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Matching in the housing market with risk aversion and savings*

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Abstract

We develop a model of the housing market that features both financial and matching frictions. In the model, risk-averse households may save or borrow in order to smooth consumption over time and finance owner housing. Each household either rents or owns its house. Some renter households become dissatisfied with rental housing and want to buy a house. Prospective sellers and buyers meet randomly and bargain over the price. We show how the outcome of the bargaining process depends on buyer's and seller's asset positions. The results also illustrate how financial frictions magnify the effects of matching frictions. For instance, because of the borrowing constraint, some matches do not result in trade and identical houses are traded at different prices.

Keywords: Housing, Matching, House prices.

JEL: E21, R21, C78.

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1 Introduction

The housing market is in many ways imperfect. Because of matching frictions, it takes time to find a trading partner. Related to this, the price is usually determined in a bargaining between a seller and a potential buyer. In addition, sellers and buyers face financial frictions. Since many households finance their housing with a mortgage, the availability of credit and households' own savings may matter a great deal for housing market outcomes. Moreover, because of uninsurable income uncertainty, households need to consider their ability to service their mortgage in case their incomes fall in the future.

In this paper, we study the interaction of matching and financial frictions in the housing market. To this end, we develop a model that combines two strands of literature. We introduce matching frictions following, for example, Wheaton (1990), Williams (1995), and Albrecht, Anderson, Smith, and Vroman (2007). In these models, potential house buyers and sellers meet randomly and bargain over the price.[1] While previous housing market matching models assume risk-neutral preferences and abstract from households' savings decisions, we embed the matching frictions into a Bewley-Huggett-Aiyagari -type framework where risk-averse households face uninsurable income shocks and make savings decisions (Huggett 1993, Aiyagari 1994). This allows us to study how borrowing constraints and households' asset positions affect housing market outcomes in the presence of matching frictions.[2]

In the model, each household either rents or owns its house. Some households prefer owner housing over rental housing and households' tenure preferences may change over time. If a renter household becomes dissatisfied with rental housing, it wants to buy a house.

[1] There is a large literature studying the role of matching frictions in the housing market. Existing models make different assumptions regarding how agents meet and how prices are determined. For instance, Díaz and Jerez (2012) consider directed search with posted prices, Carrillo (2012) considers directed search combined with a bargaining game, and Piazzesi and Schneider (2009) consider random matching with take-it-or-leave-it offers.

[2] There are many models that incorporate housing choices into a Bewley-Huggett-Aiyagari -type incomplete markets set-up with borrowing constraints. See, for instance, Ríos-Rull and Sanchez-Marcos (2008). However, to the best of our knowledge, all the previous applications abstract from matching and search frictions.

Similarly, some owner households may want to move to rental housing in which case they consider selling their house. Prospective sellers and buyers meet randomly and bargain over the price. Households may save or borrow with a financial asset but can only borrow against owner housing.

In the model, the asset distributions of potential buyers and sellers are both key equilibrium objects. For instance, when bargaining over the price, sellers need to consider the distribution of all potential buyers because it influences the value of not selling today and staying in the market. Similarly, buyers need to consider the distribution of sellers as that influences the value of not buying today. The combination of precautionary savings and matching frictions relates the present paper to recent labor market matching models with a precautionary savings motive. An example is Krusell, Mukoyama and Şahin (2010). In their model, unemployed workers and firms with vacancies are matched and bargain over the wage.[3] However, while workers are heterogeneous in their assets, all recruiting firms are identical. In our set-up, both parties of the bargaining process are heterogeneous in their assets.

As is natural in the market for owner housing, the outcome of the bargaining process between a seller and a buyer depends on the seller's and buyer's financial position. Combined with asset heterogeneity, which stems endogenously in the model, this has two realistic implications in the model. First, not all matches result in trade. Second, at any given point in time, identical houses sell at different prices.[4] These equilibrium properties arise also in some previous housing market matching models but for very different reasons. Typically, they relate to exogenous preference heterogeneity that affects the surplus from trade. The heterogeneity may be match-specific, as for instance, in Williams (1995) and Díaz and Jerez (2012). Alternatively, individuals may be inherently different as, for instance, in Carrillo

[3] An earlier example of a model that combines labor market matching frictions and precautionary savings is Costain and Reiter (2005). In their model, bargaining takes place between worker unions and firms.

[4] Merlo and Ortalo-Magné (2004) document that in the UK one third of all matches are unsuccessful. It also seems widely recognized that there is idiosyncratic dispersion in quality adjusted house prices even though it is difficult to measure it accurately. See, for instance, Leung, Leong, and Wong (2006) and the references therein.

(2012), where agents differ in their intrinsic motivation to trade.

We calibrate the model using Finnish household data. Among other things, the calibrated model features a realistic average time-on-the-market. This suggests that the degree of matching frictions is realistic in the model.

