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Inter-dealer transactions and the integration of the foreign exchange markets

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1. Introduction and summary

The present paper concludes our analysis of the quoting and position taking behaviour of an individual foreign exchange dealer. The major aim is to show how foreign exchange markets become globally integrated and efficiently organized through the private profit maximizing behaviour of individual dealers.

In the two earlier papers (Suvanto 1982b, 1983) we analyzed the dealer behaviour under isolated circumstances, where the dealer had a short-term monopoly power in the sense that he could steer his expected position by affecting the customers' net sales through changing quotations. In the first paper, there were no interactions between the dealer and the outside markets or between the dealer and any regulating agency. The major aim was to introduce a simple formal model and use it to derive the optimal pricing rule. The second paper introduced the central bank which performed the function of an exogenous (non-profit-maximizing) 'dealer of last resort'. This allowed us to study certain aspects of central bank interventions in the foreign exchange market and, more importantly, extended the analysis by introducing wholesale transactions as an alternative way of position adjustment, in addition to changing customer quotations.

The present paper, which relies entirely on the formal results of the two previous papers, takes an important step towards a more realistic description of the foreign exchange market. In particular, instead of the central bank the role of the 'dealer of last resort' is given to the outside markets in general, that is to other dealers operating in different localities and possibly in different time-zones. The resulting inter-dealer transactions serve as an integrating device in the sense that the quotations in different local markets are drawn closer each others than what would be the case if the local markets were isolated. In addition, because of the inter-dealer market the dealers themselves become more sensitive to quotation differentials, which makes the average spread narrower and tends to reduce the dispersion of quotations even more. This kind of integration becomes still closer if also the nondealer customers are able, because of lower transaction and information costs, to trade outside their local markets.

It should be noted that the dealers are defined as market makers, who stand ready to buy and sell on immediate demand at prices they are, in principle, free to choose at any moment of time (cf. Demsetz, 1968). It is this trading income the dealers are assumed to maximize, not the speculative or arbitrage profits which may arise occasionally. The source of this income results from the fact that a higher price is applied to customer purchases (dealer sales) than to customer sales (dealer purchases). The dealers' behaviour is, however, importantly conditioned by certain position constraints, in particular, by the requirement of a closed position at the end of the day, which constraint can be justified by risk considerations. While the position may fluctuate within the trading day as a result of transactions uncertainty, the end-of-day position target affects the dealers quotations during the day and determines the conditions under which it will be more profitable to send wholesale orders to other dealers in order to reduce the excessively open positions, rather than to try to attract net sales or purchases from customers by changing quotations.

This concern by the dealers about the state of the position explains why quotations are continuously changing and why prices may differ between markets at any moment of time (see Hudson, 1979, p. 44-46 for practical examples). The inter-dealer market and the sensitivity of the dealers and nondealer customers to small price differentials explain why the quotations will nevertheless remain fairly close to the each others, thus establishing conditions which approximate perfect competition. The major difference is that in the present model of the integrated dealership market no 'invisible hand' and no continuous market clearing are needed to get the efficiency properties of perfect competition, at least approximately. In this respect the present analysis resembles that by Phelps and Winter (1970) on optimal price behaviour under atomistic competition.

2. Introducing inter-dealer transactions

Assume that the foreign exchange market is composed of a large number of local markets with one dealer in each. The transaction and information costs prevent nondealer customers from trading outside their local market, which implies that each dealer can control the expected buy and sell orders of their customers by changing quotations. The dealers can, whenever they so desire, make transactions with each other. These transactions we shall call wholesale transactions, in accordance with the terminology used in the earlier paper. Note that the roles of the two dealers in any wholesale transaction differ; the dealer, at whose initiative the transaction takes place, acts in the role of a customer in relation to his partner, who in turn acts in the role of a market maker. The share of each dealer of total trade in currencies is small so that he can disregard any systematic reactions by other dealers to his own actions. The quantitative characteristics, especially those describing the local customers' buy and sell orders, may differ from one market to another, but qualitatively the situation is the same in each market. The customers' buy and sell orders depend on the prices quoted by the dealer, and each dealer maximizes his trading income during the day, subject to the constraint that the expected foreign exchange position is closed (or at some other well defined target) at the end of the day.

Under these assumptions the results of the previous paper are directly applicable. In that paper we analyzed the pricing and position taking behaviour of an isolated foreign exchange dealer, who could make wholesale transactions with the central bank. In the present paper these transactions are made in the outside market, which, from the point of view of any single dealer, performs the function of the "dealer of last resort".

Although the analysis is formally the same as in the previous paper, we repeat the main argument here in a one-period case. The expected buy and sell orders of customers in a single local market depend on the ask-rate s^a and the bid rate s^b quoted by the dealer

$$(1) \qquad p = a - bs^a \qquad \text{(buy orders)}$$

$$(2) \qquad q = -c + bs^b \qquad \text{(sell orders)} \ ,$$

where a, b, c > 0 and a - c > 0.[1] The expected trading income from the trade with customers is

$$(3) \qquad R = s^a p - s^b q$$

$$= z(\beta - \gamma z) - s(-\alpha + \gamma s)$$

where $z \equiv (s^a - s^b)/2$ is the half-spread, $s \equiv (s^a + s^b)/2$ is the mid-rate, and $\alpha \equiv a + c$, $\beta \equiv a - c$, $\gamma \equiv 2b$. The initial foreign exchange

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1) In order to reduce the notational burden, we assume that the customers are equally sensitive to the quotation both on the sell and the buy side. The stochastic terms describing the transactions uncertainty are omitted, because they do not affect the dealer's expected renevue, which he is assumed to maximize.

position is x, and the position after a wholesale transaction, if any, is x'; hence the size of the wholesale transaction is x' - x. The dealer maximizes his total revenue, that is net revenue from transactions with the customers as a market maker plus the revenue (cost) from wholesale sales (purchases) he makes at his own iniative, subject to the condition that his expected position at the end of the period is closed. This is equivalent to maximizing the Lagrangian

$$L = R - S(x' - x) + \lambda(x' - p + q), \tag{4}$$

with respect to s, z and x', where S is the price applied by the outside dealer to the wholesale transaction, and λ is the Lagrange-coefficient for the position constraint.

