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CHANGES IN TAX PROGRESSION AND LABOUR SUPPLY UNDER WAGE RATE UNCERTAINTY**

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ABSTRACT: This paper focuses on the relationship between labour supply and linear income taxation under wage rate uncertainty. It is first shown how the labour supply response to a change in the marginal tax rate can be decomposed into the income, substitution and risk effects. The risk effect results from the effect of taxation on post-tax wage rate risk and exemplifies the role of taxation as an insurance device. Second, and perhaps more importantly, it is demonstrated how increases in 'pure' progression can lead to increased labour supply thus suggesting that a widely held popular belief about the disincentive effect of increased progression may be false under wage rate uncertainty. At the welfare optimum, however, the marginal tax rate is pushed to the point where at the margin it reduces labour supply despite wage rate uncertainty and risk-sharing effect of the marginal tax rate.

KEY WORDS: Labour supply, tax progression

1. Introduction

It is a widely held popular belief that the more progressive the tax system, the greater the disincentive to work effort. Evidently, the question of how the progressive taxation affects work effort is a complex one and can be formulated in a number of different ways depending on the definition of progression and the basis on which alternative tax systems are compared. Under quite plausible assumptions, however, it can be concluded that increased linear 'progression' does decrease work effort under certainty (see e.g. Sandmo (1983)).[1)] But as has been recently indicated by Blomquist (1985), this disincentive effect of progression may not come through under nonlinear taxation because of the ambiquity of the substitution effect of wages. But what about the case where workers have imperfect information about their (real) wages at the time they choose their labour supply? The purpose of this paper is to shed light on the relationship between 'progression' and work effort under (real) wage rate uncertainty and contrast the results with those of the certainty case. In order to bring the role of uncertainty into clear focus we concentrate to analyze incentive effects of linear progressive taxation.[2)]

Turning to the question of the basis on which progressivity is evaluated we have to account for the fact that government tax revenue is stochastic in the presence of wage rate uncertainty so that it is not immediately evident what is meant by a change in tax 'progression' even though we could agree on the definition of progressivity. One possibility is to consider changes in 'progression' with the same expected tax revenue, which is particularly appealing when private risks are independently distributed.[3] This criterion emphasizes the financial restrictions faced by the government. These restrictions are not emphasized in the second route we follow, where government is assumed to be free to consider changes in 'progression', which maintain the expected utility of workers unchanged.

To anticipate results it turns out that in sharp contrast to the certainty case, there are quite plausible circumstances in situations off the welfare optimum, where the popular belief about the disincentive effect of increased tax progression may actually be false under both alternative criteria of changing progression.

An intuitive explanation of this result might go as follows: Under wage rate uncertainty, the labour supply effect of a change in the marginal tax rate can be decomposed into the positive income effect, the negative substitution effect due to the change in the expected wage rate and the a priori ambiguous risk effect due to a change in the post-tax wage rate risk. Under the compensated change in tax progression by keeping the expected tax revenues of government constant the income effect will be cancelled out so that the outcome depends on the substitution effect on the one hand and on the risk effect on the other hand. Increased progression will reduce the risk faced by the worker and provided that a rise in wage rate uncertainty discourages work effort, the risk effect is positive and runs counter to the negative substitution effect. Evidently the risk effect may even dominate the substitution effect depending on the degree of relative risk aversion and on the dependence of consumption on risky labour income. If the compensated rise in progression is carried out by keeping the expected utility of workers constant the labour supply will be higher than under the expected tax revenue criterium. A rise in the marginal tax rate increases the expected utility for risk averse workers via reducing post-tax wage rate risk. Hence, the lump-sum payment need to be raised less and the labour supply is higher when consumption and leisure are normal goods.

We proceed as follows: Analytics of labour supply under wage rate uncertainty is presented in section 2 by using two standard models of consumer choice. In section 3 these models are then used to analyze the effects of compensated changes in tax progression on labour supply. Finally, there is a brief concluding section.

2. Analytics of labor supply under wage rate uncertainty

This section presents an analysis of labor supply effects of taxation in the context of two standard models of consumer choice, namely in a static consumption-leisure choice model and in an intertemporal consumption-savings model, where consumption-leisure choice is endogenous in the first period.

