

# Keskusteluaiheita Discussion papers

Christian Edgren\*

AUTOMATIC VERSUS DISCRETIONARY

TAX-POLICY EFFECTS REVISITED

No 174

05.06.1985

\* I wish to thank Mr. Erkki Koskela,  
Jukka Lassila and Jussi Karko  
for many valuable comments on this  
topic.

ISSN 0781-6847

This series consists of papers with limited circulation,  
intended to stimulate discussion. The papers must  
not be referred or quoted without the authors'  
permission.



## CONTENTS

	page
I INTRODUCTION	1
II THEORETICAL AND CONCEPTUAL FRAMEWORK	6
1. The macro tax elasticity	6
2. The index of tax-policy	8
3. The scaling of the index	10
4. The characteristics of the index	13
5. The policy index and the cost-of-living index	16
III EMPIRICAL RESULTS	17
1. The schedule policy	17
2. The combined schedule-deduction policy	20
3. Integration of all income receivers into the tax policy analysis	23
4. The role of the deduction system	27
IV CONCLUDING REMARKS	30
References	32
Appendix 1. The scaling of the tax-policy index	34
2. Instantaneous measures for the average taxation	38
3. Data used for construction of the tax-policy index	42

## I INTRODUCTION

Both policy makers and researchers are interested in knowing the responsiveness of tax-revenue vis-à-vis the change in income. Usually we talk about the tax-revenue elasticity, which we use for forecasting and which is often necessary in studies explaining the effect of the tax policy on different variables in the economy in the past.

There have been many studies made dealing with the overall progressivity of the taxation system [1], [14], [15], [18]. They are mostly based on general formulated properties of the tax function and observed empirical income and tax distributions and give a single aggregate and momentary measure of the redistributive effect of the tax system.

The theory based on the aggregation of individual tax functions resulting in different responsiveness measures of the aggregate tax revenue function has given the frame for a more thorough study of the interaction between income growth and the tax system [11] [12]. The calculations of sources of change in tax-revenue and post-tax income elasticities has also improved the operationality needed, e.g. for forecasting. In most of the studies, however, the tax effects from changes in the taxpayer population have not been explicitly formulated. The change in the number of taxpayers affects the tax-revenue elasticity - as pointed out in [13] and [20] - but it can also be handled as a tax-policy (threshold) effect, which appears very strongly when income grows and tax rules are nominally unchanged [6] [9] [10].

Different methods has been used to clean the tax-revenue series from tax policy to obtain the residual for elasticity calculations. These methods, however, do not go very deeply into the problem and do not take into account the consequences of policy specification on the content of the elasticity. It should be noted that if we define the tax policy, we indirectly also determine the elasticity.

The problem for this paper is to construct an aggregate index for the personal progressive income taxation based on micro data. By expressing a tax-policy index, we make a distinction between the automatic and discretionary changes in taxes, explicitly taking into account the change in the taxpayer population. The equiproportionate income growth assumption frequently used for deriving meaningful formulas - a not very restrictive assumption, as pointed out later on - is used in this paper for identifying the revenue effect of changes in the taxpayer population. This effect can, however, later on be identified on the tax base side as a non-equiproportionate income growth in analyzing the income growth differences of the taxpayers and all income receivers.

First, we have to specify the reference taxation. This has to do with the tax laws. The implicit reference when we analyze the tax policy in terms of discretionarity can be the written tax law or some common idea of the real meaning behind the tax laws. The tax policy can be discretionary of zero degree if there is no change in the taxation rules - the tax schedule specified in nominal terms, for example, is the same. If in the law is included a statement of inflation indexation, the nominal schedule has to be changed according to inflation for the tax policy to be discretionary of zero degree. In



the following first we are interested in measuring the discretionarity against the written tax law, which in the Finnish case means that the discretionarity is of zero degree when no explicit amendments to the tax laws specified in nominal terms have been made. Later on we look at the situation when inflation indexation is put in as a reference for the taxation.

The progressive income taxation (the state income taxation) is the main subject for the study. The proportional income taxation (the municipal taxation and insured persons' social security fees) alone is of no interest for this matter. We also have to formulate the tax policy for the combined progressive-proportional tax function, but this is left for later studies.

The tax-policy index which is to be presented is calculated as the relation between the actual relative change in tax revenue and the change in tax revenue without a tax policy, where the policy can be defined in different modes. The operationality in the system is based on equations linking the tax-policy revenue effect to a neutral or neutral-equivalent indexation "price"-variable.<sup>1)</sup> The indexes are constructed for different aggregated tax bases to measure the policy effects separately for the schedule and the deductions. The macro tax functions behind the analysis are therefore one with the income arguments post-deduction income of taxpayers and the tax-schedule-policy parameter, one with the arguments pre-deduction income of taxpayers and a combined schedule-deduction policy parameter and one which links the taxation to the income growth of all income receivers.

---

1) Defined on page 9.

Choudhry [3] has presented a Divisia index approach for calculating a discretionary tax-policy index for a combined set of different categories of taxes (direct and indirect taxes). He, however, searched for the combined macro tax-revenue elasticity. We are going the other way round by searching for the tax-policy having derived the macro tax revenue elasticity, where the macro elasticity is a combined elasticity calculated on the micro elasticities in the taxpayer population.

The Choudhry approach gives one average estimated tax-revenue elasticity for the investigated period. We have calculated the macro elasticity for every year in the period. The elasticity changes from year to year and the change depends simultaneously on the rate of the income increase and the change in the nominally specified tax laws. The general reasoning is intuitively as follows: if tax laws - e.g. the tax schedule - are kept nominally constant and incomes grow, the macro elasticity decreases, although not so fast as otherwise, because income receivers with lower incomes who were, not previously taxed - say in period 0 - become new taxpayers in period 1. The new taxpayers have high instantaneous micro elasticities while the old taxpayers' micro elasticities as a whole go somewhat down. The change of the macro elasticity depends on the income density in the income range which includes these marginal new taxpayers. The macro elasticity, therefore, depends on the shape of the income density function of the taxpayers, which in turn depends from the policy side on the position of the nominal threshold - like the schedule's lowest income level or its "projection" on gross income level - relative to the mean or median income.

For the macro tax function and its transformation into relative changes, we are bounded to a constant set of arguments. From the

viewpoint of time new taxpayers mean an increase of arguments while a decrease in the number of taxpayers means that the number of arguments decreases. The effect of changes in the set of arguments on the tax revenue depends on the original tax policy and the question is whether the effect should be considered as a tax policy itself or as an endogenous increase or decrease of the tax revenue.<sup>1)</sup>

As a main point for the analysis we assume that the instantaneous macro tax elasticity is the same for all income receivers and taxpayers, because the sum of the taxpayers' and non-taxpayers' elasticities weighted according to tax shares is the same as the elasticity for taxpayers, the non-taxpayers' elasticity and tax share both being zero.<sup>2)</sup>

The result can be used to handle the tax-revenue effect of increasing or decreasing of incomes due to changes in the number of taxpayers as a non-equiproportionate growth problem. If the pre-deduction incomes or post-deduction incomes of all income receivers or some sub-population of them, both including the taxpayer population, grows equipro-

---

1) The problem with change in the number of arguments in the macro function can in a two-period calculation be formally solved by specifying the tax function for the union set of taxpayers in both periods. For this trick, however, we need special micro data. In the forecasting situation it is, however, of no help. In the following we will handle the tax-revenue effect as originating from the non-equiproportional income growth of the taxpayer population, although one usually thinks of it in a fixed set of arguments in the function [17].

2) It can be shown that the average marginal tax rate weighted according to income shares decreases in the same proportion as the average tax rate weighted according to income shares when we enlarge the taxpayer population to the population of all income receivers, so the elasticity does not change [8].

portionately, then taxpayers' pre-deduction and the respective post-deduction incomes grow non-equiproportionately. The tax revenue grows then in accordance with the instantaneous elasticity corrected for the non-equiproportionate growth and the tax policy by the indirect effect. The indirect effect is zero if the nominal threshold, which is a policy parameter, is changed according to the same rate as incomes grow. Only in this special case do taxpayers' incomes also grow equiproportionately, leaving the taxpayers' income share of all incomes constant.

First, we present briefly the tax revenue aggregation formula - developed in [11] - used for calculations. Next we construct the tax-policy index  $D(t)$ . The scaling of the index is done after that to obtain the original policy index  $D^*(t)$ , which does not include the new-taxpayers effect. The sources for changes in the policy index are then formulated. In the empirical section we use the index approach for the different tax bases - post-deduction and pre-deduction incomes of taxpayers and total incomes of all income receivers - for calculation of the policy effects from the schedule, from deductions and the new taxpayers, of which effects the last one can be identified as a non-equiproportionate income growth effect.

## II THEORETICAL AND CONCEPTUAL FRAMEWORK

### 1. The macro tax elasticity

The macro tax-revenue elasticity calculations used in the empirical analysis are based on the equation

$$\begin{aligned}
 (1) \quad \Delta \log T &= [\sum w_i^T e(\bar{y}_i)] \Delta \log y + [\sum w_i^T \bar{e}(y_i)] (\dot{y}_1 - \dot{y}) \\
 &= \hat{e} \Delta \log y + \text{cov}(e(\bar{y}_i), \dot{y}_1 - \dot{y}) ,
 \end{aligned}$$

where  $T$  is the total tax revenue and  $y$  is the tax base. The "individual" tax weights  $w_i^T$  are

$$w_i^T = \frac{\hat{T}_i}{\hat{T}} = \frac{L(T_i^1, T_i^0)}{L(T^1, T^0)}$$

and  $e(\bar{y}_i)$  the "individual" tax elasticities defined in income intervals  $\{y_i^0, y_i^1\}$ . In the  $\text{cov}(\ )$  term we have noted  $\hat{y}_i - \bar{y} = \Delta \log y_i - \Delta \log y$ .<sup>1)</sup>

Two simplifications:

- 1) We assume for the empirical application that the  $\text{cov}(\ )$  is zero, because we do not have the micro data for an explicit calculation. The effect on the tax revenue due to the  $\text{cov}(\ )$  can not be observed as a residual, because at the same time we have the effect of the increase in the number of arguments in the tax function - i.e. the new taxpayers continuously added to the old ones - and the tax policy. The  $\text{cov}(\ )$  zero assumption is, of course, serious if we, for example have the post-deduction income as our tax base, because even if pre-deduction incomes growth is equiproportionate we know that deductions work progressively, and hence the post-deduction incomes growth is non-equiproportionate even if the set of taxpayers is fixed.

---

1) The macro elasticity can also be calculated by first calculating the weighted average marginal tax rate ( $\bar{m}$ ) or weighted progression ( $\bar{\pi}$ ) as shown in [5]. The values of the instantaneous yearly macro parameters are reported in appendix 2.

ii) For the macro elasticity in equation (1) we make the approximation  $\hat{e} \approx \bar{e} = \frac{1}{2}(\bar{e}^0 + \bar{e}^1)$ , where  $\bar{e}^0$  and  $\bar{e}^1$  are the average instantaneous elasticities for periods 0 and 1. The approximation means that the individual elasticities are not real innerpoint estimates. Corresponding to the elasticity approximation are the weights  $w_i^T \approx \frac{1}{2}(\frac{T_i^1}{T^1} + \frac{T_i^0}{T^0})$ , so the population of taxpayers is allowed to change from period 0 to period 1.<sup>1)</sup>

## 2. The index of tax policy

From the empirical side we have a growth relation, i.e. the relation between the relative change in tax revenue and the relative change in the tax base, where the tax base can be post-deduction or pre-deduction incomes of taxpayers or the incomes of all income receivers. The time-series are calculated as five-year moving averages. We write the quotients as

$$(2) \quad \beta_t = \frac{\Delta \log T_t}{\Delta \log X_t},$$

where  $\Delta \log T$  and  $\Delta \log X$  are the log changes of the five year moving sums of the observed tax revenue and tax bases, with  $t$  noting the moving periods.

