOTES

1) See Johansson and Löfgren (1985), p. 268-275.

2) In forest economics literature the word "timber supply" has many different connotations (for various concepts of timber supply, see e.g. Duerr (1960), ch. 14). In this paper the timber supply refers to the question of when to bring harvest to the market, i.e. whether to cut now or in the future. Thus in the paper's terminology a rise in the cut now means a rise in the (current) timber supply.

3) Sometimes in this context the term "ad valorem tax" is used. In some countries, however, (e.g. in U.S.A) the "ad valorem tax" usually refers to an annual ad valorem land, or property, tax. In order to avoid confusion I use the term "yield tax" about the tax on timber revenue. It is not possible to consider land productivity tax in our two-period model to be developed since land valuation will not be built in.

4) Note also that if the 'production possibility' frontier (1) is linear (i.e. $F''(.)=0$), then (3) has no unique solution for x. Either the forest owner cuts everything today ($e>0$) or everything to-morrow ($e<0$) or is indifferent between cutting today and cutting to-morrow ($e=0$).

5) Of course, if the policy goal is to increase future harvesting $z=Q-x+F(Q-x)$, then the tax switches opposite to the ones presented here provide the correct course of action to take.

6) In the analyses we assume thoughout that government tax policy is credible in the sense that public believes in it. If government changes its policy, when public has committed itself to some choice, then policy is said to be time-inconsistent. Analyzing these time-inconsistency issues lies beyond the scope of this paper (for a seminal paper in this area, see Kydland and Prescott (1977)).

7) For a historical survey of the literature about the relationship between tax rates and government tax revenues and an empirical analysis with U.S. data, see Fullerton (1982).

8) For the seminal presentation of the optimal tax theory, see Atkinson and Stiglitz (1980).

9) The envelope theorem states that the change in the objective function (here the utility function) with respect to an exogenous parameter is the same both in the case when endogenous decision variables are adjusted to the optimum and in the case when they are not adjusted (see e.g. Varian (1984), pp. 327-329)). Therefore, the envelope theorem allows us to ignore changes in the behavioural respenses when calculating the ceteris paribus effects of the parameter changes on the utility.

10) The dominance of the lump-sum tax over the unit tax can be seen from a slightly different perspective: It can be shown that by keeping the utility level of a forest owner constant using the lump-sum tax instead of the unit tax brings about higher tax revenues for government under changing timber prices.

11) This was pointed out by Johansson and Löfgren (1985), p. 271. Notice that now the interior solution $0 < x < Q$ is obtained even with the linear 'production possibility frontier ($F'' = 0$) because risk aversion makes the harvesting decision a concave function of x.

12) We use the conventional comparative statics methodology by first substituting the behavioural functions $c_1(.)$ and $x(.)$ for c_1 and x in (10) and (11) and then implicitly differentiating the resulting identities in terms of exogenous parameters and finally solving for the partial derivatives of interest (for an account of the methodology, see e.g. Varian (1984), pp. 309-327).

13) This inefficiency of lump-sum taxation under uncertainty have been pointed out in a different context by Eaton and Rosen (1980).

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Appendix 1:

Differentiating the first-order conditions (10) and (11) with respect to a compensating change in η gives

(1) $$U^*_{c\eta} = -\beta RE[u''(c_2)(p_2-\bar{p}_2)]z(1-\tau) = -\beta Rz(1-\tau)\mathrm{cov}(u''(c_2),p_2) < 0$$

and

(2) $$U^*_{x\eta} = \beta(1-\tau)\{E[u''(c_2)e(p_2-\bar{p}_2)] - E[u'(c_2)(p_2-\bar{p}_2)](1+F')\}$$

where $e = p^*_1R-p^*_2(1+F')$, and $F' = F'(Q-x)$. The sign of $U^*_{c\eta}$ results from the fact that the non-increasing absolute risk aversion implies $U'''(c_2) > 0$, so that $\mathrm{cov}(u''(c_2),p_2) > 0$. According to the definition of e we have $p_2-\bar{p}_2 = -[(1-\tau)(1+F')]^{-1}(e-\bar{e})$, where $\bar{e} = E(e)$, and $\bar{p}_2 = E(p_2)$. Substituting this for $p_2-\bar{p}_2$ in (2) and suppressing c_2 from the utility function for convenience yields

(3) $$U^*_{x\eta} = -\beta\{z(1+F')^{-1}[E(u''e^2)-\bar{e}E(u'')] - [E(u'e)-\bar{e}E(u')]\}$$

Now $\mathrm{sgn}\,(U^*_{x\eta}) = -\mathrm{sgn}\{.\}$.

