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Keskusteluaiheita
Discussion papers

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THE SUPPLY OF INVENTIONS

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1. Exogenous and Endogenous Views

It is nowadays normal to assume that inventive activity is an economic process which absorbs scarce resources and produces, in the aggregate though not necessarily for any particular project, an output that can in principle be given a social valuation.

But it was not always so. At least up to a couple of decades ago the dominant view among economists was of invention as a process largely uninfluenced by economic forces. Exogenous invention. This view, that invention is largely exogenous, amounts to the contention that the opportunity cost of resources absorbed by inventive activity is virtually zero. Either because minimal resources are used or else because what are used have no alternative use. One way of rationalising this point of view is to see inventions arising from a kind of spontaneous concatenation of ideas, possibly containing some novel feature from the 'March of Science'. This has been called invention by serendipity, or more prosaically the "supply-push" theory of invention. If this is how inventions arise; and if the nature of the inventive process is such as to imply that the conscious pursuit of inventions, by working for them and by allocating resources to their production, is futile then the only interesting economic questions concern the development and application of inventions, i.e. innovation.

This account seems to hinge on a definition of invention that is practically synonymous with 'discovery'. It certainly relegates all connotations of contrivance, design and creation to second-order importance, leaving the principal role to chance. Moreover, it commits the fallacy of composition to argue from the futility of seeking particular inventions to the exogeneity of the inventive process in the large.

It may be noted that the exogenous invention point of view is inconsistent with the idea that the creation of a property right in inventions - through a patent system - would stimulate inventive activity. It is also difficult to reconcile it with the fact that a large amount of resources are in fact consciously devoted to exploratory research by the private enterprise sector.

In contradistinction from the exogenous invention or supply push theories, the now dominant incentive-inducement theories of invention give a central position to economic forces. Necessity is the mother of invention. The tradition includes Alfred Marshall and, more recently, Jacob Schmookler, Frederick Scherer, Zvi Griliches and Edwin Mansfield. The fundamental tenet of the endogenous invention school of thought is that inventive activity and, consequentially, the supply of inventions can and does respond to differing rewards for invention. The issue is the slope of the supply curve of inventions; and it is in the end an empirical issue.

2. A Taxonomy for the Supply of Inventions

Writing in 1962, in an article entitled "The Supply of Inventors and Inventions", Fritz Machlup gave an overview of the issues from an a priori standpoint. Starting with the statement:

> *The analysis of the supply of inventions divides itself logically into three sections: (1) the supply of inventive labor - the chief input for the production of inventions; (2) the input-output relationship - the technical production function describing the transformation of inventive labor into useful inventions; and (3) the supply and cost of useful inventions - the output obtained from the use of inventive labor. All this, of course, follows the pattern by which the supply of any economic good is analysed in modern economic theory.*

Machlup concludes with four elasticity propositions:

> *(1) The supply of inventive labor is unlikely to be infinitely elastic and quite likely to be relatively inelastic; (2) The supply of inventive labor capacity is probably even less elastic than the supply of inventive labor; (3) The supply of new raw inventions may, in certain circumstances, be even less elastic than the supply of inventive labor capacity; and (4) The supply of effective (worked) inventions is likely to be even less elastic than the supply of raw inventions.*

These decreasing elasticities at successive stages of the inventive process were ascribed to what Machlup called

> *"four potential shrinkages in the percentage increase in yield: higher rates of pay, lower quality of the personnel, smaller output of raw inventions per input of inventive capacity, and a higher rate of rejection in the selection of inventions for use."*

He goes on to state that:

> *"These shrinkages are independent of one another; but they may add up with a vengeance."*

As noted, Machlup's conclusions rest largely on a prioristic reasoning. Empirical evidence has in the past been sparse although the situation has been improving rapidly in recent years with the efforts of researchers such as Schmookler and Mansfield and with mounting official surveys of R & D. What I want to do in this paper is to examine these sources of diminishing returns, or supply inelasticity, from an empirical standpoint.

There are two distinct ways in which it is possible to approach this question - the direct way and the indirect way. The direct mode of analysis proceeds by estimating the supply function, from data on prices and output. Of course we must be careful to establish identification - in this context we need to know that it is the supply curve of inventions and not the demand curve that has been estimated.

In addition to a direct attack on the problem, it is also possible, as the quotations from Machlup imply, to infer the elasticity of supply from the production function. That is, to establish a relationship between output and inputs is equivalent to establishing the supply curve. This follows from the theory of duality.

For both the direct and indirect approaches I intend to employ evidence of a macro- and micro- kind. I shall therefore be appealing to four different sources of evidence in examining this question of the elasticity of supply of inventions.

3. The Supply of Inventions as a Function of the Level of Activity

First I wish to use the evidence assembled by Jacob Schmookler, who demonstrated convincingly that the level of patent activity is closely associated with the level of economic activity in the field to which the patent relates.

Since the value of an invention is proportional to the extent of its application, other things being equal, and assuming that extent of application can be roughly measured by the output or resource inputs of the industry utilising the invention, it can therefore be used as a proxy for the demand price of inventions. Thus, an invention that reduces unit production costs by one per cent in the coal mining industry is twice as valuable as one that reduces production costs in the leather goods industry by seven per cent since value-added in coal-mining is fourteen times that in leather goods.

The size of an industry then measures the demand price of inventions in that industry. This allows one to interpret Schmookler's correlations between patents and value-added as identifying the supply relationship directly. Schmookler reports on 16 census-year cross-sections of a selection of between 14 and 20 industries in the United States, and finds an average elasticity insignificantly different from unity. Thus Schmookler's supply elasticity is equal to one - though Schmookler himself does not make this observation, and Nathan Rosenberg in his Economic Journal review even suggests that Schmookler's findings imply an infinite supply elasticity (he calls it an "assumption" of Schmookler's).

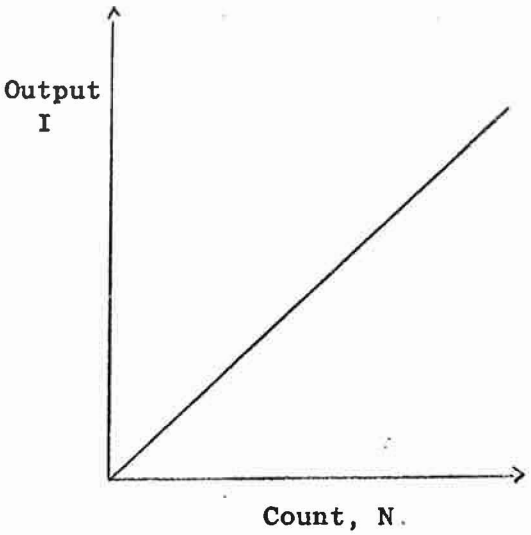
I have, however, reworked Schmookler's data, being unhappy about the methods of pooling of cross-section and time-series data he used, and the sparse information he gives for the results in which he allows differential intercept and slope coefficients for the industries.
I used the method of random-coefficients (16), (20), attempting thereby to capture the common element in the critical elasticity. I also included a couple of variables to take account of the general level of activity in the economy and the variation over time in the propensity to patent.

My results suggest that a figure much nearer to ½ than 1 would apply for the elasticity in question - in fact the 95% confidence interval for it is from ¼ to 1.

