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## Keskusteluaiheita <br> Discussion papers



* This paper is conserned with the measurement of automatic and discretionary effects on tax changes. The theoretical framework behind the study will be presented in a forthcoming publication. Edgren-Turkkila-Y. Vartia: Mathematical Analysis and Macroeconomic Modelling of Progressive Income Taxation.

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The tax-elasticity is one of alternative progressivity measures. It has, however, mostly been used in relation to a given point in the tax scale. Kakwanil), for instance, has proposed an income and tax structure based macro indicator of progressivity by using estimated concentration indices for pretax and posttax incomes. He showes how the change in posttax distribution can be decomposed into changes in pretax distribution, progressivity and tax rate. He has, however, not discussed the tax revenue effects from the automatic and discretionary point of view, a decomposition which tells us something about the yearly change of income concentration point in relation to changes in tax scale or other taxation rules. This decomposition is necessary especially when the progressivity and related measures are applicated in time series analysis.

In the theoretical model ${ }^{2)}$ we have derived at the an aggregation formula for the individual tax-elasticities to have an macroestimate for the increase in taxes due to an increase in incomes. However, we did not have explicitly clarify what we exactiy meant with the income increase on macro level in a situation where new income receivers or taxpayers are coming in and old ones are going out of the taxsystem. Nor did we define the population (total income receivers or taxpayers) for which the elasticity and the income increase have to be calculated.

We have mostly elaborated with the income variable "taxable income". The operation of the taxsystem in a changing world in conjunction with available time series of income tax-data are not very suitable for an empirical test. We are also forced to make some assumptions in the

[^0]following to attain consistency between the construction of tax-elasticity and the income tax-data series.

For the following empirical study, we have made our choices as follows:

1) the population consists only of taxpayers, which means that we only include peoples paying a positive tax in the year of consideration
2) according to the first statement the tax-elasticity has to be calculated, separately for every year, for taxpayers in respective years. Hence we have in this respect no problem with potential taxpayers outside the population
3) as income concept in the elasticity calculation we choose to use the taxed income and in fact the taxed income of the taxpayers. This means that the tax-elasticity has to include not only the progressive effect of the tax schedule but also that of the income deduction rules.

We assume that the average relative income increase or decrease is the same for every one of the taxpayers and consequently that the income distribution is from one year to another unchanged. In the progressive effect we do not allowe for the effect on taxes which is caused by an increase or decrease in income because of variation in the number of taxpayers.

When the tax schedule from one year to another is the same and especially when it is radically changed in relation to the average income increase, new taxpayers are coming in or old ones going out of the population. The variation in the number of taxpayers is normally concentrated on the low income-brackets because the tax-system itself defines the population by setting the lowest income threshold for which the tax is positive. The net number of taxpayers moving in or out of the population for other reasons than the threshold, is assumed to be rather small.

Owing to the historically high variation in the number of taxpayers, neither the increase in the total taxed or taxable income, nor the increase in average taxed or taxable income are a suitable basis for calculation of the progression effect. We therefore use as an income-variable the wage-earners wage index, which measures the relative change of an unit of income.

Of course, we do not in that case include the taxed capital incomes in our variable. However, by substracting the relative change of the wage index $(\dot{w})$ from the change in the total taxed income $(\dot{y})$, we get a new variable which measures variation in income due to the variation in the number of taxpayers and which also includes a small component indicating to which extent the change in capital and other incomes differs from the change in wages. We call this new variable the "income volume variable" L.

The tax elasticity is calculated primarily to explain the progressive effect of the tax-schedule, but due to the fact, that instead of having taxable income we have choosed taxed income, the tax-elasticity $\overline{\mathrm{e}}$ also includes the progressive effect of the main deduction rules.

First we calculate the increase in taxes due to the increase in wages $\dot{W} \bar{e}$, which can be decomposed into the relative income change effect $\dot{w}$ and the tax progression effect $\dot{w}(\bar{e}-1)$ (see table 1 and figure 1). Then we substract from the observed tax change $t$ the two effects $\dot{w}$ and $\dot{w}(\bar{e}-1)$, and we are left with an unexplained residual $R_{1}=\dot{\mathrm{t}}-\dot{w} \bar{e}$ which contains the actual changes in taxes due to changes in the tax schedule $\ell_{j}$, some contributions originating from changes in deduction rules, $\ell_{2}$ and from the variation of the income volume variable, $\ell_{3}$.

Looking at the residual $R_{1}$ we can see that for the period 1961-1974 it was positive except in years 1961, 1963, 1965 and 1974 years when the taxes were lowered through changes in the tax schedules. In years 1963-66 the income tax schedule was kept the same but in the year 1964 there was a temporarily extra tax schedule in addition to the normal schedule. In the period $1975-1982 \mathrm{R}_{1}$ is negative. In this period tax schedules and deduction rules were to a large degree adjusted according to the inflation rate.

We have made some separate calculations for the changes in taxes due to the changes in tax schedules, the effect $\ell_{1}$. So if we make correction for the residual $\mathrm{R}_{1}$ with respect to the component $\ell_{1}$, we have a new residual $R_{2}=R_{1}-\ell_{1}$ (see table 1).

