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TAX ALLOWANCES AND THE OPTIMAL INVESTMENT POLICY BY FIRMS

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Abstract

The paper studies the interaction of tax allowances and the investment policy in the Scandinavian corporate tax system with non-predetermined rates of tax depreciation. The appearance of unclaimed tax allowances is explained in terms of dividend preferences and constraints on dividends. The Bergström-Södersten inefficiency proposition concerning the investment incentives is studied and qualified in the presence of a nonincome-based local tax. The model seriously questions the current vintage of investment models based on Tobin's q. It shows that even in an expansion phase, there may be no or a negative association between the current rate of investment and the marginal valuation of capital.

I Introduction

The interaction of tax policy and investment behavior has been intensively investigated ever since the work of Hall and Jorgenson (1967) was published (see Auearbach (1983) for a review). Starting with Samuelson (1964) an understanding has been gained of the conditions under which the allocative efficiency is not destroyed by the tax system (for seminal work cf. Stiglitz (1973), King (1974) and Sandmo (1974)). Focusing on the case of corporate income tax, it is quite clear that these conditions have been hard to meet in practice. As to the deduction of the cost of depreciation, different economies have experimented with a variety of approaches. It is the purpose of this paper to study the implications of what can be called "the Scandinavian approach".

One distinctive feature which allows comparison between different types of corporate tax systems is the question of whether the tax allowances are predetermined or not. Use the most widely adopted system found in the U.S. and in the U.K. as the reference approach. One of its key features is that the tax allowances used by firms are based on predetermined depreciation rates. In contrast to this model, the Scandinavian approach (to be found in Sweden and in Norway and also adapted by Finland) is based on the principle of applying a maximum allowable rate of depreciation to the undepreciated accounting value of a firm. The research problems of the current paper can be stated as follows. First, the aim is to characterize the optimal investment and tax allowance policy of a firm operating in the Scandinavian corporate tax system. Second, questions will be raised about the potential merits or dismerits of the Scandinavian model as compared to the more widely adopted method of predetermined tax depreciation rates.

The principle of a non-predetermined tax depreciation constrained by a ceiling rate provides the firms with the possibility to adopt an accelerated depreciation scheme.[1)] The latter aspect obviously has been motivated by the aim of guaranteeing high internal financing for firms and hence a high rate of investment in the economy. What have been the effects of these tax allowances? Before this question can be answered, one must address the very astonishing but undisputable phenomenon that for some reason firms systematically leave tax allowances unclaimed for prolonged periods of time. It is not the case that this would concern only a limited number of firms. Rather, it seems to be quite a dominating feature; only expansionary firms with obvious growth opportunities are excluded. At first thought one would predict that due to a positive discount rate all tax allowances would be claimed immediately against the taxable income. Because this is not the case, an explanation is called for in terms of rational behavior. The model of our paper focuses on the role played by the dividend preferences of the owners of the firms.

The existence of unclaimed tax allowances has important implications on the working of investment incentives. In the Swedish case, Bergström and Södersten (1984) have proposed that an additional investment project might not have any effect on total tax payments, i.e. that the investment incentives are inefficient and that the tax system is neutral at the margin[2)] (for a similar argument in the Finnish case cf. Ylä-Liedenpohja (1983)). This proposition will be evaluated in this paper when the solution of our model is at hand.

One can point out a number of problems in the current collection of empirical investment studies which suggest that the underlying theories are deficient. First, the neoclassical theory is inconsistent with the strong cash-flow or acceleration effects systematically detected. Second, the tax effects have been hard to estimate. Third, the increasingly popular "Tobin's q" approach has not been as successful as predicted by the neoclassical model. Our model provides some insight into all these issues. The most important one is our conclusion that the empirical investment studies based on the q-approach may be seriously misspecified for two different reasons. We suggest it is possible that even in the expansion phase the optimal rate of investment has negative association with the marginal valuation of capital along the optimal path. Second, the marginal valuation of capital is not actually a determinant of investment demand, but they are rather jointly determined.

II THE MODEL

The intertemporal efficiency condition dictates that a firm ought to undertake all investment projects for which the marginal rate of return is at least as great as the subjective rate of time preference of the owners of the firm. To investigate whether the Scandinavian corporate tax system interferes with this condition, a model of a firm is constructed to determined the optimal policy with respect to growth, tax allowances and dividens, both along the adjustment path and in equilibrium.[3)]

Thus, consider a firm assumed to be price-taker with an output-input bundle given as $x(t) = (y(t),-f_1(t),\dots,-f_{n-1}(t),-K(t))$ where $K(t) > 0$ stands for the fixed stock of capital at point t yielding capital services strictly proportional to K(t). Let the efficiency price of capital services be denoted by $p_n(K(t))$. Let $p_0,\ldots,p_{n-1}$ denote all other prices associated with x(t). Take them as exogenous, time-invariant and hence equal to one. The firm is assumed to have a technology represented by a non-empty, convex set $T \subset R^{n+1}$ such that x(t) is said to be technologically feasible if $x(t) \subset T$. Given this constraint and K(t), the components in x(t) can be adjusted so as to generate maximum earnings for each $t \geq t_0$. Then by appealing to the duality theory, one can write the gross earnings as $\pi(K(t))$, which is assumed to be a strictly concave, strictly increasing and twice differentiable function of K(t), i.e. $\pi_K > 0$, $\pi_{KK} < 0$.

Investment in capital has to be financed. One of the strongest regularities in investment figures across different economies is the dominance of undistributed profits as a source of finance. Similarly, in contrast to the predictions of the neoclassical theory of investment, current investment outlays continue to be very closely associated with current cash flows of firms. This holds even for the case of the U.S. For more details, see Myears (1984), who discusses the strong tendency of managers to resist outside financing in terms of the so-called pecking order theory.
These observations motive our choice of abstracting from new stock issues and the credit markets. Introduction of credit markets would modify the short-run determination of investment, but it would not change our conclusions of the interaction of the dividend preferences and the tax allowances. Appelbaum and Harris (1978) is an example of earlier work where the rate of investment is entirely controlled by the internal finance. Moreover, while Steigum (1983) allows for debt issues, retained earnings play an important role in the adjustment of capital in his model.

For investment, the dividend policy matters in the current framework, i.e. the firm has to decide upon the optimal distribution of current after-tax earnings between retained earnings to be used to acquire new capital goods (I) and dividends (D) to be distributed to the owners. Thus

$$(2.1) \qquad \pi(K(t)) - T(t) = D(t) + I(t) \qquad\qquad t \geq t_0$$

where T(t) is used to denote the corporate tax liability. Suppose that the firm is allowed to decide upon its current rate of tax depreciation, say δ^*, within the following limits

$$(2.2) \qquad 0 \leq \delta^*(t) \leq \delta^{max} = \bar{\delta} .$$

If now B(t) stands for the accounting (book) value of the firm and $0 \leq \theta \leq 1$ denotes the corporate (state) income tax rate, the tax liability reads as

$$(2.3) \qquad T(t) = \theta [\pi(K(t)) - \delta^*(t)B(t)] +$$

$$\max\{\theta_0 [\pi(K(t)) - \delta^*(t)B(t)], \chi(K(t))\} .$$

The latter term {.} describes the local tax (with $\chi_K > 0$). If the local tax is levied on profits (like in Sweden), the second term disappears and reported profits are taxed at an effective tax rate of $\theta + \theta_0$. But (2.3) also allows one to study the case (for example, of Finland) where the local authorities have the discretion of levying a 'resource tax' related to the size of the firm if the firm reports abnormally low profits.

In practice, there is an expected threshold for the local tax liability when the latter type of tax levy becomes effective. We plan to focus on the case where max{.} = $\chi(K)$.

Let us abstract from the well-known agency problems and assume that the firm is owned by individuals who at the same time run the firm as managers. Assume that σ is used to denote their common rate of time preference and assume that their utility is strictly proportional to the dividend stream generated by the firm. Then regarding dividends and the rate of tax allowances as the control variables, the objective of the firm can be written as

$$(2.4) \qquad \max_{\{D(t),\delta^*(t)\}} \int_{t_0}^{\infty} D(t)\exp(-\sigma(t-t_0))dt = V(K(t_0),B(t_0)).$$

Starting with their initial values $K(t_0)$ and $B(t_0)$, the state variables evolve as

$$(2.5) \qquad \dot{K}(t) = I(t) - \delta K(t), \quad \dot{B}(t) = I(t) - \delta^*(t)B(t)$$

where δ stands for the actual (economic) depreciation.[4)]

As regards the feasible control path of dividends paid out, the firm is assumed to face the following constraints. Given the relatively free manipulation of the reported profits as specified above, the firm is constrained by the condition that the currently distributed profits are not allowed to exceed the profits reported by the firm.[5)] As it turns out, this constraint plays an important role in the formulation of an optimal policy. Other constraints for admissible dividend policies are assumed to be due to the owners of the firm. Suppose that the owners are content with a relatively low dividend (say $\underline{D} > 0$) or even with self-financing of investment projects (with negative dividends $\underline{D} < 0$) for a limited period of time, say (t_0, t_1), provided the firm 'promises' to generate a stable dividend stream, say $\tilde{D}$ ($\tilde{D} > \max(0, \underline{D})$), thereafter. It is the task of the firm to transform these preferences into a proper discount rate and to inform the owners as to the date t_1 when the dividend target is attainable.[6)]

The dividend constraints faced by the firm are hence given by[7)]

$$\text{(2.6a)} \quad \underline{D} \leq D(t) \leq (1-\theta)\left[\pi(K(t)) - \delta^*(t)B(t)\right] - \chi(K(t)) =$$

$$\bar{D}(K(t), B(t), \delta^*(t)) \qquad t_0 \leq t \leq t_1$$

$$\text{(2.6b)} \quad D(t) = \tilde{D} > \max(0, \underline{D}) . \qquad t > t_1$$

It is now very important to realize that the tax allowance decisions of the firm cannot be independent of the preferences of the owners with respect to dividends as specified above. Hence, (2.2) has to be re-written as

$$\text{(2.7)} \quad 0 \leq \delta^*(t) \leq \min\left\{\bar{\delta}, \frac{1}{B(t)}\left[\pi(K(t)) - \frac{D(t) + \chi(K)}{1 - \theta}\right]\right\} =$$

$$\min\{\bar{\delta}, \bar{\delta}^*(K(t), B(t), D(t)\} . \quad t \geq t_0$$

Accordingly, the firm will not use the maximal rate of tax allowance $\bar{\delta}$ if that would be inconsistent with the relation between current after-tax earnings and dividend preferences.