2.1. Taxation and labor supply in a static consumption-leisure model

Let $u(C,L)$ be an increasing function of consumption, C, and leisure, L, and continuous, strictly concave and at least three times differentiable. Let H be labour supply such that $H + L \equiv 1$, w a random gross wage rate, t the constant marginal tax rate applied to all income and r the lump-sum payment. The worker's budget constraint can now be written $C = \theta wH + r$, where $\theta = 1 - t$, and the worker's problem is then to maximize $Eu(C,L)$ with respect to H under the budget constraint. The first- and second-order conditions for the maximum are

$$(1) \qquad E[u_C \theta w - u_L] = 0$$

$$(2) \qquad \Delta = E[u_{CC}\theta^2 w^2 + u_{LL} + 2u_{CL}\theta w] < 0$$

where u_i, $u_{ij}(i,j = C,L)$ denote partial derivatives. Finally, let assume that both C and L are normal goods, which here means that $E[\theta wu_{CL} - u_{LL}] > 0$ and $E[u_{CL} - \theta wu_{CC}] > 0$.

Turning to comparative statics the effect of lump-sum payment r on labour supply is $H_r = \Delta^{-1}E[u_{CL} - \theta wu_{CC}] < 0$ and the effect of the marginal tax rate respectively

$$(3) \qquad H_t = \Delta^{-1}\{E[u_c w] - HE[(u_{CL} - \theta wu_{cc})w]\}$$

$$= -\bar{w}HH_r + \Delta^{-1}\{E[u_c]\bar{w} + [cov(u_c,w) + H\,cov(\theta wu_{CC} - u_{CL},w)]\}$$

where w is a random gross wage rate with finite mean $\bar{w} = E(w)$. The equation (3) can be re-expressed in a way, which turns out useful in terms of economic interpretation. First, consider the effect of an additive shift in the distribution of w, which is defined by $\tilde{w} = \varepsilon + w$, and in which changes in ε mean changes in the expected value of w with all other moments constant. This gives at $\varepsilon = 0$

$$(4) \qquad H_\varepsilon\big|_{\varepsilon=0} = H^c_\varepsilon\big|_{\varepsilon=0} + \theta HH_r$$

where $H^{c}_{\varepsilon}\big|_{\varepsilon=0} = -\Delta^{-1}\theta E[u_{c}] > 0$ is the substitution effect of a change in the expected wage rate. Second, consider the effect of a multiplicative shift in the distribution of w, which is offsetted by an additive shift to restore the mean of w to its initial value. Such a shift can be interpreted as a mean-preserving change in risk and is defined for $\tilde{w} = \varepsilon + \eta w$ by $d\varepsilon/d\eta = -\bar{w}$ at $\varepsilon = 0$, $\eta = 1$. Substituting $\tilde{\tilde{w}}$ for w in (1) and differentiating with respect to η gives the effect of a mean-preserving change in wage risk on work effort

(5) $$H_{\eta}\Big|_{\substack{\varepsilon=0\\ \eta=1}} = -\Delta^{-1}\theta\,\{\,\mathrm{cov}(u_{C},w) + H\,\mathrm{cov}\,(\theta w u_{CC} - u_{CL},w)\,\}$$

Utilizing now the expressions (4) and (5) makes it possible to rewrite the equation (3) as follows[4)]

(3') $$H_{t} = \underbrace{-\bar{w}HH_{r}}_{(+)} - \theta^{-1}\,\{\,\underbrace{\bar{w}H^{c}_{\varepsilon}\big|_{\varepsilon=0}}_{(+)} + \underbrace{H_{\eta}\Big|_{\substack{\varepsilon=0\\ \eta=1}}}_{(?)}\,\}$$

Thus we have obtained a generalization of the Slutsky equation under wage rate uncertainty, where the labour supply effect of the marginal tax rate has been decomposed into the income, substitution and risk effects.[5)] The positive income effect , $-\bar{w}HH_{r}$, - due to the normality of leisure - and the negative substitution effect, $-\theta^{-1}\bar{w}H^{c}_{\varepsilon}\big|_{\varepsilon=0}$, due to the change in the expected wage rate induced by the marginal tax change - are well-known from the certainty analysis, but the risk effect, $-\theta^{-1}H_{\eta}\Big|_{\substack{\varepsilon=0\\ \eta=1}}$, is a novel one and results from wage rate uncertainty.

