

Keskusteluaiheita Discussion papers

Erkki Koskela*

TAX CUTS, RISK-SHARING AND
CAPITAL MARKET 'IMPERFECTIONS'

No 186

18.11.1985

* Department of Economics, University of Helsinki,
Aleksanterinkatu 7, 00100 Helsinki 10, Finland.
Financial support from the Yrjö Jahnsson Foundation
is gratefully acknowledged.

ISSN 0781-6847

This series consists of papers with limited circulation,
intended to stimulate discussion. The papers must
not be referred or quoted without the authors'
permission.



Abstract:

The purpose of this paper is to study the Ricardian equivalence theorem, according to which under certain assumptions and given government expenditures the timing of taxes does not matter because people believe that current deficits will produce future surpluses and take this into account. While accepting this belief for the purposes of argument we relax the assumptions i) lump-sum taxes, ii) perfect certainty and iii) perfect capital markets and develop their consequences for the consumption behaviour. The irrelevance of the tax-debt choice no longer holds under distortionary taxation, uncertainty and "imperfect" capital markets. Moreover, and perhaps more important, while there can some exceptions, actuarially neutral tax cuts tend mostly to be stimulatory for incentive, risk-sharing and 'liquidity' reasons.

1. INTRODUCTION

Do government deficits absorb private saving? Can the burden of current government expenditures be shifted to future generations? These old questions have been recently subject to some controversy. According to the so-called Ricardian equivalence theorem, recently restated and elaborated by Barro (1974)¹⁾, the answer is negative to these questions. Given government expenditures the shift between current taxes and debt issue does not matter for the consumption behaviour, because the economic agents take the corresponding change in future tax liabilities into account. Hence, there is no long run burden of public debt; the capital stock will not be crowded out by government debt. This view lies in sharp contrast to the conventional Keynesian view according to which lowering current taxes augment aggregate demand by increasing the current disposable income. Thus while under Keynesian view tax cuts stimulate aggregate demand, under the Ricardian view²⁾ they only increase saving or decrease borrowing with no consumption effect.

The Ricardian equivalence theorem is based on the following critical assumptions: i) lump-sum taxes, ii) perfect capital markets, where individual agents can borrow and lend at equal parametric interest rate subject only to their lifetime wealth constraint, iii) absence of uncertainty and iv) certain type of bequest behavior. By assuming that the utility of the next generation is an argument of the utility function of the current generation the behaviour of each generation can be characterized as if they would have effectively infinite horizon. The analysis is usually also partial in the sense that interest rate in the capital market is assumed to be unaffected by changes in saving and/or borrowing implied by various current tax-debt choices.

It is evident that making other assumptions about the bequest behavior implies that current generation will perceive himself wealthier under debt issue than under taxation. In what follows we do not analyze the bequest issue, but assume for simplicity that the horizons of government and consumers are equal so that from this point of view the future tax liabilities implied by current taxes are accurately perceived. Granted this the purpose of the paper is to relax the assumptions of certainty, lump-sum taxes and perfect capital markets and look at the implications of neutral tax cuts - tax cuts which will keep the present value of taxes unchanged - in the partial equilibrium framework.

We proceed as follows: Section 2 develops the implications of actuarially neutral tax cuts for consumption behaviour under future income and future interest rate uncertainty with lump-sum taxes and perfect capital markets. The analysis is extended in section 3 to include distortionary taxation, which will affect consumption-leisure choices. Finally, section 4 is devoted to consider implications of various types of capital market "imperfections" on the tax-debt choice issue. To anticipate results it turns out that under quite plausible assumptions the irrelevance of tax-debt choice no longer holds under uncertainty, distortionary taxation and "imperfect" capital markets. Moreover, and perhaps more important, timing of taxes are mostly consistent with Keynesian view: for risk-sharing, incentive and 'liquidity' reasons neutral tax cuts tend to be stimulatory in terms of current consumption.

2. TAX CUTS UNDER LUMP-SUM TAXATION: RISK-SHARING2.1. Tax cuts under future income uncertainty

The consumer is assumed to have a preference ordering over present consumption c_1 and "future" consumption c_2 , which is represented by a cardinal utility function $u(c_1, c_2)$ being at least three times continuously differentiable and possessing everywhere positive marginal utilities. Present and future income y_1 and y_2 are assumed to be exogenously given and $R = 1 + r =$ the interest rate factor in the capital market. Partial derivatives are denoted by subscripts for functions with many variables (e.g. $u_1 = \partial u / \partial c_1$, $u_{12} = \partial^2 u / \partial c_1 \partial c_2$) and by primes for functions with one variable (e.g. $v(c_2)$, $v'(c_2) = \partial v / \partial c_2$ etc.). The model can be interpreted either that periods represent two halves of a single person's life or that there are two generations. Assume that in the presence of uncertainties there are no contingent claims market through which individuals could diversify away risks (i.e. government cannot provide insurance to aggregate or business cycle income or interest rate risks). Finally we assume that the utility function is strictly concave and that c_1 and c_2 are normal goods which means that

$$(1) \quad \left\{ \begin{array}{l} \text{(i)} \quad \left. \frac{\partial(u_1/u_2)}{\partial c_2} \right|_{dc_1=0} > 0 \Rightarrow u_{12} - (u_1/u_2)u_{22} > 0 \\ \text{(ii)} \quad \left. \frac{\partial(u_1/u_2)}{\partial c_1} \right|_{dc_2=0} < 0 \Rightarrow u_{11} - (u_1/u_2)u_{12} < 0 \end{array} \right.$$

After these preliminary considerations we now turn to consider the effects of actuarially neutral tax cuts - which do not change the present value of taxes - on the current consumption under various kinds of

circumstances. All the analyses are carried out on the assumption of given government expenditures and our analysis is in the similar spirit than in Bryant (1983), in which various "government irrelevance results" are demonstrated in a simple two-period model under certainty.

Before going on it might be worthwhile to point out that there are alternative ways of combining tax cuts today with future policies and thus there are alternative ways of making assumptions about peoples' beliefs about future policies. A tax cut today requires some combination of the following policy adjustments: (1) increases in future taxes, (2) decreases in future government expenditures, (3) increases in future money creation and (4) increases in future issues of interest-bearing national debt. Each of these policies may have different implications for the effects of tax cuts today. In the following we are interested in the robustness of the Ricardian equivalence theorem and assume a la Barro (1974) that tax cut today induces future tax increases. Alternatively people may believe that the policy of tax cuts today and the consequent deficit will lead to greater money creation in the future. This in turn may give rise to increased inflationary expectations so that tight money may be inflationary as have been argued by Sargent and Wallace (1981)! If people believe in (2) or (4) further possibilities will arise.

Under future income uncertainty the maximum of the expected utility $Eu(c_1, c_2)$ under the budget constraint $c_2 = q_2 \tilde{y}_2 + R(q_1 y_1 - c_1)$, where $q_1 = 1 - t_1$, and $t_i =$ taxes in period i , $i = 1, 2$ is defined by the first- and second-order conditions $U_c = E(u_1 - Ru_2) = 0$ and $U_{cc} = E(u_{11} + R^2 u_{22} - 2Ru_{12}) < 0$

The effects of current and future taxes on current consumption are

$$(2) \quad \begin{cases} (i) & (\partial c_1 / \partial t_1) = (U_{cc})^{-1} y_1 R E(u_{12} - Ru_{22}) < 0 \\ (ii) & (\partial c_1 / \partial t_2) = (U_{cc})^{-1} [\bar{y}_2 E(u_{12} - Ru_{22}) + \text{cov}(u_{12} - Ru_{22}, y_2)] = ? \end{cases}$$

where $\bar{y}_2 = E(y_2)$ the expected value of future gross income. It seems natural to assume a la Leland (1968) that along an indifference curve aversion towards an actuarially neutral bet on one variable decreases as that variable increases. This 'decreasing risk aversion to concentration' (DRAC) means that $du_{22}/dc_2|_{dU=0} > 0$ which in turn implies that $u_{222} - (u_2/u_1)u_{122} > 0$, so that under DRAC $\text{cov}(u_{12} - Ru_{22}, y_2) \geq 0$.³⁾ In the case of intertemporally additive utility $U = u(c_1) + v(c_2)$ $\text{cov}(u_{12} - Ru_{22}, y_2) \geq 0$ as $v''' \leq 0$ so that the decreasing absolute risk aversion $A' < 0$ implies, but does not necessitate $v''' > 0$, where $A(c_2) = -v''(c_2)/v'(c_2)$ is the Arrow-Pratt measure of absolute risk aversion.

