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DISEQUILIBRIUM ECONOMETRICS FOR THE FINNISH BOND MARKET*

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Disequilibrium Econometrics for the Finnish Bond Market

Recently disequilibrium econometrics has been applied to several types of markets. In this category of research the financial markets have dominated, and especially the bank loan market has been the focus of attention (Laffont and Garcia (1977), Sealey (1979) and Ito and Ueda (1982)[1]. Other markets have been analysed as well, such as the labour market and the market for investment goods. No reference can, however, be made to disequilibrium studies of the bond market, presumably due to the commonly made assumption of these markets being close to markets characterized by perfect competition.

Irrespective of which market is the basis for the disequilibrium approach, there are at least three central but unsolved issues concerning the specification of the model. These are the specification of <u>the price adjustment</u> equation (Bowden (1978), Maddala (1982) and Quandt (1982)), the problem of choosing between <u>a deterministic and a stochastic minimum condition</u> (Ginsburgh, Tishler and Zang (1980), Quandt (1982) and Sneessens (1981a)), and finally the problem of how to take into account the influences of <u>aggregation of heterogeneous markets</u> (Muellbauer (1978) and Sneessens (1981b)). All of these issues affect the choice of estimation method and the likelihood function in the case of maximum likelihood estimation, and the estimates of the structural parameters. Hence, all inference based on the results will be influenced by the way in which these issues are dealt with.

This paper deals with the effect of alternative specifications of the minimum condition in disequilibrium econometrics. Attention will be paid both to the stochastic structure of the min condition as well as to the effect of aggregation on the min condtion. Until now no empirical analysis relying on economic data has dealt with these issues. The scarce evidence of the effect of alternative stochastic structures of the disequilibrium models consists of results obtained in Monte Carlo studies only (Sneessens (1981a)). Within a simple disequilibrium model, where the sample separation is unknown, we study the effect of the use of a deterministic and a stochastic switching rule, and the aggregation of submarkets. The empirical analysis is carried out using data from the market for new government bond issues in Finland.

The outline of the paper is as follows. First, we present alternative formulations of the minimum conditions and derive the likelihood functions. Second, we discuss economic, statistic and computational aspects that speak in favour of the various specifications. Third, we report on the estimation results of the disequilibrium model using alternative specifications and estimation methods. The estimation results of the disequilibrium model are contrasted to the results of the corresponding equilibrium specification. The conclusions as to the effect of alternative specifications of the min condition and the estimation method will be based on the behavior of the residuals and on the regime indicator.

1. Alternative specifications of the minimum condition

Unlike the other parts of the single market disequilibrium model, the minimum condition is not derived from choice theoretic considerations, but the use of it is justified by referring to the principle of voluntary exchange, i.e. a buyer (seller) cannot be forced to buy (sell) more than the desired quantity (Malinvaud (1977)). Although we take the justification of the assumption of voluntary exchange at the micro level as given[2)], the appropriate specification of the minimum condition at the macro level remains open. Below we consider three specifications, of which the two first are alternatives to each other, and are related to the stochasticity of the switching rule, while the third relates to the aggregation issue. The specification issues are dealt with in a single market disequilibrium model with the sample separation unknown, because of the assumption of the price being an exogeneous variable in the model. Let the most general form of this model be given by equations (1)-(3).

(1) $D_t = a_1 + b_1'X_{1t} + u_{1t}$

(2) $S_t = a_2 + b_2'X_{2t} + u_{2t}$ $\qquad t = 1,\ldots,T$

(3) $Q_t \leq \min(D_t, S_t) + w_t$

where D_t and S_t are the unobservable demand and supply at time t, Q_t the observable quantity traded in the market, X_{1t} the vector of exogeneous variables affecting the demand, X_{2t} the vector of exogeneous variables affecting the supply, u_{1t}, u_{2t} and w_t error terms, and a_1, a_2, b_1 and b_2 the structural parameters to be estimated along with the error variances, all estimates depending on the specification chosen[3)].

The following three specifications of the minimum condition are considered in this paper:

(i) the case of a deterministic minimum condition, where the restriction $w_t = 0$ is imposed, and the equality sign in equation (3) always holds.

This case further assumes that $(u_{1t},u_{2t}) \sim N(0,\Sigma)$, where Σ is a positive definite covariance matrix of the error terms. Σ is here assumed to be a diagonal matrix with diagonal elements σ_1^2 and σ_2^2. The error terms are assumed to be serially uncorrelated[4]. This specification is below referred to as the Maddala-Nelson (MN) minimum condition[5].

(ii) the case of a stochastic minimum condition, where the restriction $u_{1t} = u_{2t} = 0$ is imposed and the equality sign in equation (3) always holds. The variance of the error term w_t, σ_w^2, can vary over the regimes, so that

$$\sigma_w^2 = \begin{cases} \sigma_{w1}^2 & \text{for } D < S \\ \sigma_{w2}^2 & \text{otherwise} \end{cases}$$

This specification is below referred to as the Ginsburgh-Tishler-Zang (GTZ) minimum condition[6].

(iii) the case where either the equality or the inequality sign in equation (3) holds. No assumptions are made of the error terms. This specification is below referred to as the Muellbauer minimum condition (Muellbauer (1978)).

For the deterministic minimum condition Maddala and Nelson (1974) derived the corresponding likelihood function, which can be written as

$$(4)\quad L = \prod_{t=1}^{T} \Big[\frac{1}{\sqrt{2\pi}\sigma_1} \exp\{-\frac{1}{2\sigma_1^2}(Q_t - a_1 - b_1'X_{1t})^2\} \{1 - \Phi(\frac{1}{\sigma_2}(Q_t - a_2 - b_2'X_{2t}))\} + \frac{1}{\sqrt{2\pi}\sigma_2} \exp\{-\frac{1}{2\sigma_2^2}(Q_t - a_2 - b_2'X_{2t})^2\} \{1 - \Phi(\frac{1}{\sigma_1}(Q_t - a_1 - b_1'X_{1t}))\}\Big]$$

where $\Phi(\cdot)$ is the distribution function of the standard normal.

