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TAXATION AND TIMBER SUPPLY UNDER UNCERTAINTY AND LIQUIDITY CONSTRAINTS

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Abstract:

The purpose of this paper is to study the effects of lump-sum, unit and ad valorem taxes on timber supply in a standard two-period consumption-saving model under future timber price uncertainty and under various assumptions concerning the way capital markets operate. The timber supply effects of taxation turn out to be very sensitive both to the introduction of uncertainty and to the question of whether liquidity constraints are operative or not. The tax policies, which will keep the expected present value of government tax revenues unchanged, will give rise to 'liquidity', 'substitution' and 'risk' effects depending on the situation. The importance of liquidity constraints can be illustrated by considering the effects of the shift in timing between the current and future lump-sum taxes; under perfect capital markets this will have no effect, while under liquidity constraints raising the current lump-sum tax and lowering the future lump-sum tax will increase cutting today.

1. Introduction

What are the determinants of timber supply? How do various kinds of tax policies affect and what is the relative effectiveness and timing effects of various taxes in terms of timber supply? These are questions with obvious relevance for the countries where the forest sector plays a major role in the economy. Unfortunately, however, these questions have thus far been subjected only to quite a few theoretical and empirical analyses (see Johansen and Löfgren (1985) for a recent survey).

As far as the theoretical analysis of timber supply is concerned, Johansen and Löfgren (1985) have made some preliminary analysis. More specifically, they have looked at the timber supply both from the point of self-employed forest farmer in a static model and from the point of view of expected utility maximizing forest owner under uncertainty about the future price of timber in an intertemporal model. They also present some analysis about the effects of various taxes, but they consider neither the relative effectiveness nor the timing effects of taxes. On the whole, while the analyses in Johansen and Löfgren (1985) are clearly an important start in the attempts to formulate 'good' theories of timber supply, these analyses suffer from some weaknesses. First, the introduction of income constraints into the static model of the self-employed forest farmer is not wholly satisfactory, mainly because it leaves open the question where the income constraints come from (see Johansen and Löfgren (1985) p. 140-149). An explicit introduction of 'imperfect' capital markets into the model provides a natural way to answer this question and moreover, it may give new insights into the role of taxes. Second, in an intertemporal model under future timber price uncertainty they analyse timber supply by assuming that forest owners try to maximize the expected utility of the present value of profits over two periods (see Johansen and Löfgren (1985), p. 268-275). Thus there is a kind of separability assumption in the sense that timber supply does not explicitly depend on the consumption preferences of the forest owner. While there are cases, where this separability assumption can be justified, it is far from a general case. In particular, by assuming the the forest owner tries the maximize the expected utility of consumption, one can see that the separability breaks down when either the uncertainty in the case of risk-averse agents and/or 'imperfect' capital markets are introduced. Moreover, by formulating the decision problem of the expected utility-maximizing forest owner is terms of maximizing the expected utility of consumption is helpful in the sense of providing a natural framework to explicitly account for liquidity constraints.

The purpose of this paper is to consider the effects of lump-sum, unit and ad valorem taxes on timber supply in the standard two-period model of consumption and saving under future timber price uncertainty. More specifically, we are interested in the following questions: First, what are the relative effectiveness and timing effects of lump-sum, unit and ad valorem taxes on timber supply? And second, do the liquidity constraints matter for the tax policy and if so, in what way?

To anticipate results it turns out that the relative effectiveness of various taxes and particularly their timing effects are very sensitive both to the introduction of uncertainty and to the way capital markets operate. We proceed and follows. Section 2 develops the effects of tax policies in the presence of perfect capital markets, while section 3 is devoted to a preliminary analysis of timber selling under liquidity constraints. Finally, there is a brief concluding section.

2. Timber supply and taxation under future timber price uncertainty and perfect capital markets

2.1. Some analytics of timber supply

Before proceeding to an analysis of timber supply under future timber price uncertainty we develop quickly some qualitative results of timber supply under certainty and perfect capital markets. This serves as a sort of finger exercise and lays ground for subsequent analyses.