The equations in (2) can be expressed in the following, perhaps more illuminating way. Consider the effect of an additive shift in the distribution of y_2 defined by $\tilde{y}_2 = y_2 + \varepsilon$, where ε can be interpreted to mean an increase in the expected value of y_2 with all other moments constant. Substituting this for y_2 and evaluating at $\varepsilon = 0$ gives

$$(\partial c_1 / \partial \varepsilon)|_{\varepsilon=0} = (U_{cc})^{-1} [-q_2 E(u_{12} - Ru_{22})] > 0 \text{ so that we have}$$

$$(2'i) \quad (\partial c_1 / \partial t_1) = -(y_1 R / q_2) (\partial c_1 / \partial \varepsilon)|_{\varepsilon=0} < 0$$

(+)

Consider next the effect of a multiplicative shift in the distribution of y_2 , which is offsetted by an additive shift to restore the mean of y_2 to its initial value. Such a shift can be interpreted to mean a mean-preserving change in risk and is defined for $\tilde{y}_2 = \varepsilon + \eta y_2$ by $d\varepsilon/d\eta = -\bar{y}_2$ at $\varepsilon = 0, \eta = 1$. Substituting this for y_2 and evaluating at $\varepsilon = 0, \eta = 1$ gives $(\partial c_1 / \partial \eta) \Big|_{\substack{\varepsilon = 0 \\ \eta = 1}} = (U_{cc})^{-1} [-q_2 \text{cov}(u_{12} - Ru_{22}, y_2)] < 0$ under DRAC or with additive utility function under decreasing absolute risk aversion. Using this expression and (2i) and (2ii) the equation (2ii) can be rewritten as

$$(2'ii) \quad (\partial c_1 / \partial t_2) = \underbrace{-(\bar{y}_2 / q_2)}_{(+)} (\partial c_1 / \partial \varepsilon) \Big|_{\varepsilon = 0} - \underbrace{(1 / q_2)}_{(-)} (\partial c_1 / \partial \eta) \Big|_{\eta = 1} = ?$$

which shows clearly the offsetting income and risk-reducing effects of futures taxes on current consumption.

Turning to government behaviour assume that government switches current and future taxes so as to keep the expected tax revenue constant.⁴⁾ The expected (present value of) tax revenue can be written as

$$(3) \quad \bar{T} = t_1 y_1 + R^{-1} t_2 \bar{y}_2$$

The tax switch, which does not change \bar{T} , is defined by $dt_2 = -(y_1 R / \bar{y}_2) dt_1$. The consumption effect of tax changes is $dc_1 = (\partial c_1 / \partial t_1) dt_1 + (\partial c_1 / \partial t_2) dt_2$ so that we have

$$(4) \quad \frac{dc_1}{dt_1} \Big|_{d\bar{T} = 0} = (y_1 R / \bar{y}_2 q_2) (\partial c_1 / \partial \eta) \Big|_{\substack{\varepsilon = 0 \\ \eta = 1}} < 0$$

which expression is obtained by using the equations (2'i) and (2'ii), and where $(\partial c_1 / \partial \eta) \Big|_{\substack{\epsilon=0 \\ \eta=1}} < 0$ under DRAC or with additive utility under decreasing absolute risk aversion.

Thus we have obtained

Proposition 1: In the presence of future income risk with lump-sum taxes and perfect capital markets the actuarially neutral tax cut will increase current consumption if DRAC holds under non-additive utility or if absolute risk aversion is decreasing under additive utility.

The non-neutrality of actuarially neutral tax cut results from the risk-reducing effect of a rise in future taxes which under most plausible assumptions tends to stimulate consumption. The risk-reducing effect at the individual level is associated with increased uncertainty of taxes at the government level so that the non-neutrality can be said to result from risk-sharing. If there is no uncertainty, then the tax cut has no effect according to the Ricardian equivalence theorem.

2.2. Tax cuts under interest rate (cost of borrowing) uncertainty

Consider next the case where y_2 is known with certainty but the interest rate r is stochastic. The comparative statics of lump-sum tax changes can be shown to be of the following form

$$(5) \quad \begin{cases} (i) \quad (\partial c_1^* / \partial t_1) = (U_{cc}^*)^{-1} y_1 [\bar{R} E(u_{12} - Ru_{22}) + \text{cov}(u_{12} - Ru_{22}, R)] = ? \\ (ii) \quad (\partial c_1^* / \partial t_2) = (U_{cc}^*)^{-1} y_2 E(u_{12} - Ru_{22}) < 0 \end{cases}$$

where $U_{cc}^* = E(u_{11} + R^2 u_{22} - 2Ru_{12}) < 0$ and $\bar{R} = E(R)$ = the expected value of the interest rate factor $1+r$. While $(\partial c_1^* / \partial t_2) < 0$ given normality of goods, the sign of $(\partial c_1^* / \partial t_1)$ is generally ambiguous because the sign of $\text{cov}(u_{12} - Ru_{22}, R)$ is equal to the sign of $(u_{122} - Ru_{222})(q_1 y_1 - c_1) - u_{22}$ which under DRAC is positive for borrowers, but can be of either sign for lenders.

With the intertemporal government budget constraint under interest rate uncertainty $T^* = t_1 y_1 + t_2 y_2 / \bar{R}$ the tax switch is now defined by $dt_2 = -(y_1 \bar{R} / t_2) dt_1$ and its consumption effect is now

$$(6) \quad \left. \frac{dc_1^*}{dt_1} \right|_{dT^*=0} = (U_{cc}^*)^{-1} \text{cov}(u_{12} - Ru_{22}, R)$$

so that $\text{sgn} (dc_1^* / dt_1) \big|_{dT^*=0} = -\text{sgn} \text{cov}(u_{12} - Ru_{22}, R)$. Now $\text{sgn} \text{cov}(u_{12} - Ru_{22}, R) = \text{sgn} [(u_{122} - Ru_{222})(q_1 y_1 - c_1) - u_{22}] > 0$ for borrowers and ambiguous for lenders under DRAC. In the case of additive utility function we have $\text{sgn} [(u_{122} - Ru_{222})S - u_{22}] = -\text{sgn} (RSv''' + v'')$, where $S = q_1 y_1 - c_1$. It can be shown that $RSv''' + v'' = v'' (1 - z(1 + R_c)) - zR_c' v'$, where $z = RS / (y_2 + RS)$ = the fraction of future consumption accounted for by saving and $R_c = -v''(c_2)c_2 / v'(c_2)$ = the Arrow-Pratt measure of relative risk aversion and R_c' its derivative with respect to c_2 . In what follows we keep to the assumption of constant relative risk aversion as the benchmark case. This gives $dc_1^* / dt_1 \big|_{dT^*=0} \gtrless 0$ as $R_c \gtrless (1/z) - 1$. Thus we have

Proposition 2: In the presence of interest rate risk with lump-sum taxes and perfect capital markets the actuarially neutral tax cut is: (i) stimulatory for borrowers if DRAC holds under non-additive utility or if absolute risk aversion is decreasing under additive utility, (ii) generally ambiguous for lenders under non-additive utility, while under additive utility more (less) likely stimulatory when relative risk aversion is low (high) and/or the fraction of future consumption accounted for by saving is low (high), ceteris paribus.