We first describe households' optimal savings policies. We then describe how the outcome of the bargaining process depends on the traders' asset positions. We show that asset positions affect the bargaining outcome mainly through the borrowing constraint. Poor sellers might be willing to sell at a relatively low price because of liquidity reasons. On the other hand, poor buyers can finance a house only if the price is relatively low. As a result, poor buyers trade only with poor sellers. Wealthier sellers, who need not sell for liquidity reasons, prefer to wait for a better match.

We then consider how changes in three different frictions, namely borrowing constraint, matching friction, and transaction tax, change the stationary equilibrium. We use price dispersion between identical houses and the average time-on-the-market as measures of market inefficiency. An important insight from the analysis is that a financial friction in the form of a borrowing constraint works to magnify the effects of matching frictions. For instance, while some type of a matching friction is needed to generate any idiosyncratic price dispersion, tightening the borrowing constraint increases price dispersion substantially. This is because the borrowing constraint makes the outcome of the bargaining process more sensitive to traders' asset positions. By the same token, tightening the borrowing constraint also increases the average time-on-the-market.

We proceed as follows. In the next section, we describe the set-up, discuss the household problem and the matching process, define the recursive stationary equilibrium, and outline our numerical solution algorithm. In section 3, we present the main results. We conclude in section 4. We discuss certain technical issues in the appendices.

2 Model

2.1 Set-up

Time is discrete and there is a continuum of households of mass one. Households live forever. In each period, households work, consume non-durables, and occupy a house. The economy is small and open to international capital markets meaning that the interest rate and the wage rate are exogenously determined.

Each household either owns or rents one house. The occupancy state of the household is denoted by $d = r, o$. In state $d = r$, the household is renting. In state $d = o$, the household owns the house it lives in. The mass of owner houses is fixed and equal to $m^o \in (0, 1)$. As a result, the share of households living in rental housing is $1 - m^o$.

Households' preferences regarding owner and rental housing change over time. Some households derive the same utility flow from rental and owner housing. However, in each period, those households may be hit by a tenure preference shock, which means that they suffer a utility cost if they continue to live in rental housing.[5] Similarly, households that suffer the utility cost related to rental housing may be hit by a tenure preference shock that makes them indifferent between rental and owner housing.

The tenure preference state is denoted by $z = 1, 2$. In state $z = 1$, the household derives the same utility from owner and rental housing. In state $z = 2$, the household suffers a utility cost if it lives in rental housing. The probability that the next period state is z' given current state z is $P(z', z)$.

Each household will therefore be in one of the following four situations: i) Those with $d = r$ and $z = 1$ are renting without suffering a utility cost. We refer to them as 'happy renters'. ii) Those with $d = r$ and $z = 2$ are renting, but suffer a utility cost relative to owning. We refer to them as 'unhappy renters'. iii) Those with $d = o$ and $z = 1$ are owning but would not suffer a utility cost from renting. We refer to them as 'unhappy owners'. iv)

[5] The shock might be related to, say, wishing to move to a single family house because of having children. Since the rental market for single family houses is almost non-existent, moving to a single family house requires buying a house.

Those with $d = o$ and $z = 2$ are owning and would suffer a utility cost if renting. We refer to them as 'happy owners'.[6]

The rental market functions perfectly in the sense that a household can always find a rental house at a fixed rental rate. This rental rate is exogenous in the model. The market for owner housing, in contrast, is characterized by matching frictions. A household that wishes to buy a house must first find a potential seller. Similarly, a household that wishes to sell must find a potential buyer.

We assume that all unhappy renters and unhappy owners participate in the housing market, that is, they search for a house to buy or put their house for sale, while all happy renters and happy owners stay out of the market. Clearly, as long as the disutility cost borne by unhappy renters is non-negligible, unhappy renters and unhappy owners have the most to gain from trade. Happy owners and happy renters, in turn, are unlikely to gain from trade.[7]

Potential buyers and sellers are randomly matched. In each period, they can meet at most one trading partner. Upon having met, the potential buyer and the potential seller bargain over the price. If there exists a price that makes trade mutually beneficial, trade takes place and the price is determined by Nash bargaining with equal bargaining power. The price will depend on the seller's and buyer's continuation values, which in turn depend on their financial wealth positions.

Each period, timing is the following. First, potential buyers (unhappy renters) and potential sellers (unhappy owners) are matched. Next, unsuccessful matches break down and the transactions of successful matches take place. Buyers move to owner housing and sellers move to rental housing. After that, renters pay a rent and owners a maintenance cost related to their current housing. Finally, all households decide on non-housing consumption and financial savings or borrowing.

The periodic utility of the household is given by $u(c, z, d)$ where c denotes non-housing

[6] It is clear that owner housing must be more expensive than rental housing. Hence, in a sense, unhappy owners, who would receive the same utility flow from rental housing, are paying too much for their housing.

