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ON THE DEFINITION

AND MEASUREMENT

OF THE FLOW CAPITAL SERVICES

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On the Definition and Measurement of the Flow of Capital Services

Introduction

Like Janus, a measure of the capital stock can look backward or forward. In the former case it estimates the productive capacity of existing assets, while in the latter measure it estimates the sum of undiscounted future outputs derived from the assets. If the lifetime of all assets were infinite, the two measures would necessarily agree. But since this condition is not met in practice, the relation between the output-producing capacity of an asset and its future productive potential depends on the service life remaining in the asset. For assets of a common type (lifetime), therefore, the current output-producing capacity depends on how many of them there are in existence at this point in time. But the future output stream depends in addition on the age distribution of the assets in question. In principle the backward-looking measure gives us the gross stock of capital whereas the forward-looking measure gives us the net stock of capital. Clearly, when assets of different types are included in the capital stock, the net stock will depend in addition on the distribution of the stock by classes of assets as well as the age distribution for each class considered separately. However, the gross stock will still depend only on the total value of the assets in existence (not yet scrapped). All this of course assumes that the productive capacity of a given asset does not change with age until it is scrapped.

Previous studies of multi-factor productivity change have used various measures of the flow of real capital services. For example, Gollop and

Jorgenson use a measure of the capital stock which they derive directly from a permanent inventory model according to the formula:

$$N_t = (1 - d).N_{t-1} + I_t$$

in which N_t is the net stock of capital at the end of year t, I_t is investment (gross fixed capital formation) in year t, and d is the rate at which the relative efficiency of capital goods is assumed to decline (geometrically) with age. The formula applies to a specific class of capital good, classified by asset type and industry of use, and the various classes of assets are distinguished by different values of the parameter d. G&J derive the parameter d from an assumption about the average lifetime of the asset in question as "twice the reciprocal of the lifetime", and they note that this results in "double declining replacement patterns for all assets". Thus d = .1 for an asset with an assumed life of 20 years. Note that in this formulation capital consumption is proportional to the capital stock, but is different from the measure of capital consumption derived in the normal manner from a perpetual inventory model of the gross capital stock with fixed lifetimes for assets. This is because the latter follows from a straight-line reducing balance formula connecting the gross concept with the net measure of the capital stock.

One problem relating to the G&J capital stock measure is that the double declining balance method "makes the ability of an individual capital good to contribute to current production drop implausibly fast. It can also reduce the average service life below what they say they accept, but this version does not clarify how, or whether, they have prevented this" (Daly, 1975).

When physical depreciation is assumed to be proportional to the stock of capital, no distinction can be drawn between the net and gross stocks. Moreover, the stock itself or, equivalently, the depreciation (capital consumption) of the stock may be used as a measure to inpute the potential capital services available. Such an imputation would be derived from the principle that, after adjusting for depreciation, all assets in existence are equivalent in productive capacity.

It can be seen therefore that an assumption of geometrically distributed depreciation makes the imputation of service flow rather straightforward. Unfortunately, establishing empirically correct patterns of depreciation is both difficult and, insofar as results are currently available, not without ambiquity - see Coen (1980) for example. One way round this problem is to use some results from renewal theory which imply that all reasonable forms of depreciation give almost the same aggregate depreciation figure when the age distribution of the capital stock is constant. This line of reasoning justifies the commonly used geometric decay assumption. The implicit assumption that the age distribution of the capital stock is in a steady-state may not, however, be to everyone's taste. Moreover, this approach implies the need to construct estimates of the capital stock which are different from officially produces estimates. The latter normally assume the lifespan of assets to be limited, often distributed round some average lifespan, and within which relatively conservative assumptions about any decline in productive efficiency with age are made. In what follows consideration is given as to how the flow of capital services shold be imputed from typical officially produced perpetual inventory estimates of the capital stock.