The control problem over an infinite horizon can now be transformed into a problem over a finite horizon

$$(2.4)^{\sim} \qquad \max_{\{D(t),\delta^*(t)\}} \int_{t_0}^{t_1} D(t)\exp(-\sigma(t-t_0))dt + (\tilde{D}/\sigma)\exp(-\sigma t_1)$$

subject to (2.6a) and (2.7). Note that the terminal condition is imbedded in the latter term in $(2.4)^{\sim}$. This problem is one with two state and two control variables. Assume hence that there exist variables λ_0, $\lambda(t)$ and $\mu(t)$, defined on (t_0,∞), and define the following Hamiltonian function (in current values)

$$(2.8) \qquad H(D,\delta^*,K,B,\lambda,\mu) = \lambda_0 D(t) + \lambda(t)\dot{K}(t) + \mu(t)\dot{B}(t), \quad t_0 \leq t \leq t_1$$

with $\lambda_0 = 1$. By introducing non-negative shadow prices $\alpha_1(t)$, $\alpha_2(t)$, $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ associated with the constraints (2.6a) and (2.7), one can form the Lagrangean

$$(2.9) \qquad L = H + \alpha_1(t)(\bar{D}(,) - D(t)) + \alpha_2(t)(D(t) - \underline{D}) + \beta_1(t)(\bar{\delta} - \delta^*)$$

$$+ \beta_2(t)\delta^*(t) + \beta_3(t)(\bar{\delta}^*(.) - \delta^*(t))$$

where $\beta_3 > 0 \Rightarrow \beta_1 = 0$, $\beta_2 = 0$ and $\beta_1 > 0 \Rightarrow \beta_3 = 0$, $\beta_2 = 0$.

This problem looks complicated, but there is a very helpful observation to be made concerning the optimal program. Due to the fact that H is linear in the control variables, the solution is basically of the bang-

bang type, though one cannot a priori rule out the possibility of a singular solution over some time invervals. As a matter of fact, the firm is solving a linear programming problem for all $t \in (t_0, t_1)$. But there is another helpful observation to be made. Since all inequality constraints include control variables, the costate variables $\lambda(t)$ and $\mu(t)$ both have to be continuous for (t_0, ∞), i.e. they cannot jump along the optimal path. The necessary conditions which any candidate for the optimal program has to satisfy, apart from the initial conditions and (2.5), read over the period (t_0, t_1) as follows

$$1 - \lambda - \mu - \alpha_1 + \alpha_2 = 0 \tag{2.10}$$

$$[(\lambda + \mu + \alpha_1)\theta - (\mu + \alpha_1)]\, B = \beta_1 - \beta_2 + \beta_3 \tag{2.11}$$

$$(1-\theta)(\lambda + \mu + \alpha_1)\pi_K - (\lambda + \mu + \alpha_1)\chi_K + \frac{\beta_3}{B(t)}\left(\pi_K - \frac{\chi_K}{1-\theta}\right) - \lambda(\sigma + \delta) = -\dot{\lambda} \tag{2.12}$$

$$[(\lambda + \mu + \alpha_1)\theta - (\mu + \alpha_1)]\,\delta^* - \frac{\beta_3}{B(t)^2}\left[\pi(K) - \frac{D + \chi(K)}{1 - \theta}\right] - \sigma\mu = -\dot{\mu} \tag{2.13}$$

$$\alpha_1 \geq 0, \qquad \alpha_1(\bar{D}(.) - D) = 0 \tag{2.14}$$

$$\alpha_2 \geq 0, \qquad \alpha_2(D - \underline{D}) = 0 \tag{2.15}$$

$$\beta_1 \geq 0, \qquad \beta_1(\bar{\delta} - \delta^*) = 0 \tag{2.16}$$

$$\beta_2 \geq 0, \qquad \beta_2\delta^* = 0 \tag{2.17}$$

$$\beta_3 \geq 0, \qquad \beta_3(\bar{\delta}^*(.) - \delta^*) = 0 \tag{2.18}$$

To interprete, (2.10) eliminates the arbitrage possibilities between dividends and retained earnings along the optimal path.

The above conditions for optimal control can be reduced to a system of four non-linear differential equations. Since these are continuously differentiable[8)] one can appeal to the existence theorem given by Hirsch and Smale (1974 p. 163) according to which there are unique functions $K(t)$, $B(t)$, $\lambda(t)$ and $\mu(t)$ that satisfy these equations.

III EQUILIBRIUM POLICY AND SUFFICIENCY OF OPTIMALITY CONDITIONS

The existence of the solution leads to the question of the existence of an equilibrium and its properties. Conditions (2.5) suggest that an equilibrium of the following type can be produced

$$(3.1) \qquad \hat{I}(t) = \delta\hat{K} = \hat{\delta}^*\hat{B}$$

by an appropriate choice of terminal investment $\hat{I}(t)$ starting at time t_1. Here $\hat{}$ has been used to denote the equilibrium values.[9)] In terms of the tax allowance policy, the conditions (3.1) suggests a crucial property of the model, i.e. that in equilibrium the firm claims a tax allowance exactly equal to the economic depreciation, no more and no less. As to the value of $\hat{\delta}^*$, one cannot yet say more because $\hat{B}$ has to be solved first.

It is another matter to convince oneself that it actually is optimal to strive to reach the equilibrium proposed above. For this to hold, the necessary conditions have to be sufficient, too. The Hamiltonian function as regards the problem of the current paper is not concave in the state and control variables. Fortunately, Arrow (1968) and Seierstad and Sydsaeter (1977) have proved that for sufficientcy, one needs to require that the maximized Hamiltonian $H^0(K,B,\lambda,\mu) = \max_{\{D,\delta^*\}} H(K,B,D,\delta^*,\lambda,\mu)$ is concave in the state variables only. We have checked that this holds for all admissible values of the control variables in all feasible policies along the adjustment path (described in the next section). Moreover, in our model, the following constraint qualification given by Seierstad and Sydsaeter is satisfied

$$\text{rank}\left[\frac{\partial h_i(\hat{K}(t),\hat{B}(t),\hat{D}(t),\hat{\delta}^*(t))}{\partial u_j}\right] \quad \begin{matrix} i \in In(t) \\ j = 1,2 \end{matrix}$$

$$= \text{number of indices in } In(t)$$

where

$$In(t) = \{\, i : h_i(\hat{K}(t),\hat{B}(t),\hat{D}(t),\hat{\delta}^*(t)) = 0 \,\}$$

with h_i, $i = 1,\ldots,4$ standing for the constraints in (2.6a) and (2.7) and $u = (D,\delta^*)$. $\hat{}$ has been used to denote the candidates for optimal values. Hence, we feel free to regard the necessary conditions also as sufficient.

Some results of the terminal policy are readily at hand. Using (2.1) and the characterization of the equilibrium (3.1), one finds that the terminal dividend requirement (2.6b) uniquely determines the terminal capital $\hat{K}$, i.e.

$$\tilde{D} = (1-\theta)(\pi(\hat{K}) - \delta\hat{K}) - \chi(\hat{K}) \qquad t \geq t_1 . \tag{3.2}$$

Those projects which do not satisfy (3.2) will not be undertaken. Moreover, given (2.1) and (3.1) the equilibrium rate of tax allowance satisfies

$$\hat{\delta}^* = \frac{1}{\hat{B}}[(1-\theta)\pi(\hat{K}) + \theta\delta\hat{K} - (\tilde{D}+\chi(\hat{K}))] . \tag{3.3}$$

Hence, it is fully determined by $\tilde{D}$, $\hat{K}$ and $\hat{B}$. Using (2.4)′ and (3.2) it is now easy to determine the terminal values of the costate variables. By the terminal condition, the marginal valuation of capital at $t = t_1$ is given by

$$\lambda(t_1) = \frac{\partial V(K(t_1),B(t_1))}{\partial K(t_1)} \tag{3.4}$$

$$= \frac{1}{\sigma}[(1-\theta)(\pi_K(K(t_1)) - \delta) - \chi_K(K(t_1))] e^{-\sigma t_1} .$$

Or viewing t_1 as the origin ($t_1 = 0$),

$$\hat{\lambda} = \frac{1}{\sigma}[(1-\theta)(\pi_K - \delta) - \chi_K] . \tag{3.4′}$$

One also obtains the intuitive result

$$\hat{\mu} = \mu(t_1) = \frac{\partial V(K(t_1),B(t_1))}{B(t_1)} = 0 . \tag{3.5}$$

We have not yet been able to determine $\hat{B}$, the terminal accounting value of the firm. But our analysis in the coming sections proves that even though different starting values $B(t_0)$ may give rise to different equilibrium values, the equilibrium is still unique for any given starting value. The initial state of the system can, in principle, be found in any part of the state space, $X = \{ (K(t_0), b(t_0))/K(t_0) > 0, B(t_0) > 0 \} = R_+^2$. By varying the initial values, one can cover the entire state space. Then uniqueness of the optimal control path associated with any given initial point in X implies that one can associate a unique pair of costate variables $\{ \lambda(t), \mu(t) | t \geq t_0 \}$ with any given path of the state variables. We now turn to consider the optimal policies in the "short run".

IV OPTIMAL CONTROL OUTSIDE THE EQUILIBRIUM: CASE I

1. Candidate Policies

The potential upper limit for $\delta^*(t)$ given in (2.7) makes it necessary to analyse two cases separately. Call it Case I when $\bar{\delta}$ is set relatively high by the tax authorities with the consequence that $\min \{ \bar{\delta}, \bar{\delta}^*(.) \} = \bar{\delta}^*(.)$. Alternatively, this case arises if the minimum dividend $\underline{D}$ required by the owners is relatively high. In terms of (2.9) this implies that one can take $\beta_1(t) = 0$ throughout Case I. What one first wants to do is to characterize the feasible, non-singular candidate policies in terms of the shadow prices, costate variables, and control variables. Then one can try to figure out what policies have to be sequentially followed before it is possible to switch to the equilibrium policy at $t = t_1$. The conditions (2.10)-(2.18) have to hold throughout. Since they include four shadow prices (when $\beta_1 = 0$), one can classify the policies with respect to active constraints as follows

Policy I	$\alpha_2 > 0,\ \beta_3 > 0$	$(\Rightarrow \alpha_1 = 0,\ \beta_2 = 0)$
Policy II	$\alpha_1 > 0,\ \beta_2 > 0$	$(\Rightarrow \alpha_2 = 0,\ \beta_3 = 0)$
Policy III	$\alpha_2 > 0,\ \beta_2 > 0$	$(\Rightarrow \alpha_1 = 0,\ \beta_3 = 0)$
Policy IV	$\alpha_1 > 0,\ \beta_3 > 0$	$(\Rightarrow \alpha_2 = 0,\ \beta_2 = 0)$.