---

1) It should be pointed out that in the approximation  $\hat{e} \approx \frac{1}{2}(\bar{e}^0 + \bar{e}^1)$  the  $\bar{e}^1$  is marginally affected through tax weights by the change of the number of arguments in the tax function and the change in the tax function itself, while the weights in equation (1) as regards period 1 are hypothetical ones which can not be observed.

The five-year moving arithmetic mean of the yearly macro elasticities - noted by  $\bar{e}$  - and the unknown tax-policy component gives the change in taxes as

$$(3) \quad \Delta \log T = \bar{e} \Delta \log x + \alpha,$$

where  $\alpha$  represents the discretionary effect of the tax policy. The content of  $\alpha$  depends on the way  $\bar{e}$  has been calculated.

The tax-policy effect  $\alpha$  can be written for the neutral indexation of tax rules and especially of the schedule as

$$(4) \quad \alpha = \log \left( 1 - \frac{(\log p) \bar{\pi}}{\bar{\theta}} \right),$$

where  $\bar{\pi}$  is the measure of the average progression - average rate responsiveness -  $\bar{\theta}$  the average tax rate and  $p$  the tax rules indexation factor.<sup>1)</sup>

The progression  $\bar{\pi} = \bar{m} - \bar{\theta}$  and the elasticity  $\bar{e} = 1 + \frac{\bar{\pi}}{\bar{\theta}}$ , where  $\bar{m}$  stands for the average marginal tax rate, so we have  $\bar{\pi} / \bar{\theta} = \bar{e} - 1$ . The parameters  $\bar{\pi}$ ,  $\bar{m}$ ,  $\bar{\theta}$  and  $\bar{e}$  are weighted in the macrofunctions: a) the tax schedule with the tax base post-deduction income and b) the combined schedule-deduction rules with the tax base pre-deduction income.

If there is no indexation of the tax rules,  $p = 1$  in equation (4) and consequently  $\alpha = 0$ . For a proportional taxation, which does not need any indexation,  $p = 1$  and  $e = 1$  so  $\alpha = 0$ .

---

1) Neutral indexation means that the upper and lower limits of the schedules' income brackets and the tax for the lowest income of the respective brackets are indexed by the same factor  $p$ , marginal tax rates unchanged. If the schedule is changed in a non-neutral fashion, we transform the effect to a neutral-equivalent indexation factor  $p$  shown in [10].

Equation (3) can be written as

$$(5) \quad \bar{e} = \frac{\Delta \log T - \alpha}{\Delta \log X}$$

Combining equations (2) and (5) we have a measure for the discretionary policy as

$$(6) \quad D(t) = \frac{\beta t}{\bar{e}} = \frac{\Delta \log T}{\Delta \log X} \cdot \frac{\Delta \log X}{\Delta \log T - \alpha} = \frac{\Delta \log T}{\Delta \log T - \alpha},$$

i.e. the actual change in tax revenue in relation to the change in tax revenue without tax policy. We have then: if  $p = 1$  (no tax policy),  $\alpha = 0$  and  $D(t) = 1$ ; if  $p > 1$ , we have  $\alpha < 0$  and  $D(t) < 1$ ; and if  $p < 1$ ,  $\alpha > 0$  and  $D(t) > 1$ . When  $D(t) > 1$  the tax policy is discretionary upwards, when  $D(t) < 1$  discretionary downwards.

### 3. The scaling of the index

Looking at the data we see that  $D(t)$  is for the whole period 1960-80 less than one,  $\alpha$  less than zero and  $p > 1$ . This means that  $D(t)$  is systematically downward biased in relation to the data-series.<sup>1)</sup> We know that the contribution to the growth in tax revenue in relation to the contribution to the growth in the tax base for new taxpayers introduced in the lower schedule brackets is less than the elasticity for the old taxpayers. We therefore scale our original index  $D(t)$  to have a new index  $D^*(t)$  by multiplying  $D(t)$  by a factor  $k$ . In the

---

1) The question of bias depends, however, on how we like to handle the effect of new taxpayers - included in  $\alpha$  - as a tax-policy effect or as an indirect endogenous effect. In every case we have to separate it from the original direct-policy effect.





We have in the chart the calculated  $\bar{e}$ , drawn from the origin in the diagram, where the axes  $\Delta \log T$  and  $\Delta \log X$  indicate the log changes of the tax revenue and the respective tax bases. For the period 1969-1973 we have the actual tax/tax base situation at point A. By shifting the elasticity line  $\bar{e}$  to the right to point A we have the line  $\bar{e} \Delta \log x + \alpha$  giving a negative intercept  $\alpha$ . For the basic period we have by definition no tax policy so we have to make the intercept  $\alpha = 0$ . The elasticity  $\bar{e}$  is calculated without correction for the new taxpayers, so  $\bar{e}$  corresponds to a smaller tax/tax base combination, say OH and OM at point C. The new taxpayers contribution to the tax increase is  $HF = \epsilon \Delta \log T$  and to the tax base increase  $ME = \epsilon \Delta \log X$ . We scale the elasticity  $\bar{e}$  with  $1/k$  to have the line  $e^* \Delta \log X = \frac{1-\bar{e}}{k} \Delta \log X$  at point A giving  $\alpha = 0$ . We still have the elasticity  $\bar{e}$  in use because the scaling means that  $\alpha = -\bar{e} \Delta \log X (1 - \frac{1}{k})$ . We rewrite equation (3) for the case of no tax policy

$$\begin{aligned}
 (7) \quad \Delta \log T &= \bar{e} \Delta \log x + \alpha \\
 &= \bar{e} \Delta \log x - \bar{e} \Delta \log x (1 - 1/k) \\
 &= e^* \Delta \log x.
 \end{aligned}$$

The  $\alpha$  for other periods are scaled in relation to the shift for the base period 1969/73. In chart 1 we have yet another situation with the same elasticity  $\bar{e}$ , that of the tax/tax base combination indicated by point B. First, we have the scaling of the  $\bar{e}$  to  $e^*$  according the base period. By shifting the line  $e^* \Delta \log x$  to point B, we have the line  $\Delta \log T = e^* \Delta \log x + \alpha^*$  with the discretionary tax-policy effect  $\alpha^*$ , not including - like  $\alpha$  - the effect of new taxpayers' contribution to the increase in tax revenue. The effect of the new taxpayers is  $\alpha^* - \alpha$ .

4. The characteristics of the index

The index  $D(t)$  can be written as

$$(8) \quad D(t) = \frac{\Delta \log T}{\Delta \log T - \alpha} = \frac{\bar{e} \Delta \log x + \alpha}{\bar{e} \Delta \log x} = 1 + \frac{\alpha}{\bar{e} \Delta \log x} .$$

The index is a relative unit-free measure, the tax-policy effect being related to the change in the tax base. The change of  $D(t)$  depends on changes in  $\alpha$ ,  $\bar{e}$  and  $\Delta \log x$ . The difference  $\Delta D$  is

$$(9) \quad \Delta D = D(1) - D(0) = \frac{\alpha_1}{\bar{e}_1 \Delta \log x_1} - \frac{\alpha_0}{\bar{e}_0 \Delta \log x_0} .$$

The index  $D^*(t)$  and its difference  $\Delta D^*$  are transformations of  $D(t)$  and  $\Delta D$ . We have

$$(10) \quad D^*(t) = kD(t) = k\left(1 + \frac{\alpha}{\bar{e} \Delta \log x}\right) = 1 + \frac{\alpha^*}{\bar{e}^* \Delta \log x}$$

and

$$(11) \quad D^*(1) - D^*(0) = \frac{\alpha_1^*}{\bar{e}_1^* \Delta \log x_1} - \frac{\alpha_0^*}{\bar{e}_0^* \Delta \log x_0} .$$

The effect  $\alpha^*$  is influenced by policy makers in their intention to correct the tax rules for inflation, and  $\Delta \log x$  is the growth in the nominal incomes or the tax base. The income growth is determined in the markets; the tax-base growth, however, depends also on the deduction system if  $x$  is post-deduction incomes. The change in  $\bar{e}$  is "structural" depending on the formation of  $\alpha^*$  and  $\Delta \log x$ . The indexation factor  $p$ , which is the measure of the degree of neutral or neutral equivalent indexation of tax rules, is defined for  $\alpha^*$  in the equation

$$(12) \quad \alpha^* = \log(1 - \log p(\bar{e}-1)).$$

For the original policy effect  $\alpha^*$  we use the elasticity  $\bar{e}$  calculated for the old taxpayers. For a given equiproportionate income growth and a positive  $p$  factor we have a solution giving a constant elasticity  $\bar{e}$ .

If  $D^*(t) > 1$ , the tax policy is discretionary upwards; if  $D^*(t) < 1$ , it is discretionary downwards. If  $D^*(t)$  increases (decreases), then the tax policy is tight (slack) in relation to the period before, and the judgement can be said to be invariant to the level of  $D^*(t)$ . The relative increase or decrease of  $D^*(t)$  indicates the degree of tightening or slacking in taxation.  $D^*(t)$  measures the tax policy against the tax rules specified in nominal terms.

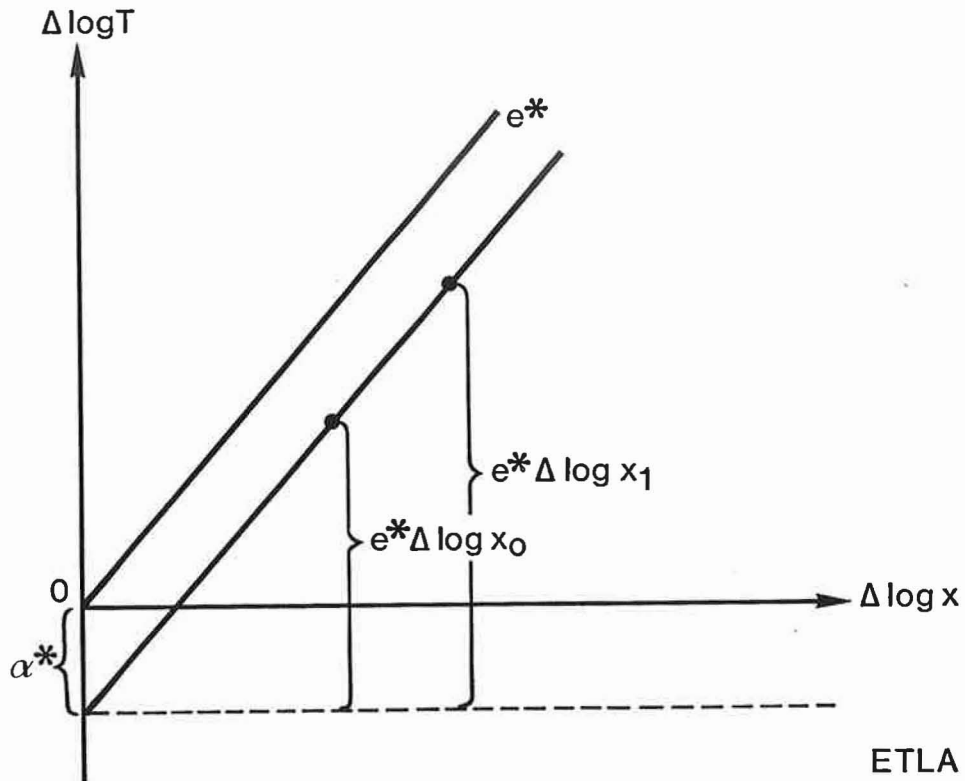
1. For fixed elasticity ( $e_0^* = e_1^* = e^*$ ) and the level of the tax-policy effect ( $\alpha_0^* = \alpha_1^* = \alpha^*$ ) we have for equation (11) the difference

$$(12) \quad \Delta D^* = \frac{\alpha^*}{e^*} \left( \frac{1}{\Delta \log x_1} - \frac{1}{\Delta \log x_0} \right).$$