As to the likelihood function of the stochastic minimum condition, it can be written as the product of the density functions of the observed quantities in the two regimes,

$$(5) \qquad L = \left[\prod_{T_1} \left(\frac{1}{\sqrt{2\pi}\sigma_{w1}}\right) \exp\left(-\frac{1}{2\sigma_{w1}^2}\left(Q_t - a_1 - b_1' X_{1t}'\right)^2\right)\right].$$

$$\left[\prod_{T_2} \left(\frac{1}{\sqrt{2\pi}\sigma_{w2}}\right) \exp\left(-\frac{1}{2\sigma_{w2}^2}\left(Q_t - a_2 - b_2' X_{2t}\right)^2\right)\right]$$

where $T_1 = \{t|D < S\}$ and $T_2 = \{t|D > S\}$. Introducing $r = 0$ for $t \in T_1$ and $r = 1$ for $t \in T_2$ and taking the log of equation (5) yields

$$(6) \qquad \ell = \sum_t \ell n \left[(1-r) f_{T1}(Q) + r f_{T2}(Q)\right]$$

where $f_{T1}(Q)$ and $f_{T2}(Q)$ are the density functions in equation (5).

Given the specifications (i)-(iii) and the likelihood functions (4) and (6), we shall next consider various arguments put forward in favour of the alternative specifications of the minimum condition. First we deal with the choice between specifications (i) and (ii). Thereafter we turn to case (iii).

Maddala-Nelson or Ginsburgh-Tishler-Zang?

The arguments presented in favour of or against the MN and the GTZ min conditions can be classified as either economic, statistical or computational.

If we stick to standard econometrics, a behavioral equation to be estimated is usually specified so as to include an error term in order to capture alternative sources that deteriorate a deterministic specification, i.e. the omission of variables, errors of aggregation and of measurement, as well as an incorrect mathematical form of the model. Thus, deterministic behavioral relationships in models to be estimated as those used by the GTZ min condition are scarce, and the behavioral functions in the MN min condition have simpler economic interpretations. The GTZ deterministic behavioral relations combined with the stochastic min condition can however be given a sound economic interpretation in terms of uncertainty of whether the demand (or supply) can be traded in the market. "An equilibrium is no longer defined as the state in which everybody knows everything, it merely means that agents have a correct appraisal of the constraints they might face. The equilibrium is stochastic, not deterministic" (Sneessens (1981b) p. 74). Despite this interpretation, stochastic behavioral relationships as those in case (i) are usually preferred in the light of standard econometrics. Another related, but more fundamental issue which concern both the MN and the GTZ specifications is the point raised by Arrow (1959) as to whether it is misleading or not to extrapolate equilibrium behavior to disequilibrium situations.

The statistical aspects concerning the choice between the MN and the GTZ min conditions relate to the problem of identification and the robustness of inferences based on the model (1)-(3). The former problem concerns only the GTZ specification, while the latter concerns the MN min condition. In the case of a GTZ min condition, where either T_1 or T_2 in equation (5) is an empty set, the structural parameters of the regime corresponding to the empty set are not identified. Identification problems are not, however, present in the MN specification due to the presence of Φ $(\cdot)$ in both terms of the likelihood function in equation (4).

Inference based on the MN specification contains however some other kinds of weak points, which surprisingly little have been noticed in empirical research on disequilibrium models. Inference on the regimes in the MN specification depends on the untestable assumption that

$$(7) \qquad \begin{pmatrix} D_t \\ S_t \end{pmatrix} \sim N \left(\begin{pmatrix} a_1 + b_1'X_{1t} \\ a_2 + b_2'X_{2t} \end{pmatrix}, \Sigma \right)$$

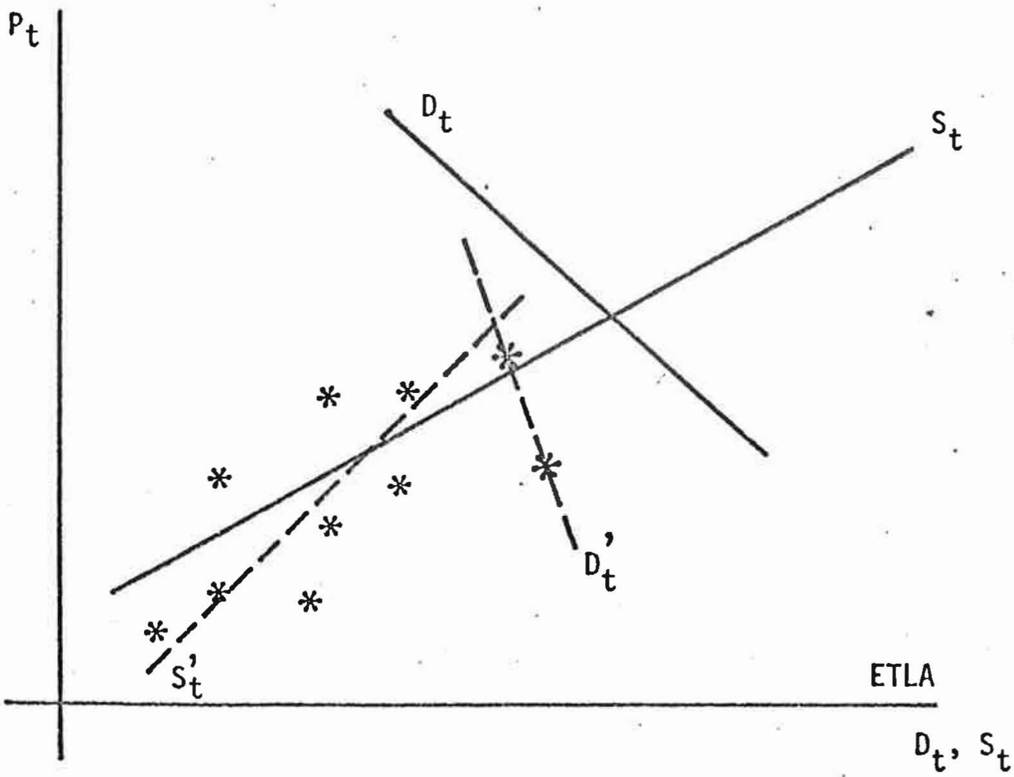
However, there exist an indefinite class of distribution functions that result in the same distribution of the minimum of the two variables, but which imply different regime probabilities (Richard (1980)). This is not the case in the GTZ specification, where the normality assumption of w_t can be tested. Thus, in evaluating the estimation result based on the MN specification and the GTZ specification, the resulting sample separation into excess demand and excess supply regimes is of interest. This comparison of the estimated sample separation is of special interest as the only existing empirical work on the choice between the MN and the GTZ min conditions - the Monte Carlo studies by Sneessens (1981a) - does not consider differences in the sample separation. Because the results obtained there were not unambiguously in favour of either specification, explicit considerations of the estimated sample separation in our study below can throw further light on the problem of the correct specification.

