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Keskusteluaiheita
Discussion papers

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OPTIMAL MONETARY POLICY AND RATIONAL EXPECTATIONS: A CASE OF RATIONED CREDIT MARKETS***

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1. INTRODUCTION

The subject of the paper is optimal monetary policy in a small open economy. We present a crude model which tries to capture some features typical of the Finnish economy, such as rationed credit markets with institutionally fixed interest rates on loans, regulated capital movements and a fixed effective exchange rate. Under these conditions the central bank does have an opportunity to exercise independent monetary policy, at least in the short run.

Our main interest lies in the formulation of optimal monetary policy in the presence of rational expectations within the private banking sector. Special interesting questions arising in this context - apart from the crucial issue of the effectiveness vs. neutrality of monetary policy - are the time-consistency of the optimal policy and the credibility of the central bank. By means of simulation techniques we present a case which shows radical differences in the course and performance of optimal policies according to whether the policy is time-consistent or not, and whether the central bank behaviour is credible or not.

The plan of the paper is as follows. Firstly, the behaviour of the credit stock and the new admitted advances is derived. The credit stock is predetermined at each point in time and the new credits is the instrument of the banks. The banks form expectations about the future monetary policy, which makes the new credits a forward-looking variable. The credit market is assumed to be an oligopolistic one, which is described by a model in which each bank maximizes its intertemporal utility function with profits and the market share as arguments.

Secondly, the behaviour of the credit market is implemented in a simple model of a small open economy. The existence of the credit rationing implies that the money market conditions are supply-side determined. Thirdly, the optimal monetary policy of the central bank will be considered. This is analyzed using the methods found e.g. in Miller and Salmon (1982) and Driffill (1982). The problem is formulated in a game theoretic setting, with the central bank acting as a Stackelberg leader and the private banking sector as the follower. The intertemporal objective function of the central bank is assumed to include the output gap and current account. The effects of time-inconsistent optimal policy rules are compared with those of a time-consistent and an unpredictable policy.

The special short run policy problem treated here is a case where the domestic price level has - for some reason or another - risen above the foreign price level resulting in a temporary loss in competitiveness, a current account deficit and unemployment.

The analysis is carried out in a deterministic world, although we implicitely assume throughout that stochastic terms are in fact present. The justification for this procedure lies in the certainty equivalence principle, as our world is Linear Quadratic Gaussian (see Simon, 1956).

2. THE BEHAVIOUR OF THE PRIVATE BANKING SECTOR

The basic assumption conserning the money market is that the lending rate is institutionally fixed and kept at a "low" level. This implies that there is a continuous excess demand for credits as the public is willing to acquire more credits than the banks are ready to admit at the going lending rate and the marginal cost of financing new credits. To simplify the analysis the possible demand side effects on the credit expansion are neglected. Thus the credit expansion is here determined solely by the optimizing decisions of the banking sector.

The behaviour of the private banking sector is analyzed by considering a representative bank. The bank minimizes an intertemporal loss function C(t):

$$(1) \qquad C(t) = \int_0^\infty \{\frac{a}{2}[L(t) - L^*]^2 + \frac{b}{2}[N(t) - k\delta L^*]^2\} e^{-rt} dt$$

$$\text{s.t.} \ \frac{dL}{dt} = N(t) - \delta L(t)$$

where $L(t)$ is the actual credit stock at time t,

L^* is the expected profit maximizing credit stock,

$N(t)$ is the actual amount of the new admitted credits,

$k\delta L^*$ is the amount of the new admitted credits corresponding to a target market share, $k \geq 1$

r is the bank's discount factor,

δ is the fixed amortization rate corresponding to the average maturity of the outstanding credit stock,

a and b are the respective positive weights attached to the profit and to the steady state new credits, which are greater than the new credits corresponding to the profit maximizing credit stock if $k > 1$.

The representative bank has two objectives at each point in time. On the one hand it maximizes its profits given the conditions of the central bank debt. The expected profit maximizing credit stock L^*, which is determined by the current and expected values of the central bank's instruments, is derived from a static maximization problem, and it is exogenous to the bank.

On the other hand it is widely recognized (cf. Creuzberg, 1982) that Finnish banks consider it important also to obtain as high a market share of deposits as possible. It is furthermore assumed that deposits are a monotonically increasing function of the bank's credits. This feature is taken into account by the target amount for the new credits in the long run $k\delta L^*$, $k \geq 1$.

To solve the optimization problem we formulate a general control problem, which is solved by using Pontryagin's maximum principle[1]:

$$(2) \qquad \min_N C = \int_0^\infty \{\frac{a}{2}(L - L^*)^2 + \frac{b}{2}(N - k\delta L^*)^2\} e^{-rt} dt$$

$$(3) \qquad \frac{dL}{dt} = \dot{L} = N - \delta L$$

$$(4) \qquad L(0) = L_0, \quad N \geq 0 \text{ and bounded for all } t.$$

The corresponding current value Hamiltonian function is (see e.g. Kamien and Schwartz, 1981, pp. 151-3):

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1) The argument t, referring to time, is dropped out to simplify the notation.

