ETLA ELINKEINOELÄMÄN TUTKIMUSLAITOS
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

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Antti Suvanto

OPTIMAL QUOTATIONS AND

WHOLESALE TRANSACTIONS

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1. Introduction

In this paper we extend our previous analysis on the behaviour of a foreign exchange dealer who has a local monopoly and as a market maker can choose his quotations and adjust his position by changing quotations during the day (Suvanto, 1982b). Here we introduce a "dealer of the last resort", e.g. the central bank who stands ready to make transactions with the private dealer at preannounced prices. We call these transactions wholesale transactions, and we assume that the customers cannot trade directly with the central bank.

The basic model is the same as in the previous paper. The dealer knows the processes generating the customers´ buy and sell orders, and he maximizes expected revenue subject to the constraint that the foreign exchange position at the end of the day must be closed. It is shown that the possibility of making wholesale transactions at preannounced prices sets limits to the open position the dealer is willing to take. The assumed short-term monopoly of the dealer affects only the spread, which is independent on the position. The position is adjusted by changing the mid-rate except in cases when wholesale transactions become profitable.

If the central bank quotation is considerably lower or higher than the quotation that would equalize the incoming buy and sell orders on the average, this gives the dealer a possibility to arbitrage between the central bank and the customers, for example, by buying relatively cheap currency from the central bank and selling it to the customers at a higher price. Active interventions on the part of the central bank become necessary if its quotation is inconsistently high or low relative to the equilibrium mid-rate and if it wants to keep the private quotation within some prespecified limits.

2. Structure of the problem

The basic model is the same as in our previous paper (Suvanto, 1982b). The customer's buy orders depend negatively on the ask-rate and the sell orders positively on the bid-rate

(1) $\quad p_t = a - bs_t^a$

(2) $\quad q_t = -c + bs_t^b$

where s_t^a is the ask-rate and s_t^b is the bid rate quoted at moment t, and where $a,b,c > 0$, $a-c > 0$. For simplicity, we assume that the customer buyers and customer sellers are equally sensitive to a small change in the quotation, $\partial p_t/\partial s_t^a = -\partial q_t/\partial s_t^b = -b$.

The dealer's foreign exchange position x_t is said to be closed if $x_t = 0$. Otherwise it is either long, $x_t > 0$, or short, $x_t < 0$. The change in the position is equal to net purchases

(3) $\quad x_{t+1} - x_t = q_t - p_t\,.$

The value of sales minus the value of purchases gives the revenue

(4) $$R_t = s_t^a p_t - s^b q_t$$

$$= z_t(q_t + p_t) - s_t(q_t - p_t)$$

$$= z_t(\beta-\gamma z_t) - s_t(-\alpha+\gamma s_t)$$

where $z_t = (s_t^a - s_t^b)/2$ is the half-spread and $s_t = (s_t^a + s_t^b)/2$ is the mid-rate, and where the new parameters $\alpha = a + c$, $\beta = a-c$, $\gamma = 2b$. It is seen that the revenue is composed of two parts one depending only on the spread and the other depending only on the mid-rate.

Maximizing (4) with respect to s_t and z_t subject to the constraint that $q_t - p_t = 0$ gives the equilibrium quotation

(5) $\quad s_t = \alpha/\gamma \equiv \hat{s}$

(6) $\quad z_t = \beta/2\gamma \equiv \hat{z}$

Maximizing the revenue subject to the constraint that net purchases must be equal to a certain target, $q_t - p_t = x_{t+1} - x_t$ gives the following mid-rate

(7) $\quad s_t = \hat{s} + (1/\gamma)(x_{t+1} - x_t)$.

The spread is the same as above and is thus independent of the position.

In the previous paper we showed that if the dealer maximizes the revenue and wants to be at closed position at the end of the day, then if he has an open position $x_{t_0} \neq 0$ at any moment to his optimal strategy is to close it gradually by applying a constant control, i.e. by keeping the mid-rate constantly above or below $\hat{s}$.

3. Introducing the "dealer of the last resort"

Assume that there is a "dealer of the last resort", e.g. the central bank, who stands ready to buy or sell foreign exchange with the private dealer at preannounced prices. The buying price of the central bank is denoted by $\underline{S}$ and the selling price by $\bar{S}$, $\bar{S} > \underline{S}$. The customers cannot trade directly with the central bank.