Using then the conditions (2.10) and (2.11), one finds that when these policies are followed by the firm the costate variables satisfy

Policy I	$1 - (\lambda + \mu) < 0$	$(\lambda + \mu)\theta - \mu > 0$
Policy II	$1 - (\lambda + \mu) > 0$	$\lambda < 1 - \theta$
Policy III	$1 - (\lambda + \mu) < 0$	$(\lambda + \mu)\theta - \mu < 0$
Policy IV	$1 - (\lambda + \mu) > 0$	$\lambda > 1 - \theta$.

For example, the result concerning Policy II follows from the requirement

$$\operatorname{sgn}\{(\lambda + \mu + \alpha_1)\theta - (\mu + \alpha_1)\} = -\operatorname{sgn}(\beta_2) < 0$$

using the conditions $\lambda + \mu + \alpha_1 = 1$, $\mu + \alpha_1 = 1 - \lambda$. Similarly, the condifion for Policy IV follows from

$$\operatorname{sgn}\{(\lambda + \mu + \alpha_1)\theta - (\mu + \alpha_1)\} = \operatorname{sgn}(\beta_1) > 0 .$$

In order to describe the candidate policies in terms of the control variables it is useful first to note that in the Hamiltonian function (2.8) the coefficient of D(t) is negative both in Policies I and III.

Hence, D(t) is chosen as small as possible. This does not restrict the set of feasible values of $\delta^*(t)$ but rather expands it. Studying the coefficient of δ^* in the Hamiltonian function, one concludes that given $\underline{D}$, δ^* is chosen to be as large as possible in Policy I while it is chosen to be as small as possible in Policy III. Hence, one obtains

Policy I $\quad D = \underline{D}, \quad \delta^* = \frac{1}{B(t)} [\pi(K) - \frac{\underline{D} + \chi(K)}{1 - \theta}]$

Policy III $\quad D = \underline{D}, \quad \delta^* = 0.$

In Policies II and IV the coefficient of D is positive and the firm has incentives to choose as large a D as possible. But this time D cannot be chosen independently of δ^* due to the constraint (2.6a). There is now a real trade-off in choosing between a low δ^* and a high D or a higher δ^* but a lower D. The payoff function is given by H, and it will be maximized with respect to D and δ^* under the constraint

$$D = (1-\theta) [\pi(K) - \delta^* B] - \chi(K)$$

or, equivalently, under

$$\delta^* = \frac{1}{B(t)} [\pi(K) - \frac{D(t) + \chi(K)}{1 - \theta}] .$$

This can be reduced to a maximization problem with respect to δ^* alone. The optimal choice is based on the sign of the derivative

$$\frac{dH}{d\delta^*} = [\lambda - (1-\theta)] B \gtrless 0 .$$

If now $\lambda < 1-\theta$, the firm chooses $\delta^* = 0$ and as large a D as possible. If $\lambda > 1-\theta$, it is optimal to choose as large a δ^* as possible which, of course, implies that D has to take the smallest acceptable value. Thus, the following holds

Policy II $\quad D = (1-\theta)\pi(K) - \chi(K) , \qquad \delta^* = 0$

Policy IV $\quad D = \underline{D}, \quad \delta^* = \frac{1}{B(t)}[\pi(K) - \frac{\underline{D} + \chi(K)}{1 - \theta}]$.

Note that in Case I, Policies I and IV are equivalent in terms of the control variables, though not in terms of the costate variables. Graphically, the control space has been given in figure 1.

It is useful to proceed by considering the costate space, given in figure 4. As long as the marginal productivity of capital is positive, one can conentrate on the sub-space $\{(\lambda,\mu)|\lambda > 0\}$. The separating lines between the different policy regimes represent the singular policies arising if the coefficients of the controls remain zero over some positive time period, see Appendix. Remembering that the equilibrium policy can be found on the λ-axis (with $\hat{\mu} = 0$), let us consider the potential policies preceding the equilibrium policy.

2. Policy IV

It is helpful to summarize Policy IV as follows

$$\alpha_2 = \beta_2 = 0, \; 1 - (\lambda + \mu) > 0, \; \lambda > 1 - \theta$$

$$D = \underline{D} , \quad \delta^* = \frac{1}{B}[\pi(K) - \frac{\underline{D} + \chi(K)}{1 - \theta}] .$$

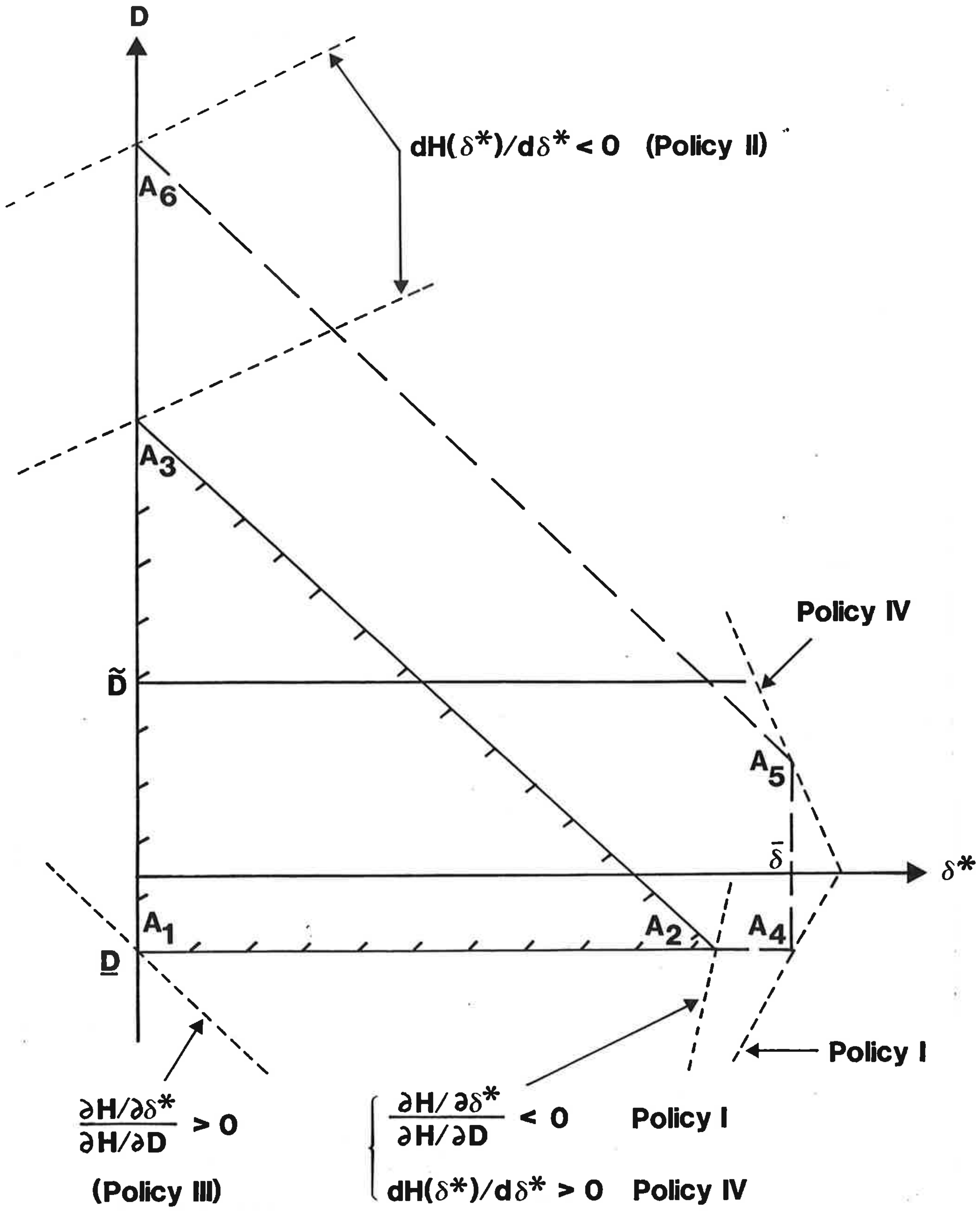

Figure 1. Control Space in Case I ($A_1A_2A_3$) and in Case II ($A_1A_4A_5A_6$) and the Iso-Payoff Lines (------).

Hence, the firm is using all the tax allowances it can without violating the dividend requirement. But it is still claiming less than allowed by the tax law, though note that $\delta^*(t)$ is not constant but is growing with $K(t)$ in Policy IV. Following this policy, the firm pays taxes as

$$T = \frac{1}{1-\theta}[\theta \underline{D} + \chi(K)]$$

while gross investment is given by

$$I = \pi(K) - \frac{1}{1-\theta}[\underline{D} + \chi(K)].$$

Thus, I is growing with K (provided χ_K is not too large) due to the larger cash flow.[10)] It is thus appropriate to call Policy IV <u>the phase of maximal and accelerating growth</u>.

