ETLA **ELINKEINOELÄMÄN TUTKIMUSLAITOS**
THE RESEARCH INSTITUTE OF THE FINNISH ECONOMY
Lönnrotinkatu 4 B, 00120 Helsinki 12, Finland, tel. 601322

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Antti Suvanto*

OPTIMAL QUOTATIONS AND FOREIGN EXCHANGE DEALING: A DEALER WITH SHORT-TERM MONOPOLY

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1. *Introduction*

In an earlier paper (Suvanto, 1982) we studied the behaviour of an individual foreign exchange dealer assuming that the dealer is a profit maximizing agent who has invested resources in non-interest bearing demand deposits with commercial banks both at home and abroad and who stands ready to buy and sell foreign exchange on immediate demand. In that paper our aim was to formulate the transactions demand for foreign exchange, or to be more accurate, the demand for money for international transactions purposes in a world where the public in each country holds only their domestic money which can immediately be converted into foreign exchange or *vice versa* with foreign exchange dealers.

The dealer was assumed to operate under competitive circumstances and therefore to have to accept the ask and bid prices given by the market. It was shown that under these circumstances the optimal size of the dealer´s trading portfolio, containing both domestic and foreign currencies, depends negatively on the opportunity cost of holding money and positively on the volume of retail transactions and the cost of a wholesale transaction according to a qubic root rule similar to that derived by Miller and Orr (1966) in their analysis of the demand for cash balances by a firm.

The results also suggested that there are likely to be economies of scale in the dealer´s currency holdings. This would imply that a dealer who has greater market share than others is able to give more favourable quotations to customer buyers and sellers, i.e. to apply a smaller ask-bid spread, thus increasing his market share still further. This, in turn, would imply that the assumption of a fully competitive market in the short-term cannot be maintained.

The present paper takes a departure from the earlier one as regards the market structure. In the following we analyze the behaviour of a foreign exchange dealer who has a short-term monopoly in the sense that he can affect the flow of expected buy and sell orders by changing quotations. We assume that the dealer has a given target for his foreign exchange position at the end of the day, *e.g.* a closed position, and that the dealer can make new quotations a finite number of times during a trading day.

It is shown that a revenue maximizing dealer will always choose such a quotation that keeps his expected position in a straight course toward the target. The price adjustment is made by changing the mid-rate, whereas the ask-bid spread is always chosen to maximize one-period return on the equilibrium volume of trade.

2. Some Preliminaries

In order to introduce the notation and to show that our problem is, in fact, a fairly straightforward application of standard microeconomic reasoning we solve the dealer's pricing problem in some simple one-period cases.

Let us write the customer's buy and sell orders of foreign currency per trading period as follows:

(1) $D(s+z) + u = a - b(s+z) + u$ (flow demand)

(2) $S(s-z) + v = -c + d(s-z) + v$ (flow supply)

where s+z is the ask-rate, s-z is the bid-rate, s is the mid-rate. and z is one half of the ask-bid spread. The parameters a,b,c, and d are all positive, and we assume that $a-c > 0$ and $ad-bc > 0$ to quarantee positive price and volume of trade in equilibrium. The exchange rate is defined as the price of one unit of foreign currency (dollar) in terms of domestic money (mark). The stochastic variables, u and v, have zero expectation and a finite variance.

Expected revenue, defined as net cash flow of domestic money, is

(3) $$R(s,z) = (s+z)D(s+z) - (s-z)S(s-z)$$

$$= \alpha s + \beta z - \gamma s^2 - \delta z^2 - 2\delta sz ,$$

where $\alpha = a+c$, $\beta = a-c$, $\gamma = b+d$, $\delta = b-d$; $\alpha,\beta,\gamma > 0$, $\delta \gtrless 0$.

The excess demand, or the dealer's net sales of dollars is defined by

(4) $$X(s,z) - w = \alpha - \gamma s - \delta z - w ,$$

Where $w = v-u$, $Ew = 0$, $Ew^2 \equiv \sigma_w^2 = Ev^2 + Eu^2 - 2Euv$.

Define the equilibrium quotation $(\hat{s},\hat{z})$ as the quotation that will maximize the expected revenue subject to the constraint that expected net sales are equal to zero. The solution to this problem gives

$$(5)\qquad \begin{cases} \hat{s} = (1/\gamma)(\alpha-\delta\hat{z}) \\ \hat{z} = (\beta\gamma-\alpha\delta)/2(\gamma^2-\delta^2)\,. \end{cases}$$

It is easily checked that both the ask-rate and the bid-rate are positive and that the proportionate ask-bid spread $0 < 2\hat{z}/\hat{s} < 1$.