The forest owner is assumed to have a preference ordering over present and future consumption c_1 and c_2, which is represented by an intertemporally additive utility function $V = u(c_1) + (1+\rho)^{-1}u(c_2)$, where $(1+\rho)^{-1}$ = rate of time preference and $u' > 0$ and $u'' < 0$. The present and future income y_1 and y_2 are assumed exogenously given and $R = 1 + r$ = the interest rate factor in the capital market. The forest owner chooses between cutting and selling 'today' x and cutting and selling 'tomorrow' z with prices p_1 and p_2 respectively. Partial derivatives are denoted by subscripts for functions with many variables and by primes for functions with one variable. Following Johansson and Löfgren (1985) (p. 268) future cutting is assumed to be a function of current cutting so that $z = f(x)$ with $f', f'' < 0$. Thus the 'production function' is strictly concave and twice continuously differentiable and in a two-period model a given level of today's cut implicitly determines the level of tomorrow's cut. Finally, denote the lump-sum, unit and ad valorem taxes in period i (i = 1,2) by T_i, t_2 and τ_2 respectively. Assuming the forest owner is a price taker in all markets the decision problem is to choose current and future consumption and current cutting so as to maximize $V = u(c_1) + (1+\rho)^{-1}u(c_2)$ under the budget constraint

$$(1) \qquad c_2 = p_2^0 f(x) + y_2 - T_2 + (I+r)(p_1^0 x + y_1 - T_1 - c_1)$$

where $p_i^0 = p_i(1-\tau_i) - t_i \quad (i = 1,2)$.

Under these conditions of perfect capital markets and certainty the timber supply decision is separable from the preferences of the forest owner and is defined implicitly by $p_2^0 f'(x) + p_1^0(1+r) = 0$. The comparative statics in terms of taxation is shown in Table 1:

Table 1: Comparative statics of timber supply under separability

	T_1	T_2	t_1	t_2	τ_1	τ_2
x	0	0	-	+	-	+

The lump-sum taxes will have no effect, while the current (future) unit and ad valorem taxes affect negatively (positively). Thus shifting the tax base in both unit and ad valorem taxation towards current taxes will increase timber supply. Moreover, unit and ad valorem taxation are equivalent in terms of their relative effectiveness so that shifting the tax base between current (or future) unit and ad valorem taxes so as to keep the present value of government tax revenues constant will have no effect on cutting decision.

Next we turn to analyze the timber supply by assuming that the future timber price is stochastic so that the forest owner determines how much to cut today, given today's known price and a probability distribution of tomorrow's uncertain price. In what follows we also assume the the forest owner is risk-averse and the utility function three times differentiable. The problem is now to choose c_1 (and c_2) and x so as to maximize $V^* = u(c_1) + (1+\rho)^{-1}E(u(c_2))$ where c_2 is now stochastic because of uncertainty associated with p_2, and where E denotes the expectations operator (see Lippman and McCall (1981) for an analysis of uncertainty effects in a standard two-period model of consumption and saving).

The first-order conditions for the expected utility maximization in terms of c_1 and x can be expressed as follows

$$(2)\quad \begin{cases} \text{(i)} \quad V_c^* = u'(c_1) - \beta E(u'(c_2)) = 0 \\ \text{(ii)} \quad V_x^* = (1+\rho)^{-1}E(u'(c_2)e) = 0 \end{cases}$$

where $\beta = (1+r)/(1+\rho)$ and $e = (p_2(1-\tau_2) - t_2)f'(x) + (p_1(1-\tau_1) - t_1)(1+r)$.

Now $V_x^* = 0$ is equivalent to $E(u'(c_2))\bar{e} + \text{cov}(u'(c_2),e) = 0$ where $\bar{e} = (E(p_2)(1-\tau_2)-t_2)f'(x) + (p_1(1-\tau_1) - t_1)(1+r)$, and where $\text{cov}(u'(c_2),e) = \text{cov}(u'(c_2),p_2)(1-\tau_2)f'(x) > 0$ because of risk aversion. This finding has two important implications: First, at the interior solution $\bar{e} < 0$ so that introducing uncertainty will have the effect of increasing cutting today, as was pointed out by Johansson and Löfgren (1985) (p. 271). Second, and more importantly, in the case of risk aversion the cutting decision is no longer separable from the consumption preferences of the forest owner.

The second-order conditions for the expected utility maximization are

$$
(3)\quad \begin{cases} \text{(i)} & V^*_{cc},\ V^*_{xx} < 0 \\ \text{(ii)} & \Delta = V^*_{cc}V^*_{xx} - V^{*2}_{cx} = (1+\rho)^{-1}u''E(u''e^2) + \beta^2 Q > 0 \end{cases}
$$

where $Q = E(u'')E(u''e^2) - (E(u''e))^2 > 0$ due to the Cauchy-Schwarz inequality.

In what follows we assume that the Arrow-Pratt measure of absolute risk aversion $A(c_2) = -u''(c_2)/u'(c_2)$ is decreasing in c_2, which can be regarded as widely plausible. On the other hand, as far as the relative risk aversion $R_c = A(c_2)c_2$ is concerned, there seems to be no consensus on how it varies in terms of its argument which is why we keep to the assumption of constant relative risk aversion as the benchmark case, (see e.g. Machina (1983) for details). It is straightforward to show that $E(u''(c_2))e) < 0$ under decreasing absolute risk aversion.