We assume that the dealer knows the average quotation in the outside markets, i.e. the average ask-rate $\bar{s}^a$ and the average bid rate $\bar{s}^b$, $\bar{s}^a > \bar{s}^b$.[2] Then he knows that he can buy foreign exchange from the outside markets at a price that is not higher than $\bar{s}^a$, and to sell foreign exchange there at a price that is not lower than $\bar{s}^b$. In Other words, the price S applied for a wholesale transaction is either $S \leq \bar{s}^a$ or $S \geq \bar{s}^b$.[3]

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2) This is a somewhat tricky assumption, because if he knows the average then he should know something about the distribution and be able to know quotations which might be more advantageous to him as a wholesale buyer or seller. We shall comment on this assumption below.

3) Note that when the dealer as a market maker sells to the customers he applies the higher price (ask-rate), but when he as a wholesale customer sells to an outside dealer, the latter as a market maker applies the lower price (bid rate), and *vice versa*.

Maximization of the Lagrangian yields

$$(5) \qquad z = \beta/2\gamma \equiv \hat{z}$$

$$(6) \qquad s = \hat{s} - (1/\gamma)x'$$

$$(7) \qquad \lambda = \hat{s} - (2/\gamma)x'$$

$$(8) \qquad x' = (\gamma/2)(\hat{s} - S),$$

where $\hat{s} \equiv \alpha/\gamma$ stands for the mid-rate that would equilibriate the expected buy and sell orders by the dealer's local customers, and it depends only on the characteristics of local customers (through the parameter α).

As seen from equation (5) the spread is chosen to maximize the return on equilibrium volume of trade and is independent of the initial position and the outside quotations. The solution for the mid-rate and the eventual wholesale transaction is illustrated in Figure 1, where we have drawn the actual mid-rate and the Lagrange multiplier, both as a function of the initial position x, as well as the average ask-rate and the average bid-rate. If the dealer's initial position is to the right of the point x'_s, at which $\lambda = \bar{s}^b$, then the dealer will make a wholesale sale by an amount $x - x'_s$, or even more if he can observe a bid-rate that is higher than the average one. If, on the other hand, the initial position is to the left from the point x'_p, at which $\lambda = \bar{s}^a$, he will make a wholesale purchase by an amount $x'_p - x$, or even more, if he can observe an ask-rate that is lower than the average.[4)]

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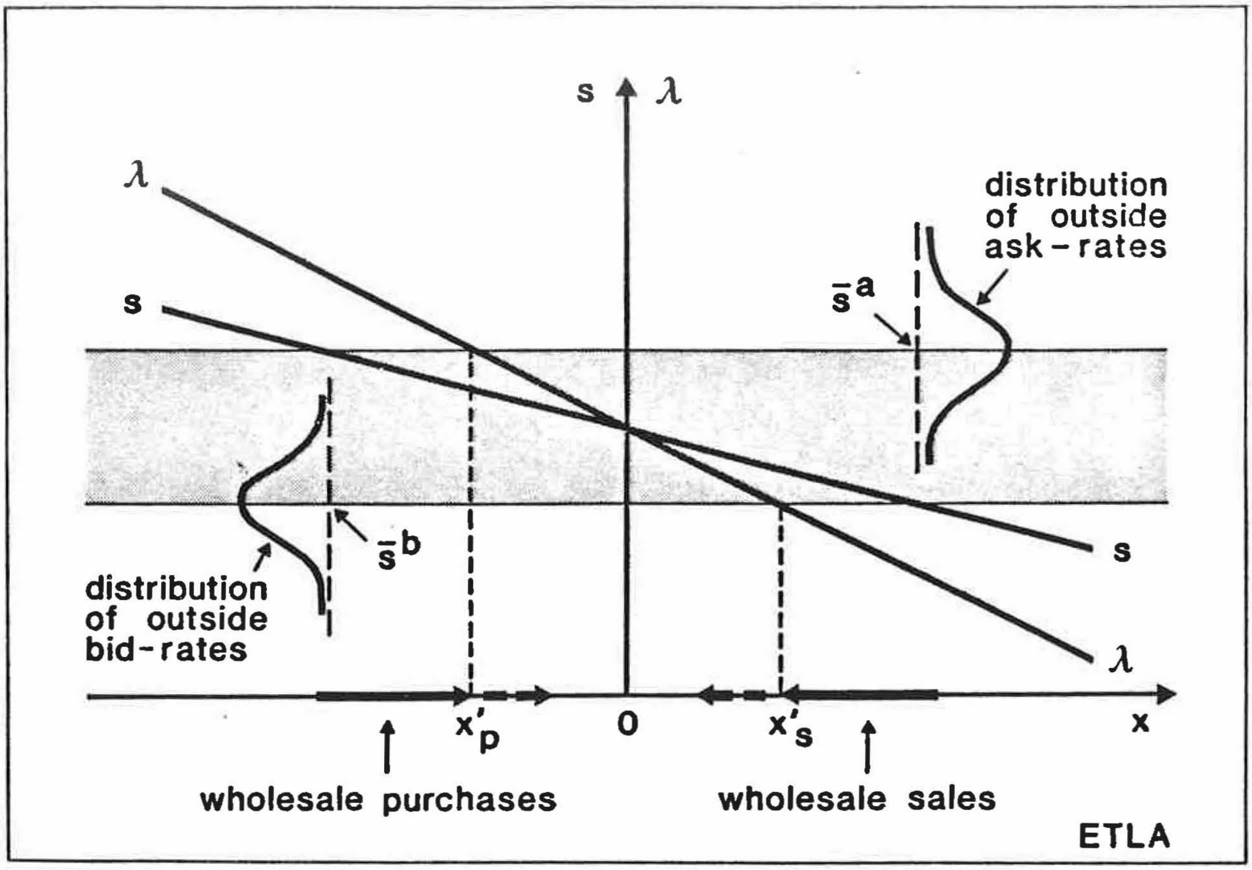
4) It is possible that he can find an ask-rate that is lower than some bid-rate implying an arbitrage profit opportunity. We return to this in Section 5.

