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SUPERIORITY COMPARISONS BETWEEN MIXED REGRESSION ESTIMATORS*

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ABSTRACT: This paper gives necessary and sufficient conditions for a mixed regression estimator to be superior to another mixed estimator. The comparisons are based on the mean square error matrices of the estimators. Both estimators are allowed to be biased.

KEY WORDS: Biased estimation, linear model, mean square error matrix,

prior information, restricted least squares.

JEL Classification: 2110

1. Introduction

Several authors have recently carried out superiority comparisons between biased estimators of the parameter vector of a linear model. A few of them have used a superiority definition based on mean square error matrices of estimators. Teräsvirta (1984, 1986) provides a framework for such comparisons. He proves a theorem which gives a necessary and sufficient condition for the superiority of a biased non-homogeneous linear estimator over another. Trenkler (1985) has an alternative formulation of this result.

In this paper I use the theorem of Teräsvirta (1986) for superiority comparisons between mixed estimators of Theil and Goldberger (1961). The results extend in several respects the ones Freund and Trenkler (1986) have recently obtained. These authors only consider the case where one of the two mixed estimators is unbiased and the rows of the two restriction matrices are linearly dependent. My results refer to the situation where both estimators can be biased. The paper covers the case where both estimators have a common subset of prior information as well as the situation where common prior information does not exist. The conditions for superiority are slightly different in these two cases. Finally, any misspecification of the model by omitting regressors is seen to invalidate the superiority results of this paper.

The plan of the paper is as follows. Section 2 introduces notation.

The superiority comparisons are in Sections 3 and 4. Section 5 concludes.

Model and notation

Consider a linear model

$$y = X\beta + \varepsilon$$
, $E\varepsilon = 0$, $cov(\varepsilon) = \sigma^2 I_n$ (2.1)

where y and ϵ are stochastic n x 1 vectors, X is a fixed n x p matrix, rank(X) = p, and β is a p x 1 parameter vector. Assume two sets of linear prior information

$$\tilde{r}_{j} = \tilde{R}_{j}\beta + \phi_{j}$$
, $E(\tilde{r}_{j} - \tilde{R}_{j}\beta) = \tilde{s}_{j}$, $E\phi_{j} = 0$, $cov(\phi_{j}) = (\sigma^{2}/k_{j})V_{j}$, $cov(\varepsilon\phi_{j}) = 0$, $j = 1,2$, (2.2)

where \tilde{r}_j and ϕ_j are stochastic $\tilde{m}_j \times 1$ vectors, \tilde{R}_j is a fixed $m_j \times p$ matrix, $rank(\tilde{R}_j) = \tilde{m}_j$, V_j is positive definite and k_j is a non-negative scale factor, j = 1,2. The mixed estimators of Theil and Goldberger (1961) based on (2.1) and (2.2) are

$$b_{j}(k_{j}) = (X'X + k_{j}\tilde{R}'_{j}V_{j}^{-1}\tilde{R}_{j})^{-1}(X'y + k_{j}\tilde{R}'_{j}V_{j}^{-1}\tilde{r}_{j}), j = 1,2$$
 (2.3)

and their mean square error matrices

$$MSE(\tilde{b}_{j}(k_{j})) = E(\tilde{b}_{j}(k_{j}) - \beta)(\tilde{b}_{j}(k_{j}) - \beta)'$$

$$= \sigma^{2}(U - U\tilde{R}_{j}^{\dagger}\tilde{S}_{j}\tilde{R}_{j}U) + U\tilde{R}_{j}^{\dagger}\tilde{S}_{j}\tilde{S}_{j}\tilde{S}_{j}\tilde{S}_{j}\tilde{R}_{j}U, j = 1,2 (2.4)$$

where
$$U = (X'X)^{-1}$$
 and $\tilde{S}_{j} = (k_{j}^{-1}V_{j}^{-1} + \tilde{R}_{j}U\tilde{R}_{j}^{-1})^{-1}$.