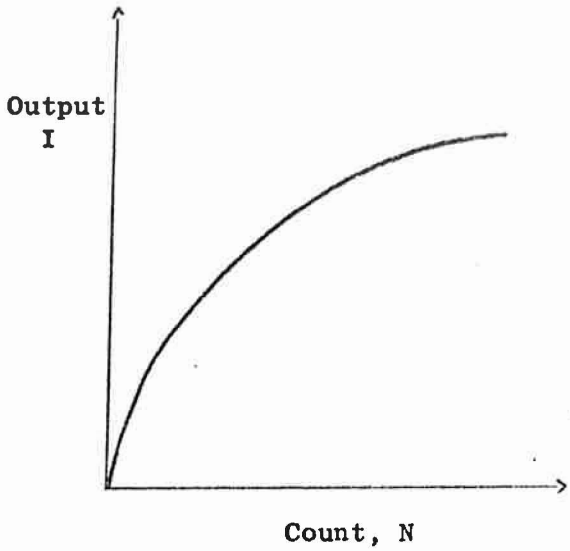
However, there could be a problem in using counts of inventions (patents) as a measure of the output of inventive activity. It could be argued that the counts should somehow be weighted by the quality of inventions. After all inventions are by definition heterogeneous, and in fact vary enormously in value. If we index the ith invention by a measure of quality, u_i, then the appropriate measure of inventive output would be:

$$I = \sum_{i=1}^{N} u_i$$

Now inventive output can be measured by the count N if N is proportional to I. This would be true in an expected value sense if researchers are unable to distinguish the quality of their invention before making them. However, if researchers are able to discriminate between inventions by quality or net value then we should expect them to work through their agenda of research projects in order of profitability. Thus we might picture the two situations:



a) No discrimination



b) Perfect discrimination

Hence this discriminating or filtering process implies that the elasticity of supply of inventions weighted by quality is less than that of pure counts of inventions. Bearing this in mind we should interpret the elasticities quoted above as upper bounds since they are based on an implicit assumption of no discrimination between inventions by inventive researchers.

4. The Distribution of Inventions by Value

Machlup asserted that diminishing returns are always due to the presence of some fixed factor, and named 'the existing stock of scientific knowledge and the state of the industrial arts at any moment of time' as one fixed factor and 'the stock of known problems' as another. Moreover he spelled-out two independent sources of diminishing returns arising from a given stock of known problems. Firstly there is duplication of effort; and secondly, with a given agenda of problems, one can think of inventive activity proceeding by attacking what are thought to be the easier or more profitable problems first, and giving a lower priority to the more difficult or less profitable ones. This is the filtering process alluded to in the previous section, and it is obviously a source of diminishing returns.

If inventors discriminate between projects by expected profitability, then the distribution of projects by expected profitability will give us the supply of inventions. Suppose that the inventor-entrepreneurs apply some external test on the profitability of their activity, where the test might be thought of as the opportunity cost of the resources used up by doing the research and development. Then, if inventors require a given rate of return to engage in inventive activity, they will produce all such inventions that exceed that rate of return according to their expectations. In other words, the invention supply curve will correspond to the cumulative frequency function derived from the expected profitability frequency distribution.

To interpret the cumulative distribution of research projects by profitability (ordered from the most profitable to the least) as a supply curve, observe that the profitability of all projects increases as the value or price of a unit of inventive output increases. Hence the profitability cut-off given by the external test moves leftwards on the diagram as the price of inventive output rises. This implies that the supply price is inversely related to the required rate of return.

Now suppose that the contrary case to that just considered holds true. That is, inventors cannot anticipate the profitability of their possible invention projects. They cannot distinguish, ex ante, good projects from bad. However they may be aware of the profitability of invention in general, so that on the average they may expect an overall rate of return to obtain even though they cannot associate particular projects with any measure of profitability. Let us suppose that inventors are expected value maximisers.[1] Then at any price lower than that corresponding to the overall average rate of return, no inventions will be supplied while at any higher price all available inventive effort will be put to use supplying inventions, which differ in value and profitability of course. Considered individually some will make losses but these will be counter-balanced by those that are profitable. The supply of inventions will be infinitely elastic at a supply price corresponding to the overall average expected rate of return.

It can be seen then that the ability to discriminate between good and bad invention projects makes the number of inventions supplied respond more inelastically with respect to price than would otherwise be the case. That is, paradoxically, it makes the invention process more exogenous or autonomous from an economic point of view. It also turns invention into a more profitable activity, considered as a whole. On this basis one can predict a derived demand for filtering service that enables a better discrimination between potential invention projects. It will pay inventors to invest resources in screening.[2] And in fact it is arguable that part of what is defined as 'Applied Research' in national surveys of research and development is in fact devoted to such screening.[3] This would make sense in terms of the typical commitment of resources to development as opposed to basic or applied research. For most inventive projects development is by far the preponderant item in costs.

Evidence

If, then inventors can sort out the potentially profitable projects, a rational attack on the set of problems at hand will give rise to a form of diminishing returns, affecting the supply curve in the sense that higher rewards will induce a greater number of inventions. The questions then arise as to the degree to which the profitability of inventions can be foreseen and the distribution of such anticipations. These are obviously empirical issues. For inventions that have already reached the development stage there is a certain amount of information available regarding one aspect of profitability forecasts, namely the estimated cost of the final article and time for completion of development projects. The most well-known such estimates, and analysis thereof, relating to American military aircraft development, is that carried out by researchers at the Rand Corporation, in the early 1960s (6). However, there has also been more recent confirmation of these results for firms in the non-aircraft business enterprise sector. The main features of this data that have been observed are: first the tendency for ex-ante estimates of production cost to be much smaller than the outcome (the average "cost factor" - ratio of outturn to estimate - is typically between 2 and 5); and secondly the very wide variations in cost factors.

Clearly, for many invention projects, and especially for radical changes as opposed to minor improvements, the ex-ante guesses about profitability in an absolute sense are likely to be wild. And yet this evidence cannot be used to argue that, at a given point in time, and faced with a particular set of potential projects, inventors are unable to screen projects and so cannot distinguish them one from another as regards profitability.[4] The value of this evidence as far as this paper is concerned is, I believe, in the shape of the distribution of cost factors. The distribution displays a remarkably fat tail. In fact a visual test suggests that the class of stable Pareto distributions may not be an unsuitable candidate for its form.[5]

If the cost factors have a Pareto distribution, it may not be far-fetched to suppose that the same kind of distribution may characterise the ex-ante expectations of profitability from inventive activity which, it was argued earlier, determines the shape of the invention supply curve. Of course expectations about profitability are not easily observable, but if they are rational or well-founded, they should be reflected in the actual, ex-post, distribution of profitability of inventive projects. Now, there is some admittedly sparse evidence on the distribution by value or profitability of successful inventions, and it does concord with the supposition that they might be described by a Pareto distribution. Since the data are not extensive, it is possible to examine briefly all that is available, in chronological order of publication:

(i) B. Sanders (12)

The results of this survey were noted by F.M. Scherer (13) to be distributed a la Pareto - from which Scherer concludes for his own study that "*it forces us to acknowledge that patent statistics are likely to measure run-of-the-mill industrial inventive output much more accurately than they reflect the occasion strategic invention which opens up new markets and new technologies. The latter must probably remain the domain of economic historians.*"7

Of course the Pareto description only applies to those inventions that made a net gain. The Sanders survey indicates however that about one half of the 281 patents reporting a numerical value were loss-making. The losers are much more tightly packed around the break-even median, however.