Signing the risk effect necessitates signing the expression $cov(u_C, w) + H\,cov(\theta w u_{CC} - u_{CC}, w)$, whose sign is equal to the sign of $X = H(\theta w u_{CCC} - u_{CCL}) + 2u_{CC}$. Since Leland's (1968) principle of 'decreasing risk aversion to concentration' (DRAC) does not help to determine the sign of X, we turn to consider the case of additive utility function for which $X = \theta w u_{CCC} + 2u_{CC} = u_{CC}(2-(1+R)f)-R'fu_C$, where $R = -u_{CC}C/u_C$ = the Arrow-Pratt measure of relative risk aversion, $R' = \partial R/\partial C$ and $f = \theta wH/C$ = the fraction of current consumption accounted for by the risky labor income. Assuming constant relative risk aversion as the benchmark case[6] gives $H_\eta\big|_{\substack{\varepsilon=0\\ \eta=1}} \gtrless 0$ as $R \gtrless (2/f)-1$. Thus increasing wage uncertainty increases (decreases) labor supply if relative risk aversion is high (low) and/or the fraction of consumption accounted for by the risky labor income is high (low).[7]

An intuitive explanation might go as follows: Workers, who are very risk averse, increase their effort when risk is increased. They are worried, as it were, about the worst possible contingencies. When risk is increased, they have to work harder to avoid these extreme contingencies, particularly if their consumption depends heavily on risky labour income. On the other hand, workers, who are less risk averse, view the return from labour supply lower as risk is increased and hence tend to work less, particularly if their consumption does not depend very much on risky labour income. The change in the marginal tax rate affects post-tax wage risk negatively so that if increased wage uncertainty decreases (increases) labor supply, then increased marginal tax rate increases (decreases) labor supply via the risk effect.

2.2. Taxation and labor supply in an intertemporal consumption-leisure model

The earlier analysis did not allow for savings behavior. Next we develop the implications of taxation for labour supply by using a standard intertemporal model, where both the current consumption-leisure choice and the consumption-savings choice are taken into account.

Let the worker's tastes be represented for simplicity by the intertemporally additive utility function $V = u(C_1,L) + v(C_2)$, which is a least three times continuously differentiable, increasing in current consumption, C_1, leisure, L, and future consumption, C_2, and exhibiting the decreasing marginal rate of substitution between C_1 and L and decreasing marginal utility in C_2. Assuming that the gross wage rate w is random, the worker's problem is to choose C_1 and H so as to maximize $EV = E(u(C_1,L) + v(C_2))$ subject to the budget constraint $C_2 = I(\theta wH + r - C_1)$, where r is the current lump-sum payment, $I = 1 + i$ = the interest rate factor and other symbols are the same as before. The first- and second-order conditions for the maximum are

$$(6)\quad \begin{cases} \text{(i)} & EV_C = E[u_C - Iv'] = 0 \\ \text{(ii)} & EV_H = E[-u_L + I\theta wv'] = 0 \end{cases}$$

and

$$(7)\quad \begin{cases} \text{(i)} & EV_{CC} = E[u_{CC} + I^2v''],\ EV_{HH} = E[u_{LL} + I^2(\theta w)^2v''] < 0 \\ \text{(ii)} & D = EV_{CC}EV_{HH} - EV_{CH}^2 > 0 \end{cases}$$

where $EV_{CH} = EV_{HC} = -E[u_{CL} + I^2(\theta w)v'']$. Finally, let us assume in order to simplify the argument that there is an intra-period separability between consumption and leisure which implies that $EV_{CH} > 0$.

Following the same procedures as in section 2.1. the effect of the marginal tax rate on labour supply can be decomposed into the income, substitution and risk effects as in (3'). The income effect, $-\bar{w}HH_r$, is positive because $H_r = -D^{-1}u_{CC}\theta E(v''w) < 0$ and the substitution effect of a change in the expected wage rate can be expressed as

(8) $$H^c_\varepsilon\big|_{\varepsilon=0} = -D^{-1}EV_{CC}E(v')I\theta > 0$$

so that the substitution effect due to the change in the expected wage rate induced by the marginal tax rate change $-\theta^{-1}\bar{w}H^c_\varepsilon\big|_{\varepsilon=0}$ is negative. And the effect of a mean-preserving change in wage rate risk turns out to be