It is easy to see that in Policy IV the tax allowance δ^*B is kept equal to gross investment. Hence, the firm keeps its book value constant in this policy, i.e. $\dot{B} = 0$, and $B(t) = B(t_0) = \hat{B}$ for $t \in (t_0, t_1)$ (if Policy IV is the only policy followed before the equilibrium policy). Furthermore, $\alpha_2 = 0 \Rightarrow \lambda + \mu + \alpha_1 = 1$ and $\mu + \alpha_1 = 1 - \lambda$. Then $\beta_1 = \beta_2 = 0 \Rightarrow \beta_3/\hat{B} = \theta - (1-\lambda)$. Now (2.12) can be reduced to $\dot{\lambda} = -\lambda[\pi_K - (\sigma + \delta + \chi_K/(1-\theta))]$ while (2.13) is reduced to $\dot{\mu} = \sigma\mu$. Since $\mu(t_1) = \partial V(.)/\partial B(t_1) = 0$, the only possibility for the firm is to keep $\mu(t) = 0$, i.e. $\dot{\mu}(t) = 0$, throughout the period (t_0, t_1). Thus Policy IV can be described by the dynamic equations

$$\dot{K} = \pi(K) - \frac{1}{1-\theta}(\underline{D} + \chi(K)) - \delta K$$

(4.1)

$$\dot{\lambda} = -\lambda[\pi_K(K) - (\sigma + \delta + \chi_K/(1-\theta))].$$

The nature of this dynamic system goes through interesting changes when one moves along a given arm, depicted in the phase diagram of figure 2. Note first that setting $\dot{\lambda} = 0$ gives an important result, i.e. the efficiency price or the cost of capital

$$(4.2) \qquad p_n(\hat{K}) = \sigma + \delta + \chi_K/(1-\theta) = \pi_K$$

where $\pi_K = \pi_K(\hat{K})$ and $\chi_K = \chi_K(\hat{K})$. (4.2) is the proper discount rate for the firm. Thus what (4.1) suggests is that the percentage adjustment of λ is proportional to the discrepancy between the marginal productivity of capital and the cost of capital. Since $d\dot{K}/dK = \pi_K - \delta - \chi_K/(1-\theta) > \pi_K - \sigma - \delta - \chi_K/(1-\theta) = 0$, one knows that the equilibrium capital $\hat{K}$ has to be less than the solution, say K^0, for the equation $\dot{K} = 0$, see figure 3. This means that $\dot{\lambda} = 0$ and $\dot{K} = 0$ have no common points. Take, for example, the arm abc∞ in figure 2. Along the section (a,b) λ is stable while K is unstable, in section (b,c) both λ and K are unstable, and in (c, ∞) K is stable and λ unstable. Yet no rest point can be found along (a,c) and no saddle-point can be found along (c, ∞).

Hence one knows that the firm can follow Policy IV only up to $\hat{K}$ given by (4.2). At $\hat{K}$ it has to switch to the equilibrium policy. Following Policy IV over an infinite time period would lead to considerable over-investment.

We have earlier solved the proper valuation of capital at the switching point and presented it in (3.4). It provides the firm with the right signal to stop the accumulation. This can be obtained by choosing the initial value $\lambda(t_0)$ on an unstable arm passing through $(\hat{K},\lambda(t_1))$ and by

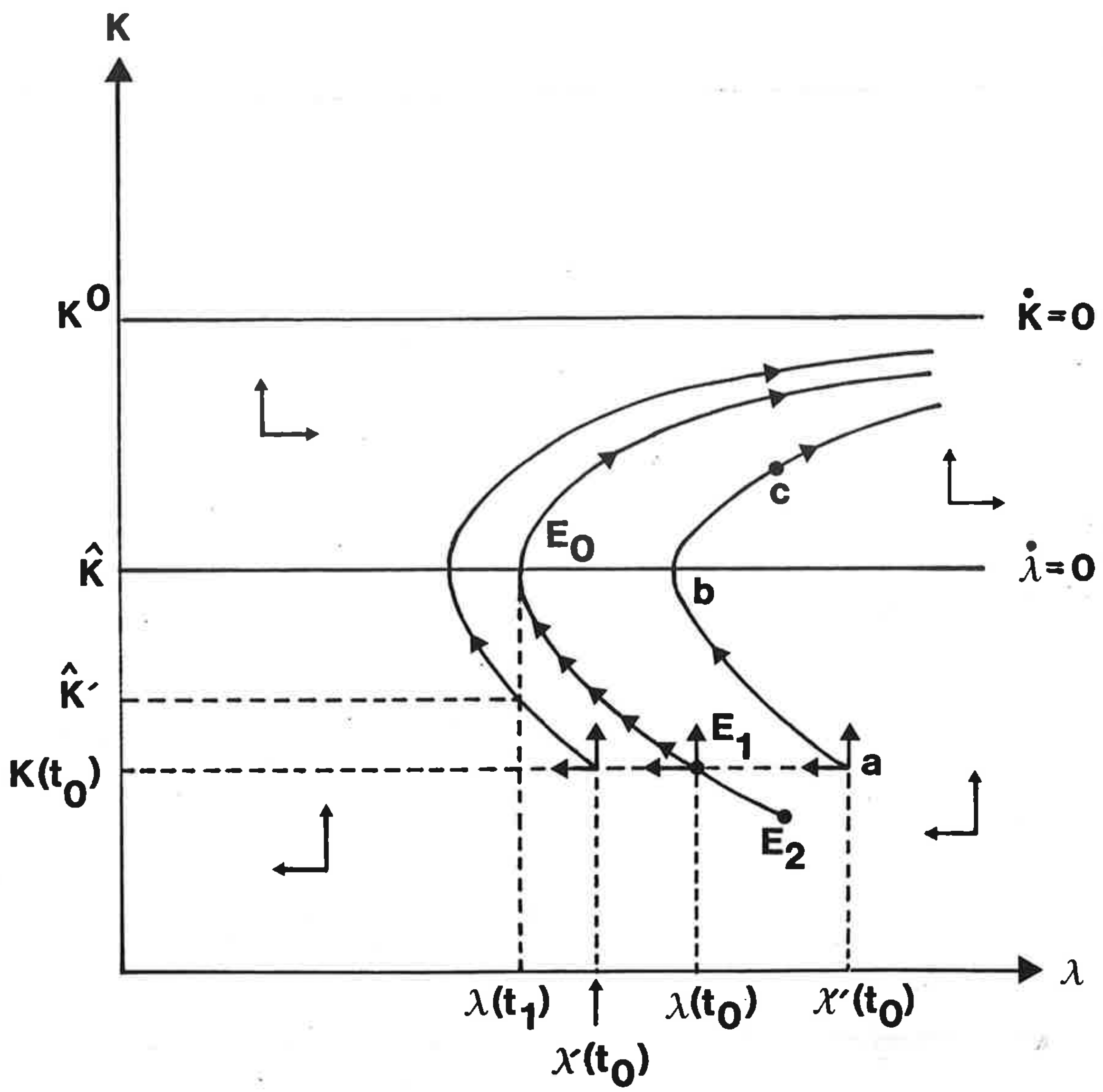

Figure 2. The Phase Diagram Associated with Policy IV.

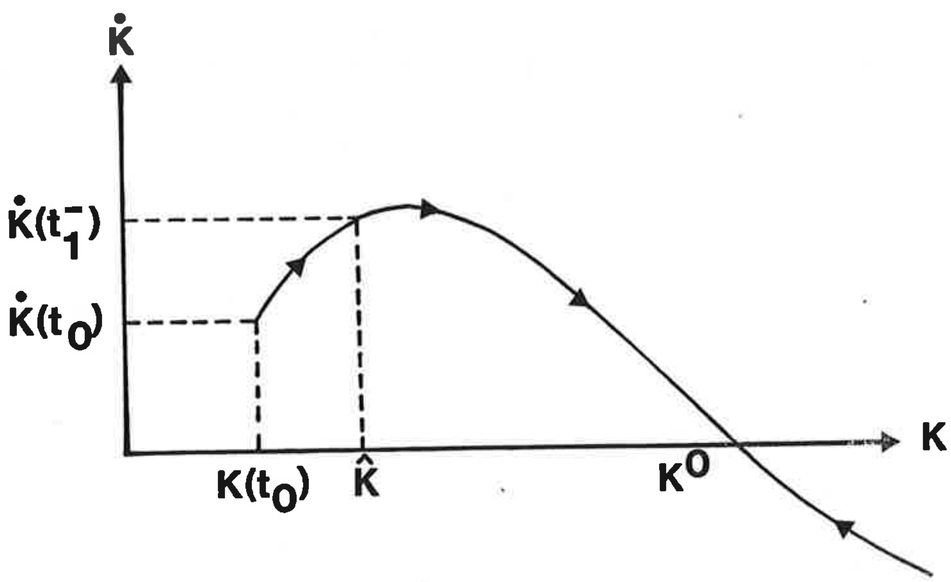

Figure 3. Adjustment of Capital in Policy IV.

letting gross investment be reduced at that point to the level corresponding to the cost of depreciation $\delta\hat{K}$. Note that if the firm chose a wrong initial value, say $\lambda'(t_0)$, it would terminate the investment phase at $\hat{K}'$, which is too early since $\lambda(t)$ would attain the value $\hat{\lambda}$ too early. This means underinvestment. If the firm in turn would start with a valuation of $\lambda''(t_0)$, $\lambda(t)$ would exceed $\hat{\lambda}$ when $K = \hat{K}$ generating overinvestment.

Having now succesfully solved the cost-of-capital equation (4.2), one can find out the precise terminal value $\lambda(t_1)$ given in (3.4). Substituting (4.2) into (3.4) one obtains

$$(4.3) \qquad \lambda(t_1) = \hat{\lambda} = 1 - \theta.$$

Hence, in our model, the tax-adjusted "Tobin's marginal q" is less than one in the long run. In the costate space, given in figure 4, the equilibrium is found at E_0, on the boundary line between Policies IV and II. In terms of figure 4, the problem of optimal control is reduced to the question of choosing $\lambda(t_0)$ properly, i.e. at a point like E_1.

3. Policy I

Suppose that the initial values of the state variables dictate that the firm initially chooses Policy I. It is summarized as

$$\alpha_1 = \beta_2 = 0, \quad 1 - (\lambda + \mu) < 0, \quad (\lambda + \mu)\theta - \mu > 0 \ ,$$

$$D = \underline{D}, \quad \delta^* = \frac{1}{B}[\pi(K) - \frac{\underline{D} + \chi(K)}{1 - \theta}]$$

As stated earlier, this policy is equivalent to Policy IV in terms of the controls and represents the maximal growth, too. This time, however, the initial value of $\lambda(t_0)$ will exceed unity (while it was less than unity in Policy IV). The dynamic equations can be shown to be the same as those given in (4.1). In terms of figures 2 and 4, the right choice of $\lambda(t_0)$ will now be higher as represented by the point E_2. Then the optimal control under Policy I is to follow the arm starting at E_2.[11)]

4. Policy II

Policy II is described by

$$\alpha_2 = \beta_3 = 0, \quad 1 - (\lambda + \mu) > 0, \quad \lambda < 1 - \theta$$

$$D = (1 - \theta)\pi(K) - \chi(K), \quad \delta^* = 0.$$

Hence, the tax liability is $T = \theta\pi(K) - \chi(K)$ and it is easy to conclude that gross investment will be equal to zero. The return on capital is thus presumably low relative to the cost of capital and the firm is in a contraction phase. In this phase the dividend constraint again has a central role in the distribution of current earnings between the owners and the tax collector. In order to maximize the distributed profits out of current earnings[12)] the firm has to report maximum profits choosing $\delta^* = 0$.