Teräsvirta (1986) defines strong superiority of a linear estimator, \tilde{b}_2 , of β over another, \tilde{b}_1 , to be equivalent to

$$E(\tilde{b}_1 - \beta)'A(\tilde{b}_1 - \beta) - E(\tilde{b}_2 - \beta)'A(\tilde{b}_2 - \beta) \ge 0$$
 (2.5)

for all loss matrices $A \ge 0$. An equivalent form of this condition is $\Delta_{12} = \text{MSE}(\tilde{b}_1) - \text{MSE}(\tilde{b}_2) \ge 0$ (Theobald, 1974). If \tilde{b}_2 is strongly superior to \tilde{b}_1 , Freund and Trenkler (1986) say that \tilde{b}_2 is not worse than \tilde{b}_1 . In what follows, superiority always refers to strong superiority if not explicitly stated otherwise.

3. Comparing mixed estimators

Freund and Trenkler (1986) have proven the following theorem on comparing mixed estimators:

Theorem 1. Assume $\tilde{s}_1 = 0$ and let P be an $\tilde{m}_1 \times \tilde{m}_2$ matrix such that $\tilde{R}_1 = P\tilde{R}_2$ and

$$cov(\phi_1) = P cov (\phi_2)P'. \qquad (3.1)$$

Then the following two assertions are equivalent:

(i)
$$\tilde{b}_2(k_2)$$
 is superior to $\tilde{b}_1(k_1)$

(ii)
$$P\tilde{s}_2 = 0$$
 and $\tilde{s}_2 \tilde{\tilde{s}}_2 \tilde{\tilde{s}}_2 \leq \sigma^2$.

This theorem gives a necessary and a sufficient condition for a biased mixed estimator $\tilde{b}_2(k_2)$ to be superior to an unbiased mixed estimator $\tilde{b}_1(k_1)$. Consider the assumption (3.1). Assume that $\tilde{m}_2 > \tilde{m}_1$ and $\tilde{R}_2 = (\tilde{T}' \; \tilde{R}_1')$ so that the \tilde{m}_1 last rows of \tilde{R}_2 equal \tilde{R}_1 , and $P = (0 \; I)$. Assume furthermore that $V_j = (\sigma^2/k_j)I_{\tilde{m}_j}$, j = 1,2. Then (3.1) implies that $k_1 = k_2$. This shows that (3.1) is a rather strong restriction. In this example it precludes the comparisons between estimators whose elements of prior information have unequal variances. Because that is

a case certainly not without interest, less restrictive results are called for.

Such results may be obtained by applying a theorem proven in Teräsvirta (1986). Write the MSE matrix of $\tilde{b}_j = D_j y + h_j$ as a sum of covariance and bias:

$$MSE(\tilde{b}_{j}) = \sigma^{2}D_{j}D_{j}' + d_{j}d_{j}'$$

where $d_j = H_j \beta + h_j$ with $H_j = D_j X - I$, j = 1,2. Set $C = D_1 D_1^1 - D_2 D_2^1$ so that $\Delta_{12} = \sigma^2 C + d_1 d_1^1 - d_2 d_2^1$. The theorem uses the following decomposition which is necessary for the superiority results:

$$C = KLK', d_j = Kf_j, j = 1,2$$
 (3.2)

where K is pxr, rank(K) = r, L is symmetric and rxr, and f_j is rx1, j = 1,2. We exclude, however, the cases where $NEf_2 = NEf_1$ or $NE\sigma^2L + f_1f_1 = NEf_2$ where $\sigma^2L + f_1f_1$ is positive semidefinite, as trivial; NEA3 is the null space of A, see Teräsvirta (1986). Now we can state

Theorem 2 (Teräsvirta, 1984, 1986). Consider linear model (2.1) and two linear estimators $\tilde{b}_j = D_j y + h_j$, j = 1,2. Assume decomposition (3.2) and furthermore that $N\mathbb{E}[f_2] \neq N\mathbb{E}[f_1]$ and $\sigma^2 L + f_1 f_1' > 0$. Then \tilde{b}_2 is strongly superior to \tilde{b}_1 if and only if

$$f_2'(\sigma^2L + f_1f_1')^{-1}f_2 \le 1.$$
 (3.3)

