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ON INSTABILITY OF A KEYNESIAN MACRO MODEL: SOME NOTES

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Abstract

The paper studies the feasibility of using monetary policy to control a Keynesian macro system exhibiting a strong acceleration effect in aggregate investment. The equilibrium of this model is - a bit surprisingly - of the saddle-point type. However, because there are no forward-looking jump variables in the model, the monetary policy is of help in controlling the system only if it is feasible to adjust the policy with the same frequency as there are demand disturbances.

I Introduction

It is a well-known textbook proposition (cf. Branson (1979) pp. 238-239) that the acceleration effect in the aggregate investment equation may give rise to global instability in a standard short-run Keynesian macro model of output determination. As studied in more detail by Buiter (1977), the equilibrium will be unstable in the sense that the Routh-Hurwitz conditions are not satisfied if the IS locus is upward-sloping and steeper than the LM-locus. Even a somewhat weaker acceleration effect may also give rise to instability depending on the relative speeds of adjustment of various markets in the case where the slope of the IS locus does not exceed that of the LM locus.

The instability problem discussed above may, for example, become relevant under the following conditions. First, the propensity to invest out of current cash flow may be quite high if the Keynesian view of destabilizing business expectations hold in the sense that current expansion of demand is interpreted as an indication of accelerating future demand. The same conclusion may follow if firms are liquidity constrained and rely on internal financing for their investments. Second, if the monetary policy is of the "Keynesian neutral type" (see Kyle (1976) for this terminology) in the sense that it aims at stabilizing the market rate of interest, this obviously tends to flatten the LM locus.

However, what has been left unnoticed is the fact that, actually, global instability is not a logical consequence of a strong acceleration effect. First, as Schinasi (1982) has shown, an introduction of a non-linear acceleration effect in a Kaldor-type investment function gives rise to a limit cycle. Second, and this is what we suggest in the current paper, in the case of a linear acceleration effect monetary policy can be used to stabilize the model. This possibility arises because actually the equilibrium studied by Buiter (1977) is a saddle-point as we show below. This suggests various ways in which monetary policy can be used to create an interest rate effect which tends to eliminate overestimation of aggregate demand by working against the acceleration effect and the expectations effect.

The standard prototype macroeconomic models of today with forward-looking behavior and rational expectations have the key property that the dynamic variables can be classified as freely determined, or "jump" variables, and as predeterminal state variables. A representative example of this class of models is Blanchard (1981). The jump variables, like stock market values or exchange rates, are "unstable" forward-looking variables while the predetermined variables are "stable", backward-looking variables. The convergence to a stationary equilibrium is guaranteed by the ability of market participants to force the jump variables to attain the "right" values.

The saddle-point property is obtained in this paper in a model which, interestingly enough, does not have any forward-looking jump variables. Moreover, the unstable variable is actually predetermined. However, convergence can not be obtained through monetary policy by directly altering any market variables but by adjusting the policy rule. The informational requirements for policy-makers are the same as those in the rational expectational models, i.e. the policy-makers are assumed to know the structure of the model or to learn it in the sense of Gottfries (1985). We present the implications of the saddle-point property in Buiter's model because this has not been done before and because versions of the model are widely used (cf. for example Blinder (1977)). It is clear that the model abstracts from the role of the flexible jump variables in the determination of the short-run equilibrium and hence departs from the today's rational expectations models. Expectations play a role in the acceleration effect, but they are of a more traditional type. The exception is the saving function which postulates full tax discounting in the sense that government debt is not regarded as private wealth. The latter assumption is, however, only made to simplify the model.

