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Keskusteluaiheita
Discussion papers

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REDUCING THE DIMENSION OF INPUT VECTOR IN TRANSFER FUNCTION MODELS: THE CASE OF QUICK INDICATORS*

No. 129 24 January 1983

* My thanks are due to Raija Leppälä for able research assistance.

Abstract. This paper is concerned with some statistical aspects of constructing a quick indicator for early estimation and short-term forecasting of the volume of industrial production in Finland. Two techniques of reducing the dimension of the input vector in single input, multiple output transfer function models used in forecasting are considered; principal components on one hand and selection of variables on the other. The prediction performance of the models is compared and neither of the two types of models is found superior to the other.

Keywords: forecasting; principal components; single input, multiple output transfer function model; time series modelling.

1. Introduction

The first preliminary value of the monthly volume of industrial production in Finland is published by the Central Statistical Office (C.S.O.) after a lag of approximately two months. In order to obtain an earlier estimate, Teräsvirta (1983) has constructed a quick indicator based on monthly time series which are related to the industrial production and published faster than the first C.S.O. preliminary values. These quick estimates as well as short-term predictions of the volume are needed at the Research Institute of the Finnish Economy when semiannual macroeconomic forecasts are being prepared. Because of the nature of the problem it is not possible to rely on structural models based on economic theory. The requirement of a short publication lag inavoidably overrules the theory when proper time series are selected. The following series have been considered:

- electricity consumption (X_{1t})
- index of advertisement space in newspapers (X_{2t})
- number of State Railways freight cars loaded (X_{3t})
- number of vacancies (X_{4t})
- number of unemployed persons (X_{5t})
- volume of exports (X_{6t}).

The first, third and last series are directly connected with the industrial output while the remaining series are more loosely related to it. All of them are available at a few weeks' notice. The quick indicator makes it possible to predict the volume of industrial production conditionally on the values of the individual series as soon as the latter become available. The indicators in Teräsvirta (1983) are based on transfer function models, and we shall now discuss some statistical aspects of the model building.

2. Reducing the dimension of the input vector

Our general model can be written as

$$D(z)y_t = \mu + \sum_{i=1}^{k} \delta_i(z)^{-1}\omega_i(z)D(z)x_{it} + \phi(z)^{-1}\Phi(z^s)^{-1}\theta(z)\Theta(z^s)a_t \tag{2.1}$$

where $a_t \sim nid(0,\sigma^2)$, μ is a constant, z is the lag operator, $zx_t = x_{t-1}$, and D(z) is polynomial operator whose roots are on the unit circle. Symbol y_t is the logarithm of the volume of industrial production and x_{it} is the logarithm of X_{it}. Furthermore,

$$\omega_i(z) = \sum_{j=0}^{u_i} \omega_{ij}z^j , \quad \delta_i(z) = 1 - \sum_{j=1}^{r_i} \delta_{ij}z^j , \quad i = 1,\dots,k$$

$$\phi(z) = 1 - \sum_{j=1}^{p} \phi_j z^j , \quad \theta(z) = 1 - \sum_{j=1}^{q} \theta_j z^j$$

$$\Phi(z^s) = 1 - \sum_{j=1}^{P} \Phi_j z^{js} , \quad \Theta(z^s) = 1 - \sum_{j=1}^{Q} \Theta_j z^{js}$$

and the roots of the above polynomials are outside the unit circle. Using the terminology of Box and Jenkins (1970), (3.1) is a single output, multiple input transfer function model with a multiplicative stationary ARMA(p,q) x $(P,Q)_s$ error process.

In the present application the number of observable inputs in (2.1) can be as high as six. Thus the specification of the model and the estimation of its parameters in particular may be costly. This is a factor to reckon with when the model is updated. Another problem is that when (2.1) is used not only for obtaining quick estimates but also for short-term prediction, a straightforward application of (2.1) would require the specification and estimation of a vector ARIMA model for the inputs or, more modestly and less efficiently, an ARIMA model for each input to generate the necessary forecasts of the input variables.

The amount of work involved would diminish if the dimension of the input vector could be reduced by replacing the original vector by a vector of lower dimension. Two ways of achieving such a reduction are considered here. The new input vector may consist of the first principal components of the original variables. Brillinger (1975, pp. 339-340) has discussed the optimality properties of this transformation, see also Rao (1965, pp. 502-504), and it has been applied in Teräsvirta (1983). Another alternative is a standard model selection approach which aims at omitting some components of the input vector altogether.

