ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

Keskusteluaiheita
Discussion papers

Juha Kinnunen

COMPARISON OF THE ARIMA-MODEL FORECASTS OF SOME FINNISH MACRO-ECONOMIC VARIABLES WITH ECONOMETRIC MACROMODEL FORECASTS*)

No 226 31.12.1986

*) The abstract of a Master's thesis by the author, see references (Kinnunen, 1978)

ISSN 0871-6847

Juha Kinnunen

COMPARISON OF THE ARIMA-MODEL FORECASTS OF SOME FINNISH MACROECONOMIC VARIABLES WITH ECONOMETRIC MACROMODEL FORECASTS

CONTENTS

1. Introduction

During recent years and decades, econometric macromodels have been built in various countries, the largest of these models including hundreds of equations. Despite the large amount of work done, however, these models have not wholly met the expectations. The economic forecasts yielded by them have proved more or less erroneous.

There are, of course, more simple alternatives to econometric macromodels to be used in explaining economic phenomena and producing economic forecasts, many of which come within the scope of time series analysis. The time series analysis methods introduced by Box and Jenkins (1976) have become particularly popular since the publication of the first edition of their book in 1970. The autoregressive integrated moving average (or ARIMA) models which they recommend, as well as the transfer function noise models, have proved to be ones well capable of competing with the econometric models. They have been widely applied in practical forecasting situations, but they have also been strongly criticized. The articles by Chatfield and Prothero (1973) and Box and Jenkins (1973) and the discussions concerning them offer a good picture of the debate going on about the Box-Jenkins models.

The ARIMA method is an autoprojective forecasting method, i.e., a method in which the only information used is information on the variables' own past. Other autoprojective methods are, e.g., exponential smoothing, Brown's method, Holt's method, Holt-Winter's method and Harrison's method (see, e.g., Kendall 1973 or Nyström 1977). Kendall (1973, p. 125) gives an account of Reid's study in which forecasts were produced by various autoprojective methods for 113 time series. In 76 (67 %) of these time series, the forecast yielded by the ARIMA method had the smallest one-step forecasting error, so that this method proved definitely the most precise. Kendall (1973, p. 127) presents the selection tree of auto-projective forecasting methods, constructed by Reid, and it is also presented in Leskinen's (1977b) article.

In statistical periodicals, articles have been published in which forecasts obtained by ARIMA models and econometric models have been compared (see, e.g., Leskinen 1977b). The ARIMA models have been found better in some studies and econometric models better in others.

Naylor, Seaks and Wichern (1972), for example, presented for the purpose of comparison a few typical equations included in Wharton's 76-equation simultaneous model of the United States economy. In the forecasts produced for four variables, the absolute forecast errors of ARIMA models were smaller in each case than the forecast errors in the forecasts yielded by Warton's model.

The econometric model considered in Prothero and Wallis's (1976) article is Hendry's model of the economy of the United Kingdom. The goodness of fit of its six behaviour equations and the goodness of fit of the ARIMA models constructed for the same variables were compared. The authors found that the residual variances of the econometric model were smaller than those of the ARIMA models in the case of each variable.

Willman (1976) compared forecasts yielded by the Bank of Finland's macroeconomic model with forecasts obtained by linear autoregressive models. He found that the forecast errors (root-mean-square error, or RMS) in the forecasts yielded by the autoregressive models were definitely smaller on average than the corresponding forecast errors of the macromodels. With the lengthening of the forecasting horizon the situation changed entirely, so that the forecast errors of the macromodels became definitely smaller than those of the autoregressive models. The macroeconomic models seemed to be better able to forecast the course of the variables in which cyclical fluctuations were clearly perceptible, whereas the autoregressive models were best able to forecast those variables the course of which was very trend-like and in which cyclical fluctuations were only weakly perceptible.

Christ (1975) compared, the RMS errors of, i.e., econometric models of the United States economy with each other and with the forecast errors of ARIMA models. The variables examined were the GDP at current prices, the GDP at constant prices and the price of the GDP. The forecasting periods were one quarter year, two quarter years, and so on, up to eight quarter years. In Christ's study, the ARIMA model forecasts were the poorest throughout.

The present study has two main objectives. First, ARIMA models will be built for ten central macroeconomic variables of the Finnish economy. Quarterly data for the longest time period possible will be used for observations, so that the last year of the estimation period will be 1973. The observations for the years 1974-76 will be used for the assessment of the predictive power of the models.

Secondly, a predictive testing will be undertaken. The forecasts for the years 1974-76 yielded by the econometric macromodel presented by Pentti Vartia (1974) in his doctoral dissertation are available. The ARIMA model forecasts and forecasts obtained by ETLA's model are compared with one another and the realized values, and conclusions will be made regarding the precision of the forecasts produced by the models in question. The absolute values of the residuals of the annual percentage changes in the variables and the mean square errors and relative (percentage) forecast errors in level forecasts will be used as the criteria for the goodness of the forecasts. It will be examined, further, whether the more sophisticated forecasting methods are able to yield more accurate forecasts in this situation than the simple, mechanical methods. Therefore, a naive model and the forecasting errors in the forecasts produced by it will be presented to form a point of comparison.

The theory of the ARIMA models, with numerous applications to practice, has been presented in Box and Jenkins's (1976) study referred to above. Concise Finnish-language accounts of the building of ARIMA models and their use in forecasting have been given respectively in two articles by Esko Leskinen (1977a and 1977b). The building of the models and the computer programs used in forecasting by them have been described in Leskinen (1973). Kanninen and Suvanto have considered Box's and Jenkins' methods particularly from an economist's point of view and discussed the merits and drawbacks of the traditional econometric method and this new time-series analysis method.

The present article includes an outline of the theory of the ARIMA models. The ARIMA models built in Kinnunen's (1978) study are briefly described and the main attention will be devoted to a comparison of the forecasts produced by these models and by ETLA's econometric model (Vartia, 1974).

2. On the building of and forecasting with ARIMA models

2.1. On the theory of the ARIMA models

Let us assume that the time series $z_1, z_2, \ldots, z_N$ has been observed, as a realization of the stochastic process $\underline{z}_t$

$$(2.1) \qquad \underline{z}_t = \mu + \psi(B)\underline{a}_t$$

$$= \mu + \underline{a}_t + \psi_1\underline{a}_{t-1} + \psi_2\underline{a}_{t-2} + \cdots ,$$

Which has been produced by the linear filter $\psi(B)$, and in which the backward operator (or lag operator) B is defined by

$$B\underline{a}_t = \underline{a}_{t-1} \text{ and } B^k\underline{a}_t = \underline{a}_{t-k}$$

$\underline{a}_t$ being white noise. White noise can be interpreted as independent observations on a probability distribution with zero mean and constant variance, thus $a_t \sim nid(0,\sigma_a^2)$.

The linear filter $\psi(B)$ may be approximated by the rational polynomial

$$(2.2) \qquad \psi(B) = \frac{\theta(B)}{\phi(B)} ,$$

where $\phi(B)$ is a pth degree stationary autoregressive process, or AR(p) process, and $\theta(B)$ is a qth degree invertible moving average process, or MA(q) process.

For non-stationary time series involving seasonal variation, Box and Jenkins (1976) propose, as an approximation to the filter $\psi(B)$, the multiplicative seasonal model, or ARIMA$(p,d,q)x(P,D,Q)_s$ model,

(2.3) $$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D \underline{z}_t = \phi_0 + \theta(B)\Theta(B^s)\underline{a}_t ,$$

where $\Phi(B^s)$ is a seasonal AR(P) process and $\Theta(B^s)$ is a seasonal MA(Q) process, and

$$\nabla^d = (1 - B)^d \text{ and } \nabla_s^D = (1 - B^s)^D$$

are difference operators of order d and of order D, s being the number of seasons per year.

In the present study, the model type (2.3) was used in building the ARIMA models.

2.2. On the building of ARIMA models

In building ARIMA models of the model family (2.3), the intention is to find a adequate but parsimonious model, by means of which the process that has generated the time series under study can be described and which can be used in forecasting the future values of the process.

The building of the model consists of three main phases: the identification of the model, the estimation of its parameters and diagnostic checking.

In identifying the model, the values of p, d, q, P, D and Q have to be determined. The most important tools, additional to graphical analyses of the time series, are the autocorrelation function (acf) and the partical autocorrelation function (pacf) computed from the time series. Thus the identification of the model is based on autocorrelation analysis. Since the generally used estimators of autocorrelations are biased, though consistent, the number of observations for the building of reliable models should be at least 50 and preferably 100 (Box and Jenkins, 1976 p. 18).

If the observed time series is non-linear, an attempt may be made to linearize it by means of a logarithmic or some other transformation, e.g. Box-Cox transformation (Nelson and Granger, 1979). The logarithmic transformation has in several economic applications the merit that, by means of it, the variance of the original series, which often changes with time, can be rendered invariable (see, e.g., Leskinen and Teräsvirta 1976). If the transformed time series is nonstationary, it will be differenced as many times as it is necessary to obtain a stationary series. The smallest number of differencing operations sufficient to give a stationary series will be performed, which means, in practice, differencing the series once or twice (see e.g. Prothero and Wallis, 1976, p. 473-474).

Not only the graph of a time series but also its correlation functions can be used in finding out whether the series is stationary, as the acf and pacf of a stationary process rapidly decays to zero with the lengthening of the lag.

When a stationary series has been reached, the behaviour of the estimated acf and pacf are compared with the corresponding theoretical correlation functions in order to determine the values of p, q, P and Q; see Box and Jenkins (1976), p. 176.

In uncertain cases it will be advisable first to identify and estimate a number of models, starting with simple ones. Identification on the basis of the acf does not generally lead to an unambiguous result, and therefore, a few tentative models may be included in further analysis.