Assume next that the dealer has a given inventory of dollars, x, and that he wants to go to some target level, x*, during the trading period. This implies that when $x \neq x^*$ the dealer has to give a quotation that will generate expected net sales or purchases by the amount x-x*.

Maximizing R(s,z) with respect to s and z and subject to the constraint $x - x^* - X(s,z) = 0$ leads to the following state-dependent quotation:

$$(6)\qquad \begin{cases} \hat{s}(x-x^*) = (1/\gamma)[\alpha-\delta\hat{z} - (x-x^*)] = \hat{s} - (1/\gamma)(x-x^*) \\ \hat{z}(x-x^*) = \hat{z} \end{cases}$$

It is seen that only the mid-rate is state-dependent, whereas the spread is independent on the initial position. This result holds also when the dealer's transaction costs or the customers' transaction costs are taken into account. In the former case the spread becomes larger and in the latter case smaller than in the present case. In the latter case the effect of customers' transaction costs is similar to the effect of a sales tax on price and the volume of sales in standard microeconomic analysis. In both cases the spread is constant irrespective of the initial position and the mid-rate is adjusted to generate the desired expected net sales. The proofs are straightforward and are omitted here.

The target level x^* for the dealer's dollar holdings at the end of the period can be interpreted as the closed position. This interpretation is also formally correct if we make an assumption that the dealer has borrowed an amount x^* of dollars, which appears on the liability side of his balance sheet. On the asset side there is an amount x of dollars held in liquid form, say, in non-interest bearing demand deposits with foreign commercial banks. In this interpretation $x - x^* > 0$ implies a long position and $x - x^* < 0$ a short position.

3. Sequential pricing decision under transactions uncertainty

Following Zabel (1981) we assume that the trading day is divided into T trading periods. The dealer starts with a given position $x_0 - x^*$ and he wants to be at a closed position at the end of the day. In the beginning of each trading period (t,t+1) the dealer is free to choose a new quotation at which he is ready to trade with incoming buy and sell orders. The dealer's objective is to maximize his expected trading income during the day subject to the system constaint that describes how the system evolves in discrete time and subject to the constraint that the expected position at the end of the day is closed. In other words, the dealer seeks a feedback controller of the form $\{f_0(x_0), f_1(x_1), \ldots f_{T-1}(x_{T-1})\}$, *i.e.* a sequence of functions which tells that when at moment t the state is x_t then the control $f_t(x_t)$ should be applied. The control $f_t(x_t)$ is a mapping from the state space $x_t \in \mathbb{R}$ into the control space $(s_t, x_t) \in \mathbb{R}^2$. The system function describing the evolvement of the state is $x_{t+1} = x_t - X(s_t, z_t) - w_t$, where $X(\cdot)$ is defined above by equation (4) and w_t is a realization in period (t,t+1) of a random variable with given probability distribution (stochastic net sales).

This formulation of the problem leads to a straightforward application of the dynamic programming technique (cf. Bertsekas, 1981, Ch. 2). Assuming a finite horizon, perfect state information and uncorrelated disturbances the dynamic programming algorithm takes the following form:

(7.1) $\quad J_T(x_T) = 0$

(7.2) $\quad J_t(x_t) = \max\limits_{s_t, z_t} \{R(s_t, z_t) + E_t[J_{t+1}(x_{t+1})]\}$

(7.3) $\quad x_{t+1} = x_t - X(s_t, z_t) - w_t \qquad$ (system constraint)

(7.4) $\quad E_t\, x_T = x^* \qquad$ (control constraint)

where $t = 1,2,\ldots,T-1$, $R(\cdot)$ is the expected revenue function and E_t denotes expectation made at moment t. The value of the function $J_t(x_t)$ gives the expected revenue from moment t until the end of the day when optimal control is applied at each moment. Note that no cost is attached to the possible remaining open position at the end of the day, which makes the terminal value $J_T(x_T)$ equal to zero. This assumption drops out without affecting the results when the horizon is extended indefinitely provided that the requirement that the expected position must be closed at the end of each trading day is maintained.