The former reduction was performed by using two first principal components as inputs. In the original input variable space $\{\nabla_{12}x_{jt}\}$ these principal components represent 78 per cent of the standardised total variation in the sample consisting of differences from 1970(i) till 1980(xii) while the remaining components are clearly less important. Note that the degree of differencing is not unique. In the present application, no theory tells us in which variable space the principal component transformation should be performed. For instance, stationarising the variables is no prerequisite to the transformation. Nonetheless, to curtail the number of alternatives experiments were made with stationary variables only. It turned out that the use of $\{\nabla\nabla_{12}x_{jt}\}$ and two principal components, in which case the output variable has to be $\nabla\nabla_{12}y_t$, gives results not very different from having $\nabla\nabla_{12}y_t$ explained by two differenced principal components of $\{\nabla_{12}x_{jt}\}$ and their lags. Another problem is how to define the dimension of the space. It may be argued that the set of variables spanning the space should also include lags of differenced input variables. However, model (2.1) allows a rational distributed lag structure for each input, and that seems to ensure sufficient flexibility in modelling at least in this application.

Deleting components from the input vector can be carried out at the specification stage of the model. Although each indicator variable as such has prediction power, some of them may become redundant when taken together. Since the number of potential inputs was high, frequency domain techniques proposed by Box and Jenkins (1970, pp. 415-416), and recently evaluated by Pukkila (1982), were applied to specifying u_i and r_i, $i = 1,\dots,6$, and deleting unnecessary inputs. It appeared that the outcome was sensitive to the degree of differencing. If the model was specified for seasonally differenced variables ($D(z) = \nabla_{12}$), four indicators were omitted whereas the number was two or three when doubly differenced variables ($D(z) = \nabla\nabla_{12}$) were used. The former degree of differencing is obviously preferable to the latter from the point of view of parsimony, in particular as the differences in the quality of fit of the models are minor.

3. Results

Models were estimated from data extending from 1970(i) to 1978(xii), 1979(xii) and 1980(xii), respectively, using a full maximum likelihood procedure (Mellin, 1980). The only exception were the transfer function models with more than two observable inputs for which a least squares approximation (Box and Jenkins, 1970, pp. 388-389) was employed. The twelve months following the end of the estimation period were predicted *ex post* to obtain an idea of the prediction performance of the models. Models based on seasonally differenced data fitted better than those based on double differencing, and principal component models were in that respect superior to models with non-transformed input variables. The AIC criterion also favoured principal component models. For comparison, we also estimated very simple models with the first principal component without lags as the only input, together with a very parsimonious error process. Our intention was to see how much is actually gained by meticulous modelling and aiming at the "best" possible model.

When the prediction performance of the models is considered, the situation is not so clear-cut as far as the choice of transformation is concerned. Table 1 contains statistics of the prediction errors and, for comparison, the residual standard deviations of the models. The median of absolute prediction errors is included, since both in 1979 and in 1980 there is one exceptionally large prediction error, for details see Teräsvirta (1983), and the root mean square error alone would have given a distorted view of the situation. Following general conclusions emerge from Table 1:

(i) The ARIMA models typically fail at turning-points, and even here they are clearly inferior to the transfer function models in predicting the upswing in 1979 and the slowdown in 1981.

(ii) The principal component models are not superior to the ordinary transfer function models. However, when models based on double differencing are considered, the principal component models have just two observable input variables while the ordinary transfer function models have three or four.

(iii) Although the principal component models based on only seasonally differenced variables fit the data better than the models based on double differencing, the forecasts from the former are not consistently more accurate than those from the latter models. No clear pattern can be distinguished in the ordinary transfer function models either.

(iv) Predictions from single input principal component models are on average only slightly less accurate than the models with a more generous parameterisation. The payoff from careful specification is obviously positive as it ought to be but in this example hardly substantial.

The estimated models and data are in the appendix.