Because the parameters of the ARIMA models are non-linear, they have been estimated by means of a non-linear least squares method, used to approximate the maximum likelihood estimate of the parameter vector $\beta = (\mu,\phi,\theta,\Phi,\Theta)$ (Box and Jenkins 1976, pp. 500-505; Leskinen 1973, pp. 17-27).

Following the identification and the estimation of the parameters, diagnostic cheeks are then applied to the fitted model, which means the controlling of the adequacy of the estimated model, or, in other words, the verification of the assumptions made concerning the theoretical model, through additional parametrization and through an analysis of the autocorrelations $r_k(\hat{a})$ of the estimated residuals (Box and Jenkins 1976, pp. 285-299; see also Prothero and Wallis 1976, pp. 480-481).

The adequacy of an estimated model can be examined by considering the acf as a whole by means of the Portmanteau test statistic Q(v),

$$(2.4) \qquad Q(v) = n \sum_{k=1}^{N} r_K^2(\hat{a}) .$$

When the residuals consist of white noise, $Q(v)$ follows the χ^2 distribution at the degrees of freedom $v = K - M - p - q - P - Q$ in the case of the model type (2.3). In that case, $n = N - d - D$ and $M = 0$, if the expectation μ_W of the differenced process is not estimated, and $M = 1$ if it is estimated (Box and Jenkins 1976, pp. 290-291; Leskinen 1973, pp. 26-27).

If the residuals of the model are autocorrelated and if, in particular, some individual lags have high values in the acf of the residuals, an attempt can be made to improve the model through additional parametrization. A model of their own will be identified from the acf of the residuals and the model thus obtained will be incorporated in the original, inadequate model. Thus the acf of the residuals reveals the inadequacy of the model and provides, in general, clues as to how the model should be correlated.

2.3. On forecasting by means of ARIMA models

Following the diagnostic checks, the model can be used for forecasting. In accordance with its mathematical structure, the model gives the minimum mean square error estimates based on the earlier observations of the process. Since the model is autoprojective and, consequently, does not receive information from outside the time series, the forecasts it yields tend to follow the previous course of the process. If the nature of the process changes after the estimation period, the model will not be able to forecast the new course of the process. Thus, as a forecasting model, an ARIMA model is a typical short-term model (Box and Jenkins, pp. 126-170).

3. The models used as points of comparison

3.1. ETLA's econometric model

The econometric macromodel of the Finnish economy presented by Pentti Vartia in his doctoral dissertation (Vartia 1974) is a short-term Keynesian demand-oriented simultaneous annual model, with features of a so-called Scandinavian model, built using primarily the annual model of the Dutch Central Planning Bureau as a prototype. The foundation of the model was formed by the aggregate balance of resources and expenditure, or the equality between total demand and total supply.

The model is a difference model, the variables included being percentage year-on-year changes. The model was rendered dynamic by using lags and quasi-multipliers. The longest lag used in the model is two years.

The dissertation version of the model involves a total of 57 variables, of which 42 are endogenous and 15 are exogenous. The model contains 42 equations, of which 12 are estimated stochastic behaviour equations. The estimation period of the model consisted of the years 1951-70, and the equations were mainly estimated one at a time by the least squares method and, partly, also by the two-stage least squares method.

The model presented in the doctoral dissertation is one of the early versions of Vartia's model but it is the version which is best documented. The model is subject to experimenting and further elaboration at the Research Institute of the Finnish Economy (ETLA), where it is used as a tool in analysing and forecasting short-term and medium-term economic fluctuations. Some of the behaviour equations of the model have been re-estimated and, in addition, disaggregation of its variables has been used to arrive at an increasingly accurate and detailed description of the interrelations between the variables. The version used by the present author's study included 61 equations and 92 variables, the number of estimated equations being 12 as in the original version (Kinnunen 1978, p. 86-92).

3.2. The naive model

To base the comparisons upon a wider foundation, forecasts were produced not only by sophisticated methods but also by a mechanistic naive model. In the naive model chosen, the change during each quarter year is assumed to be equal to the relative change during the preceding year's corresponding quarter:

$$\hat{z}_{t+4}/z_t = z_t/z_{t-4}$$

so that, for the forecast $\hat{z}_{t+4}$ for the quarter t+4 at time t, we have

$$(3.1) \quad \hat{z}_{t+4} = z_t^2/z_{t-4} .$$

4. Construction of ARIMA models for macroeconomic variables

4.1. Choice of the variables

ARIMA models were constructed for ten central variables describing the Finnish economy, which also occupy a central position in ETLA's model. The variables chosen were: gross domestic product, the volume of private consumption expenditure, the cost of living index (corresponding to private consumption prices in ETLA's model), the volume of private investment, the labour input of salary and wage earners, unemployment rate, the index of salary and wage earnings, the price index of commodity exports, the volume of multilateral commodity exports and the volume of commodity imports.

All of the above variables, except gross domestic product, are dependent variables of the estimated structural equations in ETLA's model. In ETLA's model, GDP is defined as the difference between total demand and imports, whereas both the components of total demand and imports were to be estimated. Since GDP is an important variable in describing macroeconomic developments, its inclusion in the group of variables for which ARIMA models were to be built was called for.

In ETLA's model, the GDP variable used is GDP at market prices. However, quarterly data on it are not available and the annual observations were not numerous enough for building a reliable ARIMA model, and therefore, use was made of GDP at factor cost, published also quarterly.

Of the estimated variables involved in ETLA's model, inventory changes, the price of total investment and the price of public demand were omitted, since they were not considered to be as central as the variables included.

4.2. ARIMA models of the macrovariables

For each of the ten variables an ARIMA model was built that passed the diagnostic checks. For some variables, a number of different adequate models were obtained, and from among these the one was finally chosen to describe the time series concerned that the statistical checks showed to be the best. The principal criteria used were the portmanteau test statistic Q(v) and the size of the estimated residual variance $\hat{\sigma}_a^2$.

The ARIMA models are summarized in Table 1. The figures below the parameters are these parameters' estimated standard errors. In addition, the estimated residual variance $\hat{\sigma}_a^2$ and the coefficient of explanation R^2 of the model

$$R^2 = 1 - \hat{\sigma}_a^2/s_w^2 ,$$

where s_w^2 is the variance of the differenced time series, are given.

5. Comparison of the ARIMA model forecasts with the forecasts yielded by ETLA's model and the naive models

5.1. General remarks on forecasts

In this section the ARIMA model forecasts will be compared with the forecasts yielded by ETLA's econometric model and the naive models chosen for points of comparison. Since ETLA's is an annual model, the quarterly forecasts of the ARIMA models were aggregated into annual forecasts, as sums of quarterly forecasts (or, in the case of indices, as their means). The comparison of the forecasts relates to periods not included in the estimation periods of either the ARIMA models or ETLA's model.

Two kinds of forecasts were computed. First, quarterly three-year forecasts for the years 1974-76 were computed from the ARIMA models and naive models by chaining. In this case, the model computed the first forecast (for the first quarter of 1974) from the estimation period observations and, the following forecasts, from the observations and the previously computed forecasts. It should thus be noted that the forecast for 1974 is a one-year forecast and the forecast for 1975 is a two-year forecast and that only the forecast for 1976 is a three-year forecast. By chaining the forecasts the medium-term predictive performance of the models can be measured.

Secondly, one-year forecasts were computed for the years 1974, 1975 and 1976, in which case the set of observations still contained the observations for the year preceding the forecast year. Since the data consists of quarterly observations, the forecast for the first quarter of the forecast year was computed exclusively on the basis of the set of observations and the forecasts for the other three quarters were then obtained by chaining. One-year forecasts are used to measure the short-term predictive performance of the models.

The forecasts yielded by ETLA's model were obtained from ETLA in the form of percentage year-on-year changes, from which the levels of the forecasts were computed with the help of the previous year's levels. In this ex post forecasting, the exogenous variables' realized values