II The Conditions for the Saddle-point Property in Buiter's Model

Buiter's macro model with a non-adjusting capital stock can be presented as

(1) $\dot{Y} = \gamma_1[I - S] \quad \dot{R} = \gamma_2[L - M] \quad \gamma_1 > 0, \gamma_2 > 0,$

where $\dot{Y}$ stands for the rate of adjustment of output per unit of time and $\dot{R}$ similarly for the rate of interest on bonds. Moreover,

(2a) $I = I[E(R,Y,K) - p_K K] \quad E_R < 0, E_Y > 0 < E_K < I,\ I' > 0$

(2b) $S = S[\mu Y - M - E(R,Y,K)] \quad S' > 0,\ S(0) = 0, \mu > 0$

(2c) $L = L(R,Y) \quad L_R < 0,\ L_Y > 0$

Investment (I) is modelled as depending on the difference between the market value of claims against the existing capital stock (E) and the reproduction cost of capital ($p_K K$). Saving (S) depends positively on the remainder of the desired wealth (proportional to current income) after subtracting existing wealth. L stands for the conventional liquidity preference function and M for the supply of nominal money balances. In anticipation of the subsequent analysis we augment Buiter's model by a money supply rule

$$(3) \qquad M = \theta_o + \theta(R) \qquad\qquad \theta_o,\ \theta_R \geq 0$$

which makes the money supply interest-elastic. There are a number of mechanisms in the determination of the supply of money which justify this assumption.

The equilibrium loci of (1) have their slopes as

$$(4a) \qquad \left.\frac{dR}{dY}\right|_{IS} = - \frac{I'E_Y - S'(\mu - E_Y)}{(I' + S')E_R + S'\theta_R} \gtrless = 0$$

$$(4b) \qquad \left.\frac{dR}{dY}\right|_{LM} = \frac{-L_Y}{L_R - \theta_R} > 0 .$$

Write the linearized version of (1)-(2) as follows

$$(5) \qquad \begin{pmatrix} \dot{Y} \\ \dot{R} \end{pmatrix} = J \begin{pmatrix} Y - \tilde{Y} \\ R - \tilde{R} \end{pmatrix}$$

where $\tilde{Y}$ and $\tilde{R}$ denote the equilibrium values (assumed to exist) and J denotes the Jacobian evaluated at the equilibrium. The Routh-Hurwitz stability conditions read as

(6a) $$\gamma_2/\gamma_1 > -\frac{I'E_Y - S'(\mu - E_Y)}{L_R - \theta_R}$$

and

(6b) $$I'E_Y - S'(\mu - E_Y) < \frac{L_Y((I'+S')E_R+S'\theta_R)}{L_R - \theta_R} .$$

In stydying the stability conditions one observes that a sufficiently high interest rate sensitivity of the monetary policy rule (θ_R) may give rise to an upwardly sloping IS curve. The mechanism is the following. An increase in income expands investment demand via the acceleration effect. Hence, the interest rate ought to increase to leave investment demand unchanged. But an increase in income also gives rise to an increase in saving and this would necessitate a reduction in the rate of interest to keep saving unchanged. If the effect of income on saving is larger than on investment, one needs a reduction in the rate of interest to maintain the equilibrium. The monetary policy rule (3), however, counteracts this reduction and if sufficiently strong may actually lead to an upwardly sloping IS curve.

Since in this paper we are interested in an upwardly sloping IS curve due to the acceleration effect and not due to the monetary policy stance, we work with the assumption

(7) $$\theta_R < \frac{(I' + S')|E_R|}{S'}$$

which means an upper bound to the interest sensitivity of monetary policy and which maintains the negative slope of the IS curve in the absence of the acceleration effect.

Turning to the stability conditions one notes that the negativity of the slope of the IS curve, i.e.

$$(8) \qquad I'E_Y - S'(\mu - E_Y) < 0$$

is sufficient for (6a) and (6b) to hold. The equilibrium of system (5) is a stable node. Then the Hartman-Grobman theorem implies that the non-linear system (1)-(2) is also locally stable. But if (8) does not hold, the right-hand side of (6a) is positive and the ratio γ_2/γ_1 has to be high "enough". In the latter case, also the IS curve has to be less steep than the IM curve for (6b) to hold as obtained by Buiter (1977), too.