Richard (1980) also points to the fact that the resulting sample separation in the MN specification will be dependent on how the observations fit respective the demand and the supply curve. The estimation of case (i) with the maximum likelihood method will force to either regime at least as many observations as there are unknown parameters in the respective equation. This will be the result even if only one of the two regimes has been operating during the sample period. Assume that D and S in figure 1 represent the true model, and that the observations available thus all come from the excess demand regime, and belong to the supply curve. Nevertheless, the fitted model is likely to be given by D' and S'. A consequence of this phenomena is "that the estimated variances and the corresponding regime probabilities might be very misleading as a consequence of the near perfect fit of one of the two regimes" (Richard (1980)). This problem is present particularly when the sample period is short (Portes and Winter (1980)).

Computational aspects influencing the choice between the two specifications (i) and (ii) relate to how easily the likelihood function converges to a global or local optimum.

Although the functions in both cases, when one of the variances go to zero, are unbounded, in which case the maximization breaks down[7)], the GTZ likelihood function is usually more easy to handle. On the other hand, if a stochastic dynamic price adjustment function, e.g. $p_t = p_{t-1} + \gamma(D_t - S_t) + u_{3t}$ is implemented into the model, the GTZ likelihood function looses its computational attractiveness.

Figure 1. The MN minimum condition



Heterogeneous submarkets

Next we turn to the Muellbauer minimum condition. The relevance of this case is given by the observation that when dealing with disequilibrium models, the focus of analysis is usually a market, which actually consists of several, not neccessarily homogeneous submarkets, and thus the aggregation of excess demand and excess supply submarkets yields a situation, where the minimum condition $Q_t = \min(D_t, S_t)$ must be replaced by $Q_t \leq \min(D_t, S_t)$. The more heterogeneous the submarkets are, the more biased is the former minimum condition, where D_t and S_t now must be interpretated as aggregates of the corresponding quantities from the submarkets j. For the jth submarket holds $D_j = D_t + e_{1j}$ and $S_j = S_t + e_{2t}$. If the minimum condition $Q_j = \min(D_j, S_j)$ holds for every submarket, and the submarkets are heterogeneous, then for every value of the price p, $Q_t \leq \min(D_t, S_t)$.

This question relates to the assumption of market by market efficiency, i.e. only one side of the market is rationed, in the MN and GTZ minimum conditions. If we, however, account for the situation that at the ruling price, some submarkets are in excess demand, while others are in excess supply, the equality sign in the minimum condition cannot be justified. Estimation of a model consisting of a demand function, a supply function and a Muellbauer min condition is, however, not easily handled. Nevertheless, the inefficiency and heterogeneity of the market can be implemented into a quantity rationing model using a friction function. The model will in such case consist of equations (1) and (2) combined with a minimum condition into which the friction function is implemented, e.g. as in equation (8)[8]

$$(8) \qquad Q_t = \min(D_t, S_t) - \frac{a}{1 + b(D_t - S_t)} + w_t$$

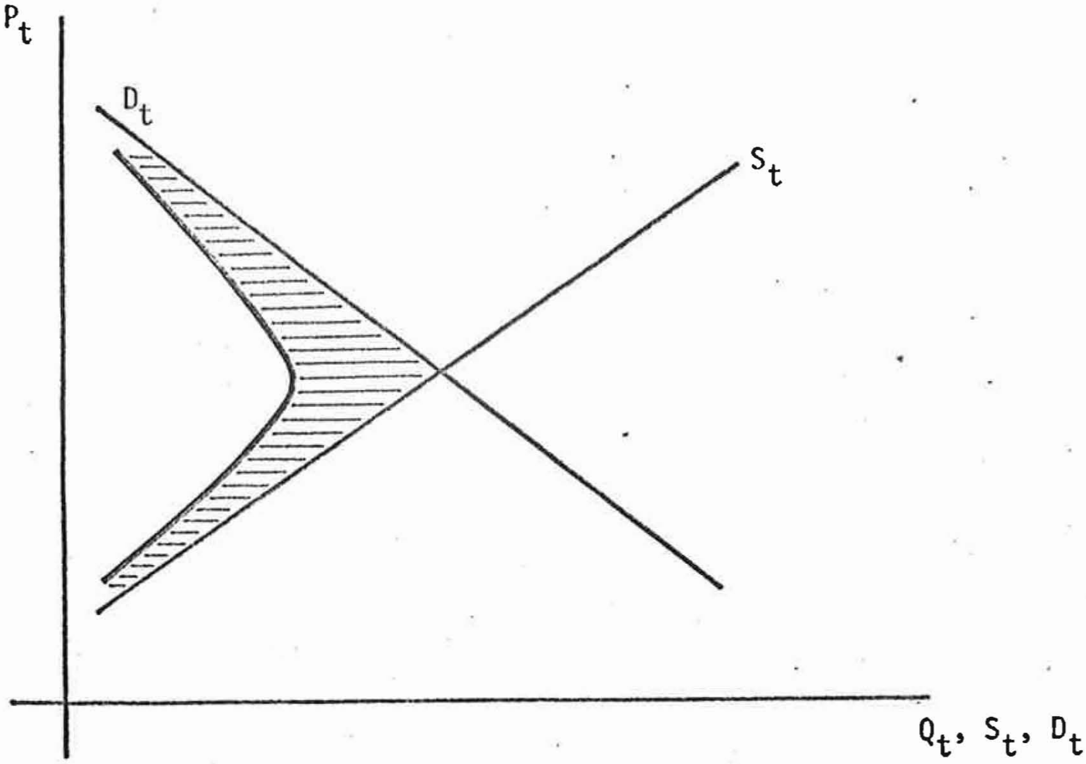
where the parameters a and b determine the extent of heterogeneity of the market. The maximum inefficiency is given when $D_t = S_t$, while the friction due to inefficiency reduces as $/D_t - S_t/$ increases. Graphically, the model is given by figure 2. The likelihood function of the most general form of this friction-augmented disequilibrium model is however complicated. Even so is the case where the MN min condition is enlarged with a friction function. If, however, the GTZ assumptions of the stochastic nature of the model is used, the resulting likelihood function can be handled relatively easily. The model is hence given by