(9) $$H_\eta\Big|_{\substack{\varepsilon=0\\ \eta=1}} = -D^{-1}I\theta\,\{\,EV_{CC}[\text{cov}(v',w) + IH\theta\,\text{cov}(v''w,w)] + EV_{CH}IH\,\text{cov}(v'',w)\,\}$$

The plausible assumption that absolute risk aversion is decreasing implies $\text{cov}(v'',w) > 0$ so that $EV_{CH}\,\text{cov}(v'',w) > 0$. The sign of $\text{cov}(v',w) + IH\theta\,\text{cov}(v''w,w)$ is equal to the sign of $X^o = IH\theta wv''' + 2v'' = v''(2-(1+R)f^o - R'v'f^o$, where $f^o = I\theta wH/C_2$ = the fraction of future consumption accounted for by the future value of risky labour income. If we again assume the constant relative risk aversion $(R' = 0)$ as the benchmark case, then we have $X^o \gtrless 0$ as $R \gtrless (2/f^o)-1$. Thus $X^o \leq 0$ provide a sufficient, but not a necessary condition for the negative labour supply effect of a mean-preserving change in wage rate risk. This means that if relative risk aversion is low and/or future consumption does not depend very much on risky labour income, then the mean-preserving change in wage rate uncertainty tends to affect labour supply negatively and the risk effect of the marginal tax rate tends to be positive and run counter to the negative substitution effect. We should notive, however, that high relative risk aversion and/or strong dependence of future consumption on risky labour income do not any more guarantee the positive risk effect in an intertemporal consumption-leisure model.[8)]

It would be tempting to say that a generalized Slutsky equation (3') conveys all that can be said about the effects of increased progression. But (3') is based on a simultaneous change in the marginal as well as the average rate of tax. In order to isolate the 'pure' tax progression effect it is desirable to keep the average tax rate constant in some sense. Next we turn to analyze this question.

3. Changes in Progression and Labour Supply under Wage Rate Uncertainty

As we indicated in the introduction the effect of changes in 'pure' progression is analyzed in what follows under two alternative criteria: the compensated changes in the marginal tax rate are conducted to keep either the expected tax revenue of government or the expected utility of workers unchanged.

Under the assumptions used above the expected tax revenue of government is $ET = t\bar{w}h - r$.[9] In a static consumption-leisure model keeping the expected tax revenue or the expected utility constant respectively defines the following relationship between changes in the lump-sum payment r and the marginal tax rate t

$$(10) \qquad dr = \bar{w}H dt + t\bar{w} dH \qquad \text{as } ET = T^*$$

and

$$(11) \qquad dr = \{ \bar{w}H + (E[u_C])^{-1} H \operatorname{cov}(u_C, w) \} dt \qquad \text{as } Eu = u^*$$

where T* and u* stand for constant levels of ET and Eu. In an inter-temporal consumption-leisure model the expressions are the same with the exception that (11) is of the form

$$(11') \qquad dr = \{ \bar{w}H + (E[v'])^{-1} H \operatorname{cov}(v', w) \} dt \qquad \text{as } EV = V^*$$

where V* stands for constant level of EV.

The labour supply effect of the policy of changing t so as to keep ET unchanged can be obtained by substituting the right hand side of (10) for dr in $dH = H_t dt + H_r dr$, which yields

$$(12) \qquad \left(\frac{dH}{dt}\bigg|_{ET=T^*}\right) = \beta^{-1}(H_t + \bar{w}HH_r)$$

$$= \underbrace{-[\beta\theta]^{-1}}_{(-)} \{ \underbrace{\bar{w}H^c_\varepsilon\big|_{\varepsilon=0}}_{(+)} + \underbrace{H_\eta\big|_{\substack{\varepsilon=0\\ \eta=1}}}_{(?)} \}$$

where $\beta \equiv 1 - t\bar{w}H_r > 0$ and where we have utilized the equation (3').

Finally, the labour supply effect of the policy of changing the marginal tax rate t compensated by a change in the lump-sum payment r so that the expected utility Eu of workers remains unchanged can be obtained by substituting the right hand sides of (11) and (11') for dr in $dH = H_t dt + H_r dr$. This yields

$$(13) \qquad \left(\frac{dH}{dt}\bigg|_{Eu=u^*}\right) = H_t + \bar{w}HH_r + HH_r\{(E[u_C])^{-1}\operatorname{cov}(u_C,w)\}$$

$$= \beta\left(\frac{dH}{dt}\bigg|_{ET=T^*}\right) + \underbrace{HH_r(E[u_C])^{-1}\operatorname{cov}(u_C,w)}_{(+)}$$

and respectively

$$(13) \qquad \left(\frac{dH}{dt}\bigg|_{EV=V^*}\right) = \beta\left(\frac{dH}{dt}\bigg|_{ET=T^*}\right) + \underbrace{HH_r(E[v'])^{-1}\operatorname{cov}(v'w)}_{(+)}$$

where we have utilized the first-order conditions for the expected utility maximization.