When income or the tax base accelerates,  $\left( \frac{1}{\Delta \log x_1} - \frac{1}{\Delta \log x_0} \right) < 0$ . If  $\alpha^* < 0$ , the difference  $\Delta D^*$  is positive. Also the partial derivative  $\partial D^* / \partial \Delta \log x$  depends on the sign of  $\alpha^*$ . This means that if, for example, the policy makers fixed  $\alpha^*$  through the indexation factor  $p$  for a forecast income growth or inflation and the income growth happens to be somewhat bigger than forecast, the downward discretionarity is smaller than assumed. The index  $D^*(t)$  therefore accelerates. In chart 2 we see that the relative tax policy is tightening because the discretionary policy effect  $\alpha^*$  is of lesser significance when the tax base accelerates. We however know that when the tax base accelerates,

the elasticity  $e^* = \frac{1}{k} \bar{e}$  is marginally dropping. To have  $\alpha^*$  constant, when  $\bar{e}$  changes, we have to marginally change the indexation factor  $p$ .

Chart 2. The change in the tax-policy index due to accelerated income growth



2. For fixed elasticity ( $e_0^* = e_1^* = e^*$ ) and tax-base growth ( $\Delta \log x_0 = \Delta \log x_1 = \Delta \log x$ ) the equation (11) is

$$(13) \quad \Delta D^* = \frac{\alpha_1^* - \alpha_0^*}{e^* \Delta \log x} = \frac{1}{e^* \Delta \log x} \Delta \alpha^* .$$

By increasing the indexation factor  $p$ , we receive  $\Delta \alpha^* < 0$  and therefore  $\Delta D^*$  decreases. The partial derivative  $\partial D^* / \partial \alpha^* > 0$  and  $\partial D^* / \partial p < 0$ . The effect  $\alpha^*$ , determined by  $p$ , is independent in the short run of the income growth. In the long run  $\alpha^*$  of course depends on the income growth through the factor  $p$ .

3. If  $\alpha_0^* = \alpha_1^* = \alpha^*$  and  $\Delta \log x_0 = \Delta \log x_1 = \Delta \log x$  and  $e_1^* > e_0^*$ , we have that

$$(14) \quad \Delta D^* = \frac{\alpha^*}{\Delta \log x} \left( \frac{1}{e_1^*} - \frac{1}{e_0^*} \right)$$

is positive if  $\alpha^* < 0$  and negative if  $\alpha^* > 0$ . The partial derivative  $\partial D^* / \partial e^*$  is positive if  $\alpha^*$  is negative and negative if  $\alpha^*$  is positive.

The change in the structure, which can be observed in the change of the elasticity  $\bar{e}$ , means that a specific tax policy effect  $\alpha^*$  requires a bigger indexation factor  $p$  the smaller the elasticity is. Also we have that a specific  $p$  gives a smaller  $\alpha^*$  the bigger the elasticity is.

#### 5. The tax-policy index and the cost-of-living index

As well as having the "doing nothing" as a discretionary policy norm we can choose the official index of cost of living as norm to construct the tax-policy index.<sup>1)</sup> Using a five-year moving cost-of-living index  $P_n$  and putting the price changing factor  $p$  corresponding to  $P_n$  into equation (4), we receive the normative policy effect  $\alpha_n$  and also the normative tax-policy index  $D_n$ , which depends on the change in the cost of living  $p_n$ , the elasticity  $\bar{e}$  and the increase in the tax base  $\Delta \log x$ . The tax-policy index, based on the norm of the cost of living can be calculated as

---

1) It is, however, with reservation we use the average cost-of-living index (see, however, [19]) because, for example, in the beginning of 1960' about 40 % of the income receivers - the richer part - were payers of state income tax and their respective income share was 70 %. The average commodity basket is perhaps not representative for the taxpayers so we have not a "fair-norm" in mind when we make our measures. In the year 1981 the taxpayer share was 60 % and the income share 90 %.

$$(15) \quad \Delta \hat{D}(t) = D^*(t)/D_n(t).$$

If  $\hat{D} < 1$ , the tax policy is discretionary downwards, i.e. the tax rules (e.g. the tax schedule) has been adjusted more than the inflation requires, because  $p > p_n$  and  $p/p_n > 1$ . If  $D > 1$ , the schedule is indexed less than the inflation and we have  $p < p_n$  and  $p/p_n < 1$ .

### III EMPIRICAL RESULTS

The foregoing method for calculation of a tax-policy index can be used both for the tax-schedule policy and the combined schedule-deduction policy.

#### 1. The schedule policy

We have the unscaled tax-policy index  $\dot{D}_1$ , and its scaled version  $D_1^*$ . Both measure the degree of tax policy discretionarity for the schedule parameter. They relate the observed change in tax revenue to the hypothetical change in taxes, which does not include the tax policy. They are unit-free from the level of change in tax revenue. If  $D_1$  or  $D_1^*$  are one, the discretionarity is zero. The scaling  $D_1 \rightarrow D_1^*$  is made to exclude from the policy measure the effect of new taxpayers - the non-equiproportionate growth effect. This is an indirect effect of the schedule policy itself.<sup>1)</sup> Corresponding to  $D_1^*$  we have the schedule-policy effect  $\alpha_1^*$  - given in log-change points - which does not include the effect of the new taxpayers.

---

1) The deduction policy is only indirect because it affects the growth of post-deduction incomes.

$\alpha_1^*$  is zero if  $D_1^* = 1$ , negative if  $D_1^* < 1$  and positive if  $D_1^* > 1$ . The policy effect  $\alpha_1^*$ , which is not to be considered as a residual, is independent of the change in the tax base. It is in fact the weighted sum of the individual changes in taxes due to changes in the schedule, the individual tax bases being kept constant. The effect  $\alpha_1^*$  has an operational counterpart in the schedule indexation factor  $p_1$ . If  $p_1 = 1$ , the schedule is nominally kept constant and gives  $\alpha_1^* = 0$ . If  $p_1 > 1$  the schedule is nominally indexed forwards - neutral or neutral-equivalent - giving a negative policy effect ( $\alpha_1^* < 0$ ), if  $p_1 < 1$ , it is indexed backwards giving  $\alpha_1^* > 0$ . The  $\alpha_1^*$  is, however, dependent on the average schedule progressivity  $\bar{\pi}_1 = \bar{e}_1 - 1$ . Given the forward shift in the nominal schedule ( $p_1 > 1$ ), the downward discretionary effect is bigger the steeper the progressivity is.

Using the cost of living as an indexation factor for a normative shift in the schedule, we can calculate the normative effect  $\alpha_1^*(n)$  and the index  $D_1^*(n)$ . Thus  $\hat{D}_1 = D_1^*/D_1^*(n)$  is a new index for the discretionary tax policy. If  $\hat{D}_1 = 1$  the schedule is on average shifted according to inflation; if  $\hat{D}_1 > 1$  or  $\hat{D}_1 < 1$  the schedule has been shifted respectively more or less than the inflation.  $\hat{D}_1$  has a counterpart in the indexation factor  $\hat{p}_1 = p_1/p(n)$ .

We present in the following a qualitative-numerical comparison for the schedule tax policy in table 1.



Table 1. The index points for  $D_1^*$  for five-year periods and a qualitative indication of changes in the nominal schedule within the five-year periods.

	Qualitative indication <sup>1)</sup> years in the period					$D_1^* \cdot 100$
	1	2	3	4	5	
1961-65	-	0	-	+	-	74
62-66	0	-	+	-	0	77
63-67	-	+	-	0	+	93
64-68	+	-	0	+	0	94
65-69	-	0	+	0	0	94
66-70	0	+	0	0	0	100
67-71	+	0	0	0	0	100
68-72	0	0	0	0	0	95
69-73	0	0	0	0	0	100
70-74	0	0	0	0	-	90
71-75	0	0	0	-	+	97
72-76	0	0	-	+	-	76
73-77	0	-	+	-	-	66
74-78	-	+	-	-	-	53
75-79	+	-	-	-	-	54
76-80	-	-	-	-	-	49

1) + schedule shifted nominally backwards  
 - schedule shifted nominally forwards  
 0 schedule not changed at all

It should be pointed out that the index based on the written tax law has for the periods the reference  $D_1^* = 1$ ,  $\alpha_1^* = 0$  and  $p_1 = 1$ , while the index  $\hat{D}_1$  based on the inflation  $p(n)$  gives a reference  $D_1^*(n)$ , which changes over the period. The comparisons to the two references are done in comparative static manner, and they are not based on any chained hypothetical references. We cannot give answers, for example, to the question: how would the actual tax policy work today if the schedule had previously been indexed yearly according to inflation? These types of questions can not be answered because i.a. the inflation of the past has been caused by a tax policy of no change of the schedules.

From the index  $D_1^*$  we can see that with the nominally specified tax law as a norm the tax policy was discretionary downwards, especially in the beginning of 1960's and after the period ending with 1975, when the schedule was indexed according to inflation. Looking at the policy in relation to the inflation, i.e.  $\hat{D}_1$ , we see that the policy was mostly discretionary upwards except in the two first periods and the last period when the inflation indexation of the schedule even exceeded the inflation (see chart 3). In 1976-80 the schedule indexation was made with about 1.7 log percent higher rate than the inflation.

The tightening and slacking of the tax policy between the periods can be calculated as log changes in the indices  $D_1^*$  and  $\hat{D}_1$  marked  $\dot{D}_1^*$  resp.  $\dot{\hat{D}}_1$  (see appendix 3).

## 2. The combined schedule-deduction policy

The combined schedule-deduction policy can formally be analyzed in the same way as the schedule policy. The deduction system works slightly progressively so the combined tax-revenue elasticity  $\bar{e}_2$ , with respect to the taxpayers' pre-deduction incomes is somewhat bigger than that for the schedule alone which was defined according to the taxpayers' post-deduction incomes. We have the index  $D_2$ , the scaled  $D_2^*$ ,  $\alpha_2^*$  and  $p_2$ . The scaling factor  $k_2$  is a little bit higher than that in the schedule case ( $k_1$ ). The index means  $\bar{D}_1^*$  and  $\bar{D}_2^*$  are very much the same. The combined tax policy has been slightly discretionary upwards for the periods 1964/68-1967/71 ( $D_2^* > 1$ ). From the period 1971/75 forward the combined policy has been more downwards discretionary than the schedule alone, i.e.  $D_2^* < D_1^*$ .