If L>0, then the strong superiority is equivalent to

$$\sigma^{-2} \left\{ f_{22} - f_{21}^2 (\sigma^2 + f_{11})^{-1} \right\} \le 1 \tag{3.4}$$

where $f_{ij} = f_i^t L^{-1} f_j$, j = 1,2. On the other hand, if L < 0 then \tilde{b}_2 is strongly superior to \tilde{b}_1 if and only if L is a scalar and, in obvious notation,

$$\sigma^2 \ell_{11} + f_1^2 - f_2^2 \ge 0$$
.

Theorem 2 is suitable for comparing estimators of type (2.3). Assume $V_j = I_{\widetilde{m}_j}$, j = 1,2. This does not mean any loss of generality but simplifies the exposition. Let $\widetilde{R}_j = (R_j' R_3')'$, j = 1,2. R_3 is thus a common block in both \widetilde{R}_1 and \widetilde{R}_2 . Assuming $m_1 + m_2 + m_3 \le p$ we have

$$c = c_3^2 R_2^1 D_{22 \cdot 3}^{-1} R_2 c_3^2 - c_3^1 R_1^1 D_{11 \cdot 3}^{-1} R_1 c_3^1$$
(3.5)

where

$$D_{jj\cdot3} = (S_3^{(j)})^{-1} - R_3 U R_j S_j^{(j)} R_j U R_3^{'} = k_j^{-1} I + \sigma^{-2} R_3 C_j^{j} R_3^{'}, j = 1,2$$

with $S_i^{(j)} = (k_j^{-1}I + R_jUR_j^i)^{-1}$, and $C_i^j = U - UR_i^iS_i^{(j)}R_j^iU$. The latter matrix equals σ^{-2} times the covariance matrix of $b_i(k_j) = (X'X + k_jR_i^iR_i)^{-1}(X'y + k_jR_i^ir_i)$.

Furthermore,

$$d_{j} = C_{3}^{j} R_{j}^{j} D_{11 \cdot 3}^{-1} s_{j} + (U - C_{3}^{j} R_{j}^{i} D_{jj \cdot 3}^{-1} R_{j} U) R_{3}^{i} S_{3}^{(j)} s_{3}, j = 1, 2.$$
 (3.6)

It is seen from (3.5) and (3.6) that in this case the decomposition (3.2) does not exist and Theorem 2 does not apply. Assume $s_3 = 0$ (the information common to both sets is unbiased) and $k_1 = k_2 = k$ so that $s_3^{(1)} = s_3^{(2)}$ and $s_3^{(1)} = s_3^{(2)} = s_3^{(2)}$ and $s_3^{(1)} = s_3^{(2)} = s_3^{(2)} = s_3^{(2)}$. Now the choice

$$K = C_{3}R', L = diag \{-D_{11}^{-1} \cdot 3, D_{22}^{-1} \cdot 3\}$$

$$f_{1} = \begin{bmatrix} D_{11}^{-1} \cdot 3 \\ 0 \end{bmatrix} s_{1} \text{ and } f_{2} = \begin{bmatrix} 0 \\ D_{12}^{-1} \cdot 3 \end{bmatrix} s_{2}$$
(3.7)

defines the desired decomposition. L is generally indefinite, rank(L) = $m_1 + m_2$, and the number of negative eigenvalues equals m_1 . Following Teräsvirta (1986), write

$$\sigma^2 L + f_1 f_1' = \text{diag} \left\{ -\sigma^2 D_{11 \cdot 3}^{-1} + D_{11 \cdot 3}^{-1} s_1 s_1' D_{11 \cdot 3}^{-1}, \sigma^2 D_{22 \cdot 3}^{-1} \right\}$$
 (3.8)

and choose $m_1 = 1$. The necessary and sufficient condition for (3.8) to be positive definite is

$$\sigma^{-2} s_1^{\dagger} D_{11 \cdot 3}^{-1} s_1 > 1. \tag{3.9}$$

This is a necessary and sufficient condition for the unbiased mixed estimator $b_3(k)$ to be superior to $b_1(k)$. If $m_1 > 1$, (3.9) is no longer sufficient.