Figure 1. *Determination of wholesale transactions*



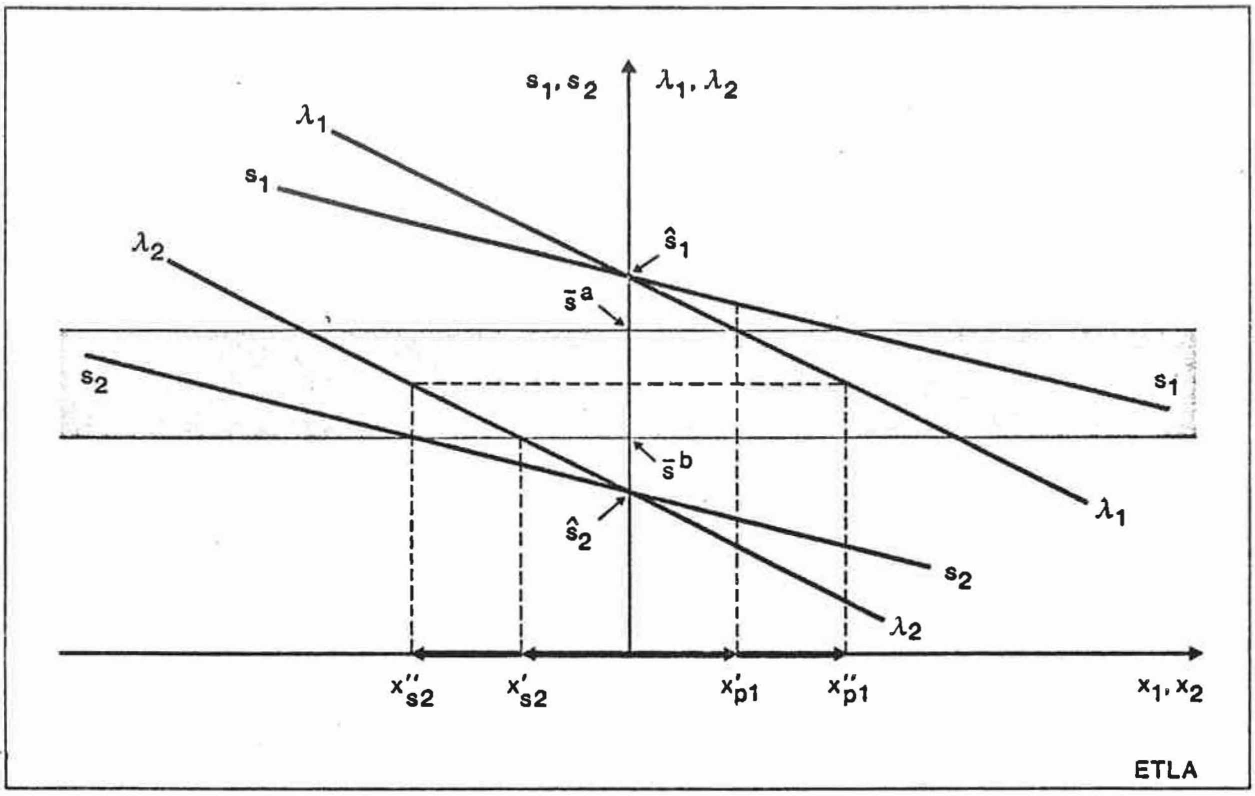
Explanation: A wholesale transaction becomes profitable when λ(x) is below the highest observed outside bid-rate or above the lowest observed outside ask-rate.

These results are essentially the same as in the previous paper, in which the dealer could make wholesale transactions with an exogenous "dealer of last resort" (the central bank), and are based on the interpretation of the Lagrange-multiplier as a shadow price of the position constraint. The major difference is qualitative and follows from the fact that instead of the preannounced quotation of the central bank the dealer now, when considering a wholesale transaction, has an opportunity to choose the best one from a number of quotations. As a consequence, the wholesale transactions tend to be larger and more frequent.

The most important new result, however, is the integration of local markets created by the inter-dealer transactions. As Figure 1 illustrates, the possibility of inter-dealer transactions tends to equalize the quotations of different dealers even if there were significant differences in local characteristics. Consider, for example, two local markets, each having only a small share of global trade in currencies, and assume that in the first market one currency is relatively scarce, in the sense that the local customers' demand is high relative to what local customers supply, and that in the second market this currency is relatively abundant. In the isolated case, the price of this currency would be high in the first market and low in the second one. Applying Figure 1, the λλ-line in the first market, $\lambda_1\lambda_1$, would be significantly above the average mid-rate at x = 0, and the same line in the second market, $\lambda_2\lambda_2$, would be significantly below it at this point, as illustrated in Figure 2. Assuming a closed initial position for both dealers, Dealer 1 would then be willing to buy an amount x'_{p1} at the average ask-rate $\bar{s}^a$, or more if he can observe a lower price. Similarly, Dealer 2 would be willing to sell an amount

$-x'_{s2}$ at the average bid-rate $\bar{s}^b$, or more if he can observe a higher price. As a result, both dealers would quote close to the average one, and the first one would be a net-seller to his local customers (net buyer from outside), and the second one would be a net buyer in his local market (net seller to outside). One can also imagine situations where the two dealers, who are symmetrically related to the 'average market', find it profitable to exchange positions bilaterally at a price which is somewhere between the average ask-rate and the average bid-rate. In such a case the transaction would be a pure swap, and the market maker/customer roles could not be distinguished.

Figure 2. *Wholesale transactions with differential equilibrium mid-rates and initial closed positions*



Explanations: With a closed position initially Dealer 1 buys an amount x'_{p1} of dollars at the average ask-rate and Dealer 2 sells an amount x'_{s2} of dollars at the average bid-rate. If they are aware of each other's situation and able to communicate directly, a pure swap at a price close to the average mid-rate will be more profitable. In a symmetric case Dealer 1 gets on amount $x''_{p1} = -x''_{s2}$ of dollars from Dealer 2, the implicit price of the transaction being equal to the average mid-rate.

3. A dynamic extension and integration between time-zones

An extension into a dynamic (multiperiod) framework, in which the dealer can change his quotations along the day, is formally similar to that in the previous paper. Assume that the trading day is divided into a large number of short trading periods, and that in the beginning of each such period the dealer observes his position and makes the decision on his quotation and on the eventual wholesale transaction using his information on the average quotation in the outside markets. Then the dealer's decision described in Section 2 can be regarded as his optimal decision in the beginning of the last trading period of the day, i.e. just before closing. Once we know how the dealer behaves in the beginning of the last period, we can solve for his optimal decision in the beginning of the second last period, and proceeding backwards we can, by applying the dynamic programming algorithm, solve for his optimal decision at any moment of the day.[5] As long as at any moment of time the dealer excepts that the average quotation in the outside markets remains constant during the rest of the day, the dynamic decision problem does not in any formal sense differ from the situation analyzed in the previous paper, in which the dealer could use the preannounced quotations of the central bank for his decision on whether or not to make a wholesale transaction. As a consequence, the results of that paper are directly applicable. In particular, it

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5) See Suvanto (1982b and 1983) for the technical details of the solution.