If a stock figure is used as a proxy for the flow of real capital services, it is not obvious that it should be the net stock. Kendrick (1973) used a net stock concept, noting that because "an increase in the average age of depreciable assets will tend to be associated with decreases in productivity" because of "physical deterioration, increased downtime for repair, and creeping obsolescence which may result in shifts of the assets to less productive uses", the assumption implicit in the gross stock that "the full value of the depreciable capital inputs are carried until they are discarded from stock" is not fully appropriate. However, Kendrick agrees with Daly about the declining balance method of estimating depreciation, stating that "it is not plausible that the output-producing capacity (as distinct from the present value of the future net income stream) declines more in early years than in later years". Hence Kendrick's net stock figures are based on straight-line depreciation, but even so he admits that "it probably tends to overstate the overall decline in output-producing capacity of depreciable assets as they age". He also notes the preference of both Goldsmith and Denison for a (weighted) average of real gross and net stocks. In a similar vein, Domar (1961) argues that whether the capital stock should be defined gross or net of depreciation depends on the the relation between the productive qualities of capital and its age, and that this may vary among different countries, industries and kinds of assets: "In the absence of relevant information, some deduction from the value of the gross stock of capital should be made, though I suspect that, at least in the advanced countries, it should be below conventional depreciation, heavily weighted as the latter is with tax considerations".

Measuring the flow of capital services

The basic measurement difficulty stems from the fact that the size of the capital stock and the rate of flow of capital services from that stock must be measured by imputation rather than observation. This arises because normally there is no separation between ownership and use of this factor of production, and hence no observable market in capital services. In addition, the fact that an item of capital is not (by definition) used-up in the statistically-defined production period, implies that the capital services needed in different years cannot be purchased separately. Hence the need to distribute the productive services of a capital asset over its lifetime by some method of imputation. Such an imputation is needed to derive national income estimates, in which case it is called "capital consumption". What is the most sensible pattern of capital consumption, from the standpoint of the flow of real productive output services?

It is not correct to measure the rate of flow of capital services by the decline in value of the capital asset, or "economic depreciation", because that reflects in large part the discounted value of the shortening remaining lifespan of the asset. Even if an asset's productive qualities remain unchanged throughout its life, it will decline in value as it ages for two reasons: first because it is older, and hence can produce less in the remainder of its life; and secondly because at any moment of time future output is worth less than current output - ie. is discounted to the present. Neither of these factors is valid if we are examining ex post the relation between output and inputs.

If we measure the depreciation of an asset by the decline in the undiscounted future productive potential over its remaining lifespan, as in the "straight-line" method of calculating depreciation, then we are implicity distributing the flow of capital services evenly over the asset's life. For any given asset this may not be appropriate, but, as Hibbert et al (1977) observe: "When we are concerned with estimating the volume of capital services provided each year by the capital stock of a given industry, it may be reasonable to adopt the assumption that for the stock as a whole - even if not for any particular capital good - the services provided can be regarded as flowing at a constant rate over the lifetime of each part of the stock". The authors recogize, however, that this argument is strictly only valid for an industry which has its capital stock in a more or less steady-state configuration (considering the distribution of the capital stock by asset type and by age). But for industries that are growing or declining rapidly the assumption may be questionable. This may be of some importance if, as Barna (1957) and Stuvel (1955) have argued, the survival curve is linear declining as opposed to rectangular. However, let us accept the "straight-line" assumption for capital consumption as a first approximation to what is required.

Accepting for the moment the validity of the straigh-line depreciation method as a measure of the flow of capital services, there is nevertheless a problem in using common perpetual inventory estimates of this measure, even when they embody the straight-line assumption. The problem arises because of the way in which assets of different durabilities are aggregated together. The constant price replacement cost value of an asset reflects not only its current (and future, steady) output potential, but also its length of life. In other words, the price weights that are used in forming the gross stock estimates include an element of undesired discounting.

To clarify the way in which the normal estimates distort the desired measure of capital services, consider how two assets, A and B, which are otherwise identical except that the latter lasts twice as long as the former, enter the capital stock. Assume that A's lifetime is n years, and let A and B represent the total undiscounted volume of capital services over their respective lifetimes as will as labelling the assets. Then the appropriate measure of capital services per year from these two assets is:

$$K = \frac{A}{n} + \frac{B}{2n}$$

with both assets still in production. But annual capital consumption from a perpetual inventory estimate of the gross stock with market price weights is:

$$K' = \frac{A'}{n} + \frac{B'}{2n}$$

where

$$A' = \frac{A.(1 - e^{-rn})}{rn} \text{ and } B' = \frac{B.(1 - e^{-2rn})}{2rn}$$

By assumption, both assets produce the same volume of capital services each year of their lives, but because future output is discounted at rate r, the annual service flow from the longer-lasting asset appears to be smaller when aggregated together with the other asset using market price weights. Ideally B should enter the capital stock with twice the weight of A, to reflect its doubled longevity, but will do so only if the discount rate r is zero. As r tends to zero, so the expression $\frac{1}{r}(1 - e^{-rm})$ tends to m, the length of life, and the perpetual inventory figure for capital consumption converges on the concept desired for an historical analysis of production[1].