Chart 3. The schedule-policy and schedule-deduction-policy indices  $D_1^*$ ,  $D_2^*$  specified against nominal rules and  $\hat{D}_1$ ,  $\hat{D}_2$  specified against inflated rules, five-year moving periods.

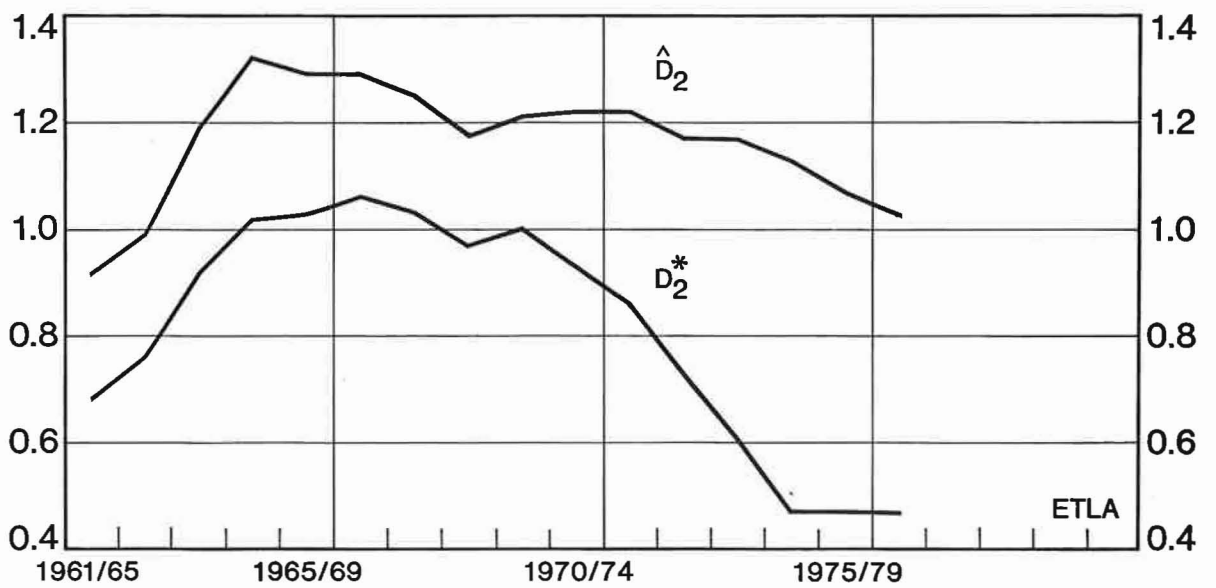
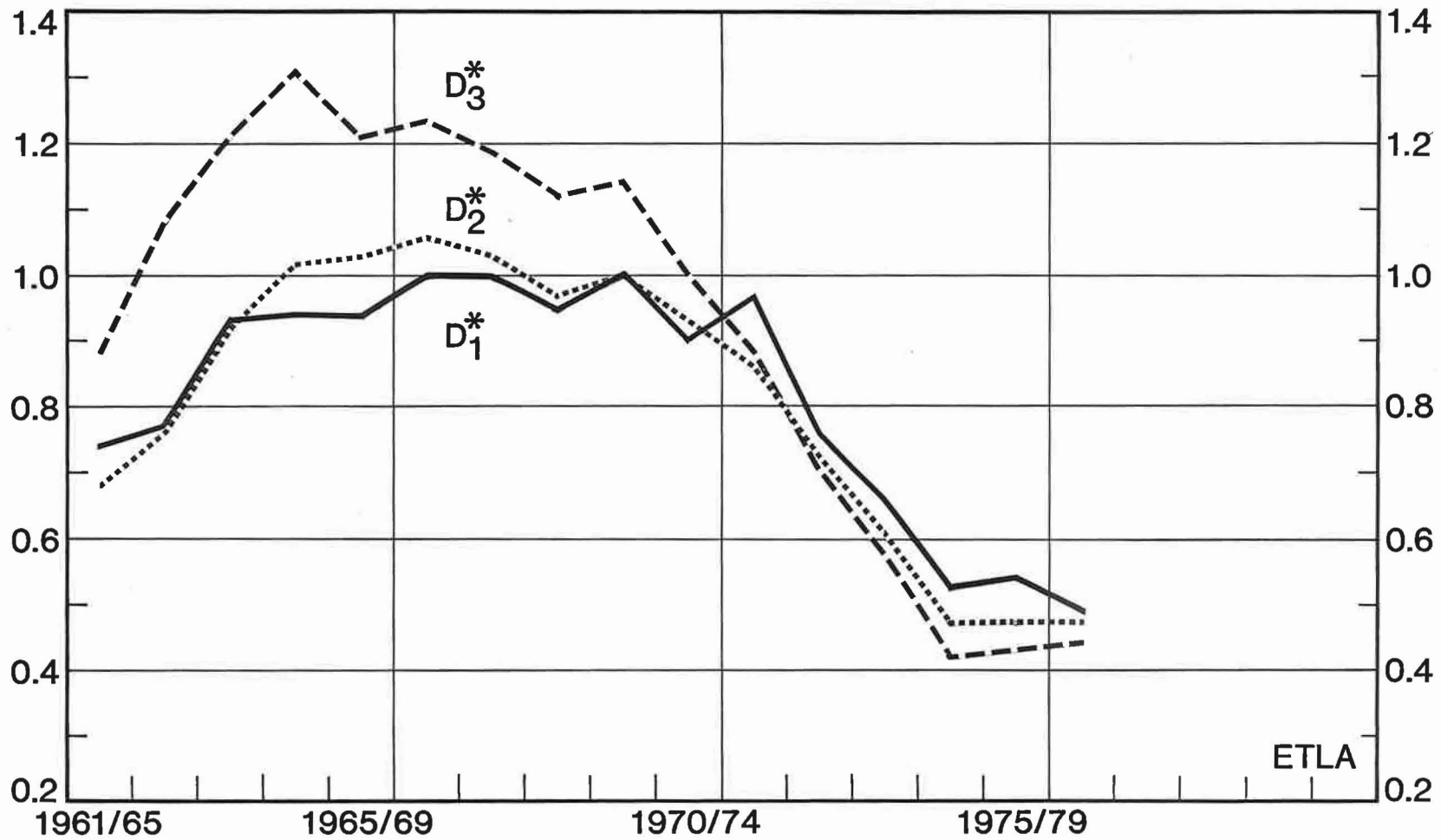


Chart 4. The schedule-policy index  $D_1^*$ , the combined schedule-deduction-policy index  $D_2^*$  and the policy index including the tax-revenue effect from the non-equiproportionate income growth  $D_3^*$ , five-year moving periods.



Regarding the schedule indexation factor  $p_1$  and the factor for the combined schedule-deduction indexation  $p_2$ , we can observe in chart 5 that mainly they do not differ from each other except in the five last periods. The combined indexation for these periods was somewhat smaller, i.e.  $p_2 < p_1$ . However, the effect on the tax revenue was bigger, i.e.  $|\alpha_2^*| > |\alpha_1^*|$ , because the average elasticity for the deduction system increased for that time. The marginal policy effect from the deduction system is counted as  $\Delta\alpha_2^* = \alpha_2^* - \alpha_1^*$ . It was negative for the periods 1961/65-1963/67, positive for 1964/68-1970/74 and again negative for 1971/75-1976/80.

By comparing  $\hat{D}_1$  with  $\hat{D}_2$  and  $\hat{p}_1$ , with  $\hat{p}_2$ , we can observe that the deduction policy marginally increased the schedule's upwards discretionarity and decreased the schedule's downwards discretionarity.

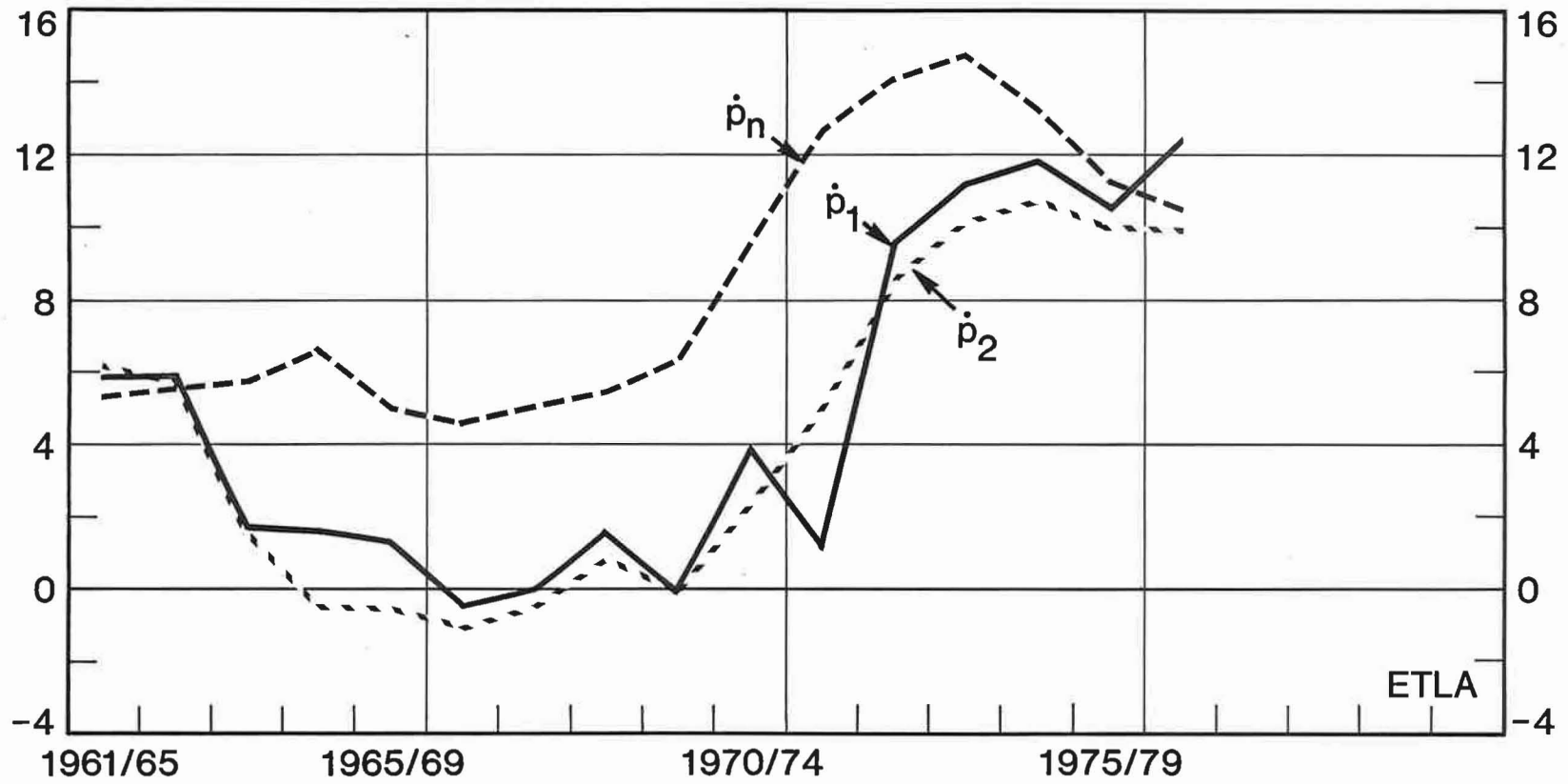
### 3. Integration of all income receivers into the tax-policy analysis

We have above defined the tax policy and the effect of the new taxpayers - the non-equiproportionate income growth effect - with regard to the tax bases' post- and pre-deduction incomes. In the following we further assume the equiproportionate growth for all income receivers, an assumption which is not very restrictive.<sup>1)</sup>

---

1) To have equiproportionate growth we can even loosen the individual identity between periods 0 and 1. Given an income vector  $y_0$  with  $n$  income elements in ascending order, we have after, say, a 10 % increase of every element a new vector  $y_1$  with  $n!$  permutations giving the same income distribution as the original  $y_0$ . All permutations of  $y_1$  gives the same total tax revenue.