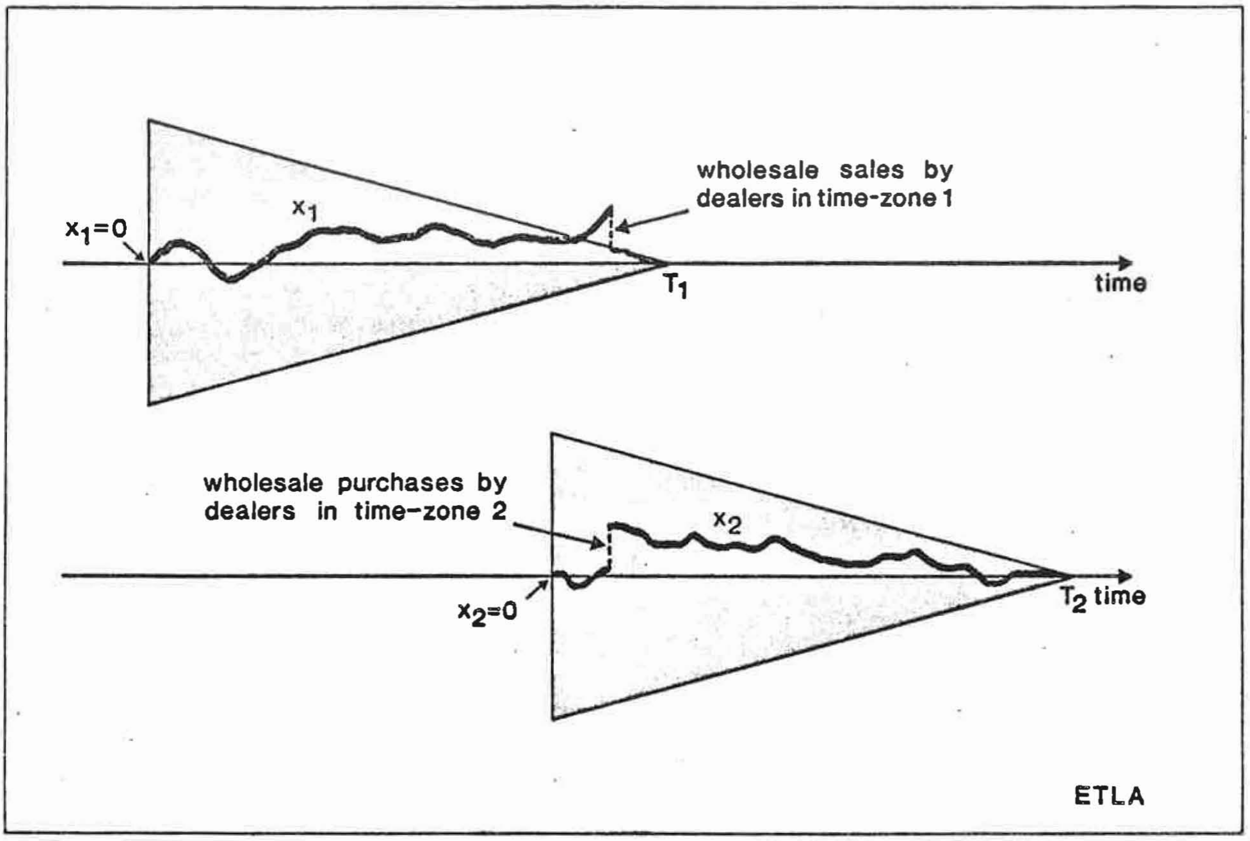
still holds that the dealer is willing to accept larger open positions in the morning, without recourse to the wholesale facility, than towards the end of the day. This is because the end-of-day position constraint is less binding when he still has many trading periods left to go, and hence enough time to steer his position by net sales to customers.

Allowing for the fact that different local markets are located in different time-zones implies that the operating hours of different dealers differ, that is some dealers are closing when some other dealers are just opening. Applying the above mentioned result, it follows that the former are likely to be those who send wholesale orders to the latter in order to get rid of the excessively open positions before the closing time. In this way the dealers in the former time-zone are able to end the day with their positions approximately closed, and the dealers in the latter time-zone may find their positions open as a result of wholesale transactions they have made as a market maker. But as the latter have plenty of time to steer their positions by affecting the customers' net sales, they are unlikely to send wholesale orders immediately back to the closing markets. This is also the mechanism by which the shocks are transmitted between the local markets and between the time-zones. Assume, for instance, that the dealers in one time-zone have unexpectedly in the late afternoon bought a large amount of dollars in net terms so that their positions generally are excessively long (in dollars). Applying the results of the previous paper they in this situation are likely to sell dollars to the dealers in the opening time-zones and reduce the price of the dollar in their local markets only marginally in order to generate some net sales to their local customers. The dealers in the opening time-zones find their positions excessively long, but as they have the whole day to go they most likely choose to reduce the price of that currency somewhat in order to get a constant flow of net sales to their local customers in each trading period so that the expected position only gradually moves towards the target. This kind of a situation is illustrated in Figure 3.

At this stage it is useful to briefly comment on our assumption that each dealer knows the average quotation and at each moment of time expects this to remain constant during the rest of the trading day. First, the dealer himself, applying the rule outlined above, will not change his quotations unless there is any further surprises during the rest of the day in the customers' buy and sell orders. If other dealers behave similarly, nobody has any chance to predict somebody else's future quotations, not even his own. In this situation the expected average quotation will remain constant and be the same for each dealer. Of course, the dealers' positions will change unexpectedly, as a result of the stochastic timing of the customers's transactions, but as long as these are unpredictable and not correlated between the markets, the average quotation should not change much, even though the local quotations are constantly changing.

Secondly, we may assume that in order to keep in touch with the market and to be informed on the quotations elsewhere, the dealer invests in information by constantly asking quotations from other dealers, even if he had no need to make any wholesale transaction. This investment in information pays off in the sense that when the dealer decides to make a wholesale sale or purchase he can immediately use the best price available. As long as the cost of such investment does not depend on the dealer's quotation, and the information is

Figure 3. *Wholesale transactions between time-zones*



Explanation: The shaded are shows the limits within which wholesale transactions are not profitable assuming closed end-of-day position target and constant expected average quotation.

collected as a constant flow throughout the day, it can be regarded as a fixed cost, which does not affect the dealer's optimal pricing rule. If anything, the investment in information tends to integrate the local markets more closely, because profitable wholesale transactions are more frequent, and the dispersion of quotations is reduced even further.