Finally, it is useful to reiterate how the results differ depending on the question of whether the tax switch is conducted in the presence of future income, or interest rate uncertainty. Under income uncertainty the tax cut will tend to stimulate current consumption via the risk-reducing effect of raising future taxation. Under interest rate uncertainty the stimulatory effect will tend to hold for borrowers, while in the case of lenders for this to materialize relative risk aversion and/or the fraction of future consumption accounted for by saving has to be low.

3. TAX CUTS UNDER DISTORTIONARY TAXATION: INCENTIVE VERSUS RISK-SHARING

3.1. Tax cuts under distortionary taxation: no uncertainty

Given the problem posed, the analyses of tax cuts in the earlier section abstracted from two major issues: taxes were assumed to be of lump-sum type and capital markets perfect. The idea was to emphasize that even under those circumstances tax cuts are quite likely non-neutral in the presence of uncertainty concerning future income or interest rate. This section relaxes the assumption of lump-sum taxes and explores its

implications when timing of taxes may affect the present value of income and thereby the present value of taxes. In order to get a feel for the kind of results to be expected we start by analysing tax cuts under certainty. This serves as a sort of finger exercise and lays ground for subsequent models where various kinds of uncertainties are introduced.

Assume that consumer tastes may be represented by the utility function $U^0 = u(c_1, L) + v(c_2)$ which is at least three times continuously differentiable, increasing in consumptions c_1 and c_2 and decreasing in hours worked L and exhibiting decreasing marginal utilities of consumptions and increasing marginal disutilities of hours worked so that $u_1, v' > 0$, $u_2 < 0$, $u_{11}, u_{22}, v'' < 0$. Moreover, it is assumed that the marginal rate of substitution between c_1 and L under certainty is increasing. The budget constraint can now be written as $c_2 = q_2 y_2 + R(q_1 w L - c_1)$, where L = hours worked, w = the wage rate, $q_i = 1 - t_i$, ($i = 1, 2$) describes taxes for labour income and future lump-sum income respectively. Before going on we make finally the natural assumption that current consumption and leisure are normal goods so that

$$(7) \quad \left\{ \begin{array}{l} \text{(i)} \quad \left. \frac{\partial(-u_2/q_1 w u_1)}{\partial c_1} \right|_{dL=0} > 0 \implies u_{12} + q_1 w u_{11} < 0 \\ \text{(ii)} \quad \left. \frac{\partial(-u_2/q_1 w u_1)}{\partial L} \right|_{dc_1=0} > 0 \implies u_{22} + q_1 w u_{12} < 0 \end{array} \right.$$

The comparative statics of current consumption and labour supply with respect to current and future taxes is of usual type

Correspondingly, the consumption effect of tax changes is $dc_1 = (\partial c_1 / \partial t_1) dt_1 + (\partial c_1 / \partial t_2) dt_2$ and after substitution we end up with

$$(10) \quad \left. \frac{dc_1}{dt_1} \right|_{dT=0} = \underbrace{(\partial c_1 / \partial t_1)}_{(-)} dU^0 = 0 - \underbrace{(\partial c_1 / \partial t_2)}_{(-)} (wLR/y_2) \underbrace{\left. \frac{dL}{dt_1} \right|_{dT=0}}_{(-)} < 0$$

where $(\partial c_1 / \partial t_1)_{dU^0=0} = D^{-1}(-Rwv'U_{CL}^0) = D^{-1}(-Rwv'(u_{12} - R^2 q_1 wv''))$. Notice that the negativity of (10) does not necessitate the net substitutability between consumption and leisure. Anyway we have, not surprisingly,

Proposition 3: An actuarially neutral tax cut which puts more weight on future lump-sum taxation and less weight on current distortionary taxation will increase labour supply and stimulates consumption both directly and indirectly. The direct stimulus results from Hicks-Allen net substitutability between consumption and leisure, while indirect stimulus is due to the positive labour supply effect of tax cut.

Thus dropping the assumption of lump-sum taxes makes the equivalence theorem, according to which timing of taxes does not matter, invalid. In what follows we introduce various kinds of uncertainties and develop their implications in the presence of endogenous labour supply.

3.2. Distortionary taxation under uncertainty: incentives versus risk-sharing

3.2.1. Future non-labour income uncertainty

Under our assumptions this case is relatively straightforward to analyze. The only difference from the analysis in section 3.1. is that y_2 is now

One would expect that the question of how changing 'correlation' between y_2 and R affects consumption and labour supply is significant in evaluating the effects of tax policies. This conjecture turns out to be correct as the following indicates. Moreover, it turns out to be important to make a distinction between borrowers and lenders because changing 'correlation' between future income and interest rate (cost of borrowing) tends to affect these groups differently.

Let us begin by putting forward the comparative statics of present and future taxes in terms of current consumption and labour supply

$$(15) \quad \left\{ \begin{array}{l} \text{(i)} \quad (\partial \hat{c}_1 / \partial t_1) = \hat{D}^{-1} \{ -wLE(v'' R^2)(u_{22} + q_1 w u_{12}) - wE(v'R) \hat{U}_{cL} \} < 0 \\ \text{(ii)} \quad (\partial \hat{L} / \partial t_1) = \hat{D}^{-1} \{ wLE(v'' R^2)(q_1 w u_{11} + u_{12}) + wE(v'R) \hat{U}_{cc} \} = ? \\ \text{(iii)} \quad (\partial \hat{c}_1 / \partial t_2) = \hat{D}^{-1} \{ -y_2 E(v'' R)(u_{22} + q_1 w u_{12}) \} < 0 \\ \text{(iv)} \quad (\partial \hat{L} / \partial t_2) = \hat{D}^{-1} \{ y_2 E(v'' R)(q_1 w u_{11} + u_{12}) \} > 0 \end{array} \right.$$

where $\hat{D} = \hat{U}_{cc} \hat{U}_{LL} - \hat{U}_{cL}^2 > 0$, $\hat{U}_{cc} = u_{11} + E(v'' R^2) < 0$, $\hat{U}_{LL} = u_{22} + (q_1 w)^2 E(v'' R^2) < 0$ and $\hat{U}_{cL} = \hat{U}_{Lc} = u_{12} - q_1 w E(v'' R^2) > 0$. Signs result from the assumptions of risk aversion and normality of current consumption and leisure.

The effect of the tax switch - which keeps the expected present value of taxes $T = t_1 w L + t_2 y_2 / \bar{R}$ unchanged - on the supply of labour becomes now

$$(16) \quad \frac{d\hat{L}}{dt_1} \Big|_{d\bar{T}=0} = \left[1 + (t_1 w R / y_2) \frac{\partial \hat{L}}{\partial t_2} \right]^{-1} \left\{ \frac{wL(q_1 w u_{11} + u_{12}) \text{cov}(v'' R, R)}{\hat{D}} + \frac{wE(v'R) \hat{U}_{cc}}{\hat{D}} \right\}$$

(+)

(-)

Analogously, the consumption effect of the tax switch can be expressed as

$$(17) \quad \frac{d\hat{c}_1}{dt_1} \Big|_{d\bar{T}=0} = \left\{ \frac{-wL(u_{22} + q_1 w u_{12}) \text{cov}(v'' R, R)}{\hat{D}} - \frac{wE(v' R) \hat{U}_{cL}}{\hat{D}} \right\} -$$

$$- \left(\frac{t_1 w \bar{R}}{y_2} \right) \left(\frac{\partial \hat{c}_1}{\partial t_2} \right) \frac{dL}{dt_1} \Big|_{d\bar{T}=0}$$

(-)

The substitution (incentive) effects of tax cut stimulate consumption and increase labour supply (the terms $-\hat{D}^{-1} wE(v' R) \hat{U}_{cL}$ and $\hat{D}^{-1} wE(v' R) \hat{U}_{cC}$) so that signing the equations (16) and (17) necessitates signing the term $\text{cov}(v'' R, R)$.