$$(9) \qquad D_t = a_1 + b_1'X_{1t}$$

$$(10) \qquad S_t = a_2 + b_2'X_{2t}$$

$$(11) \qquad Q_t = \min(D_t, S_t) - \frac{a}{1 + b(D_t - S_t)} + w_t$$

Figure 2. Aggregation of heterogenous submarkets



Because the switching rule is non-stochastic we can write the whole model as the sum of two submodels, each of them premultiplied by the binary variable r or (1-r). The observed quantity Q_t can thus be written as $Q_t = (1-r)D_t + r_tS_t - FR_t + w_t$. Assuming $w_t \sim N(0,\sigma_w^2)$, where $\sigma_w^2 = (1-r)^2\sigma_{w1}^2 + r^2\sigma_{w2}^2$ the density function of the observed variable Q_t is given by $h(Q) = \frac{1}{\sqrt{2\pi\sigma_w^2}} \exp\{-\frac{1}{2\sigma_w^2}(Q_t - (1-r)D_t - rS_t + FR_t)^2\}$.

Taking logs of h(Q) yields

$$\log h(Q) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\sigma_w^2 - \frac{1}{2\sigma_w^2}(Q_t - (1-r)D_t - rS_t + FR_t)^2$$

The log likelihood function is then given by equation (12)

$$(12) \qquad L = \sum_{t=1}^{T} \log h(Q) .$$

The problem of the likelihood function in (6) or (12) being non-continuously differentiable can be dealt with using some smoothing procedure (Tishler and Zang (1979))[9]. Non-linear least squares can also be used to estimate the model in equations (9)-(11).

2. Empirical results

The last section of this paper contains some results of disequilibrium econometrics for the Finnish bond market. Beside throwing light upon the usefulness of the disequilibrium approach per se to this market, our aim is to give some empirical results of the effect of alternative specifications of the min condition.

To begin with, one can of course question the appropriateness of the underlying data for disequilibrium econometrics, as the bond market is usually referred to as the prototype of a market with high speed of adjustment to changes in market conditions, and a market, where prices are assumed to reflect all available information. There are, however, several arguments that support a disequilibrium approach to the Finnish bond market. First, empirical evidence suggests that modelling the bond market as a market being always in equilibrium may not be appropriate. On the one hand, the semistrong hypothesis of market efficiency is not unambiguously supported by the Finnish data (Stenius (1981)). On the other hand, preliminary empirical results indicate that we cannot straight off reject the hypothesis of two regimes ruling in the Finnish bond market. Although the methods, used in obtaining this latter empirical evidence, can be questioned (Stenius (1982)), we cannot, taking these result as given, rule out that the bond market in Finland is a market in temporary disequilibrium.

Not only empirical results point to the need of a more exhaustive analysis of the relevance of the disequilibrium framework for the Finnish bond market. On the supply side, government can control the entry to the market for new issues, while the price formation on the new bond issues lacks competitive elements, as the bonds are distributed to investors not through underwriting or competitive bidding, but through financial intermediaries on a 'best effort' principle (Friend (1967)).

On the demand side, slow adjustment giving rise to temporary disequilibrium in the bond market cannot be excluded (Rantala (1976)). This can be due, among other things, to spillover effects from other financial markets in the economy, e.g. the bank loan market, being in disequilibrium.

The data used in the estimations consist of seasonally unadjusted, monthly data from the market for new government bond issues in Finland during the period 1969-1977. All new issues are included in the study. The demand for and supply of bonds are formulated in real terms.

The estimated model is a slightly modified version of that presented in Stenius (1982). The static (flow) demand and supply equations to be estimated are given in equations (13) and (14)

$$(13) \qquad D = \alpha_0 + \alpha_1 RPP + \alpha_2 YD + \alpha_3 ARP + \alpha_1 RDP$$

$$(14) \qquad S = \beta_0 + \beta_1 RPP + \beta_2 DPP + \beta_3 ARP + \beta_4 FRP$$

where

D is the demand for new government bonds, in real terms
S is the supply of new government bonds, in real terms
RPP is a weighted average of the real yield on government bonds in the market for new issues. The weights are given by the supply of bonds at the beginning of the period.
RDP is the real yield on bank deposits
ARP is the sum of interest and redemption payments on government bonds, in real terms
FRP is the average real yield on dollar denominated bonds with medium life in the international bond market
YD is real income
DPP is government total borroqing, in real terms

The model is closed either by the equilibrium condition Q = D = S or by the Maddala-Nelson minimum condition as specified in case (i) above, or by the Ginsburgh-Tishler-Zang minimum condition as specified in case (ii) above.

Equilibrium Models

First, the equilibrium conditions was imposed on the model. Table 1 shows the estimates obtained using various estimation methods. The ordinary least squares estimates are inconsistent, while the 2SLS are not assymptotically efficient. Systems method of estimation (3SLS) yields however assymptotically efficient estimates. The use of estimation methods for systems of equation underlines our view that the Finnish bond market cannot be modelled without explicitly modelling the supply of bonds. One important result in Table 1 must be noticed. On the assumption that the equilibrium specification is correct and the estimation is carried out with appropriate methods in order to take into account simultaneity effects, there exists a clear substitution effect between bonds and bank deposits. This result can be contrasted to previous results e.g. Rantala (1976)) rejecting the substitution hypothesis, a conclusion which can be due either to the use of inconsistent estimates or to a misspecification of the demand function.