Let us develop the argument step by step. Denote by $K_{i,t,T}$ the amount of capital good i existing in the current period, t, which had been installed T periods before. Let d_i represent the lifetime or durability of asset type i. Then the gross stock of asset type i in the current period is given by

$$G_{i,t} = \sum_{T=0}^{d_i} K_{i,t,T}$$

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1) The argument outlined above owes its origins to Johansen and Sørsveen (1967), who in turn used a theoretical underpinning provided by Haavelmo (1960). It is also used by Hibbert et al (1977), who consider not only that it provides a sounder basis for examining the relation between capital and output, compared for example with the gross or net stock, but also make the point that the calculation of the flow of capital services for any given rate of discount by the above methods would be a relatively simple task when tabulations of the gross stock classified by original expected length of life already exist.
A related, but slightly different exposition is given by Ward (1976) who bases his account on the use of a factor income identity.

and the gross stock of all types of assets is:

$$G_t = \sum_i G_{i,t}$$

But, assuming that there is no discounting in the value of assets of varying durabilities, so that an asset that lasts 20 years is double the price of one that lasts 10 years, then the annual flow of capital services is proportional to annual capital consumption, C_t:

$$H_t(r=0) = C_t = \sum_i \frac{K_{i,t}}{d_i}$$

However, in general, market prices <u>do</u> reflect the discounted value of future capital services, and hence the above formula will tend to under-value the annual productive services of more durable components of the capital stock.

To allow for the fact that market price weights incorporate a discount factor, the flow of capital services is more generally given by:

$$H_t(r) = \sum_i \frac{r}{(1 - \exp(-rd_i))} . K_{i,t}$$

$$= \sum_i Z(r,d_i).K_{i,t}$$

For a derivation of the formula, consider two capital goods each producing a constant and equal amount of real capital services per year, denoted by y. The assets differ in their durability - the first lasts d_1 years while the second lasts d_2 years. The present value of good 1 is:

$$V_1 = \int_0^{d_1} y.e^{-rt}\,dt$$

$$= y.\frac{(1 - e^{-rd_1})}{r}$$

$$= \frac{y}{Z(r,d_1)}$$

and that of good 2 is, similarly:

$$V_2 = \frac{y}{Z(r,d_2)}$$

so we would expect the prices of the two capital goods to stand in the ratio $V_1:V_2$ when new.

However, from the standpoint of an historical analysis of the output-producing capacity of capital as a factor of production, the way in which these assets should enter the capital stock would reflect their output-producing capacity over the whole span of their lives, and would not be discounted. Ideally, therefore, the gross stock is given by:

$$G^{\S} = y.d_1 + y.d_2$$

and the annual flow of real capital services is equal to annual (straight-line) capital consumption:

$$H = 2y\ .$$

But aggregation by market values gives the measured capital stock as:

$$G = {}^{y}/Z(r,d_1) + {}^{y}/Z(r,d_2)$$

(which yields the ideal gross stock if $r = 0$) .

Now the annual flow of capital services can be recovered as:

$$H = V_1.Z(r,d_1) + V_2.Z(r,d_2)$$

The function $Z(r,d_i)$ is non-linear, but a reasonable approximation may be achieved by a linear function of the form:

$$\hat{Z} = a(r) + b(r).\frac{1}{d_1}$$

for a given value of r. Of course for a different r the intercept and slope of the approximation will be different - ie. a and b are functions of r.

The approximation implies that the annual flow of capital services can be approximated by:

$$\hat{H} = \sum_i (a + b.\frac{1}{d_i}).K_i$$

$$= \sum_i a.K_i + \sum_i b.\frac{K_i}{d_i}$$

$$= a.\sum_i K_i + b.\sum_i \frac{K_i}{d_i}$$

$$= a.G + b.C$$

allowing capital services to be approximated by a weighted sum of the gross stock and capital consumption.

The next question is: what rate of discount should be assumed?