Chart 5. Inflation rate  $\dot{p}_n$ , the indexation rates of the schedule  $\dot{p}_1$  and the schedule-deduction rules  $\dot{p}_2$ , five-year moving periods, %



For the changing taxpayer population we do not have equiproportionate growth even if for the original taxpayers it is equiproportionate, because for new taxpayers the income growth is infinity.<sup>1)</sup> The non-equiproportionate growth which indirectly depends on the tax policy can be measured by the income growth difference between taxpayers and all income receivers. We calculate the contribution to the growth of tax revenue corresponding to the income growth difference.

From the empirical-data series we have the growth relations

$$(16) \quad \beta_3 = \frac{\Delta \log T}{\Delta \log x_3},$$

where  $T$  is the tax revenue and  $x_3$  is the income variable for all income receivers. If we calculate the index

$$(14) \quad D_3 = \beta_3 / \bar{e}_2$$

and scale it by  $k_2$ , we have

$$(18) \quad D_3^* = k_2 D_3 = \beta_3 / e_2^* = \frac{\Delta \log T}{\Delta \log T - \alpha_3^*} = 1 + \frac{\alpha_3^*}{e_2^* \Delta \log x_3},$$

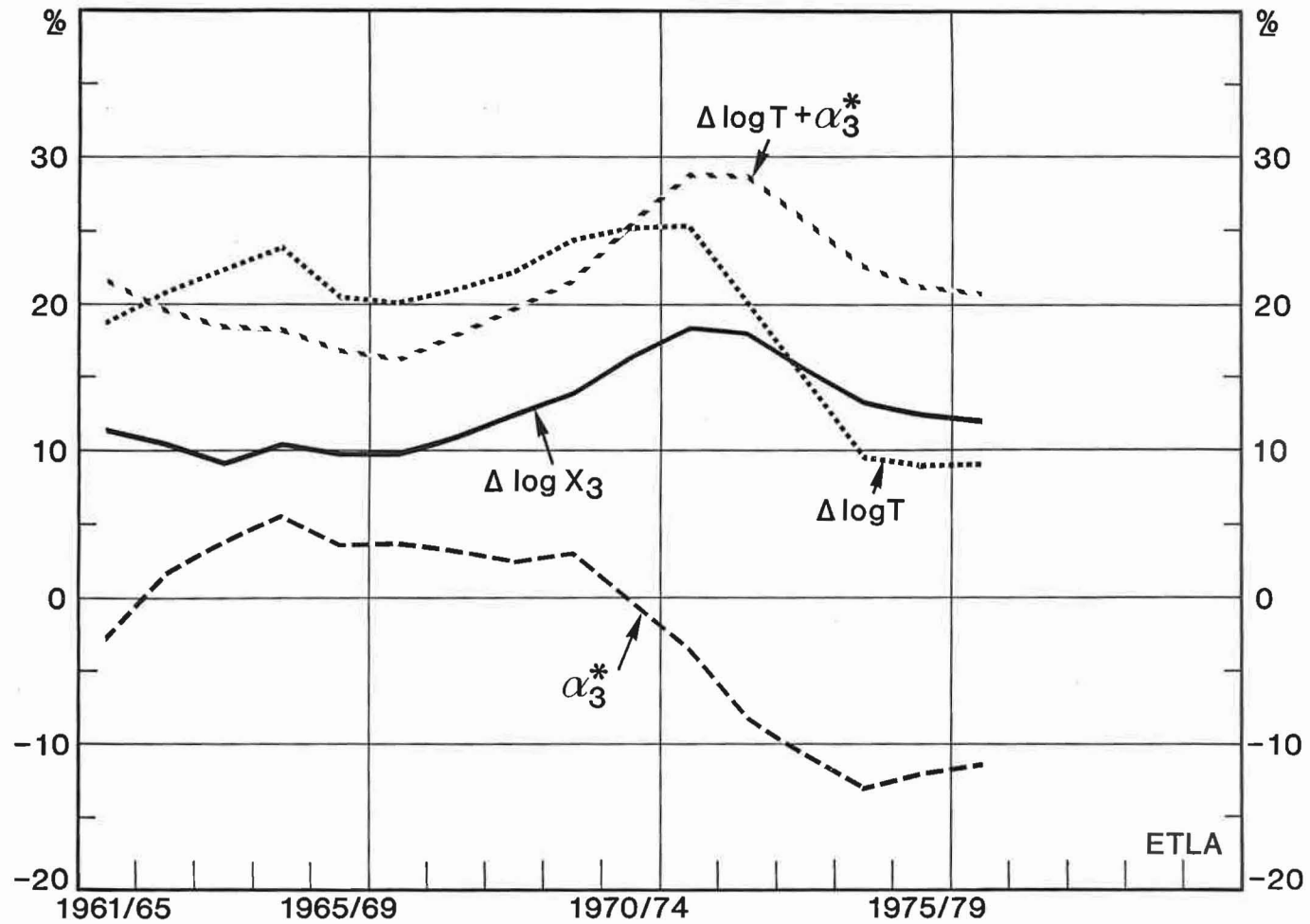
where  $\alpha_3^*$  is the total tax-policy effect defined on the income growth of all income receivers. The difference

$$(19) \quad \Delta \alpha_3^* = \alpha_3^* - \alpha_2^*$$

---

1) The income of new taxpayers in period 0 is zero in terms of the taxpayer population.

Chart 6. The income growth (all income receivers)  $\Delta \log X_3$ , the growth of tax revenue without tax policy  $\Delta \log T + \alpha_3^*$  and including tax policy  $\Delta \log T$ . The total tax-policy effect is  $\alpha_3^*$ , i.e. the sum  $\alpha_1^* + \Delta \alpha_2^* + \Delta \alpha_3^*$





gives us the contribution of new taxpayers to the growth of tax revenue. This effect is caused by the non-equiproportionate income growth of the changing taxpayer population. As we can see from the data,  $\Delta\alpha_3^*$  disappears or is small when the tax rules are indexed according to inflation or the growth of incomes. When the tax rules are indexed according to inflation i.e.  $\hat{p}_2 = 1$ ,  $\Delta\alpha_3^*$  is relative small compared with  $\alpha_2^*$ ; when  $p_2$  is one, the effect  $\Delta\alpha_3^*$  is big in relation to the policy effect  $\alpha_2^*$ .

The tax-revenue effect  $\Delta\alpha_3^*$  relative to the non-equiproportionate income growth depends on the history of the lowest threshold for the tax function relative to the mean or median income and the shape of the money income density function. The residuals for the estimated equation

$$(20) \quad \Delta\alpha_3^* = -0.0004 + 1.7848 (\Delta \log x_2 - \Delta \log x_3),$$

(0.000) (0.034)

$R^2 = 0.995$   
D-W = 0.264

where  $\Delta \log x_2 - \Delta \log x_3$  is a proxy for the non-equiproportionate growth of taxpayers income, behave therefore in a systematically way.

#### 4. The role of the deduction system

The deduction system works slightly progressively, strenghtening the growth of post-deduction incomes relative to the growth of pre-deduction incomes. Also the deduction policy works in the same way so the effects from the deduction policy are transmitted to the tax revenue through the change in post-deduction incomes. The deductions tax-policy effect  $\Delta\alpha_2^* = \alpha_2^* - \alpha_1^*$  is therefore to be considered as marginal.

The schedule's lowest threshold and its "projection" through deduction rules on the pre-deduction income variable - giving the pre-deduction threshold  $y^*$  - is the policy instrument determining the non-equiproportionate growth of taxpayer income.<sup>1)</sup>

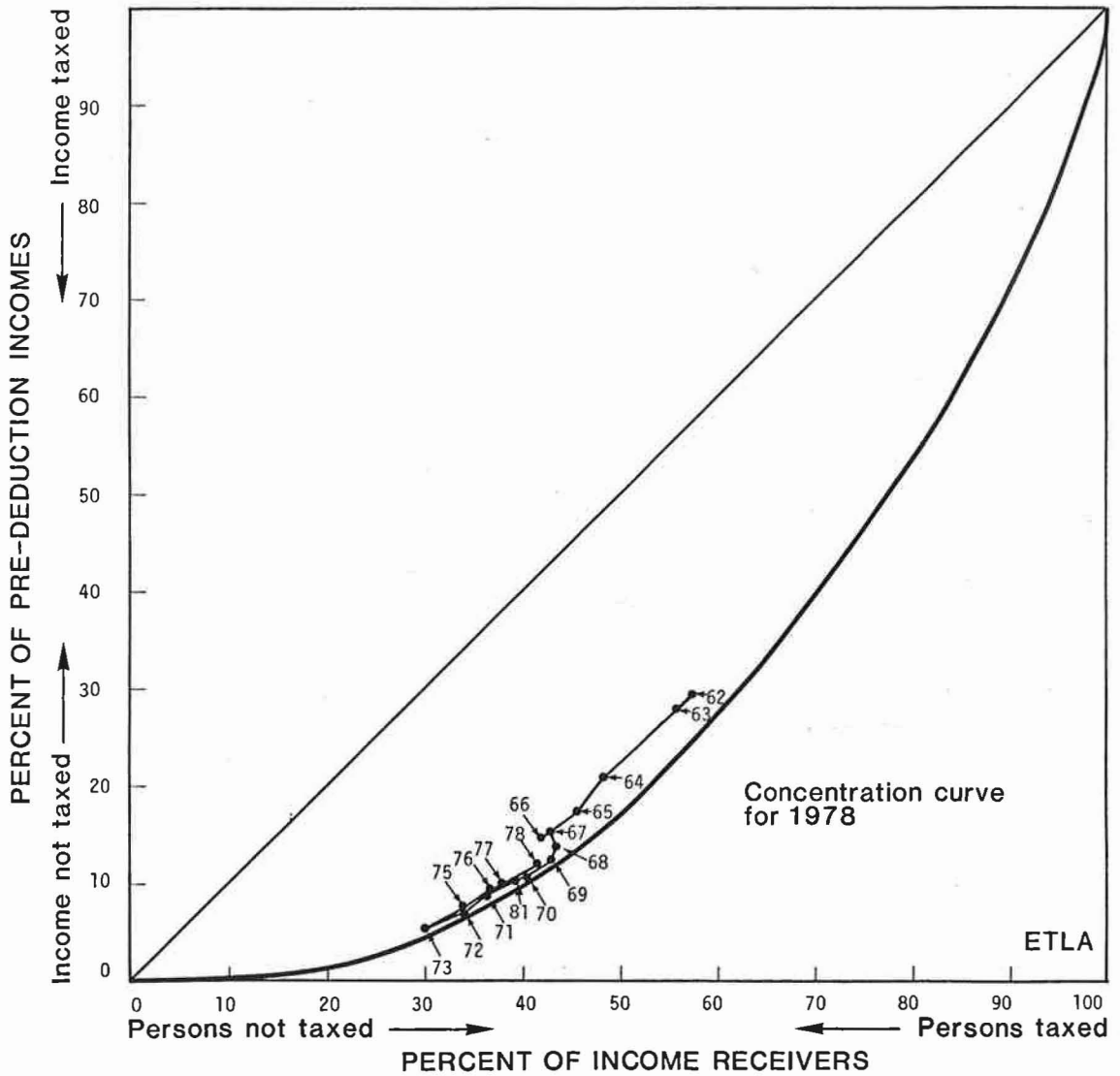
The effect of this combined threshold instrument, working in a bounded income range, can be quite different from the schedule-deduction-policy effect on bigger income levels. It is therefore not surprising that the effect of the scaling (non-equiproportionate effect) on the tax revenue  $\alpha_1^* - \alpha_1$  and  $\alpha_2^* - \alpha_2$  is positive and quite big because the threshold  $y^*$  was not changed at all in years 1964-66 and 1969-73. In some years the schedules have been indexed forwards, but in a non-neutral fashion leaving the nominally threshold  $y^*$  unchanged.