If (3.9) holds and $m_1=1$, Theorem 2 applies. The necessary and sufficient condition for $\tilde{b}_2(k)$ to be superior to $\tilde{b}_1(k)$, given (3.9), is

$$\sigma^{-2} s_2' D_{22 \cdot 3}^{-1} s_2 \le 1 . \tag{3.10}$$

This is equivalent to the superiority of $\tilde{b}_2(k)$ over $b_3(k)$ and we have $\underline{\text{Corollary 2.1.}}$. Consider linear model (2.1) and two mixed estimators $\tilde{b}_1(k)$ and $\tilde{b}_2(k)$ with m_3 common elements of prior information. Assume that $\text{rank}(R_1^l \ R_2^l \ R_3^l)^l = m_1 + m_2 + m_3 \le p$ and that $s_3 = 0$. If $m_1 = 1$, $\tilde{b}_2(k)$ is superior to $\tilde{b}_1(k)$ if and only if (i) the unbiased estimator $b_3(k)$ is superior to $\tilde{b}_1(k)$ and (ii) $\tilde{b}_2(k)$ is superior to $b_3(k)$. If $m_1 > 1$, no superiority condition exists.

Corollary 2.1 generalizes the result of Freund and Trenkler (1986) in that both estimators under comparison may be biased. However, we have maintained the restrictive assumption $k_1 = k_2 = k$. (Letting $k \to \infty$ yields the corresponding result on RLS estimators in Teräsvirta (1986)). Next, we shall discuss a special case in which it can be relaxed. Assume $m_3 = 0$, i.e., there is no common set of prior information: the rows of \widetilde{R}_1 and \widetilde{R}_2 are mutually linearly independent. This implies $C_3 = U$ and $D_{jj\cdot3} = S_j^{(j)} = S_j^{(j)}$, j = 1,2. The appropriate decomposition is

$$K = UR', L = diag\{-S_1, S_2\}, f_1 = \begin{bmatrix} S_1 \\ 0 \end{bmatrix} s_1, f_2 = \begin{bmatrix} 0 \\ S_2 \end{bmatrix} s_2.$$
(3.11)

Another special case with practical interest is the one where the restriction matrices are identical: $\tilde{R}_1 = \tilde{R}_2$, $\tilde{m}_1 < p$; see also Teräsvirta (1984). Then $C = U\tilde{R}_1'(\tilde{S}_2 - \tilde{S}_1)\tilde{R}_1U$ and $d_j = U\tilde{R}_1'\tilde{S}_j\tilde{s}_j$. We need not assume $\tilde{Er}_1 = \tilde{Er}_2$ ($\tilde{s}_1 = \tilde{s}_2$) but in practice the interest may most often focus on the problem of what happens when only the variance of the prior information changes. Now, $\tilde{S}_2 - \tilde{S}_1$ is either positive definite ($k_1 < k_2$), a null matrix ($k_1 = k_2$) or negative definite ($k_1 > k_2$). Thus we may choose

$$K = U\widetilde{R}_{1}^{\dagger}$$
, $L = \widetilde{S}_{2} - \widetilde{S}_{1}$, $f_{j} = \widetilde{S}_{j}\widetilde{S}_{j}$, $j = 1,2$.

If $k_1 < k_2$, then L > 0 and

$$L^{-1} = (\tilde{S}_2 - \tilde{S}_1)^{-1} = k_1 k_2 (k_2 - k_1)^{-1} (\tilde{S}_1 \tilde{S}_2)^{-1}.$$

Theorem 2 now yields the result which is not stated in detail here. Letting $k_2 \to \infty$ leads to a necessary and sufficient condition for a RLS estimator to be superior to a mixed estimator based on the same prior information $(\tilde{r}_1, \tilde{R}_1)$ with positive variance.

