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SAVING, INCOME RISK AND INTEREST RATE WEDGE: A NOTE**

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ABSTRACT: This note uses a simple two-period model of consumption and saving as the vehicle to discuss saving behavior under future income uncertainty and the (exogenous) interest rate wedge between the borrowing and lending rate. The interest rate wedge will affect saving positively, if there is a positive probability that consumers will end up with a borrowing situation in the future as a result of future income realization. Furthermore, the perfect capital market result, according to which a rise in future income risk will increase saving if and only if the marginal utility is convex in terms of future income, ceases to be valid under the interest rate wedge.

KEY WORDS: saving
income risk
interest rate wedge

I. INTRODUCTION

The theoretical analysis of saving under uncertainty about future income and/or investment returns has been developing rapidly during the last fifteen years (for a reasonably up-to-date survey of the literature, see e.g. Lippman and McCall (1981)). A common simplification in this literature has been the assumption of perfect capital markets in the sense that subject to their intertemporal budget constraint individuals have seen supposed to be able to borrow and lend freely at one interest rate, common to borrowers and lenders. While this assumption is not literally true, one might justify its use as a simplifying device to the extent that results obtained under the perfect capital market assumption still hold under various kinds of capital market 'imperfections'.

The purpose of this note is to relax the assumption of one interest rate, common to borrowers and lenders, and look at some of the implications of the (exogenous) interest rate wedge between the borrowing and lending rate for saving behavior under future income uncertainty. More specifically, by using a two-period model of consumption and saving as the framework we study the saving effects of ceteris paribus changes in (1) the interest rate wedge and (2) future income "risk" under a kinked budget constraint. It turns out that the interest rate wedge will affect saving positively, if there is a positive probability that individuals will end up with a borrowing situation in the future as a result of future income realization. Furthermore, the result, according to which in the presence of perfect capital markets a rise in future income "risk" will increase saving if and only if the marginal utility is convex in terms of future income, ceases to hold under the (exogenous) interest rate wedge.

The model is presented in section 2, while the above mentioned issues are studied in section 3.

2. THE MODEL

A 'representative' consumer is assumed to have a preference ordering over the present and future consumption c_1 and c_2, which is represented for simplicity by an intertemporally additive, three times differentiable and strictly concave utility function $U = u(c_1) + (1+\rho)^{-1}u(c_2)$, where $1+\rho$ = rate of time preference factor and $u' > 0$, $u'' < 0$ for $u(c_i)$, $i = 1,2$ so that consumer is risk-averse.[1)] Partial derivatives are denoted by primes for functions with one argument (e.g. $u'(c_1) = \partial u(c_1)/\partial c_1$ etc.) and by subscripts for functions with many arguments (e.g. $F_y(y,\sigma) = \partial F(y,\sigma)/\partial y$ etc.). We abstract from bequests and inheritances and assume that labor is supplied inelastically.

The present and future income is denoted by I and $\tilde{y}$ respectively. The future income is a random variable, whose range is $[\underline{y},\bar{y}]$. Let σ denote a "riskiness parameter" such that increases in σ represent increases in risk with the mean of a random variable being constant and let $f(\tilde{y},\sigma)$ and $F(\tilde{y},\sigma)$ be the density and distribution functions of future income $\tilde{y}$. Now a pure increase in risk for $\tilde{y}$ is given by

$$(1) \qquad \int_{\underline{y}}^{\tilde{y}} F_\sigma(\theta,\sigma)d\theta \geq 0 \qquad \text{for all } \tilde{y}$$

and

$$(2) \qquad \int_{\underline{y}}^{\bar{y}} F_{\sigma}(\theta,\sigma)d\theta = 0$$

where (1) implies a shift of the weight to the tails of the distribution when σ increases and (2) describes the constancy of the mean of $\tilde{y}$ as σ changes (for more details, see Diamond and Stiglitz (1974)).