4. Customer flows and competition

Above we have analyzed the behaviour of a single foreign exchange dealer and presented the conditions under which it becomes profitable for him to send wholesale orders to other dealers. For the sake of symmetry we should also to allow for the possibility that the same dealer may receive wholesale orders from outside, and that the outside dealers may, like himself, be eager to find the best price available for such transactions. This means that the equations (1) and (2) describing the customers' buy and sell orders may no more be valid and they must be generalized in order to capture the reactions of outside dealers to his quotation. We can go even further and relax the assumption that transaction and information costs prevent local customers from trading outside. Instead of prohibitively high transaction costs we assume that the local customers face greater transaction costs when trading outside than with the local dealer, and that the customers differ with respect to these costs in the sense that for some of them the relative transaction cost is higher than for others. In fact, we can assume a continuum of nondealer customers ordered according to their relative transaction costs. At the one end there are those for whom the relative transaction cost is low so that they choose to go outside once they observe a quotation that is only marginally better than the one announced by the local dealer. At the other end there are those customers for whom it hardly ever is possible to approach outside dealers when they need to convert one currency into another.

These plausible, though admittedly vaguely defined, economic frictions leave each local dealer with some freedom to choose his own quotations and hence to affect the expected net sales by his own decision. Accordingly, we can analyze the behaviour of an individual dealer as if he had a short-term monopoly.[5)] As a market maker the dealer announces his quotation and stands ready to trade with incoming market orders, without any knowledge about the characteristics of the customers who send those order. He does not even know whether the customer asking for a quotation is a potential seller or buyer. By this assumption the possibility of price discrimination is excluded.

Let us write the incoming buy and sell orders received by a single local dealer i as follows:

$$(9) \qquad p_i = a_i - bs_i^a - \varepsilon(s_i^a - \bar{s}^a)$$

$$(10) \qquad q_i = -c_i + bs_i^b + \varepsilon(s_i^b - \bar{s}^b),$$

where (s_i^b, s_i^a) is the price quoted by the dealer i, and $(\bar{s}^b, \bar{s}^a)$ is the average quotation. We assume that the customers are equally sensitive to the local quotation as well as to the difference between this and the global average, i.e. b and ε are the same for all i. The parameter ε describes how sensitive the customers on the average are to small quotation differentials, that is how easily the transaction and information costs allow them to transact outside their local market. The parameter b describes how sensitive the nondealer customers are to a uniform change in the exchange rate everywhere.[6)]

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5) Note that this assumption is similar to that used by Phelps and Winter (1970) in their analysis of atomistic competition with slow diffusion of information on prices between local markets.

6) This is seen by adding the equations (9) and (10) respectively over all N markets, i = 1,...,N, and taking the average.

If the customers' information and transaction costs are low, then ε is large relative to b. It will be even larger if the dealer does not regard wholesale orders from other dealers as pure random events, but instead recognizes the fact that he is more likely to receive wholesale buy orders if his ask-rate is below the average one and similarly more likely to receive wholesale sell orders if his bid-rate is above the average one.

The dealer's trading income per period is the value of sales minus the value of purchases, that is

$$
\begin{aligned}
(11) \qquad R_i &= s_i^a p_i - s_i^b q_i \\
&= z_i(q_i + p_i) - s_i(q_i - p_i) \\
&= z_i\,(\beta_i - \delta z_i + 2\varepsilon\bar{z}) - s_i(-\alpha_i + \delta s_i - 2\varepsilon\bar{s}),
\end{aligned}
$$

where $\alpha_i \equiv a_i + c_i$, $\beta_i \equiv a_i - c_i$, $\delta = 2(b + \varepsilon)$, $z_i = (s_i^a - s_i^b)/2$, $\bar{z} = (\bar{s}^a - \bar{s}^b)/2$, $s_i = (s_i^a + s_i^b)/2$, and $\bar{s} = (\bar{s}^a + \bar{s}^b)/2$.

Any wholesale transaction made at the dealer's own initiative contributes to the total revenue by the amount $- S(x_i' - x_i)$, where x_i is the initial position, x_i is the position after a wholesale transaction, and S is the price applied by an outside dealer for this transaction. As above we can assume that the dealer can find a price $S \leq \bar{s}^a$, when he wants to make a wholesale purchase $(x_i' - x_i > 0)$, and a price $S \geq \bar{s}^b$, when he wants to make a wholesale sale $(x_i' - x_i < 0)$.

As is seen, the structure of the model remains the same as above. Assuming that the dealer maximizes the one-period revenue subject to the condition that the end-of-period position is closed leads to the maximization of the following Lagrangian:

$$(12)\qquad L_i = R_i - S(x_i' - x_i) + \lambda_i(x_i' - p_i + q_i).$$

After some manipulation the first order conditions for the maximum can be written as follows[7]:

$$(13)\qquad z_i = (\beta_i/2\delta) + (\varepsilon/\delta)\bar{z}$$

$$(14)\qquad s_i = (\alpha_i/\delta) + (2\varepsilon/\delta)\bar{s} - (1/\delta)x_i'$$

$$(15)\qquad \lambda_i = s_i - (1/\delta)x_i' = (\alpha_i/\delta) + (2\varepsilon/\delta)\bar{s} - (2/\delta)x_i$$

$$(16)\qquad -S + \lambda_i = 0\ .$$

Equations (13) and (14) show that the local quotation (s_i and z_i) depends directly on the quotations elsewhere ($\bar{s}$ and $\bar{z}$), in addition to the local market characteristics (α_i and β_i), and equations (15) and (16) determine the position after an eventual wholesale transaction.