The residual $R_{2}$ is for the period 1961-1974 positive except in years 1961 and 1967. The income volume component $L$ has almost the same sign as the residual $R_{2}$ (except in year 1967), and $R_{2}$ can in a way be explained by the component $L$ as demonstrated in figure 2. In the period 1961-1974 there was no significant changes in the nominal values of the deduction rules so for this period $R_{3}=R_{2}-\ell_{2}=R_{2}$.

The contribution of the income volume component $\dot{L}$ to the increase in taxes has been quite big. In relation to the increase in taxes $\dot{t}, R_{2}$ has been on the average $15 \%$ and in relation to tax increase corrected with the changes in schedules $\left(\dot{t}^{+} \ell_{1}\right)$ in average $12 \%$. The volume component $i$ in relation to the total income increase $\dot{Y}$ of the taxpayers has been bigger, on the average $30 \%$. New taxpayers coming into the population are mostly small income receivers and therefore their contribution to the increase in taxes in relation to their
increase in incomes is smaller than that of the taxpayers continuously staying in the population. Over the period 1960-1973 the share of taxpayers to all income receivers has in fact increased 22 percentage points and the corresponding income share 28 percentage points.

For the period 1975-1982 of inflation indexation with a two years' remont of the taxsystem in 1975 and 1976, the residual $\mathrm{R}_{1}$ is negative. In 1975 the former three tax-classes (I, II and III, into which population was classified according socioeconomic situation) was reduced to two classes ( A and B ). According to our estimate the taxation sharpened in year 1975 with regards to the schedule component $\ell_{1}$. On the other hand, the deduction rules were easened troughout. We estimate that the taxable income in 1975 was on average 6.8 log percentage points smaller when using deduction rules for 1975 than using rules for 1974. What the change in rules means as a change in taxes is a rather difficult question. The relative decrease in taxes could have been the double of the relative decrease in taxable income. The residual $R_{3}=R_{2}-l_{2}$ is in year 1975 negativ like the income volume component $L$.

In year 1976 the two tax classes were combined to only one tax class and many of the income deductions were substituted by deductions from the tax. The situation in 1976 was quite different from that in 1975, so a comparative analysis is rather complex to be done especially with respect to the deductions. For the period 1976-1982 we have made a tentative correction in the increase in taxes due to the income volume component and defined the difference $R_{1}-R_{3}$ as the effect of changes in the tax schedule and deduction rules $\left(\ell_{1}+\ell_{2}\right) \cdot \ell_{1}$ is however calculated so $\ell_{2}$ is left as a resisidual.

The elasticity $\bar{e}$ has been calculated in two stages: for tax schedules and for the main income deductions. The elasticity for tax schedules indicates the average increase in taxes due to the increase in taxable income and the elasticity effect for deductions indicates the average increase in taxable income due to the increase in taxed income.

The "individual" elasticities are separately estimated by incomeclasses (taxable income and taxed income respectively). In the schedule elasticity calculation we have taken as income class means the ratio of the marginal $m(\bar{y} i)$ and average $\theta(\bar{y} i)$ taxes. For the taxable income elasticity the average slopes $\pi_{i}^{*}=\Delta \theta_{i}^{*} / \Delta \log Y_{i}^{*}$ for the income-class $i$ where $\Delta \theta_{j}^{*}$ is the average change in the taxable income ratio and $\Delta \log Y_{i}^{*}$ the $\log$ differences of the taxed income in income class i. The average tax schedule elasticity is calculated by aggregating using tax weights $w_{i}^{\top}: \bar{e}^{-1}=\sum w_{i}^{\top} e_{i} .{ }^{1)}$ The average taxable income-elasticity measuring the progressive effect of deductions from income is calculated by aggregating first the average slopes $\Pi_{j}^{*}$ with the income-weights $\bar{\Pi}^{*}=\Sigma w_{i}^{Y} \Pi_{i}^{*}$ and then by making the transformation $\overline{\mathrm{e}}^{-2}=1+\frac{\overline{\bar{\Pi}}^{*}}{\overline{\mathrm{\theta}}^{*}}$. The overall elasticity $\overline{\mathrm{e}}$ is the product of $\bar{e}^{-1}$ and $\overline{\mathrm{e}}^{2}$.

The average elasticity for the tax schedule, if calculated on individual level using the schedule function gives us an exact measure. The accuracy of calculations made on income class level, however depends on how representative the class means are by which the "individual elasticity" is calculated. The schedule elasticity

[^1]variates in the schedules taxable income intervales because the marginal tax rate is constant but the average tax rate is rising with the income in the intervale. Because the taxable income classes in income tax statistics do not match the schedule income intervales the class means are not necessarely well representative .