(ii) J.L. Enos (1)

Enos presents data for the profitability of inventions in the petroleum refinery industry. Since his data was presented as the personal returns to individual inventors of whom more than half in his sample were salaried and presumably assigned their inventions to their employers, only five data points remain. These span a period of almost 30 years, over which time the output of the industry was growing strongly, and presumably the demand price for such inventions along with it. Even so, the impression that the cumulative distribution is linear on the log-log scale would remain if adjustment were made for the varying demand price.

It must of course be pointed out that Enos's sample is in no way random. The inventions that he selected for scrutiny were among the more dramatic in the industry. He was obviously examining the upper tail of the distribution.

(iii) E. Mansfield et al (5)

Mansfield and his collaborators attempted to measure both the private and social rates of return from 17 industrial inventions that were put into production. Of these innovations, four were process innovations, ten were new producer goods and three were new consumer goods. They occurred in a "wide variety of industries, and in firms of quite different sizes." The authors go on to say that most were of "average or routine importance, not major breakthroughs", and "although the sample cannot be regarded as randomly selected, there is no obvious indication that it is biased toward very profitable inventions (socially or privately) or relatively unprofitable ones".

For 9 of the 17 innovations the authors obtained "data concerning the approximate private rate of return expected from the innovation by the innovator when it began the project". "In 5 of the 9 cases, this expected private rate of return was less than 15 per cent (before taxes), which indicates that these five projects were quite marginal from the point of view of the firm.... Yet the average social rate of return from these 5 innovations was over 100 per cent." ....."Among the innovations for which we have data, there is no significant correlation between an innovation's expected private rate of return and its social rate of return."

One is much less inclined to view this data as deriving from a Pareto distribution. Possibly for the social rate of return, but certainly not for the private rate of return. What we seem to have there is a more tightly packed distribution, except for the outlier. Speculating freely, it would not be hard to give a rationalisation for this observation. It might be supposed, for example, that most of these innovations are routine, run-of-the-mill improvements as indeed the authors claim all of them are. But a few may in fact be of a more radical, "state of the art" variety.

In most studies of individual inventions there is a strong "success bias". Since it is difficult, if not by definition impossible, to observe the output of inventive effort that did not result in an invention; and since in practice most research has concentrated on the 'noteworthy' inventions that have thrust themselves on the researchers, what tends to be measured is in the upper tail of the distributions of inventions by value. When attempting to examine the supply response of inventions to inducements on the demand side, however, it is important to gather evidence on marginal inventions and, if possible, inframarginal inventions.

As Sander's data seems to suggest, most inventions are just about marginal one way or the other. That is, the distribution seems to be clustered around the break-even point of profitability. This impression seems to be corroborated by the figures for lapsed patents and patent renewals in the UK. The UK data are interesting because a tax is levied on the patent monopoly according to the length of duration of the patent. About one fifth of patents survive for the maximum duration of 16 years, and these are presumably in the main protecting economically valuable inventions. For the remainder, the survival of a patent for another year depends on the patent-holder's willingness to pay the renewal fee, and this in turn must reflect his expectation as to the invention's value. In addition of course it reflects a process of learning as the patent holder becomes more aware of the invention's qualities as it is being developed and appraised.

Concluding this section on the distribution of inventions by value, if the distribution of expected profitability mirrors that of actual profitability (which Mansfield's evidence casts doubt on), then despite the fact that the upper tail suggests a low elasticity ($-\frac{1}{2}$), since most of the distribution is about the margin, we must conclude that this aspect of supply will not confer much inelasticity on the supply curve for numbers of innovations.

5. The Input-Output Relationship : Micro

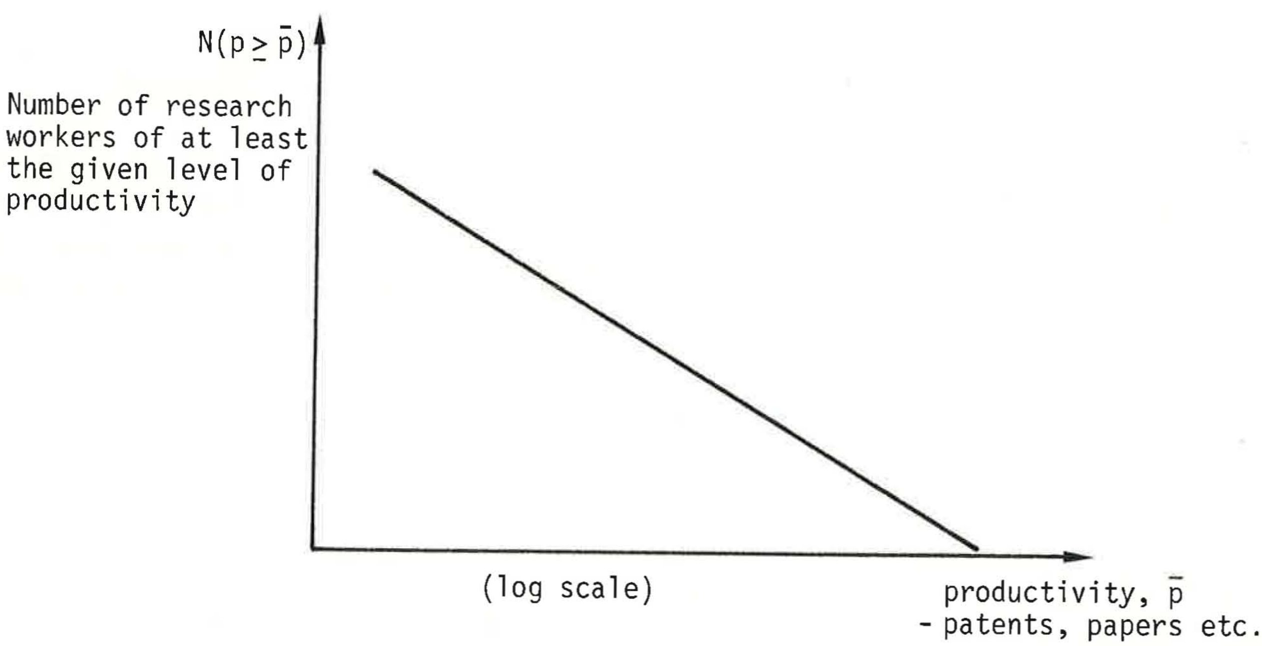
Another way to approach the question of diminishing returns, and hence inelasticity in the supply of inventions, is to examine the production function directly. To examine the input-output relation.

I shall offer two variations on this theme - one micro and one macro. The micro evidence relates to the productivity of the individuals in the invention labour force, while the macro evidence examines the relation between research resources and research output in the aggregate.

The idea behind what I want to present as the micro evidence on the input-output relation for inventive activity is rather similar to that presented previously as the distribution of inventions by value. It is to examine the distribution of inventors by productivity.