Turning to the analysis of the relationship between timber supply and taxation consider first the lump-sum taxes. Their effects are

$$
(4)\qquad x_{T_1} = \Delta^{-1}\, u''E(u''e) > 0
$$

and $x_{T_2} = (1+r)^{-1}x_{T_1} > 0$. In contrast to the certainty case lump-sum taxes will have a positive effect on timber cutting today; e.g. a rise in T_1 will decrease c_2, ceteris paribus, so that risk-taking in the sense of postponing timber cutting for tomorrow will decrease.[1)]

In the case of unit taxation the timber supply effects can be expressed as

$$(5)\quad \begin{cases} \text{(i)} \quad x_{t_1} = \underset{(+)}{x x_{T_1}} + \underbrace{\Delta^{-1} V^*_{cc} \, E(u^{\prime\prime})}_{(-)} = ? \\[2ex] \text{(ii)} \quad x_{t_2} = \underset{(+)}{f(x) x_{T_2}} + \underbrace{\Delta^{-1}(1+\rho)^{-1} V^*_{cc} E(u^{\prime\prime}) f^{\prime\prime}(x)}_{(+)} > 0 \end{cases}$$

Raising the unit taxes will tend to increase timber supply via the 'wealth effect', but the 'substitution effect' runs counter to it in the case of current unit tax and reinforces it in the case of future unit tax.

In the case of ad valorem taxes we have $x_{\tau_1} = p_1 x_{t_1} = ?$ because of offsetting 'wealth' and 'substitution' effects. Before developing the implications of a change in the future ad valorem tax it will turn out to be convenient to look first at the effect of a change in the degree of uncertainty about future timber price.

Consider the effect of a multiplicative shift in the distribution of p_2, which is offsetted by an additive shift to restore the mean of p_2, $E(p_2)$, to its initial value. Such a shift can be interpreted as a mean-preserving change in risk and is defined for $\tilde{p}_2 = \varepsilon + \eta p_2$ by $d\varepsilon/d\eta = -E(p_2)$ at $\varepsilon = 0, \eta = 1$. By utilizing the result according to which for two random variables k and l $E(kl) = E(k)E(l) + \text{cov}(k,l)$ the effect of a change in risk can be expressed as

$$(6)\qquad x_\eta \Big|_{\substack{\varepsilon=0 \\ \eta=1}} = \Delta^{-1} [-V^*_{cc} V^*_{x\eta} + V^*_{cx} V^*_{c\eta}]$$

where

$$
(7)\quad \begin{cases} \text{(i)} & V^{*}_{c\eta} = -\beta(1-\tau_2)f(x)\mathrm{cov}(u''(c_2),p_2) < 0 \\ \text{(ii)} & V^{*}_{x\eta} = (1+\rho)^{-1}(1-\tau_2)\{f'(x)\mathrm{cov}(u'(c_2),p_2) + \\ & \qquad f(x)\mathrm{cov}(u''(c_2)e,p_2)\} \end{cases}
$$

and where $V^{*}_{cx} = -\beta E(u''e) > 0$. Thus the sign of the equation (6) depends on the sign of $V^{*}_{x\eta}$. In an appendix it is shown that

$$
(8)\qquad V^{*}_{x\eta} \gtreqless 0 \quad \text{as} \quad R_c \lesseqgtr (2/\alpha) - 1
$$

where

$$
(9)\qquad \alpha \equiv a[1 + (1/f'(x))(p^0_1(1+r)/p^0_2)]
$$

with $a = p^0_2 f(x)/c_2$ = the fraction of future consumption accounted for by future net sales revenue from timber. The interior solution (2ii) implies that $1 > 1 + (1/f'(x))(p^0_1(1+r)/p^0_2) > 0$. As far as the relative risk aversion is concerned, these estimates, while varying widely, all come up with the conclusion that it is well above one (see e.g. Machina (1983) for further details).

Thus the effect of a change in risk associated with future timber price on cutting today remains ambiguous a priori so that the proposition 12.5. (p. 273) in Johansson and Löfgren (1985) does not necessarily carry over to an intertemporal setting where the forest owner faces not only the cutting decision, but also the saving decision. The economic reason for the ambiguity of a change in risk might go as follows: a rise in future timber price uncertainty may affect current and future consumption and current cutting and it is not possible to say which way the effects go at this level of generality. $V^*_{x\eta} \leq 0$, which means that $R_c \geq (2/\alpha) - 1$, is a sufficient, but not a necessary condition for the negative effect of a change in risk on cutting today. If relative risk aversion is high (low) and/or the fraction of future consumption accounted for by the future net sales revenue from timber supply is high (low), then e.g. an increase in risk will tend to decrease (to affect ambiguously) the current timber supply.