First let us observe that the argument is expressed entirely in real terms, so the appropriate rate of discount should not reflect actual or anticipated inflation. A real rate of interest is called for. There seem to be two kinds of possible candidates: first a rate that corresponds in some sense to time preference, and secondly a rate that reflects the rate of return to capital in the industry. These rather different concepts are also likely to be numerically distinct - the former of the order of 2 or 3 per cent per annum, and the latter between 6 and 12 per cent per annum for most industries (see Fraumeni and Jorgenson (1980) for a recent estimate for the U.S.). As far as the measurement of capital services is concerned, the former concept would yield a measure close to capital consumption, while the latter would correspond more nearly to the gross stock in its relative movements.

The price of an asset should not depend on the industry of use - the price will be the same in all industries. But the rate of return to capital does vary across industries, reflecting different profit opportunities, different risk factors, differential intensity of competition and so on. This suggests that the rate of return concept is not the appropriate one for the discount factor to apply to differing durabilities of a given asset. In fact, considering that the argument depended only on the time factor inherent in a longer life, it would seem more suitable to use a discount factor that is very close to a pure time-preference concept.

The table below shows how different measures of the flow of capital services imply different relative changes for the four quinquennia from 1960 to 1980 for the food, drink and tobacco industry in the U.K.. Of course these figures should not be taken as representing a "typical" industry - the comparison might be rather different for an industry with a different composition of physical assets by durability. In particular, this industry has a large proportion of its assets in the

Table 1. MEASURES OF THE FLOW OF CAPITAL SERVICES

U.K. Food, Drink and Tobacco Industry
Average annual rate of growth (%)

	1960-65	1965-70	1970-75	1975-80
Net Stock	5.57	4.76	3.69	1.63
Gross Stock	4.83	4.57	3.97	2.41
Cap Consumptn.	5.50	5.07	4.02	2.35
Cap. Services[1]	5.16	4.82	3.96	2.36

1. Calculated on the basis of a discount rate of .03.

form of building and structures - about 48 % compared to an average of 38 % for manufacturing industry as a whole in 1970. This implies that capital consumption will probably diverge further from the gross stock in its relative changes than what is typical for manufacturing industry. On the other hand a highish rate of discount of 3 % p.a. is assumed, which to some extent may overweight the influence of durable assets like buildings and structures.

Consider now how th function $Z(r,d_i)$, the weight function for capital goods of varying durability, behaves as a function of durability. Or, more precisely, how it behaves as a function of the replacement rate which is the inverse of durability. In practice the durability of fixed assets is assumed to lie between 10 years[2)] (the assumed length of life of vehicles in the U.K. perpeptual inventory model) and 80 years (buildings and structures). Within those limits the function's deviation from linearity is barely perceptible at the low discount rates that correspond to a time-preference concept. This is shown in diagram 1., in which a discount rate of 2½% p.a. is assumed. The diagram also shows capital consumption, which is of course equivalent to a weight function incorporating a zero rate of discount, and is strictly equal to the replacement rate. The diagram, however, has been drawn with the slope of the capital consumption curve adjusted so that it cuts the capital services curve at a replacement rate corresponding to durability of 30 years. If in fact 30 years was the average durability of fixed assets in a particular industry, then comparison between the two schedules would indicate the degree to which the two measures differ in the importance accorded do assets with differing lifespans.

Also shown on the diagram is a horizontal straight line drawn through the cutting point of the other two curves. It represents the gross capital stock measured in the conventional manner, which of course