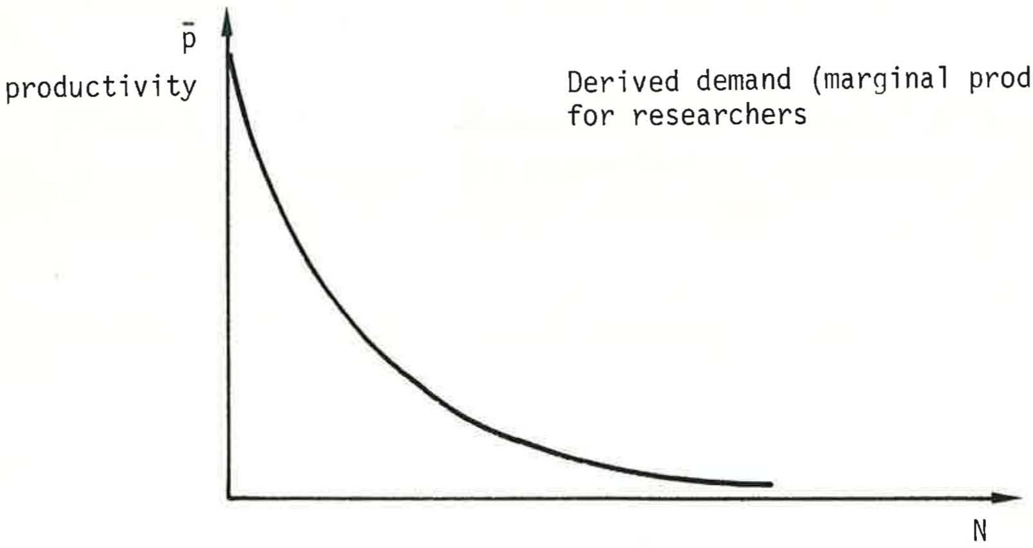
Again, the evidence is not exactly plentiful, but one must make do with what one can get. The data, where it exists, measures the output of research workers by a count of inventions or research papers. But it does not value the items of that count. It is therefore assumed that the average value of an invention from a prolific inventor or research team is the same as that from a typical one of the numerous one-off inventors. The productivity or value of an inventor or inventive team is then measured by how many inventions he or they produce.

Evidence on researcher output in government and business laboratories was presented in the mid-1950s by William Shockley (15), a physics Nobel Prize winner. What comes through quite clearly is that the cumulative distribution of research workers by productivity, as measured by the number of patents or research publications, is linear in the logarithm of productivity.



For example, in one large industrial laboratory employing about 280 researchers, roughly 100 had filed no patents in a six year period, about 30 filed one patent, while the most productive worker filed approx. 100 patents. The same kind of pattern was repeated in the four other laboratories' experience catalogued by Shockley.

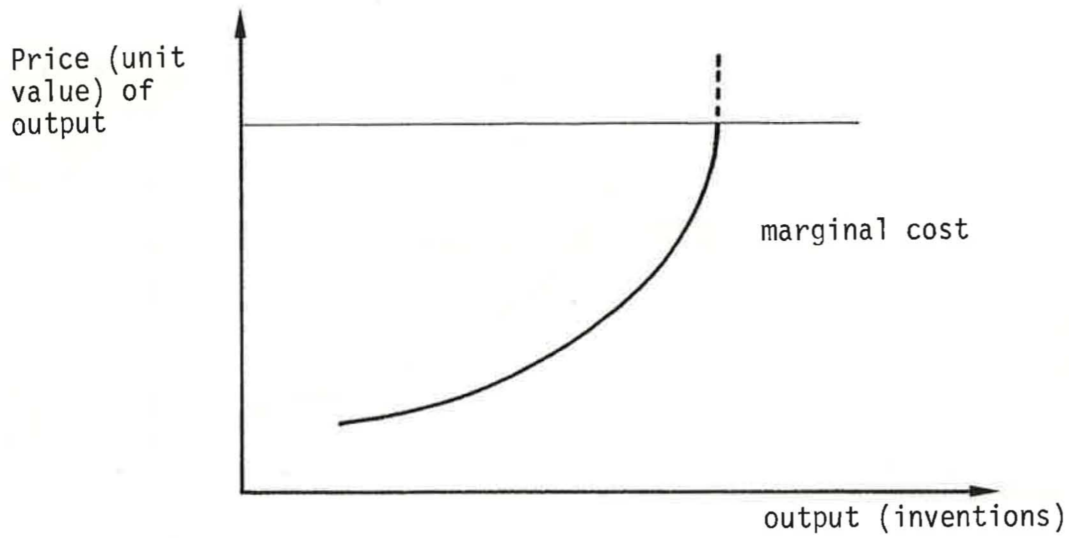
If we ignore interaction effects between research workers, it seems possible to interpret this diagram as representing a marginal productivity schedule. For this case, in which the productivity of any individual researcher is independent of the presence or absence of other researchers, we need simply to reverse the axes and convert the log scale of productivity into an arithmetic scale to get the derived demand for research workers. Of course, it is implicit for this operation that the unit value of research output is held constant.



Since the curve is based on observation, it is presumably the case that the researchers whose productivities were measured are all supra-marginal. So, to interpret the curve as derived demand requires us to label the vertical axis as the positive deviation of researcher salaries from the current norm. In which case it tells us how many researchers would be employed at higher wage rates. However, the point of the diagram is mainly impressionistic. It indicates high elasticity of demand for researcher services, at current costs of such services, with the elasticity diminishing sharply as costs increase.

Because of the duality between cost and production (in particular, $MC = \frac{S}{mp}$), it is also possible to interpret the curve from the point of view of marginal cost, or supply. To do so it is necessary to hold the remuneration of researchers constant, and consider how variations in the price or unit value of their output (inventions) will induce changes in that output. In this interpretation the marginal cost of an invention is the cost of the marginal researcher divided by his output. The margin in question is the "extensive margin", to use Ricardian terminology. That is, it does not take into account variations in individual researcher productivity that are affected by incentives. This does not seem unreasonable for the data provided by Shockley, since variations in researcher salaries were much less spread out than those of productivity, and were only weakly correlated with it.

The picture that emerges in this case would look something like this:



that is the marginal cost or supply curve is much more inelastic at the current margin of output (i.e. at the current observed price or value of inventions) than it would be at lower levels of output.

The contention here relating to increasing marginal cost is based on the observed wide disparities in the productivity of research workers. This is very much in the spirit of Machlup's "second shrinkage" which is attributed to lower quality of the personnel. Machlup is implicitly considering the effects of an expansion of inventive activity when he says:

> *"....a point must exist beyond which further transfers to the research and development work force cannot possibly be of the same quality."*7

6. A Macro Production Function for Inventions

At this point it is useful to have a glimpse at the mounting volume of data of a more highly aggregated nature that is becoming available. An obvious possibility is to think in terms of production function approach similar to and analogous with those that are applied to industrial output. What would be looked for then is some relation between inputs of research and development resources or scientific and technical manpower and a measure of output of that activity. The data tends to be measured at the industry level, which dictates the minimal feasible level of aggregation to be applied.

What I have to report is in the nature of a pilot study of the association between R & D expenditures (input) and counts of international patents (output) across 15 industries and 7 countries. The patents were counted as applications made in 1975 for a wide, but not exhaustive, selection of technology classes (19), and the R & D expenditures are those reported by the OECD for 1972 (8). The R & D expenditures are denominated in US $ - no attempt was made to adjust the exchange rates for differing real costs of scientific and technical manpower across countries.

A great advantage of using international data is the fact that it makes possible a standardisation across industries for the wide variations in: technological opportunity; the propensity to patent; and in the rough concordance between technological classification for patents and the industrial classification of expenditures.