Finally we are now in a position the develop the comparative statics of timber supply in terms of future ad valorem tax τ_2, which can be expressed as

$$(10) \qquad x_{\tau_2} = \underset{(+)}{E(p_2)x_{t_2}} - (1-\tau_2)^{-1} \underset{(?)}{\left(\frac{\partial x}{\partial \eta}\bigg|_{\substack{\varepsilon=0 \\ \eta=1}}\right)} = ?$$

The future ad valorem taxation will affect cutting today via the positive 'wealth' and 'substitution' effects, while the 'risk' effect is ambiguous for the reasons explained above. The latter 'risk' effect is a novel one and results from the fact that under uncertainty the level of taxation and risk will have a negative relationship.

For convenience, we summarise the results in Table 2.

Table 2: Comparative statics of timber supply under future price uncertainty.

	T_1	T_2	t_1	t_2	τ_1	τ_2
x	+	+	?	+	?	?

Comparing Tables 1 and 2 with each other reveals how introducing uncertainty modifies the channels of influence of taxation. Timber supply now depends on lump-sum taxation, while with the exception of future unit tax all other taxes affect ambiguously because of offsetting or ambiguous 'wealth', 'substitution' and 'risk' effects.

2.2. Relative effectiveness and timing effects of lump-sum, unit and ad valorem taxes

Turn to consider the tax policy questions. Now we have to take into account that under future timber price uncertainty government tax revenue is stochastic so that it is not immediately evident what is meant by a change in the tax rate or in the timing of taxes. One possibility, which we follow here, is to consider changes in taxes which will keep the expected present value of tax revenues unchanged. This is particularly appealing when private risks are independently distributed.[2] In the presence of taxes we have analyzed the expected present value of taxes is

$$E(T) = T_1 + (t_1 + p_1\tau_1)x + (1+r)^{-1}[T_2 + (t_2 + E(p_2)\tau_2)f(x)] \tag{11}$$

Consider first the relative effectiveness of various taxes. The tax switch between T_1 and t_1 so that $E(T)$ does not change is defined by $dt_1 = -x^{-1}dT_1 - x^{-1}mdx$, where $m = t_1 + p_1\tau_1 + (1+r)^{-1}(f_2 + p_2\tau_2)f'(x)$. Substituting this for dt_1 in the expression $dx = x_{T_1}dT_1 + x_{t_1}dt_1$ and assuming that changes in t_1 and T_1 are positively related yields

$$\left.\frac{dx}{dT_1}\right|_{dE(T)=0} \underbrace{(1 + mx^{-1}x_{t_1})^{-1}}_{(+)} \underbrace{(x\Delta)^{-1}[-V^*_{cc}E(u')]}_{(+)} > 0 \tag{12}$$

Changing the tax base towards the lump-sum tax will increase timber supply and this holds also when the current lump-sum tax and the current ad valorem tax are compared. The reason for this is that the unit and ad valorem taxes t_1 and τ_1 are equivalent; the tax switch between them, which does not change E(T), will have no timber supply effect.[3)]

As far as the timing effects of taxes are concerned, it is easy to see that changing the tax base between T_1 and T_2 so that E(T) does not change, will have no timber supply effect; this is because the tax switch will bring about no 'wealth' effect. As far as the timing in unit taxation are concerned, using the similar procedures as above and provided that dt_1 and dt_2 are negatively related we can end upwith

$$(13) \qquad \left.\frac{dx}{dt_1}\right|_{dE(T)=0} = \underbrace{(1+m(1+r)f^{-1}x_{t_2})^{-1}}_{(+)}\ \underbrace{\beta(\Delta^{-1}\ V^*_{cc}E(u''))}_{(-)}\underbrace{(1-xf'f^{-1})}_{(+)} < 0$$

so that quite naturally changing the tax base in unit taxation toward the current tax will tend to decrease timber supply. This is because 'substitution' effects reinforce each other.

One might be tempted to conclude that the similar result holds for the timing effects in ad valorem taxation. This is not necessarily the case, however. This is simply because in the case of the future ad valorem taxation we have a generalized Slutsky equation according to which the effect of a change in τ_2 can be decomposed into the 'wealth', 'substitution' and 'risk' effects, which last effect is due to the negative relationship between the level of taxation and risk. (see the equation (10)).