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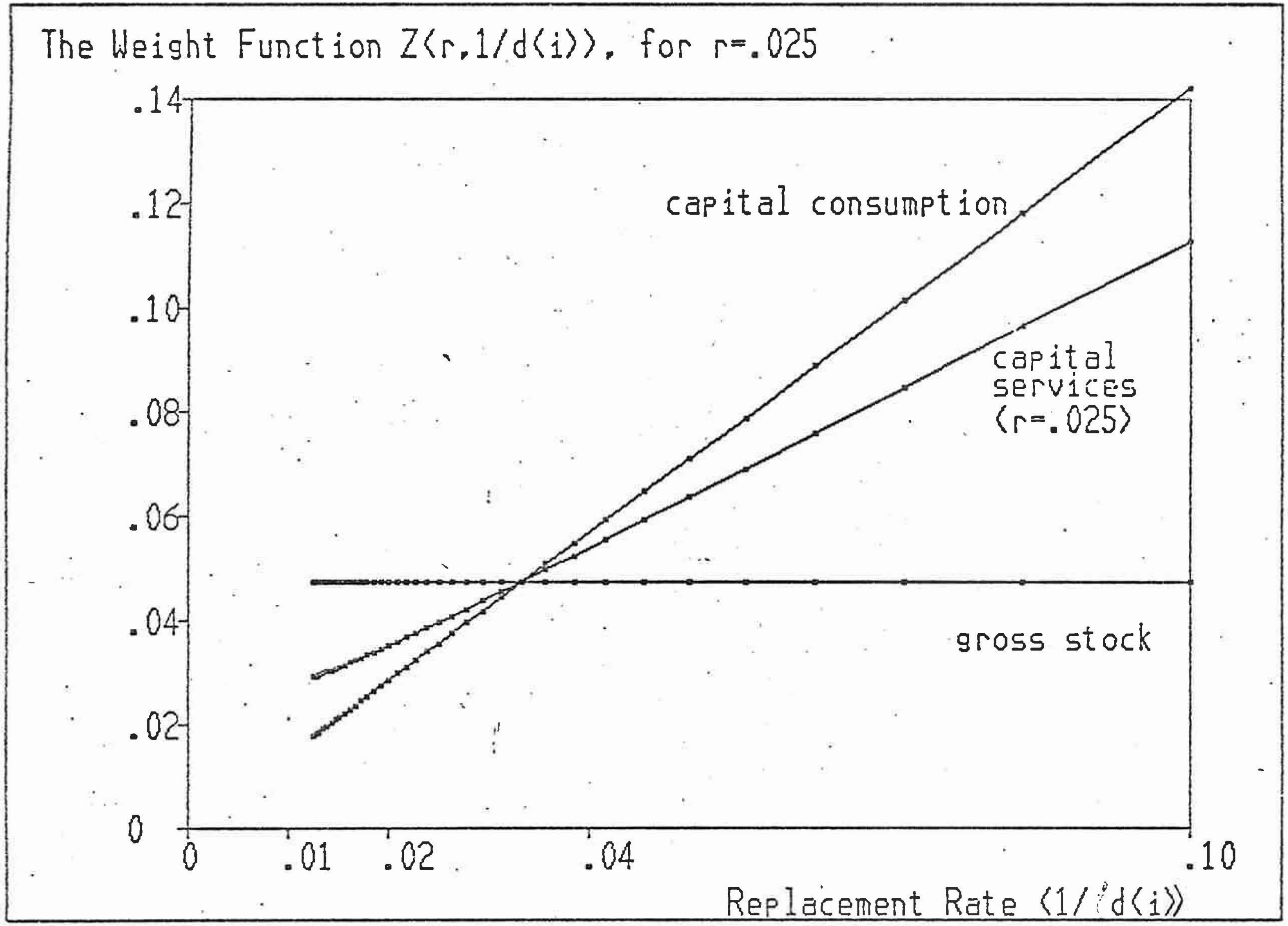
2) Though in the motor vehicle industry itself, certain tools, representing about one third of total expenditure on plant and machinery, are assumed to have a life of only five years, see Hibbert et al (1977).

Diagram 1.


The Weight Function Z(r,1/d(i)), for r=.025
.14
.12
.10
.08
.06
.04
.02
0
0
.01
.02
.04
.10
Replacement Rate (1/d(i))
capital consumption
capital
services
(r=.025)
gross stock

implicitly assigns an equal weight to assets of varying durability when used as a proxy measure of the flow of capital services. It is positioned at the common intersection of the curves to indicate the fact that it emphasises the longer-lasting assets at the cost of the less durable assets in relation to the other measures.

The diagram suggests very clearly that the approximation of capital services by a weighted average of the other two measures may in practice be rather successful. Since the gross stock and capital consumption are routinely produced estimates, an average of the two that accurately approximates a more awkwardly derived statistic has much to commend it. The approximation is good enough, over the range of interest for simple interpolation to be sufficient to calculate the slope (0.945) and intercept (0.0168). These imply the following weighted average:

$$H = 0.0174G + 0.9826C$$

This may be compared to the suggestion made by Ward (1976), which gives:

$$H^{W} = 0.0244G + 0.9756C$$

on the basis of a 2½% interest rate[3]. The difference between these estimators in most applications would be negligible.

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3) Ward suggests measuring the "real annual flow of capital services" by C + r.G. He suggests, however, that r may vary between industries and time periods to reflect risk, uncertainty etc - which would appear to imply a discount rate rather higher than a pure time-preference rate. This would weaken the approximation proposed above. But Ward also asserts, later, that r should represent a time preference rate. Be that as it may, the foundation of his argument is rather different from the above.