It can be immediately seen that as $\varepsilon \to 0$, $z_i \to \beta_i/2\gamma$ and $s_i \to (\alpha_i/\gamma) - (1/\gamma)x_i'$, where $\gamma = 2b$, which situation corresponds to our earlier case discussed in Section 2. On the other hand, as $\varepsilon \to \infty$, $z_i \to \bar{z}/2$ and $s_i \to \bar{s}$, that is the mid-rate becomes the same everywhere and the spread approaches zero. The latter follows from the fact that $z_i \to \bar{z}/2$ must hold for each i, which is possible only if $\bar{z} \to 0$. This can be

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7) Equation (13) follows directly from $\partial L_i/\partial z_i = 0$, equation (14) is obtained from the position constraint and equation (15) by solving $\partial L_i/\partial s_i = 0$ for λ_i and using (14). Equation (16) is equal to $\partial L_i/\partial x_i = 0$.

shown also by using equation (13) and solving for the average (half) spread:

$$\bar{z} = \bar{\beta}/2(\gamma + \varepsilon) \quad , \tag{17}$$

where $\bar{\beta} = \Sigma\, \beta_i/N$. It is seen that the average spread is the smaller the greater is the customers' sensitivity to quotation differentials. This means that the competition created by customer flows reduces the degree of monopoly of each local dealer in the same way as the degree of monopoly is reduced by the existence of close substitutes in standard microeconomic analysis.

The dealer's decision concerning the possible wholesale transaction is determined by his initial position and the outside quotations exactly in the same manner as in the previous case with $\varepsilon = 0$. In terms of Figure 1 the $\lambda\lambda$-line is now less steep which fact tends to reduce the likelihood of wholesale transactions, because an excessively open position can be more easily closed by a marginal change in the quotation. On the other hand, the average spread is smaller, which tends to increase the likelihood of wholesale transactions, because the dealer can more easily observe profitable outside quotations. In the limiting case, as $\varepsilon \to \infty$, $x_i^!$ and hence the size of the wholesale transaction becomes indeterminate, which is obvious because, with zero spread and the quotations exactly the same everywhere, it does not matter on which side of the market the dealer operates. In fact, the whole notion of a dealer as an individual profit maximizing and price-setting agent loses its *raison d'être*.[8)]

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8) Note that if the costs of producing dealer services are taken into account the spread will remain positive even with infinite price elasticity of customers' buy and sell orders. This situation is analyzed in Suvanto (1982a), in which paper the emphasis is on the transactions demand for foreign currency by a single foreign exchange dealer.

5. Arbitrage and uncertainty

As long as the above mentioned economic frictions (transaction and information costs) exist, each local dealer is left with some price-setting power. With each dealer quoting prices independently and simultaneously, the quotations are bound to differ from each others at any moment of time, implying the possibility of arbitrage profit opportunities to arise occasionally. These opportunities arise when one dealer quotes an ask-rate that is lower than the bid-rate quoted by some other dealer. In this situation any third party, who is quick enough to observe the inconsistent quotations is able to make an immediate profit by buying from the former and simultaneously selling to the latter. The arbitrator can be any third dealer or a nondealer customer with sufficiently low transaction costs.

The two dealers, who quoted inconsistently in the first place, will lose, because their positions move far from the desired path and at the next moment they either have to make wholesale transactions at unfavourable prices or to make a relatively large adjustment to their quotations. In each case the two dealers will change their quotations in the direction that will eliminate, or at least reduce, the probability of inconsistent quotations thereafter.

In the following we examine how the behaviour of an individual dealer is affected if he attaches a cost to the probability of quoting inconsistently. Figure 4 presents the frequency distribution of outside quotations together with some alternative quotations by the individual dealer under consideration. A comparison of the four panels of Figure 4 shows that for a given mid-rate the probability of an inconsistent quotation is the smaller the broader is the spread and approaches zero as the spread grows indefinitely. Similarly, for a given spread this probability is positively related to the deviation of the mid-rate from the average one (in either direction).

These observations suggest that allowing for the cost of the risk of inconsistency will affect the spread positively and reduce the dispersion of quotations globally. This can be shown formally by postulating the cost function

$$(18) \qquad C_i = C_i(z_i, s_i - \bar{s})$$

with the following properties

$$\partial C_i/\partial z_i < 0, \quad \partial^2 C_i/\partial z_i^2 \geq 0 \, ,$$

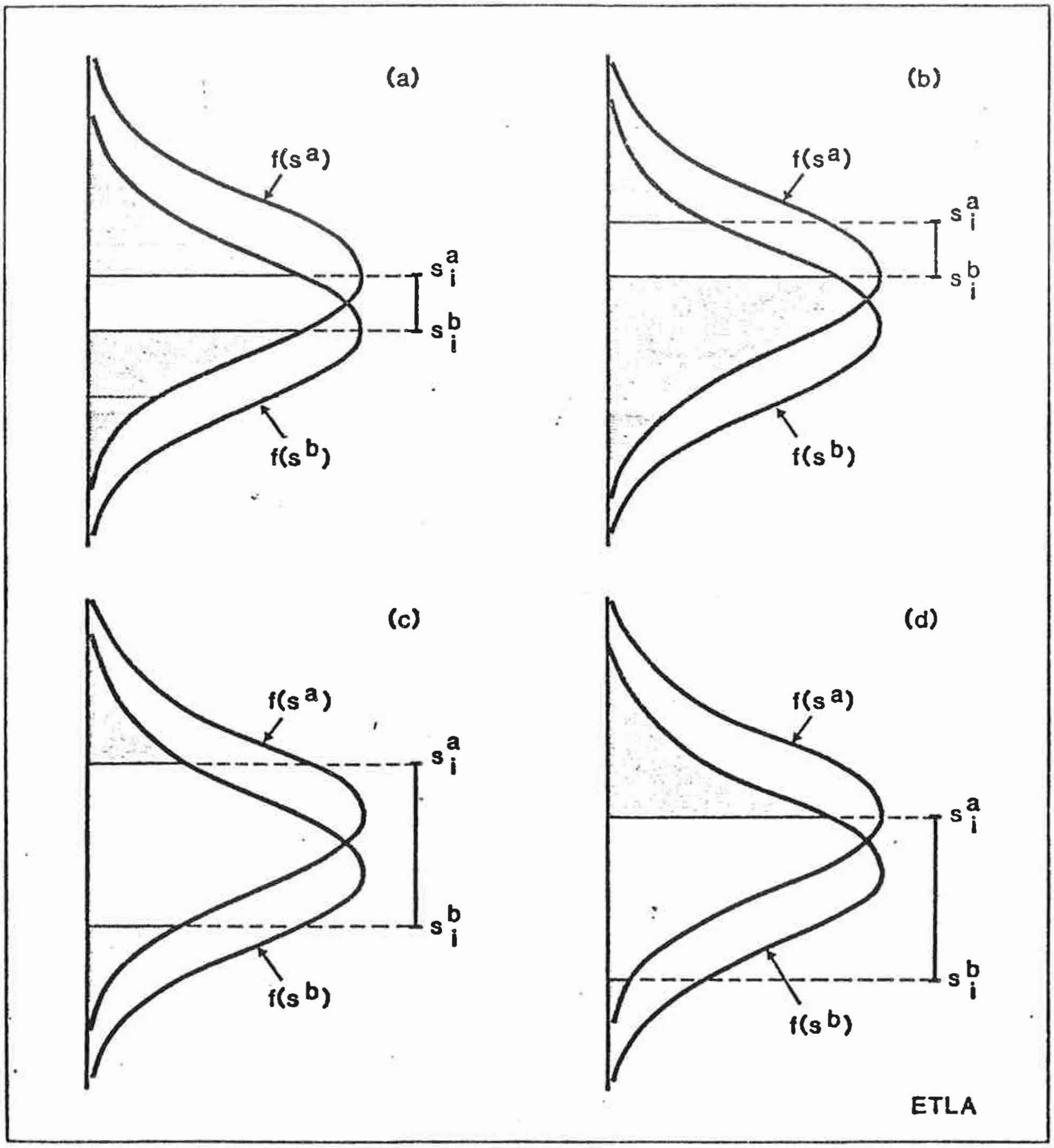
$$\partial C_i/\partial(s_i - \bar{s}) \gtrless 0 \iff s_i - \bar{s} \gtrless 0 \, ,$$