(5) $$H = \frac{a}{2}(L - L^*)^2 + \frac{b}{2}(N - k\delta L^*)^2 + \tilde{\mu}(N - \delta L) ,$$

where $\tilde{\mu}$ is the current value costate variable. Assuming an interior solution the necessary conditions for optimum are:

(6) $$\frac{\partial H}{\partial N} = b(N - k\delta L^*) + \tilde{\mu} = 0$$

(7) $$-\frac{\partial H}{\partial L} = - a\,(L - L^*) + \tilde{\mu}\delta = \dot{\tilde{\mu}} - \tilde{\mu}r$$

(8) $$\frac{\partial H}{\partial \mu} = N - \delta L = \dot{L}$$

Instead of solving the optimal closed loop control for N as a function of L (cf. Intriligator, 1971, ch. 14) we differentiate equation (6) with respect to time:

(6') $$b\dot{N} + \dot{\tilde{\mu}} = 0$$

Using (6') and equations (6)-(8) we can write the system under optimal control in a state space representation with L and N as state variables:

(9) $$\begin{bmatrix} \dot{L} \\ \dot{N}^{1)} \end{bmatrix} = \begin{bmatrix} -\delta & 1 \\ a/b & \delta+r \end{bmatrix} \begin{bmatrix} L \\ N \end{bmatrix} + \begin{bmatrix} 0 \\ -[a/b+k\delta(\delta+r)] \end{bmatrix} L^* = U \begin{bmatrix} L \\ N \end{bmatrix} + VL^*$$

Because L is a stock variable, it is predetermined and its time path is governed by N. On the other hand the bank is continuously free to choose the amount of the new credits so that its losses are minimized. This means that N is a forward-looking variable: it can make discrete jumps in response to the news concerning the expected future values of the forcing variable L*.

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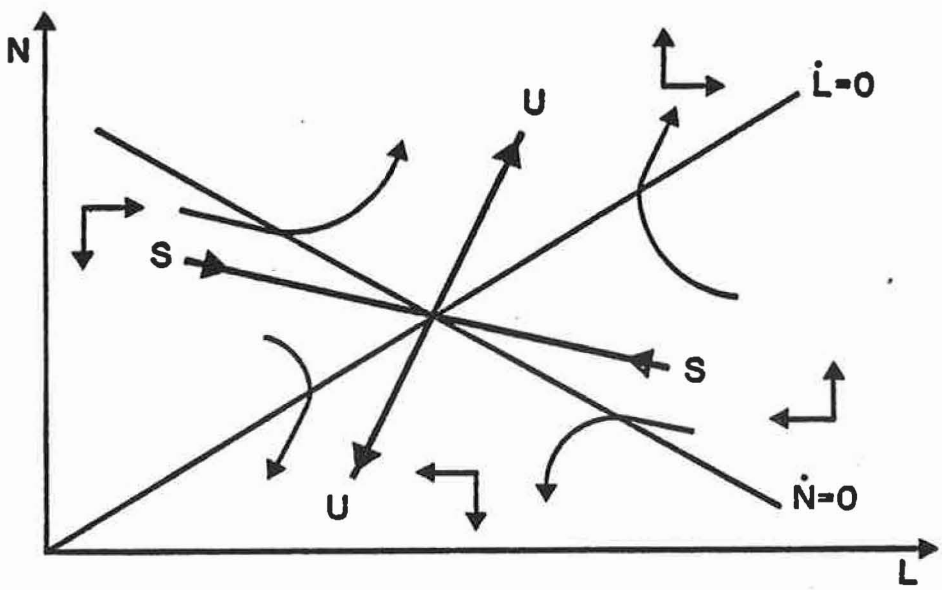
1) See footnote on page 11.

The determinant of the matrix U is $-\delta(\delta+r) - a/b$, which is negative. This together with the implicit transversality condition guarantees the existence of a unique convergent saddle path solution to (9). In what follows it is assumed that the bank has a perfect foresight about the current and future values of L* and it chooses N so that the solution is restricted to the stable manifold. For the method for solving linear differential equation systems with forward-looking variables see e.g. Buiter, 1982.

The dynamic behaviour of the system is illustrated in figure 1.

Figure 1.



The locus of the stationary values of L is obtained by setting $\dot{L} = 0$ in (9):

$$(11) \qquad N \Big|_{\dot{L} = 0} = \delta L .$$

Correspondingly we get

$$(12)\qquad N\Big|_{\dot{N}=0} = -\frac{a/b}{\delta+r}L + \frac{a/b+k(\delta+r)}{\delta+r}L^*$$

The arrows in figure 1 show the directions of movement in L-N -space. The unique convergent saddle path SS is downward sloping. The unstable root defines the unstable manifold UU.

When the solution is restricted to the stable manifold, the dynamics of the system can be described as a function of the stable eigenvalues and their associated eigenvectors. This observation will be utilized later when we consider the case with no forward looking behaviour in the banking sector.

By setting $\dot{L}$ and $\dot{N}$ equal to zero in (9) we obtain the long run values for L and N, denoted by $\hat{L}$ and $\hat{N}$:

$$(13)\qquad \begin{bmatrix}\hat{L}\\ \hat{N}\end{bmatrix} = [a/b+\delta(\delta+r)]^{-1}\begin{bmatrix}a/b+k\delta(\delta+r)\\ \delta a/b+k\delta^2(\delta+r)\end{bmatrix}.L^*$$

It can be seen that $\hat{N} = \delta\hat{L}$, which implies that the long run values of L and N lie on the $\dot{L} = 0$ locus in figure 1.

Some obvious results are readily seen:

$$(14)\qquad \lim_{k\to 1}\hat{L} = L^* \qquad a,b \geq 0$$

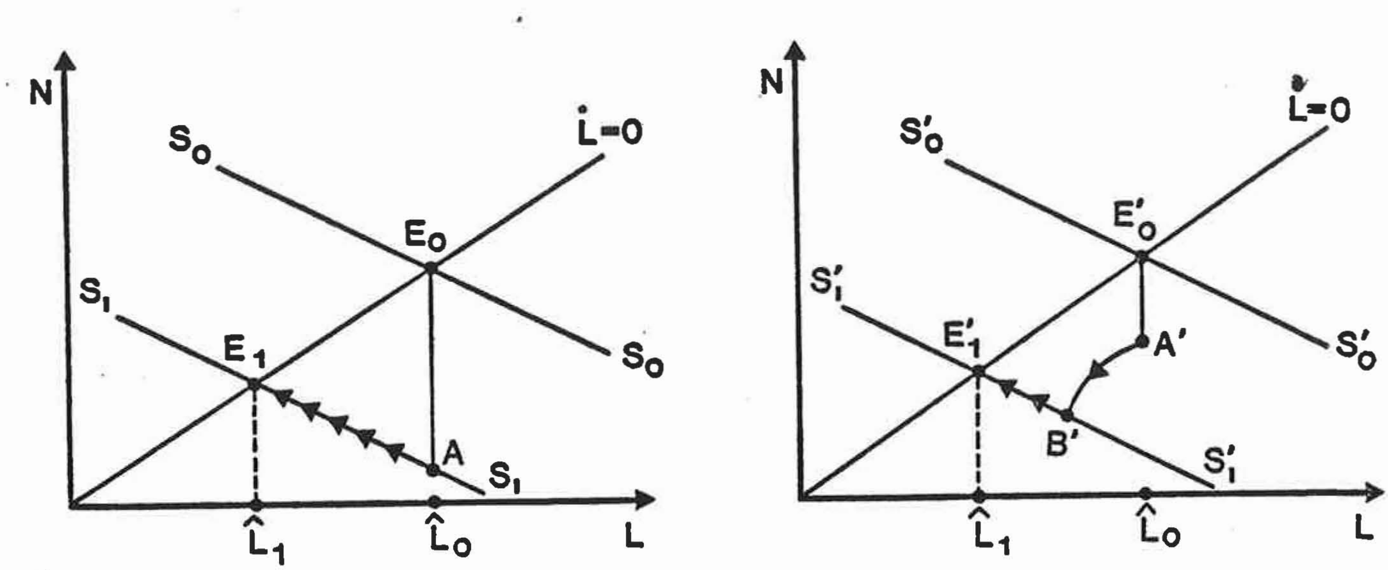
$$(15)\qquad \lim_{a\to 0}\hat{L} = kL^* \qquad b>0,\ k\geq 1$$

$$(16)\qquad \lim_{b\to 0}\hat{L} = L^* \qquad a>0,\ k\geq 1$$

As the parameter k, which measures the desire to have a greater credit stock than the profit maximizing one L* in the steady state, approaches to unity, the resulting long run credit stock converges to L*. This is also the case when the weight attached to the "steady state market share" (b) tends to zero. At the other polar case when no weight is given to the profits ($a \to 0$) the steady state credit stock equals the given market share target kL*. It may also be noted that when only the profits are given weight in the loss function ($b \to 0$) and when N is bounded the optimal control is a bang-bang solution.

Figures 2a and 2b illustrate the the effects of a discrete tightening of the monetary policy. Two cases are considered: the change in the L* is unanticipated (2a) or anticipated (2b).

Figures 2



2a: An unanticipated reduction of L*

2b: An anticipated reduction of L*

In both cases the steady state point is shifted from the initial equilibrium E_0 to E_1, corresponding to a reduction in the long run value of $\hat{L}$ from $\hat{L}_0$ to $\hat{L}_1$. If the change in the monetary policy is unanticipated, the amount of the new credits, N, is immediately reduced and the system jumps from E_0 directly to the new stable manifold S_1S_1 at point A, and thereafter converges to the new steady state point E_1.

If the policy change is anticipated N jumps to a point like A' in figure 2b. To ensure the stability of the system, this initial jump must be such that the system will reach a point like B' on the new stable manifold $S_1'S_1'$ at the time of the implementation of the new tighter policy.

It can be seen that in both cases N "overshoots" in the sense that, during the adjustment process towards the new optimal credit stock, N will reach a lower level than its new steady state level will be. If the news is unanticipated, the initial jump in N will always be greater than the change in the long run N. In the anticipated case the size of the initial jump will depend negatively on the time period between the announcement day and the implementation day of the new policy.[1)]

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1) Qualitatively the same dynamic behaviour as in (13) will result from a profit maximizing behaviour with quadratic adjustment costs on the credit expansion, except that the stable manifold may be upward - sloping and the long run credit stock is - naturally - the profit maximizing one.

3. OPTIMAL MONETARY POLICY

3.1. The model

We are considering a small open economy with regulated money market and a fixed effective exchange rate. The country faces an infinitely elastic demand for its exports at any given level of relative prices. We have assumed the foreign price level to be constant, as is the fixed effective exchange rate which is scaled to be equal to one. The domestic price level can thus be interpreted as a measure of discompetitiveness and a rise in it affects negatively the demand for the domestic production.

As the domestic lending rate is assumed to be fixed all the time, the relevant interest rate variable in the goods market equilibrium equation is the call money rate r_c. It is assumed that the firms cover themselves from individual currency valuation changes when using foreign loans, and that the covered foreign interest rate moves in line with r_c. It follows that a rise in r_c will reduce investments.[1)]

Because of the rationing of the credit market and capital flows the effective domestic demand in the goods market is the notional demand corrected by the spillover effects of the unsatisfied demand due to the credit rationing (see Alho (1982), Ito (1980) and Willman (1981)). This

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1) Underlying the direct adverse effect of the rise in the call money rate r_c on investment activity are the following stylized facts typical of the Finnish economy. As the capital movements, especially the long run ones, are strongly regulated by the Bank of Finland, only the firms have an access to the foreign credit markets. In the absence of speculators the forward exchange rates do not reflect expected movements of any single exchange rate. In practice the banks set the covered foreign interest rate very close to the marginal cost of the central bank debt (see Suvanto (1983)).

allows us to enter the new admitted credits into the IS-equation with a positive sign.

Finally the inclusion of the inflation in the IS-equation captures the effects of the real interest rate on the notional demand and also on the demand for the foreign credits.