A consumer decides about current consumption c_1 and saving S in the face of stochastic future income. The future consumption $\tilde{c}_2$ for given saving is stochastic and depends on the future income realization. In particular, if $\tilde{y} = \hat{y} = \hat{c}_2$, then $S = I - c_1 = 0$, while $S > 0$ for $\tilde{y} \geq \hat{y}$ and $S < 0$ for $\tilde{y} < \hat{y}$. Denoting the lending rate and borrowing rate factors respectively by $1 + r$ and $1 + r\gamma$, where $\gamma \neq 1$ we can write the intertemporal budget constraint as a kinked function of $\tilde{y}$ as follows

$$(3) \qquad \tilde{c}_2 = \begin{cases} \tilde{c}_2^S = \tilde{y}_2 + (1+r)(I-c_1) & \text{for } \tilde{y} \geq \hat{y} \\ \tilde{c}_2^B = \tilde{y}_2 + (1+r\gamma)(I-c_1) & \text{for } \tilde{y} < \hat{y} \end{cases}$$

A consumer allocates resources between c_1 and c_2 so as to maximize the expected utility

$$(4) \qquad U^* = u(c_1) + (1+\rho)^{-1} \left\{ \int_{\underline{y}}^{\hat{y}} u(\tilde{c}_2^B) F_y(\tilde{y},\sigma)d\tilde{y} + \int_{\hat{y}}^{\bar{y}} u(\tilde{c}_2^S) F_y(\tilde{y},\sigma)d\tilde{y} \right\}$$

subject to the intertemporal budget constraint (3). In the expression (4) $F_y(\tilde{y},\sigma) = \partial F(\tilde{y},\sigma)/\partial\tilde{y} = f(\tilde{y},\sigma)$. The first-order condition for this problem is

$$(5) \qquad U_c^* = 0 = u'(c_1) - (1+\rho)^{-1}\left\{(1+r\gamma)\int_{\underline{y}}^{\hat{y}} u'(\tilde{c}_2^B)\, F_y(\tilde{y},\sigma)d\tilde{y} + (1+r)\int_{\hat{y}}^{\bar{y}} u'(\tilde{c}_2^S)\, F_y(\tilde{y},\sigma)d\tilde{y}\right\}$$

where $U_c^* = \partial U^*/\partial c_1$ and where we have used the facts that $\partial\hat{y}/\partial c_1 = -1$ at $S = 0$ and that $u(\tilde{c}_2^S) = u(\tilde{c}_2^B) = u(\hat{c}_2)$ as $\tilde{y} = \hat{y}$. In what follows we assume that the second-order condition for the expected utility maximization is satisfied so that $U_{cc}^* < 0$.[2)] According to (5) a consumer maximizes the expected utility by equating the marginal utility of current consumption to the expected marginal utility of future consumption.

3. SAVING, INTEREST RATE WEDGE AND INCOME RISK

Let denote the interior solution of (5) as a function of the (exogenous) interest rate wedge γ and risk parameter σ by $c_1^*(\gamma,\sigma)$. Differentiating the first-order condition with respect to γ gives

$$(6) \qquad \text{sgn}\ (c_{1\gamma}^*) = \text{sgn}\ (U_{c\gamma}^*)$$

where $c_{1\gamma}^* = \partial c_1^*/\partial\gamma$, and where $U_{c\gamma}^* = \partial U_c^*/\partial\gamma$ can be written as

$$(7) \qquad U_{c\gamma}^* = -(1+\rho)^{-1} r\left\{\int_{\underline{y}}^{\hat{y}} u'(\tilde{c}_2^B)\, F_y(\tilde{y},\sigma)d\tilde{y} + (1+r\gamma)\, S\int_{\bar{y}}^{\hat{y}} u''(\tilde{c}_2^B)\, F_y(\tilde{y},\sigma)d\tilde{y}\right\} \gtrless 0$$

if $\hat{y} > \underline{y}$ is possible because $(1+r\gamma)S < 0$ for $\tilde{y} < \hat{y}$. Thus a rise in interest rate wedge will decrease current consumption and increase saving under future income uncertainty if there is a positive probability that an individual will end up with a borrowing situation. This is of interest because it suggests that saving should be positively associated with a degree of capital market 'imperfection' measured by the wedge between the borrowing and lending rate.[3)]