It is true that more direct measures of technical progress than patents are conceivable - such as the rate of advance of total factor productivity or decline in unit production costs or diversification of products - but these all suffer a disadvantage compared to patents in that technology is international and so these measures of progress may in fact have been produced in large part elsewhere. A virtue of patents is that they are unambiguously related to the inventive activity that produced the invention. But a lengthy discussion of the strengths and weaknesses of the data is not justified for a pilot study, which is looking for indications and suggestions in the data rather than attempting to test hypotheses or estimate parameters carefully.

It being a pilot study, I used the methods of Exploratory Data Analysis, the results of which are summarised in the graphs. What these graphs show is that:

i Plotting logarithm of patent counts against the logarithm of R & D expenditures ($ million) reveals an upward drift with a lot of variability around it;

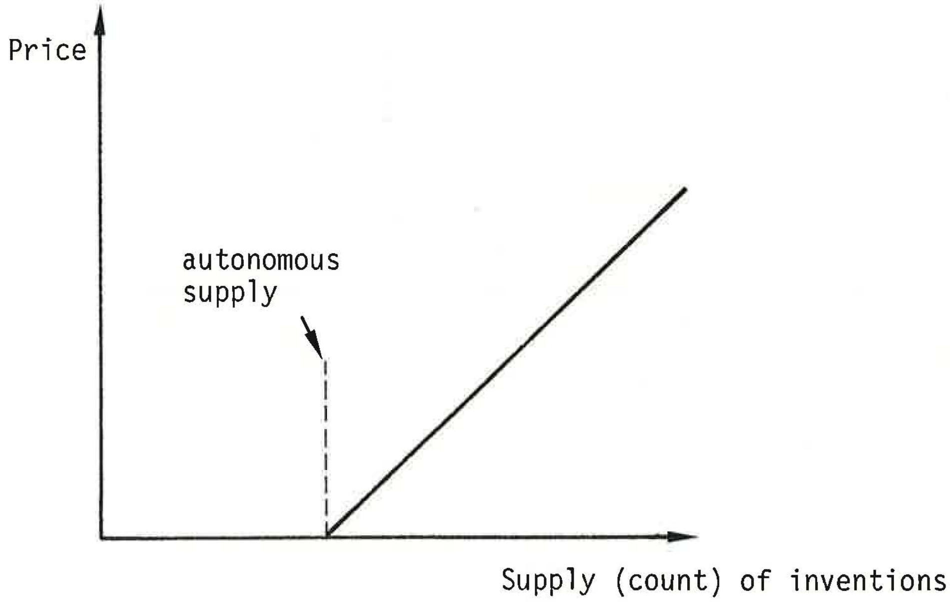
ii Country and industry effects seem to account for much of the noise, leaving comparatively little residual variability around the general upward tendency when they are removed (see table for country/industry effects);

iii The curve is linear in log-logs with a slope of about ½, suggesting the application of a square-root law for the production function.[8] Or an elasticity of inventive output with respect to R & D inputs of a half.

iv The spread of observations around the regression line tends to increase at lower levels of R & D. This is a slightly unusual form of heteroskedasticity, but one that can be explained by supposing that an additive constant is present in the underlying relationship. If this were the case it could be described as the autonomous component of invention.

If inventions are partly autonomous and partly generated by research and development activity by a square-root law, what does that imply for the supply curve? Of course the autonomous component implies a perfectly inelastic supply, while the induced component suggests a linear supply curve.[9]

The picture that emerges then is qualitatively of this kind:



with supply everywhere inelastic, but with the elasticity of supply increasing as the price of inventions increases.

7. Conclusions

This paper was written in the belief that an empirically eclectic view of the supply of inventions might help one arrive at a decision as to whether inventions should be seen as exogenous or endogenous. I believe that the indications to be drawn from the evidence reported and interpreted here are not unambiguously in favour of either side. However there is a broad consistency in the various approaches, suggesting that the elasticity of supply may well be less than unity, even for the run-of-the-mill inventions that represent the bulk of patent counts, and is almost certainly even more inelastic for important and path-breaking inventions. It follows that, despite the presently dominant view that inventors are motivated by expectation of profit, policy proposals designed to operate through this endogenous motivation, for example variations in the patent term, are unlikely to have strong effects on the supply of inventions. That is, not unless drastic action is taken.

Notes

1. Whether this supposition is correct is not known - on the whole one supposes that people are risk averse, but this may not be true for inventors as a class. They may be "plungers" rather than "hedgers". In any case the argument does not depend on the supposition.

2. An important literature has been developed in recent years in the economics of screening, particularly with respect to the labour market. See, for example (17).

3. In surveys of research and development the activity is often divided into the trinity: Basic Research; Applied Research; Experimental Development. The OECD's "Frascati Manual" (9), which is a proposal for standardising such surveys, describes Applied Research as:

   "...undertaken either to determine possible users for the findings of basic research or to determine new methods or ways of achieving some specific and pre-determined objectives. It involves the consideration of the available knowledge and its extension in order to solve particular problems. In the Business Enterprise sector the distinction between basic and applied research will often be marked by the creation of a new project to explore any promising results of a basic research programme. .....Applied research develops ideas into operational form."

   While this description, and the examples given in the same publication to distinguish the three categories, makes clear the fact that applied research as measured in the surveys is wider than a screening process of potential invention research projects for the investment of resources at the development stage, it is also clear that this is an important part of such activity.

4. In fact the very careful statistical analysis by R Summers (6) which examines this data demonstrates that if the initial estimates for two development projects stand in as low a ratio as 5:3 then what appears initially to be the more costly will turn out so four times out of five.

5. If the variable, u, is distributed with a distribution function (cumulative frequency) F(u), then the Pareto distribution holds if log (1 - F(u)) is linearly related to log u. Mandelbrot (4) has pointed out that a visual test of approximate linearity is valid if the line has a shallow slope.

   One of the features of the Pareto distribution that makes it uncomfortable to analyse or use in statistical methodology is the fact that for $\alpha < 1$ it possesses neither mean nor variance and for $\alpha < 2$ the variance does not exist.

6. Nordhaus (7) noted Scherer's observation that the exponent of the Pareto distribution was "equal to" (in fact Scherer says "less than") ½, and that this further implies that it is not possible to reduce risk by carrying a suitably diversified portfolio of research projects. He also noted that Machlup had come to a similar conclusion by an unspecified route. Machlup says:

"Contrary to other industries where the probable errors are larger for individual producers than for the industry as a whole, the "invention industry" is apt to present smaller dispersions in the probability distributions for the individual producers."

7. In this context of inventor productivity it is interesting to note some of the observations in the parallel field of researcher productivity in science. Taking the basic measure of scientific output to be a count of publications, a remarkable regularity was discovered in the 1920s by the biomathematician Lotka (2). Lotka's law relates the number of authors A(n) who publish exactly n papers by an inverse square proportionally:

$$A(n) \quad \alpha \quad \frac{1}{n^2}$$

Lotka's law relates to whole disciplines of science. However, it resembles in character the laboratory worker productivity figures given by Shockley, though it depicts an even more skewed distribution.