Consider first the term $a = E(u''e^2)-\bar{e}E(u'')$. $E(u''e^2) < 0$ because of risk aversion. Moreover, it is easy to show that $E(u''e) \leq 0$ under non-increasing absolute risk aversion. On the other hand $\bar{e} < 0$ due to the first-order condition (12) so that $a < 0$. As for the term $b = E(u'e)-\bar{e}E(u')$, the first-order conditions (11) and (12) imply that $b > 0$. Thus $U^*_{c\eta} < 0$ and $U^*_{x\eta} > 0$. □

Appendix 2:

This appendix indicates how to derive the equations (18) and (20) of the text. Let us first consider the equation (18). The first-order conditions (10) and (11) implicitly define c_1 and x as a function of T and t among others. Substituting the behavioural functions for c_1 and x yields the (expected) indirect utility function $V^*(T,t,..) = u^o$ with $V_T^* = -\beta(1+R)E(u') < 0$ and $V_t^* = (Rx+z)(1+R)^{-1}V_T^* < 0$. Inverting now V*(.) for T as a function of t and u^o gives $T = G(t,..,u^o)$ and substituting this for T in V* gives the (expected) compensated indirect utility

(1) $$V^*(G(t,..,u^o),t,..) = u^o$$

(see e.g. Diamond and Yaari (1972)). This implicitly defines the necessary compensation through the permanent lump-sum tax T, which will keep the maximized expected utility of the forest owner unchanged, when the permanent unit tax is changed. Through (1) we can isolate the wealth effect associated with T from the substitution effect associated with t. Differentiating (1) with respect to t gives $V_t^* + V_T^*G_t = 0$ so that $G_t = -V_t^*(V_T^*)^{-1} = -(Rx+z)(1+R)^{-1} < 0$. To the uncompensating behavioural functions, which result from maximizing the expected utility, there corresponds the compensated behavioural functions, which result from the highest T that can be charged and still obtain the utility level u^o given other parameters. Denoting the compensating function by superscript c and substituting G(.) for T in the uncompensated function gives the following relationship between the uncompensated and compensated timber supply

(2) $$x(G(t,..,u^o),t,..) = x^c(t,..,u^o)$$

Differentiating (2) with respect to t gives $x_T G_t + x_t = x_t^c$ so that

(3) $$x_t^c = x_t - (Rx+z)(1+R)^{-1}x_T$$

The direct calculation yields (18) and the expression for x_t^c.

Let us next sketch how the equation (20) of the text can be derived. The direct comparative statics technique gives

(4) $$x_\tau = \Delta^{-1}(-U^*_{x\tau}U^*_{cc} + U^*_{c\tau}U^*_{cx})$$

where subscripts refer to partial derivatives, and where

(5) $$U^*_{c\tau} = \beta R E(u'')(Rp_1x+\bar{p}_2z) + \beta R\,\mathrm{cov}(u'',p_2)z$$

and

(6) $$U^*_{x\tau} = -\beta E(u''e)(Rp_1x+\bar{p}_2z) - \beta\,\mathrm{cov}(u''e,\ p_2)z$$

Using the partial derivatives U^*_{cT}, $U^*_{c\eta}$, U^*_{xT} and $U^*_{x\eta}$ the equations (5) and (6) can be rewritten as

(5') $$U^*_{c\tau} = (Rp_1x+\bar{p}_2z)(1+R)^{-1}U^*_{cT}-(1-\tau)^{-1}U^*_{c\eta}$$

and

(6') $$U^*_{x\eta} = (Rp_1x+\bar{p}_2z)(1+R)^{-1}U^*_{xT}-(1-\tau)^{-1}U^*_{x\eta} + \beta E(u')\bar{e}^0$$

where $\bar{e}^0 = p_1R-\bar{p}_2(1+F') < 0$ due to the first-order condition (12). Substituting these results into (4) and noting the equations (16) and (17) of the text gives $x_\tau = \Delta^{-1}U^*_{cc}\beta E(u')\bar{e}^0 - (1-\tau)^{-1}(x_\eta|_{\varepsilon=0,\ \eta=1}) + (Rp_1x+\bar{p}_2z)(1+R)^{-1}x_T$.

Using the similar techniques as in the derivation of the Slutsky equation (3) the first term of the x_τ-expression can be rewritten as

(7) $$\Delta^{-1}U^*_{cc}\beta E(u')\bar{e}^0 = S = -(1-\tau)^{-1}(p_1x^c_{p_1} + \bar{p}_2x^c_{\varepsilon|\varepsilon=0}) > 0$$

where $x^c_{p_1}$ and $x^c_{\varepsilon|\varepsilon=0}$ refer to the compensated timber supply in terms of the current timber price and the expected future timber price respectively, when compensation is done through T. This establishes the equation (20) of the text. □

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