Consider first the case where the dealer has an open position $x_t \neq 0$, and he wants to close it in a single trading period, i.e. $x_{t+1} = 0$. If he does not make a wholesale transaction his mid-rate will be, according to equation (7), $s_t = \hat{s} - (1/\gamma)x_t$, and the revenue will be $R_t = \hat{R} + (\hat{s} - (1/\gamma)x_t)x_t$, where $\hat{R}$ is the equilibrium revenue, i.e. the maximum revenue when net purchases are equal to zero. Closing the position by a wholesale transaction alone and quoting the equilibrium mid-rate, will give a revenue $\hat{R} + \underline{S}x_t$ if the initial long position $x_t > 0$ is sold to the central bank, or $\hat{R} + \bar{S}x_t$ if the initial position is short, $x_t < 0$. A comparison of these two alternatives, closing the position by customer transactions alone or by a wholesale transaction alone, shows that a wholesale transaction is profitable if $x_t > \gamma(\hat{s}-\underline{S})$ or if $x_t < \gamma(\hat{s}-\bar{S})$. This comparison does not, however, tell anything about the optimal size of a wholesale transaction.

Denote the size of the wholesale purchase by y_t and the position after the wholesale transaction by $x_t' = x_t + y_t$. The objective function must now be reformulated in such a way that it takes the contribution of an eventual wholesale transaction into account. Given an initial position x_t and maintaining the requirement that the position must be closed at the end of the period leads to the following problem

$$
(8) \quad \begin{cases} \max L \quad \text{w.r.t.} \quad s_t,\ z_t,\ x_t',\ \text{and } \lambda_t \\ \\ L = R_t - S(x_t' - x_t) + \lambda_t(x_t' - p_t + q_t) \end{cases}
$$

where $S = \bar{S}$ if $y_t = x_t' - x_t > 0$ (a wholesale purchase) or $S = \underline{S}$ if $y_t < 0$ (a wholesale sale), and λ_t is the Lagrange multiplier for the end-of-period position constraint. The first order conditions are