Because $I = \delta^* = 0$ in Policy II, it means that $\dot{B} = 0$. Hence, the firm keeps its book value unchanged in this phase. The actual capital decays as

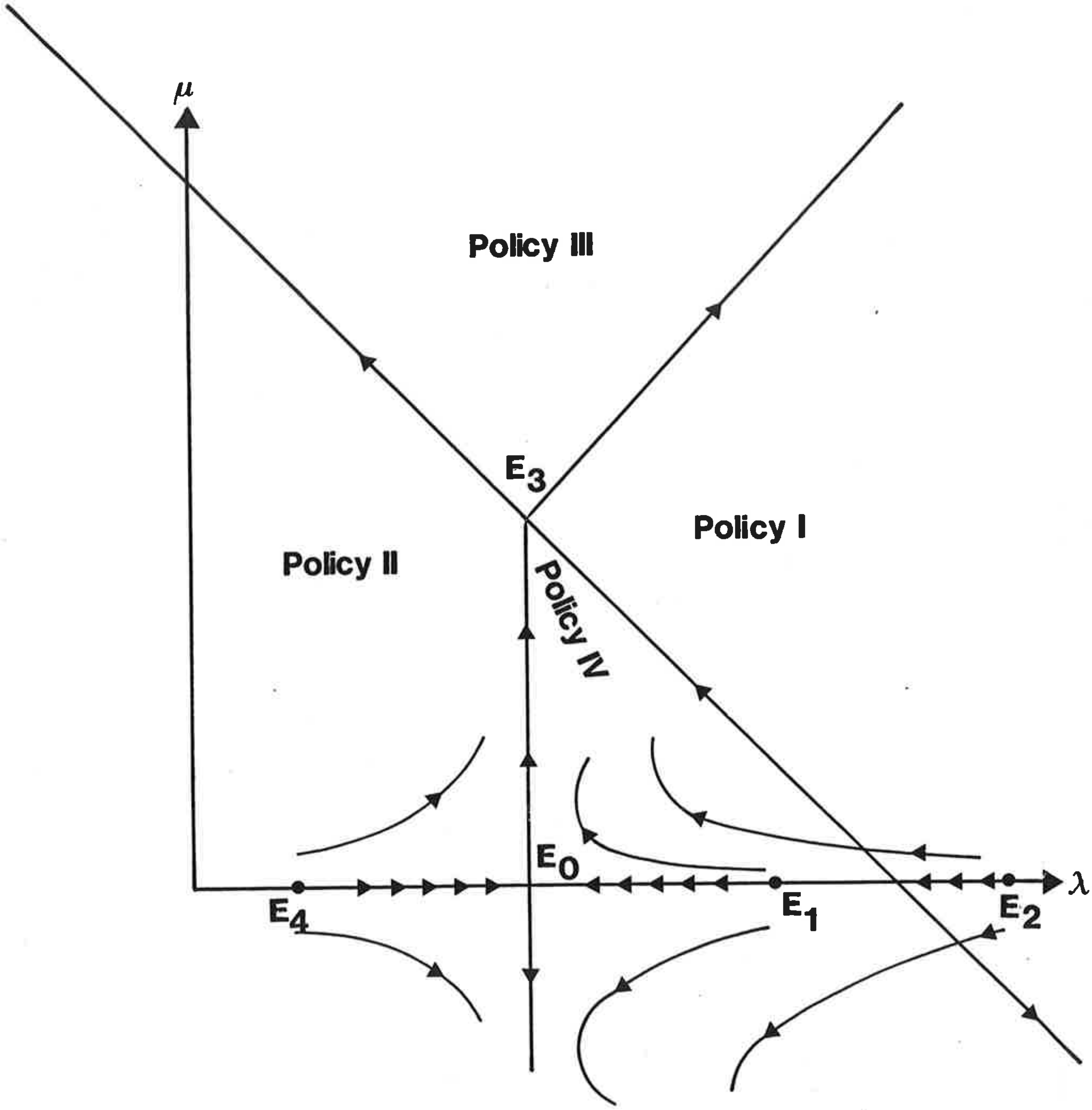

Figure 4. Costate Space and the Feasible Candidate Policies, Case I.

$\dot{K} = -\delta K$. Having also $\delta^* = \beta_3 = 0$ implies (see (2.13)) that μ obeys $\dot{\mu} - \sigma\mu = 0$.

To limit the range of optimal paths under Policy II it is time to propose that in Case I, at least, Policy III can never belong to the sequence of optimal policies. This is because continuity of μ prevents any switches from Policy III to Policy I, because a direct switch to Policy IV is impossible and because the singular policy between E_2 and E_0 (see figure 4) is upwards. Hence, Policy II can only be followed by the equilibrium policy. Having so determined the terminal value $\lambda(t_1) = 0$, one knows that the initial value of μ must be located on the λ-axis and that $\mu(t) = 0$ throughout Policy II. But $\alpha_2 = \mu = 0 \Rightarrow \lambda + \alpha_1 = 1$ and (2.12) allows one to solve the equation for $\dot{\lambda}$. Thus in Policy II one obtains

$$\text{(4.4)} \qquad \begin{aligned} \dot{K} &= -\delta K \\ \dot{\lambda} &= \lambda(\sigma + \delta) - (1-\theta)\pi_K + \chi_K . \end{aligned}$$

This suggests (imposing $\dot{\lambda} = 0$) that the equilibrium marginal valuation reads as

$$\text{(4.5)} \qquad \hat{\lambda} = (\sigma + \delta)^{-1} [(1-\theta)\pi_K - \chi_K] .$$

Since this must be consistent with (3.4)´, it follows that $\hat{\lambda} = 1-\theta$, guaranteeing uniqueness of the equilibrium regardless of the preceding policy. Then from (4.4) one can see that again $\hat{K}$ is determined by the condition (4.2). The phase diagram is given in figure 5. Given $K(t_0)$ the task is reduced to the problem of picking up the right initial

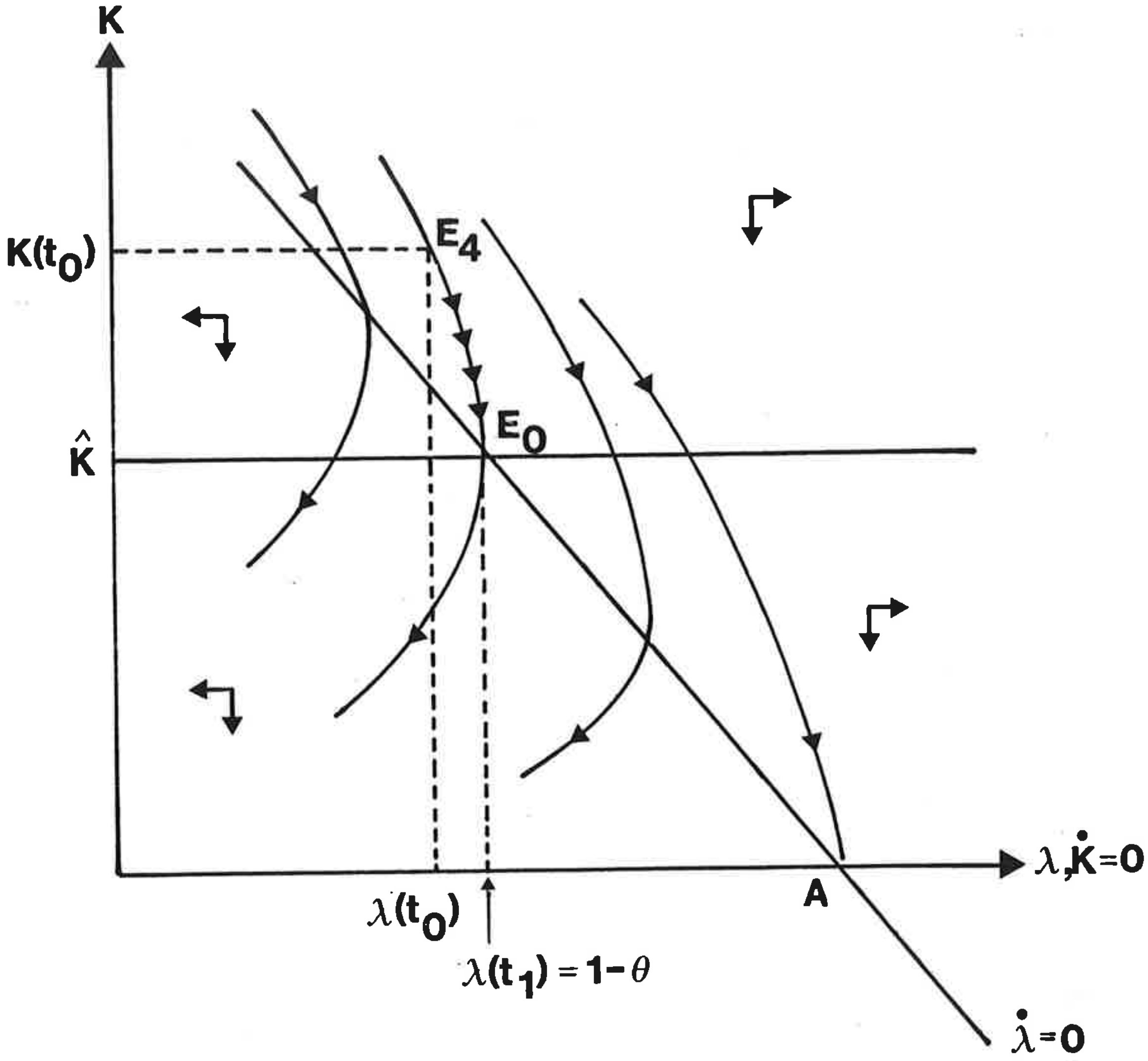

Figure 5. The Phase Diagram Associated with Policy II.

value $\lambda(t_0)$ that is located on the unstable arm going through point E_0. The right starting value is unique and is given by point E_4 in figure 5 and is also depicted in figure 4.[13)]

5. Case I: Conclusions

The analysis in this part of the paper has shown that the puzzle of unclaimed tax allowances can be explained in terms of the dividend preferences of the owners. Unclaimed tax allowances arise both in the phase of expansion, in the phase of contraction and in equilibrium. Moreover, at the equilibrium the tax allowance corresponds to the economic depreciation as a result of rational choice. In the absence of the size-related local tax, the Bergström-Södersten proposition of the allocative neutrality of the corporation tax is valid under unclaimed tax allowances even if no explicit allowance is made for the deduction of interest in the tax base. However, the size-related local tax creates a wedge between the return on capital and the return to the owners. This wedge can only be partially eliminated by making the local tax deductible before the state tax is levied. In thus appears that the local tax may be a source of inefficiency.