Testing of the hypothesis that no interest rate effects are of importance yields for the supply equation irrespective of the estimation method an unambiguous answer. We cannot reject the hypothesis of both interest rates simultaneously being zero. As to the demand equation the joint hypothesis of α_1 and α_4 being zero can on the contrary clearly by rejected. The value of the F-statistic of the latter hypothesis is given in table 1.

Table 1. Equilibrium models

	OLS		2SLS	3SLS
	Reduced form*	Structural form		
α_0	-2.872 (1.242)**	4.220 (1.473)	5.952 (1.780)	2.148 (1.062)
α_1	-0.079 (0.865)	0.622 (4.236)	0.760 (3.790)	0.346 (2.502)
α_2	0.149 (1.629)	-0.015 (1.565)	-0.021 (1.861)	-0.007 (0.986)
α_3	0.406 (8.986)	0.501 (10.60)	0.510 (10.599)	0.481 (10.439)
α_4	0.008 (1.216)	-0.681 (3.558)	-0.857 (3.318)	-0.352 (2.067)
β_0	2.013 (5.384)	0.206 (1.548)	0.170 (1.262)	0.460 (3.895)
β_1		0.050 (0.820)	0.011 (0.171)	0.087 (1.821)
β_2		1.979 (5.182)	2.049 (5.332)	0.665 (2.791)
β_3		0.404 (8.881)	0.401 (8.801)	0.444 (10.216)
β_4		-0.022 (0.378)	0.014 (0.224)	-0.054 (1.198)
F		11.684	10.411	4.860
DW		1.750/1.885	1.728/1.893	1.765/1.716
R^2		0.578/0.617	0.574/0.616	0.560/0.567

* Reduced form: $Q = \alpha_0 + \alpha_1$ RDEP $+ \alpha_2$ FRP $+ \alpha_3$ ARP $+ \alpha_4$ YD $+ \beta_0$ DPP

** Absolute value of the t-statistic is given in parenthesis. The corresponding critical values of the t-statistic at 5 per cent (10 per cent) level of significance are 1.645 (1.282).

*** The covariance matrix Σ of the disturbance terms is

$\begin{bmatrix} 0.676 & 0.656 \\ 0.656 & 0.664 \end{bmatrix}$, (Teil (1971)).

Disequilibrium models

Next we imposed alternative min conditions on the model. These disequilibrium models were estimated using on the one-hand maximum likelihood estimation and on the other hand ordinary least squares. For each estimation method two min conditions were used. First, the disequilibrium model was estimated with the maximum likelihood method with either the MN or the GTZ min condition. Numerical optimization of the likelihood function was accomplished with the Davidon-Fletcher-Powel (DFP) algorithm (Powell (1971)) and the quadratic hill-climbing (GRADX) alogrithm (see Goldfeld and Quandt (1972)). Second, to make possible comparison with methods used in some recent Finnish empirical studies least squares estimation was applied. The GTZ specification of the model was estimated using nonlinear least squares. Ordinary least squares was finally used to estimate a disequilibrium model, which in the terminology of Quandt (1978) is actually not a genuine disequilibrium model, in the sense that the estimation is not based on a model which explicitly incorporates a minimum condition. Instead, digression analysis is used. Sample separation is thus not based on the min condition. Although this method is not really comparable to the other disequilibrium methods used in this paper, we report the results here, because previous work on disequilibrium in various markets in Finland has relied mostly on this method[10)]. Our results below, however, indicate that the results obtained using digression analysis considerably differ from estimations of genuine disequilibrium models, and hence a cautious attitude towards the use of it in the context of disequilibrium models should be taken[11)].

Of the four cases, the estimation by nonlinear least squares was the most inefficient, as it turned out to be very time consuming, because the use of analytical first and second derivatives was not possible. The nonlinear least squares estimation was carried out by the Hooke-Jeeves algorithm (Goldfeld and Quandt (1972)).

Table 2 gives the parameter estimates of the four models. Columns (1) and (2) refer to the maximum likelihood estimation. Here numbers in parenthesis below the parameter estimates are the parameter estimates divided by the assymptotic standard errors. The assymptotic standard errors were obtained by taking the square root of the diagonal elements of the negative inverse of the Hessian matrix of the log-likelihood function evaluated at the optimum.

Column (3) refers to the nonlinear least squares estimation, while column (4) yields the results of digression analysis. A comparison of columns (2) and (3) is informative, since the same model specification is estimated in two different ways.

Consider first the structural parameters of the demand function in the equilibrium models (3SLS) versus those of the disequilibrium models. (MN and GTZ). Most of the parameters in the disequilibrium models are significant at the 5 per cent level of significance. The estimate of the coefficient of the own rate has a higher absolute value in the disequilibrium specification. This holds also for the parameter estimate of the interest rate on bank deposits in the MN and the GTZ cases. The income variable obtains a significant estimate with a correct sign in most of the disequilibrium specifications; in the equilibrium case YD is insignificant and has a negative sign.

Contrary to the equilibrium model, the price variables in the supply function of the disequilibrium specifications are small and insignificant, while the quantity variable have a large influence. The result that the value of the coefficient of the total government borrowing requirement in the dis-

Table 2. Disequilibrium models*

$$D = \alpha_0 + \alpha_1 \, RPP + \alpha_2 \, YD + \alpha_3 \, ARP + \alpha_4 \, RDP$$
$$S = \beta_0 + \beta_1 \, RPP + \beta_3 \, DPP + \beta_4 \, ARP + \beta_5 \, FRP$$
$$Q = \min \, (D,S)$$