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Table 1. Statistics on the prediction accuracy of estimated models in 1979, 1980 and 1981: (a) Median of prediction errors, (b) Median of absolute prediction errors, (c) Root mean square error of prediction, (d) Standard deviation of residuals

The year forecast		ARIMA model		Transfer function model		Two PC model		Single PC model	
	D(z)	∇_{12}	$\nabla\nabla_{12}$	∇_{12}	$\nabla\nabla_{12}$	∇_{12}	$\nabla\nabla_{12}$	∇_{12}	$\nabla\nabla_{12}$
1979	(a)	0.048	0.058	0.037	0.013	-0.000	-0.001	0.003	-0.001
	(b)	0.048	0.058	0.037	0.023	0.026	0.023	0.026	0.023
	(c)	0.083	0.088	0.052	0.044	0.049	0.048	0.046	0.046
	(d)	0.044	0.044	0.038	0.037	0.034	0.035	0.036	0.037
1980	(a)	0.018	0.010	0.021	-0.023	-0.025	-0.026	-0.026	-0.032
	(b)	0.046	0.037	0.031	0.035	0.034	0.035	0.035	0.041
	(c)	0.063	0.065	0.053	0.050	0.051	0.056	0.055	0.056
	(d)	0.048	0.048	0.037	0.037	0.036	0.037	0.038	0.038
1981	(a)	-0.035	-0.044	-0.018	-0.023	-0.020	-0.015	-0.023	-0.012
	(b)	0.035	0.044	0.019	0.023	0.024	0.017	0.026	0.019
	(c)	0.039	0.055	0.020	0.025	0.028	0.024	0.031	0.026
	(d)	0.050	0.050	0.039	0.038	0.037	0.038	0.040	0.040

Appendix: Models and data

This appendix contains the estimated models with which the results in Table 1 have been obtained and the data used in the estimation of parameters.

List of symbols:

s	standard deviation of residuals
AIC	Akaike's Information Criterion
AIC*	AIC adjusted for the number of observations (one observation is lost in doubly differenced series)
$Z(\cdot)$	Box-Pierce test statistic for testing whether the error process is white noise. The figure in parentheses after the value of the statistic is the corresponding value of the asymptotic c.d.f.
p_t^j	j^{th} principal component of $\{\nabla_{12}x_{it}\}$.

Nearly all models have been estimated using a full maximum likelihood routine of the SURVO 76 statistical data processing system, see Mellin (1980). The only exceptions are the transfer function models (A8), (A10) and (A12) with more than two observable inputs for which a least squares approximation (Box and Jenkins, 1970, pp. 388-389) has been used. Consequently, no AIC value is available for these three models.

A1. ARIMA models

Period of estimation: 1970(i) - 1978(xii):

$$\nabla_{12}y_t = \underset{(0.020)}{0.047} + (1 - \underset{(0.09)}{0.35}z - \underset{(0.10)}{0.13}z^2 - \underset{(0.09)}{0.36}z^3)^{-1}(1 - \underset{(0.09)}{0.17}z^{12})\hat{a}_t \qquad (A.1)$$

$s = 0.0443$, AIC $= -358.5$, $Z(8) = 5.9$ (0.34)

$$\nabla\nabla_{12}y_t = (1 - 0.19z^{12})(1 - 0.70z)\hat{a}_t \qquad \text{(A.2)}$$
$$\qquad\qquad (0.09) \qquad\qquad (0.06)$$

s = 0.0442, AIC = - 361.8, Z(10) = 7.6 (0.34)

Period of estimation: 1970(i) - 1979(xii):

$$\nabla_{12}y_t = 0.054 + (1 - 0.32z - 0.16z^2 - 0.30z^3)^{-1}\hat{a}_t \qquad \text{(A.3)}$$
$$(0.019) \qquad (0.09) \quad (0.09) \quad (0.09)$$

s = 0.0480, AIC = - 379.1, Z(9) = 7.8 (0.44)

$$\nabla\nabla_{12}y_t = (1 - 0.21z^{12})(1 - 0.70z)\hat{a}_t \qquad \text{(A.4)}$$
$$\qquad\qquad (0.11) \qquad\qquad (0.06)$$

s = 0.0477, AIC = - 385.6, Z(10) = 5.1 (0.12)

Period of estimation: 1970(i) - 1980(xii):

$$\nabla_{12}y_t = 0.056 + (1 - 0.25z - 0.17z^2 - 0.34z^3)^{-1}\hat{a}_t \qquad \text{(A.5)}$$
$$(0.017) \qquad (0.08) \quad (0.09) \quad (0.08)$$

s = 0.0498, AIC = - 408.3, Z(9) = 8.2 (0.49)

$$\nabla\nabla_{12}y_t = (1 - 0.12z^4)(1 - 0.71z^{12})\hat{a}_t \qquad \text{(A.6)}$$
$$\qquad\qquad (0.09) \qquad\qquad (0.06)$$

s = 0.0502, AIC = - 410.5, Z(10) = 8.0 (0.37)