Another tantalising suggestion is made by N. Rescher in his book "Scientific Progress" - that it may be possible to infer something about the distribution of the quality of scientific results from the development of their quantity. In particular, he argues that while there has been an exponential growth in the number of total scientific findings, the number of first-rate findings has been growing approximately linearly. He says that, with the total number of findings Q, the volume of "λ - quality findings stands at Q^λ (for $0 < \lambda < 1$)." And he gives labels for the various values of λ:

$\lambda = 1$: routine

$\lambda = \frac{1}{2}$: important ("Rousseau's law")

$\lambda = \frac{1}{4}$: very important

$(\lambda = 0)$ log Q: first rate

If it were possible to measure scientific quality, then on this basis we should observe a double-logarthmic rank-size correlation (Zipf's law). The evidence of the distribution by value of inventions suggesting a possibly Pareto distribution would be consistent with this suggestion of a quality-quantity relationship - if one interprets importance as profitability.

8. If one dares to interpret this according to Rescher's schema outlined in the previous note, then it suggests that internationally patented inventions represent R & D output of "important" quality level, while the national income accounting convention of measuring output by inputs - as is done for all service sectors including R & D - is implicitly measuring "routine" inventive output.

9. Let I(R) denote the volume (count) of inventions induced by research and development expenditure R:

$$I(R) = \alpha . R^{\frac{1}{2}}$$

then the marginal product of R & D is $I'(R) = \frac{\alpha}{2} . R^{-\frac{1}{2}}$

the inverse of which gives the marginal cost of invention since the cost of an increment in R is definitionally unity:

$$\text{marginal cost} = \frac{2}{\alpha} R^{\frac{1}{2}}$$

Expressing marginal cost as a function of I derives the linear supply curve for induced invention.

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Random Coefficients Regression for Schmookler's Data

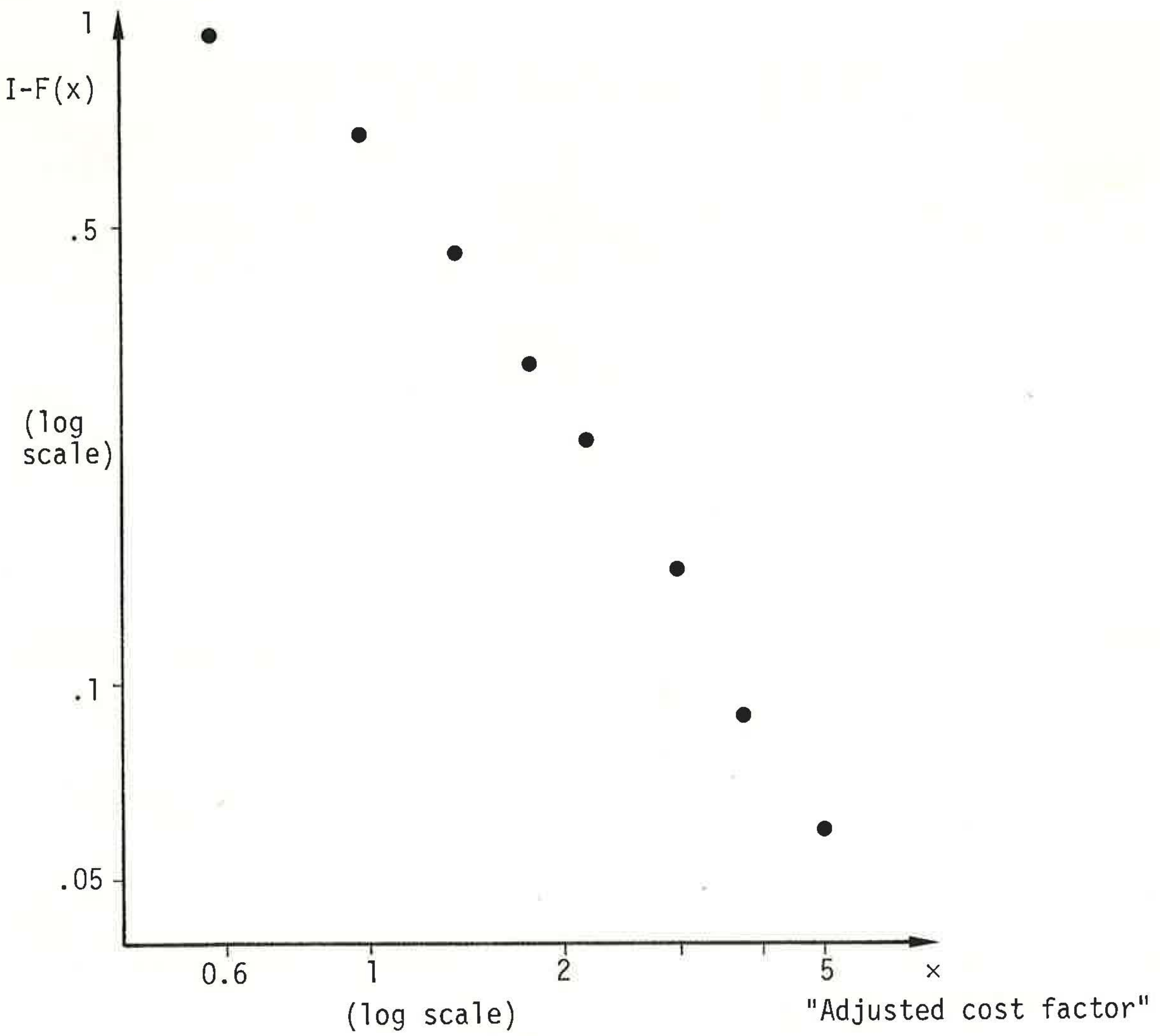
$$\ln P_{it} = 0.01 + 0.57 \ln V_{it} + 1.27 \ln (TP_t - P_{it}) - 1.12 \ln (TV_t - V_{it})$$

(2.16)	(0.20)	(0.26)	(0.26)
[6.05]	[0.18]	[0.68]	[0.57]

χ^2 (d.f. = 52) = 746.9

Where P_{it} is patents in industry i and period t, V is value added, TP is total patents, TV is total value added for all industries. All coefficients are assumed drawn from a random distribution across industries - the means of those distributions are reported in the equation, with their standard errors below, and with the estimated standard deviations of the distributions in square brackets. The χ^2 statistic indicates rejection of the hypothesis that the coefficients for different industries are equal.

Distribution of Cost Factors for 68 cost estimates for U.S. Military Flight Vehicles