As far as the mean-preserving change in risk parameter is concerned, it is well-known that in the presence of perfect capital markets the convexity of the expected marginal utility in terms of future income is a necessary and sufficient condition for saving to increase and current consumption to decrease. The plausible assumption of non-increasing Arrow-Pratt absolute risk aversion $A(c_2) = -u''(c_2)/u'(c_2)$ implies, but does not necessitate this. Differentiating the first-order condition with respect to the risk parameter σ gives under the interest rate wedge ($\gamma \neq 1$) for the consumption effect

$$(8) \qquad \operatorname{sgn}(c^*_{1\sigma}) = \operatorname{sgn}(U^*_{c\sigma})$$

where $c^*_{1\sigma} = \partial c^*_1/\partial\sigma$ and where $U^*_{c\sigma} = \partial U^*_c/\partial\sigma$ can be written as

$$(9) \qquad U^*_{c\sigma} = -(1+\rho)^{-1}\left\{(1+r\gamma)\int_{\underline{y}}^{\hat{y}} u'(\tilde{c}_2^B)\, F_{y\sigma}(\tilde{y},\sigma)d\tilde{y} + (1+r)\int_{\hat{y}}^{\bar{y}} u'(\tilde{c}_2^S)\, F_{y\sigma}(\tilde{y},\sigma)d\tilde{y}\right\}$$

where $F_{y\sigma}(\tilde{y},\sigma) = \partial F_y(\tilde{y},\sigma)/\partial\sigma$. Now taking into account that $F_\sigma(\bar{y},\sigma) = F_\sigma(\underline{y},\sigma) = T(\bar{y},\sigma) = T(\underline{y},\sigma) = 0$, where $T(y,\sigma) = \int_{\underline{y}}^{y} F_\sigma(\theta,\sigma)d\theta$, (see the equations (1) and (2) fora mean-preserving change in risk parameter σ) and integrating (9) by parts twice, we end up with[4)]

$$(10) \qquad U^*_{c\sigma} = -(1+\rho)^{-1}\left\{ r(\gamma-1)[u'(\hat{c}_2)F_\sigma(\hat{c}_2,\sigma) - u''(\hat{c}_2)(\int_{\underline{y}}^{\hat{y}} F_\sigma(\theta,\sigma)d\theta)] + (1+r\gamma)\int_{\underline{y}}^{\hat{y}} u'''(\tilde{c}_2^B)(\int_{\underline{y}}^{\tilde{y}} F_\sigma(\theta,\sigma)d\theta)d\tilde{y} + (1+r)\int_{\hat{y}}^{\bar{y}} u'''(\tilde{c}_2^B)(\int_{\underline{y}}^{\tilde{y}} F_\sigma(\theta,\sigma)d\theta)d\tilde{y} \right\}$$

In the absence of interest rate wedge the equation (10) collopses to

$$(10') \qquad U^*_{c\sigma|\gamma=1} = -(1+\rho)^{-1}(1+r)\int_{\underline{y}}^{\bar{y}} u'''(\tilde{c}_2)(\int_{\underline{y}}^{\tilde{y}} F_\sigma(\theta,\sigma)d\theta)d\tilde{y}$$

whose sign depends on the third derivative of the utility function. Clearly, $U^*_{c\sigma|\gamma=1} < 0$ under non-increasing Arrow-Pratt absolute risk aversion, while $U^*_{c\sigma|\gamma=1} = 0$ if the utility function is quadratic. In the former case a rise in income risk will decrease current consumption and increase saving, while it has no effect in the latter, rather implausible, case. Under the interest rate wedge the first term in curly brackets

$$(11) \qquad k = r(\gamma-1)[u'(c_2)\ F_\sigma(\hat{c}_2,\sigma) - u''(\hat{c}_2)(\int_{\underline{y}}^{\hat{y}} F_\sigma(\theta,\sigma)d\theta)]$$

is also relevant. In (11) $\hat{c}_2 = \hat{y}$ = the realization of $\tilde{y}$ under which $S = 0$ and thus $F_\sigma(\hat{c}_2,\sigma)$ describes how a change in risk parameter affects the probability of ending up with a borrowing situation. Now $F_\sigma(\hat{c}_2,\sigma) \geq 0$ implies that $-(1+\rho)^{-1}k < 0$ while $-(1+\rho)^{-1}k$ is ambiguous a priori when $F_\sigma(\hat{c}_2,\sigma) < 0$. Hence, in the presence of interest rate wedge, we have