The indirect tax-policy effect on the tax revenue  $\Delta\alpha_3^*$ , i.e. the non-equiproportionate income growth effect, where we use the income growth rate of all the income receivers as a "norm" for the taxpayer population, is positive and diminishing from the period 1961/65 to, the period 1971/75 and after that negative. The taxpayers' share of all pre-deduction incomes rose from about 70 % in the beginning of 1960's to 95 % in the year 1973 and fell to 88 % in 1978. The taxpayers share of the number of all income receivers respectively rose from about 40 % in 1960 to 70 % in 1973 and fall to 60 % in 1978 (see chart 7).

---

1) This transmission problem and role of the schedule's lowest threshold has been analyzed in a forthcoming paper, Edgren: The personal state income taxation and its dependence on deductions and the lower limit of the schedule income in Finland.

Chart 7. The concentration curve for pre-deduction incomes in the year 1978 and shares of persons taxed and not taxed in relation to pre-deduction incomes taxed and not taxed in the years 1962-81, percent.



## IV CONCLUDING REMARKS

Tax policy measurement is first of all a references problem. The momentary one period or one year tax-revenue elasticities aggregated from micro data have to be considered as ceteris paribus elasticities, i.e. as such not suitable in a dynamic context. The elasticity has to be scaled to suit an unknown income growth. The tax-policy effect can be separated into a direct effect with an operational tax-rules indexation factor and an indirect effect (the effect of new taxpayers) - a form of non-equiproportionate income growth effect. This income growth pattern is generated from the tax system's discontinuity properties in the lower income brackets and is not to be considered as growth differences, for example, in the labour market. Because the pre-tax income distribution in Finland for a long time has been rather stable while the taxpayer population has increased relative to income receivers, we think that the discontinuity of the tax system is a factor which dominates tax revenue more than, for example, than the effect of feedback from non-equiproportionate growth within the tax system itself.

The threshold variable  $y^*$ , which is an integrated part of the tax rules, can be used for the distributional analysis. If all income receivers incomes grow equiproportionately and the threshold grows at the same rate - i.e. tax rules are shifted in a neutral fashion - leaving the taxpayers' share of all incomes constant, the distribution of taxes and after-tax incomes is unchanged both regarding all income receivers and taxpayers.

If the threshold is moved faster than the average income growth the tax burden is left on a smaller income share so the after-tax distribution

changes, thus diminishing income differences. If threshold and tax rules are nominally fixed or are changed more slowly than the incomes grow, the distribution effect works in the opposite direction.

## References:

- [1] Alchin, Teray M.: A New Measure of Tax Progressivity. Public Finance, Vol. 39, No. 1, 1984.
- [2] Barro, Robert J. and Chaipat Sahasakul: Measuring the Average Marginal Tax Rate from the Individual Income Tax. Journal of Business, Vol. 56, No. 4, 1983.
- [3] Choudhry, Nurun N.: Measuring the Elasticity of Tax Revenue, A Divisia Index Approach. International Monetary Fund Staff Papers, Vol. 26, No. 1, 1979.
- [4] Dorrington, John C.: A Structural Approach to Estimating Built in Flexibility of United Kingdom Taxes on Personal Income. Economic Journal, Vol. 84, No. 335, 1974.
- [5] Edgren, Christian: En analys av sambandet mellan förändring i statlig inkomstskatt och förändring i beskattad inkomst. Discussion Paper No. 31. The Research Institute of the Finnish Economy, 1979.
- [6] Edgren, Christian: En analys av den undre gränsen för beskattad inkomst som skattepolitiskt instrument. Discussion Paper No. 41. The Research Institute of the Finnish Economy, 1979.
- [7] Edgren, Christian: Vähennysjärjestelmän merkitys verotuksessa. Discussion Paper No. 42. The Research Institute of the Finnish Economy, 1979.
- [8] Edgren, Christian: Tuloverotuksen automatiikan kvantifiointi. Discussion Paper No. 74. The Research Institute of the Finnish Economy, 1980.
- [9] Edgren, Christian: En systematisk granskning av skattedata. Discussion Paper No. 78. The Research Institute of the Finnish Economy, 1981.
- [10] Edgren, Christian: The Tax Elasticity - An Empirical Application. Discussion Paper No. 166. The Research Institute of the Finnish Economy, 1984.
- [11] Edgren-Turkkila-Vartia Y.: Tuloverotuksen analysoinnin matemaattisista ongelmista. Discussion Paper No. 17. The Research Institute of the Finnish Economy, 1978.
- [12] Hutton, John P. and Peter S. Lambert: Evaluating Income Tax Revenue Elasticities. Economic Journal, Vol. 90, No. 360, 1980.
- [13] Hutton, John P. and Peter S. Lambert: Modelling the Effects of Income Growth and Discretionary Change on the Sensivity of U.K. Income Tax Revenues. Economic Journal, Vol. 92, No 365, 1982.
- [14] Jakobsson, Ulf: On the Measurement of the Degree of Progression. Journal of Public Economics, Vol. 5, No. 1,2, 1976.

- [15] Kakwani, Nanak C.: *Income Inequality and Poverty, Methods of Estimation and Policy Applications*, Washington D.C. 1980.
- [16] Kay, John A. and Christopher N. Morris: *The Gross Earning Deflator*. *Economic Journal*, Vol. 94, No. 374, 1984.
- [17] Lambert, Peter. J.: *Non-equiproportionate Income Growth, Inequality, and the Income Tax*. *Public Finance*, Vol. 39. No. 1, 1984.
- [18] Suits, D.: *Measurement of Tax Progressivity*. *American Economic Review*, Vol. 70, No. 4, 1977.
- [19] Turkkila, Juhani: *Valtion tuloveroasteikkojen reaalin kirs-tyminen Suomessa vuosina 1949-1979*. ETLA B:27, Helsinki 1980.
- [20] Wales, Terence J.: *Analysis of the Constancy of the Effective Tax Rate*. *Review of Economics and Statistics*, Vol. 50, No. 1, 1968.

Appendix 1.

The scaling of the tax-policy index

The formal equation for the tax change is

$$(A) \quad \Delta \log T = \bar{e} \Delta \log x + \alpha,$$

where  $x$  is the tax base,  $\bar{e}$  the tax-revenue elasticity and  $\alpha$  an intercept measured in log terms. The "overestimating" of the elasticity with respect to the data - i.e. in the case of an increasing number of arguments in the tax function  $T = f(x_1, \dots, x_n, t)$  where  $t$  is a tax-policy parameter - or the "underestimating" - in the case of a decreasing number of arguments - gives us an other decomposition for the tax change

$$(B) \quad \Delta \log T - \alpha^* = \hat{w} \Delta \log T + (1 - \hat{w}) \Delta \log T.$$

In the above identity  $\hat{w}$  indicates the fraction of the tax-revenue change for which the elasticity is calculated and  $\alpha^*$  the tax-policy effect we are searching for.

Further, we have the decomposition

$$(C) \quad \Delta \log T - \alpha^* = e^* \Delta \log x,$$

where  $e^* = \frac{1}{k} \bar{e}$  is the scaled elasticity. The scaled term  $e^* \Delta \log x$  in equation (C) can be written as



$$(D) \quad e^* \Delta \log x = [\bar{e}w^X + \hat{e}(1-w^X)] \Delta \log x,$$

where  $w^X + (1-w^X) = 1$ .

$w^X$  is the fraction of the tax base increase which suits the calculated elasticity  $\bar{e}$ . In the case of  $\alpha^* = 0$  in equation (C) we have using equation (A),  $e^* \Delta \log x = \bar{e} \Delta \log x + \alpha$  for the scaling. By making  $\alpha = 0$ , we have to transform  $e$  to  $e^*$ . For the "contribution" elasticity  $\hat{e}$  we have

$$(E) \quad \hat{e} = \frac{1 - w^X}{1 - kw^X} \bar{e}.$$

In the case of no scaling ( $k=1$ ),  $e^* = \frac{1}{k} \bar{e} = \bar{e}$ . In equation (E)  $\hat{e} = \bar{e}$  and in equation (D) we have

$$\begin{aligned} (F) \quad e^* \Delta \log x &= [\bar{e}w^X + \bar{e}(1-w^X)] \Delta \log x \\ &= \bar{e}[w^X + (1-w^X)] \Delta \log x \\ &= \bar{e} \Delta \log x. \end{aligned}$$

If the scaling factor  $k = 1.1520$  as in the schedule case, we receive, for example, for a fraction  $w^X = 0.8$  and a calculated elasticity  $\bar{e} = 2.1393$  a value of  $\hat{e} = 0.728$ . The contribution  $\hat{e}(1-w^X) = 0.146$  and for a growth in the tax base of 0.10 the effect  $\hat{e}(1-w^X) \Delta \log x = 0.015$ . When the elasticity drops to 1.5,  $\hat{e}$  falls to 0.511 and the effect to 0.010. We do not know the fraction  $w^X$ . However, in case of increasing arguments in the tax function - i.e. when the schedule specified in nominal terms is not corrected at all or less than the increase in nominal incomes - we have  $w^X < \frac{1}{k}$  and the new arguments giving a positive contribution to the automatic tax-revenue increase. This happens ordinarily when  $e$  is decreasing.

In the case of a decreasing number of arguments, the elasticity is underestimated in relation to the data and  $w^X > \frac{1}{k}$ . This gives a negative contribution because  $\hat{e}$  is then negative according to equation (E) and we have  $\hat{e}(1-w^X) \Delta \log x < 0$ . This happens when elasticity  $\bar{e}$  is increasing due to the fact that the schedule specified in nominal terms is corrected faster than the increase of nominal incomes.