Table 1. A summary of the ARIMA models of macroeconomic variables

Variable	Model and estimation period		
1. GDP (at factor cost), volume	Model 1: ARIMA $(0,1,3)\times(2,1,0)_4$ Period : 1949 I - 1973 IV		
$(1 + \underset{(0.10)}{0.55}B^4 + \underset{(0.09)}{0.53}B^8)\nabla\nabla_4 \ln z_t = (1 - \underset{(0.10)}{0.40}B - \underset{(0.11)}{0.09}B^2 + \underset{(0.10)}{0.32}B^3)\hat{a}_t$		0.00088	0.41
2. Private consumption expenditure, volume	Model 2: ARIMA $(0,0,3)\times(0,1,2)_4$ Period : 1958 I - 1973 IV		
$\nabla_4 \ln z_t = \underset{(0.002)}{0.047} + (1 + \underset{(0.14)}{0.24}B + \underset{(0.13)}{0.48}B^2 + \underset{(0.14)}{0.27}B^3)(1 - \underset{(0.13)}{0.37}B^4 - \underset{(0.12)}{0.51}B^8)\hat{a}_t$		0.00074	0.36
3. Cost-of-living index	Model 3: ARIMA (1,1,0) Period : 1952 I - 1973 IV		
$(1 - \underset{(0.09)}{0.59}B)\nabla \ln z_t = \underset{(0.003)}{0.013} + \hat{a}_t$		0.00012	0.33
4. Private investment, volume	Model 4: ARIMA $(0,1,1)\times(4,1,0)_4$ Period : 1958 I - 1973 IV		
$(1 + \underset{(0.12)}{0.39}B^4 + \underset{(0.10)}{0.69}B^8 + \underset{(0.11)}{0.51}B^{12} + \underset{(0.12)}{0.27}B^{16})\nabla\nabla_4 \ln z_t = (1 - \underset{(0.11)}{0.57}B)\hat{a}_t$		0.00460	0.51
5. Salary and wage earner's labour input	Model 5: ARIMA $(0,1,1)\times(0,1,2)_4$ Period : 1954 I - 1973 IV		
$\nabla\nabla_4 \ln z_t = (1 - \underset{(0.11)}{0.43}B)(1 - \underset{(0.12)}{0.57}B^4 - \underset{(0.11)}{0.31}B^8)\hat{a}_t$		0.00026	0.46
6. Unemployment rate	Model 6: ARIMA $(2,0,0)\times(3,1,0)_4$ Period : 1958 I - 1973 IV		
$(1 - \underset{(0.11)}{1.17}B + \underset{(0.11)}{0.31}B^2)(1 + \underset{(0.10)}{0.53}B^4 + \underset{(0.10)}{0.49}B^8 + \underset{(0.10)}{0.47}B^{12})\nabla_4 z_t = \hat{a}_t$		0.106	0.84
7. Index of salary and wage earnings	Model 7: ARIMA $(0,1,0)\times(0,1,1)_4$ Period : 1954 I - 1973 IV		
$\nabla\nabla_4 \ln z_t = (1 - \underset{(0.02)}{0.91}B^4)\hat{a}_t$		0.00032	0.37
8. Price index of commodity exports	Model 8: ARIMA (3,1,0) Period : 1950 I - 1973 IV		
$(1 - \underset{(0.10)}{0.43}B - \underset{(0.11)}{0.18}B^2 + \underset{(0.10)}{0.25}B^3)\nabla \ln z_t = \underset{(0.008)}{0.014} + \hat{a}_t$		0.00278	0.22
9. Multilateral commodity exports, volume	Model 9: ARIMA $(1,0,0)\times(3,1,0)_4$ Period : 1948 I - 1973 IV		
$(1 - \underset{(0.08)}{0.61}B)(1 + \underset{(0.07)}{0.56}B^4 + \underset{(0.07)}{0.55}B^8 + \underset{(0.08)}{0.23}B^{12})\nabla_4 \ln z_t = \underset{(0.009)}{0.072} + \hat{a}_t$		0.00669	0.60
10. Commodity imports, volume	Model 10: ARIMA $(0,1,1)\times(0,1,1)_4$ Period : 1949 I - 1973 IV		
$\nabla\nabla_4 \ln z_t = (1 - \underset{(0.10)}{0.24}B)(1 - \underset{(0.07)}{0.72}B^4)\hat{a}_t$		0.0143	0.30

were used in ETLA's model, which of course facilitates forecasting and is of help in finding the turning-points when examining short-term fluctuations. This procedure can be justified by the fact that, in the case of ETLA's model, interest centred on its structural characteristics, and the best picture possible of these can be obtained by means of ex post forecasts.

5.2. Measuring the forecast errors

Thus, the present study was concerned with an ex post forecasting situation in which the forecasts yielded by various models could be compared with the realized values in order to compute forecast errors and to judge the predictive performance of the various models.

First, as a measure of the forecast error was used the size of the absolute values of the residuals of the percentage year-on-year changes, whereby the residual is the percentage difference between the true value and the forecast computed from the model. Mean absolute error MAE computed for each variable from these residuals,

$$\text{(5.1)} \qquad MAE = \frac{1}{n} \sum_{t=1}^{n} |x_t - \hat{x}_t| ,$$

where x_t = the realized value (%) of x in year t and
$\hat{x}_t$ = the forecast value (%) of x in year t.

These MAE's were computed both the chain forecasts and for the one-year forecasts (Table 2).

Secondly, relative forecasts errors FE_{REL}, expressed as percentages of the true values in the level units, were computed for the chain forecasts of the variables for the third years, or the year 1976,

(5.2) $$FE_{REL} = 100 \cdot \left(\frac{\hat{X}_t - X_t}{X_t}\right),$$

where $\hat{X}_t$ = the forecast level value of X in year t and

X_t = the realized level value of X in year t.

The FE_{REL}'s are presented in Table 2.

The square root of the mean-square-error $\sqrt{MSE}$, or the root-mean-square (RMS) error, computed for each variable was chosen as a third measure of the absolute forecast error,

(5.3) $$RMS = \sqrt{MSE} = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (\hat{x}_t - x_t)^2},$$

where $\hat{x}_t$ = the forecast value (%) of x in year t and

x_t = the realized value (%) of x in year t.

The predictive performance of a model, in comparison with another alternative model, can be measured by the ratio of the RMS errors RM,

(5.4) $$RM = \frac{RMS_A}{RMS_E},$$

where the subscripts A and E refer to the forecasting methods. If $0 < RM < 1$, then the forecasts A are better than the forecasts E; and if $RM > 1$, then the forecasts E are better than the forecasts A.

In this paper, RMS_A refers to the RMS error of the ARIMA model, RMS_E to the RMS error of ETLA's model and RMS_N to the RMS error of the naive model. The RM ratios for the variables examined are given in Table 4.

In ETLA's model the GDP variable is GDP at market prices, whereas the corresponding variable in the ARIMA model is GDP at factor cost; and, to the prices of private consumption goods, involved in ETLA's model, there corresponds, in the case of ARIMA models, the cost-of-living index. The differences in level between these pairs of variables have been taken into account in computing the RM ratios.

5.3. Comparison of forecasts

In the case of the average change percentages for the years 1974-76 (Table 2), the naive model yielded the most accurate forecast only of the course of the earnings level. In 1973, a result of international inflation, the level of salary and wage earnings began to rise far more rapidly than it had risen in previous years, and this rapid rise continued throughout the forecasting period. The mechanical naive model succeeded in forecasting this continued rise, whereas the forecasts produced by ETLA's model and the ARIMA model tended to follow the trend of the earnings level variable. In the case of one-year forecasts, too, the ARIMA model was quite well able to find a new path characterized by a growth rate faster than before.

Of the ten three-year forecasts obtained by chaining - i.e., the forecasts for which the mean of the forecast errors of the first, second and third years was computed (Table 2 A) - those yielded by the naive

Table 2. Mean absolut errors* of the forecasted annual percentage changes computed from the naive models, ETLA's model and from the ARIMA models for the years 1974-76 (percentage points)**

	Naive model	ETLA's model	ARIMA model	Naive model	ETLA's model	ARIMA model
	A. three-year chain forecasts (mean of forecasts for the first, second and third years)			B. one-year forecasts (mean for three years)		
1. GDP	4.09	4.04	3.55	1.95	4.94	1.69
2. Private consumption expenditure	4.32	1.68	2.88	2.37	3.38	1.30
3. Cost-of-living index (Consumption prices)	4.66	9.12	2.95	3.04	8.59	2.33
4. Private investment	10.41	2.81	11.47	8.84	4.45	7.40
5. Labour input	3.45	1.12	3.01	2.27	1.33	1.94
6. Unemployment rate	1.00	0.50	0.70	0.80	0.40	0.50
7. Earnings level	3.63	9.40	8.27	4.42	9.19	4.56
8. Export prices	14.77	4.85	13.98	23.39	4.40	8.65
9. Multilateral commodity exports	19.39	10.86	19.91	27.03	10.25	17.24
10. Commodity imports	12.25	13.47	6.96	5.33	14.36	2.73

* Mean absolut error is the absolute value of the percentage difference between the actual change and the change forecast by the model.

** To facilitate the comparison between ETLA's model and the ARIMA models, the forecast error which is the smaller of the two has been underlined in the case of each variable. In cases where the forecast error of the naive model is the smallest, it has been underlined by a dashed line.

model were the most inaccurate in five cases, those yielded by ETLA's model in three cases and those yielded by the ARIMA model in the remaining two cases. The naive model was consequently clearly inferior to the more sophisticated methods. A comparison of ETLA's model and the ARIMA models reveals that in the former the forecast error was smaller in six variables and, in the latter, in four variables.

In the one-year ARIMA model forecasts - i.e. forecasts for which the mean one-year forecast error was computed for the years 1974-76 (Table 2 B) - the mean forecast error was smaller in each of the ten variables than was the forecast error in the corresponding three-year forecasts. By contrast, the one-year forecasts yielded by ETLA's model were more inaccurate than the three-year forecasts in the case of as many as five variables. The ARIMA model compared unfavourable with the naive model only in the case of the earnings level variable, whereas ETLA's model was inferior to the naive model in the case of five variables. The comparison between ETLA's model and the ARIMA models ended in a draw: each was superior to the other in five cases and inferior to the other in another five cases.

When the predictive performance of the various models were examined in terms of relative forecast errors, the results were in the same direction (Kinnunen 1978, p. 76). When the forecasting horizon becames longer, the forecasts yielded by ETLA's model were more accurate than those produced by the ARIMA and naive models. In seven cases the relative forecast errors in the forecasts obtained from ETLA's model by chaining for the third year, i.e., the year 1976, were smaller than the relative errors in the corresponding ARIMA model forecasts (Table 3).

Table 3. The relative forecast errors in the forecasts for the third year (1976) obtained by chaining from the naive models, ETLA's model and the ARIMA models(%)*

Variable	Naive model	ETLA's model	ARIMA model
	%	%	%
1. a. GDP (at factor cost)	12.6	-	9.4
b. GDP (at market prices)	-	8.2	-
2. Private consumption expenditure	13.3	-0.6	1.7
3. a. Cost-of-living index	-11.5	-	-7.4
b. Consumption prices	-	-21.9	-
4. Private investment	37.3	1.0	41.3
5. Labour input	8.0	3.3	5.2
6. Unemployment rate	-57	-40	-43
7. Earnings level	-8.0	-21.9	-19.4
8. Export price index	-12.0	-1.0	-17.6
9. Multilateral commodity exports	56.5	-7.0	43.8
10. Commodity imports	41.0	-34.4	13.5

* The relative forecast error, computed from the level values of the variables, per cent of the true value.
To facilitate the comparison between ETLA's model and the ARIMA models, the relative forecast error which is the smaller of the two has been underlined in the case of each variable. In cases where the relative forecast error of the naive model is the smallest it has been underlined by a dashed line.