Actually, doing the now familiar computations and assuming and $d\tau_1$ and $d\tau_2$ are negatively related implies

$$(14) \qquad \left.\frac{dx}{d\tau_1}\right|_{dE(T)=0} = \underbrace{(1+m(1+r)\,[E(p_2)f]^{-1}x_{\tau_2})^{-1}}_{(+)} \cdot$$

$$\underbrace{\Delta^{-1}\beta p_1 V^*_{cc}E(u')(1-xf'f^{-1})}_{(-)} + h\underbrace{\left(\left.\frac{\partial x}{\partial \eta}\right|_{\substack{\varepsilon=0\\ \eta=1}}\right)}_{(?)} = ?$$

where $h = p_1x(1+r)/E(p_2)f(1-\tau_2)$. Changing the tax base in ad valorem taxation towards the current tax will have an a priori ambiguous timber supply effect. The non-positive 'risk' effect is a sufficient, but not a necessary condition for the negative timber supply effect of the policy switch.

3. Timber supply, future price uncertainty and liquidity constraints

3.1. On timber supply under credit rationing

In earlier analyses we have used a simple two-period model under uncertainty as a vehicle to discuss the channels of influence of various kinds of taxes in the context of timber supply. Undoubtedly, the framework can be extended in various ways, but the analyses have assumed perfect capital markets which can be regarded as a particularly unrealistic feature of the model used thus far. It is not difficult to argue both on theoretical and empirical grounds against the perfect capital market assumption. Relatively recently, the nature and working of capital markets, particularly bank loan markets, have been subject to a number of theoretical analyses. As a result, justifications to various kinds of capital market 'imperfections' have been presented. These include the non-linear interest rate schedule in borrowing (see e.g. Keeton (1979)), an endogenously determined wedge betwe- borrowing and lending rates (see e.g. King (1984)) and credit rationing in the form of binding quantitative limits on the amount of borrowing (see e.g. Stiglitz and Weiss (1981)).

In what follows we do not carry out an extensive analysis of the implications of various kinds of capital market 'imperfections' for the timber supply decisions of the forest owners. Instead, in order to see capital market 'imperfections' in a sharp focus, we look at the implications of binding quantitative credit rationing for the cutting decisions. The analysis that will follow is also unsatisfactory in the sense that credit rationing is postulated to be a part of the model, but it is not explained in it.

Assume that the forest owner is subject to an upper limit on borrowing $\bar{B}$, which it is not possible to exceed. In this case the current period liquidity constraint is of the form $p_1^0 x + y_1 - T_1 - c_1 \geq -\bar{B}$. The decision problem is now to choose c_1 (and c_2) and x so as to maximize V^* subject to this liquidity constraint and subject to the intertemporal budget constraint (1). Forming the Lagrangian $L = V^* + \lambda[p_1^0 x + y_1 + B - T_1 - c_1)]$, where $c_2 = p_2^0 f(x) + y_2 - T_2 + (1+r)(p_1^0 x + y_1 - T_1 - c_1)$, the first order conditions for the expected utility maximization can be written as

$$
(15)\quad \begin{cases} \text{(i)} \quad L_c = 0 = u'(c_1) - \beta E(u'(c_2)) - \lambda \\ \text{(ii)} \quad L_x = 0 = (1+\rho)^{-1} E(u'(c_2)e) + \lambda p_1^0 \\ \text{(iii)} \quad L_\lambda = 0 = p_1^0 x + y_1 + \bar{B} - T_1 - c_1 \end{cases}
$$

The second-order conditions are respectively

$$(16)\quad \begin{cases} \text{(i)}\quad V^*_{cc},\ V^*_{xx} < 0 \\ \text{(ii)}\ \Omega = -(V^*_{xx} + 2p^0_1 V^*_{cx} + (p^0_1)^2 V^*_{cc}) > 0 \end{cases}$$

where $V^*_{cc} = u'' + (1+r)E(u'')$, $V^*_{xx} = (1+\rho)^{-1}E(u''e^2) + p^0_2 f''E(u')$ and $V^*_{cx} = -\beta E(u''e) > 0$.

From the first-order conditions it is clear that the separation does not hold; the timber supply decision depends on the consumption preferences of the forest owner. Moreover, binding credit rationing does not only distort the intertemporal allocation of consumption, but also increases the current timber supply, which will partially alleviate the credit constraint facing the forest owner.