$$(9) \qquad \alpha - 2\gamma s_t + \gamma\lambda_t = 0$$

$$(10) \qquad \beta - 2\gamma z_t = 0$$

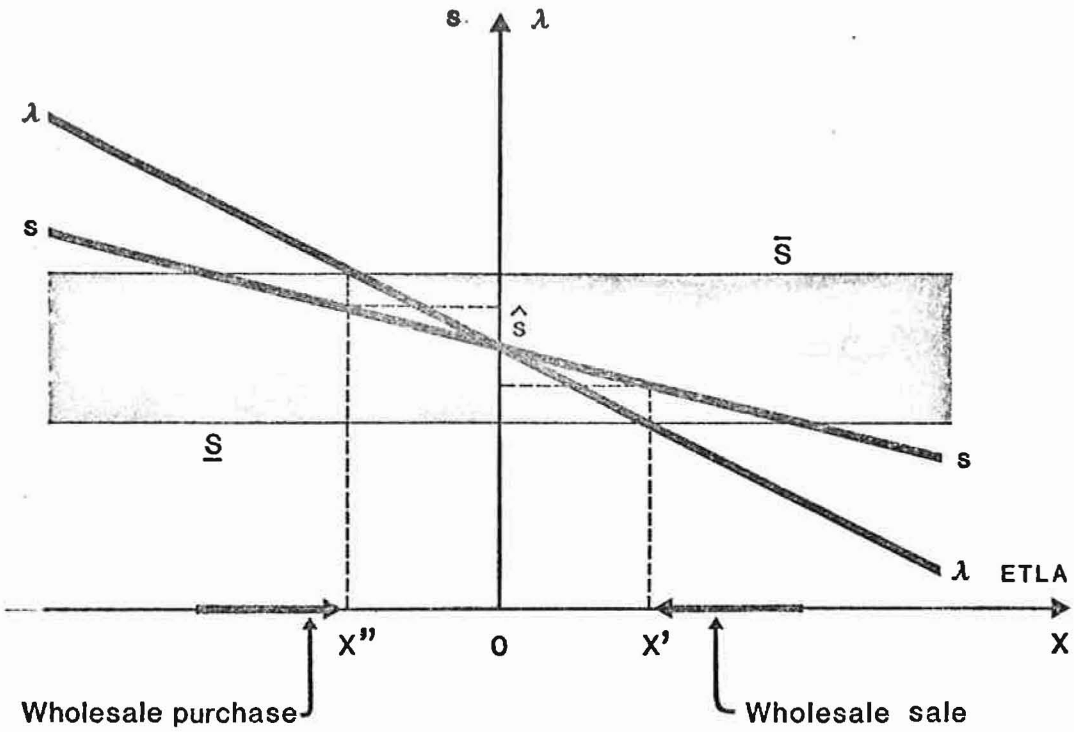
$$(11) \qquad \lambda_t - S = 0$$

$$(12) \qquad x_t' - \alpha + \gamma s_t = 0 \,.$$

From equation (10) it is seen that the possibility of wholesale transactions does not affect the spread, $z_t = \hat{z}$. The mid-rate is adjusted to close the position after the wholesale transaction, if that is made, $s_t = \hat{s} - (1/\gamma)x_t'$. From equation (9) it is seen that $\lambda_t = \hat{s} - (2/\gamma)x_t'$, which together with equation (11) gives the position after an eventual wholesale transaction, $x_t' = (\gamma/2)(\hat{s}-S)$. Whether or not a wholesale transaction is profitable depends on the initial position and on the central bank quotation $(\underline{S},\bar{S})$

$$
(13) \qquad y_t = \begin{cases} x_t' - x_t < 0 & \text{if} \quad x_t > (\gamma/2)(\hat{s}-\underline{S}) \\ \\ x_t' - x_t > 0 & \text{if} \quad x_t < (\gamma/2)(\hat{s}-\bar{S}) \\ \\ 0 \quad \text{if} & (\gamma/2)(\hat{s}-\bar{S} \leq x_t \leq (\gamma/2)(\hat{s}-\underline{S}) \,. \end{cases}
$$

Figure 1. *The mid-rate and the shadow price of the position constraint as a function of the position*



This result is illustrated in Figure 1, where the ss-line shows the mid-rate as a function of the the initial position that must be closed, and the $\lambda\lambda$-line shows the Lagrange multiplier similarly as a function of the initial position. If $\lambda(x) < \underline{S}$, then a wholesale sale by the amount x-x' is profitable, and if $\lambda(x) > \bar{S}$, then a wholesale purchase by the amount x"-x is profitable.

At this point it is useful to remind of the interpretation of the Lagrange multiplier as a shadow price. Without the possibility of making wholesale transactions the shadow price indicates the marginal change in the objective function if the net purchase requirement is changed marginally. In other words, it tells the price the dealer would be willing to pay for a unit of foreign exchange in order to reduce his short position marginally, or the price he would require for a unit of foreign exchange in order to reduce his long position marginally. Assuming a central wholesaler there is a market in which the open position can be sold at preannounced prices. A positive spread in the central bank quotation implies that this alternative is not always profitable.

From Figure 1 it is obvious that the higher $\bar{S}$ and the lower $\underline{S}$ relative to the equilibrium mid-rate the less likely it is that the dealer finds himself in a situation where wholesale transactions become profitable. In the limiting case, when $\bar{S} = \underline{S} = \hat{s}$, the dealer would always quote the equilibrium mid-rate and the revenue maximizing spread and use the central bank to eliminate immediately any open position that may result because of transactions uncertainty. In this case the dealer would cease to perform the dealing function in the proper sense of the word, that is accepting transactions uncertainty by holding inventories of currencies (cf. Suvanto, 1982a). He would simply transfer the transactions uncertainty to the central bank and still make profit by selling to the customers at a higher price than he buys. In section 5 we shall briefly discuss the case in which the central bank quotation differs considerably from the mid-rate that would equalize the incoming buy and sell orders.