If, however, properly estimated the average elasticity is an exact measure for big hypothetical relative changes in income. When we in the elasticity include other than to the income increase strictly functionally determined deductions, which depends on the behavior of the individuals, the measure and its use is more bounded to the structure and the environment of the actual outcome. On the other hand, we are not interested in knowing what would the increase in taxes have been if the incomes had increased $20 \%$ instead of the actual $10 \%$, the behavioral element following and calculated for an income increase of $10 \%$.

The income tax statistics do not in later years give the distribution of taxable income. We can only find the taxable income according taxed income classes. Also the taxes before deductions from taxes for taxpayers are missing, so we propose an indirect method for estimations of the elasticities which include the behavioral i.e. not strictly functional determined components. The method is indirect because the schedule elasticity is not estimated using the schedule itself.

Table 1. Tax bases, tax elasticity and effect of changes in tax instruments, log percent change (\%) and log percent points change (\%-points).

| Year | Wageindex $\dot{w}$ \% | Taxed income $\dot{Y}$ \% | Incomevolume component L \%-points | Taxelasticity e | Progressioneffect $\begin{array}{r} \dot{w}(e-1) \\ \underline{\%} \text {-points } \end{array}$ | $\begin{gathered} \text { Tax } \\ \dot{\mathrm{t}} \\ \underline{\%} \end{gathered}$ | Residua 1 $R_{1}$ | Effect of change in schedule $\ell_{1}$ | Resi- <br> dual <br> $\mathrm{R}_{2}$ | Effect of change in deduction rules $\varepsilon_{2}$ | Residual <br> $R_{3}=$ income <br> volume <br> effect $\ell_{3}$ | Effect of change in instruments $l_{1}+l_{2}$ | Inflation indexation \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | \%-points |  |  |  |  |  |  |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| 1961 | 6.8 | 5.6 | -1.2 | 2.35 | 9.2 | 0.8 | -15.2 | -14.1 | -1.1 | - | -1.1 | -14.1 |  |
| 62 | 6.4 | 13.2 | 6.8 | 2.26 | 8.1 | 17.3 | 2.9 | 0 | 2.9 | - | 2.9 | 0 |  |
| 63 | 8.2 | 11.4 | 3.2 : | 2.23 | 10.1 | 14.5 | -3.8 | -5.2 | 1.4 | - | 1.4 | -5.2 |  |
| 64 | 12.7 | 23.8 | 11.1 | 2.20 | 15.2 | 38.9 | 11.0 | +8.7 | 2.3 | - | 2.3 | +8.7 |  |
| 1965 | 8.6 | 15.4 | 6.8 | 2.11 | 9.6 | 15.1 | -3.1 | -5.5 | 2.4 | - | 2.4 | -5.5 |  |
| 66 | 6.2 | 11.4 | 5.2 | 2.05 | 6.5 | 19.1 | 6.4 | 0 | 6.4 | - | 6.4 | 0 |  |
| 67 | 9.1 | 7.7 | 1.4 | 2.04 | 9.5 | 24.5 | 6.0 | +8.1 | -2.1 | - | -2.1 | +8.1 |  |
| 68 | 10.4 | 12.1 | 1.7 | 1.97 | 10.1 | 25.0 | 4.5 | 0 | 4.5 | - | 4.5 | 0 |  |
| 69 | 6.9 | 11.7 | 4.8 | 1.91 | 6.3 | 16.9 | 3.7 | 0 | 3.7 | - | 3.7 | 0 |  |
| 1970 | 8.2 | 13.1 | 4.9 | 1.88 | 7.2 | 17.5 | 2.1 | 0 | 2.1 | - | 2.1 | 0 |  |
| 71 | 12.0 | 16.0 | 4.0 | 1.83 | 10.0 | 23.3 | 1.3 | 0 | 1.3 | - | 1.3 | 0 |  |
| 72 | 10.8 | 17.1 | 6.3 | 1.81 | 7.6 | 26.2 | 6.7 | 0 | 6.7 | - | 6.7 | 0 |  |
| 73 | 14.4 | 18.1 | 3.7 | 1.76 | 10.9 | 30.5 | 5.2 | 0 | 5.2 | - | 5.2 | 0 |  |
| 74 | 18.0 | 24.8 | 6.8 | 1.87 | 15.7 | 25.0 | -8.7 | -16.3 | 7.6 | - | 7.6 | -16.3 |  |
| 1975 | 19.7 | 15.9 | -3.8 | 1.98 | 19.3 | 23.1 | -15.9 | +2.6 | -18.5 | -12.2(-17.3) | -6.3(-1.2) | -19.4(-14.7) |  |
| 76 | 14.0 | 12.9 | -1.1 | 1.99 | 13.9 | 7.5 | -20.3 | -13.4 | -6.9 | -5.8 | -1.1 | -19.2 |  |
| 77 | 8.2 | 7.2 | -1.0 | 2.02 | 8.4 | 0.7 | -15.9 | -11.1 | -4.8 | -3.1 | -1.2 | -14.7 |  |
| 78 | 6.5 | 4.3 | -2.2 | 2.07 | 7.0 | -0.9 | -14.4 | -9.7 | -4.7 | -1.2 | -3.5 | -10.9 | 12.2 |
| 79 | 10.9 | 14.1 | 3.2 | 2.05 | 11.5 | 17.1 | -5.3 | -7.7 | 2.4 | -0.3 | +2.7 | -8.0 | 7.7 |
| 80 | 11.4 | 15.2 | 3.8 | 2.11 | 12.7 | 18.2 | -5.9 | -6.0 | 0.1 | -2.7 | 2.8 | -8.7 | 7.2 |
| 81 | 11.9 | 14.3 | 2.4 | 2.10 | 13.1 | 13.8 | -11.2 | -8.8 | -2.4 | -4.4 | 2.0 | -13.2 | 10.4 |
| 82 | 10.2 | 13.0 | 2.8 | 2.10 | 11.2 | 13.5 | -7.9 | -9.6 | 1.7 | -0.5 | 2.2 | -10.1 | 11.3 |