The price level is assumed to be a sticky variable and inflation is determined by an augmented Phillips curve, with output gap and the credit expansion, which stands for the core rate of inflation, as arguments.

Credit market conditions are determined by the optimization procedure of the banking sector, outlined in the previous chapter. The banks view the expected profit maximizing credit stock as a function of the central bank's instrument, r_c, only. The banking sector's responses to the current and anticipated future monetary policy are modelled in a perfect foresight equivalency of rational expectations. We have used here a slightly different loss function when deriving the banks' behaviour, namely we have exogenized the target for the new credits to N* and assumed that the discount factor r is equal to zero. These changes do not, however, affect the qualitative results we obtained in chapter one.

The model is in the structural form:

$$(17) \qquad \dot{L} = N - \delta L$$

$$(18) \qquad \dot{P} = \emptyset(Y - \bar{Y}) + \dot{L}$$

$$(19) \qquad \dot{N} = aL + \delta N - aL^* - \delta N^*$$ [1]

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1) Here $\dot{N} = \lim_{\substack{h \to 0 \\ h > 0}} \{ \frac{E[N(t+h)|\Omega(t)] - E[N(t)|\Omega(t)]}{h} \}$, and the information set $\Omega(t)$ includes all current and past values of endogenous and exogenous variables, the model structure and current anticipations of future exogenous variables.

(20) $Y = \beta + \eta\dot{P} - \xi r_c + \varepsilon N - \sigma(P - \bar{P})$

(21) $L^* = \alpha - \gamma r_c$

where N is new credits

L is the outstanding credit stock

$\dot{N}$ and $\dot{L}$ are their respective time derivatives

$\dot{P}$ is the rate of inflation

Y is output

$\bar{Y}$ is the exogenously given full employment level of output

P is the price level

$\bar{P}$ is the fixed foreign price level

N^* is the exogenously given target for new credits

L^* is the profit maximizing credit stock

r_c is the call money rate

$a, \alpha, \beta, \emptyset, \delta, \eta, \xi, \varepsilon, \sigma, \gamma$ are positive parameters

The weights in the banks' loss function are scaled so that a is the relative weight given to the profits by the banks.

All the variables in the model are expressed in levels. Because of this the Phillips curve equation (18) cannot, in our linear formulation, be expressed in relative changes. Instead we make use of the assumption that P and L are scaled to be initially equal. This guarantees the approximate equivalence of relative and absolute changes in our formulation.

The trend or the core rate of inflation stands for the factors that give inertia to trends in prices (see Buiter and Miller, 1982), and it is assumed to be equal to the credit expansion. Equation (20) is the goods market equilibrium condition, and equation (21) determines the profit maximizing credit stock. The constant β in equation (20) captures the effects of exogenous demand shocks, including the government fiscal policy. The dynamics of the model is determined by equations (17)-(19). The model is neutral in the long run as $\hat{Y} = \bar{Y}$, but the domestic price level and credit stock are determined endogenously also in the long run.

The model can be expressed in a state space representation as:

$$(22)\quad \begin{bmatrix} \dot{P} \\ \dot{L} \\ \dot{N} \end{bmatrix} = \begin{bmatrix} -(1-\emptyset\eta)^{-1}\emptyset\sigma & -(1-\emptyset\eta)^{-1}\delta & (1-\emptyset\eta)^{-1}(\emptyset\varepsilon+1) \\ 0 & -\delta & 1 \\ 0 & a & \delta \end{bmatrix} \begin{bmatrix} P \\ L \\ N \end{bmatrix} +$$

$$\begin{bmatrix} -(1-\emptyset\eta)^{-1}\emptyset\xi & 0 & 0 & -(1-\emptyset\eta)^{-1}\emptyset & (1-\emptyset\eta)^{-1}\emptyset & (1-\emptyset\eta)^{-1}\emptyset\delta \\ 0 & 0 & 0 & 0 & 0 & 0 \\ a\gamma & -a & -\delta & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_c \\ \alpha \\ N^* \\ \bar{Y} \\ \beta \\ \bar{P} \end{bmatrix}$$

or $\dot{x} = Ax + Bz$.

The determinant of the matrix A is $(1-\emptyset\eta)^{-1}\emptyset\sigma(a+\delta^2)$ and its trace is $-(1-\emptyset\eta)^{-1}\emptyset\sigma$. As we have assumed that all the parameters are positive, the trace and the determinant are necessarily of opposite sign. Assuming that $\emptyset\eta < 1$ there are two stable roots and one unstable root[1], and there exists a unique convergent saddle path solution. As the model was constructed mainly for illustrative purposes, we skip further analysis conserning the model (see closer Mustonen & Salonen, 1983).

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1) Recall that the determinant is the product and the trace is the sum

3.2. Derivation of optimal monetary policy

The objectives of the central bank are assumed to be a full employment level of output and balanced current account, which we simply assume to be a linear function of the difference between the domestic and foreign price levels. In our model these goals can always be obtained in the long run, so the problem of optimal policy is a short run one.

The nature of the optimization problem presented here has been described aptly in Buiter (1981, p. 664): "Traditional dynamic programming techniques do not allow for the impact of future policy measures on the current state through changes in current and past behaviour induced by anticipation of these future policy measures. This does not matter for causal or backward-looking models... It matters greatly in non-causal or forward-looking models... in which, in the structural model, the current state depends on the anticipated future state(s) as well as, possibly, on the past state and the current values of forcing variables."

In the framework of this paper the central bank can be viewed as a Stackelberg leader who knows the structure and dynamics of the economy and its reactions to changes in the monetary policy. The central bank can select the optimal monetary policy rule so that it allows for the banking sector's response to the choice of instruments. Because we are dealing with a linear-quadratic problem with additive white noise in the state variables, the certainty equivalence principle holds. This results in a closed loop rule for the instrument (see Buiter, 1981). The other "player", the banking sector, acts as a Stackelberg follower in the sense that it optimizes its intertemporal objective function given the current and expected future values of the central bank's instrument, the call money rate r_c.