A2. Transfer function models

Period of estimation: 1970(i) - 1978(xii)

$$\nabla_{12}y_t = \underset{(0.009)}{0.019} + \underset{(0.08)}{0.42}\nabla_{12}x_{1t} + \underset{(0.04)}{0.13}\nabla_{12}x_{3t}$$

$$+ (1 - \underset{(0.10)}{0.38}z - \underset{(0.10)}{0.23}z^2)^{-1}(1 - \underset{(0.11)}{0.19}z^{12})\hat{a}_t \quad \text{(A.7)}$$

s = 0.0376, AIC = - 389.4, Z(9) = 9.4 (0.60)

$$\nabla\nabla_{12}y_t = \underset{(0.07)}{0.30}\nabla\nabla_{12}x_{1t} + \underset{(0.062)}{0.092}\nabla\nabla_{12}x_{2t} + \underset{(0.04)}{0.13}\nabla\nabla_{12}x_{3t}$$

$$+ \underset{(0.016)}{0.049}\nabla\nabla_{12}z^2x_{4t} + (1 - \underset{(0.07)}{0.80}z)(1 - \underset{(0.11)}{0.37}z^{12})\hat{a}_t \quad \text{(A.8)}$$

s = 0.0367, Z(10) = 4.6 (0.09)

Period of estimation: 1970(i) - 1979(xii)

$$\nabla_{12}y_t = \underset{(0.009)}{0.019} + \underset{(0.08)}{0.47}\nabla_{12}x_{1t} + \underset{(0.04)}{0.16}\nabla_{12}x_{3t}$$

$$+ (1 - \underset{(0.09)}{0.34}z - \underset{(0.09)}{0.24}z^2)^{-1}(1 - \underset{(0.10)}{0.24}z^{12})\hat{a}_t \quad \text{(A.9)}$$

s = 0.0374, AIC = - 434.8, Z(9) = 7.1 (0.38)

$$\nabla\nabla_{12}y_t = \underset{(0.07)}{0.38}\nabla\nabla_{12}x_{1t} + \underset{(0.06)}{0.11}\nabla\nabla_{12}x_{2t} + \underset{(0.04)}{0.14}\nabla\nabla_{12}x_{3t}$$

$$+ \underset{(0.013)}{0.048}\nabla\nabla_{12}z^2x_{4t} + (1 - \underset{(0.06)}{0.85}z)(1 - \underset{(0.10)}{0.33}z^{12})\hat{a}_t \qquad (A.10)$$

s = 0.0369, Z(10) = 6.6 (0.23)

Period of estimation: 1970(i) - 1980(xii)

$$\nabla_{12}y_t = \underset{(0.009)}{0.030} + \underset{(0.08)}{0.36}\nabla_{12}x_{1t} + \underset{(0.04)}{0.20}\nabla_{12}x_{3t}$$

$$+ (1 - \underset{(0.09)}{0.29}z - \underset{(0.09)}{0.19}z^2 - \underset{(0.09)}{0.19}z^3)^{-1}(1 - \underset{(0.10)}{0.22}z^4)\hat{a}_t \qquad (A.11)$$

s = 0.0387, AIC = - 469.4, Z(8) = 6.7 (0.43)

$$\nabla\nabla_{12}y_t = \underset{(0.07)}{0.37}\nabla\nabla_{12}x_{1t} + \underset{(0.04)}{0.16}\nabla\nabla_{12}x_{3t} + \underset{(0.01)}{0.05}\nabla\nabla_{12}z^2x_{4t}$$

$$+ (1 - \underset{(0.05)}{0.90}z)(1 - \underset{(0.10)}{0.35}z^4)\hat{a}_t \qquad (A.12)$$

s = 0.0377, Z(10) = 3.7 (0.04)