Table 4. The RM ratios for the annual forecasts for the years 1974-76 yielded by the naive models, ETLA's model and the ARIMA models

Variable	$\frac{RMS_A}{RMS_E}$	$\frac{RMS_A}{RMS_N}$	$\frac{RMS_E}{RMS_N}$	$\frac{RMS_A}{RMS_E}$	$\frac{RMS_A}{RMS_N}$	$\frac{RMS_E}{RMS_N}$
	A. Three-year forecasts			B. One-year forecasts		
1. GDP	0.73	0.71	0.97	0.47	1.01	2.15
2. Private consumption expenditure	1.49	0.30	0.20	0.41	0.48	1.18
3. Cost-of-living index	0.36	0.66	1.83	0.26	0.65	2.50
4. Private investment	7.46	1.16	0.15	1.78	0.67	0.37
5. Labour input	1.14	0.67	0.59	1.14	0.76	0.67
6. Unemployment rate	0.91	0.71	0.79	1.00	0.62	0.62
7. Earnings level	0.91	2.16	2.37	0.49	0.85	1.74
8. Export price index	4.41	0.87	0.20	2.29	0.44	0.19
9. Multilateral commodity exports	5.11	0.86	0.17	1.63	0.75	0.46
10. Commodity imports	0.30	0.31	1.05	0.18	0.47	2.55

The influence of the lengthening of the forecasting horizon on the forecast errors appears very clearly from the RM ratios of the ARIMA models and ETLA's model (Table 4). In the case of eight variables, this RM ratio was lower for the one-year forecasts than for the three-year forecasts, which is to say that the relative forecasting performance of ETLA's model in comparison with the ARIMA models improved when the length of the forecasting horizon grew from one to three years. The ARIMA-model forecasts of only one single variable (unemployment rate) improved, in relative terms, with the lengthening of the forecasting horizon.

In terms of the absolute RMS errors, ETLA's model produced more accurate three-year forecasts for five variables and the ARIMA models for the remaining five variables. Of the one-year forecasts, those obtained from the ARIMA models were more accurate in five cases and those yielded by ETLA's model in four cases, whereas in the forecasts of the unemployment rate the RMS errors were equal. In the case of ETLA's model, the RMS errors were larger in the one-year forecasts than in the three-year forecasts of three variables whereas the RMS errors in one-year ARIMA models forecasts were smaller than the RMS errors in the corresponding three-year ARIMA models forecasts in all ten cases (Kinnunen 1978, p. 74).

The mechanical naive model proved to be definitely inferior to the ARIMA model. Likewise, it proved inferior to ETLA's model in the case of three-year forecasts, whereas the one-year forecasts were in five cases less accurate and in five cases more accurate than those produced by ETLA's model. The RMS error in each of the one-year naive-model forecasts was smaller than the RMS error of the corresponding three-year forecasts

(Kinnunen 1978, p. 74), so that the naive models, like the ARIMA models, were invariably able to yield increasingly accurate forecasts when the length of the forecasting horizon was reduced. Thus their relative predictive performance, in comparison with ETLA's model, also improved as the length of the forecasting horizon decreased.

The level forecasts obtained from the ARIMA models and ETLA's model are represented in Figures 1 - 10 in the Appendix. Particularly the features characteristic of the ARIMA-model forecasts are revealed quite clearly by graphical analysis. The ARIMA models were especially poorly able to forecast the course of private investment (Figure 4), export prices (Figure 8) and multilateral commodity exports (Figure 9). The international depression and fast inflation caused by the 1973 oil crisis affected these three variables particularly clearly, and their course was therefore markedly different in 1974-1976 than it had been in previous years. The autoprojective ARIMA method did not succeed in forecasting the new directions of these variables, especially not in the chain-forecasts, but continued to produce trend-like predictions.

On the other hand, ETLA's model succeeded particularly poorly in forecasting commodity imports (Figure 10), prices of consumer goods (Figure 36) and the level of earnings (Figure 7). One of the explanatory variables involved in its equation for commodity imports in import prices (the weight for the same year being two-thirds and the weight for the preceding year being one-third), whose coefficient (elasticity) is -0.59. Because of the sharp rise in oil prices, import prices rose by as much as 44 per cent in 1974, and thus the contribution of import prices to this equation was big and negative in 1974 and 1975. However, the volume of imports did not respond to this exceptional rise in the price of crude oil but decreased only in 1975, as a result of the fact that world trade shrank due to the international depression. Another variable whose contribution to the commodity imports equation was big and negative was the change in stocks, the course of which was exceptional in the depression years 1974-76. Moreover, this variable is inaccurately measured, because it includes, in national accounting, also the statistical discrepancy.

The forecasts obtained from ETLA's model that have been presented in this paper, just as the ARIMA-model forecasts, were computed mechanically, without making use of any information external to the model. In a normal forecasting situation an effort will of course be made to utilize all the available information. The impact of temporary factors (such as the sharp rise in import prices caused by the rise in the price of oil in 1974 and its influence on the volume of imports in an econometric model) may be taken into account through, e.g., manipulating the error terms of the model or by exogenizing the variable concerned (Vartia 1975).

6. Summary

Ten central macro-variables of the Finnish economy were made the subject of this study, and ARIMA models for them were constructed (Table 1). From these models, three-year chain forecasts and one-year forecasts were computed for the years 1974-1976. The forecasts thus obtained were compared with the corresponding forecasts yielded by ETLA's econometric model (Vartia 1974), in order to find out which of the two methods produced more accurate forecasts (Tables 2-4). The realized values of the exogenous variables were used in ETLA's model.

Of the three-year forecasts computed by chaining, those yielded by the ARIMA models were superior in average accuracy, in terms of the RMS error, as often as those obtained from ETLA's model. When the length of the forecasting horizon was increased, so that only the relative forecast errors of the third year (1976) were examined, the forecasts produced by ETLA's model were definitely more accurate than the corresponding ARIMA model forecasts. Variables very difficult to forecast for the years 1974-76 from ETLA's model were commodity imports, prices of consumer goods and the level of earnings, and when the forecasting horizon was lengthened, these forecasts became also inferior to the corresponding ARIMA model forecasts.

In terms of the RMS error, the one-year ARIMA-model forecasts were superior in accuracy to the corresponding forecasts obtained from ETLA's model slightly more often than the latter were superior to the former. The one-year ARIMA-model forecast of each variable was more accurate than the corresponding three-year forecast. On the other hand, ETLA's model was not always at its best in one-year forecasts, for the three-year forecasts of several variables obtained from it were more accurate than the corresponding one-year forecasts.

The results lend support to the view that ARIMA models are useful as tools of short-term forecasting and that the accuracy of the forecasts produced by them clearly deteriorates with an increase in the length of the forecasting horizon. Being autoprojective in nature, the ARIMA models tend to forecast that the course of the processes concerned will continue in the same direction as earlier and thus they cannot easily find the turning points of the trade cycle, essential for the study of economic developments. These models are thus helpful particularly in the forecasting of the future course of trend-like time series. An ARIMA model is generally will able to describe the seasonal variation displayed by the process concerned.

In the present study, the forecasting period consisted of the years 1974-76, during which the course of economic developments slowed down or actually turned downwards in comparison with the early 1970s. Such developments are not particularly favourable for the application of autoprojective forecasting techniques. Nevertheless, the ARIMA models compared relatively favourably with ETLA's econometric model particularly as far as short-term one-year forecasts were concerned. ETLA's model is a simultaneous one, and when structural changes occurred in the relevant time series in 1974, this strongly affected the various variables and their interrelationships.

Contrary to the case with econometric models, simulation experiments cannot be made with ARIMA models, nor can they be used to explore the effects of various economic policy measures. It can be concluded that the use of the ARIMA models is confined mainly to short-term forecasting, and the forecasts obtained from them can be used as points of comparison for forecasts produced by econometric models.

REFERENCES

BOX, G.E.P. and JENKINS, G.M. (1973): *Some Comments on a Paper by Chatfield and Prothero and on a Review by Kendall*. Journal of the Royal Statistical Society A 136, 337-352.

BOX, C.E.P. and JENKINS, G.M. (1976): *Time Series Analysis, Forecasting and Control*. Revised Edition. San Francisco: Holden Day.

CHATFIELD, C. and PROTHERO, D.L. (1973): *Box-Jenkins Seasonal Forecasting Problems in Case-study (with Discussion)*. Journal of the Royal Statistical Society A 136, 295-336.

CHRIST, C.F. (1975): *Judging the Performance of Econometric Models of the U.S. Economy*. International Economic Review 16, 54-74.

KANNIAINEN, V. and SUVANTO, A. (1976): *Ekonometrinen menetelmä uuden aikasarja-analyysin valossa*, (The Econometric Method in the Light of the New Time Series Analysis), in Finnish. University of Helsinki, Department of Economics, Discussion Papers No. 52.

KENDALL, M.G. (1973): *Time Series*. London: Charles Griffin & Co.

KINNUNEN, J. (1978). *Eräiden kokonaistaloudellisten muuttujien ARIMA-mallit: Mallien rakentaminen ja ennusteiden vertaaminen ekonometrisen mallin ennusteisiin*, (The ARIMA Models of Some Macroeconomic Variables: Building of the Models and Comparison of the Forecasts with a Econometric Model Forecasts), in Finnish. University of Helsinki, a Master's thesis.

LESKINEN, E. (1977a): *Aika-alueen keinoista mallien rakentamisessa: Auto- ja ristikorrelaatioanalyysista*, (On Manners of the Time Area in the Building of the Models: On the Autocorrelation and the Crosscorrelation Analysis), in Finnish. Edited by E. Blåfield, E. Leskinen and T. Teräsvirta, 78-231, Helsinki.