The objective function of the central bank is defined as

$$(23) \qquad V(t) = \frac{1}{2}\int_0^\infty \{uC^2 + v(Y-\bar{Y})^2\}\, e^{-\rho t}\, dt$$

$$= \frac{1}{2}\int_0^\infty [C\ Y\ \bar{Y}] \begin{bmatrix} u & 0 & 0 \\ 0 & v & -v \\ 0 & -v & v \end{bmatrix} \begin{bmatrix} C \\ Y \\ \bar{Y} \end{bmatrix} e^{-\rho t}\, dt$$

$$= \frac{1}{2}\int_0^\infty \tilde{x}'\tilde{Q}\tilde{x}\, e^{-\rho t}\, dt ,$$

where $C = k(P - \bar{P})$ can be interpreted as a measure of loss of competitiveness, or - as in the present context - the current account, ρ is the discount factor[1)] and u and v are the positive weights attached to current account and output gap, respectively, in the loss function (23) and ' denotes a transpose. The variables of the objective function $\tilde{x}' = (C, Y, \bar{Y})'$ can be solved from equations (17)-(21) as a linear function of the vector of the state variables $x' = (P,L,N)$, the control variable r_c, and the vector of the other exogenous variables $\tilde{z}' = (\alpha, N^*, \bar{Y}, \beta, \bar{P})'$:

$$(24) \qquad \tilde{x} = G \begin{bmatrix} x \\ r_c \\ \tilde{z} \end{bmatrix}$$

The integrand in (23) (excluding the discount factor) can be written as

1) It is implicitely assumed that the central bank has a more myopic objective function than the banking sector. This is reflected by the omission of a discounting factor in the banks' behavioural equations.

(25) $$[x', r_c, \tilde{z}'] G' \tilde{Q} G \begin{bmatrix} x \\ r_c \\ \tilde{z} \end{bmatrix}$$

$$= [x', r_c, \tilde{z}'] \underbrace{\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{bmatrix}}_{\Gamma} \begin{bmatrix} x \\ r_c \\ \tilde{z} \end{bmatrix},$$

where the matrix $\Gamma = G' \tilde{Q} G$ is partioned conformably with the vector $[x', r_c, \tilde{z}']'$.

The central bank's optimization problem is now

(26) $$\min_{r_c} \int_0^\infty [x' \; r_c \; \tilde{z}'] \Gamma \begin{bmatrix} x \\ r_c \\ \tilde{z} \end{bmatrix} e^{-\rho t} dt$$

(27) $$\dot{x} = Ax + Bz = Ax + br_c + \tilde{B} \tilde{z}$$

(28) $L(0) = L_0$, $P(0) = P_0$,

$N \geq 0$ and bounded for all t

$r_c \geq 0$

The optimization problem is solved by using Pontryagin's maximum principle. The relevant current value Hamiltonian is:

$$(29)\qquad H = \frac{1}{2}\{x'\Gamma_{11}x + r_c\Gamma_{21}x + \tilde{z}'\Gamma_{31}x + x'\Gamma_{12}r_c + r_c\Gamma_{22}r_c$$

$$+ \tilde{z}'\Gamma_{32}r_c + x'\Gamma_{13}\tilde{z}' + r_c\Gamma_{23}\tilde{z} + \tilde{z}'\Gamma_{33}\tilde{z}\} + \tilde{\mu}'(Ax + br_c + \tilde{B}\tilde{z})$$

where $\tilde{\mu}$ is the vector of the current value costate variables or the current value shadow prices (i.e. $\tilde{\mu} = e^{\rho t}\mu$, where μ is the vector of the costate variables corresponding to the original Hamiltonian).

Assuming an interior solution the first order conditions for optimum are

$$(30)\qquad \frac{\partial H}{\partial r_c} = x'\Gamma_{12} + \Gamma_{22}r_c + \tilde{z}'\Gamma_{32} + \tilde{\mu}'b = 0$$

$$(31)\qquad -\frac{\partial H}{\partial x} = \dot{\tilde{\mu}} \Rightarrow \dot{\tilde{\mu}} = -\Gamma_{11}x - \Gamma_{12}r_c - \Gamma_{13}\tilde{z} - (A' - \rho I)\tilde{\mu}$$

$$(32)\qquad \frac{\partial H}{\partial \tilde{\mu}} = \dot{x} = Ax + br_c + \tilde{B}\,\tilde{z}$$

The system under optimal control can be written as

$$(33)\qquad \begin{bmatrix} \dot{P} \\ \dot{L} \\ \dot{N} \\ \hline \dot{\tilde{\mu}}_P \\ \dot{\tilde{\mu}}_L \\ \dot{\tilde{\mu}}_N \\ \hline 0 \end{bmatrix} = \begin{bmatrix} A & b & 0 \\ -\Gamma_{11} & -\Gamma_{12} & -(A'-\rho I) \\ \Gamma_{21} & \Gamma_{22} & b' \end{bmatrix} \begin{bmatrix} P \\ L \\ N \\ \hline r_c \\ \hline \tilde{\mu}_P \\ \tilde{\mu}_L \\ \tilde{\mu}_N \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ -\Gamma_{13} \\ \Gamma_{23} \end{bmatrix} \begin{bmatrix} \alpha \\ N^* \\ \bar{Y} \\ \beta \\ \bar{P} \end{bmatrix}$$

The boundary conditions for the solution are:

(34) $P(0) = P_0$

(35) $L(0) = L_0$

(36) $\tilde{\mu}_N(0) = 0$

(37) $\lim_{t\to\infty} N(t) = \delta L(t)$

(38) $\lim_{t\to\infty} \tilde{\mu}_P(t) = 0$

(39) $\lim_{t\to\infty} \tilde{\mu}_L(t) = 0$

As the problem was formulated with the current value shadow prices $\tilde{\mu}$ the stability of the the system requires that $\rho > \left(\frac{\dot{\mu}_i(t)}{\mu_i(t)}\right)$ as $t \to \infty, i = L,P$ This guarantees that $\tilde{\mu}_i$ tends to zero as t goes to infinity.