4. A dynamic extension

An extension into a dynamic framework is straightforward. Applying the dynamic programming procedure of our previous paper (Suvanto, 1982b), assume that at some moment t_0 the dealer has $T-t_0$ trading periods left to go and he considers how to quote and whether or not to make a wholesale transaction now. The quotation and the wholesale transaction are obtained by solving.

$$(14) \qquad J_{t_0}(x_{t_0}) = \max_{s_{t_0}, z_{t_0}, x'_{t_0}} \{R_{t_0} - S(x'_{t_0} - x_{t_0}) + J_{t_0+1}(x_{t_0+1})\}$$

where x_{t_0} is the position at moment t_0, and x'_{t_0} is the position after the wholesale transaction, and $J_{t_0+1}(x_{t_0+1})$ is the maximum revenue from the moment t_0+1 until the end of the day assuming that no further wholesale transactions are made.

From the previous paper we know that the optimal strategy without wholesale transactions implies a gradual elimination of the open position by applying a constant quotation, which strategy gives

$$(15) \qquad J_{t_0+1}(x_{t_0+1}) = (\hat{s} - \frac{1}{\gamma}\frac{1}{T-t_0-1}\, x_{t_0+1})x_{t_0+1} + A_{t_0+1}$$

where A_{t_0+1} is a constant independent of the position. The position at moment t_0+1 depends on the quotation at moment t_0

$$\text{(16)} \qquad x_{t_0+1} = x'_{t_0} - \alpha + \gamma s_{t_0} \, .$$

We are now able to solve the problem as described by equation (14). This gives

$$\text{(17)} \qquad s_{t_0} = \hat{s} - \frac{1}{\gamma}\frac{1}{T-t_0} x'_{t_0}$$

$$\text{(18)} \qquad x'_{t_0} = (\gamma/2)(T-t_0)(\hat{s} - S) \, .$$

The spread is again the one that maximizes the trading period revenue on the equilibrium volume of trade. Whether or not a wholesale transactions is profitable depends, as above, on the initial position and on the central bank quotation

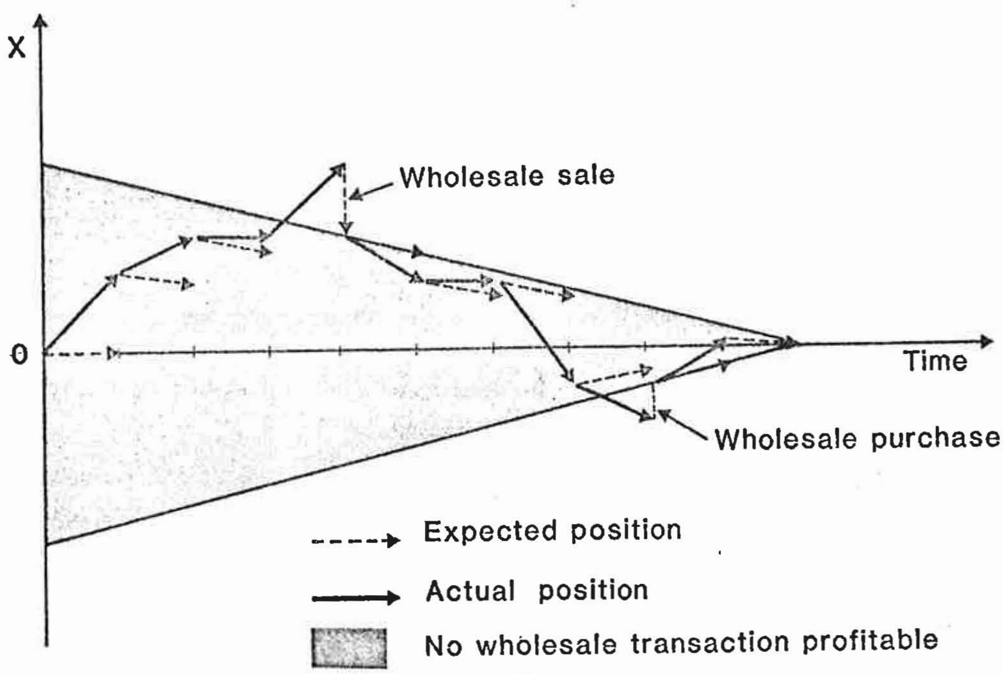
$$\text{(19)} \qquad y_{t_0} = \begin{cases} x'_{t_0} - x_{t_0} < 0 & \text{if} \quad x_{t_0} > (\gamma/2)(T-t_0)(s-\underline{S}) \\ x'_{t_0} - x_{t_0} > 0 & \text{if} \quad x_{t_0} < (\gamma/2)(T-t_0)(s-\bar{S}) \\ 0 & \text{otherwise.} \end{cases}$$