	Maximum likelihood		Least squares	
	MN (1)	GTZ (2)	Nonlin	Digression
α_0	-3.283 (3.160)	-3.291 (3.291)	-2.095 (1.243)	0.072 (0.060)
α_1	0.905 (1.877)	0.920 (1.638)	0.182 (1.891)	0.360 (6.62)
α_2	0.009 (1.783)	0.009 (2.184)	0.007 (1.273)	-0.001 (0.31)
α_3	1.696 (3.542)	1.702 (1.704)	1.664 (6.380)	0.749 (28.10)
α_4	-0.836 (1.611)	-0.847 (1.424)	-0.123 (1.000)	-0.362 (5.07)
β_0	0.214 (1.164)	0.209 (0.358)	-0.099 (1.320)	-0.533 (4.99)
β_1	-0.106 (1.232)	-0.103 (1.047)	0.007 (0.208)	-0.707 (0.64)
β_2	3.065 (4.391)	3.046 (3.089)	9.409 (21.712)	6.481 (18.98)
β_3	0.304 (5.840)	0.307 (2.059)	-0.032 (0.993)	0.095 (3.35)
β_4	0.121 (1.492)	0.117 (1.466)	-0.011 (0.309)	0.704 (10.15)
σ^2	0.022 (2.793)	0.027 (1.444)	0.119	0.073
σ^2	0.654 (5.943)	0.632 (5.948)		0.126

* t-statistics in parenthesis

equilibrium framework is four times as large as in the equilibrium model is of special interest, since it can be taken as an indication of government's possibilities to influence the market. Here can only a more detailed analysis of the factors underlying government borrowing in the domestic bond market throw more light on the issue. Taken as a whole, the results in tables 1 and 2 suggest some weaknesses in the specification of the supply function.

As to the maximum likelihood estimates of the MN and GTZ specifications, the parameter estimates strongly conform to each other. The MN specification suggests a somewhat larger interest rate sensitivity both of the demand and the supply than does the GTZ specification. The data seem to fit the specification of the supply function in the GTZ case very poorly. In contrast, the estimation results based on digression analysis suggest a strong interest rate sensitivity of the supply, while the demand seem to react more modestly to changes in both the own interest rate and the interest rate on bank deposits. This result from digression analysis stands in sharp contrast with both the MN and the GTZ results, where interest rate sensitivity of demand is strong relative to that of the supply.

The estimates of the nonlinear least squares estimation method differ from all results above. Only the own interest rate in the demand function and the government borrowing requirement in the supply function obtained significant estimates.

Likelihood surfaces

When maximum likelihood estimation is used, the sensitivity of the model to small changes in the value of some specific parameters can be examined by the use of likelihood surfaces. For a given sample and for fixed values of all but two of the parameters, the likelihood surface can be plotted by varying the values of the two parameters chosen. In figure 3 one likelihood surface map is shown. This corresponds to the maximum likelihood estimation of the MN specification using the DFP optimization algorithm. In this special case all other parameters than those of the own interest rate α_1 and the interest rate on deposits α_4 in the demand function were held constant. The contour is relatively flat over a wide range of the parameters, indicating that the value of the likelihood function is insensitive to a fairly wide range of both α_1 and α_4. The range over which $\alpha_1(\alpha_4)$ was allowed to vary in figure 3 was given by $\alpha_1 \pm \sigma_{(\alpha_1)}$ $(\alpha_4 \pm \sigma_{(\alpha_4)})$.

Predicted versus observed values of Q

In addition to the parameter estimates, the comparison of alternative stochastic structures and the estimation methods can be viewed from the characteristics of the calculated $\hat{Q}$, i.e. from $\hat{Q} = \min(\hat{D},\hat{S})$, where $\hat{Q}$, $\hat{D}$, and $\hat{S}$ represent calculated values of Q, D, and S respectively. If there is no relevant differences between the specifications and the estimation methods used, there should not either be a clear divergence between the various $\hat{Q}$´s obtained on the one hand, and on the other hand the lag structure of $\hat{Q}$ should resemble that of the observed Q.

In figure 4 the autocorrelation function of Q and $\hat{Q} = \min(\hat{D},\hat{S})$ using alternative specifications are given. Some statistics in table 3 gives additional information on Q. As far as the lag structure is concerned,

Figure 3. Likelihood surface of MN specification.