Clearly $\text{sgn } \text{cov}(v'' R, R) = \text{sgn } (v'' + v''' R(q_1 w L - c_1))$ and moreover,

$$v'' + v''' R(q_1 w L - c_1) = Z = v'' (1 - m(1 + R_C)) - m v' R'_C, \text{ where}$$

$m = R(q_1 w L - c_1) / (q_2 y_2 + R(q_1 w L - c_1)) =$ the fraction of future consumption accounted for by the current saving.

For borrowers $Z < 0$ so that $\text{cov}(v'' R, R) < 0$ and $(dL/dt_1) \Big|_{d\bar{T}=0} = ?$ according to (16). As far as the consumption effect is concerned the first terms (describing risk and incentive effects respectively) are both negative, while the second part of the expression is positive so that the total effect is ambiguous. If the tax switch, however, is calculated from a given labour supply, then only the first term is relevant and total effect is unambiguously negative.

For lenders, the sign of $\text{cov}(v'' R, R)$ can be positive, negative (or zero) depending on the relative risk aversion, its change when c_2 varies and the fraction of future consumption accounted for by the current saving.

More specifically, with constant relative risk aversion $\text{cov}(v'' R, R) \gtrless 0$ as $R_C \gtrless (1/m) - 1$. Thus we have

Proposition 5: In the presence of uncertainty about interest rate (cost of borrowing) and additive intertemporal and non-additive intratemporal utility: (i) the actuarially neutral tax cut for borrowers has an ambiguous effect on labour supply while it stimulates consumption if the tax cut is calculated from a given labour supply, (ii) for lenders, on the other hand, the effects are of the same sign than for borrowers, if their relative risk aversion is low and/or the fraction of future consumption accounted for by the current saving is low ceteris paribus. Otherwise, tax cut affects consumption ambiguously, while depresses labour supply.

Because labour supply has been assumed to be a discommodity, the effects of tax policies affecting it via changing 'correlation' between y_2 and R are mirror images (and thus of different sign) of those affecting consumption.

3.2.3. Wage rate uncertainty and the tax switch

Finally, we analyze the implications of tax policies in a situation, where consumer-workers do not know the real return on their labour effort when deciding how to allocate their time between labour and leisure and their resources between current and future consumption. The analysis of labour supply under uncertainty has been developing rapidly in recent years. Papers by Eaton and Rosen (1980), Stiglitz (1982) and Tressler and Menezes (1980) have all presented analyses of this problem.⁶⁾ We follow along these lines with the exception that the labour supply under wage uncertainty is analyzed in an intertemporal consumption-saving model, which gives rise to some additional effects not found in static models.

The maximum of the expected utility $U^{**} = u(c_1, L) + E v(c_2)$ subject to the budget constraint $c_2 = q_2 y_2 + R(q_1 \tilde{w} L - c_1)$, where w is stochastic, in terms of c_1 and L gives the first-order conditions which, given the second-order conditions, define the current consumption c_1^{**} and the labour supply L^{**} in terms of parameters. Take first the case of lump-sum taxation of future income y_2 . If we define income effects of t_2 in terms of wages and denote an increase in the expected value of w by ϵ with all other moments being constant and evaluate at $\epsilon = 0$, then we have

$$(18) \quad \begin{cases} (i) & (\partial c_1^{**} / \partial t_2) = -(\partial c_1^{**} / \partial \epsilon) \Big|_{\epsilon=0} A + D^{**^{-1}} E(v') U_{cL}^{**} A R q_1 \\ (ii) & (\partial L^{**} / \partial t_2) = -(\partial L^{**} / \partial \epsilon) \Big|_{\epsilon=0} A - D^{**^{-1}} E(v') U_{cc}^{**} A R q_1 \end{cases}$$

where $A = y_2 / R q_1 L^{**} > 0$, $D^{**} > 0$ according to the second-order conditions, $U_{cL}^{**} = u_{12} - R^2 q_1 E(v'' w) > 0$ and $U_{cc}^{**} = u_{11} + R^2 E(v'' w) > 0$ (see appendix 1). The effects of lump-sum taxation can be expressed in terms of expected values of wages corrected by the substitution effects, to which changes in wages will give rise.

The comparative statics of wage taxation can be worked up in terms of income effects (in terms of expected changes of wages) and in terms of total effects of risk changing. Denoting a mean-preserving change in risk (offsetted by an additive shift ϵ to restore the mean of w to its initial value) by η and evaluate at $\epsilon = 0$, $\eta = 1$ we get

$$(19) \quad \begin{cases} (i) & (\partial c_1^{**} / \partial t_1) = -(1/q_1) \left[(\partial c_1^{**} / \partial \epsilon) \Big|_{\epsilon=0} \bar{w} + (\partial c_1^{**} / \partial \eta) \Big|_{\eta=1} \right] \\ (ii) & (\partial L^{**} / \partial t_1) = -(1/q_1) \left[(\partial L^{**} / \partial \epsilon) \Big|_{\epsilon=0} \bar{w} + (\partial L^{**} / \partial \eta) \Big|_{\eta=1} \right] \end{cases}$$

where $\bar{w} = E(w) =$ the expected value of wage rate. According to (19) wage taxation will give rise to both income effects (the first terms in parentheses) and to risk-changing effects (the second terms in parentheses). The tax switch - which will keep the present value of the expected tax revenue $\bar{T}^{**} = t_1 \bar{w} L^{**} + t_2 y_2 / R$ constant - is defined by $dt_2 = -(\bar{w} L R / y_2 dt_1 - (t_1 \bar{w} R / y_2) dL^{**})$ and its labour supply and consumption effects are

$$(20) \quad \left. \frac{dL^{**}}{dt_1} \right|_{d\bar{T}^{**} = 0} = -B_0^{-1} q_1 \left[(\partial L^{**} / \partial \eta) \right]_{\substack{\varepsilon = 0 \\ \eta = 1}} - w R q_1 E(v') U_{CC}^{**} / D^{**} \quad (-)$$

and

$$(21) \quad \left. \frac{dc_1^{**}}{dt_1} \right|_{d\bar{T}^{**} = 0} = - (1/q_1) \left[(\partial c_1^{**} / \partial \eta) \right]_{\substack{\varepsilon = 0 \\ \eta = 1}} + w R q_1 E(v') U_{CL}^{**} / D^{**} \quad (+)$$

$$- B_1 \left. \frac{dL^{**}}{dt_1} \right|_{d\bar{T}^{**} = 0}$$

where $B_0 = 1 + (t_1 \bar{w} R / y_2) (\partial L^{**} / \partial t_2)$ and $B_1 = (t_1 \bar{w} R / y_2) (\partial c_1^{**} / \partial t_2)$. B_0 is most likely positive and B_1 negative. The intra-period separability of preferences ($u_{12} = 0$) is a sufficient, but not a necessary condition for these to hold.

Like in the certainty case the substitution effects of tax cut will increase labour supply and stimulates consumption so that the total effect depends on the risk-reducing effect of tax cut. The condition that a rise (fall) in wage rate uncertainty will increase (decrease) neither consumption nor labour supply is a sufficient, but not a necessary condition for the conclusion that both the labour supply and consumption will be stimulated

by tax cut. In order to see the uncertainty effects in terms of risk aversion behavior we write out the equations which show the effects of a change in uncertainty on consumption and labour supply. They are

$$(22) \quad \left. \frac{\partial c_1^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} = D^{**^{-1}} \{ Rq_1 [RLU_{LL}^{**} \text{cov}(v'', w) + U_{CL}^{**} W] \} \quad (-)$$

and

$$(23) \quad \left. \frac{\partial L^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} = D^{**^{-1}} \{ -Rq_1 [U_{CC}^{**} W + RLU_{CL}^{**} \text{cov}(v'', w)] \} \quad (+)$$

where $W = \text{cov}(v', w) + RLq_1 \text{cov}(v''w, w)$ and $\text{sgn } W = \text{sgn} (2RLq_1 v'' + (RLq_1)^2 v'''w = \text{sgn} (2v'' + RLq_1 v'''w)$. The same kind of expression $2v'' + RLq_1 v'''w$ shows up in the analysis of wage rate uncertainty in Eaton and Rosen (1980). In our model signing $M = 2v'' + RLq_1 v'''w$ does not necessarily help to sign the uncertainty effects. Now $M = v'' (2 - j(1 + R_c)) - jR_c' v'$, where $j = Rq_1 wL/c_2 =$ the fraction of future consumption accounted for by the current wage income.