[^2]Figure 1. The tax change $t$ explained by the relative change in wageindex $\dot{\mathbf{w}}$ and the taxprogression effect $\dot{\mathbf{w}}(\overline{\mathrm{e}}-1)$


Figure 2. The effect of changes in tax instruments, the income-volume component and its effect on tax change.


1. A demonstration of the method of estimating the tax elasticity from structural data

In the following we estimate the combined elastictty of state and municipal income tax for households in year 1981. For the state tax we have two separate elasticities, one for the tax schedule and one for the hole state income taxation. Having estimated these two elasticities, we are also able to calculate the contribution of the deduction rules to the total elasticity in state income taxation.

The tax elasticity in municipal taxation on the other hand is very close to 1.0 both with respect to taxable and taxed income, because the taxation is relative for the taxable income and the deductions do not on the average work progressively.

The calculations are made for the population of taxpayers. For this purpose we constructed the taxpayers income (taxed income) by the income classes. For every income class 1 we calculated the average tax rates, taxes per taxed income $\theta_{\mathcal{j}}$ and taxes per taxable income $\Theta_{\mathfrak{i}}^{*}$. The ratios are representative for the average taxed income $\bar{Y}_{\mathbf{i}}$ and average taxable income $\bar{Y}_{i}^{*}$ respectively.

The average slopes $\pi_{i}$ and $\pi_{j}^{*}$, of which the former is for the whole state income taxation and the later with asterix for the schedule, are calculated as follows:

$$
\begin{aligned}
& \pi_{i}=\frac{\theta_{i+1}-\theta_{i-1}}{\log Y_{i+1}^{*}-\log Y_{i-1}^{*}}=\frac{\Delta \theta}{\Delta \log Y} \text { and } \\
& \pi_{i}^{*}=\frac{\theta_{i+1}^{*}-\theta_{i-1}^{*}}{\log Y_{i+1}^{*}-\log Y_{i-1}^{*}}=\frac{\Delta \theta^{*}}{\Delta \log \gamma^{*}} .
\end{aligned}
$$

The macro slopes are the weighted sums of the slopes for income classes

$$
\bar{\pi}=\Sigma w_{i}^{Y} \quad \pi_{i} \text { and } \bar{\pi}^{*}=\Sigma w_{i}^{Y^{*}} \pi_{i}^{*},
$$

where $W_{j}^{Y}$ and $W_{j}^{Y^{*}}$ are the relative shares of taxed and taxable incomes.

The tax rates are on the macro level

$$
\bar{\theta}=T / Y \text { and } \bar{\theta}^{\star}=T / Y^{*},
$$

where $T$ is total taxes, $Y$ total taxed income and $Y^{*}$ total taxable income. On macro leve1 we have the average marginal tax rates

$$
\begin{aligned}
& \bar{m}=\bar{\theta}+\bar{\pi} \text { and } \\
& \bar{m}^{\star}=\bar{\theta}^{*}+\bar{\pi}^{*} \text { respectively, }
\end{aligned}
$$

and the taxelasticities

$$
\begin{aligned}
& \overline{\mathrm{e}}=\overline{\mathrm{m}} / \bar{\theta}=1+\bar{\pi} / \bar{\theta} \text { and } \\
& \overline{\mathrm{e}}_{2}^{\star}=\overline{\mathrm{m}}^{\star} / \bar{\theta}^{\star}=1+\bar{\pi}^{\star} / * \bar{\theta} .
\end{aligned}
$$

The contribution of the tax income deduction rules to total tax elasticities $\bar{e}_{1}=\overline{\mathrm{e}} / \overline{\mathrm{e}}_{2}^{\star}$ so the total $\overline{\mathrm{e}}=\overline{\mathrm{e}}_{1} \overline{\mathrm{e}}_{2}^{\star}$.

The relevant calculations for 1981 are presented in tables 2 and 3, in table 2 for the state income taxelasticities $\bar{e}, \bar{e}_{1}$ and $\bar{e}_{2}^{*}$, and in table 3 the relevant data for aggregation of the two taxation forms, i.e. state and municipal income taxation.