In other words, the requirement that the end-of-day position must be closed together with the assumption of a preannounced central bank quotation determine the limits outside which an wholesale transaction becomes profitable. As seen from Figure 2, the dealer is willing to accept greater open positions in the morning than towards the end of the day. The possibility of making wholesale transactions at preannounced prices also set the limits to the deviation of the quoted mid-rate from the equilibrium one (cf. Figure 1).

(20) $\quad (\underline{S}-\hat{s})/2 \leq s_t - \hat{s} \leq (\bar{S}-\hat{s})/2\,.$

These limits do not depend on the position during the day.

Figure 2. *Illustration of the path of the position with optimal quotations and wholesale transactions*



If there is a proportionate cost associated with a wholesale transaction then this cost is added or subtracted to the selling or buying price of the central bank, respectively. The above results would not change except the widening of the limits outside which wholesale transactions become profitable.

If, on the other hand, there is a lump sum cost associated with a wholesale transaction, this would alter the results in the sense that now the dealer would allow the position to exceed the above mentioned limits by a certain amount. If the position goes beyond these higher limits then the dealer would by a single wholesale transaction bring the position back to the earlier limits, see Figure 3.

A lump sum cost of a wholesale transaction may have an affect that the dealer wants to avoid a further wholesale transaction in a later trading period. In this case, when a wholesale transaction becomes profitable, it would be greater than above so that the probability of a further wholesale transaction is reduced, see Figure 4.

These implications are obvious and could be shown formally.

5. Inconsistent central bank quotations and interventions

The above model can be applied to situations where the preannounced prices of the central bank are considerably higher or lower than the mid-rate that would equilibriate the incoming buy and sell orders of the customers on the average. Consider, for instance, the case where both $\bar{S}$ and $\underline{S}$ are below the equilibrium mid-rate as illustrated in Figure 5. Applying the results of Section 3, it is seen that the dealer would be willing to make a wholesale purchase at price $\bar{S}$ even if his initial position were closed. After a wholesale purchase his position would be long, and he would then quote below the equilibrium mid-rate thus generating net sales to the customers. In other words, the dealer would arbitrage relatively cheap foreign

Figure 3. *Illustration of the path of the position assuming a lump sum cost of wholesale transactions*