A3. Transfer function models: two principal components

Period of estimation: 1970(i) - 1978(xii)

$$\nabla_{12}y_t = \underset{(0.003)}{0.039} + (\underset{(0.004)}{0.018} + \underset{(0.004)}{0.009}z^3)p_t^1 + (\underset{(0.0036)}{0.0086} + \underset{(0.0029)}{0.0033}z^6)p_t^2$$

$$+ (1 - \underset{(0.10)}{0.15}z)^{-1}(1 - \underset{(0.11)}{0.18}z^4)\hat{a}_t \qquad (A.13)$$

s = 0.0338, AIC = - 411,2, Z(10) = 4.1 (0.06)

$$\nabla\nabla_{12}y_t = (\underset{(0.004)}{0.018} + \underset{(0.004)}{0.009}z^3)\nabla p_t^1 + (\underset{(0.004)}{0.007} + \underset{(0.003)}{0.003}z^6)\nabla p_t^2$$

$$+ (1 - \underset{(0.05)}{0.90}z)(1 + \underset{(0.10)}{0.17}z^4)^{-1}\hat{a}_t \qquad (A.14)$$

s = 0.0350, AIC* = -403.2, Z(10) = 5.4 (0.13)

Period of estimation: 1970(i) - 1979(xii)

$$\nabla_{12}y_t = \underset{(0.004)}{0.047} + (\underset{(0.004)}{0.023} + \underset{(0.0041)}{0.0051}z^3)p_t^1 + (\underset{(0.0037)}{0.0081} + \underset{(0.0032)}{0.0047}z^6)p_t^2$$

$$+ (1 - \underset{(0.09)}{0.14}z)^{-1}\hat{a}_t \qquad (A.15)$$

s = 0.0357, AIC = - 447.2, Z(11) = 5.6 (0.10)

$$\nabla\nabla_{12}y_t = \underset{(0.004)}{0.028}\nabla p_t^1 + \underset{(0.003)}{0.006}\,\nabla p_t^2 + (1 - \underset{(0.04)}{0.91}z)(1 - \underset{(0.10)}{0.22}z^4)\hat{a}_t \qquad (A.16)$$

s = 0.0369, AIC* = - 440.9, Z(10) = 5.0 (0.11)

Period of estimation: 1970(i) - 1980(xii)

$$\nabla_{12}y_t = \underset{(0.003)}{0.049} + \underset{(0.002)}{0.027}p_t^1 + (\underset{(0.003)}{0.008} + \underset{(0.003)}{0.004}\,z^6)\,p_t^2 + (1 - \underset{(0.09)}{0.18}z^4)\hat{a}_t \qquad (A.17)$$

s = 0.0374, AIC = - 483.1, Z(11) = 5.8 (0.11)

$$\nabla\nabla_{12}y_t = (\underset{(0.004)}{0.030} - \underset{(0.004)}{0.006}z)\nabla p_t^1 + (\underset{(0.003)}{0.005} + \underset{(0.003)}{0.004}z^6)\nabla p_t^2$$

$$+ (1 - \underset{(0.05)}{0.87}z)(1 - \underset{(0.09)}{0.24}z^4)\hat{a}_t \qquad (A.18)$$

s = 0.0380, AIC* = - 475.2, Z(10) = 2.5 (0.01)

A4. Transfer function models: one principal component

Period of estimation: 1970(i) - 1978(xii)

$$\nabla_{12}y_t = \underset{(0.004)}{0.042} + \underset{(0.002)}{0.027}p_t^1 + (1 - \underset{(0.10)}{0.21}z)^{-1}\hat{a}_t \qquad (A.19)$$

s = 0.0363, AIC = - 403.4, Z(11) = 9.1 (0.39)

$$\nabla\nabla_{12}y_t = \underset{(0.003)}{0.026}\nabla p_t^1 + (1 - \underset{(0.04)}{0.91}z)\hat{a}_t \qquad (A.20)$$

s = 0.0372, AIC* = - 399.0, Z(11) = 13.8 (0.76)

A5. Data

(1) Volume of industrial production, "1975" = 100
(2) Electricity consumption, "1975" = 100
(3) Index of advertisement space in newspapers, "1975" = 100
(4) State Railways freight cars loaded, "1975" = 100
(5) Number of vacancies, 1000's
(6) Number of unemployed persons, 1000's
(7) Volume of exports, "1975" = 100