In tables 4 and 5 the aggregations of tax elasticities for state and municipal taxation are made both through weighting marginal and average taxratios and directly through weighting elasticitis. According our figures taxes increase due to increase in taxable incomes (with constant taxbase ratios for the two taxforms) in relation 1.36 and due to increase in taxed income in relation 1.48. The contribution of deduction rules to the elasticity of total income taxation is in average 1.09.
2. The calculation of effects of changes in tax schedules

The estimated average macro slope $\bar{\pi}^{*}$ (as in table 2 ) can be used to construct an approximative tax rate function $\theta_{0}^{*}=\left(\log y_{1}-\log \alpha_{0}\right)_{\pi}^{*}$, where $\alpha_{0}$ is some threshold income. The inflation indexation of the tax schedule causes a shift in the slope of the tax function with respect to $\log y$, so we have after indexation according to inflation rate $p$ a new tax rate function $\theta_{1}^{*}=\left(\log y_{j}-\log \alpha_{1}\right) \pi^{*}$, where $\log \alpha_{1}=\log \alpha_{0}+\log p$ and $\alpha_{1}=p \alpha_{0}$. The change in the average tax rate is

$$
\begin{aligned}
\Delta \theta^{*}=\theta_{1}^{*}-\Theta_{0}^{*} & =\bar{\pi}^{*}\left(\log \alpha_{0}-\log \alpha_{1}\right) \\
& =(-\log \mathrm{p})_{\pi^{*}} .
\end{aligned}
$$

The average change in taxes is when the income is kept constant,

$$
\log \left(T_{1} / T_{0}\right)=\log \left(\frac{T_{1} / Y_{0}}{T_{0} / Y_{0}}\right)=\log \left(\theta_{1}^{*} / \theta_{0}^{*}\right)
$$

and further we get

$$
\log \left(T_{1} / T_{0}\right)=\log \left(1-\frac{(\log p) \pi^{*}}{\theta_{0}^{*}}\right) .
$$

In year 1981 the tax schedule of year 1980 was index corrected by $11 \%$, so according to the above formula the effect of the schedule change on the state tax was about $-8.8 \underline{\%}(-8.5 \%) .1$ ) On the state and municipal tax together the effect was in average about -3.9 \%.

The effects of the tax system on the disposable income of households can be calculated by using the tax elasticity and the effect of tax schedule indexation. An income increase of e.g. $10 \%$ results in an increase in disposable income of about $8.4 \%$. The indexation of the tax schedule by $11 \%$ results in an increase of the disposable income by $1.3 \%$, which is independent of the increase in income.

[^3]Table 2. Data for calculation of tax elasticities in state income taxation in year 1981.

| Income class (taxed income) FMK | Average taxed income, FMK $\bar{Y}$ | Average taxable income, FMK $\bar{Y}^{*}$ | Average tax-ratios |  | Average slopes |  | Relativ share of income $\begin{array}{ll}\text { taxed } & \text { taxable } \\ W^{Y}, \% & W^{Y *}, \%\end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\theta$, \% | $\theta^{\star}, \%$ | $\pi$ | $\pi *$ |  |  |
| 10-3000 | 1291 | 656 |  |  |  |  | 0.007 | 0.005 |
| $3000-6000$ | 4502 | 2705 | 6.159 | 10.249 |  |  | 0.018 | 0.015 |
| $6000-10000$ | 7897 | 5081 | 5.829 | 9.058 | -0.0312 | -0.0506 | 0.036 | 0.031 |
| $10000-15000$ | 12427 | 9295 | 2.993 | 4.002 | -0.0624 | -0.0891 | 0.106 | 0.109 |
| 15000-20000 | 17437 | 12188 | 0.882 | 1.262 | -0.0093 | -0.0526 | 1.375 | 1.315 |
| $20000-25000$ | 22431 | 14677 | 2.441 | 1.597 | 0.0410 | 0.0781 | 2.485 | 2.223 |
| $25000-30000$ | 27534 | 17859 | 2.754 | 4.246 | 0.0458 | 0.1315 | 3.679 | 3.263 |
| $30000-35000$ | 32614 | 21161 | 4.157 | 6.407 | 0.0923 | 0.1332 | 6.075 | 5.390 |
| $35000-40000$ | 37498 | 24638 | 5.606 | 8.531 | 0.1279 | 0.1437 | 8.511 | 7.647 |
| $40000-50000$ | 44829 | 31003 | 8.227 | 11.896 | 0.1441 | 0.1391 | 19.856 | 18.777 |
| $50000-60000$ | 54556 | 39648 | 11.010 | 15.150 | 0.1520 | 0.1189 | 16.966 | 16.860 |
| $60000-80000$ | 68098 | 53817 | 14.583 | 18.453 | 0.1021 | 0.1586 | 18.567 | 20.066 |
| $80000-100000$ | 88535 | 62879 | 15.954 | 22.463 | 0.1395 | 0.1491 | 9.466 | 9.193 |
| 100 000-200 000 | 127404 | 104706 | 23.322 | 28.377 | 0.2090 | 0.1203 | 10.429 | 11.721 |
| $200000-$ | 282985 | 256978 | 40.243 | 39.395 | 0.2120 | 0.1227 | 2.424 | 3.386 |
| $\bar{\pi}=\Sigma w_{i}^{Y} \pi_{i}=0.1325$ |  | $\overline{\mathrm{e}}=1+\bar{\pi} / \bar{\theta}=2.103$ |  |  | $\overline{\mathrm{m}}=0.2527$ |  |  |  |
| $\bar{\pi} *=\Sigma w_{i}^{Y^{*}} \pi_{i}^{*}=0.1333$ |  | $\bar{e}_{2}^{*}=1+\bar{\pi}^{*} / \bar{\theta}^{*}=1.811$ |  |  | $\overline{\mathrm{m}}^{\star}=0.2977$ |  |  |  |
| $\bar{\theta}=T / Y=0$. | $=0.12017$ | $\bar{e}_{1}=\overline{\mathrm{e}} / \mathrm{e}_{0}^{*} \quad=1.161$ |  |  |  |  |  |  |
| $\bar{\theta}^{\star}=T / Y^{\star}=0$. | $=0.16432$ |  |  |  |  |  |  |  |