The fact that N is a jump variable requires that the corresponding costate variable $\tilde{\mu}_N$ is a predetermined variable. As the costate variable measures the contribution of a marginal change in the value of the corresponding state variable to the optimal value of the objective function, $\tilde{\mu}_N$ is required to be equal to zero in equation (36) at the starting point $t = 0$ (see Driffil, 1982, p. 8). As $\tilde{\mu}_N$ is a predetermined variable it can, however, deviate from its initial value zero when the system under optimal control proceeds from its initial state.

The solution to the system (33) is obtained by restricting it to the stable manifold and the optimal closed-loop control for r_c is obtained from (31).

4. THE EFFECTS OF ALTERNATIVE MONETARY POLICY RULES

As the analysis of a linear system of differential equations of sixth order is obviously extremely complicated and awkward, we consider the effects of different kinds of policies of the central bank by simulation techniques. The simulations are carried out by using a programme for solving continuous time linear rational expectations models (Austin and Buiter, 1982). In our simulations we have used the following values for the structural form parameters and the exogenous variables:

$\delta = 0.10$	$a = 0.20$	$\emptyset = 0.20$	$\sigma = 0.75$
$\varepsilon = 0.50$	$k = 0.33$	$\eta = 0.35$	$\xi = 0.50$
$N^* = 20$	$\bar{Y} = 100$	$\beta = 101.5$	$\bar{P} = 100$
$r_c = 13$	$u = 1$	$v = 1$	$\rho = 0.02$
$\gamma = 5$	$\alpha = 160$		

The resulting long run values for the state and output variables are:

$P = L = Y = 100$

$N = 10$

$L^* = 95$

$C = Y - \bar{Y} = 0$

The optimization problem of the central bank is the following. The domestic price level has - for some reason - risen above the foreign price level to 103. This results in a temporary current account deficit ($C = -0.5$) and a negative output gap ($Y - \bar{Y} = -2.4$).

The first case considered is when the central bank follows the optimal policy rule derived above. If this rule is announced and the central bank sticks to it, the banking sector is able to acquire a "genuine" perfect foresight, in the absence of exogenous shocks, of the future values of the central bank's instrument, r_c. Because the sluggish costate variable $\tilde{\mu}_N$ will deviate from zero as the system proceeds from its initial state, this policy is called time-inconsistent (no-cheating) optimal policy. This policy rule requires an immediate step reduction in the call money rate followed by an increase above the initial level. After that the call money rate is brought gradually back to its initial level. At the time the policy is implemented the central bank has one extra instrument at hand: the surprise element of its actions (see figure 3).

The central bank will, however, have an incentive to reoptimize as $\tilde{\mu}_N$ deviates from zero. If this reoptimization is done continuously $\tilde{\mu}_N$ can be set equal to zero all the time. This kind of policy is called time consistent optimal policy. As the announced policy and the actual policy will then differ from each other, the banking sector cannot trust the announcements. If the central bank pursues the time consistent optimal policy systematically, the banking sector will eventually learn the central bank's rule and form rational expectations about the future values of r_c.[1)] The central bank doesn't in fact take properly into account the endogeneity of expectations. The resulting call money rate path is quite the opposite as in the previous case: r_c will have to be gradually lowered at first

1) Should this not be the case the central bank could actually use two instruments: the money rate and banks' expectations. This policy is called perfect cheating (see Hämäläinen, 1981).

and after a while brought back to the inivial level, possibly with an oscillating behaviour. The central bank tries to cheat in vain; by assumption of rational expectations it will not succeed.

We also examine a case when the time consistent policy is successful in the sense that the banks do believe in the policy announcements and do not learn the true policy rule. This successful cheating policy would, no doubt, be the best policy considered here. As noted in the time-consistent policy case, perfect cheating will most probably not succeed. On the other hand it might be possible for the central bank to reoptimize from time to time, provided that it can somehow preserve its credibility and assure the banks that every change in the policy rule will be the final one. In our simulations we have assumed that the central bank reoptimizes at periods 1,2,...,10 and sticks to the last announced policy thereafter. In our example this requires a volatile behaviour for the r_c and new credits, but the variability of the credit stock is reduced and the resulting loss is the smallest of the cases considered here.

Finally we consider the case when the central bank drives an unsystematic or unpredictable policy. In this case the banking sector does not have the relevant information needed to form rational expectations about the future values of r_c. Because there are now no news concerning the future, in the point of view of the banks, it is only the current value of r_c that affects the optimisation behaviour of the banking sector. Effectively this means that the banks' instrument N jumps always straight to the stable manifold (see picture 2a) and that the forward-looking equation for new credits (19) must be dropped out and substituted by an equation where the new credits depend only on the current variables N, L* and L (see page 7):

$$(41) \qquad N = [\delta - (\delta^2 + a)^{\frac{1}{2}}] L + (\delta^2 + a)^{-\frac{1}{2}} \delta N^* + a(\delta^2 + a)^{\frac{1}{2}} L^*$$

In our simulation the time path of the call money rate is the same as in the successful cheating case up to period 10, after which it is set equal to its initial value. The myopic behaviour of the banks results in a very wild jumps in new credits. This policy comparison is naturally a bit clumsy, but it serves to illustrate the importance of the continuity and credibility of the central bank policy as viewed by the banks.