LESKINEN, E. (1977b): *Aikasarjojen ennustaminen aika-alueessa: ARIMA- ja siirtofunktio-kohinamallit*, (Forecasting of the Time Series in the Time Area: The ARIMA Models and the Transfer Function Noise Models), in Finnish. Edited by E. Blåfield, E. Leskinen and T. Teräsvirta, 200-231, Helsinki.

LESKINEN, E. (1973): *Stokastisten mallien rakentamiseen ja malleilla ennustamiseen liittyviä toimintaohjeita ja tarvittavien aikasarja-analyyttisten tietokoneohjelmien esittely*, (Recommendations for Construction of and Forecasting with Stocastic Models and a Presentation of Relevant Time Series Programs), in Finnish. University of Jyväskylä, Department of Statistics, Publications 1/1973.

LESKINEN, E. and TERÄSVIRTA, T. (1976): *Forecasting the Consumption of Alcoholic Beverages in Finland: A Box-Jenkins Approach*. European Economic Review 8, 349-369.

NAYLOR, T.H., SEAKS, T.G. and WICHERN, D.W. (1972): *Box-Jenkins: An Alternative to Econometric Models*. International Statistical Review 40, 123-137.

NELSON, H.L. and GRANGER, C.W.J. (1979): *Experience with Using the Box-Cox Transformation when Forecasting Economic Time Series*. Journal of Econometrics 10, 57-69.

NYSTRÖM, A. (1977): *Aikasarjojen ennustaminen aika-alueessa: Autoprojektiivisia menetelmiä*, (Forecasting of the Time Series in the Time Area: Autoprojective Methods), in Finnish. The Finnish Statistical Society, Publications 4: Aikasarja-analyysin menetelmiä, (Methods of the Time Series Analysis). Edited by E. Blåfield, E. Leskinen and T. Teräsvirta, 186-199, Helsinki.

PROTHERO, D.L. and WALLIS, K.F. (1976): *Modelling Macroeconomic Time Series*. Journal of the Royal Statistical Society A 139, 468-485.

VARTIA, P.L.I. (1974): *An Econometric Model for Analyzing and Forecasting Short-Term Fluctuations in the Finnish Economy*. The Research Institute of the Finnish Economy, Serie A 2, Helsinki.

VARTIA, P. (1975): *Malli ei ole mekaano*, (A Model is not a Mechanical Machine), in Finnish. The Finnish Economic Journal 1975:1, 66-70.

WILLMAN, A. (1976): *Ekonometrinen tutkimus finanssipolitiikan vaikutuksista*, (A Econometric Research of Influences of the Fiscal Policy), in Finnish. Bank of Finland, D:38.

APPENDIX

Figure 1 A: Volume of gross domestic product (at factor cost), at 1964 prices

A: ARIMA model forecasts

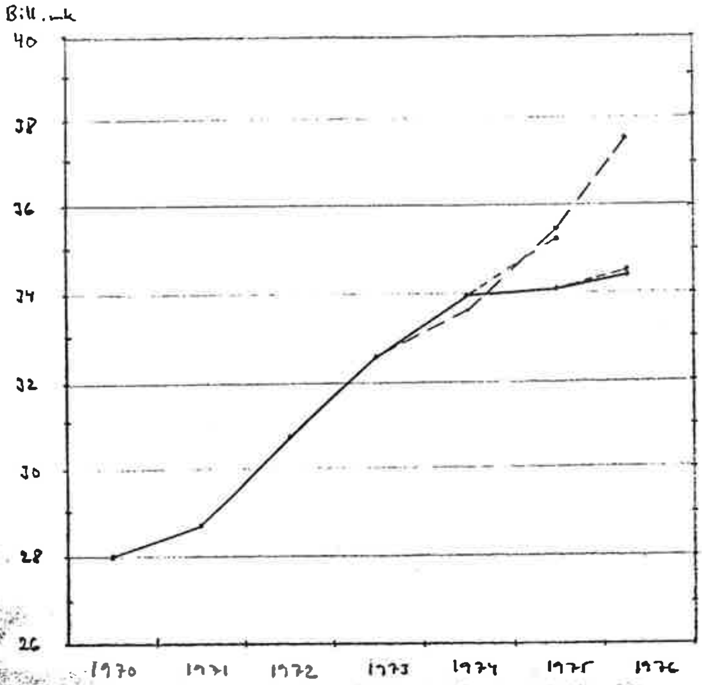


Figure 1 B: Volume of gross domestic product (at market prices), at 1964 prices

B: ETLA's model forecasts

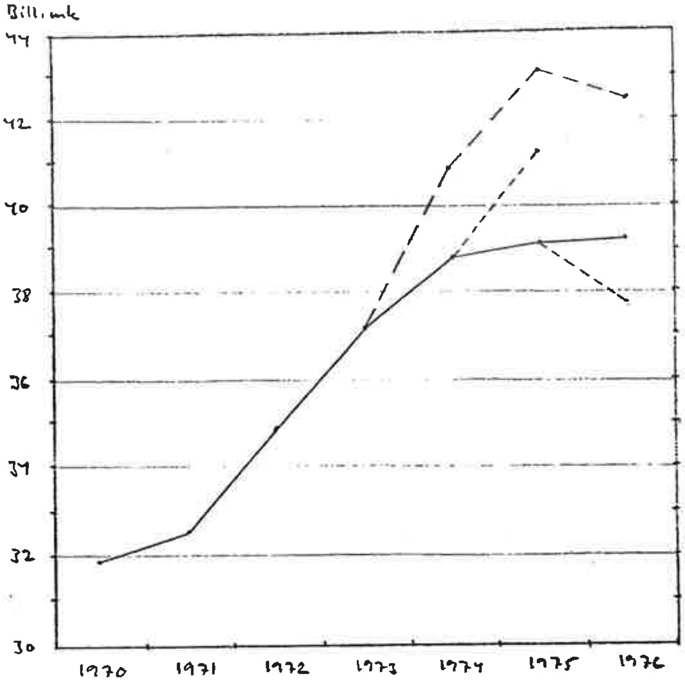
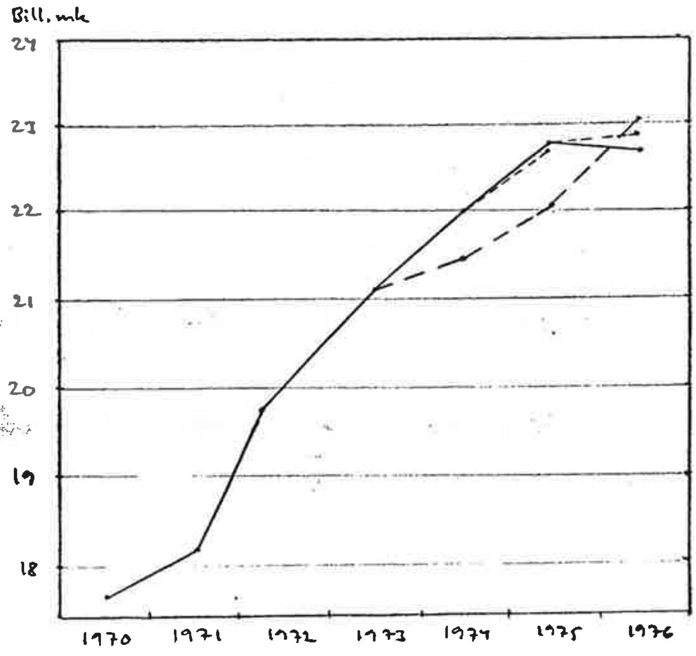
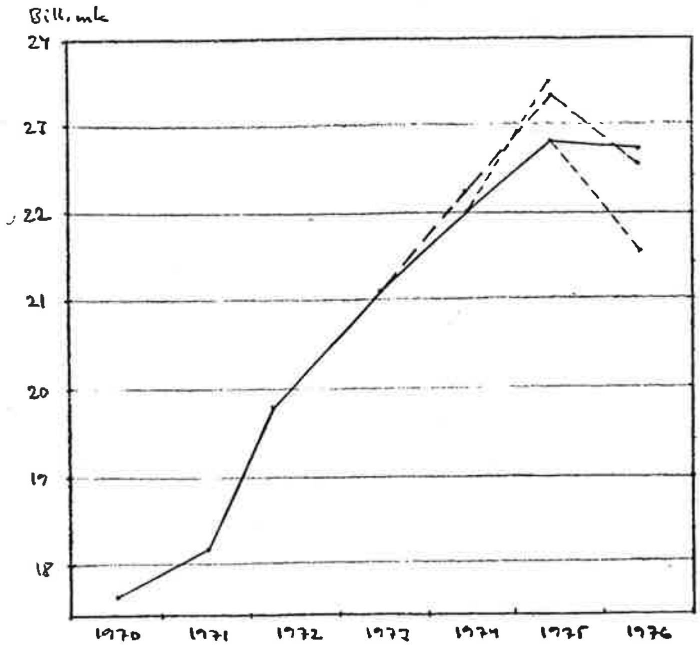


Figure 2: Volume of private consumption expenditure, at 1964 prices

A: ARIMA model forecasts



B: ETLA's model forecasts



——— = the realized values

Figure 3 A: Cost-of-living index, (1951:10 = 100)