Table 3. Data for calculation of the combined state and municipal income tax elasticity in year 1981

| Item | State <br> income <br> taxation <br> m111.FMK | Municipal income taxation mill.FMK |
| :---: | :---: | :---: |
| Total income of income-receivers | 122567.7 | 122771.9 |
| Taxable income of income receivers | 87152.4 | 107250.0 |
| Taxable income of taxpayers | 80345.6 | 106869.1 |
| - of which taxed both in state and municipal taxation <br> - which only in municipal taxation |  | $\begin{array}{r} 98511.5 \\ 8357.6 \end{array}$ |
| Taxable income, not taxed | 6806.8 | 380.9 |
| Taxed income of taxpayers | 109906.4 |  |
| Income taxes | 13207.1 | 16944.5 |
| - of which related to income both in state and municipal taxation | 13207.1 | 15596.7 |
| - of which related to income taxed only in municipal taxation |  | 1347.8 |
| Municipal tax rate, \% |  | 15.86 |
| - related to income in state and municipal taxation <br> - related to income in municipal taxation only |  | 15.83 16.13 |

Table 4. Calculation of combined elasticity with respect to taxable income in year 1987

| Item | mill.FMK | weight, \% | average tax rate, \% | average <br> marginal <br> tax rate, \% |
| :---: | :---: | :---: | :---: | :---: |
| Taxable income in state and municipal taxation | 80345.6 | 75.18 | 32.26 | 45.60 |
| municipal taxation due to bigger tax base in municipal tax | 18165.9 | 17.00 | 15.83 | 15.83 |
| Income taxed only in municipal taxation | 8357.6 | 7.82 | 16.13 | 16.13 |
| Total | 106869.1 | 100.00 | 28.20 | 38.23 |
| Combined tax-elastic with respect to tax | $=\text { ity }=\frac{0.38}{0.28}$ <br> ble income | $=1.3558$ |  |  |
| Item | elasticity | tax <br> share, \% |  |  |
| Elasticity in state income taxation | 1.811 | 43.80 |  |  |
| Elasticity in municipal taxation | 1.000 | 56.20 |  |  |
| Weighted elasticity | 1.3552 | 100.00 |  |  |

Table 5. Calculation of the combined elasticity with respect to taxed income.

| Item | mill.FMK | weight, <br> $\%$ | average <br> tax rate, <br> $\%$ | average <br> marginal <br> tax rate, $\%$ |
| :--- | :---: | :---: | :---: | :---: |
| In state- and <br> municipal taxation | 109906.4 | 92.05 | 26.21 | 39.46 |
| In municipal <br> taxation only | 9497.6 | 7.95 | 14.19 | 14.19 |
| Total | 119404.0 | 100.00 | 25.25 | 37.45 |

Combined tax-elasticity $=\frac{0.3745}{0.2525}=1.4832$
with respect to taxed income

| Item | elasticity | tax- <br> share, \% |
| :--- | :--- | :---: |
| Elasticity in state <br> income taxation | 2.1029 | 43.80 |
| Elasticity in <br> municipal taxation | 1.000 | 56.20 |

3. The breakdown of the tax elasticity into instrumental parts

For the total state income taxation the relevant measures are according to the previous section on average:

| progression | $\bar{\pi}=0.1325$ |
| :--- | :--- |
| average tax rate | $\bar{\theta}=0.1202$ |
| the marginal tax rate | $m=0.2527$ |
| the tax elasticity | $e=2.103$ |

The total tax elasticity depends - given the income structure - on the different taxation rules: the deductions from income, the tax schedule and the deductions from taxes. If we like to have an Idea of how the "taxation instruments" work for themselves or in relation to total taxation, it is necessary to eliminate from the total tax elasticity the contribution of the deductions from tax (tax before the final taxes).