Before considering the modus operandi of taxes under credit rationing it is worthwhile to see how changes in the credit constraint affect the cutting decision. We have

$$(17)\qquad \hat{x}_{\bar{B}} = \Omega^{-1}\{p^0_1 u''(c_1) - f'(x)E(u''(c_2)p^0_2)\} < 0$$

so that a fall in credit limit will increase timber supply and vice versa when credit markets become 'less tight' in the sense that $\bar{B}$ goes up. Thus if credit markets are 'tight', the timber supply tends to be high, ceteris paribus and vice versa.

The effects of the lump-sum taxation can be shown to be

(18)
$$\begin{cases} \text{(i)} \quad \hat{x}_{\tau_1} = \Omega^{-1}\{-p_1^0 u''(c_1)\} > 0 \\ \text{(ii)} \quad \hat{x}_{T_2} = \Omega^{-1}(1+\rho)^{-1}\{-E(u''(c_2)p_2^0)f'(x)\} < 0 \end{cases}$$

A rise in the current lump-sum taxation will increase timber supply, while a rise in future lump-sum taxation will decrease it. The positive current lump-sum taxation effect on timber supply is due to the 'liquidity' effect, not to the decreasing absolute risk aversion as in the case of perfect capital markets; a rise in T_1 will decrease 'liquidity' and this will be partially alleviated by cutting more today. The negative future lump-sum taxation effect on timber supply is in turn due to the 'future liquidity' (or 'precautionary') effect; a rise in T_2 requires more income tomorrow which is why the forest owner tends to cut less today.

As for the unit taxation we have

(19)
$$\begin{cases} \text{(i)} \quad \hat{x}_{t_1} = \underset{(+)}{x\hat{x}_{T_1}} + \underset{(-)}{\Omega^{-1}(-u'(c_1))} = ? \\ \text{(ii)} \quad \hat{x}_{t_2} = \underset{(+)}{f(x)\hat{x}_{T_2}} + \underset{(+)}{[\Omega(1+\rho)]^{-1}(-E(u'(c_2))f'(x))} = ? \end{cases}$$

so that the signs are ambiguous a priori because of offsetting 'liquidity' and 'substitution' effects.

Finally, the effect of the current ad valorem tax is $\hat{x}_{\tau_1} = p_1\hat{x}_{t_1} = ?$. Under the liquidity constraint the effect of a change in risk associated with the future timber price can - following the procedures presented in section 2.1. - be expressed as

$$(20) \qquad \left.\frac{\partial \hat{x}}{\partial \eta}\right|_{\substack{\varepsilon=0 \\ \eta=1}} = \Omega^{-1}[\, p_1^0 V^*_{c\eta} + V^*_{x\eta} \,]$$

where $V^*_{c\eta}$ and $V^*_{x\eta}$ have been defined in equations (7i) and (7ii). Thus $p_1^0 V^*_{c\eta} + V^*_{x\eta} = (1+\rho)^{-1}(1-\tau_2)\{ f'(x)\underset{(+)}{cov(u',p_2)} + f(x)[\,cov(u''e,p_2) + p_2^0 f'(x)cov(u'',p_2)\,]\}$. After some manipulations it can be shown, using the procedures analogous to those in the appendix, that

$$(21) \qquad p_1^0 V^*_{c\eta} + V^*_{x\eta} \gtreqless 0 \quad \text{as } R_c \lesseqgtr (2/\alpha^0) - 1$$

where

$$(22) \qquad \alpha^0 = a\,[\,2 + (1/f'(x))(p_1^0(1+r)/p_2^0)\,] > 0$$

with $a = p_2^0 f(x)/c_2$ = the fraction of future consumption accounted for by the future net sales revenue from timber. The interior solution for x implies that $2 > 2 + (1/f'(x))(p_1^0(1+r)/p_2^0) > 0$. If the relative risk aversion is low (high) and/or the fraction of future consumption accounted for by the future net sales revenue from timber low (high), then a rise in risk will affect timber supply positively (ambiguously). The reason for the greater likelihood of a positive risk effect under the liquidity constraint than in the presence of perfect capital market lies in the inability in the former case for economic agents to react to a change in risk by adjusting saving.

Now we can develop the expression for the timber supply effect of the future ad valorem tax, which is

$$(23) \qquad \hat{x}_{\tau_2} = \underbrace{E(p_2)\, f(x)\, x_{T_2}}_{(-)} + \underbrace{[\Omega(1+\rho)]^{-1} - E(u'(c_2))E(p_2)f'(x)}_{(+)}$$

$$- \underbrace{(1-\tau_2)^{-1} \left(\frac{\partial \hat{x}}{\partial \eta}\bigg|_{\substack{\varepsilon=0\\ \eta=1}}\right)}_{(?)} \gtreqless ?$$

Like in the presence of perfect capital markets the future ad valorem tax effect has been decomposed into three effects with the exception that now the negative 'future liquidity' effect substitutes for the positive 'wealth' effect with a consequence that even without taking the risk effect into account, the sign remains ambiguous.