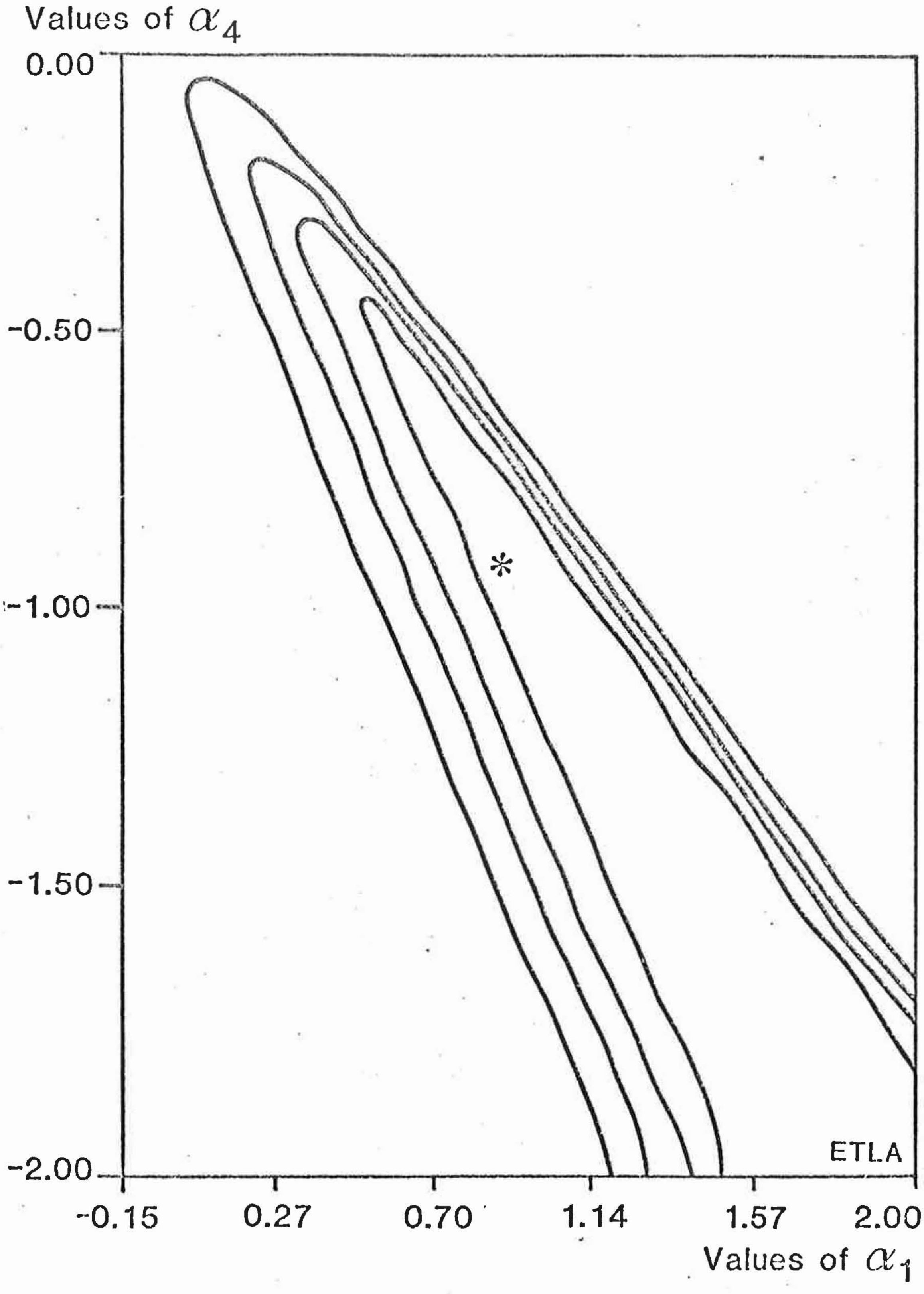
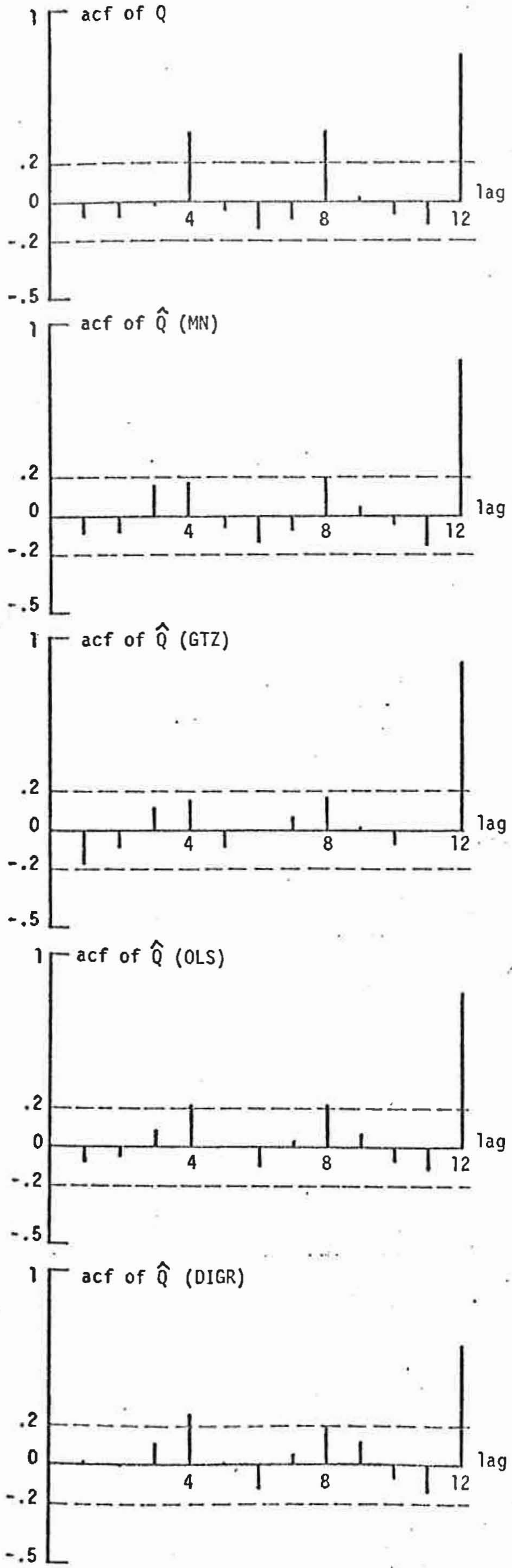


Figure 4. Autocorrelation Function of Q and $\hat{Q}$.



the autocorrelation function of the observed Q, displaying a very clear pattern, is also reflected in the various $\hat{Q}$'s. Only the GTZ specification deviates, in that significant autocorrelation occurs to a larger extent. The Q calculated from digression analysis, on the other hand, yields a range of variation which differs clearly from that of the observed Q, thus implying that digression analysis produces biased results, which together with other problems makes the use of digression analysis questionable. The minimum of the observed Q is zero, while that of Q calculated from digression analysis is -0.842. The corresponding maxima are 5.149 and 2.839 respectively.

Table 3. Some descriptive statistics of Q and $\hat{Q} = \min(\hat{D},\hat{S})$

	Q	MN	GTZ	Nonlin	Dig-ression
Mean	0.853	0.969	0.851	0.951	0.423
Standard deviation	1.194	0.978	1.156	1.231	0.767
Minimum	0.000	-0.068	-0.583	-0.146	-0.842
Maximum	5.149	4.203	4.680	5.109	2.839

Regime indicators

An equilibrium specification of the bond market (e.g. Rantala (1976)) with the supply exogeneous, assumes a priori that the market is always on the demand curve. Government sets the interest rate and the market determines the quantity traded. If we take a closer look at the above estimated regime indicators, given by $(\hat{D} - \hat{S})$, the results strongly contradict this view. The regime indicator obtained in the MN specification showed that nearly 75 per cent of all observations belonged to the supply regime[12]. Thus, there has been an excess demand for government bonds at the prevailing interest rate, which contradicts the equilibrium hypothesis.

As to the estimated regime indicators, those obtained from the MN and the GTZ specifications differed from each other only slightly. Barely 15 per cent of all observations were classified differently by the two model specifications. Hence, not even the estimated regime indicator can decisively separate these specifications from each other. The differences in the sample separation obtained by the maximum likelihood estimation (the MN or the GTZ specification) and the digression analysis were however clearly larger; about 40 per cent of the observations were classified differently. Finally, a comparison of the estimation of models with the same stochastic structure (the additive error term) but with different estimation procedures (maximum likelihood or nonlinear least squares) shows that the sample separation is sensitive to the estimation method used; nearly 35 per cent of the observations differed in classification. Hence, our results show that if maximum likelihood estimation is used the sample separation is not sensitive to the stochastic specification of the disequilibrium model. However, on the contrary, given the stochastic specification, the choice of estimation method (maximum likelihood or nonlinear least squares) affects clearly the sample separation.