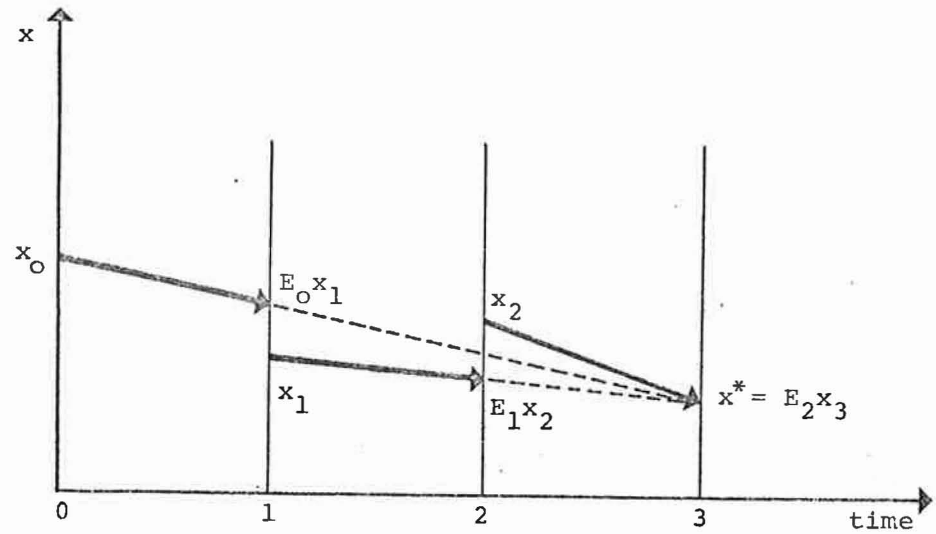
The optimal quotations are determined in a usual manner starting with the last period and proceeding recursively backward in time. In this particular case the solution is fairly simple and it is presented in the appendix. The resulting feedback controller or the sequence of functions expressing the optimal quotation for any period (t,t+1) as a function of the current state has the form:

$$(8) \quad \begin{cases} \hat{s}_t = (1/\gamma)(\alpha - \delta\hat{z} - \frac{1}{T-t}(x_t - x^*); \quad t = 1,2,\ldots,T-1 \\ \hat{z}_t = \hat{z}\,. \end{cases}$$

This quotation inserted into the system function implies that at any moment t with the open position $x_t - x^*$ the dealer quotes in such a way that during the next trading period (t,t+1) he can expect to close one (T-t)th part of the remaining open position. In other words, the quotations are adjusted to keep the expected position in a straight course toward the target during the rest of the day (see Figure 1).

As in the one-period case only the mid-rate is adjusted to steer the position, and the spread is always chosen to maximize one-period return on equilibrium volume of trade.

Figure 1. Illustration of the development of the actual and expected foreign exchange position with optimal quotations



The gradual adjustment toward the target x* implies that the fluctuations in quotations within a day become smaller than in the case where the dealer would always attempt to keep his position closed or if there were an auction mechanism that would equilibriate the sales and purchases of foreign currency in each short time period. In other words, the dealer makes Walrasian price adjustments even though the market is Non-Walrasian in the sense that it need not clear in each market period. This is the essential role of a *market maker* who quotes prices and then is ready to trade at these prices and to absorb any excess demand which comes out as a reaction of the market. To be able to perform the role of a market maker the dealer must have inventories of both foreign and domestic money or a ready access to borrow both of them. The monopoly position of the dealer is reflected only in the spread and does not affect his price adjustments.

Note that extending the horizon indefinitely while maintaining the requirement that the expected position must be closed at the end of each day does in no way change the results. This is shown in the appendix for the case of two periods, but the procedure can be repeated *ad infinitum*.

4. *Conclusions*

The results replicate some features which are already familiar from the literature analysing the dealer behaviour in organized securities markets such as the New York Stock Exchange and the U.S. Over-To-Counter market. In fact, the basic formulation of the problem is borrowed from one such analysis (Zabel, 1981), the results of which also imply the independence of the spread on the state as well as the state-dependent price adjustment. Ho and Stoll (1981) analyze the optimal pricing behaviour of a specialist at New York Stock Exchange. They assume return uncertainty in addition to transactions uncertainty and use a Poisson jump process to describe the arrivals of buy and sell orders, which both assumptions make the mathematical treatment rather complicated. The price adjustment is similar to that in Zabel´s study and the spread does not depend on the dealer´s inventory of securities, neither on the degree of transactions uncertainty. The degree of return uncertainty, on the other hand, affects the spread positively. Amihud and Mendelson (1980) also assume that the arrival rates of market orders evolve as price-dependent Poisson processes. They find the similar kind of price adjustment behaviour as the other studies, but in their model the spread depends positively on the deviation of the inventory from the preferred level. This result follows from the fact that they, by assumption, limit the permissible stock inventory level.