$$\partial^2 C_i/\partial(s_i - \bar{s})^2 > 0,$$

and subtracting it from the revenue function and maximizing the resulting net revenue with respect to s_i, z_i and x_i' Without any loss of genevality we assume that C_i is linear with respect to z_i, $\partial C_i/\partial z_i = -\phi_i$, and quadratic with respect to $s_i - \bar{s}$, $\partial C_i/\partial(s_i - \bar{s}) = \theta_i(s_i - \bar{s})$, where ϕ_i, $\theta_i > 0$. Differentiating the Lagrangian

$$(19) \qquad L_i = R_i - C_i - S(x_i' - x_i) + \lambda_i(x_i' - p_i - q_i)$$

Figure 4. *Probability of an inconsistent quotation*



Explanations: f(·) show the frequency distributions of the outside ask-rate and bid-rate. The sum of the shaded areas in each panel illustrates the probability of inconsistency for a given quotation (s_i^a, s_i^b) by the Dealer i.

with respect to s , z_i, x_i' and λ_i and setting the partial derivatives equal to zero yields

$$(20) \qquad \beta_i - 2\delta z_i + 2\varepsilon\bar{z} + \phi_i = 0$$

$$(21) \qquad \alpha_i - 2\delta s_i + 2\varepsilon\bar{z} - \theta_i(s_i - \bar{s}) + \delta\lambda_i = 0$$

$$(22) \qquad -S + \lambda_i = 0$$

$$(23) \qquad x_i' - \alpha_i + \delta s_i - 2\varepsilon\bar{s} = 0 \ .$$

The first condition gives the (half) spread

$$(24) \qquad z_i = (\beta_i + \phi_i)/2\delta + (\varepsilon/\delta)\bar{z} \ ,$$

which is greater than above but still remains independent of the initial position. The last two conditions are the same as above, in particular, the ss-line (equation (23)) remains untouched. Solving (21) for the Lagrange coefficient gives the new $\lambda\lambda$-line

$$(25) \qquad \hat{\lambda}_i = s_i - (1/\delta)x_i' + (\theta_i/\delta)(s_i - \bar{s}) \ ,$$

which together with the best observed outside quotation, S, determines whether or not the wholesale transaction is profitable.[9] As compared

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9) The hat '^' has been added in order to differentiate between the new $\lambda\lambda$-line and that of the previous exercise.

to the previous exercise the $\lambda\lambda$-line how declines more steeply[10)] and intercepts the ss-line at the point where $s_i = \bar{s}$, cf. Figure 5. The steeper $\lambda\lambda$-line implies that the range at which the position is allowed to fluctuate without recourse to the wholesale facility is how narrower than in the previous case, that is wholesale transactions are more likely. Furthermore, the fact that the whole $\lambda\lambda$-line shifts to the right or to the left, depending on whether the mid-rate that would keep the expected customer orders in balance[11)] is higher or lower than the global average, tends to keep the quotation relatively close to the average one.

Note that the spread approaches infinity and the mid-rate converges to the average mid-rate as the cost attached to the inconsistency risk grows indefinitely. The former result is seen from (24) by letting $\phi_i \to \infty$, and the latter result is seen by solving (21) - (23) for s_i,

$$(26) \qquad s_i = (2\delta + \theta_i)^{-1} [\alpha_i + (2\varepsilon + \theta_i)\bar{s} + \delta S]$$

and letting $\theta_i \to \infty$. In both cases the dealer would lose his role as a market maker; with infinite (or very large) spread there would be no customers, and with $s_i = \bar{s}$ there would be no price setters.