The effect of increased progression on work effort under the policy of keeping the expected tax revenue constant thus depends on the negative substitution effect and on the a priori ambiguous risk effect. Under certainty only the first effect will be operative so that in conformity with a popular belief increase progression will discourage work effort (see also fn 1). Under uncertainty, however, the risk effect of taxation is also operative reinforcing the substituion effect if a rise in wage rate risk increases labour supply, while running counter to the substitution effect if a rise in wage rate risk reduces labour supply. In particular if relative risk aversion is very low and/or consumption does not depend very much on risky labour income, then the risk effect of the marginal tax rate may dominate the negative substitution effect in which latter increased progression would increase labour supply (see discussion in section 2).

What about the effect of the policy of increasing progression under the expected utility criterium? The expressions (13) and (13') imply that if the policy of increasing progression by keeping the expected tax revenue of government unchanged will increase labour supply, then the policy of increasing progression by keeping the expected utility of workers unchanged does the same. Moreover, even though the former policy would discourage work effort, the latter may not. Hence, increasing progression under the expected utility compensation will more likely encourage labour supply than under the expected tax revenue compensation. A rise in the marginal tax rate will increase the expected utility of risk-averse workers via reducing post-tax wage rate risk. To keep the expected utility constant, the lump-sum payment need to be raised less and labour supply is higher when consumption and leisure are normal goods.

Thus far we have proceeded under the assumption that hours of work H is a choice variable. Alternatively, and perhaps more realistically at least in the short run, one might assume that primary job contractually calls for H_1 hours of work at the wage rate w_1 and the worker's problem is to choose H_2 secondary hours of work at the wage rate w_2. If we adopt this interpretation, which have been used in a certainty context e.g. by Shishko and Rostker (1976), and assume that w_2 is stochastic, then all our earlier analyses go through in terms of H_2 with inessential modifications. Finally, one might want to analyze the case, in which H_1 and H_2 are both choice variables so that e.g. w_1 is known for certainty, while w_2 is stochastic. While the analysis of this case turns out to be a bit messy, there seems to be a tendency for increased progression for the total labour income to lead the allocation of hours of work toward risky activities, but the total labour supply effect remains ambiguous (see also Cowell (1981)).

The above arguments hold for any linear income tax. But what about if the linear income tax is required to be optimal in the sense of maximizing the expected utility of a representative worker? Let us now restrict our attention to the static consumption-leisure model. Analogous considerations go through in the case of the intertemporal consumption-leisure model. The government chooses t and r so as to maximize $Eu(c,1-H)$, where $c = \theta wH+r$ subject to the government budget constraint $T^* = t\bar{w}h-r$ with T^* given exogenously. Let the Lagrangian function be $K = Eu(\theta wH+r,1-H)$ $-\mu(T^* - (1-\theta)\bar{w}H+r)$, where μ is the Lagrangian multiplier associated with the constraint. The first-order conditions for the welfare optimum can be written

$$(14)\quad \begin{cases} \text{(a)}\quad K_t = 0 = -E(u_C w)H + \mu\bar{w}H + \mu t\bar{w}H_t \\ \text{(b)}\quad K_r = 0 = Eu_C - \mu\beta \end{cases}$$

where $\mu > 0$ and $\beta \equiv 1-t\bar{w}H_r > 0$. Utilizing the equation (14b) and the equation (3') for labour supply makes it possible to write (14a) as follows

$$(15)\qquad K_t = 0 = -\mathrm{cov}(u_C,w)H\theta + Eu_C\beta^{-1}(1-\theta)\bar{w}\{-[\bar{w}H^C_\varepsilon\big|_{\varepsilon=0} + H_\eta\big|_{\substack{\varepsilon=0\\ \eta=1}}]\}$$