Certain similarities between our results and those presented in finace literature are to be expected, because the dealership market has the same characteristics irrespective of a particular empirical reference. All of the studies referred to above analyzed a situation where the specialist has a monopoly in trading with a given stock. This is a reasonable assumption as far as the New York Stock Exchange is concerned but may be rather unrealistic as far as the closely integrated foreign exchange market is concerned. We hope to be able to deal with the inter-dealer competition in the foreign exchange market in a separate paper. Anyway, we feel that this analysis gives a useful starting point for such an extension. We know that in reality the foreign exchange rate quotations are not exactly the same everywhere and that dealers frequently trade with each other and that those dealers who have overbought or oversold some currency do change quotations in order to make market maker transactions with other dealers and in this way to steer their position in the desired direction (cf. Hudson, 1979, 44-46). Referring, again, to finance literature one could expect that introducing the inter-dealer competition would affect the spread but not necessarily the nature of the price adjustment mechanism itself (cf. Ho and Stoll, 1980).

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Appendix

The dealer's optimal control law is derived by solving the dynamic programming algorithm as described by equations (7.1)-(7.4). The solution is arrived recursively starting with the last period and proceeding recursively backward in time.

The last period problem can be written as follows

$$\max_{s_{T-1}, z_{T-1}} \{R(s_{T-1}, z_{T-1})\}$$

$$\text{subject to } 0 = x_{T-1} - x^* - X(s_{T-1}, z_{T-1})$$

where the function R(.) and X(·) are defined above by equations (3) and (4). Define the Lagrangian

$$L(s_{T-1}, z_{T-1}, \lambda_{T-1}) = R(s_{T-1}, z_{T-1}) + \lambda_{T-1}[X(s_{T-1}, z_{T-1}) - (x_{T-1} - x^*)]$$

and differentiate this with respect the control variables and the Lagrange multiplier and set the partial derivatives equal to zero

$$0 = \alpha - 2\gamma s_{T-1} - 2\delta z_{T-1} - \gamma\lambda_{T-1}$$

$$0 = \beta - 2\delta s_{T-1} - 2\gamma z_{T-1} - \delta\lambda_{T-1}$$

$$0 = \alpha - \gamma s_{T-1} - \delta z_{T-1} - (x_{T-1} - x^*) .$$

It is seen that the first order conditions for the optimal quotation are (cf. eq. 6)

$$\text{(A.1)} \quad \begin{cases} \hat{s}_{T-1} = (1/\gamma)[\alpha - \delta\hat{z} - (x_{T-1} - x^*)] \\ \hat{z}_{T-1} = (\beta\gamma - \alpha\delta)/2(\gamma^2 - \delta^2) \equiv z . \end{cases}$$

The Hessian matrix of the second order partial derivatives of the Lagrangian with respect to the control variables is always negative definite given the assumptions on the signs of the basic model, and hence the second order conditions for maximum are satisfied. This is seen also from the fact that for given spread the expected revenue function is concave (parabola opening downward).

Inserting the optimal quotation (A.1) into the revenue function gives

$$\text{(A.2)} \quad J_{T-1}(x_{T-1}) = R(\hat{s}_{T-1},\hat{z})$$
$$= (1/\gamma)[\alpha(x_{T-1} - x^*) - (x_{T-1} - x^*)^2] + A_1 ,$$

where A_1 is a constant determined by the parameters α,β,γ and δ. The expected value of $J_{T-1}(\cdot)$ at moment T-2 is

$$\text{(A.3)} \quad E_{T-2}[J_{T-1}(x_{T-1})] = (1/\gamma)\{\alpha E_{T-2}(X_{T-1} - x^*)$$
$$- [E_{T-2}(x_{T-1} - x^*)]^2 - \sigma_w^2\} + A_1 .$$

Recall that $x_{T-1} = x_{T-2} - X(s_{T-2},z_{T-2}) + w_{T-2}$, which gives $E_{T-2}(x_{T-1} - x^*) = x_{T-2} - x^* - X(s_{T-2},z_{T-2})$. Insert this into (A.2) and solve the second period problem

$$\text{(A.4)} \qquad J_{T-2}(x_{T-2}) = \max_{s_{T-2},z_{T-2}} \{R(s_{T-2},z_{T-2}) + E_{T-2}[J_{T-1}(x_{T-1})]\}$$

which gives the optimal quotation at moment T-2

$$\text{(A.5)} \qquad \begin{cases} \hat{s}_{T-2} = (1/\gamma)[\alpha - \delta\hat{z} - \frac{1}{2}(x_{T-2} - x^*)] \\ \hat{z}_{T-2} = \hat{z}\,. \end{cases}$$

The Hessian matrix can again be shown to be negative definite so that the second order conditions for maximum are satisfied.