What about the case where (6b) does not hold, i.e. where the IS curve is steeper than the IM curve? One finds that det(J) < 0, i.e. the equilibrium is a saddle-point.[1] Note that in this case the $\dot{Y}$ equation in (1) is unstable while the $\dot{R}$ equation is stable, i.e. $E_Y > S'\mu/(I'+S')$ and $L_R < \theta_R$.

The eigenvalues of matrix J, say λ_1 and λ_2 are found as roots of the following equation

$$(9) \qquad \lambda^2 - \lambda\phi_1 + \phi_2 = 0$$

where

$$(10) \qquad \begin{aligned} \phi_1 &= \gamma_2(L_R - \theta_R) + \gamma_1[I'E_Y - S'(\mu - E_Y)] \\ \phi_2 &= \gamma_1\gamma_2(L_R-\theta_R)(I'E_Y-S'(\mu-E_Y))-\gamma_1\gamma_2L_Y[(I'+S')E_R+S'\theta_R]. \end{aligned}$$

Since det(J) < 0, we know that the roots are real and of opposite sign. Choose $\lambda_1 < 0, \lambda_2 > 0$.

The time paths for Y(t) and R(t) are now given by

$$
(11) \qquad \begin{aligned} Y(t) &= \theta_{11} e^{\lambda_1 t} + \theta_{12} e^{\lambda_2 t} + \tilde{Y} \\ R(t) &= \theta_{21} e^{\lambda_2 t} + \theta_{22} e^{\lambda_2 t} + \tilde{R} . \end{aligned}
$$

The transversality condition requires setting $\theta_{12} = \theta_{22} = 0$. Then the slope coefficient of the saddle-path

$$
(12) \qquad R - \tilde{R} = (\theta_{21}/\theta_{11})(Y - \tilde{Y})
$$

can be found by solving the right-hand eigenvector $x = [x_1 \quad x_2]'$ corresponding to the stable root from

$$
(13) \qquad J \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} .
$$

One obtains

$$
(14) \qquad \theta_{21}/\theta_{11} = x_1/x_2 = \frac{\gamma_1[I'E_Y - S'(\mu - E_Y)] - \lambda_1}{-\gamma_1[(I'+S')E_R + S'\theta_R]} .
$$

One can immediately conclude the following. First, the slope of the saddle-path is always positive. Second, the saddle-path is always steeper than the IS curve. Finally, one may ask what happens to the slope of the saddle-path when the interest elasticity of the money supply is allowed to change. Note that this slope given in (14) depends on θ_R both directly and indirectly via λ_1. It is tedious but straightforward to show that with a probability of "almost" one it holds that

$$(15) \qquad \partial\lambda_1/\partial\theta_R < 0 \; .$$

III <u>Policy Analysis</u>

Consider a one-shot unanticipated increase in aggregate demand due to, say, "animal spirits". In terms of figure 1, this means that the $\dot{Y} = 0$ locus moves to the left from the original equilibrium denoted by E_0. Two questions can now be raised. First, is there any option left for policy-makers who want to stabilize the economy? Second, can a policy response be found which stabilizes the economy at, say, the target level of income?

The easiest way to guarantee the stability of the economy within the current framework is to originally choose the interest sensitivity of the money supply θ_R in the policy rule in such a way that the resulting LM curve actually is steeper than the IS curve. This provides an upper limit for θ_R and it can be shown that in this case one must have

$$(16) \qquad \theta_R < \frac{-L_Y(I'+S')E_R+L_R[I'E_Y-S'(\mu-E_Y)]}{I'E_Y-S'(\mu-E_Y) + S'L_Y} \; .$$

Note that this may actually reguire that θ_R has to be set less than zero. If the θ_R parameter is chosen to approach the interest sensitivity of the demand for money L_R, this would at the limiting case make the LM curve vertical. This choice generates a strong interest rate effect in aggregate demand which tends to counteract the acceleration effect.