For the tax schedule including deductions from taxes we have also the following measures wich relate the final taxes to the taxable income of taxpayers:

| progression | $\bar{\pi}^{*}=0.1333$ |
| :--- | :--- |
| average tax rate | $\bar{\theta}^{\star}=0.1643$ |
| the marginal tax rate | $\bar{m}^{\star}=0.2977$ |
| tax elasticity | $\bar{e}_{2}^{\star}=1.811$ |

The deductions from taxes were in year 1981 for all income receivers 1041 mill. FMK. In the tax income statistics the deductions in lower income classes were bigger than the final taxes so there can be noted in the total some overflow, i.e. not really used deductions. We have no exact data concerning deductions for taxpayers, but we assume them to amount to about 600 mill . FMK. The share of deductions per cent of the final tax is bigger in low income classes and smaller in higher classes. ${ }^{1)}$

The level of the deductions of the taxpayers are, however, on average bigger in high income classes and smaller in low income classes because parents making the child allowance are more frequent in upper

The share of deductions from taxes according to income classes, \%

|  | Income class <br> (toductions in ratio <br> tinal tax, $\%$ |
| :---: | :---: |
| $30000-35000$ |  |
| $35000-40000$ | 18.4 |
| $40000-45000$ | 11.7 |
| $45000-50000$ | 8.9 |
| $50000-55000$ | 8.1 |
| $55000-60000$ | 7.6 |
| $60000-65000$ | 6.4 |
| $65000-70000$ | 5.8 |
| $70000-80000$ | 5.1 |
| $80000-90000$ | 4.3 |
| $90000-100000$ | 3.6 |
| $100000-150000$ | 2.6 |
| $150000-200000$ | 1.5 |
| $200000-$ | 0.6 |
|  |  |

[^4]income classes than in lower classes. The children-allowances also constitute the main part of the deductions from taxes.

We have done an approximative estimation using a deduction of 250 FMK in income class $30000-35000$ FMK and of 670 FMK in class 100000 150000 FMK. The deductions are 275 FMK in average for taxpayers to make in total the assumed 600 milj . FMK.

If we use the approximative average tax rate functions $\theta=\theta(Y, \alpha, \pi)$ and $\Theta^{\star}=\Theta^{\star}\left(Y^{*}, \alpha^{\star}, \pi^{\star}\right)$ and calculate new slopes for taxes without deductions from taxes, we receive a more appropriate elasticity measure for the deductions from income. ${ }^{1)}$

According to table 6 the schedule tax elasticity is smaller without than with deductions from taxes and total elasticity is also smaller without than with deductions from taxes. A $10 \%$ increase in income would result in a $11.6 \%$ increase in taxable income and a $20.4 \%$ increase in taxes before deductions from taxes. Including these deductions the final taxes would increase on average $21.0 \%$.

From a marginal income of 100 FMK 25 FMK goes on average to state income taxes leaving the income receiver with 75 mk . The average state income tax rate is $12 \%$.

Taking also the municipal taxation into consideration, from a marginal income of 100 FMK 40 FMK is going to taxes (to the state 25 FMK and to

[^5]the municipals 15 FMK). The income receiver is left on the margin with an average of 60 FMK . On average the tax rate is $27 \%$ and the disposable income $73 \%$ of the original income.

Table 6. State income taxation, decomposition of the relative effect of the tax-instruments. ${ }^{1)}$

|  |  | $\bar{\pi}$ | $\bar{\theta}$ | mi | $\overline{\text { e }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Total taxation | 0.1325 | 0.1202 | 0.2527 | 2.103 |
| 2 | Total taxation without deductions from tax | 0.1310 | 0.1256 | 0.2566 | 2.043 |
| 3 | Schedule with. deductions from tax | 0.1333 | 0.1643 | 0.2977 | 1.811 |
| 4 | Schedule without deduction from taxes | 0.1301 | 0.1718 | 0.3019 | 1.757 |
| 5a | The system of deductions from income | 0.1192 | 0.7310 | 0.8502 | 1.163 |
| 5b | The system of deductions from income without no correction for deductions from taxes | 0.1177 | 0.7310 | 0.8487 | 1.161 |

[^6]4. The effect of inflation indexation on taxation once more

We can make som recalculation of the effects of the inflation indexation with the new parameters in table 6 . Using the formula on page 14 with the parameters for schedule the indexation effect is a little bit smaller, i.e. $-8.2 \%$ instead of $-8.8 \%$. We have in table 7 also made a check of our estimate for the schedule indexation by using the formula

$$
\Delta \log T=\Sigma W_{i}^{\top} \Delta \log T_{i}
$$

where $w_{i}^{\top}$ are the tax weights (in final taxes) according to taxed income classes $i$ and $\Delta \log T_{i}$ the relative change in taxes - taxable mean income unchanged - due to different schedules. We have in fact calculated the schedule tax for the different taxable incomes $y_{i}$ using the schedules for year 1980 and for year 1981. The average relative change in tax is the weighted sum of the "individual" changes.