Thus we have with $R_c' = 0$ that $M \geq 0$ as $R_c \geq (2/j) - 1$. This means that $R_c \leq (2/j) - 1 \Rightarrow \left. \frac{\partial c_1^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} < 0$ and $\left. \frac{\partial L^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} < 0$. Moreover, if

$\left. \frac{\partial c_1^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} > 0$ and $\left. \frac{\partial L^{**}}{\partial \eta} \right|_{\substack{\epsilon=0 \\ \eta=1}} > 0$, then $R_c > (2/j) - 1$. Utilizing (22)

and (23) in the expressions (20) and (21) leads up to

Proposition 6: In the presence of uncertainty about the wage rate and additive intertemporal and intratemporal utility the actuarially neutral tax cuts have stimulatory substitution effects on labour supply and consumption and either stimulatory or contractionary effect via risk-increasing induced by tax cut under wage uncertainty. High relative risk aversion and/or the high ratio of

wage income to future consumption are necessary, but not sufficient conditions for risk-sharing effects to reinforce the incentive effects. On the other hand if relative risk aversion and/or the ratio of wage income to future consumption tend to be low, then the risk-sharing effects run counter to substitution effects and one cannot say whether tax cut is stimulatory or contractionary.

Finally, before going on, we briefly summarize the results obtained under endogenous current labour supply. In the presence of future income uncertainty an actuarially neutral tax cut from a given labour supply will most likely stimulate current consumption as it is also the case under interest rate uncertainty for borrowers. In the case of lenders, however, the stimulatory current consumption effect of the tax cut calculated from a given labour supply presupposes that relative risk aversion and/or the fraction of future consumption accounted for by saving is low. Under current wage rate uncertainty the low relative risk aversion and/or the low fraction of future consumption accounted for by saving imply that substitution and risk-increasing effects of tax cut run counter to each other with the total effect being ambiguous. In this particular case the stimulatory effect on consumption of an actuarially neutral tax cut necessitates, but is not guaranteed by, high relative risk aversion and high fraction of future consumption accounted for by saving.

4. TAX CUTS UNDER CAPITAL MARKET "IMPERFECTIONS"

4.1. Background

Earlier sections have developed the implications of the actuarially neutral tax cut under various circumstances concerning the nature of taxation (lump-sum versus distortionary) and the type of uncertainty (future income.

interest rate and wage rate uncertainties) but assuming perfect capital markets. The purpose of this section is to relax the assumption of perfect capital market and work up implications of various kinds of capital market "imperfections" to the intertemporal tax policy we have been interested in.

It is not difficult to argue both on theoretical and empirical grounds against the perfect capital market assumptions. Relatively recently, the nature and working of capital markets, particularly bank loan markets, have been subject to a number of theoretical analyses. As a result justifications to various kinds of capital market "imperfections" have been presented. First, allowing for the fact that beyond certain amount loans to individual borrowers decrease in 'quality' from the point of view of banks because of a rise in the probability of default, the equilibrium may be characterized by the credit rationing in the sense that at the given borrowing rate the borrower gets less amount of loan he (she) desires. This is due to the non-linear interest rate schedule. In the presence of default risk, the borrowing rate is simply not parametric to economic agents as we supposed in earlier analyses (see Keaton (1979) for a detailed analysis). Second, one might argue that for moral hazard and/or sorting (adverse selection) reasons a rise in the borrowing rate may not increase the rate of return on loans to banks. If so, the banks may not want to raise the borrowing rate even though there is an excess demand for loans. Hence, credit rationing in the form of binding quantitative limits might materialize (see Stiglitz and Weiss (1981)). Finally, the difference between borrowing and lending rate is one aspect of capital market "imperfections" and may be even more pervasive than quantitative borrowing constraints. Actually, by

dispensing with the assumption that banks are able to observe the total amount of loans consumers borrow so that only price contracts are feasible, it can be shown that under informational asymmetries between borrowers and banks one may end up, not with quantitative credit rationing, but with an endogenously determined wedge between borrowing and lending rates (see King (1984)).

In what follows we analyze the implication of all these capital market "imperfections". Section 4.2. introduces the wedge between borrowing and lending rates and non-linear interest rate schedule, while the implications of quantitative borrowing constraints are taken up in section 4.3. Because allowing for endogenous labour supply does not bring any new insights, in what follows we abstract from labour supply decisions for simplicity.

4.2. Tax cuts, interest rate wedge and non-linear interest rate schedule: non-neutralities via 'income' effects

Consider first the case where there is a wedge between borrowing rate r_B and lending rate r_L such that $r_B > r_L$ (see Flemming (1973) for a preliminary analysis of this case). For simplicity we abstract from uncertainty and endogenous loan supply. Now with additive intertemporal preferences the utility function is of the form $V^B = u(c_1) + v(q_2 y_2 + (1 + r_B)(q_1 y_1 - c_1))$ if $q_1 y_1 - c_1 < 0$ and $V = V^L = u(c_1) + v(q_2 y_2 + (1 + r_L)(q_1 y_1 - c_1))$ if $q_1 y_1 - c_1 \geq 0$ and the change in the utility with respect to a change in c_1 can be expressed respectively as $V_C^B = u' - (1 + r_B)v'$ for $q_1 y_1 - c_1 < 0$ and $V_C^L = u' - (1 + r_L)v'$ for $q_1 y_1 - c_1 \geq 0$. If we denote by c_1^0 the optimal consumption $c_1^0 = q_1 y_1$, then $V^B(c_1^0) = V^L(c_1^0)$. We can distinguish between three cases concerning optimal c_1^*

$$(24) \quad \left\{ \begin{array}{l} \text{(i)} \quad V_C^B(c_1^0) < 0 < V_C^L(c_1^0) \Rightarrow c_1^* = c_1^0 \\ \text{(ii)} \quad 0 < V_C^L(c_1^0) < V_C^B(c_1^0) \Rightarrow c_1^* > c_1^0 \\ \text{(iii)} \quad V_C^B(c_1^0) < V_C^L(c_1^0) < 0 \Rightarrow c_1^* < c_1^0 \end{array} \right.$$

In the first case the consumer is at the corner solution which is similar to total credit rationing; the consumer wants to lend at the the borrowing rate and to borrow at the lending rate. Clearly, the time pattern of net incomes, the interest rate wedge and time preference matter from the point of view of whether the consumer chooses the corner solution, the borrowing or lending position. It is worthwhile to stress that under the interest rate wedge the consumer may choose to consume at the kink in the budget constraint; it is then the optimal response to nonlinear budget constraint and has nothing to do with 'borrowing constraint'. There is a range of interest rates within which $c_1^* = q_1 y_1$ is the optimal policy.