The high variability in the market rate of interest which is the unavoidable consequence of the above policy may not be socially desirable. Moreover, though it makes the policy task easy by allowing the policy-makers to stick to a fixed policy rule, it does not allow them to stabilize income at some targeted level (unless the policy-makers have additional instruments available). If the choice of θ_R such that (16) is satisfied is for some reason ruled out, it is clear that no fixed policy rule like (3) can provide the desired convergence to a new equilibrium. But within the policy rules of type (3) the policy-makers may have the option of varying the parameters θ_o and θ_R so as to guarantee the convergence. This follows from the observation that the equilibrium is a saddle-point.

Hence, suppose that policy-makers stick to the interest sensitivity of the money supply provided by the policy rule (3) but in face of an expansionary demand disturbance use contractive monetary policy by reducing θ_o. Then the LM curve moves up. But note that this time there is also a related shift in the IS curve (see equation (2b)) since an increase in money balances reduces the gap between the target wealth and the actual wealth. Note that this is a stabilizing mechanism in the model because it counteracts the original demand shock. If the money supply is reduced sufficiently so that the new LM

Fig. 1 Adjustment of income and interest rate under money stock control

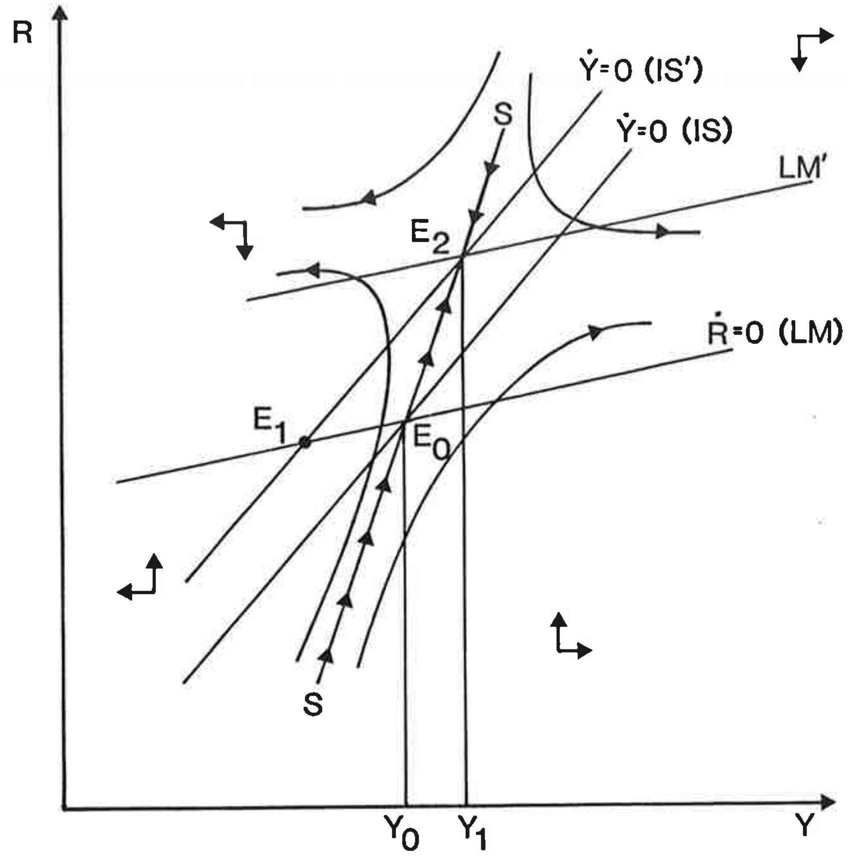
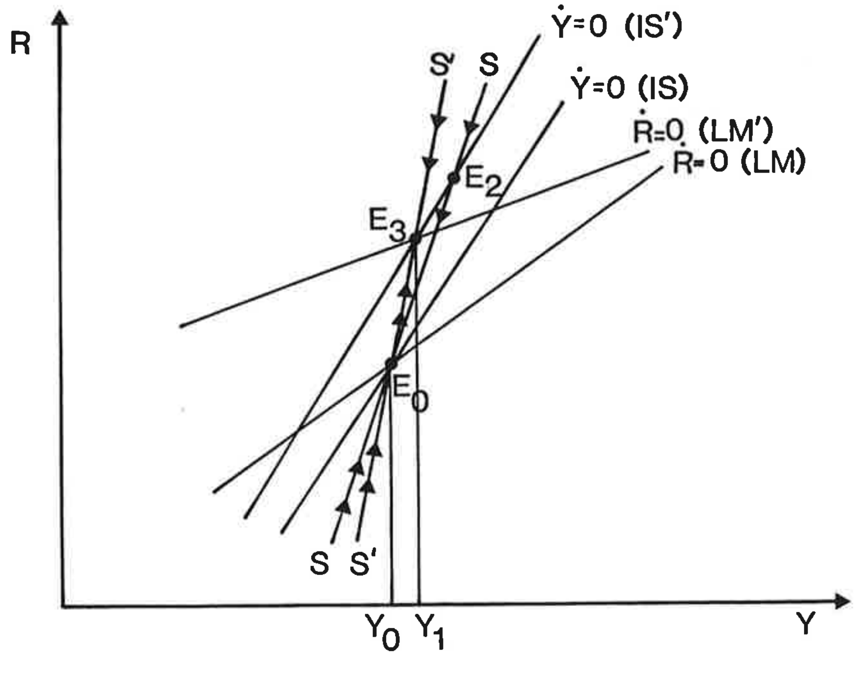