where $\theta \equiv 1-t$. Now it is easy to see that $K_{t|t=0} > 0$ and that $K_{t|t=1} < 0$ so that at the optimum the marginal tax rate t is between one and zero.[10)] The equation (15) can be re-expressed as

$$(16) \qquad (1-\theta)\theta^{-1} = \mathrm{cov}(u_c,w)H\beta/\{Eu_c[-[\bar{w}H_\varepsilon^c|_{\varepsilon=0} + H_\eta|_{\substack{\varepsilon=0\\ \eta=1}}]]\}$$

where the numerator $\mathrm{cov}(u_c,w)H\beta < 0$ and where $0 < t < 1$ presupposes that denumerator is negative so that $-(\bar{w}H_\varepsilon^c|_{\varepsilon=0} + H_\eta|_{\substack{\varepsilon=0\\ \eta=1}}) < 0$. In other words at the welfare optimum progression is pushed to the point where at the margin it reduces labour supply despite wage rate uncertainty and risk-sharing effects of the marginal tax rate.

4. Concluding Remarks

We have analyzed the relationship between labour supply and linear wage taxation under wage rate uncertainty in a static consumption-leisure and in an intertemporal consumption-leisure model respectively. It has first been shown how the labour supply response to a change in the marginal tax rate can be decomposed into the income effect, the substitution effect and the risk effect. The risk effect results from a negative effect of a change in the marginal tax rate on the post-tax wage rate risk and can be of any sign. Moreover, and related, it has been demonstrated how increases in pure progression - by keeping either the expected government tax revenue or the expected utility of worker unchanged - can lead to increased work effort, which lies in sharp contrast to a widely held popular belief about the disincentive effect of increased progression. Particularly, this may happen if workers' relative risk aversion is not very high and if their consumption does not depend too strongly on risky labour income. Finally, however, it is shown that at the welfare optimum the marginal tax rate is pushed to the point where at the margin it reduces labour supply despite wage rate uncertainty and risk-sharing effects of the marginal tax rate.

In the light of these results and noting that in the case of nonlinear taxation not even the pure substitution effect can be unambiguously signed (see Blomquist (1985)) it can be concluded that in situations off the welfare optimum a widely held popular belief about the disincentive effect of increased progression is far from being generally true. Signing labour supply effects of changes in progression is potentially very sensitive to precise details of the tax code and to risk attitudes of workers; both are the areas where more empirical research is urgently needed.

Footnotes:

1) This can be seen as follows. Let $u(C,L)$ be an increasing function of consumption and leisure L and continuous and strictly quasi-concave. Let H be labour supply such that $H+L=1$, w wage rate, t constant marginal tax rate and r lump-sum payment. Maximizing u subject to the budget constraint $C = \theta wH+r$, where $\theta = 1-t$, yields the Slutsky decomposition $H_t = H_t^c - wHH_r$, where the substitution effect H_t^c is negative and the income effect $-wHH_r$ positive given normality of leisure. Raising t so that u does not change is just H_t^c, while raising t so that tax revenue $T^* = twH-r$ does not change implies $(dH/dt)_{T^*} = (1-twH_r)^{-1}H_t^c < 0$. Nevertheless, it has been argued in the literature that even under certainty there may be the case against the popular belief that the more progressive the tax system, the greater the disincentive to work effort (see Barlow and Sparks (1964), Chatterji (1979) and Hemming (1980)). The argument relies on the different effect of t and r. Let us suppose that r is changed and compensated by a change in t so that either u or T will remain unchanged.

In the former case we have $(dH/dr)_{u^*} = H_t^c/wH$ and in the latter case

(1) $(dH/dr)_{T^*} = H_t^c/w(H+tH_t) > 0 \quad \text{if } H+tH_t < 0$

From the tax function $T = twH - r$ we have, however,

(2) $$wH(dt/dr) = 1 - tw(dH/dr)_{T^*} = 1 - tH_t^c/(H+tH_t)$$
$$= (H+tH_t)^{-1}(H(1-twH_r))$$

which is positive with negative $H+tH_t$ only when leisure is an inferior good. If leisure is assumed to be a normal good then $H+tH_t < 0$ implies that in the new equilibrium t and r are not positively, but negatively related. But this hardly constitutes a proper definition of increased progression when a rise in r is associated with a fall in the marginal tax rate! In the papers referred above the tax function to be used is not $T = twH - r$, but $T = t(wH - e)$ if $wH - e > 0$ and 0 otherwise so that the linear income tax is applied to all incomes above the exemption level e. All the analyses, however, go through under this assumption as well. Sandmo (1983) has extended this analysis to a model with many workers under certainty, in which case in order for increased progression to reduce labour supply, the negative substitution effects must dominate the income effects resulting from redistribution of incomes over workers. For this to be true would not seem to require any peculiar assumptions about preferences.