Based on the dynamic theory of the firm (formulated by Abel (1979), (1982), Yoshikawa (1980) and Hayashi (1982)) the q theory of investment (originally advanced by Tobin (1969)) has gained increasing popularity in empirical work (von Furstenberg (1977), Abel (1980), Hayashi (1982), Poterba-Summers (1983) and others). The key assumptions in this approach, which gives rise to a positive association between the current rate of investment and the current marginal valuation of capital, are that there are strictly convex costs of adjusting long-term capital and that the capital markets are perfect. Our results suggest that if these two assumptions are relaxed the q approach breaks down. Of course the marginal valuation of capital continues to work as the right signal as to whether the marginal project satisfies the rate-of-return requirement. Yet in the expansion phase the association between the current rate of investment and the marginal valuation of capital is negative. This result is easy to explain. While the marginal q provides information on the profitability of the investment, the rate of investment is determined by the financial resources available, i.e. current cash flow. Note that it is hence not possible to augment the investment equation based on q by a liquidity variable. Note also that the relationship between the marginal valuation and the rate of investment depends on the policy regime. In the contraction phase, there is no relationship.

These conclusions emerge strikingly from our model, which describes a cash-flow constrained firm operating in imperfect credit markets. For those who favour the model of an adjustment cost constrained firm operating in perfect credit markets, one should point out that even in that framework there is no causal relationship between the marginal q and the rate of investment. Rather these variables are jointly determined along the optimal path of the firm. Finally, the empirical studies seem to reveal several puzzles associated with the q approach (cf. Ueda and Yoshikawa (1986)).

The view emerging from our model is roughly as follows. While the long-run capital is determined by the cost of capital, as predicted by the neoclassical model, the current rate of investment is heavily influenced by the current cash flow of the firm. This view seems to be quite consistent with the bulk of empirical investment studies available which emphasize the acceleration, profit or cash-flow variables as determinants of the flow of investment. (For a recent study see Mairesse and Dormont (1985)). While it is not well understood why the firms even in the absence of binding constraints favour internal sources of finance, it is possible that this has something to do with the control aspects and the principal agent relations between the firms and the outside sources of finance.

V OPTIMAL CONTROL OUTSIDE THE EQUILIBRIUM: CASE II

1. Candidate Policies

Turn now to consider the case where $\min\{\bar{\delta}, \bar{\delta}^*(.)\} = \bar{\delta}$. This case arises if either $\bar{\delta}$ or the dividend requirement is relatively low. Since the optimal policy is different than in Case I, Case II has to be studied separately. In terms of (2.9) one now has $\beta_3 = 0$ throughout. Since four shadow prices are left, the candidate policies can be characterized during the interval (t_0, t_1) as follows

Policy I	$\alpha_2 > 0,\ \beta_1 > 0$	$(\Rightarrow\ \alpha_1 = \beta_2 = 0)$
Policy II	$\alpha_1 > 0,\ \beta_2 > 0$	$(\Rightarrow\ \alpha_2 = \beta_1 = 0)$
Policy III	$\alpha_2 > 0,\ \beta_2 > 0$	$(\Rightarrow\ \alpha_1 = \beta_1 = 0)$
Policy IV	$\alpha_1 > 0,\ \beta_1 > 0$	$(\Rightarrow\ \alpha_2 = \beta_2 = 0)$

Policy I	$1 - (\lambda+\mu) < 0,$	$(\lambda+\mu)\theta - \mu > 0$
Policy II	$1 - (\lambda+\mu) > 0,$	$\lambda < 1-\theta$
Policy III	$1 - (\lambda+\mu) < 0,$	$(\lambda+\mu)\theta - \mu < 0$
Policy IV	$1 - (\lambda+\mu) > 0,$	$\lambda > 1-\theta$

Policy I	$D = \underline{D}, \quad \delta^* = \bar{\delta}$
Policy II	$D = (1-\theta)\pi(K) - \chi(K), \quad \delta^* = 0$
Policy III	$D = \underline{D}, \quad \delta^* = 0$
Policy IV	$D = (1-\theta)[\pi(K) - \bar{\delta}B] - \chi(K), \quad \delta^* = \bar{\delta}$

Note that this time Policies I and IV differ with respect to the dividend policy if $(1-\theta)[\pi(K) - \bar{\delta}B] - \chi(K) > \underline{D}$ along the path in Policy IV. For Case II, the control space is depicted in figure 1 as the area $A_1A_4A_5A_6$. When we now move to determine the optimal control in Case II, it should be remembered that the equilibrium results (3.2)-(3.5) continue to be valid. It is helpful again to start with Policy IV.

2. Policy IV

To summarize, Policy IV is given by

$$\alpha_2 = \beta_2 = 0, \quad 1 - (\lambda+\mu) > 0, \quad \lambda > 1-\theta$$

$$D = (1-\theta)[\pi(K) - \bar{\delta}B] - \chi(K), \quad \delta^* = \bar{\delta}.$$

In contrast to Case I, by following this policy the firm is deferring tax payments using maximal tax allowances to generate internal financing for investment. The growth rate need not be, however, the maximum possible because the firm may be distributing some profits. It is then appropriate to call this policy a phase of relatively intensive investment activity. Investment can be solved to be

$$(5.1) \qquad I(t) = \bar{\delta}B(t)$$

and hence it is constrained by the maximum rate of tax allowance. Note that due to (5.1) $\dot{B} = 0$ and hence the book value is kept unchanged in Policy IV. Since this will not be followed by any other policy except by the equilibrium policy, the book value will be at its final level $\hat{B}$. Thus, net investment is given by $\dot{K} = \bar{\delta}\hat{B} - \delta K$. Since $\alpha_2 = 0 \Rightarrow \lambda + \mu + \alpha_1 = 1$, $\mu + \alpha_1 = 1 - \lambda$, one can use (2.12) to write down the dynamics of λ and use (2.13) for μ. One obtains

$$\dot{K} = \bar{\delta}\hat{B} - \delta K$$

$$(5.2) \qquad \dot{\lambda} = \lambda(\sigma + \delta) - (1 - \theta)\pi_K + \chi_K$$

$$\dot{\mu} = \sigma\mu - [\lambda - (1 - \theta)]\bar{\delta}$$

To analyse this system it is again helpful to notice that the equilibrium has to be characterized by the conditions $\dot{\lambda} = \dot{\mu} = 0$. This implies that $\hat{\lambda} = (\sigma + \delta)^{-1}[(1 - \theta)\pi_K - \chi_K]$ which, when coupled with the condition (3.4)´, again gives the cost of capital (4.2) and the marginal valuation of capital in equilibrium $\hat{\lambda} = 1 - \theta$. (Hence, (5.2) satisfies $\hat{\mu} = 0$).

Since (5.2) is recursive, let us first concentrate on the first two equations. Their phase diagram is given in figure 6. The $\dot{K} = 0$ and $\dot{\lambda} = 0$ loci have a unique intersection at (λ^0, K^0). In the neighbourhood of this point, the linearized system behaves as

$$(5.3) \quad \begin{pmatrix} \dot{K} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} -\delta & 0 \\ -(1-\theta)\pi_{KK} + \chi_{KK} & \sigma+\delta \end{pmatrix} \begin{pmatrix} K - K^0 \\ \lambda - \lambda^0 \end{pmatrix} .$$

Since the determinant of the coefficient matrix $-\delta(\sigma+\delta) < 0$, this intersection is a saddle-point by nature (with the saddle-path denoted by ss).[14)] As to the nature of the optimal path, one has to recall that the equilibrium capital $\hat{K}$ has to be obtained in finite time and that the accumulation process has to be stopped before K^0. But assume that this is not too far away from K^0 and that one can use figure 6 for a qualitative analysis. Then optimality requires choosing $\lambda(t_0)$ on an unstable arm (point E_1) that leads the adjustment to E_0, which will be reached by t_1.[15)] This necessitates a reduction in investment activity at $\hat{K}$. But this gives an important conclusion according to which the firm does not claim all the potential tax allowances at the equilibrium. (Comparison of (5.1) and (3.1) helps one to see that this is indeed the case). This is a remarkable result based on the intrinsic idea of the model that the owners of the firm expect it to reach the equilibrium in finite time. Figure 7 gives the adjustment in the costate space.

3. Policy I

In Policy I the firm behaves as follows

$$\alpha_1 = \beta_2 = 0, \quad 1 - (\lambda+\mu) < 0, \quad (\lambda+\mu)\theta - \mu > 0$$