Month	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1969/01	71.9	71.9	57.7	116.1	5.2	96	78.1
1969/02	72.5	65.8	58.8	113.9	5.5	92	65.1
1969/03	74.0	69.2	78.0	132.1	6.4	86	82.5
1969/04	76.8	61.5	82.0	121.9	8.0	81	73.3
1969/05	77.1	60.7	88.1	124.2	11.4	62	99.2
1969/06	69.6	53.8	66.0	117.5	8.6	52	92.0
1969/07	51.9	59.2	48.4	127.6	8.1	49	96.2
1969/08	69.6	66.8	61.0	127.2	10.7	46	90.7
1969/09	74.2	68.6	82.2	138.0	11.0	37	93.5
1969/10	76.5	72.6	89.5	144.8	9.6	38	111.9
1969/11	79.3	75.6	84.2	124.6	9.1	43	89.4
1969/12	79.7	76.2	83.8	116.7	8.6	61	103.5
1970/01	83.9	83.2	68.7	121.8	8.8	59	90.8
1970/2	83.1	75.7	71.6	116.9	9.2	57	70.0
1970/03	83.1	74.6	88.1	121.2	10.3	54	85.7
1970/04	88.0	72.8	101.3	136.4	12.2	52	95.4
1970/05	88.0	68.4	102.7	118.2	16.8	38	96.4
1970/06	79.0	61.4	75.9	128.6	14.8	39	94.9
1970/07	59.5	64.3	58.3	129.6	12.5	32	92.5
1970/08	78.2	70.7	70.9	130.8	17.1	35	98.0
1970/09	82.3	75.6	93.2	136.7	18.1	28	103.4
1970/10	82.3	80.1	103.4	131.6	15.2	29	106.9
1970/11	84.7	82.7	86.9	125.2	12.8	30	99.0
1970/12	85.6	85.3	90.4	116.2	11.4	38	107.4
1971/01	90.4	88.1	70.9	115.4	9.9	50	102.9
1971/02	89.9	77.1	75.2	98.1	8.4	58	78.4
1971/03	88.6	82.8	87.6	109.5	8.2	62	78.4
1971/04	91.3	74.5	96.9	121.7	11.5	57	83.0
1971/05	91.3	75.6	104.0	110.2	16.9	50	93.1
1971/06	85.6	65.8	73.7	122.0	12.4	39	99.4
1971/07	61.1	68.6	57.2	112.5	11.0	45	101.1
1971/08	80.7	76.1	70.7	120.0	16.5	42	96.8
1971/09	87.2	82.0	87.8	124.7	13.9	40	102.0
1971/10	88.8	87.9	96.0	121.4	12.0	41	108.9
1971/11	88.0	93.5	86.8	124.8	10.4	47	112.9
1971/12	88.0	93.5	95.4	110.5	10.0	53	118.3
1972/01	94.6	104.6	76.0	113.7	10.1	70	100.2
1972/02	94.2	97.3	77.8	120.1	10.5	72	109.4
1972/03	96.7	98.9	94.8	129.7	11.8	72	118.7

Period of estimation: 1970(i) - 1979(xii)

$$\nabla_{12}y_t = \underset{(0.004)}{0.048} + \underset{(0.002)}{0.029}p_t^1 + (1 - \underset{(0.09)}{0.19}z)^{-1}\hat{a}_t \tag{A.21}$$

s = 0.0375, AIC = - 441.1, Z(11) = 7.2 (0.22)

$$\nabla\nabla_{12}y_t = \underset{(0.003)}{0.028}\nabla p_t^1 + (1 - \underset{(0.04)}{0.92}z)\hat{a}_t \tag{A.22}$$

s = 0.0384, AIC* = - 436.0, Z(11) = 11.5 (0.60)

Period of estimation: 1970(i) - 1980(xii)

$$\nabla_{12}y_t = \underset{(0.004)}{0.046} + \underset{(0.002)}{0.028}p_t^1 + (1 - \underset{(0.09)}{0.14}z)^{-1}\hat{a}_t \tag{A.23}$$

s = 0.0396, AIC = - 472.0, Z(11) = 11.6 (0.61)

$$\nabla\nabla_{12}y_t = \underset{(0.004)}{0.28}\nabla p_t^1 + (1 - \underset{(0.09)}{0.91}z)\hat{a}_t \tag{A.24}$$

s = 0.0404, AIC* = - 467.4, Z(11) = 10.9 (0.55)

