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ERROR CORRECTION MECHANISM: AN ECONOMIC INTERPRETATION

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ABSTRACT: The error correction mechanism is shown to come out naturally from a simple cost minimisation problem. Different representations of the solution help interpret the behavioral dynamics of the mechanism. Particularly the importance of the relevant target of adjustment is directly related to the marginal relative cost of being off from the corresponding target. It is hoped that the given economic interpretation of the error correction mechanism will help economists in recognising this powerful dynamic specification as a natural and theoretically justifiable formulation in empirical work.

Key Words: Construction and use of econometric models,
Cost minimisation, dynamic specification, equilibrium,
relative marginal costs, target of adjustment.

1. INTRODUCTION

The past ten years or so have experienced a strong march of analysis of dynamic econometric models called error correction mechanisms. Since the seminal work of Sargan (1964), it took until Davidson et. al (1978) before these models started to gain a growing interest among econometricians. The obvious appeal of these models is their ability to incorporate both short run dynamics and long run equilibrium conditions between variables in a single equation specification, thereby offering a synthesis between difference models and levels models.

The natural environment of the error correction mechanisms is in modelling nonstationary time series which can be made stationary by suitable differencing. These type of series are commonly called integrated time series. If a linear combination of two or more integrated series is found stationary, these series are called cointegrated. Engle and Granger (1987) demonstrate the interplay between error correction formulations and the existence of the cointegrating relationship. In particular, the statistical analysis of the error correction mechanisms is relevant and of interest only when the cointegrating relationship exists. The statistical establishment of these relationships is a nontrivial task, since the well known inferential procedures of stationary time series do not apply here. Stock and Watson (1986) develop formal statistical procedures to find cointegrating relationships among integrated time series. In fact, a whole new asymptotic theory of analysing nonstationary series has emerged. In econometric context a good exposition is Phillips and Durlauf (1986). An excellent overview of various conceptual and technical issues in modelling nonstationary time series is Hendry (1986), which contains an informative list of references to the existing literature.

While econometricians and statisticians have been busy in developing a framework wherein to analyse thoroughly both the short run and the long run relationships between nonstationary variables, it is my impression that models, which incorporate the error correction mechanism, have not gained a widespread acceptance among economists. The notion of being able to combine different kinds of dynamic aspects into a single equation may be the strongest reason for the statistician, who should take the data seriously, to apply these models. The economist, on the other hand, would like to derive the models to be estimated from relevant economic theory, which describes the behavior of economic agents. I doubt that very few economists dared to include a lagged dependent variable into their regression equations until Feige (1967) demonstrated that the already well known partial adjustment mechanism could be derived as representing the optimising behavior of a cost minimising economic agent. Today lagged dependent variables are routinely thrown into regressions and justified by economists almost unanimously through some cost minimising behavior, which leads to partial adjustment toward the equilibrium.

The purpose of this note is to give a simple interpretation of the basic error correction mechanism by deriving it from a cost minimisation problem in the similar fashion as Feige (1967) derived the partial adjustment mechanism. The error correction term will be directly attributable to the partial adjustment mechanism and the (possibly lagged) differences of the independent variables arise naturally from changes in the equilibrium value of the dependent variable.

Due to these explicit economic interpretations, old-fashioned as they may be, the author hopes that applied economists would find the error correction mechanisms as viable models to be entertained in modelling time series data. These hopes apply particularly to those economists, who frequently estimate partial adjustment models in levels form and typically obtain unreasonably slow adjustment speeds.

2. THE ECONOMICS OF THE ERROR CORRECTION MECHANISM

Let y_t represent the actual value of the variable of interest in period t and let y_t^* represent its equilibrium value in the same period. Feige (1967) demonstrated that if quadratic costs are involved, when deciding on the actual value of y_t, such that costs arise both from being away from the equilibrium y_t^* and from changing y_t from its previous period value y_{t-1}, then the cost minimiser would choose the value of y_t according to the partial adjustment rule.

Specifically, let $a(y_t-y_t^*)^2$ be the cost of not being in equilibrium, and $d(y_t-y_{t-1})^2$ be the cost of changing the actual value, where a and d are fixed parameters. If the total cost function is

$$C_t = a(y_t-y_t^*)^2 + d(y_t-y_{t-1})^2, \qquad (1)$$

then minimising C_t with respect to y_t gives

$$y_t - y_{t-1} = (a/(a+d))(y_t^* - y_{t-1}) \ . \qquad (2)$$

This is the familiar partial adjustment mechanism, which yields an estimable equation once y_t^* has been replaced by a function of observable variables, typically a linear function of explanatory variables selected from relevant economic theory. If we denote these variables by X_t, then equation (2) is usually estimated in the levels form

$$y_t = b_0 + py_{t-1} + b'X_t + u_t \ ,$$

where b_0, p and b' are parameters and u_t is the error term.

The cost function (1) is quite sensible in many respects, but it totally neglects all cost aspects arising from (possibly abruptly) changing economic conditions, which drive the development of y_t^*. For example, consider a situation where y_t^* suddenly rises due to some exogenous shock, which is largely believed to be only temporary. Now, for decision making purposes, the previous equilibrium, y_{t-1}^*, might be as relevant target as y_t^*, or maybe even more so, since the uncertainty about the relevance of y_t^* has increased.