$$D = \underline{D}, \quad \delta^* = \bar{\delta}.$$

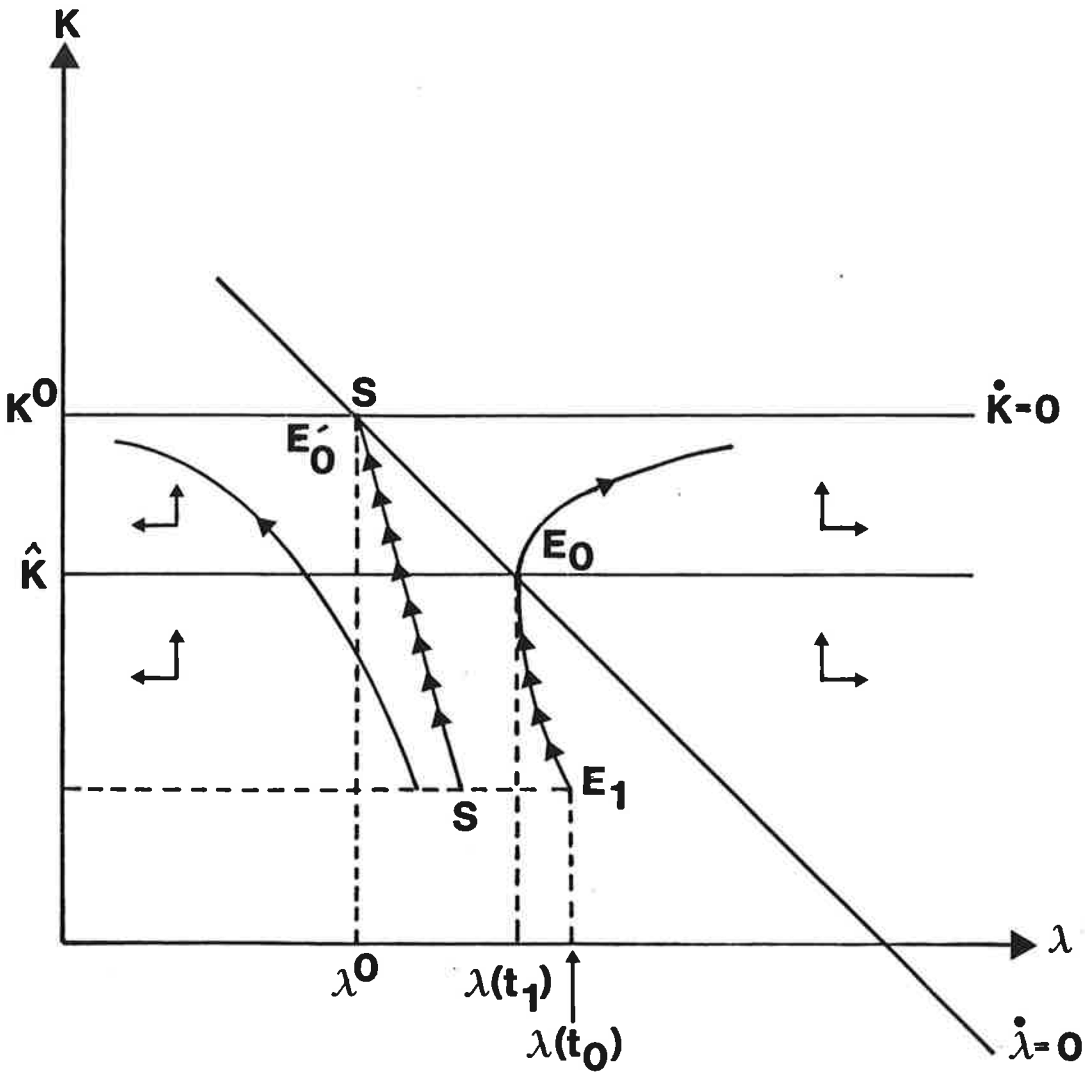

Figure 6. Case II: Phase Diagram Associated with Policy IV.

The firm is inclined to choose Policy I at an early stage of adjustment if the current valuation of marginal capital is relatively high compared to the marginal valuation in equilibrium. Consequently, the firm minimizes both taxes by claiming all potential tax allowances and pays minimum dividends (or even sells new shares to the existing owners) in order to build up maximal financial resources for investing. This can be called <u>the phase of maximal growth</u>. In this regime, investment is given by the after-tax cash flow

$$I(t) = (1-\theta)\pi(K) + \theta\bar{\delta}B(t) - \chi(K) - \underline{D}$$

and there is again a strong flow effect of current tax payments on the flow of investment. In Policy I there is nothing which would require that $I(t) = \bar{\delta}B(t)$. Instead, investment allows the firm to expand its book value, which serves as the base for its tax allowances. This feature distinguishes Policy I from the other regimes studied so far. Equations (2.12) and (2.13) suggest that Policy I is given by the dynamic system

$$
\begin{aligned}
\dot{K} &= (1-\theta)\pi(K) + \theta\bar{\delta}B(t) - \chi(K) - \underline{D} - \delta K \\
\dot{B} &= (1-\theta)\pi(K) + \theta\bar{\delta}B(t) - \chi(K) - \underline{D} - \bar{\delta}B(t) \\
\dot{\lambda} &= \lambda(\sigma + \delta) - (1-\theta)(\lambda + \mu)\pi_K - (\lambda + \mu)\chi_K \\
\dot{\mu} &= \sigma\mu - [(\lambda + \mu)\theta - \mu]\bar{\delta}
\end{aligned}
\qquad (5.4)
$$

This is quite a complicated system. But it is easy to see that it must have a rest point. First, $d\dot{B}/dB < 0$ and with the growth of K, $d\dot{K}/dK$ will become negative, too. Second, choosing the initial values for λ and μ properly one ends up with $\dot{K} = \dot{B} = \dot{\lambda} = \dot{\mu} = 0$. Since the last two equations

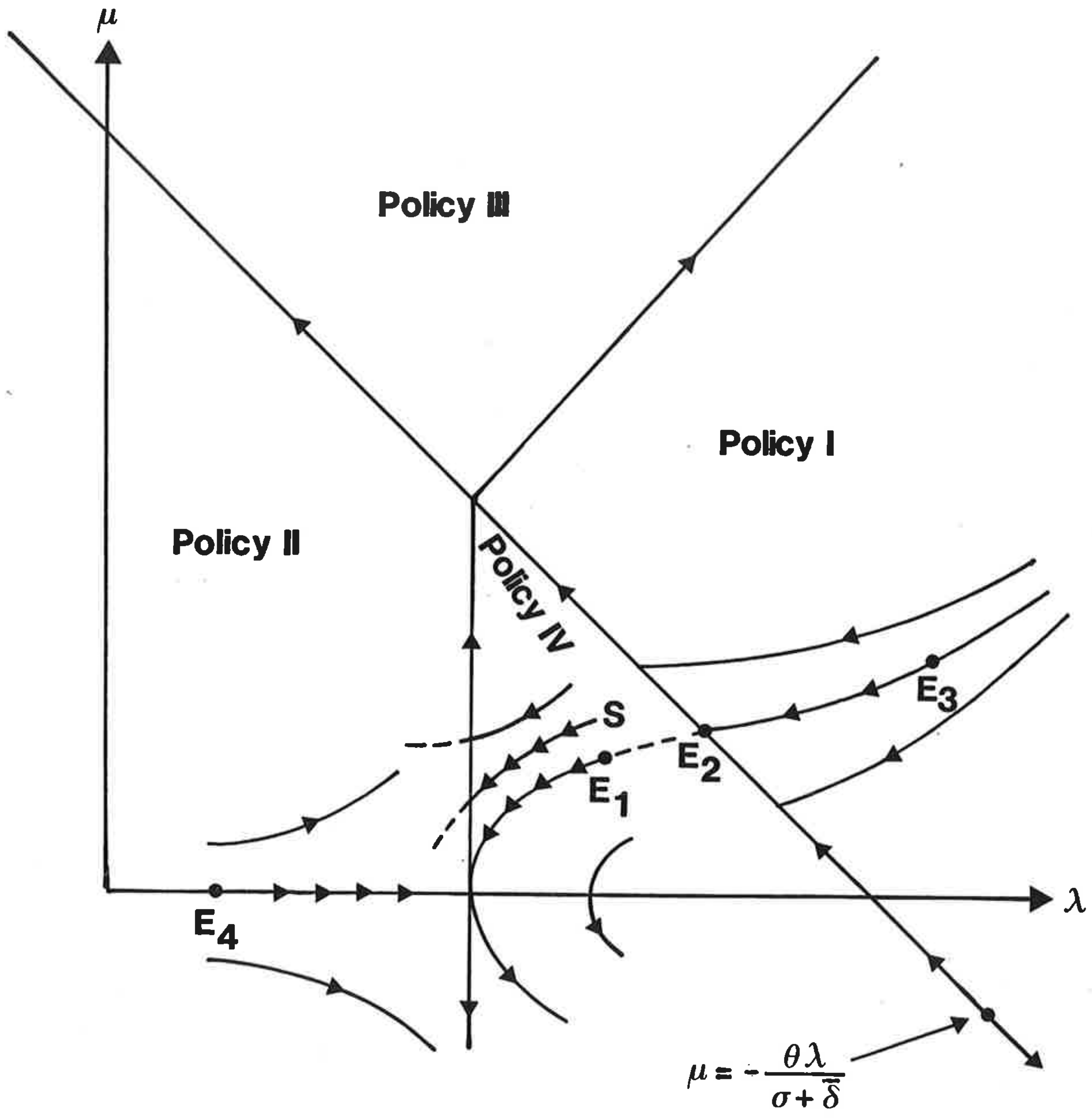

Figure 7. Costate Space and the Feasible Policies, Case II.

are inherently unstable, this observation suggests that the rest point is of the saddle-point type. One also knows that around this point, say $(K^0, B^0, \lambda^0, \mu^0)$, the linearized version of the system (5,4) behaves as

$$(5.5) \quad \begin{bmatrix} K(t) - K^0 \\ B(t) - B^0 \\ \lambda(t) - \lambda^0 \\ \mu(t) - \mu^0 \end{bmatrix} = F \begin{bmatrix} \phi_1 \exp(\omega_1 t) \\ \phi_2 \exp(\omega_2 t) \\ \phi_3 \exp(\omega_3 t) \\ \phi_4 \exp(\omega_4 t) \end{bmatrix}$$

where $\phi_1, \ldots, \phi_4$ are arbitrary constants, $\omega_1, \ldots, \omega_4$ are the eigenvalues of the Jacobian matrix, say J, of (5.4) evaluated at $(K^0, B^0, \lambda^0, \mu^0)$ and F is the matrix of the characteristic vectors of the Jacobian corresponding to the eigenvalues (i.e. $F^{-1}JF = \mathrm{diag}(\omega_1, \ldots, \omega_4)$). At this point it is reasonable, however, to notice that the firm probably gives up Policy I long before it approaches point $(K^0, B^0, \lambda^0, \mu^0)$. Hence it may not be a good idea to rely too much on the eigenvalues of the linearized system.[16)] Second, the arm on which the initial values for $\lambda(t_0)$ and $\mu(t_0)$ are to be chosen does not need to satisfy the saddle-path property but instead the following condition. Recall that the firm has to switch from Policy I to Policy IV at time, say t^1, $t_0 < t^1 < t_1$. Hence, there is a unique point, call it E_2, on the boundary of these policies (see figure 7) where the system has to be at $t = t^1$. Thus, for Policy I, there is the terminal condition at $t = t^1$, $\mu(t^t) = 1 - \lambda(t^1)$. To pick up the optimal path, the firm has hence to choose $\lambda(t_0)$, $\mu(t_0)$ (given $K(t_0)$, $B(t_0)$) on the arm (which presumably is unstable) that satisfies this terminal condition. With a bit of imagination, it could be a point like E_3 in figure 7.