For convenience we again recapitulate the results in Table 3.

Table 3: Comparative statics of timber supply under liquidity constraint.

	T_1	T_2	t_1	t_2	τ_1	τ_2
$\hat{x}$	+	-	?	?	?	?

Comparing tables 1 and 3 reveals strikingly the importance of both uncertainty and liquidity constraints for the operation of tax policy; all the signs can be different. Comparing tables 2 and 3 indicates that under the liquidity constraint only the lump-sum taxes can be unambiguously signed. Moreover, now in the case of future unit and ad valorem taxes the 'liquidity' and 'substitution' effects offset each other, while the 'wealth' and 'substitution' effects reinforce each other under perfect capital markets.

3.2. Do liquidity constraints matter for tax policy?

Earlier the policies, which did not change E(T) had no 'wealth' effects; only the 'substitution' and sometimes the 'risk' effects were operative. Now the policies will in addition have 'liquidity' effects, which work up to the scale factor like a change in credit supply, and which has some interesting implications.[4)]

First, the tax switch between T_1 and T_2 with unchanged E(T) is defined $dT_2 = -(1+r)dT_1$ so that

$$(24) \qquad \left.\frac{d\hat{x}}{dT_1}\right|_{dE(T)=0} = \hat{x}_{T_1} - (1+r)\hat{x}_{T_2} = -\hat{x}_B > 0$$

where we have assumed that government is not subject to credit rationing and where we have utilized the equations (17) and (18i) and (18ii). Thus the lump-sum policy switch, which does not change the expected present value of tax revenues of government, is equivalent under credit rationing to a change in credit supply. In particular, under liquidity constraint a change in the lump-sum tax base towards the current lump-sum tax will increase timber supply! In the presence of perfect capital markets this kind of operation will have no effect. Second, the effect of the tax switch between t_1 and t_2 under the assumption that dt_1 and dt_2 are negatively related can be written as

$$(25) \qquad \left.\frac{d\hat{x}}{dt_1}\right|_{dE(T)=0} = \underbrace{(1 + m(1+r)f^{-1}\hat{x}_{t_2})^{-1}}_{(+)} \{ \underbrace{-\hat{x}\hat{x}_B}_{(+)} + \Omega^{-1}.$$

$$\underbrace{[-u'(c_1) + \beta E(u'(c_2))xf'(x)f^{-1}]}_{(-)}\} = ?$$

In contrast to the perfect capital market case, the tax switch affect timber supply ambiguously because of offsetting 'liquidity' and 'substitution' effects. It is quite well possible that changing the tax base towards the current unit tax will increase cutting today.

Third, the tax switch between τ_1 and τ_2, when $d\tau_1$ and τ_2 are negatively related, is shown in

$$(26) \quad \left.\frac{d\hat{x}}{d\tau_1}\right|_{dE(T)=0} = \underbrace{(1+m(1+r)(E(p_2)f)^{-1}\hat{x}_{\tau_2})^{-1}}_{(+)} \{ \underbrace{-p_1\hat{x}\hat{x}_B}_{(+)} + p_1\Omega^{-1}[-u'(c_1) + \underbrace{E(u'(c_2))xf'(x)f^{-1}}_{(-)}] + \underbrace{h\left(\left.\frac{\partial\hat{x}}{\partial\eta}\right|_{\substack{\varepsilon=0\\ \eta=1}}\right)}_{(?)} \}$$

where $h = p_1x(1+r)/E(p_2)f(1-\tau_2)$. Changing the timing in taxes in ad valorem taxation will now have positive 'liquidity' effect, negative 'substitution' effect and ambiguous, but quite likely positive 'risk' effect. Again it is quite possible that changing the tax base toward the current ad valorem taxation will increase cutting today.

4. Concluding Remarks

We have analyzed the effects of taxation on timber supply in a standard two-period model of consumption and saving, where the forest owner also decides, on the basis of the maximization of the intertemporal utility of consumption, how much to cut timber today which, given the 'production' (or growth) function, determines how much to cut tomorrow.