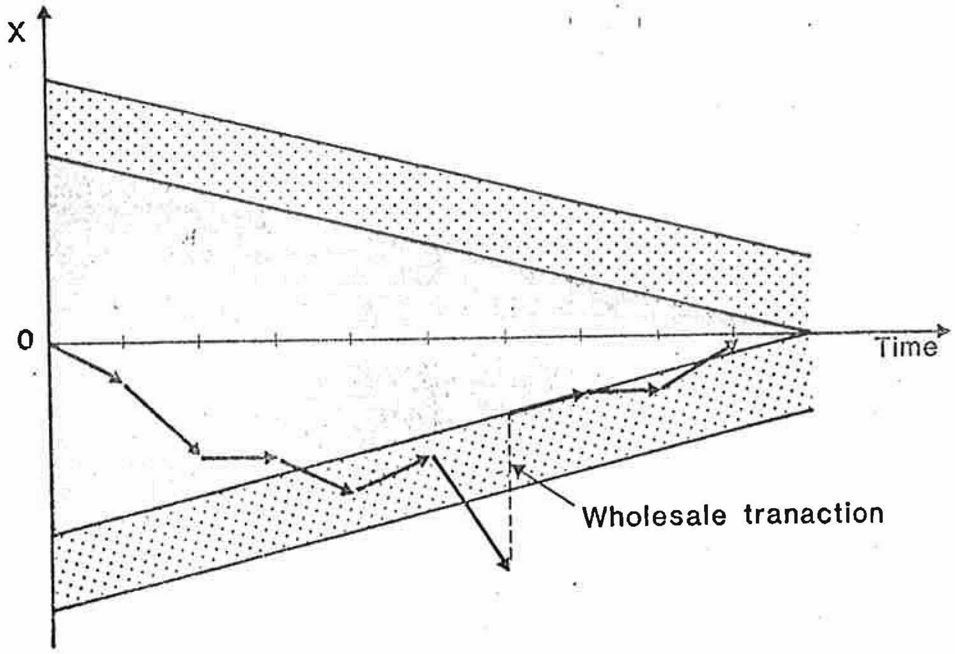
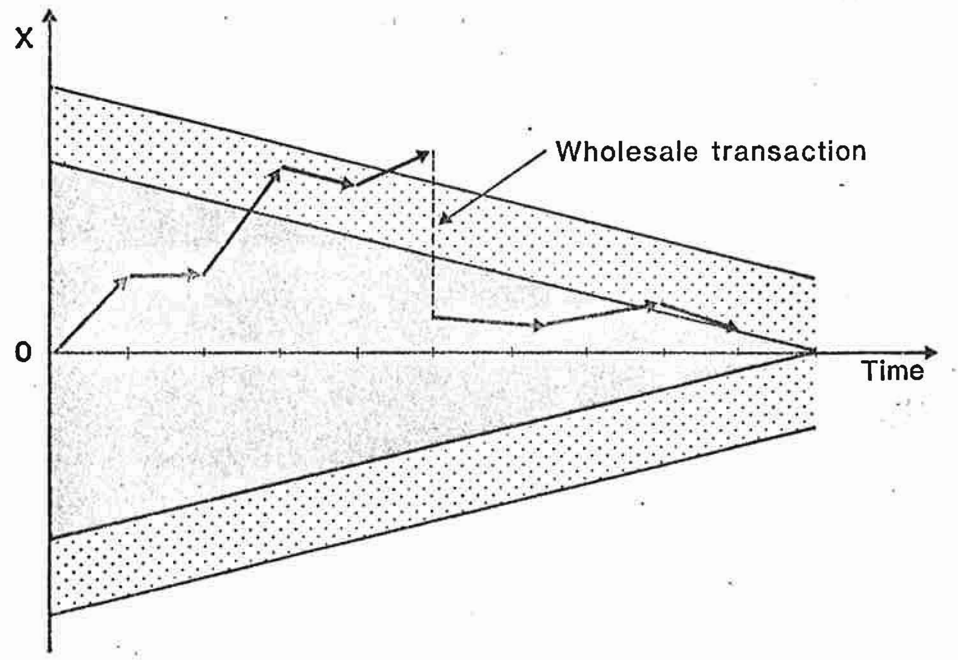


Figure 4. *Illustration of the path of the position taking into account a cost of possible future wholesale transactions*



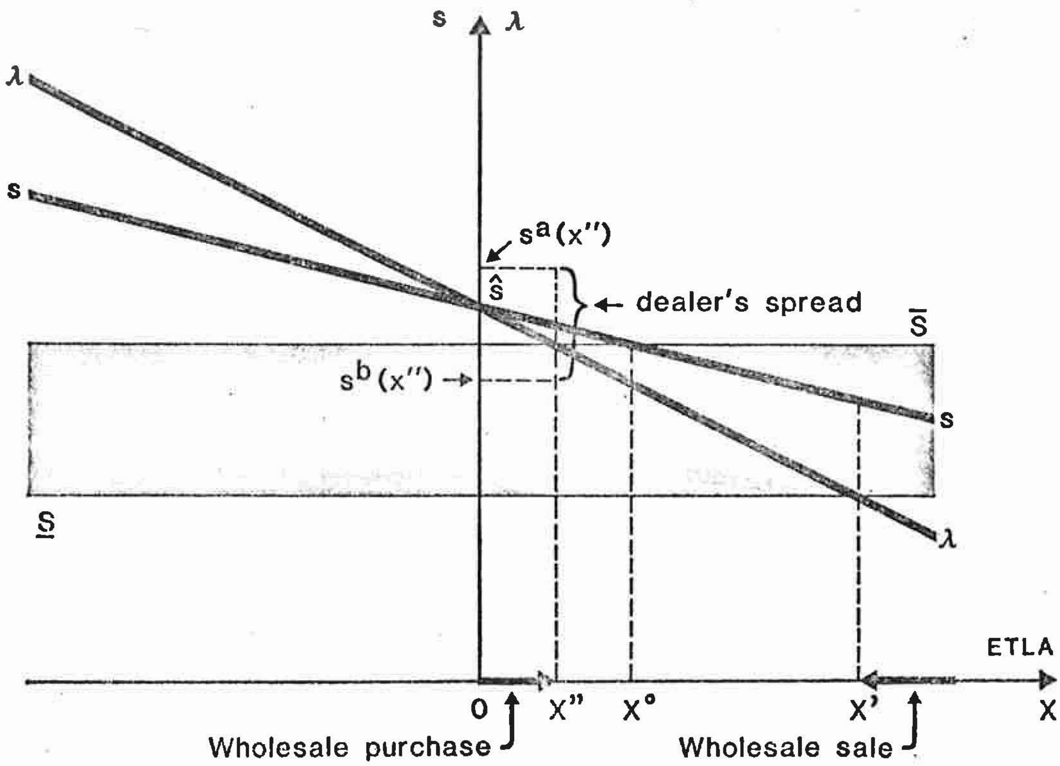
exchange from the central bank to the customers at somewhat higher price. A wholesale sale on the part of the dealer would in this situation be a rare event.