Fig. Adjustment of income and interest rate under combination policy



curve cuts the new IS curve (say IS') at point E_2 in figure 1, one knows that the system will converge to E_2 because the initial point E_0 now is located on the saddle-path of the new system (labelled ss in figure 1). Since the saddle-path has a positive slope, one knows that the level of income is higher at E_2 as compared to the initial state. The expansionary effect on income is measured by the distance Y_0 Y_1 in figure 1.

The policy response described above makes the system dynamically stable, but it does not completely stabilize output. Moreover, depending on the slope of the saddle-path the adjustment in the rate of interest required may also be substantial in this case. Both of these problems can be reduced to some extent if the policy-makers have the option of adopting a combination policy, i.e. a policy of adjusting both the interest elasticity of money supply and the outstanding money stock simultaneously. It is noteworthly that this time both the variability of income and the variability of the interest rate in face of a demand disturbance can be reduced as compared to the above policy if the authorities increase the interest rate sensitivity of the money supply while reducing the outstanding money supply at the same time. The stabilizing effect on income results from the steeper slope of the saddle-path as presented in figure 2. The system is again under control because the initial equilibrium E_0 is located on the stable manifold of the new system. Note finally that the combination policy can be used to move towards - though perhaps not completely achieve - the targeted level of income.

IV Final Remarks

The above analysis has taken place on the "Keynesian" continent, which traditionally has abstracted from the role of forward-looking "jump" variables in the determination of short-run equilibrium. This type of model, which was formalized by Buiter (1977) and utilized by Blinder (1977), for example, continues to be utilized in macroeconomic theorizing. Recently, Schinasi (1981) and (1982) has used this model to show in which way a non-linear acceleration effect generates limit cycles. The current paper has utilized the linear acceleration effect and asked whether the saddle-point instability can be utilized to control the system in the absence of freely determined jump variables. The answer is yes, but this necessitates a very active stabilization policy in the sense that the money supply has to be adjusted at least with the same frequency as there are disturbances in the aggregate demand. We do not know whether this model is closer to "reality" than are the models where the "jump" variables help to attain the convergent path. But this model provides a case against any fixed policy rules and even suggest that the stability of the economy can be increased by adopting a combination type of policy. Note that the informational requirements for policy-makers are the same as those in the rational expectation perfect foresight models.

From the point of view of the feasibility of the required policy responses, one notes that though the interest sensitivity of the money supply can be frequently altered by a variety of policy instruments available there may be fixed costs of using these instruments, which limits the frequency of their use. But in principle the daily open market operations can, of course, be directed to guarantee the desired convergence.

Footnotes

1. The equality sign in (6b) implies either that no equilibrium exists or that the number of equilibrium points is infinite. This case is left out of the subsequent discussion.

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