The effect of the actuarially neutral tax cut can now be shown to be

$$(25) \quad \left. \frac{dc_1^*}{dt_1} \right|_{dT=0} = - [y_1/q_1(1+r_i)] (\partial c_1^*/\partial y_1) (r_i - r_g)$$

where r_g is the interest rate at which government can borrow and $r_i = r_L$ for lenders and $r_i = r_B$ for borrowers. In the presence of perfect capital market the actuarially neutral tax policy has no consumption effect in this case with lump-sum taxation and no uncertainty because $r_i = r_g$. If the government can borrow at the lending rate⁷⁾, then for lenders there is no effect on consumption from the tax policy because the relevant part of the budget constraint remains unchanged. But if the consumers are at the

kink both before and after the tax switch, tax cut is stimulatory by the amount of increase in the current net income (i.e. $dc_1^*/dt_1|_{dT=0} = -y_1$). Finally, for borrowers the tax cut is stimulatory, but under normality of c_1 and c_2 by the smaller amount than for the consumers who are at the kink. The intertemporal tax policy here affects via altering the budget constraints facing consumers.

It may be the case not only that there is an interest rate wedge but that the borrowing rate depends on the amount borrowed e.g. in such a way that the interest rate is an increasing function of the loan/current net income ratio, i.e. $r_B = G(L/q_1y_1) = G(c_1 - q_1y_1/q_1y_1) = G(K)$ with $G' > 0$ and $G'' \geq 0$. In this case we have

$$(26) \quad (dc_1^*/dt_1)|_{dT=0} = (V_{cc})^{-1} [v'(2G' + KG'')/q_1y_1] < 0$$

where $V_{cc} < 0$ because of the second-order conditions. The policy of changing the timing of taxes towards future is stimulatory because consumers want to borrow less and hence the marginal borrowing rate is lower.⁸⁾ For convenience we summarize the results in the

Proposition 7: In the presence of the positive interest rate wedge between the borrowing and lending rate and/or with nonlinear borrowing rate schedule with increasing marginal cost of borrowing, the timing of taxes towards future with no change in their present value stimulates consumption to the extent that government can borrow at the lower (and parametric) interest rate than consumers.

4.3. Tax cuts under credit rationing: 'liquidity' effects

Finally we analyze the tax policies in the presence of credit rationing where consumers are subject to binding borrowing constraints. This is a special case of the interest rate wedge with $r_B = \infty$ at the constraint point.

Above we have abstracted from uncertainty considerations and one may wonder how does this affect results? Under the regime, where consumers are presently subject to credit rationing, future uncertainties do not clearly matter since they have no bearing on liquidity. But even through current credit rationing is stochastic, economic agents express their Walrasian demands and supplies in the market so that they 'ignore' the potential constraint and the intertemporal allocation will remain unchanged. Hence, in order to get some implications of credit rationing under uncertainty we have to consider the situation where consumers expect in the future to be subject to credit rationing with some probability. Let us now turn to analyze this possibility.

4.3.1. Timing of taxes under stochastic future credit rationing: the expected 'liquidity'

Consider a three-period model where in period 2 the consumer is subject to credit rationing with probability θ . Other potential uncertainties are ignored. Under the intertemporally additive utility and exogenous incomes and interest rates the consumer's decision problem is the following one

where λ_1 , λ_2 and λ_3 denote the Lagrange multipliers for the (inter-temporal) budget constraints (i) and (ii) and for the asset constraint (iii) respectively.

In order to find out qualitative behaviour of consumption we differentiate totally (29.i-viii) with respect to c_1, c_2, d_2, c_3 and d_3 and with respect to exogenous variables B_2, t_1, t_2 and t_3 we are interested in. The effect of a change in future credit limit \bar{B}_2 on the current consumption can be shown to be

$$(29) \quad (\partial \hat{c}_1^* / \partial \bar{B}_2) = \Omega^{-1} (\theta R_2 u_2''(d_2)) > 0$$

$$= \begin{cases} 0 & \text{if } \theta = 0 \\ \frac{R_2 u_2''(d_2)}{R_3 u_1''(c_1) + u_2''(d_3)} > 0 & \text{if } \theta = 1 \end{cases}$$

$$\text{where } \Omega = R_3 u_1''(c_1) + \theta u_2''(d_2) + \left\{ \frac{(1-\theta)u_2''(c_2)u_3''(c_3)}{[R_3 u_2''(c_2) + u_3''(c_3)]} \right\} < 0.$$

Thus if the probability of becoming rationed in the future is positive, then consumers increase saving and decrease consumption when they expect credit market to become 'tighter' in the sense that credit limit falls with given θ .

Consider the following actuarially neutral tax policy. Tax rates in period 1 and 2 are equal $t_1 = t_2 = t$ and there is the tax switch between first two periods taken together and the third period which does not change the present value of tax revenues. The comparative statics of taxes becomes

$$(30) \quad (\partial \hat{c}_1^* / \partial t) = -\Omega^{-1} \left[\theta u_2''(d_2) + \frac{(1-\theta)u_2''(c_2)u_3''(c_3)}{[R_3 u_2''(c_2) + u_3''(c_3)]} \right]$$

and

$$(31) \quad (\partial \hat{c}_1^{**} / \partial t_3) = -\Omega^{-1} y_3 R_3 \left[\frac{(1-\theta) u_2''(c_2) u_3''(c_3)}{[R_3 u_2''(c_2) + u_3''(c_3)]} \right]$$

where $Y = y_1 + y_2 R_2$. If the probability of rationing is below one, then both taxes affect current consumption negatively, while under certain credit rationing t_3 has no effect on current consumption; under certain credit rationing in period 2 the consumer behaves as if its planning horizon were two periods so that changes of exogenous variables beyond that period has no bearing on behaviour.

The actuarially neutral tax switch is now defined by $dt_3 = -(Y/y_3 R_3) dt$ so that the tax policy which keeps $T = ty_1 + ty_2 R_2 + t_3 Y_3 R_3$ unchanged implies

$$(32) \quad \left. \frac{d\hat{c}_1^{**}}{dt} \right|_{dT=0} = (\partial \hat{c}_1^{**} / \partial t) - (Y/y_3 R_3) (\partial \hat{c}_1^{**} / \partial t_3)$$

$$= -(Y/R_2) (\partial \hat{c}_1^{**} / \partial \bar{B}_2) \begin{cases} 0 & \text{if } \theta = 0 \\ < 0 & \text{if } \theta > 0 \end{cases}$$

which follows from (29)-(31). This model produces the Ricardian equivalence theorem if the the probability of being subject to credit rationing in the future is zero. But if consumers do not know the rationing scheme and expect to be subject to credit rationing in the future with some probability, then the actuarially neutral tax cut is stimulatory; decreasing current taxes works like a increase in future credit limit. Shifting taxes towards future raises expected 'liquidity' and consumption goes up. Fiscal policy matters because even a small chance of credit rationing makes the effective planning horizon of consumers shorter than the horizon over which taxes

are smoothed. If the tax switch is carried out between periods 1 and 2, then it has no consumption effect even with credit rationing in period 2 because the tax smoothing and effective planning horizons of consumers coincide. Thus credit rationing per se does not procedure non-neutralities. As a recapitulation we collect the results from sections 4.3.1. and 4.3.2. in

Proposition 8: In the presence of credit rationing shifting taxes towards future will stimulate (have no effect on) consumption if the horizon over which taxes are smoothed is shorter (equal to or longer) than the effective planning horizon of consumers implied by credit rationing. Even a small chance of credit rationing during the tax smoothing horizon produces non-neutralities basically because timing of taxes works like changes in credit supply which in turn change the actual (or expected) 'liquidity'.

5. CONCLUDING REMARKS

According to the so-called Ricardian equivalence theorem the timing of taxes with given government expenditures does not matter in terms of consumption behaviour. In this paper we have relaxed the assumptions of certainty, lump-sum taxes and perfect capital markets and looked at their implications. Allowing for uncertainty, distortionary taxes and 'imperfect' capital markets of various kinds will have the effect of giving rise to the fact that actuarially neutral tax cuts - which will the expected (present value of) government tax revenue constant - bring about risk-sharing, substitution (incentive) and liquidity effects. These effects will induce non-neutralities. In the case of future income uncertainty and 'imperfect' capital markets the tax cuts tend to be stimulatory in terms of

current consumption under quite weak assumptions, while in the presence of interest rate and wage rate uncertainties more knowledge e.g. about the risk aversion behaviour and the position of economic agents in the capital market are required in order to be able to sign the effects of policies.