There are no tests for discriminating between maximum likelihood and non-linear least squares methods. The results concerning the parameter estimates and the regime indicator imply, however, clearly conclusions which depend on the estimation methods used. Computational ease, nevertheless, clearly discriminates between the two methods in favour of the maximum likelihood method.

Residual analysis

The alternative specifications of the disequilibrium model are next compared with each other by focusing on the residuals of the model, i.e. on $Q - \min(\hat{D},\hat{S})$. All models have in common that the seasonality of Q could not be properly taken into account, and as a result the error term contains the same seasonality as Q. The value of the autocorrelation function of the residuals in all the models differs significantly from zero at a lag length of 12 months. Hence, none of the residuals are neither independent of the observed quantity Q, not even the models where the error term appears in an additive form in the model. Eventual explanations to this could either be a misspecification of the underlying behavioral equations of the disequilibrium model, or the exclusion of a friction function from the specification (e.g. of the form in equation (11)). Further empirical analysis is here needed to enlighten the issue.

As far as the MN specification is concerned, the question of the residual is not easily dealt with, as the error term appears in the model in the switching rule[13)].

Our results above suggested that basing the inference on the sample separation, too much attention need not to be paid to the correct stochastic structure of the model, as the MN and the GTZ specifications yield very similar sample separations. Next we consider whether the residuals obtained from the two specifications can yield information in favour of either specification. If the stochastic structure of the disequilibrium model does not matter, there should not either be any systematic difference between the residuals of the two specifications. The correlation between the two residuals is high, 0.964. A closer look at the difference between RES(MN) and RES(GTZ), however, suggests that there is a systematic difference, and that this difference is not only due to differences in the levels of the residuals, but also to the fluctuations in the variable Q[14)]. The higher the value of Q is, the larger is the difference between the residuals. This might stem from the relatively strong correlation between RES(MN) and Q. Thus, in this respect the stochastic min condition in the GTZ specification appears to be able to handle the strong seasonality in Q better than the deterministic min condition in the MN specification.

Conclusions

In this paper alternative stochastic structures of the disequilibrium model have been dealt with. The two main alternatives, i.e. the the stochastic and the deterministic minimum condition were compared within a single market disequilibrium model of the Finnish bond market. The empirical evidence showed no clear differences in the parameter estimates, and the lag structure of the calculated $\hat{Q}$ compared to the observed Q could not either play a decisive role in the choice of the correct specification. The two stochastic specifications yielded also approximately the same sample separation. Autocorrelation in the error terms is however more easily analysed in the stochastic min condition case.

Hence the empirical results presented here cannot be used as evidence clearly favouring either specification. Thus it confirms results obtained from Monte Carlo studies (Sneessens (1981a) which indicate that one should perhaps not pay too much attention to the true stochastic specification. Our results however suggest that the use of the GTZ specification may be an easier way to handle a variety of problems, such as aggregation of heterogeneous submarkets, serial correlation of error terms, seasonality and computational difficulties.

Nevertheless, in the case of a more sophisticated model, where e.g. a stochastic price adjustment is implemented, the choice of the stochastic structure will most probably be based on the same criteria as those used in earlier studies, i.e. on the ease of deriving the likelihood function and on the computational ease of finding the optimum.

FOOTNOTES

1) The housing market is studied in Fair and Jaffee (1972) and the FHLBB market by Goldfeld, Jaffee and Quandt (1980).

2) The possibility of the realized outcome in the market being between the quantity demanded and the quantity supplied is touched on in Rosen and Quandt (1979).

3) Because of its static nature, the model ignores the effect of past unsatisfied demand and supply. Spillover effects, past or contemporaneous, are also excluded from the model.

4) The case of non zero covariances is dealt with in Goldfeld and Quandt (1978). The likelihood function for the simple single market disequilibrium model with autocorrelated errors is derived in Quandt (1981).

5) This specification is also referred to as the stochastic switching rule.

6) This specification is also referred to as the deterministic switching rule.

7) This case can be circumvented by restricting for the MN case $\sigma_1^2 = k\sigma_2^2$, and for the GTZ case $\sigma_{w1}^2 = k\sigma_{w2}^2$.

8) For details see Sneessens (1981b).

9) In estimating the GTZ specification with the maximum likelihood method, the following smoothing procedure was used:

$$r_t = \begin{cases} 0 & \text{if } E_t \leq -\varepsilon \\ -\frac{3}{16}\left(\frac{E_t}{\alpha}\right)^5 - \frac{5}{8}\left(\frac{E_t}{\alpha}\right)^3 + \frac{15}{16}\left(\frac{E_t}{\alpha}\right) + \frac{1}{2} & \text{if } -\varepsilon \leq E_t \leq \varepsilon \\ 1 & \varepsilon \leq E_t \end{cases}$$

where ε is an arbitrarily small number and $E_t = a_1 + b_1'X_{1t} - a_2 - b_2'X_{2t}$.

10) Observations are allocated in two clusters corresponding to two models (the demand and the supply equation) according to the 'digression criterion' (Mustonen (1979)). For an application of this estimation method to the bank loan market see Tarkka (1979), to the labour market Virén (1981) and to the bond market Stenius (1982).

11) The estimation was performed by using the CLUSTREG module of SURVO'76.

12) The GTZ specifications classifies slightly more than 60 per cent of the observations as belonging to supply regime.

13) The difficulty in testing for serial correlation of the error terms in the demand and the supply equations has been noticed in Fair and Keleijan (1974). No explicit empirical analysis of the residuals can be found in the literature.

14) A regression of the difference between the residuals on Q yields the following result:

$$(RES(MN) - RES(GTZ)) = \underset{(21.02)}{-0.371} + \underset{(8.34)}{0.093}\,Q \qquad R^2 = .42$$

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