A: ARIMA model forecasts

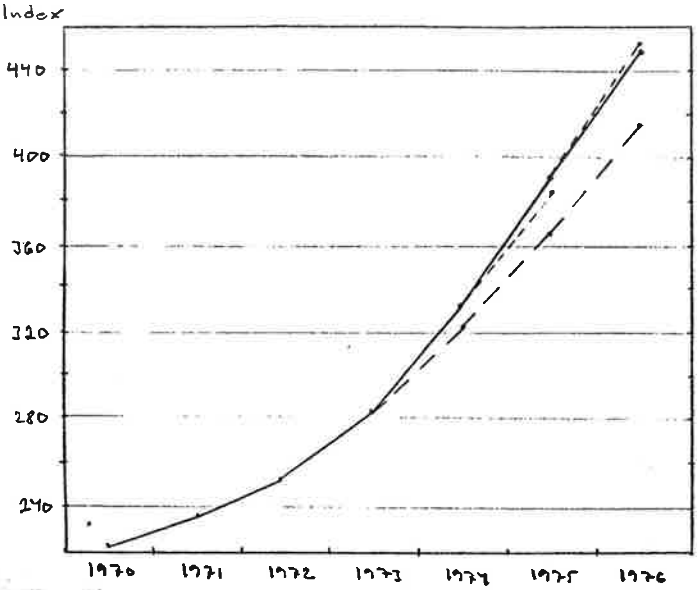


Figure 3 B: Price index of private consumption expenditure, (1964=100)

B: ETLA's model forecasts

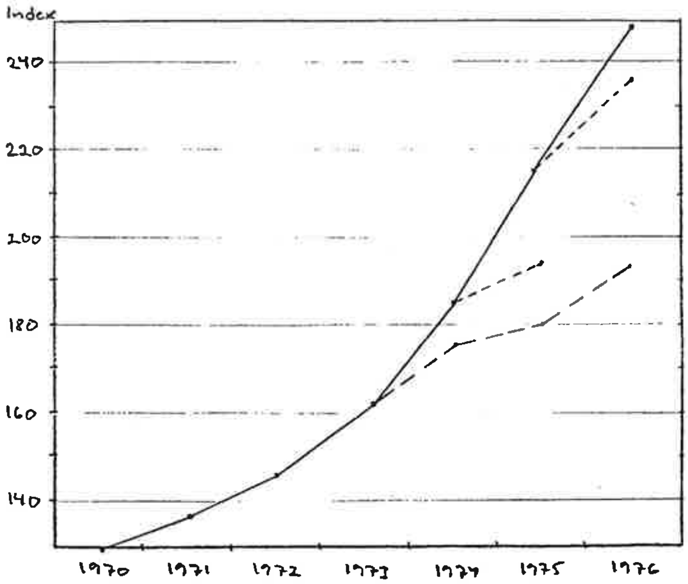
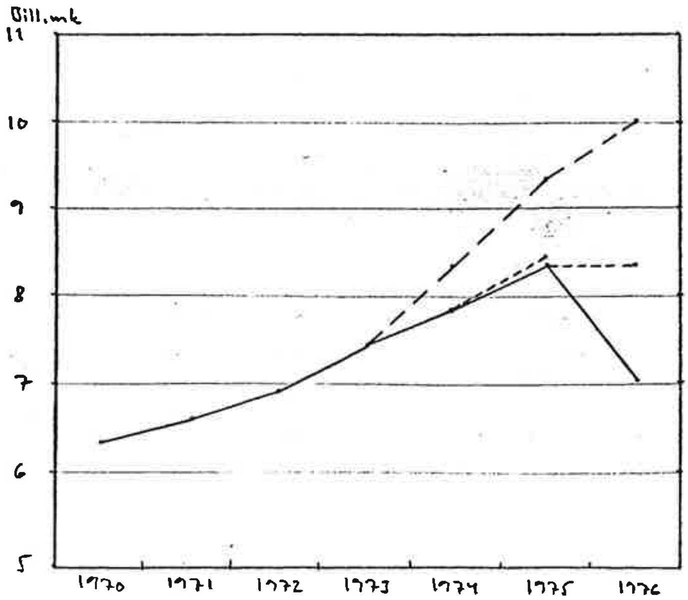
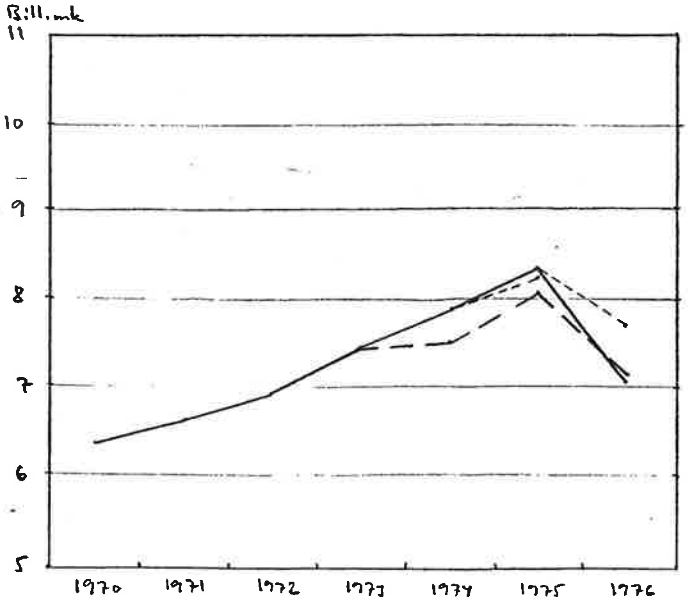


Figure 4: Volume of private investment

A: ARIMA model forecast



B: ETLA's model forecasts



——— = the realized values
– – – – = one year forecasts 1974,1975,1976
— — — = three years forecasts 1974-76

Figure 5: Salary and wage earner's labour input, (1964=100)

A: ARIMA model forecasts

B: ETLA's model forecasts

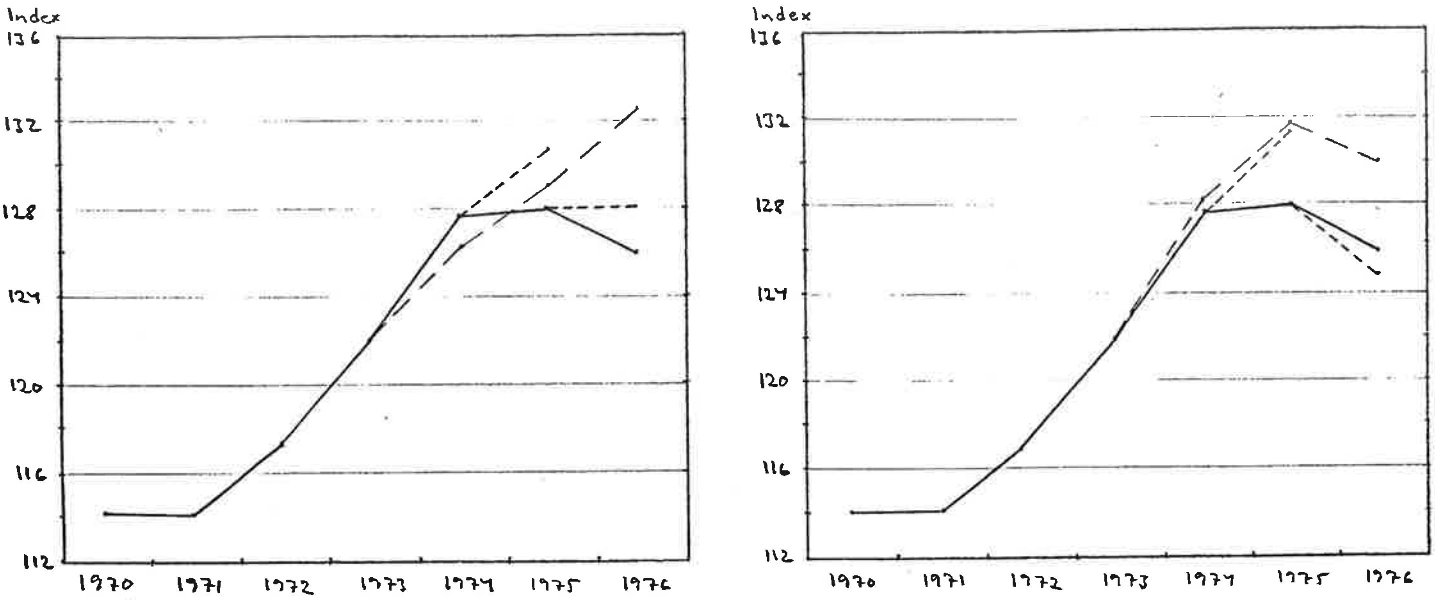
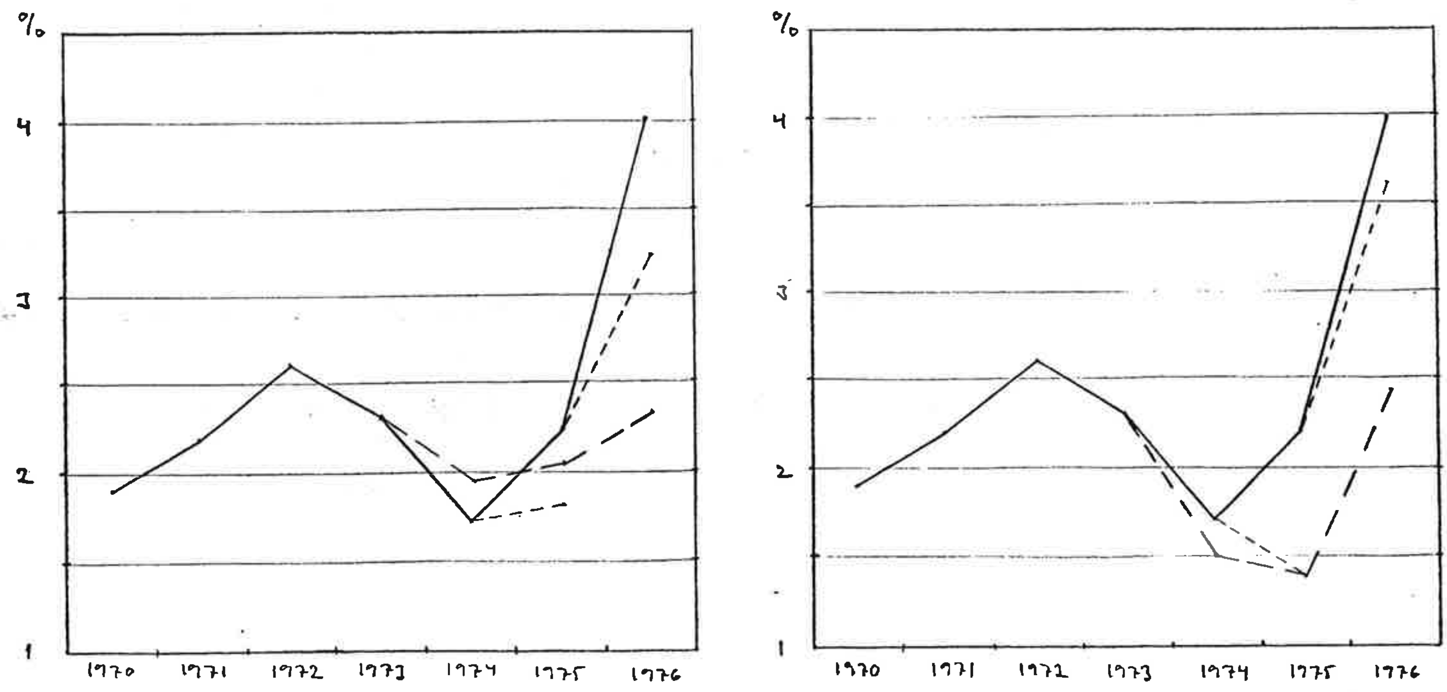