Using a formula developed elsewhere ${ }^{1)}$ the change in deductions from the tax reduced the taxes on average $1.6 \%$ primarily due to the increase of child-allowance by about $14.5 \%$. ${ }^{2}$ )

For the deductions from income we use for the moment an estimate taken from the situation in 1977-1978. In 1978 the maximal amount of deduction from wage income (in finnish "työtulovähennys") and respectively the top income level under which it can be made were increased by $12 \%$. The calculated effect of the change in this special deduction - taxed incomes constant - on the taxable income was on average -1.48 \%. ${ }^{3}$ ) In year 1981 the maximal amount of this deduction and the top income level was increased by $11.8 \%$. The effect on taxable income is assumed to be about the same as in year 1978.

[^7]Using the average schedule elasticity to transform the taxable income effect to taxes we end to a final effect of about -2.6 \%.

The calculated effects are summarized as follows:

| Indexated <br> "tax-instrument" | the inflation <br> indexation, $\%$ | the effect on <br> taxes $\%$ |
| :--- | :---: | :---: |
| the schedule | 11.0 | -8.2 |
| the deductions from <br> taxes | min | 14.5 |
| the special deduction <br> from income | 11.8 | -1.6 |
| residual | 11.9 | -2.6 |
| Total |  | -0.8 |

We have in the table added a residual of -0.8 to match the calculated instrument-effect $\left(\ell_{1}+\ell_{2}\right)$ in table 1.

Table 7. The change in schedule tax due to change in schedule $1980 \rightarrow$ 1981.

| Taxed income classes FMK | Average taxable income, \% | $w_{i}^{\top}$ | $\Delta \log _{\%} T_{i},$ |
| :---: | :---: | :---: | :---: |
| $0-15000$ | 1) | 0.0006 |  |
| 15000-20000 | 12188 | 0.0010 | -40.2 |
| $20000-25000$ | 14677 | 0.0033 | -38.9 |
| $25000-30000$ | 17859 | 0.0084 | -30.0 |
| $30000-35000$ | 21161 | 0.0210 | -22.4 |
| $35000-40000$ | 24638 | 0.0397 | -18.0 |
| $40000-45000$ | 28702 | 0.0606 | -14.3 |
| $45000-50000$ | 33173 | 0.0754 | -11.0 |
| $50000-55000$ | 37635 | 0.0801 | -8.8 |
| $55000-60000$ | 42089 | 0.0754 | -7.3 |
| $60000-65000$ | 46563 | 0.0690 | -7.3 |
| $65000-70000$ | 50989 | 0.0604 | -7.3 |
| $70000-80000$ | 57301 | 0.0960 | -6.0 |
| $80000-90000$ | 66436 | 0.0699 | -6.7 |
| $90000-100000$ | 75610 | 0.0558 | -5.5 |
| 100 000-150 000 | - 97248 | 0.1441 | -5.1 |
| 150 000-200 000 | 146021 | 0.0583 | -3.5 |
| 200000 - | 256978 | 0.0812 | -2.5 |

1) Smaller than tax schedules threshold income

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[^0]:    1) N.C. Kakwani: Income Inequality an Poverty. Methods of Estimation and Policy Applications. Washington D.C. 1980.
    2) This empirical study is to some extent based on the results in Edgren-Turkkila-Y. Vartia: Tuloverotuksen analysoinnin matemattisista ongelmista. ETLA DP No 17, 1978 and a forthcoming publication "Mathematical Analysis and Macroeconomic Modelling of Progressive Income Taxation". The author alone is responsible for the application of the theory.
[^1]:    1) We could also alternatively estimate the individual marginal tax rates or -progressivities and aggregate them using income-weights.
[^2]:    $(3)=(2)-(1)$
    $(7)=(6)-(1)-(5)$
    (9) $=(7)-(8)$
    $(11)=(9)-(10)$
    $(12)=(8)+(10)$

[^3]:    1) The effect depends on the indexation rate $p$ and through the average slope $\bar{\pi}^{*}$ on the schedules parameters $m(y i)$ and $\theta(y i)$ and the income structure.
[^4]:    1) In upper classes the number of taxpayers is equal to that of income recetvers.
[^5]:    1) If we had used the slope $\bar{\pi}^{*}$ in calculating the effect of deductions from taxes the deductions had been in income class 30000-35000 162 instead of 250 FMK and in class $100000-150000810$ instead of 670 FMK.
[^6]:    1) Parameters $\bar{\pi}$ in points 1 and 3 are estimated from tax income structure data, in points 2 and 4 calculated with corrections in the average tax rate functions. Parameters in $5 a$ and $5 b$ are calculated from points 2 and 4 and points 1 and 3 respectively.
[^7]:    1) Edgren: Verolainsäädännössä tehtyjen muutosten vaikutuksista valtion tuloveron tuotossa. ETLA DP No 104, 1982, page 24.
    2) The child-allowance from taxes was in year 1980480 FMK for every child and in year 1981550 for the first 650 for the second, 750 for the third and 1000 FMK for the forth child. The estimate of the effect is therefore a minimum.
    3) Edgren: Vähennysjärjestelmän merkitys verotuksessa, DP No 42, ETLA 1979, page 45.