Month	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1972/04	97.1	85.3	98.8	114.1	14.3	63	96.3
1972/05	98.0	81.3	106.7	128.2	20.5	49	99.7
1972/06	90.8	74.4	86.4	126.0	16.6	50	125.2
1972/07	62.9	73.4	60.2	111.6	18.1	52	110.4
1972/08	84.8	86.8	80.3	122.4	23.6	49	92.3
1972/09	93.6	92.2	93.7	124.7	19.7	43	128.1
1972/10	99.0	95.1	100.9	138.5	15.5	44	110.0
1972/11	99.3	102.1	95.5	135.3	14.2	48	121.0
1972/12	102.6	101.4	96.3	118.7	14.5	53	124.6
1973/01	100.6	112.3	81.3	132.0	14.9	72	111.4
1973/02	104.6	102.6	88.9	122.4	16.7	69	113.2
1973/03	104.8	108.9	113.1	137.4	18.8	63	121.9
1973/04	103.1	93.0	106.8	115.3	24.3	60	97.5
1973/05	100.4	91.4	109.4	125.4	20.6	49	138.0
1973/06	95.2	76.0	90.2	102.6	22.6	50	92.8
1973/07	66.2	80.6	65.8	121.9	24.1	46	106.7
1973/08	93.3	94.4	88.8	135.9	32.9	42	115.5
1973/09	102.2	100.5	101.0	124.7	29.5	38	127.8
1973/10	105.4	112.3	109.0	146.7	24.8	41	141.9
1973/11	107.2	116.7	105.7	133.0	22.9	40	142.2
1973/12	109.8	116.7	101.0	101.8	21.9	48	116.5
1974/01	108.2	117.0	86.4	128.7	22.0	53	129.7
1974/02	110.3	103.9	87.2	119.6	24.0	53	119.4
1974/03	111.0	111.8	112.8	133.4	24.7	44	124.7
1974/04	110.2	93.0	110.4	127.1	31.6	45	133.5
1974/05	107.9	95.1	121.5	133.4	37.3	36	133.6
1974/06	102.9	83.2	92.2	111.4	31.9	35	109.3
1974/07	67.7	86.5	70.3	116.8	33.2	38	123.0
1974/08	100.0	95.8	92.0	120.5	38.8	37	103.2
1974/09	104.6	100.3	104.8	119.2	35.4	32	115.5
1974/10	108.2	112.2	112.0	125.3	30.3	30	141.7
1974/11	109.6	113.1	103.6	121.0	26.3	32	111.9
1974/12	108.2	110.5	102.3	101.4	22.4	39	103.7
1975/01	110.8	122.1	88.2	118.7	20.7	49	120.3
1975/02	104.0	109.5	93.6	108.3	20.5	48	90.0
1975/03	103.0	109.5	104.8	101.9	20.8	47	91.9
1975/04	113.2	102.0	122.0	115.7	23.5	48	101.7
1975/05	104.1	93.2	112.5	99.2	26.1	40	93.5
1975/06	90.4	71.7	93.8	86.2	20.4	45	81.2
1975/07	61.1	72.9	80.5	80.3	19.9	48	97.5
1975/08	97.4	92.2	92.7	92.1	23.1	48	67.3
1975/09	102.7	92.1	110.7	97.2	16.0	49	101.4
1975/10	109.9	104.4	106.2	108.7	12.6	52	109.0
1975/11	105.7	112.2	97.4	99.1	10.0	61	99.1
1975/12	97.8	118.1	97.6	92.7	8.9	72	140.5
1976/01	102.4	128.4	79.8	91.8	9.3	94	92.7
1976/02	99.0	122.8	83.3	89.5	10.1	96	87.6
1976/03	112.6	125.3	96.4	112.1	11.0	84	115.8