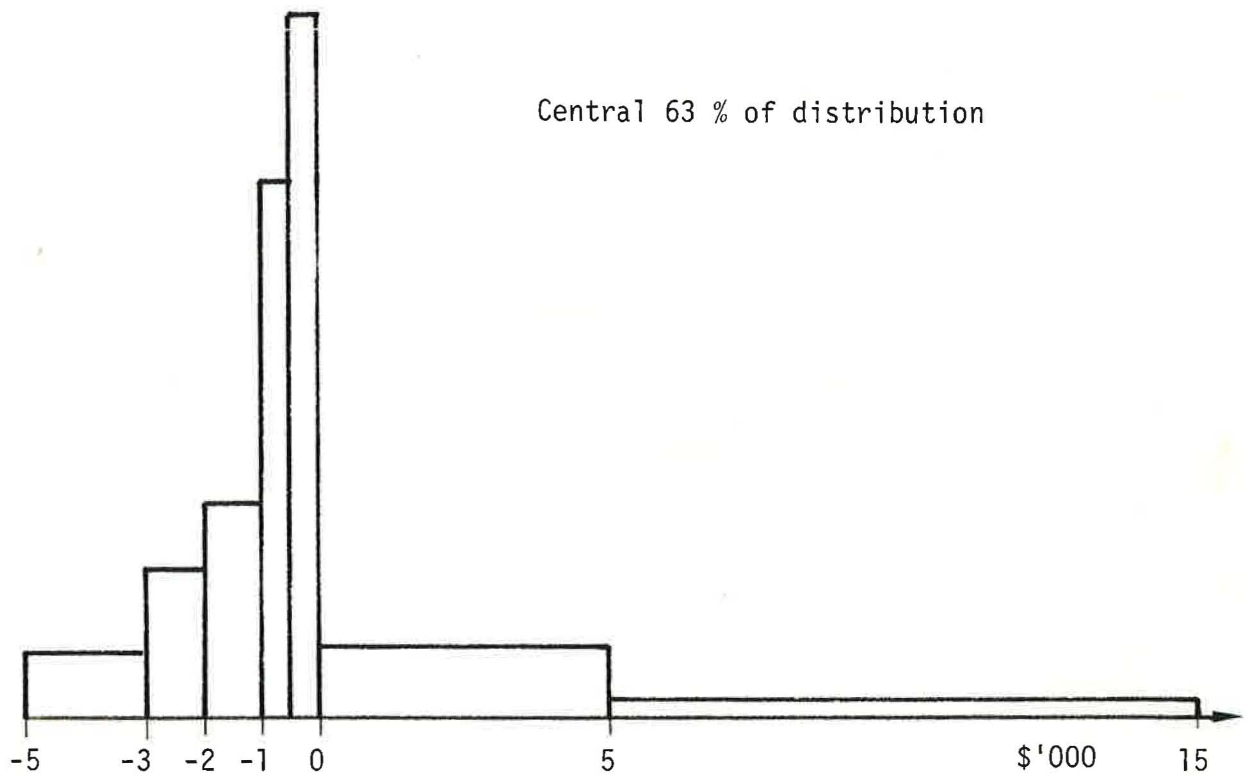


Source: T. Marshak et al. (6).

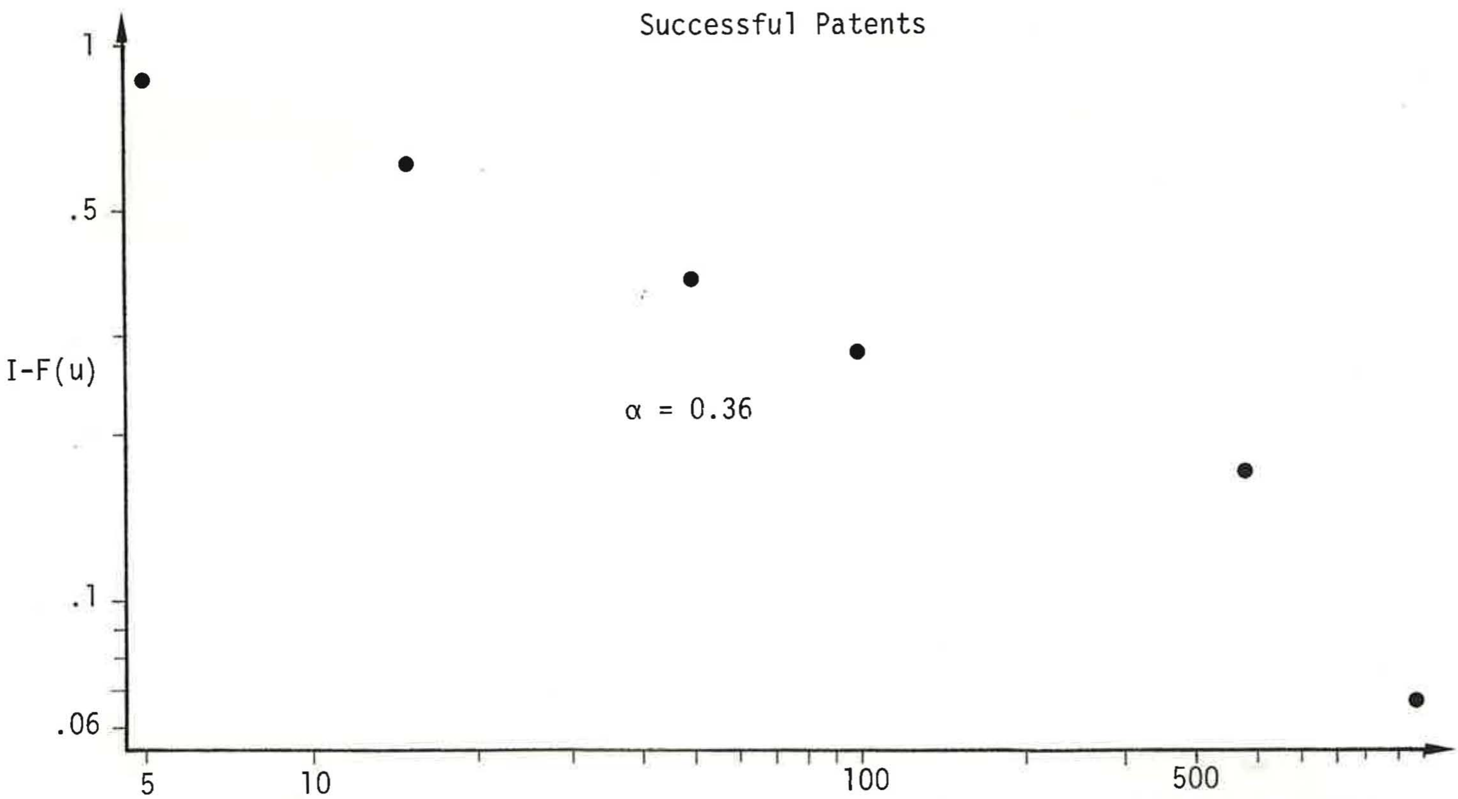
The cost factor is the ratio of estimated cost to actual (outturn) cost. The adjustments, carried out by Summers, relate to deflation for changes in price levels and to adjustment for learning curve effects due to procurement-quantity differences between the estimate and the outturn.