In figure 3 are the time paths for the call money rate under different policies and in figure 4 the cumulative losses are compared. We have also presented the loss resulting from a passive policy, i.e. when the dynamic forces of the system are allowed to bring the system back to the steady state without any policy actions.

Figure 3. The time paths of the call money rate under different policies.

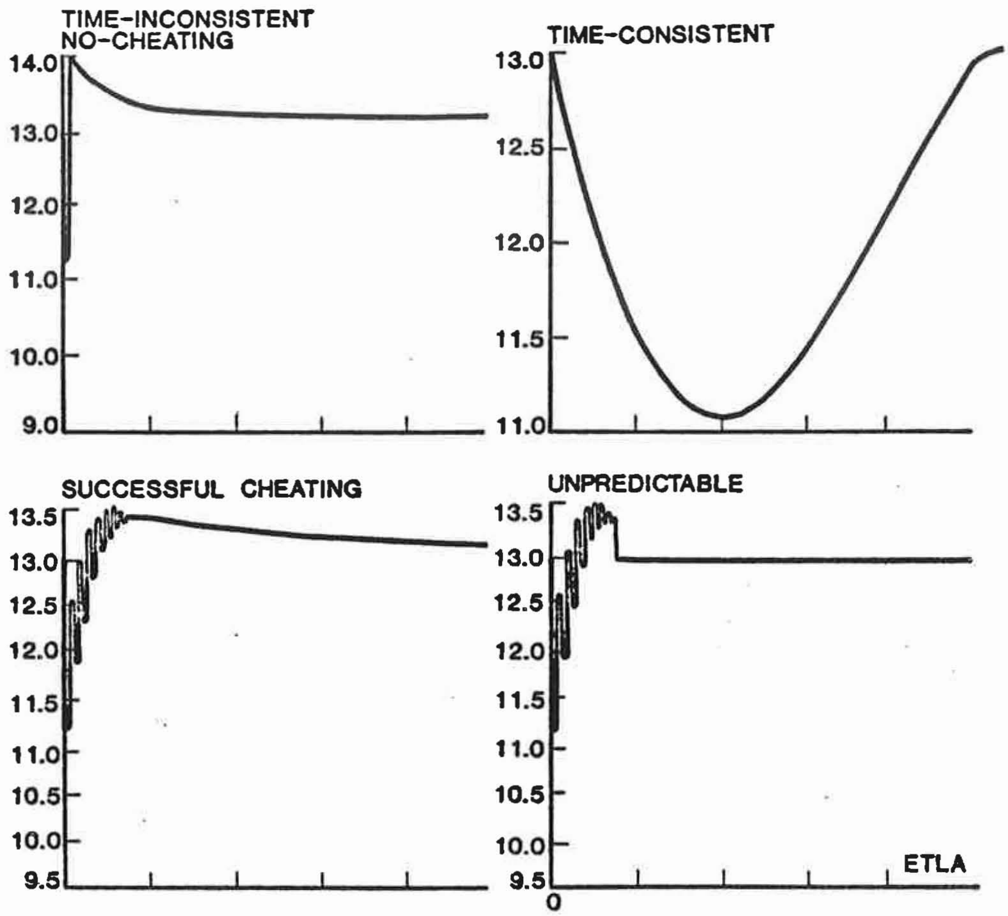
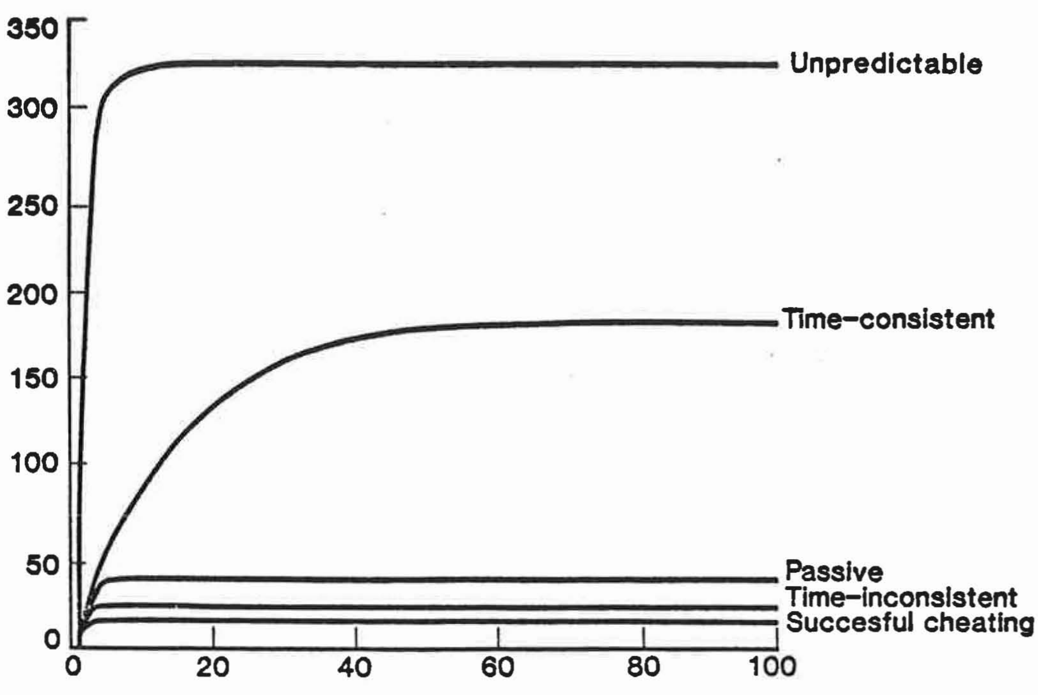


Figure 4. The cumulative losses of different policies.