Throughout the whole paper we have kept to the assumption that economic agents believe that tax cuts today induces future tax increases. An obvious area for further research is to study the implications of alternative ways of linking tax cuts today with future policies and compare various policies in terms of their behavioral and welfare effects.⁹⁾

FOOTNOTES:

- 1) Barro's contribution is to show how mortal households can effectively have infinite horizons thus suggesting that under certain additional assumptions timing of taxes do not affect the intertemporal budget constraint. Each generation is assumed to include in its utility function, along with its own consumption, the utility of the next generation. Via the chain of overlapping generations, connected by bequests, a shift in timing of taxes with given present value of taxation does not change the relevant budget constraint of the current generation (see Barro (1974)).
- 2) The term 'Ricardian view' has become established to mean the neutrality theorem according to which the tax-debt choice does not matter, even though Ricardo himself did not believe in the actual relevance of the neutrality theorem (see e.g. O'Driscoll (1977)). Thus the term 'Ricardian view' is a misnomer. For simplicity we follow the present usage.
- 3) Sandmo (1970) assumes that $A(c_1, c_2) = -u_{22}(c_1, c_2)/u_2(c_1, c_2)$ is decreasing in c_2 and increasing in c_1 and shows that this "decreasing temporal risk aversion" implies $U_{cy_2y_2} < 0$. These characterizations give sufficient conditions for the $U_{cy_2y_2}$ sign of future income risk. The sign can alternatively be characterized in terms of how income risk affects the marginal rate of substitution between c_1 and c_2 . In fact, the effect of income risk on marginal rate of substitution is both a necessary and sufficient condition to characterize how future income risk affects consumption and saving (see Menezes and Auten (1978)).
- 4) This need not imply that government is risk neutral. To the extent that risks are independent across individuals and the number of individuals is large, then the law of large numbers will guarantee government a constant total revenue despite uncertainty at the individual level. Under these circumstances government is simply a more efficient risk-pooler than individuals. To the extent that the law of large numbers does not work e.g. because of the dominance of 'business cycle risks', then the assumption that government is risk neutral is presupposed.
- 5) Chan (1983) has presented in a slightly different model a formula which essentially comes to the same than (13).
- 6) Eaton and Rosen (1980) were mainly interested in the effects on wage taxation on labour supply and the 'optimal' use of lump-sum and wage taxation, while Stiglitz (1982) argues that under not too implausible circumstances randomization of wage taxes is desirable. The idea is roughly that even though randomization of taxes imposes risk on individuals, it may so strongly increase labour supply for 'hedging' reasons and thereby tax base so that the average tax rate can be reduced as a result of randomization. The benefit from this reduction of average taxes may exceed the loss from induced risk accompanied by the randomization! Usually, effects of increasing risk on behaviour have been characterized in terms of risk aversion parameters. Tressler and Menezes (1980) on the other hand, show that the question of how wage

risk affects the marginal rate of substitution between foregone leisure and consumption provides a necessary and sufficient condition to unambiguously sign the effect of increased wage uncertainty on labour supply. Actually, in contrast with Stiglitz (1982) they argue that increased wage uncertainty decreases labour supply.

- 7) One can argue that under 'normal circumstances' this will likely be the case; taxing power of government tend to make default risk of less importance than in the case of individual borrowers (see e.g. Webb (1981)).
- 8) The justification of the non-linear interest rate schedule relies on change in loan 'quality' with the amount of loans via default risk changes. Therefore, one might want to analyze consumer behaviour by taking the bankruptcy possibility into account. This could be easily introduced by assuming that there is some institutionally determined minimum level of consumption below which consumers are not allowed to fall in the case of bad realization of e.g. future income (see King (1984)). For a general analysis of bankruptcy in an intertemporal context see Hellwig (1977).
- 9) A step in this direction has been recently taken by Stiglitz (1983).

Appendix 1: Consumption, labour supply and taxation in an intertemporal model with wage uncertainty

Maximizing $U^{**} = u(c_1, L) + E v(c_2)$ subject to $c_2 = q_2 y_2 + R(q_1 \tilde{w} L - c_1)$ where w is stochastic, in terms of c_1 and c_2 gives the following first-order and second-order conditions

$$(1) \quad \begin{cases} (i) & u_1 - RE(v') = 0 = U_C^{**} \\ (ii) & u_2 + Rq_1 E(v'w) = 0 = U_L^{**} \end{cases}$$

and

$$(2) \quad \begin{cases} (i) & U_{CC}^{**} = u_{11} + R^2 E(v'') < 0, \quad U_{LL}^{**} = u_{22} + (Rq_1)^2 E(v'' w^2) < 0 \\ (ii) & D^{**} = U_{CC}^{**} U_{LL}^{**} - U_{CL}^{**2} > 0 \end{cases}$$

where $U_{CL}^{**} = U_{LC}^{**} = u_{12} - R^2 q_1 E(v'' w) > 0$. Denote

$$(3) \quad \begin{cases} (i) & a_0 = -R^2 L E(v'' w) \\ (ii) & a_1 = R^2 L q_1 E(v'' w^2) + RE(v'w) \\ (iii) & b_0 = -R y_2 E(v'') \\ (iv) & b_1 = R y_2 E(v'' w) \end{cases}$$

The effects of taxes can now be expressed as

$$(4) \quad \begin{pmatrix} U_{CC}^{**} & U_{CL}^{**} \\ U_{LC}^{**} & U_{LL}^{**} \end{pmatrix} \begin{pmatrix} \partial c^{**} \\ \partial L^{**} \end{pmatrix} = \begin{pmatrix} a_0 & b_0 \\ a_1 & b_1 \end{pmatrix} \begin{pmatrix} \partial t_1 \\ \partial t_2 \end{pmatrix}$$

where $Q = E(v'')E(v'' w^2) - E((v'' w)^2) > 0$ due to Cauchy-Schwartz inequality (see e.g. Mood-Graybill-Boes (1974), p. 162).

Let ε be change in the the expected value of w with all other moments constant (i.e. an additive shift in the distribution of $\tilde{w} = w + \varepsilon$ evaluated at $\varepsilon = 0$) and let η be a mean-preserving change in risk of w (i.e. a multiplicative shift in the distribution $\tilde{w} = \varepsilon + \eta w$ offsetted by an additive shift ε to keep the mean constant and evaluated at $\varepsilon = 0, \eta = 1$).

Denote

$$(8) \quad \left\{ \begin{array}{l} (i) \quad e_0 = R^2 q_1 L E(v'') \\ (ii) \quad e_1 = -R^2 q_1^2 L E(v'' w) - R q_1 E(v') \\ (iii) \quad f_0 = R^2 q_1 L E(v'' (w - \bar{w})) \\ (iv) \quad f_1 = -R^2 q_1^2 L E(v'' w)(w - \bar{w}) - R q_1 E(v' (w - \bar{w})) \end{array} \right.$$