If the central bank wants to keep at least the mid-rate within the prespecified limit ($\underline{S}$, $\bar{S}$) an active intervention on the part of the central bank becomes necessary. Assume that the dealer has opened his position by buying an amount x" of foreign exchange from the central bank and is quoting accordingly. Then the central bank could intervene by selling to the dealer at prices $s^b(x'')$. If this intervention lengthens the dealer's postion to the range (x^0, x'), this would reduce the dealer's quotation sufficiently to make it consistent with the limits ($\underline{S}$, $\bar{S}$). Alternatively, the central bank could reduce its selling price $\bar{S}$ in order to make a greater wholesale purchase for the dealer more desirable. In each case there would be an outflow of reserves from the central bank first to the dealer and then to the customers and finally, most probably, out of the country. In the longer-run, if this situation is repeated many times, it would become necessary for the central bank either to increase its own quotation ($\underline{S}$,$\bar{S}$) or to use other measures that affect the market orders of the customers.

6. Conclusions

Above we have analyzed the behaviour of a foreign exchange dealer as a market maker in a situation where he can affect the expected market orders of the customers by changing quotations and where, in addition, he has a possibility to adjust his foreign exchange position by dealing with the

Figure 5. *Optimal mid-rate and wholesale transactions with an inconsistently low central bank quotation*



"dealer of the last resort", which role we have given to the central bank. The analysis was carried out without an explicit treatment of uncertainty, even though one of the basic assumptions was that there is uncertainty as regards the timing of the customers' buy and sell orders. Other kind of uncertainty is taken into account by the assumption that the dealer wants to have a closed position at the end of the day. A justification for this assumption is that even though the dealer may have relatively good information about the conditions today he does not know what the conditions will be tomorrow, i.e. what the equilibrium mid-rate will be tomorrow.

The requirement that the position must be closed at the end of the day affects the dealer's quotations at every moment of the day. An open position constraints the revenue the dealer can achieve by trading with the customers. The presence of the "dealer of the last resort" opens a market in which the dealer can buy and sell his excess open position independently of the customers' reactions. It was shown that the central bank quotation sets the limits to the mid-rate the dealer is willing to quote and hence to the net purchases or net sales the dealer is willing to make with the customers. It was also shown that whatever the position at some moment of the day the dealer plans to close the position, after a possible wholesale transaction with the central bank, only gradually by applying a constant quotation during the rest of the day. In other words, from the point of view of a customer the expected quotation at some later moment of the day is equal to the current quotation and therefore there is no reason for him to delay the transaction he needs to make until some later moment.

If the central bank's quotation does not differ much from the quotation at which the buy and sell orders of the customers will balance on the average then the central bank can assume a passive role and respond on demand to the wholesale orders of the private dealer at preannounced prices. Active interventions become necessary if the central bank quotation is inconsistently high or low relative to the equilibrium quotation and if the central bank does not allow the private quotation differ much from the official one.

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