5. CONCLUDING REMARKS

We have considered a model in which the rational expectations hypothesis is assumed to be valid, not in the goods and labor market as the proponents of the New Classical School would claim, but only in the money market. As the money market is rationed in this model, the optimisation decisions of the private banks determine the credit stock and credit expansion according to the current and future values of the central banks instruments, which in turn determine the profit maximizing credit stock. The model is neutral in the long run, so the optimizing problem of the central bank is a short run one. We do not claim that the model itself should have much relevance as a description of the true world; its purpose is to highlight some crucial aspects of the policy rule selection.

We treat the central bank as a Stackelberg leader and the banks as a follower - perhaps not a very unrealistic assumption. The behaviour of the banks and the optimisation procedure of the central bank are derived from intertemporal maximisation problems with the certainty equivalence version of the rational expectations. This yields us a nice outcome of avoiding the ad hoc assumptions usually imposed on the stability of the solution to the forward-looking differential equations.

We considered a case where the central bank has to react to a temporary exogenous shock and bring the system back to equilibrium with as small a loss as possible by using one instrument, the call money rate. Our simulation results, which we although believe to be rather general, show that the central bank indeed can stabilize the economy in the short run. If we believe that the central bank cannot fool the banks continuously by announcing something and doing something else, the resulting optimal policy rule is a time-inconsistent closed loop rule. The time-consistent rule is certainly inferior to time-inconsistent rule provided that the rational expectations hypotheses holds, and - at least in our simulation experiment - it was found to be inferior to passive policy.

Our example suggests that the central bank policy stance should be state-contingent and clearly announced so that the central bank is able to exploit the forward looking nature of the banking sector's behaviour. Unsystematic or unpredictable policy will result in a volatile behaviour of the credits, as the banks accelerate and decelerate their lending wildly every time the current call money rate is changed.

Also the credibility of the central bank is of crucial importance. As we noted earlier the central bank will be tempted to cheat the banks to gain a short run advantage stemming from the surprises. If the cheating could be done continuously, all the better, but as in a poker game, you are able to bluff successfully only once: after that there usually will be somebody who always calls your hand.

Finally we make a few remarks on the admittedly restrictive policy formulation problem we studied. We saw that the optimal policy rule was a time-inconsistent, but a credible (and binding) commitment made at time $t_0 = 0$, the starting point of the planning horizon. Actually we took it for granted that there must have been some other policy rule in effect before the recognized disequilibrium situation which triggered off the optimizing procedure. Time-inconsistency arised from the fact that this new policy is suboptimal from the vantage point of any time $t_0 + h$, $h > 0$. Now the determination of our starting time $t_0 = 0$ is by no means a trivial matter (see e.g. Calvo, 1978). If the "first" optimizing calculation can be timed arbitrarily we are back in the crucial dilemma of credibility. The policy-maker should, therefore, be ready to preserve its credibility and resist any temptations of short term gains by reoptimizing only when facing fundamental structural changes, such as totally new institutional arrangements or other permanent exogenous shocks. The feasibility of a monetary policy which is able to resist short term gains seems to call for a strong and independent monetary authority, which should, however, be willing to promote effective forecasting schemes of the private banking sector by stating its policy rule clearly and avoiding unnecessary and esoteric mysticism.

REFERENCES:

ALHO K. (1982): Rahoitusmarkkinamallin rakenne ja yhtälöiden spesifikaatio. Osa I: Mallin rakenne, ETLA Discussion Paper No. 122, (In Finnish).

AUSTIN G.P. and BUITER W.H. (1982): "Saddlepoint", A programme for solving continuous time linear rational expectations models, LSE Econometrics Programme Discussion Paper: A.37.

BUITER W.H. (1981): "The Superiority of Contingent Rules Over Fixed Rules in Models with Rational Expectations", The Economic Journal.

BUITER W.H. (1982): Saddlepoint Problems in Continuous Time Rational Expectations Models: A General Method and Some Macroeconomic Examples, Centre for Labour Economics, London School of Economics, Discussion Paper No. 114.

BUITER W.H. and MILLER M (1982): "Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation", European Economic Review.

CALVO G.A. (1978): On the Time Consistency of Optimal Policy in a Monetary Economy", Econometrica.

CREUTZBERG A.F. (1982): An Empirical Study on Bank Lending in Finland, Economic Planning Centre.

DRIFFILL E.J. (1982): Optimal Money and Exchange Rate Policies, Mimeo: Dept. of Economics, University of Southampton.

HÄMÄLÄINEN R. (1981): "On the Cheating Problem in Stackelberg Games", International Journal of System Science.

INTRILLIGATOR M.D. (1971): Mathematical Optimization and Economic Theory, Prentice-Hall, U.S.A.

ITO T. (1980): "Methods of Estimation for Multi-Market Disequilibrium Models", Econometrica.

KAMIEN M.I. and SCHWARTZ N.L. (1981): Dynamic Optimization: The Calculus of Variations and Optimal Control in Economies and Management, North Holland, U.S.A.

MILLER M. and SALMON M. (1982): Optimal Control and Economic Dynamics: An Investigation of Coordinated and Decentralised Policy Formation in a Multi-Country Setting with Flexible Exchange rates, Mimeo, University of Warwick.

MUSTONEN J. and SALONEN I. (1983): Optimal Monetary Policy in a Small Open Economy with Regulated Money Markets, a mimeo.

SIMON H.A. (1956): "Dynamic Programming Under Certainty with a Quadratic Criterion Function", Econometrica.

SUVANTO A.(1982): Valuuttakauppa ja ulkomaisen rahoituksen välitys suomalaisten pankkien toiminnassa (Foreign exchange dealing and financial intermediation by the Finnish banking sector), ETLA B 36, (in Finnish with English summary).

WILLMAN A. (1981): The Effects of Monetary and Fiscal Policy in an Economy with Credit Rationing, The Bank of Finland B:52.