The separability of timber supply from consumption preferences of the forest owner breaks down when either the uncertainty with risk aversion is introduced and/or there are liquidity constraints. The particular kind of uncertainty we have taken into account is uncertainty about the future price of timber. The effects of various taxes turned out to be very sensitive both to the introduction of uncertainty and to the way capital markets operate. So e,g. in the presence of future price uncertainty the lump-sum taxes affect timber supply positively, while they have no effect under certainty. Moreover, if the forest owners are currently subject to liquidity constraints, then the current lump-sum tax affects timber supply positively, but the future lump-sum tax negatively. These effects are due to a change in liquidity resulting from lump-sum tax changes and they have nothing to do with 'wealth' effects arising from decreasing absolute risk aversion.

Turning to tax policy considerations the policies have - depending on whether there is uncertainty or not and whether capital markets are perfect or not - 'substitution','risk' and 'liquidity' effects. So e.g. lowering the current lump-sum tax and increasing the future lump-sum tax so that the expected present value of government tax revenues does not change will increase timber supply under credit rationing.

Obviously, these preliminary findings are only a beginning and far from exhausting even the timber supply modelling problems. Finally, we mention some areas for further reasearch.(1) modelling timber supply in the presence of multiple sources of risk and under other kinds of capital market 'imperfections' than quantitative credit rationing, (2) modelling the self-active farmer's timber supply in an intertemporal framework, which would make it possible to condider the relationship between forest and wage taxation, and (3) empirical reasearch about timber supply, while increasing, is still quite scanty. In particular, the role of uncertainty and credit markets should be examined.

FOOTNOTES:

1) This result has been shown in Johansson and Löfgren (1985) (Proposition 12.6., p. 273) in a model, where the timber supply is the only decision variable and the forest owner maximize the expected utility of the present value of profits.

2) This need not imply that government is risk neutral. To the extent that risks are independent across the forest owners and the number of forest owners is large, the law of large numbers will guarantee government a constant total revenue despite uncertainty at the individual level. Under these circumstances government is simply a more efficient risk-pooler than individuals. To the extent that the law of large numbers does not work e.g. because of the presence of 'business cycle risks', then the assumption that government is risk neutral is presupposed. For a discussion of various criteria of changing the tax base under uncertainty, see Atkinson and Stiglitz (1980), lecture 4).

3) Similarly we can compare the tax switches between T_2, t_2 and τ_2 so that E(T) does not change. This is left as an exercise to the interested reader. We mention only briefly that changing the tax base towards T_2 and away from t_2 will decrease timber supply, while changing the tax base towards T_2 and away from τ_2 will affect timber supply ambiguously because of the ambiguous 'risk' effect associated with τ_2. Therefore, it is no longer necessarily true that t_2 and τ_2 equivalent in terms of their relative effectiveness. All this presupposes that in the tax switches the tax changes are negatively related.

4) The major implication of liquidity constraints in the kind of tax policy analyses we have carried out in this paper is to bring about 'liquidity' effects which are due to the changes in the timing of taxes. If the timing will remain unchanged, there are no 'liquidity' effects, so that the conclusions about the relative effectiveness of various taxes, which were obtained in section 2.2., will apply here as well.

Appendix:

We have to derive the expression which shows how the sign of

(1) $$V^*_{x\eta} = (1+\rho)^{-1}(1-\tau_2)\{ f'(x)\mathrm{cov}(u'(c_2),p_2) + f(x)\mathrm{cov}(u''(c_2)e,p_2)\}$$

depends on some key factors. Now $\mathrm{sgn}\, V^*_{x\eta} = \mathrm{sgn}\,[f'\mathrm{cov}(u',p_2) + f\mathrm{cov}(u''e,p_2)] =$ $\mathrm{sgn}\,[f'(1-\tau_2)fu'' + f'f(1-\tau_2)u'' + f(1-\tau_2)u'''ef]$ so that

(2) $$\mathrm{sgn}\, V^*_{x\eta} = \mathrm{sgn}\,[2f'(x)u''(c_2) + f(x)eu'''(c_2)]$$

The assumption of constant relative risk aversion $R_c = -u''(c_2)c_2/u'(c_2)$ implies that $u'''(c_2) = -u''(c_2)(1+R_c)/c_2$ so that we get

(3) $$\mathrm{sgn}\,[2f'u'' + feu'''] = \mathrm{sgn}\,[2f'u'' + feu''']$$
$$= \mathrm{sgn}\,(2 - \alpha(1+R_c))$$

where $\alpha = a\,[1 + (1/f'(x))p_1^0(1+r)/p_2^0]$ with $a = p_2^0(x)/c_2$. Thus we have

(4) $$V^*_{x\eta} \gtreqless 0 \quad \text{as } R_c \lesseqgtr (2/\alpha)-1$$

which establishes the expression (8) of the text.

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