Figure 6: Unemployment rate

A: ARIMA model forecasts

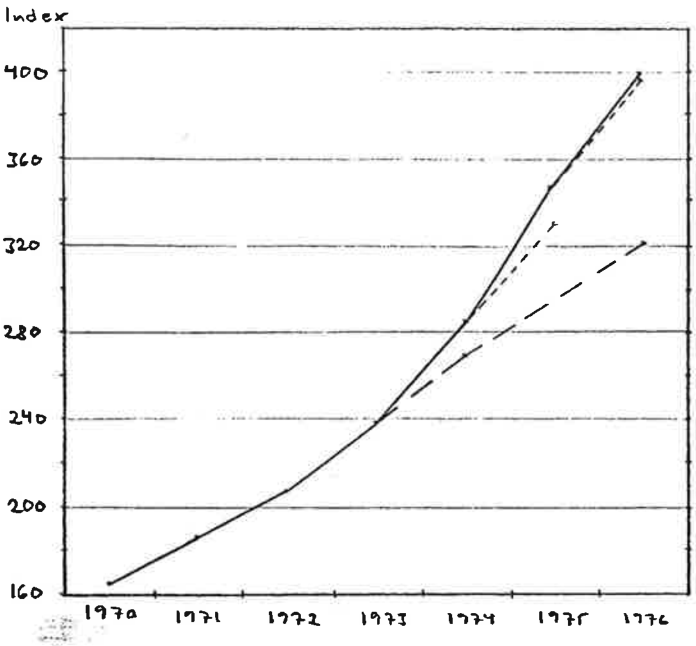
B: ETLA's model forecasts



——— = the realized values
- - - one year forecasts 1974 1975 1976

Figure 7: Index of salary and wage earnings, (1964=100)

A: ARIMA model forecasts



B: ETLA's model forecasts

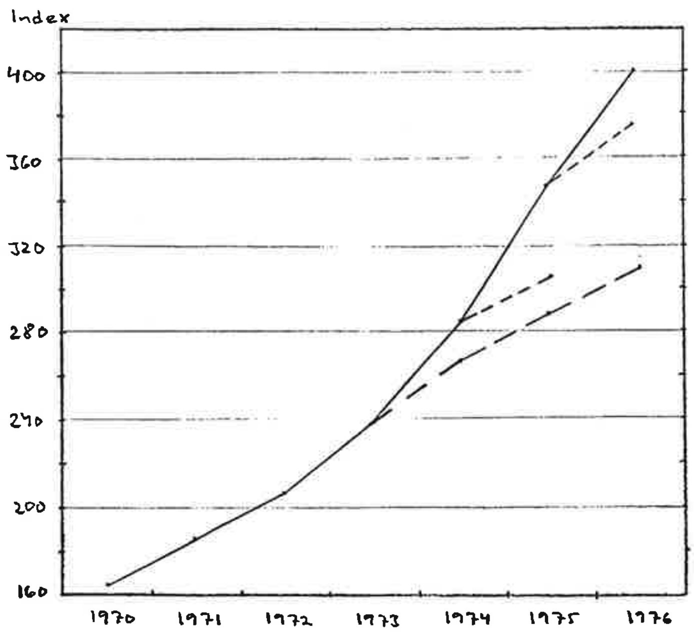
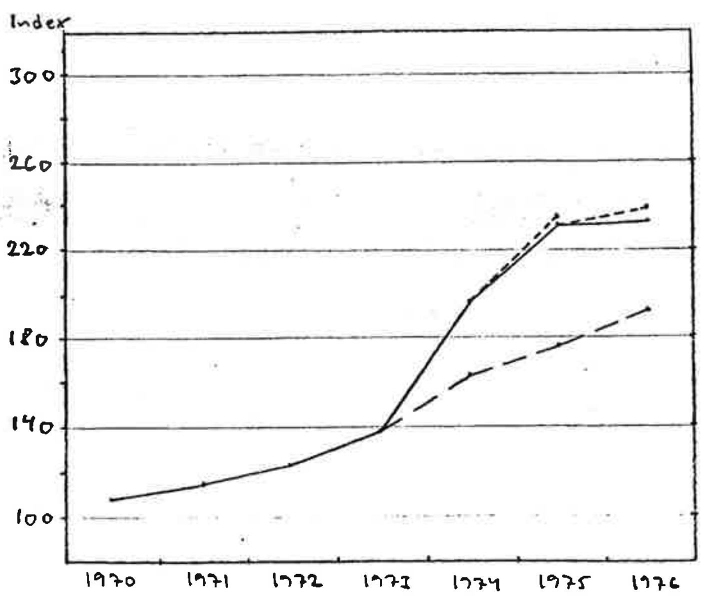
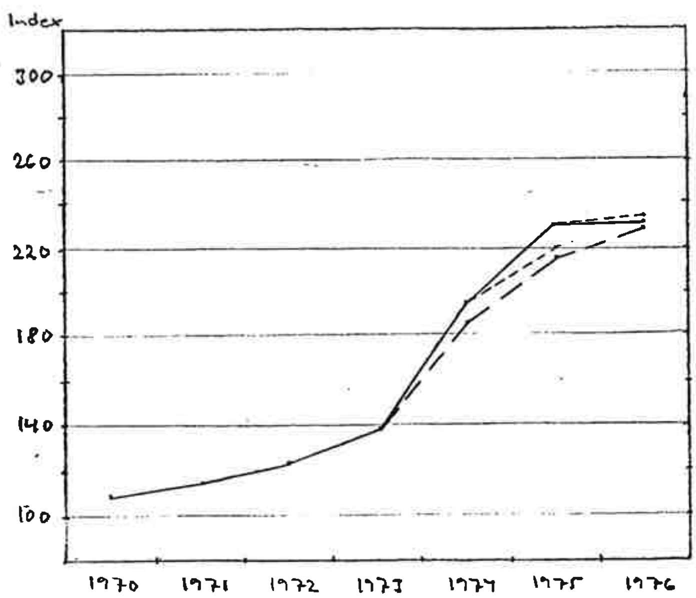


Figure 8: Price index of commodity exports, (1964=100)

A: ARIMA model forecasts



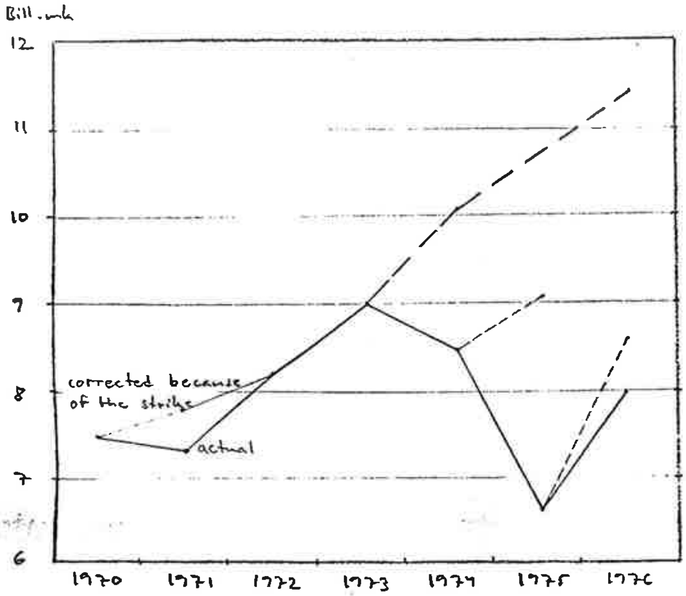
B: ETLA's model forecasts



———— = the realized values
- - - - = one year forecasts 1974,1975,1976
— — — = three years forecasts 1974-76