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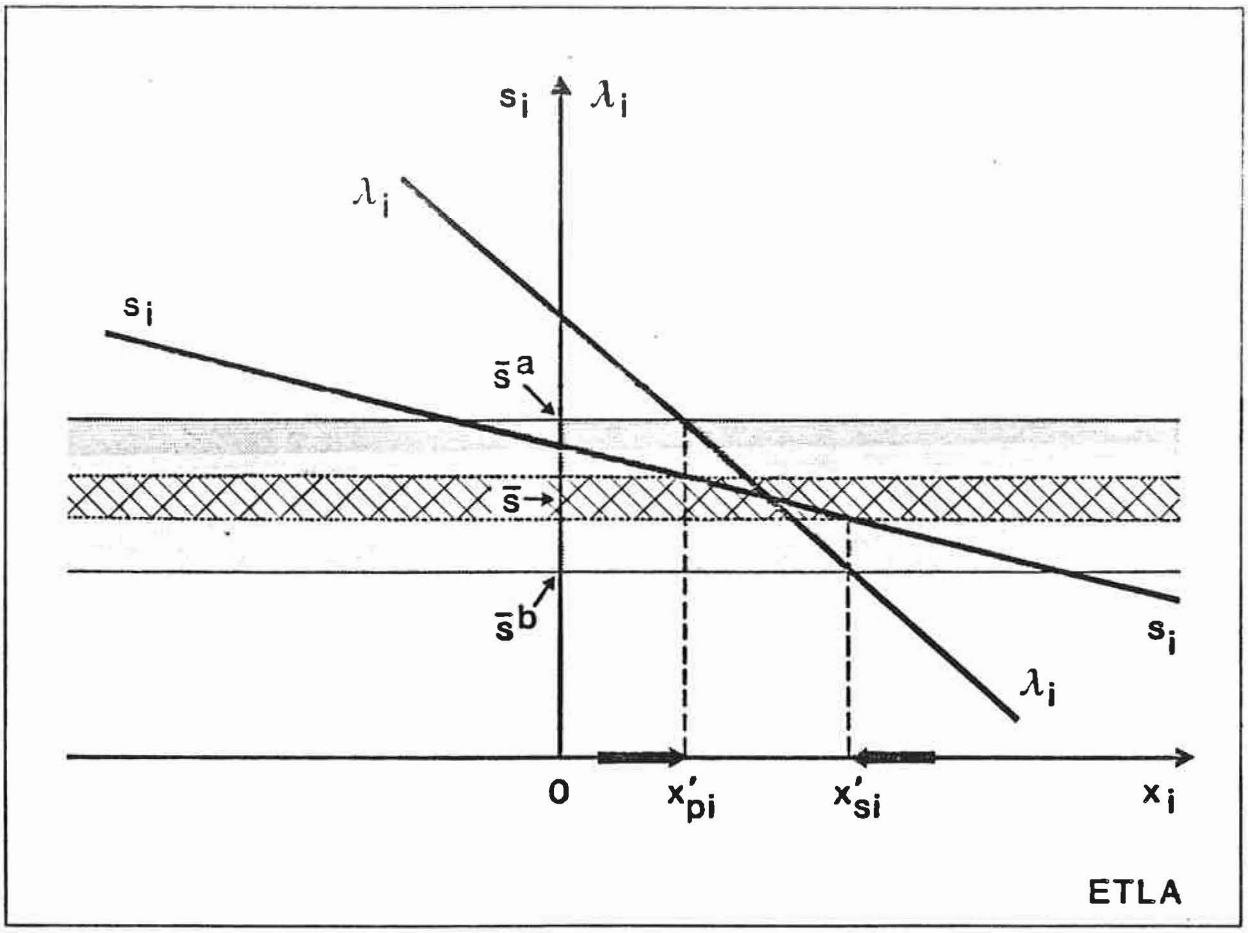
10) Comparing equations (15) and (25) shows that $\hat{\lambda}_i = \lambda_i + (\theta_i/\delta)(s_i - \bar{s})$ and $\partial\hat{\lambda}_i/\partial x_i^1 = -(2 + \theta_i)/\delta < \partial\lambda_i/\partial x_i = -2/\delta$.

11) That is $s_i = (\alpha_i/\delta) + (2\varepsilon/\delta)\bar{s}$, which gives $p_i = q_i$.

Figure 5. *Illustration of the effects of inconsistency risk*



Explanations: A wholesale transaction is profitable when λλ-line is outside the shaded area, cf. Figure 1. The open position is allowed to fluctuate in the range $[x'_{pi}, x'_{si}]$, which limits the dealer's mid-rate to a narrow range around the average mid-rate (lined area).

What has been said above is based on a partial equilibrium analysis of the behaviour of an individual dealer who takes the distribution of outside quotations as given. Assuming that all other dealers behave accordingly would reduce the dispersion of quotations, which fact in itself would reduce the risk of inconsistent quotations. Moreover, while each dealer may lose by quoting inconsistently, he can equally well find himself in a situation where he observes inconsistent prices. Knowing this possibility would give each dealer an incentive to actively seek for arbitrage profit opportunities by continuosly asking quotations from others, even without any immediate need to make any wholesale transaction for the position adjustment purposes. As a result the market participants would be better informed, which would increase their sensitivity to small quotation differentials and thus intensify competition and integration and reduce the average spread. Hence the general equilibrium consequences, as regards the effects of the inconsistency risk on the average spread, are not unambiguously positive. Nevertheless, it would appear that there is no mechanism that would eliminate the inconsistency risk completely, as long as the dealers are basically price setters and quote independently with incomplete information on how others are quoting at the same moment.

6. Concluding remarks

Above we have shown how the integration of the foreign exchange markets is brought about by the profit maximizing behaviour of individual foreign exchange dealers. Ultimately, the integration, which manifests itself in a reduced dispersion of quotations between various local markets, results from inter-dealer transactions, and it is further intensified by the increased sensitivity of customers, dealer and nondealer customers alike, to small quotation differentials. The latter also improves the efficiency of the market in the sense that the average spread is reduced. The possibility of arbitrage opportunities to arise occasionally draws local quotations even closer to each other, while at the same time it may broaden the average spread because the dealers try to avoid costs brought about by inconsistent quotations.

Although the role of uncertainty was explicitly taken into account only in connection with the inconsistency risk, it should be kept in mind that the whole analysis has in a more fundamental way been based on uncertainty and ways of reducing costs related to it. First, the *raison d'être* of the dealership market follows from transaction uncertainty with dealers standing ready to buy and sell on immediate demand. There is very little the dealers can do about this source of uncertainty except using the inter-dealer market in order to adjust excessively open positions. Secondly, a rather strict element of risk aversion is involved in the closed position requirement at the end of the day. A closed position will leave the dealer safe against any changes in the average quotation, which may occur overnight, while the optimal pricing rule and wholesale transactions keep the open position during the day within certain relatively narrow limits.

Of course, there may be 'news', economic or other, which change the market participants' views about the average quotation in the immediate future. These affect the dealers' behaviour in the manner which tends to justify their beliefs in advance. Those dealers, who believe in a certain change, say an appreciation of the dollar, raise their dollar quotations in order to attract net purchases from customers, or they buy dollars in the inter-dealer market thus forcing others to raise the price of a dollar in order to restore their positions. This means that even those, who do not know the 'news' or do not believe in them, are forced to follow the suit in order to avoid excessively short positions. Formally, the 'news' can be incorporated in the preceding analysis by letting the expected average mid-rate $\bar{s}$ to change, or by adjusting the dealer's end-of-day position target to be different from zero. Otherwise the formal analysis would remain unchanged.

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