2) The effect of taxes on labour supply under uncertainty have also been analyzed by Eaton and Rosen (1980) and by Cowell (1981). While the focus of interest in those papers is slightly different, neither Eaton and Rosen (1980) nor Cowell (1981) utilizes the decomposition of comparative statics into the income, substitution and risk effects displayded in equation (3') of the text. Instead the substitution and risk effects are mixed up in terms $E[u_C w]$ and

$H\,cov(\theta wu_{CC}-u_{CL},w)$ with a consequence that the possibility for compensated increases in progression to encourage labour supply becomes underestimated.

3) This need not imply that government is risk neutral. To the extent that risks are independent across workers and the number of workers is large, the law of large numbers will guarantee government a constant total revenue despite uncertainty at the individual level. Under these circumstances government is simply a more efficient risk-pooler than individuals. To the extent that the law of large numbers does not work e.g. because of the dominance of 'business cycle risks', then the assumption that government is risk neutral is presupposed. In the latter case we have to assume that taxes are spent on public goods which enter the utility function in a separable way. Thus the variability of the supply of public goods conseguent on the variability of government tax revenue has no effect on labour supply.

4) The expressions (4) and (5) were first presented in a labour supply context by Block and Heineke (1973).

5) This kind of decomposition has been utilized in Koskela (1984) in an analysis of the effect of income taxation on asset choice under rate of return uncertainty.

6) There seems to be no consensus on how the relative risk aversion varies so that the constant relative risk aversion might be regarded as the good benchmark case. As far as the size of the relative risk aversion is concerned, the estimates, while varying widely, all come up with the conclusion that it is well above one (see Machina (1983) for further details).

7) Another way of winding up in X and in (5) is to ask whether the first-order condition (1) is a strictly convex or strictly concave function of w. In the former case X is positive and in the latter case X is negative respectively. This Rothschild-Stiglitz criterion have been used e.g. by Eaton and Rosen (1980). Uusually, and here as well, effects of changing risk on behaviour have been characterized in terms of risk aversion parameters. Tressler and Menezes (1980) have shown that the question of how wage risk affects the marginal rate of substitution between leisure foregone and consumption provides a necessary and sufficient condition to unambiguously sign the effect of wage risk on labor supply. Actually, Tressler and Menezes (1980) argue that increased wage uncertainty will decrease labour supply.

8) The effect of taxes on behaviour under uncertainty has been subject to research at least since the seminal paper by Domar and Musgrave (1944), in which it was shown that taxation may actually encourage risk-taking, if government is sharing the risk. In a general equilibrium capital market asset pricing model Leccisotti (1976) has also emphasized the potential return and risk effects of non-lump sum taxes. In particular, he has shown how the effects of taxes on asset prices are sensitive both to the type of tax and to the question of whether and to what extent loss offset provisions are operative. Finally, in a slightly different context of tax evasion with endogenous labour supply Sproule (1985) has analysed tax evasion and labour supply effects of change in

uncertainty associated with individual parameters of the tax system like with the tax rate, the probability of detection and the penalty rate. Thus in terms of this paper's notation he studies the expression $H_\eta|_{\substack{\varepsilon=0\\ \eta=1}}$ under various (and more general) circumstances than what has been done in this paper. Neither of the paper,however, develops the decomposition (3') nor studies compensated changes in progression.

9) This tax schedule is progressive under the following now conventional definitions of progressivity, suggested by Musgrave and Thin (1948): (i) the average tax rate is increasing with income before tax, (ii) the elasticity of the tax function with respect to income before tax is greater than one and (iii) the elasticity of income after tax with respect to income before tax is less than one. The implications of their fourth possible definition of progressivity as the increasing marginal tax rate is not considered here. All the analyses and properties go through under the tax function $T = t(wH-e)$ as well, where the linear income tax t is applied to all incomes above the exemption level e.

10) This has been pointed out in Eaton and Rosen (1980).

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