4. A more general case

Freund and Trenkler (1986) have an example which at first may appear to resemble the situation in Corollary 2.2. However, in our notation their two mixed estimators are

$$\tilde{b}_{j}(K_{j}) = (X'X + \tilde{R}_{j}'K_{j}\tilde{R})^{-1}(X'y + \tilde{R}_{j}'K_{j}\tilde{r}_{j}), j = 1,2$$
 (4.1)

where K_j = diag(k_j I_{m_j} , $k_3 I_{m_3}$). They are thus more general than $b_j(k_1)$ and $b_j(k_2)$ in that the scale factors of the two blocks in the covariance matrices are not the same. On the other hand, common prior information r_3 = $R_3\beta + \phi_3$ does have the same covariance matrix, $(\sigma^2/k_3)I_{m_3}$, in both estimators. Thus

$$C = C_3 R_2^{1} D_{22 \cdot 3}^{-1} R_2 C_3 - C_3 R_1^{1} D_{11 \cdot 3}^{-1} R_1 C_3$$

and

$$d_{j} = C_{3}R_{j}^{\dagger}D_{jj\cdot 3}^{-1}s_{j} + (U - C_{3}R_{j}^{\dagger}D_{jj\cdot 3}^{-1}R_{j}U)R_{3}^{\dagger}S_{3}s_{3}, j = 1,2$$

where

$$c_3 = U - UR_j S_3 R_j U. \tag{4.2}$$

Assuming $s_3 = 0$, the relevant decomposition is (3.7) where C_3 is now defined as in (4.2). We have

Corollary 2.3. Consider linear model (2.1) and two mixed estimators $\tilde{b}_1(K_1)$ and $\tilde{b}_2(K_2)$ as in (4.1), with m_3 common elements of prior information with the same variance σ^2/k_3 . Assume that $\text{rank}(R_1' R_2' R_3') = m_1 + m_2 + m_3 \le p$ and that $s_3 = 0$. If $m_1 = 1$, $\tilde{b}_2(\tilde{K}_2)$ is superior to $\tilde{b}_1(K_1)$ if and only if (i) the unbiased estimator $\tilde{b}_3(k_3)$ is superior to $\tilde{b}_1(K_1)$ and (ii) $\tilde{b}_2(K_2)$ is superior to $\tilde{b}_3(k_3)$. If $m_1 > 1$, no superiority condition exists.

Corollary 2.3 generalizes the corresponding results in Teräsvirta (1981) and Freund and Trenkler (1986). As a special case, suppose $R_3 = [0\ I]$, $k_3 \rightarrow \infty$, $m_1 = 0$, and $m_2 + m_3 < p$. Furthermore, let $R_2 = [R_{21}\ 0]$ where the block division conforms to that of R_3 . Then $\tilde{b}_1(K_1)$ is the OLS estimator of a misspecified model where the m_3 regressors corresponding to β_3 have been omitted. Estimator $\tilde{b}_2(K_2)$ is a mixed estimator of the parameters of the

same model. Corollary 2.3 then implies that no condition for strong superiority of the mixed estimator over OLS exists unless $s_3 = 0$, i.e., $\beta_3 = 0$. Teräsvirta (1981) already indicates this result. Mittelhammer (1981) has derived a weak superiority result for this situation. If $m_1 = 1$, so that the comparison is between two mixed estimators, misspecification continues to destroy the conditions for strong superiority.

5. Final remarks

The paper has concentrated on mixed estimators but results where one or both estimators are RLS estimators are obtained as special cases by letting the scale parameter(s) k_j approach infinity. As to the situation where both estimators are mixed estimators, it is seen that the presence of common restrictions only allows very strict superiority conditions. The existence of a superiority condition requires either equal scale factors or at least equal scale factors for common restrictions. If we treat the case $\tilde{R}_1 = P\tilde{R}_2$ this translates into very strict restrictions on the covariance matrices of the prior information or matrix P. Because I have wanted to stress the role of the scale factors, I have used simple covariance structures and common restrictions. The corresponding results for the apparently more general case $\tilde{R}_1 = P\tilde{R}_2$ are, however, obtainable without difficulty.

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