Sensitivity of Productivity Estimates to the Definition of Capital

In this section estimates of multifactor productivity change are presented for 12 Finnish industries over the period 1960 to 1980. The estimates have been calculated by subtracting a Törnqvist index of relative changes in inputs from the relative change in real gross output:

$$z = q_t - \sum_i w_{it} . x_{it}$$

in which q represents gross output, x_L represents labour input (hours worked), x_M represents inputs of materials and intermediate products, and x_K represents inputs of capital services. The variables are measured as relative changes, ie. as differenced logarithms.

$$x_{it} = \ln(X_{it}) - \ln(X_{it-1})$$

The w_{it} are the corresponding income (cost) shares, calculated as the average share for the years t and t-1.

The calculations were carried out with two alternative definitions of the capital services variable: first the officially produced net stock figures, and secondly a weighted sum of the gross stock and capital consumption according to the formula given in the previous section. Results are presented in table 2. Since the purpose of multifactor productivity estimation is usually to examine longer-term trend behaviour, which captures technical changes and economies of scale and which might be expected to evolve fairly smoothly over time, decenniel and annual variability are also examined. The results are summarised below under two headings: average productivity change, and the variability of productivity change.

Table 2. MULTIFACTOR PRODUCTIVITY MEASURES AND THE DEFINITION OF CAPITAL

log percentage changes in productivity§

	1961 - 1980				Change in average	
	Annual Average		Standard Deviation		1961-'70 to 1971-'80	
Capital	Net Stock	Services	Net Stock	Services	Net Stock	Services
SIC						
2	1.181	1.521	4.56	4.92	-1.789	-2.268
3	0.916	0.885	1.12	1.06	-0.348	-0.312
31	0.335	0.278	0.57	0.56	-0.173	-0.083
32	1.427	1.360	1.10	1.09	-0.391	-0.203
33	0.664	0.698	2.16	2.07	-0.374	-0.349
34	0.685	0.596	2.04	2.07	-0.101	-0.062
341	0.670	0.564	2.07	2.13	-0.153	-0.125
342	0.462	0.473	1.69	1.64	0.424	0.384
35	1.407	1.301	2.04	2.03	-1.862	-1.907
36	2.127	2.005	2.30	2.15	-1.788	-1.911
37	0.950	0.927	2.34	2.33	0.390	0.106
38	1.408	1.473	1.39	1.35	-0.574	-0.469
39	2.727	2.699	3.33	3.23	-3.716	-3.319
4	1.595	1.441	2.21	2.06	-2.029	-1.794

§ The log percentage change is defined as $100.(\log_e(C_t) - \log_e(X_{t-1}))$, and it is symbolised by %. It takes values close to the normal percentage change for small relative changes, say less than 10 %, but has the desirable property that sums and differences are consistent.

1) Average productivity change

The net stock measure results in a faster estimate of productivity change in 8 out of 12 distinct industries (ignoring SIC 3, Manufacturing, and SIC 34, Paper and Printing, which are aggregates of other industries) for the full period 1961 to 1980. The same is true for each decade of the period, though there are differences in the set of industries under is different between the decades.

2) The variability of productivity change

a) Comparing the decades

For all industries other than Printing & Publishing (SIC 342) and Metal Manufacture (SIC 37) the average rate of productivity change in the period 1971-1980 is less than that for 1961-1970. The median decline is 0.48% on the net stock definition and 0.41% on the "services" definition of the capital stock.

The absolute change in annual average productivity growth between the decades (ie. disregarding the sign of the change) is less for the services definition in all but two industries: SIC 35 (Chemicals) and SIC 36 (Stone, Clay, Glass etc.). The median absolute change for the net stock definition is 0.50% whereas that for the "services" definition is 0.43%.

b) Annual variability

Using the standard deviation of annual log percentage change as the definition of variability, and considering the whole period 1961 to 1980, the variability of productivity changes is greater under the net stock definition than under the "services" definition for all industries except SIC 2 (Mining & Quarrying) and SIC 341 (Paper and Board).

However, the standard deviation is not the only possible measure of variability, and it coud be argued that, since the average growth in productivity tends to be greater under the net stock definition, it might be preferable to use the coefficient of variation (standard deviation divided by the mean). Under this definition the situation is more even, with six industries showing greater variability for the net stock definition and six for the "services" definition.

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