A simple way to take account of this type of uncertainty about the relevant target is to add a cost component

$$b(y_t - y_{t-1}^*)^2$$

to the cost function (1). The minimisation of the new cost function

$$C_t = a(y_t-y_t^*)^2 + b(y_t-y_{t-1}^*)^2 + d(y_t-y_{t-1})^2 \quad (3)$$

with respect to y_t is straightforward. This quickly leads to the expression

$$y_t-y_{t-1} = (a+b)/(a+b+d))(y_t^*-y_{t-1}) - (b/(a+b+d))(y_t^*-y_{t-1}^*). \quad (4)$$

After rearranging we get

$$y_t-y_{t-1} = (a+b)/(a+b+d))(y_{t-1}^*-y_{t-1}) + (a/(a+b+d))(y_t^*-y_{t-1}^*) \quad (5)$$

Equation (5) is in the form of the basic error correction mechanism, where the first term on the right hand side is the error correction term. It is interesting to look at both (4) and (5) simultaneously and interpret them as partial adjustment mechanisms. We clearly see how the relevant target becomes an integral part of the interpretation.

Equation (4) can be interpreted as representing the case, where y_t^* is thought as the target. Here large changes in the target level moderate the actual adjustment, since the second term on the right hand side of (4) has a negative coefficient. Equation (5), on the other hand, represents the case, where y_{t-1}^* can be thought as the target. Here large changes in the target level add to the actual adjustment, since the coefficient of the second term on the right hand side of (5) is positive.

Equations (4) and (5) demonstrate that error correction mechanisms have extremely natural interpretations as adjustment mechanisms. Both y_t^* and y_{t-1}^* are monitored as targets and the relative importance of these targets is in direct relation to the relative marginal costs a/(a+b+d) and b/(a+b+d) of being away from the respective target. In the extreme case, when b=0 the adjustment is solely towards y_t^*, since the marginal cost of being away from the previous equilibrium level is zero. Symmetrically, when a=0, the adjustment is solely towards y_{t-1}^*, since the marginal cost of being away from the current equilibrium is zero. It is an empirical issue to determine the magnitudes of the relative cost components. These could be directly estimated for instance from (5) if the levels of y* were directly observable. Formal statistical tests of their magnitudes could also be constructed in this case. More commonly, however, the equilibrium level is expressed as a parametric function of explanatory variables. In this case the relative cost parameters are typically mixed with the other parameters of the model in an unidentifiable form to facilitate easy estimation of the whole model, usually by least squares.

3. FURTHER COMMENTS

The equilibrium level y_t^* of y_t is often related to other variables through a linear relationship

$$y_t^* = b^{*'} X_t$$

Upon inserting this into (5) we get

$$(1-L)y_t = -f(y_{t-1}-b^{*'}X_{t-1}) + g'(1-L)X_t \,, \qquad (6)$$

where L is the lag operator, f=(a+b)/(a+b+d) and g=b*a/(a+b+d).

Equation (6) is the standard parameterisation of the error correction mechanism, which appears in the literature. Unless b* is known an a priori grounds, Engle and Granger (1987) suggest its estimation from the regression of y_t on X_t. Then inserting the estimated error correction variable, $y_t-\hat{b}^{*'}X_t$, into (6) the estimation can be completed by estimating the resulting linear model, where -f and g are the parameters to be estimated. Banerjee et al. (1986) have, however, warned using this method, due to serious finite sample bias in the estimate of b*. This can be a serious defect from the relative marginal cost component estimation point of view, too. Namely, if b* could be estimated accurately from the static regression of Engle and Granger, then we could also insert that estimate into g and run a regression of $(1-L)y_t$ on the estimated error correction variable and $\hat{b}^{*'}(1-L)X_t$. From estimates so obtained we could easily infer the values of the relative marginal cost components a/(a+b+d), b/(a+b+d) and d/(a+b+d). If the estimate of b* is strongly biased in finite samples, this procedure may not be reasonable unless the sample sixe is extremely large. Therefore nonlinear estimation methods should be employed.

A final note for the need to be careful when interpreting the estimates of the error correction mechanisms. Suppose the economic theory suggests that the elements of b* should be positive. This implies that g in (6) should be positive, too and negative estimates would be treated as suspect.

The situation under study may be such that, when the target level y_t^* is being formed, the actual value of X_t is not available. In such instances X_t is usually interpreted as an expectation, X_t^e, say. To simplify the exposition, let us assume that X_{t-1} is the best estimate of X_t^e. (If the elements of X_t are random walks, X_{t-1} is the optimal forecast of X_t). Now, $y_t^* = b^{*'} X_{t-1}$. In this kind of circumstances the estimable error correction mechanism comes from equation (4) as

$$(1-L)y_t = -f(y_{t-1}-b^{*'} X_{t-1}) + h'(1-L)X_{t-1}, \qquad (7)$$

where h = -b*b/(a+b+d) and the rest is as in (6).

The first notable deviation in (7) compared to (6) is the <u>lagged</u> difference, $(1-L)X_{t-1}$, of the explanatory variables. A remarkable deviation is that the coefficient vector of this lagged difference term is <u>negative</u>. Thus, on the first sight, the estimated version of (7) may look counter to the economic theory, which postulates a positive relationship between y and X. Therefore, care is needed when interpreting models with expectations in them, in order not to throw away estimated models which may make complete sense.

4. CONCLUSION

The error correction mechanism comes naturally out of a simple cost minimisation problem. Different representations of the solution help interpret the behavioral dynamics of the mechanism. Particularly the importance of the relevant target of adjustment is directly related to the marginal relative cost of being off from the corresponding target. It is hoped that the given economic interpretation of the error correction mechanism will help economists in recognising this powerful dynamic specification as a natural and theoretically justifiable formulation in empirical work.

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