The effects of ε and adjusted η can now be expressed as

$$(9) \quad \begin{pmatrix} \partial c_1^{**} \\ \partial L^{**} \end{pmatrix} = \frac{1}{D^{**}} \begin{pmatrix} U_{LL}^{**} & -U_{CL}^{**} \\ -U_{LC}^{**} & U_{CC}^{**} \end{pmatrix} \begin{pmatrix} e_0 & f_0 \\ e_1 & f_1 \end{pmatrix} \begin{pmatrix} \partial \varepsilon \\ \partial \eta \end{pmatrix}$$

and the comparative statics becomes

$$(10) \quad \begin{aligned} (i) \quad (\partial c_1^{**} / \partial \varepsilon) \Big|_{\varepsilon=0} &= D^{**^{-1}} \{ R^2 q_1 L [U_{LL}^{**} E(v'') + q_1 U_{CL}^{**} E(v'' w)] \\ &\quad + R q_1 E(v') U_{CL}^{**} \} \\ (ii) \quad (\partial L^{**} / \partial \varepsilon) \Big|_{\varepsilon=0} &= D^{**^{-1}} \{ -R^2 q_1 L [q_1 U_{CC}^{**} + U_{LC}^{**} E(v'')] \\ &\quad - R q_1 E(v') U_{CC}^{**} \} \end{aligned}$$

$$(iii) \left. \frac{\partial c_1^{**}}{\partial \eta} \right|_{\substack{\epsilon = 0 \\ \eta = 1}} = D^{**^{-1}} \{ R^2 q_1 L [U_{LL}^{**} E(v''(w - \bar{w})) + \\ q_1 U_{cL}^{**} E(v'' w(w - \bar{w}))] + R q_1 U_{cL}^{**} E(v'(w - \bar{w})) \}$$

$$(iv) \left. \frac{\partial L^{**}}{\partial \eta} \right|_{\substack{\epsilon = 0 \\ \eta = 1}} = D^{**^{-1}} \{ -R^2 q_1 L [q_1 U_{cc}^{**} E(v'' w(w - \bar{w})) + \\ U_{Lc}^{**} E(v''(w - \bar{w}))] - R q_1 U_{cc}^{**} E(v'(w - \bar{w})) \}$$

Comparing the equations (6iii) and (10i) on the one hand and the equations (6iv) and (10ii) on the other hand gives the equations (30i,ii) in the text. Similar comparison of (6i) with (10i) and (10iii) gives the equation (31i) and comparing (6ii) with (10ii) and with (10iv) yields finally the equation (31ii) of the text. This last step utilizes the result according to which for two random variables x and y $E(xy) = \bar{x}\bar{y} + \text{cov}(x,y)$.

REFERENCES:

- Barro, R.J.: "Are Government Bonds Net Wealth?", Journal of Political Economy, 1974, 1095-1117.
- Bryant, J.: "Government Irrelevance Results: A Simple Exposition", American Economic Review, 1983, 758-761.
- Chan, L.K.C.: "Uncertainty and the Neutrality of Government Financing Policy", Journal of Monetary Economics, 1983, 351-372.
- Eaton, J. and Rosen, H.S.: "Labor Supply, Uncertainty and Efficient Taxation", Journal of Public Economics, 1980, 365-374.
- Flemming, J.S.: "The Consumption Function when Capital Markets are Imperfect: The Permanent Income Hypothesis Reconsidered", Oxford Economic Papers, 1973, 160-172.
- Hanson, D.L. and Menezes, C.F.: "The Effect of Capital Risk on Optimal Saving Decisions", Quarterly Journal of Economics, 1978, 653-670.
- Hellwig, M.F.: "A Model of Borrowing and Lending with Bankruptcy", Econometrica, 1977, 1879-1905.
- Keeton, W.: Equilibrium Credit Rationing, Garland Publishing Company, New York, 1979.
- King, M.: "Capital Market 'Imperfections' and the Consumption Function", unpublished mimeo, April, 1984.
- Koskela, E. and Virén, M.: "Credit Rationing and Consumer Intertemporal Choice", Oxford Economic Papers, 1984, 241-247.
- Leland, H.E.: "Saving Under Uncertainty: the Precautionary Demand for Saving", Quarterly Journal of Economics, 1968, 465-473.
- Menezes, C.F. and Auten, G.: "The Theory of Optimal Saving Decisions under Income Risk", International Economic Review, 1978, 253-258.
- Mood, A.M. and Graybill, F.A. and Boes, D.C.: Introduction to the Theory of Statistics, McGraw-Hill, 1974, 3rd edition.
- O'Driscoll, G.P.: "The Ricardian Nonequivalence Theorem", Journal of Political Economy, 1977, 207-210.

- Sargent, T.J. and Wallace, N.: "Some Unpleasant Monetarist Arithmetic", Federal Reserve Bank of Minneapolis Quarterly Review, 1981, 1-17.
- Stiglitz, J.E.: "Utilitarianism and Horizontal Equity: the Case for Random Taxation", Journal of Public Economics, 1982, 1-33.
- Stiglitz, J.E.: "On the Relevance or Irrelevance of Public Financial Policy: Indexation, Price Rigidities, and Optimal Monetary Policies", in Dornbusch, R. and Simonsen, M.H. (eds): Inflation, Debt, and Indexation, the MIT Press, Cambridge, Massachusetts, 1983, 184-222.
- Stiglitz, J.E. and Weiss, A.: "Credit Rationing in Markets with Imperfect Information", American Economic Review, 1981, 393-410.
- Tressler, J.H. and Menezes, C.F.: "Labour Supply and Wage Rate Uncertainty", Journal of Economic Theory, 1980, 425-436.
- Webb, D.C.: "The New Wealth Effect of Government Bonds When Credit Markets are Imperfect", Economic Journal, 1981, 405-414.

ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)
The Research Institute of the Finnish Economy
Lönrotinkatu 4 B, SF-00120 HELSINKI 12 Puh./Tel. (90) 601 322

KESKUSTELUAIHEITA - DISCUSSION PAPERS ISSN 0781-6847

No 174 CHRISTIAN EDGREN, Automatic versus Discretionary Tax-Policy
Effects Revisited. 05.06.1985. 44 p.

No 175 TAUNO KALLINEN, Työn tuottavuuden taso ja ansiotaso Suomen,
Ruotsin, Norjan ja Tanskan tehdasteollisuudessa vuosina 1973 ja
1982. 11.06.1985. 44 s.

No 176 J. LESKELÄ - J. MOILANEN, A Comparison of Recent Medium-Term
Forecasts. 17.06.1985. 34 p.

No 177 PAAVO SUNI, Efektiivisen nimellisen valuuttakurssin mittaaminen
ja mittarien tulkinta. 20.06.1985. 42 s.

No 178 MARKKU LAMMI, Alkoholi- ja polttoaineverojen kertymästä.
27.09.1985. 49 s.

No 179 YRJÖ O. VARTIA, Tunnetuissa pisteissä epäsäännöllisen funktion
numeerinen integrointi. 21.10.1985. 8 s.

No 180 ERKKI KOSKELA, On Monopolistic Tax Evasion under Direct and
Indirect Taxation. 23.10.1985. 17 p.

No 181 PEKKA ILMAKUNNAS, Stochastic Constraints on Cost Function
Parameters: Mixed and Hierarchical Approaches. 31.10.1985.
26 p.

No 182 HIROKI TSURUMI - HAJIME WAGO - PEKKA ILMAKUNNAS, Gradual
Switching Multivariate Regression Models with Stochastic
Cross-Equational Constraints and an Application to the Klem
Translog Production Model. 01.11.1985. 28 p.

No 183 HANNU TÖRMÄ, Panossubstituution rakennemuutos Suomen
teollisuudessa. 06.11.1985. 23 s.

No 184 FREDRIK NYGÅRD, Lifetime Incomes in Finland - Desk Calculations
Based on Civil Servant Salaries 1985. 18.11.1985. 28 p.

No 185 FREDRIK NYGÅRD, Relative Income Differences in Finland 1971-1981.
18.11.1985. 22 p.

No 186 ERKKI KOSKELA, Tax Cuts, Risk-Sharing and Capital Market
'Imperfections'. 18.11.1985. 40 p.

Elinkeinoelämän Tutkimuslaitoksen julkaisemat "Keskusteluaiheet" ovat
raportteja alustavista tutkimustuloksista ja väliraportteja tekeillä
olevista tutkimuksista. Tässä sarjassa julkaistuja monisteita on rajoi-
tetusti saatavissa ETLAn kirjastosta tai ao. tutkijalta.

Papers in this series are reports on preliminary research results and on
studies in progress; they can be obtained, on request, by the author's
permission.

0033A/18.11.1985