While in Case I Policy III could be eliminated from the sequence which made up the optimal control solution due to continuity of the costate variables, the same argument cannot, to be strict, be used here. One way to exclude Policy III here might be to argue that a really exceptional disturbance is needed to locate the initial point in the regime represented by this policy. This is not very convincing, however. In the absence of a mathematical proof that the line $\mu = \theta\lambda/(1-\theta)$ could not serve as the initial state for Policy I, we disregard Policy III simply because it is very counterintuitive and implausible by nature.

4. Policy II

Summarize Policy II as

$$\alpha_2 = \beta_1 = 0, \quad 1 - (\lambda + \mu) > 0, \quad \lambda < 1 - \theta$$

$$D = (1-\theta)\pi(K) - \chi(K), \quad \delta^* = 0.$$

This is again <u>a contraction phase</u>. Since the dynamics can be described by the same system as given in (4.4) with the associated phase diagram in figure 5, these are not repeated here. It is sufficient to state in terms of figure 7 that along the λ-axis there is a unique point, say E_4, where the initial value for λ has to be chosen to satisfy the optimality of the resulting path.

5. Case II. Conclusions

In contrast to the previous case, we have shown that for firms operating under the conditions of Case II, no unclaimed tax allowances arise during the expansion phase. However, the equilibrium policy is very much the same as it was for firms operating under the conditions of Case I with unclaimed tax allowances. Consequently, the conclusions of the tax effects remain unchanged. The current results support the view presented in Case I that the relationship between the marginal valuation of capital and the current rate of investment may be quite complex. In Policies IV and II there is no relationship (because investment is kept constant or zero) while it is negative in Policy I (because investment is accelerating along the adjustment path).

Earlier the Bergström-Södersten inefficiency proposition was qualified in the presence of a nonprofit-based local tax. Another qualification arises from Case II. Since for firms in the expansion phase it is optimal to use all the possible tax allowances, these allowances have a cash-flow effect on the current rate of investment even if there is no cost-of-capital effect. (Note that these tax effects are imbedded in the cash-flow variable used successfully by Bergström and Södersten in their empirical work).

VII FINAL REMARKS

All the results in the traditional theory of corporate taxes rest on the assumption that there always exist intramarginal profits against which firms claim all of their potential tax allowances. The evidence in the U.K. and in the U.S. has, however, suggested that tax exhaustion has been an important phenomenon. The recent theoretical literature Mayer (1986) and Auerbach (1986)) suggests in turn that the tax effects need a new evaluation. In the Scandinavian tax system unclaimed tax allowances are, partly, the Scandinavian counterpart of tax exhaustion. Again the traditional results need not apply, as discussed in the current paper. Leaving tax allowances unclaimed may be a rational policy when there are losses because then these losses can be fully claimed against future profits later on. This may indeed be a sensible alternative to relying on the loss-offset provisions which are always incomplete. A more symmetric tax treatment of profits and losses (apart from the interest factor) is one advantage of the Scandinavian tax system with non-predetermined rates of tax allowances. We have excluded this aspect from our model, which has focused on the dividend preferences and the dividend constraints in the determination of unclaimed tax allowances.

Given that firms tend to rely on internal sources of finance, a tax system with a non-predetermined tax allowance rates better guarantees financial resources for investments when the firms want to invest than a system with predetermined rates of tax allowance. And this is obtained without distortion in the long-term capital intensity (under a profits tax). This may be important if the incomplete loss-offset provisions tend to raise the effective tax rates on capital. But it should not lead to a situation with abnormally low effective tax rates on profits as it apparently has led to in practice . Moreover, from the stabilization point of view it is possible that the non-predetermined tax allowance rates make investments more cyclical than the predetermined rates do (no empirical evidence on this, however, has been presented).

Relatively free adjustment of reported profits has led in practice to a situation where actual and reported profits have very little to do with each other. If filtering of actual profit figures provides misleading signals to the stock market, this potentially leads to welfare losses. This is one of the aspects in the Scandinavian allowance system where further evaluation is urgently needed. It is plausible that analysts and experts are able to judge the real situation of the firms. But it is very obvious that less-informed ordinary investors cannot. Whether and to what extent the stock prices are influenced by the latter group determines how seriously this problem ought to be taken.

Footnotes:

1) Overdepreciation relative to the economic depreciation gives rise to deferred tax payments. This is, in effect, an interest-free loan from the government. As pointed out by Södersten (1980), government's participation in financing in this way can be justified due to an incomplete allowance for the cost of internal equity. Note that using accelerated depreciation rates will diminish the tax allowances in the later stages of an asset's life.

2) This follows because due to the stock of unclaimed allowances the taxable profits generated by the marginal project can be adjusted to zero. Bergström and Södersten go on to argue that the main impact of the Swedish tax system has been in providing a questionable intramarginal subsidy for capital.

3) The current model abstracts from several issues. For example, by focusing on the corporation tax it does not deal with the differential tax treatment of distributed and undistributed profits in personal or corporate taxation. But as it stands, the model is complicated enough to justify the simplifications chosen.

4) It is most sensible to formulate the control problem without imposing $K(t_0) = B(t_0)$. This is because $K(t)$ and $B(t)$ usually differ along the adjustment path. Thus, a reformulation of the optimal policy after any disturbance, say at point t', has to start with the assumption $K(t') \neq B(t')$. But we make the assumption that $K(t_0)$ and $B(t_0)$ are large enough to justify our focus on the case where both $K(t) \geq 0$ and $B(t) \geq 0$ for all $t \geq t_0$.

5) With some minor qualifications, this restriction is part of the Scandinavian corporate tax system. In the case of the U.K. tax system a dividend constraint has been discussed earlier by Edwards and Keen (1985).

6) Note that these types of preferences may spur the growth of the firm in the absence of direct borrowing opportunities while maintaining the net present value of the marginal project equal to zero. Our formulation is not actually different from neoclassical models. While the latter implicitely often assume negative dividends in the initial state, we are explicit about this. Second, while the neoclassical models based on adjustment costs imply a constant stream of dividends in equilibrium attained only after on infinite amount of time, the equilibrium in our model is attained in finite time.

7) Note that the upper limit for D(t) actually implies that the standard irreversibility condition for investment is automatically satisfied.

8) These functions are continuously differentiable within all policy regimes which sequentially form the optimal path if $\alpha_1(t)$, $\beta_3(t)$ and the instruments are continuously differentiable within each regime. This is indeed the case. For example, in the regimes where $\alpha_2(t) > 0$ one has $\alpha_1(t) = 0$, while in the other regimes $\alpha_1(t) = 1 - \lambda(t) - \mu(t)$. Finally, if $\beta_3(t) \neq 0$, one has $\beta_3(t) = [(\lambda(t) + \mu(t) + \alpha_1(t))\theta - (\mu(t) + \alpha_1(t))] B(t)$.

9) Since the equilibrium can be produced, it must, trivially, exist. But it is very crucial to observe that in the current model it is not obtained as the limit process $\lim_{t\to\infty} K(t)$, $\lim_{t\to\infty} B(t)$. Instead, one expects from (2.4)´ that there is a finite time $(t = t_1)$ when a discontinuous jump in investment takes place with the consequence that the firm switches to the equilibrium policy.

10) Note that this result stands in contrast to the standard neoclassical model which claims that the gap between the targeted and actual capital stock will be eliminated over time via diminishing gross investment.

11) Given the initial values, the dynamic system also determines the time when the switch to Policy IV takes place. But due to nonlinearities, it cannot be solved explicitely.

12) It is not our intent to go into a detailed discussion of the policy in the bankruptcy case. We concentrate on the case where current earnings are sufficient so as to satisfy $D \geq \underline{D}$ (where $\underline{D} \gtrless 0$). But note that now the case of tax exhaustion arises if the earnings generated both by the marginal and intramarginal profits are such that $\pi \leq 0$. Then taxes are regarded as a subsidy. Hence, the model assumes perfect symmetry between the tax treatment of positive and negative profits.

13) Nothe that the system actually has a stable equilibrium at point A with zero capital. But it is not on the optimal path if $\pi_K(0) > \sigma + \delta + \chi_K(0)/(1-\theta)$, as it can be assumed here.

14) The negative eigenvalue $-\delta < 0$ is associated with the stable equation for $\dot{K}$ while the unstable eigenvalue $\sigma + \delta > 0$ is associated with the unstable equation for $\dot{\lambda}$. There is a third eigenvalue $\sigma > 0$ associated with the last equation of (5.2). Since none of these is zero or purely imaginary, the linearized model provides information on the behavior of the non-linear model around E_0^{-} (cf. the Hartman-Grobman theorem in Guckenheimer-Holmes (1983) p. 13).

15) If our system (5.2) were completely simultaneous, there would be a surface in the (K,λ,μ)-space where the starting values ought to be chosen. Or, given K_0, there would be a curve on the plane spanned by (λ,μ). But in the case of (5.2), $\lambda(t_0)$ can be obtained by integrating backwards with the terminal condition $\lambda(t_1) = 1-\theta$. Similarly, the initial value $\mu(t_0)$ can be obtained by integrating backwards with the terminal value $\mu(t_1) = 0$.

16) Based on Dockner (1985) one can solve these eigenvalues. He also gives necessary and sufficient conditions for the saddle-point property.

Appendix. Derivation of the singular policies (figure 4 in the text).

Consider the limiting case of Policy I with $\alpha_2 = 0$ (in addition to having $\alpha_1 = \beta_2 = 0$). Then (2.10) gives $\lambda + \mu = 1$ and this policy represents movement along the $\mu = 1 - \lambda$ line. From (2.13) μ obeys $\dot{\mu} - \sigma\mu = 0$, which has an unstable equilibrium at $\mu = 0$. Hence, the arrows point away from the λ-axis. As another limiting case of Policy I consider instead the case where $\beta_3 = 0$. Then both sides of (2.11) are equal to zero, implying that the policy represents movement along the line $\mu = \theta\lambda/(1-\theta)$. (2.13) tells that μ again obeys $\dot{\mu} - \sigma\mu = 0$ providing the direction of the movement. Finally, consider the limiting case of Policy IV with $\beta_3 = 0$. Then (2.10) and (2.11) imply that the movement takes place along $\lambda = 1-\theta$. Since from (2.13) μ has to satisfy $\dot{\mu} - \sigma\mu = 0$, the movement is away from the $\mu = 0$ line.

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