We have on one hand the index

$$(G) \quad D^* = kD(t) = k\left(1 + \frac{\alpha}{e \Delta \log x}\right) = k + \frac{\alpha}{e^* \Delta \log x} .$$

On the other hand is

$$(H) \quad D^* = 1 + \frac{\alpha^*}{e^* \Delta \log x} .$$

Therefore,  $\alpha = (1-k)e^* \Delta \log x + \alpha^*$  and  $\alpha^* = \alpha - \left(\frac{1-k}{k}\right)\bar{e} \Delta \log x$ . If there is no scaling,  $k=1$  and  $\alpha^*=\alpha$ . The scaling trick and its formal meaning in the form of new arguments in the tax function use the assumption of equiproportional change for the individual tax bases, which means that the cov( ) is zero in equation (1). The effect on the elasticity of an increase (or decrease) of arguments have in a way been taken into account for the calculatedde by taking the mean elasticity for the period 0 and 1, because in period 1 we have for the calculation included the new taxpayers which affect the tax weight  $w_1^T$ . The new taxpayers however disturb the equiproportional assumption, so we can think new taxpayers affect the cov-term by causing a non-equiproportional change in the tax base. In this respect the cov depends on the tax policy. We can intuitively imagine that the faster the tax base changes, the bigger the distance between the mean

income and the lowest nominally specified limit for the taxable income (the income level which gives a positive tax) is going to be and the more new taxpayers are being integrated into the taxpayer population. The effect depends, however, on the history, because a yearly indexed nominal tax scale, for example, including its lowest threshold maintains its relative position over time to the income distribution, disturbing not at all or very little the equiproportional assumption, while the scale nominally fixed for a long time has emptied its stock of new potential new taxpayers in the end, so the problem of non-equiproportionate tax base growth disappears.

In the proportional taxation the new taxpayers contribution to the increase in tax revenue is equivalent to the contribution of the increase in the tax base. The more progressive taxation, is the smaller the contribution to the tax increase is in relation to the tax base increase for new taxpayers in the lower tail of the income distribution. For a new taxpayer jumping to an income level corresponding to the average tax rate, the relation between the contributions is one.

Appendix 2.

The average or macro parameters in income taxation, instantaneous yearly values

The meaning of the parameters:

1. The average tax rate indicates the tax share of total incomes.
2. The marginal tax rate gives on average the increase in tax revenue in monetary terms for an increase of incomes of one monetary unit.
3. The average progression is the difference between the average marginal tax rate and the average tax rate.
4. The average tax elasticity gives the relative increase of the tax revenue on average due to the relative increase of incomes.
5. The average post-tax elasticity gives the relative increase of post-tax incomes due to the relative increase of incomes.
6. The "gross earnings deflator" indicates how much the gross income or tax base has to increase on average to have a one-unit increase of post-tax income or base (see [16]).

Values of macro parameters; income concept: taxpayers' pre-deduction incomes

	1977	1978	1979	1980
<b>State income taxation</b>				
- marginal tax rate	0.252	0.256	0.252	0.256
- average tax rate	0.122	0.118	0.119	0.121
- progression	0.130	0.138	0.133	0.135
- tax elasticity	2.066	2.169	2.118	2.116
- post-tax elasticity	0.852	0.844	0.849	0.846
- gross-earnings deflator	1.174	1.185	1.178	1.181
<b>State income taxation, municipal taxation and secured persons social security fees</b>				
- marginal tax rate	0.4064	0.3989	0.3978	0.4032
- average tax rate	0.2864	0.2717	0.2764	0.2794
- progression	0.1200	0.1272	0.1214	0.1238
- tax elasticity	1.4190	1.4682	1.4392	1.4431
- post-tax elasticity	0.8318	0.8253	0.8322	0.8282
- gross-earnings deflator	1.2022	1.2116	1.2016	1.2074

Values of macro parameters in state income taxation; income concept:  
taxpayers' post-deduction incomes

Year	Marginal tax rate	Average tax rate	Pro- gression	Tax elasticity	Post- tax elasticity	Gross- earnings deflator
1960	0.1537	0.0703	0.0834	2.1863	0.9107	1.0985
61	0.1581	0.0700	0.0881	2.2586	0.9053	1.1046
62	0.1612	0.0742	0.0870	2.1725	0.9060	1.1037
63	0.1656	0.0774	0.0882	2.1395	0.9044	1.1057
64	0.1897	0.0895	0.1002	2.1196	0.8900	1.1237
1965	0.1776	0.0885	0.0891	2.0068	0.9022	1.1083
66	0.1840	0.0944	0.0896	1.9492	0.9011	1.1098
67	0.2170	0.1117	0.1053	1.9427	0.8815	1.1345
68	0.2282	0.1216	0.1066	1.8766	0.8786	1.1381
69	0.2313	0.1274	0.1039	1.8155	0.8809	1.1352
1970	0.2402	0.1344	0.1058	1.7872	0.8778	1.1392
71	0.2513	0.1443	0.1070	1.7415	0.8750	1.1429
72	0.2682	0.1571	0.1111	1.7072	0.8682	1.1518
73	0.2844	0.1747	0.1097	1.6279	0.8671	1.1533
74	0.28*	0.1765	0.1035	1.5864	0.8743	1.1438
1975	0.3177	0.1958	0.1219	1.6226	0.8484	1.1787
76	0.3076	0.1751	0.1325	1.7567	0.8394	1.1914
77	0.2925	0.1655	0.1270	1.7674	0.8478	1.1795
78	0.2860	0.1573	0.1287	1.8182	0.8473	1.1803
79	0.2885	0.1603	0.1282	1.7998	0.8473	1.1802
1980	0.2969	0.1643	0.1326	1.8071	0.8413	1.1886

Values of macro parameters in state income taxation, municipal taxation and insured persons social security fees; income concept: taxpayers' post-deduction incomes

Year	Marginal tax rate	Average tax rate	Pro- gression	Tax elasticity	Post- tax elasticity	Gross- earnings deflator
1960	0.2297	0.1778	0.0519	1.2919	0.9369	1.0674
61	0.2192	0.1709	0.0483	1.2826	0.9417	1.0619
62	0.2238	0.1751	0.0487	1.2781	0.9410	1.0627
63	0.2275	0.1780	0.0495	1.2781	0.9398	1.0641
64	0.2563	0.1949	0.0614	1.3150	0.9237	1.0826
1965	0.2631	0.2047	0.0584	1.2853	0.9266	1.0793
66	0.2764	0.2159	0.0605	1.2802	0.9228	1.0836
67	0.3052	0.2347	0.0705	1.3004	0.9079	1.1015
68	0.3241	0.2491	0.0750	1.3011	0.9001	1.1110
69	0.3312	0.2567	0.0745	1.2902	0.8998	1.1114
1970	0.3470	0.2700	0.0770	1.2852	0.8945	1.1179
71	0.3627	0.2822	0.0805	1.2853	0.8879	1.1263
72	0.3809	0.2956	0.0853	1.2886	0.8789	1.1378
73	0.4046	0.3181	0.0865	1.2719	0.8731	1.1453
74	0.4233	0.3252	0.0981	1.3017	0.8546	1.1701
1975	0.4226	0.3347	0.0879	1.2626	0.8679	1.1522
76	0.4248	0.3241	0.1007	1.3107	0.8510	1.1751
77	0.4095	0.3147	0.0948	1.3012	0.8617	1.1605
78	0.3940	0.2997	0.0943	1.3146	0.8653	1.1556
79	0.3997	0.3045	0.0952	1.3126	0.8631	1.1586
1980	0.4097	0.3102	0.0995	1.3208	0.8558	1.1685

Appendix 3. Data used for construction of the tax-policy index

Notation:

- |     |             |   |
|-----|-------------|---|
| 1.  | $T$         | Total tax revenue   |
| 2.  | $X_1$       | Total post-deduction incomes of taxpayers   |
| 3.  | $X_2$       | Total pre-deduction incomes of taxpayers  |
| 4.  | $X_3$       | Total pre-deduction incomes of all income receivers   |
| 1-4 |             | Empirical data-series, 5-year moving sums   |
| 5.  | $p(n)$      | Factor of change of the official cost-of-living index   |
| 6.  | $\bar{e}_1$ | The macro (non-scaled) tax-revenue elasticity with respect to post-deduction incomes of taxpayers |
| 7.  | $\bar{e}_2$ | The macro (non-scaled) tax-revenue elasticity with respect to pre-deduction incomes of taxpayers  |
| 6-7 |             | The elasticities are calculated on yearly micro data and transformed to 5-year arithmetic means   |



Transformations:

$i$  refers to tax bases:

$i = 1$  post-deduction incomes of taxpayers

$i = 2$  pre-deduction incomes of taxpayers

$i = 3$  pre-deduction incomes of all income receivers

8.  $\beta_1, \beta_2, \beta_3$   $\beta_i = \Delta \log T / \Delta \log X_i$   
Empirical ratios of log changes
9.  $D_1, D_2, D_3$   $D_1 = \beta_1 / \bar{e}_1, D_2 = \beta_2 / \bar{e}_2, D_3 = \beta_3 / \bar{e}_2$   
The (non-scaled) tax-policy indices
10.  $\alpha_1, \alpha_2, \alpha_3$   $\alpha_i = (D_i - 1 / D_i) * \Delta \log T$   
The (non-scaled) policy effects
11.  $D_1^*, D_2^*$   $D_i^* = k_i * D_i$  Scaled policy indices  
 $k_1 = 1 / D_1(1969/73) = 1.1520$   
 $k_2 = 1 / D_2(1969/73) = 1.1762$
- $D_3^*$   $D_3^* = k_2 * D_3$
12.  $e_1^*, e_2^*$   $e_i^* = \frac{1}{k_i} \bar{e}_i$  Scaled elasticities
13.  $\alpha_1^*, \alpha_2^*, \alpha_3^*$   $\alpha_i^* = (D_i^* - 1 / D_i^*) * \Delta \log T$   
Scaled policy effects
14.  $p_1, p_2$   $p_i = \exp\{-\exp\{(\alpha_i^* - 1) / (\bar{e}_i - 1)\}\}$   
Neutral (neutral-equivalent) indexation factor  
of tax rules: the schedule  $i=1$ , the schedule +  
deductions from income  $i=2$

15.  $\alpha_1^*, \alpha_2^*$   $\alpha_i^* = \log(1 - (\log p_i)(\bar{e}_i - 1))$
16.  $\alpha_1^*(n), \alpha_2^*(n)$   $\alpha_i^*(n) = \log(1 - \log p(n))(\bar{e}_i - 1)$   
The reference policy effect according to the official cost-of-living index
17.  $D_1^*(n), D_2^*(n)$  The reference tax-policy indices
18.  $\hat{D}_1, \hat{D}_2$   $\hat{D}_i = D_i^*/D_i^*(n)$   
Measures of tax policy with respect to the cost-of-living index
19.  $\hat{p}_1, \hat{p}_2$   $\hat{p}_i = p_i/p(n)$   
The indexation of tax rules relative to changes in cost of living
20.  $\Delta\alpha_2^*$   $\Delta\alpha_2^* = \alpha_2^* - \alpha_1^*$   
Deduction policy marginal effect
- $\Delta\alpha_3^*$   $\Delta\alpha_3^* = \alpha_3^* - \alpha_2^*$   
Policy effect of "new taxpayers"
21.  $\dot{D}$  Log changes of tax-policy indices