Estimated α = 1.3

The Distribution of Patents by Value

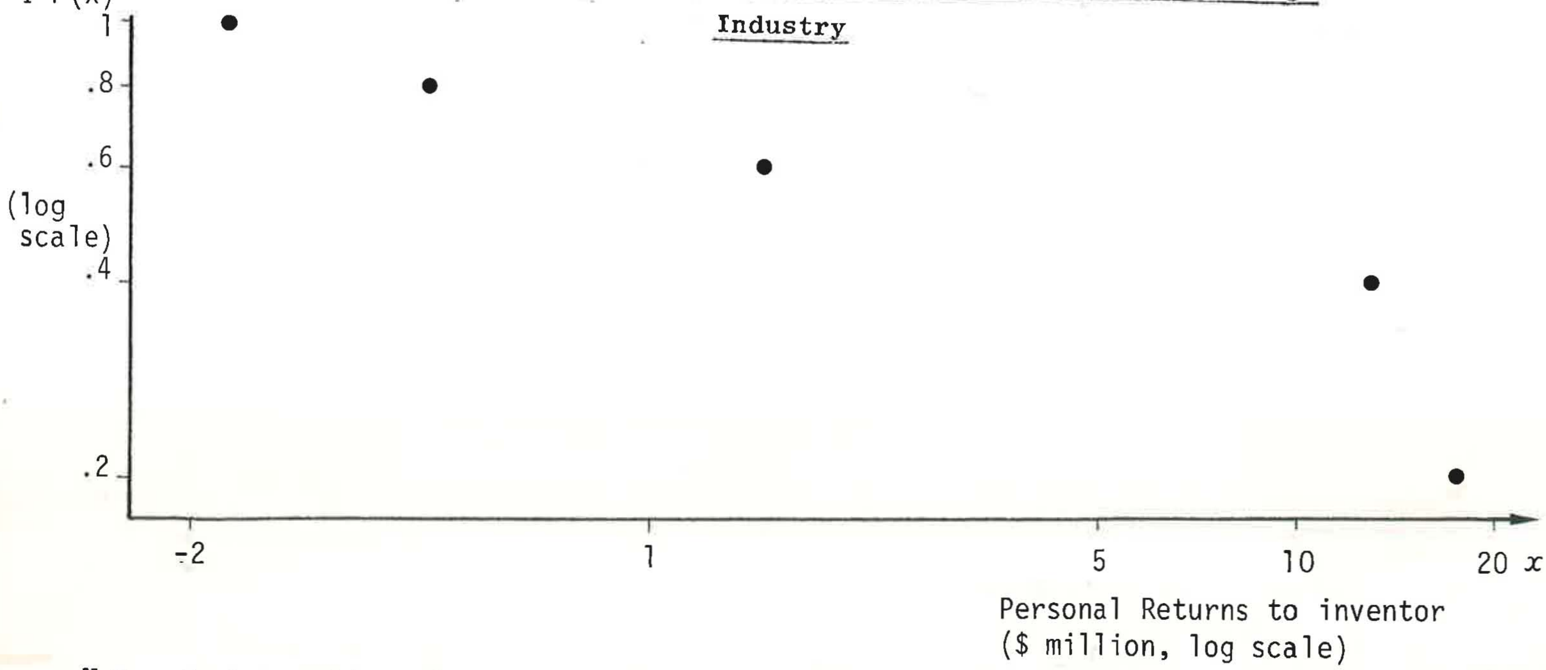


N.B. Allocating the spike at zero equally between adjacent cells. (About 1/5 of the sampled patents reported "no gain and no loss".)



Source: B.S. Sanders et al (12)

Profitability of Successful Inventions in the Petroleum Refining Industry

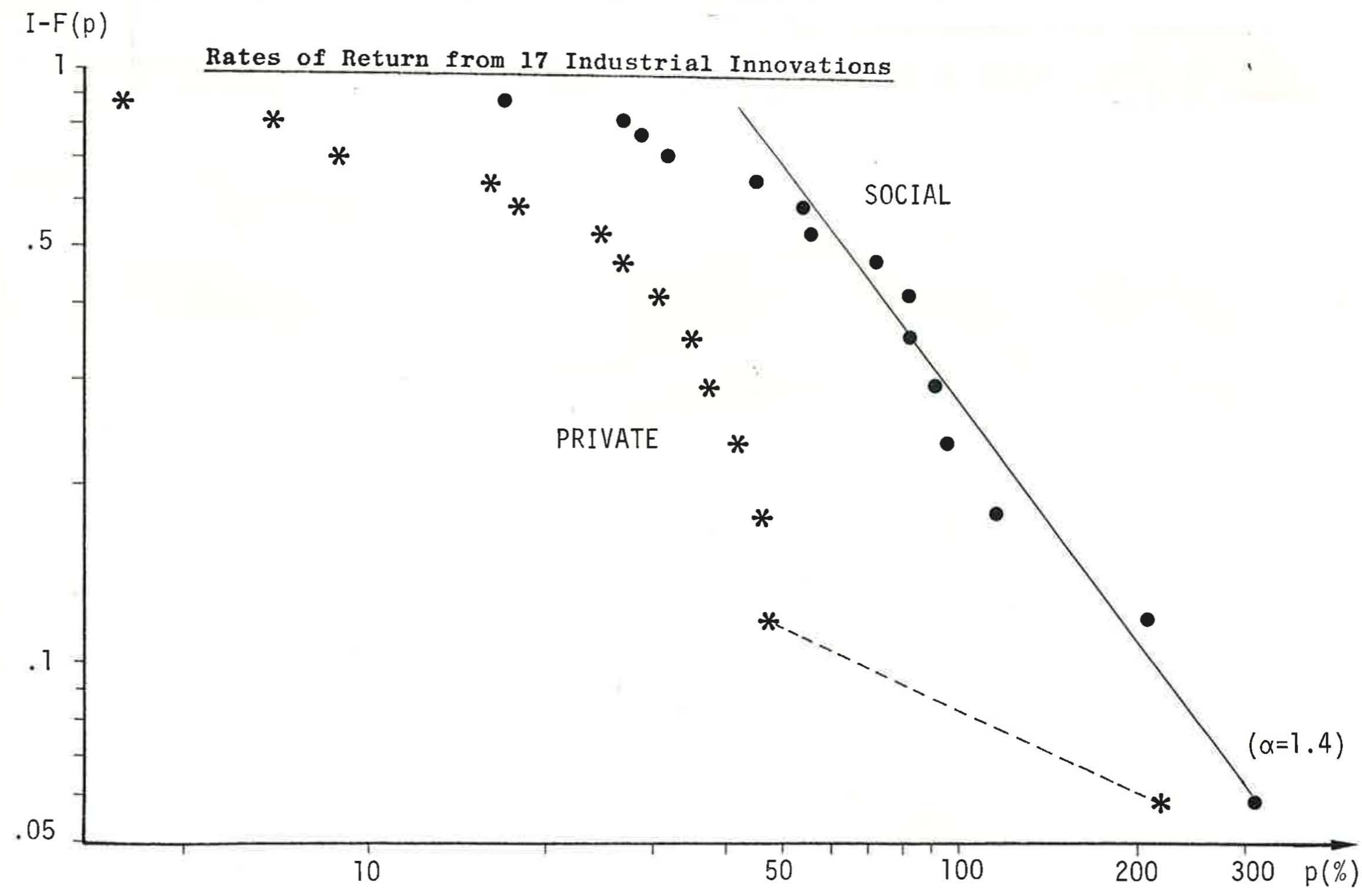


N.B. 5 observations (excludes salaried inventors from Enos' list)

Source: J.L. Enos (1)

Guess for α = between .25 and .35 (ignoring one of the last two points plotted.)

Rates of Return from 17 Industrial Innovations



Source: E. Mansfield et al (5)

Systemmatic Industry and Country Factors in the Relation between International Patents and R & D Expenditures

INDUSTRY

ISIC			
1	Agriculture	5.1	
2	Mining	.4	
5	Elect. Machinery and Equipment	1.5	
9	Chemicals, Drugs, Petrol	.6	
14	Cars, Ships, Other Transport	1.6	
18	Metals and Metal Products	.9	
19	Instruments	3.1	
20	Machinery	2.5	
22	Food, Drink and Tobacco	.4	
24	Rubber and Plastic Products	(1.0)	median
26	Stone, Clay and Glass	.3	
27	Paper and Printing	1.2	
28+ 29	Wood etc. and Other Manufacturing	4.0	
31	Total Manufacturing	.5	
33	Construction	3.2	

COUNTRY

Belgium	.3	
Netherlands	.6	
Canada	.5	
France	(1.0)	median
U.K.	1.3	
Germany	2.6	
U.S.A.	1.3	