$$(12) \quad \left.\begin{array}{l} (a)\ u'''(c_2) \geq 0 \\ (b)\ F_\sigma(\hat{c}_2,\sigma) \geq 0 \\ (c)\ \gamma > 1 \end{array}\right\} \Rightarrow c^*_{1\sigma} < 0$$

and

$$(13) \quad \left.\begin{array}{l} (a)\ u'''(c_2) = 0 \\ (b)\ F_\sigma(\hat{c}_2,\sigma) \geq 0 \\ (c)\ \gamma < 1 \end{array}\right\} \Rightarrow c^*_{1\sigma} > 0$$

Thus if the borrowing rate exceeds the lending rate, if the marginal utility is not concave, and if the rise in income risk does not decrease the probability that individuals end up with a borrowing situation, then saving is positively associated with income risk. Under these circumstances saving is increased even though the utility function were quadratic ($u'''(c_2) = 0$). The assumptions in (12) are certainly not implausible and thus it is tempting to suggest that allowing for interest rate wedge ($\gamma > 1$) tends to increase the range of circumstances, under which the mean-preserving rise in income risk increases saving.

But as the conditions in (13) remind us, the reverse might as well be the case. The sufficient, but not necessary conditions for the unconventional negative relationship between savings and the income risk are that the marginal utility is linear (the quadratic utility function $u'''(c_2) = 0$), the rise in income risk does not decrease the probability of ending up with a borrowing situation and that the lending rate is higher than the borrowing rate. The quadratic utility is generally regarded as an unappealing way of describing consumers' behavior towards risk so that (13a) seems somewhat less plausible than (12a). Moreover, one might regard (13c) as a particularly implausible assumption. But this may not be the case, however. The relevant interest rate wedge for consumers the post-tax interest rate wedge. If e.g. capita income is tax-free and borrowing expenses are tax-deductible, then the positive pre-tax interest rate wedge ($\gamma > 1$) may well be compatible with the negative post-tax interest rate wedge ($\gamma < 1$).[5)] Thus the unconventional result, according to which a mean-preserving change in income risk will affect saving negatively, is by no means excluded on a priori grounds.[6)]

Footnotes:

1) Results reported below can be generalized under fairly plausible assumptions to cover the case of non-additive utility function $U = u(c_1, c_2)$. For an account of how this can be done in the absence of interest rate wedge, see Lippman and McCall (1981).

2) It should be noticed, however, that in the case, where the borrowing rate exceeds the lending rate, the assumption that the consumer is risk-averse is not sufficient to guarantee that the second order condition is satisfied.

3) This would be true if there were neither taxes on income from saving nor deductions from borrowing expenses. Naturally, in empirical implementations both taxes and deductions should be taken into account. In these matters there are differences not only across countries, but also over time. Empirical implementations are also rendered difficult by the fact that in the presence of progressive taxation the post-tax interest rate wedge varies across consumers so that it is not quite obvious what should be used as a proxy for the post-tax interest rate wedge.

4) The analysis of consumption and saving under interest rate wedge and income uncertainty belongs to a general class of problems in decision-making under uncertainty, where the objective function has a kink at a critical value of the random variable. This class of problems has been analyzed in Kanbur (1982) and the expression (10) of the text is an application of the equation (11) in Kanbur (1982) p. 223.

5) Progressive taxation, where marginal taxes vary with the tax base e.g. piecewise linearly, gives also rise to decision-making problems under uncertainty with a kink in the objective function at some critical value of a random variable. See Kanbur (1983) for an analysis of labor supply under uncertainty with piecewise linear tax regimes.

6) The consumption behavior under the positive interest rate wedge ($\gamma > 1$) and stochastic future income has been also analyzed in Hassin and Lieber (1982). They develop neither comparative statics of the interest rate wedge nor comparative statics of a mean-preserving change in risk, which have been the major focus of interest in this note. Hassin and Lieber compare the total effect of uncertainty on consumption and stress the possibility than even with the positive interest rate wedge income uncertainty does not necessarily lead to a rise in saving compared with the perfect certainty case. To illustrate this they construct an example, in which saving increases when income uncertainty is eliminated.

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