Figure 9: Volume of multilateral commodity exports, at 1964 prices

A: ARIMA model forecasts



B: ETLA's model forecasts

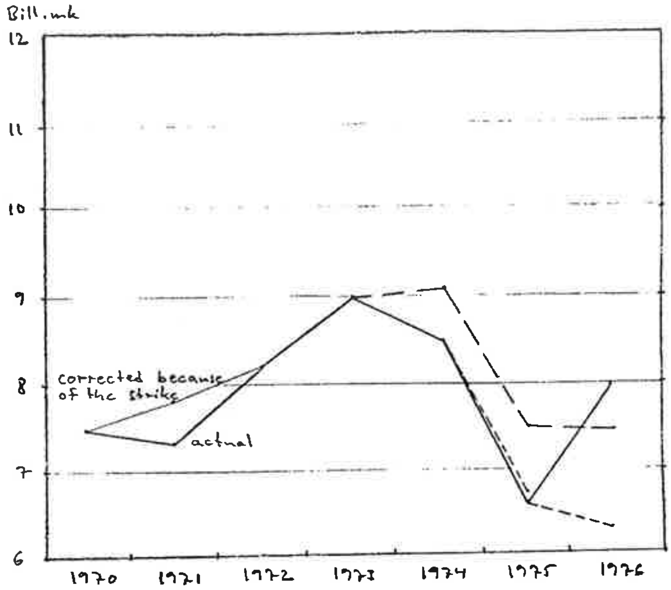
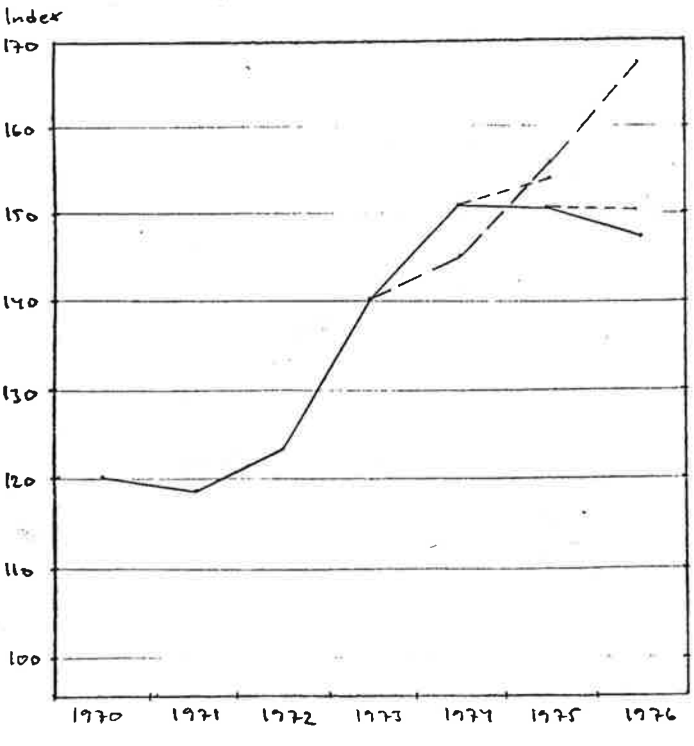
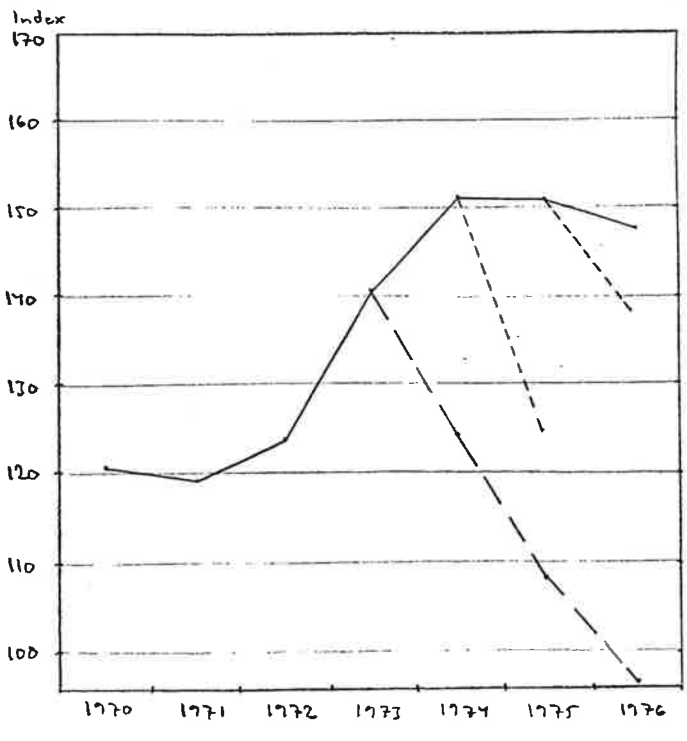


Figure 10: Volume of commodity imports, (1969=100)

A: ARIMA model forecasts



B: ETLA's model forecasts



——— = the realized values

one year forecasts 1974 1975 1976

ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)
The Research Institute of the Finnish Economy
Lönnrotinkatu 4 B, SF-00120 HELSINKI Puh./Tel. (90) 601 322

KESKUSTELUAIHEITA - DISCUSSION PAPERS ISSN 0781-6847

No 200 JUKKA LESKELÄ, Välitysvaluutat ja ulkomaankaupan laskutus. 10.03.1986. 22 s.

No 201 VESA KANNIAINEN - HANNU HERNESNIEMI, Asset Structure, Indebtedness, and the Rate of Return on Capital in a Sample of Finnish Manufacturing Firms in 1961 - 1983. 11.03.1986. 31 s.

No 202 ANTTI RIPATTI, Teollisuus- ja ulkomaankauppatilaston yhdisteen hyödyntäminen. 20.03.1986. 31 s.

No 203 SYNNÖVE VUORI, Returns to R & D in Finnish and Swedish Manufacturing Industries. 20.03.1986. 23 p.

No 204 VESA KANNIAINEN, On the Effects of Inflation: The Debtor-Creditor Hypothesis Reconsidered. 20.03.1986. 15 p.

No 205 PEKKA ILMAKUNNAS, Aggregation of Micro Forecasts. 01.04.1986. 17 p.

No 206 JUSSI RAUMOLIN, Recent Trends in the Development of the Forest Sector in Finland and Eastern Canada. 04.04.1986. 40 p.

No 207 VESA KANNIAINEN - JUHA VEHVILÄINEN, On Instability of a Keynesian Macro Model: Some Notes. 08.04.1986. 14 p.

No 208 PEKKA YLÄ-ANTTILA, Investment Structure, Productivity and Technical Change - Implications for Business Organizations and Management. 17.04.1986. 19 p.

No 209 JUHA AHTOLA, Consequences from Improper Use of Ordinary Least Squares Estimation with Time Series Data. 12.05.1986. 11 p.

No 210 TIMO AIRAKSINEN, Vertaileva analyysi pääomatulojen verotuksesta Suomessa ja Ruotsissa vuonna 1986. 29.05.1986. 36 s.

No 211 JUSSI RAUMOLIN, Kaivos- ja metallituotteiden maailmantalous. 18.06.1986. 40 s.

No 212 TARMO VALKONEN, Vakuutusyhtiöiden sijoitustoiminnan puitteet ja sijoitusten jakautuminen Suomessa vuosina 1962-1984. 19.06.1986. 68 s.

No 213 TIMO TERÄSVIRTA - GANG YI - GEORGE JUDGE, Model Selection, Smoothing and Parameter Estimation in Linear Models under Squared Error Loss. 17.07.1986. 21 p.

No 214 MARKKU RAHIALA - TIMO TERÄSVIRTA, Formation of Firms' Production Plans in Finnish Manufacturing Industries. 18.07.1986. 30 p.

No 215 SEIJA ILMAKUNNAS, The Monopoly Union Model with Endogenous Price Expectations. 15.08.1986. 15 p.

No 216 VESA KANNIAINEN - HANNU HERNESNIEMI, The Cost of Holding Inventories, and the Demand for Labor and Capital under Corporate Taxation: Another Look. 06.10.1986. 24 p.

No 217 TIMO AIRAKSINEN, Pääomaverotuksen teoriaa. 12.11.1986. 63 s.

No 218 VESA KANNIAINEN, Tax Allowances and the Optimal Investment Policy by Firms. 04.12.1986. 45 p.

No 219 JUSSI RAUMOLIN, The Role of Education in the Development of the Mining Sector in Finland. 04.12.1986. 83 p.

No 220 MARKKU RAHIALA - TIMO TERÄSVIRTA - VESA KANNIAINEN, Factors Affecting Firms' Employment Plans in Finnish Manufacturing Industries. 15.12.1986. 30 p.

No 221 TIMO TERÄSVIRTA, Incomplete Ellipsoidal Restrictions in Linear Models. 16.12.1986. 9 p.

No 222 OSMO FORSSELL, Panos-tuotos-laskelmat Suomen Neuvostoliiton-viennistä. 30.12.1986. 119 s.

No 223 OLLI-TAPIO MATTILA, Suomen Neuvostoliiton-kaupan kehitys, kuvioliite.

No 224 PEKKA ILMAKUNNAS, Survey Expectations vs. Rational Expectations in the Estimation of a Dynamic Model: Demand for Labor in Finnish Manufacturing. 30.12.1986. 22 p.

No 225 PEKKA SPOLANDER, Kapitalmarknader och ägarförhållanden i Finlands näringsliv. 31.12.1986. 42 s.

No 226 JUHA KINNUNEN, Comparison of the Arima-Model Forecasts of Some Finnish Macroeconomic Variables with Econometric Macromodel Forecasts. 31.12.1986. 33 p.

No 227 ERKKI KOSKELA, Personal Savings and Capital Income Taxation: A Differential Incidence Analysis. 12.01.1987. 16 p.

Elinkeinoelämän Tutkimuslaitoksen julkaisemat "Keskusteluaiheet" ovat raportteja alustavista tutkimustuloksista ja väliraportteja tekeillä olevista tutkimuksista. Tässä sarjassa julkaistuja monisteita on rajoitetusti saatavissa ETLAn kirjastosta tai ao. tutkijalta.

Papers in this series are reports on preliminary research results and on studies in progress; they can be obtained, on request, by the author's permission.

0033A/13.01.1987