	$\beta_1$	$\bar{e}_1$	$D_1$	$D_1^*$	$\alpha_1^*$	$\Delta \log T$
1961/1965	1.36858	2.1393	.63973	0.73695	-.06726	.18844
1962/1966	1.38229	2.0775	.66536	0.76648	-.06385	.20956
1963/1967	1.63453	2.0315	.80459	0.92687	-.01765	.22369
1964/1968	1.61216	1.9789	.81467	0.93848	-.01566	.23895
1965/1969	1.57034	1.9181	.81870	0.94311	-.01229	.20375
1966/1970	1.63011	1.8742	.86976	1.00194	.00039	.20089
1967/1971	1.59004	1.8327	.86759	0.99944	-.00012	.21108
1968/1972	1.46722	1.7856	.82170	0.94657	-.01257	.22273
1969/1973	1.50681	1.7358	.86808	1.00000	-.00000	.24547
1970/1974	1.34092	1.7127	.78293	0.90191	-.02765	.25426
1971/1975	1.41333	1.6797	.84142	0.96929	-.00805	.25404
1972/1976	1.10893	1.6828	.65898	0.75913	-.06464	.20372
1973/1977	0.97088	1.6948	.57286	0.65992	-.07633	.14812
1974/1978	0.79336	1.7329	.45782	0.52740	-.08531	.09520
1975/1979	0.81519	1.7529	.46505	0.53572	-.07919	.09137
1976/1980	0.75709	1.7898	.42300	0.48728	-.09671	.09191

	$p_1$	$p(n)$	$\alpha_1^*(n)$	$D_1^*(n)$	$\hat{D}_1$	$\hat{p}_1$
1961/1965	1.05876	1.0533	-.06098	.75550	0.97546	1.00518
1962/1966	1.05908	1.0557	-.06018	.77689	0.98659	1.00320
1963/1967	1.01711	1.0576	-.05950	.78989	1.17342	0.96171
1964/1968	1.01600	1.0658	-.06441	.78767	1.19146	0.95328
1965/1969	1.01339	1.0500	-.04583	.81638	1.15525	0.96514
1966/1970	0.99956	1.0456	-.03976	.83477	1.20026	0.95596
1967/1971	1.00014	1.0504	-.04181	.83468	1.19739	0.95215
1968/1972	1.01603	1.0544	-.04251	.83974	1.12721	0.96361
1969/1973	1.00000	1.0630	-.04600	.84219	1.18738	0.94073
1970/1974	1.03901	1.0955	-.06722	.79092	1.14033	0.94843
1971/1975	1.01186	1.1270	-.08476	.74983	1.29268	0.89784
1972/1976	1.09601	1.1413	-.09458	.68294	1.11156	0.96032
1973/1977	1.11157	1.1472	-.10028	.59630	1.10669	0.96894
1974/1978	1.11803	1.1324	-.09555	.49907	1.05676	0.98731
1975/1979	1.10641	1.1124	-.08360	.52222	1.02585	0.99462
1976/1980	1.12380	1.1047	-.08191	.52878	0.92153	1.01729

	$D_3^*$	$\alpha_3^*$	$\Delta\alpha_3^*$	$\dot{D}_1^*$	$\hat{D}_1$	$\dot{D}_2^*$
1961/1965	0.87879	-.02599	.06207			
1962/1966	1.07729	.01504	.08210	.03928	.01135	.10579
1963/1967	1.20895	.03866	.05691	.19001	.17342	.19924
1964/1968	1.30686	.05611	.05093	.01245	.01526	.10032
1965/1969	1.21661	.03628	.03056	.00493	-.03087	.00658
1966/1970	1.22862	.03738	.02690	.06051	.03822	.02511
1967/1971	1.17673	.03170	.02644	-.00250	-.00239	-.02837
1968/1972	1.12048	.02395	.03131	-.05435	-.06040	-.05775
1969/1973	1.14047	.03023	.03023	.05491	.05200	.03253
1970/1974	0.99805	-.00050	.01890	-.10324	-.04042	-.07352
1971/1975	0.87999	-.03464	.00691	.07205	.12539	-.07798
1972/1976	0.70965	-.08335	-.00792	-.24439	-.15095	-.16352
1973/1977	0.58184	-.10645	-.01269	-.14006	-.00440	-.17539
1974/1978	0.42315	-.12977	-.02391	-.22416	-.04616	-.25727
1975/1979	0.43200	-.12014	-.01778	.01566	-.02968	-.00386
1976/1980	0.44359	-.11529	-.00993	-.09477	-.10725	-.01221

	$\hat{D}_2$	$D_3^*$	$\Delta\alpha_2^*$	$\alpha_1^* - \alpha_1$	$\alpha_2^* - \alpha_2$	$(\alpha_2^* - \alpha_1^*) - (\alpha_2 - \alpha_1)$
1961/1965			-.02080	.03886	.04873	.00987
1962/1966	.07754	.20366	-.00322	.04155	.04875	.00720
1963/1967	.18357	.11530	-.00050	.03668	.04264	.00596
1964/1968	.10288	.07787	.02084	.03869	.04120	.00251
1965/1969	-.02879	-.07156	.01801	.03283	.03490	.00207
1966/1970	.00047	.00983	.01009	.03047	.03356	.00309
1967/1971	-.02702	-.04316	.00537	.03210	.03627	.00418
1968/1972	-.06385	-.04898	.00521	.03576	.04055	.00479
1969/1973	.03278	.01768	.00000	.03730	.04326	.00596
1970/1974	.00178	-.13339	.00826	.04284	.04823	.00539
1971/1975	.00425	-.12589	-.03351	.03983	.05209	.01227
1972/1976	-.04115	-.21514	-.01079	.04078	.04920	.00841
1973/1977	-.00164	-.19857	-.01742	.03411	.04263	.00852
1974/1978	-.03831	-.31847	-.02056	.02743	.03544	.00800
1975/1979	-.05608	.02069	-.02317	.02592	.03414	.00822
1976/1980	-.03790	.02648	-.00865	.02867	.03477	.00610

	$\beta_2$	$\bar{e}_2$	$D_2$	$D_2^*$	$\alpha_2^*$	$p_2$
1961/1965	1.29206	2.23	.57940	0.68151	-.08806	1.07094
1962/1966	1.39759	2.17	.64405	0.75756	-.06707	1.05701
1963/1967	1.67427	2.13	.78604	0.92457	-.01825	1.01613
1964/1968	1.79882	2.07	.86899	1.02214	.00518	0.99516
1965/1969	1.76695	2.02	.87473	1.02889	.00572	0.99439
1966/1970	1.76703	1.97	.89697	1.05505	.01048	0.98920
1967/1971	1.68273	1.93	.87188	1.02554	.00526	0.99435
1968/1972	1.54715	1.88	.82295	0.96799	-.00737	1.00837
1969/1973	1.56431	1.84	.85017	1.00000	.00000	1.00000
1970/1974	1.44553	1.83	.78991	0.92912	-.01940	1.02342
1971/1975	1.35171	1.85	.73065	0.85942	-.04155	1.04905
1972/1976	1.16641	1.88	.62043	0.72978	-.07543	1.08607
1973/1977	0.99960	1.92	.52062	0.61238	-.09375	1.10216
1974/1978	0.80103	1.99	.40253	0.47347	-.10587	1.10680
1975/1979	0.80997	2.02	.40098	0.47164	-.10236	1.10009
1976/1980	0.81202	2.05	.39611	0.46592	-.10536	1.09992

	$\alpha_2^*(n)$	$D_2^*(n)$	$\hat{D}_2$	$\beta_3$	$D_3$	$\hat{p}_2$
1961/1965	-.06600	.74059	0.92022	1.66607	0.74712	1.01674
1962/1966	-.06552	.76182	0.99441	1.98746	0.91588	1.00124
1963/1967	-.06537	.77384	1.19479	2.18925	1.02781	0.96079
1964/1968	-.07062	.77187	1.32425	2.29988	1.11105	0.93372
1965/1969	-.05105	.79966	1.28666	2.08933	1.03432	0.94704
1966/1970	-.04422	.81960	1.28727	2.05774	1.04454	0.94606
1967/1971	-.04681	.81849	1.25296	1.93080	1.00042	0.94664
1968/1972	-.04774	.82350	1.17546	1.79088	0.95260	0.95635
1969/1973	-.05268	.82330	1.21462	1.78405	0.96959	0.94073
1970/1974	-.07872	.76358	1.21679	1.55277	0.84851	0.93420
1971/1975	-.10717	.70331	1.22197	1.38406	0.74814	0.93083
1972/1976	-.12365	.62230	1.17271	1.13425	0.60332	0.95161
1973/1977	-.13506	.52305	1.17079	0.94976	0.49467	0.96074
1974/1978	-.13136	.42019	1.12678	0.71591	0.35975	0.97739
1975/1979	-.11502	.44272	1.06532	0.74189	0.36727	0.98893
1976/1980	-.11043	.45424	1.02571	0.77311	0.37713	0.99567

ELINKEINOELÄMÄN TUTKIMUSLAITOS (ETLA)  
The Research Institute of the Finnish Economy  
Lönrotinkatu 4 B, SF-00120 HELSINKI 12 Puh./Tel. (90) 601 322

KESKUSTELUAIHEITA - DISCUSSION PAPERS ISSN 0781-6847

No 161 ESA-JUKKA KÄÄR, A Survey on Borderline Problems of Institutional Sectoring in the National Accounts of OECD Countries. 31.07.1984. 37 p.

No 162 KARI ALHO, Korkopolitiikan itsenäisyydestä Suomessa. 31.08.1984. 22 s.

No 163 PAAVO OKKO, Suhteellisten hintojen muutokset Suomen teollisuudessa 1960-82. 12.09.1984. 25 s.

No 164 JORMA VANNINEN, Näkökohtia teollisuuden energian, erityisesti sähkön kysynnästä. 01.10.1984. 29 s.

No 165 KARI ALHO, A Model for the Banking Sector in Finland. 03.10.1984. 24 p.

No 166 CHRISTIAN EDGREN, The Tax-Elasticity - an Empirical Application. 01.11.1984. 25 p.

No 167 JUHA KETTUNEN, Sairausrakuutuskorvauksien ja niiden rahoituksen kehityksestä. 12.11.1984. 36 s.

No 168 OSMO FORSSELL, Changes in the Structure of the Finnish Economy 1970-1980. 20.11.1984. 17 p.

No 169 JUHANI RAATIKAINEN - KARI TAKALA, Kausaalisuustestejä suomalaisilla rahamarkkinamuuttujilla. 11.02.1985. 53 s.

No 170 ROBERT A. HART, Wage Supplements through Collective Agreement or Statutory Requirement? 04.02.1985. 26 p.

No 171 AKINARI HORII, Financial Innovation and its Implications for the Japanese Financial System. 18.03.1985. 8 p.

No 172 PAUL D. McNELIS, Indexing and Macroeconomics: A Survey of Theoretical Developments and International Experience. 02.04.1985. 44 p.

No 173 EERO PYLKKÄNEN - PENTTI VARTIA, Some Comments on Fine-Tuning Macro-Model Forecasts. 29.05.1985. 22 p.

No 174 CHRISTIAN EDGREN, Automatic versus Discretionary Tax-Policy Effects Revisited. 05.06.1985. 44 p.

Elinkeinoelämän Tutkimuslaitoksen julkaisemat "Keskusteluaiheet" ovat raportteja alustavista tutkimustuloksista ja väliraportteja tekeillä olevista tutkimuksista. Tässä sarjassa julkaistuja monisteita on rajoitetusti saatavissa ETLAn kirjastosta tai ao. tutkijalta.

Papers in this series are reports on preliminary research results and on studies in progress; they can be obtained, on request, by the author's permission.

0033A/05.06.1985