Month	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1976/04	98.8	97.0	99.1	84.2	12.1	104	84.9
1976/05	108.5	101.6	103.1	107.3	15.7	89	128.5
1976/06	96.2	81.7	83.8	94.5	13.1	91	128.4
1976/07	54.9	74.3	65.0	79.6	13.4	100	128.6
1976/08	101.6	98.4	80.6	97.6	15.7	99	99.2
1976/09	110.7	110.8	100.1	105.4	12.2	71	118.5
1976/10	110.3	121.8	106.8	104.1	9.2	73	132.7
1976/11	112.1	124.2	90.7	79.5	7.2	84	135.2
1976/12	107.0	128.2	91.9	97.9	5.9	102	141.5
1977/01	107.5	133.8	76.2	88.2	5.8	129	132.2
1977/02	104.1	124.8	78.4	84.9	6.1	133	101.4
1977/03	104.8	116.1	97.9	95.3	7.1	132	136.3
1977/04	94.2	97.5	91.8	95.4	7.7	137	106.5
1977/05	107.7	103.8	99.5	104.0	9.1	134	112.2
1977/06	100.9	89.2	83.4	83.6	7.7	138	126.2
1977/07	53.7	74.3	64.6	67.1	7.3	148	139.1
1977/08	105.5	103.6	77.1	89.3	7.9	141	108.8
1977/09	113.0	117.4	94.6	94.0	5.8	122	139.8
1977/10	112.1	123.4	101.7	97.5	4.4	126	143.0
1977/11	114.9	124.6	94.5	95.5	4.1	145	140.3
1977/12	103.3	131.5	99.1	83.0	3.8	159	145.7
1978/01	110.8	141.0	82.3	88.2	4.0	192	113.0
1978/02	104.9	134.6	82.0	82.2	4.2	186	110.8
1978/03	110.4	130.6	92.7	89.9	4.8	188	119.5
1978/04	109.4	116.9	100.7	95.3	6.2	168	124.6
1978/05	114.4	108.0	104.6	95.6	7.9	155	125.4
1978/06	102.4	93.5	84.2	89.9	6.8	179	119.5
1978/07	55.3	80.9	63.0	66.1	6.3	175	112.6
1978/08	110.9	108.5	80.0	93.2	6.6	155	100.0
1978/09	114.1	121.5	98.1	93.5	5.2	147	135.9
1978/10	121.2	130.2	98.2	100.1	4.7	153	144.8
1978/11	122.2	134.7	93.9	96.1	4.7	163	154.0
1978/12	107.5	151.8	92.6	84.9	4.5	164	140.7
1979/01	122.6	159.5	80.3	96.3	5.3	185	142.2
1979/02	114.8	142.9	86.5	89.5	5.6	181	115.0
1979/03	129.3	145.7	111.1	107.9	7.3	162	138.6
1979/04	114.6	123.2	103.4	99.6	9.3	149	132.5
1979/05	129.2	118.8	108.8	110.9	11.0	135	154.2
1979/06	111.9	100.5	85.1	94.2	10.4	135	131.4
1979/07	72.6	101.5	69.9	87.6	8.9	141	129.4
1979/08	123.5	118.7	89.4	106.2	10.1	120	120.8
1979/09	119.7	124.6	105.5	106.2	9.6	110	119.9
1979/10	134.6	139.0	108.5	122.8	7.7	116	155.3
1979/11	134.3	142.9	106.5	118.3	7.4	106	154.6
1979/12	112.2	147.8	99.6	92.4	7.3	131	140.1
1980/01	132.2	163.4	93.2	114.3	8.5	134	159.9
1980/02	126.2	153.1	100.7	108.2	9.6	118	148.0
1980/03	133.1	152.1	121.4	117.4	10.2	127	183.0

Month	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1980/04	126.1	127.4	116.5	111.0	13.5	115	145.1
1980/05	129.2	121.4	120.5	119.5	16.7	98	158.3
1980/06	124.5	109.4	100.2	112.1	15.7	113	169.9
1980/07	89.5	110.6	80.8	113.5	13.2	119	163.1
1980/08	125.5	123.6	100.8	107.2	16.5	106	146.2
1980/09	135.3	129.3	120.9	116.9	13.5	91	168.6
1980/10	143.7	144.8	131.3	126.3	10.7	121	168.7
1980/11	133.1	154.3	119.7	117.4	9.0	107	161.2
1980/12	127.8	156.8	103.5	124.3	9.8	115	199.1
1981/01	130.3	167.7	101.0	108.7	11.3	126	176.7
1981/02	125.0	151.4	105.4	111.8	13.3	126	155.5
1981/03	136.9	163.4	123.6	128.1	14.1	119	175.4
1981/04	128.4	131.6	121.8	120.6	16.8	113	160.8
1981/05	135.6	127.9	128.0	120.5	19.7	91	189.8
1981/06	123.7	110.2	99.3	111.5	15.0	100	179.8
1981/07	87.8	114.8	83.6	103.6	14.3	111	142.2
1981/08	129.5	130.2	101.6	109.1	14.4	99	158.2
1981/09	138.6	137.7	117.8	122.3	11.6	107	168.8
1981/10	143.6	149.5	127.2	123.7	9.5	117	186.3
1981/11	136.7	157.6	113.2	120.1	8.4	126	187.0
1981/12	131.8	168.9	113.7	109.6	7.8	150	162.7