This quotation gives the value function at moment T-2

$$\text{(A.6)} \qquad J_{T-2}(x_{T-2}) = (1/\gamma)[\alpha(x_{T-2} - x^*) - \frac{1}{2}(x_{T-2} - x^*)^2]\,,$$

where A_2 is a given constant depending on the parameters of the system including the variance of w, σ_w^2.

This suggest that the general solution for the optimal quotation at an arbitrary moment t, t = 1,2,...,T-1, is

$$\text{(A.7)} \qquad \begin{cases} \hat{s}_t = (1/\gamma)[\alpha - \delta\hat{z} - \frac{1}{T-t}(x_t - x^*)] \\ \hat{z}_t = \hat{z} \end{cases}$$

and that the value function is

$$\text{(A.8)} \qquad J_t(x_t) = (1/\gamma)[\alpha(x_t - x^*) - \frac{1}{T-t}(x_t - x^*)^2] + A_{T-t}\,.$$

Assume that this is the case. The optimal quotation at moment t-1 is then received by maximizing

$$(A.9)\qquad R(s_{t-1},z_{t-1}) + (1/\gamma)\{\alpha E_{t-1}(x_t - x^*) - \frac{1}{T-t}[E_{t-1}(x_t - x^*)]^2 - \frac{1}{T-t}\sigma_w^2\} + A_{T-t} ,$$

where $E_{t-1}(x_t - x^*) = x_{t-1} - x^* - X(s_{t-1},z_{t-1})$. Differentiation with respect to s_{t-1} and z_{t-1} gives the first order conditions

$$(A.10)\qquad 0 = \alpha - 2\gamma s_{t-1} - 2\delta z_{t-1} + (1/\gamma)[\alpha\gamma - \frac{2}{T-t}(x_{t-1} - x^* - \alpha + \gamma s_{t-1} + \delta z_{t-1})\gamma]$$

$$0 = \beta - 2\delta s_{t-1} - 2\gamma z_{t-1} + (1/\gamma)[\alpha\delta - \frac{2}{T-t}(x_{t-1} - x^* - \alpha + \gamma s_{t-1} + \delta z_{t-1})\delta] .$$

Multiplying the first equation by δ and the second equation by γ and subtracting directly gives the optimal spread, $\hat{z}_{t-1} = \hat{z}$, which is equal to the equilibrium spread of the monopoly dealer. Use the first equation to solve for the optimal mid-rate, which is

$$(A.11)\qquad \hat{s}_{t-1} = (1/\gamma)(\alpha - \delta\hat{z} - \frac{1}{T-(t-1)}(x_{t-1} - x^*)) .$$

The second order condtions can be shown to be satisfied.

Finally, insert this quotation (s_{t-1},z) into (A.9) to see that

$$(A.12)\qquad J_{t-1}(x_{t-1}) = (1/\gamma)[\alpha(x_{t-1} - x^*) + \frac{1}{T-(t-1)}(x_{t-1} - x^*)^2] + A_{T-(t-1)} ,$$

which completes the proof that the optimal quotation is, in fact, the one expressed by equation (8) (and (A.7)).

Extending the horizon into two days gives the same solution as above for the second day, and the dynamic programming algorithm for the first day becomes

$$J_T(x_T) = (1/\gamma)[\alpha(x_T - x^*) + \frac{1}{T+1}(x_T - x^*)^2] + A_{T+1}$$

$$J_{T-1}(x_{T-1}) = \max_{s_{T-1},z_{T-1}} \{R(s_{T-1},z_{T-1}) + \rho E_{T-1}[J_T(x_T)]\}$$

$$J_t(x_t) = \max_{s_t,z_t} \{R(s_t,x_t) + E_t[J_{t+1}(x_{t+1})]\} ,$$

$$t = 0,1,2,\dots,T-2$$

where ρ is the discount factor and where the system and the control constraints are the same as above. The value function at the end of the first day is no more equal to zero but its expected value is constant and therefore it does not affect the optimal quotations within the first day. The same procedure can be repeated indefinitely.