[7] The participation decision could be endogenized in the model by assuming that there is a fixed cost of entering the market.

consumption. Households supply one unit of labor in each period. The wage rate is given by εw, where $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$ is an i.i.d. income shock and w the average wage rate. Probability of income shock ε_i is given by φ_i. Households can save and borrow with a financial asset. The interest rate on savings or borrowing is $R - 1$. The direct cost of housing services is either rent, υ, or, in the case of owner housing, a maintenance cost, κ.

Using s to denote financial saving or borrowing, the financial asset position of the household, a, evolves as

$$a' = Rs + \varepsilon' w,$$

where primes indicate next period values. In what follows, we refer to a as 'financial wealth'.

Borrowing is limited by a borrowing constraint. We require that $s \geq \underline{s}^d$ for $d = r, o$. We will later assume that $\underline{s}^r = 0$ and $\underline{s}^o < 0$, that is, only owners can borrow.

If a household does not buy or sell a house, its current non-housing consumption is

$$c = a - s - g, \tag{1}$$

where $g = \upsilon$ for renters and $g = \kappa$ for owners.

If a renter buys a house with price p, its current non-housing consumption is

$$c = a - s - \kappa - (1 + \tau)p, \tag{2}$$

where $\tau \geq 0$ denotes a transaction tax.

Finally, if an owner sells a house, its current non-housing consumption is given by

$$c = a - s - \upsilon + p. \tag{3}$$

Of course, all households must be able to afford strictly positive non-housing consumption. If we denote a minimum consumption level by $c_{\min} > 0$ (close to zero), the maximum price that a buyer can afford to pay can be written as

$$p = \frac{a - \underline{s}^o - \kappa - c_{\min}}{1 + \tau}. \tag{4}$$

Similarly, the minimum price that a seller needs to receive is

$$p = \underline{s}^r + \upsilon - a + c_{\min}. \tag{5}$$

2.2 Household problem and bargaining

We now define the household optimization problem recursively. Let $V^d(a,z)$ denote the value function before current period matches and $v^d(a,z)$ the value function conditional on the household not trading. The latter value function is determined as:

$$\begin{aligned} v^d(a,z) &= \max_{s \geq \underline{s}^d} \left\{ u(c,z,d) + \beta \sum_{j=1}^{2} P(j,z) \sum_{i=1}^{n_\varepsilon} \varphi_i V^d(Rs + w\varepsilon_i, j) \right\} \\ & \quad \text{subject to (1)} \end{aligned} \tag{6}$$

where $\beta < 1$ is the subjective discount factor, $z = 1,2$ and $d = r,o$. We use $s^d(a,z)$ to denote the associated optimal savings policy.

Happy renters and happy owners do not participate in the housing market. Therefore, for them, the value function before current period matches equals the value function conditional on not trading, that is

$$V^r(a,1) = v^r(a,1) \tag{7}$$

$$V^o(a,2) = v^o(a,2). \tag{8}$$

In the case of unhappy renters and unhappy owners, who participate in the housing market, we have to consider the value of being matched with a potential trading partner. Let $W^b(a,\widetilde{a})$ denote the value of a potential buyer (unhappy renter) with financial wealth a matched with a potential seller (unhappy owner) with financial wealth $\widetilde{a}$. Similarly, let $W^s(\widetilde{a},a)$ denote the value of a potential seller with financial wealth a matched with a potential buyer with financial wealth $\widetilde{a}$.

Finally, let us denote the population of households with financial wealth a, tenure state d, and tenure preference state z, by $\mu^d(a,z)$. The mass of potential sellers is denoted by $m^s = \int \mu^o(a,1)\,da$ and the mass of potential buyers by $m^b = \int \mu^r(a,2)\,da$.

We can now define $V^r(a,2)$ as

$$V^r(a,2) = \phi^s \int W^b(a,\widetilde{a}) \frac{\mu^o(\widetilde{a},1)}{m^s} d\widetilde{a} + (1-\phi^s)\, v^r(a,2), \tag{9}$$

where ϕ^s denotes the probability of meeting a potential seller. The first term is the expected value of a match weighted by the probability of being matched with a potential seller. The second term is the value of a renter not trading weighted by the probability that the household is not matched.

Similarly, we define $V^o(a,1)$ as

$$V^o(a,1) = \phi^b \int W^s(\widetilde{a},a)\frac{\mu^r(\widetilde{a},2)}{m^b}d\widetilde{a} + \left(1-\phi^b\right)v^o(a,1), \tag{10}$$

where ϕ^b denotes the probability of meeting a potential buyer.

In order to determine $W^b(.)$ and $W^s(.)$ for all possible matches, we need to find out whether the match leads to trade and if so, at what price. Consider a potential buyer with financial wealth a who has met a potential seller and contemplates buying the house with price p. If the household buys the house, it becomes a happy owner with financial wealth equal to $a-(1+\tau)p$. If it decides not to buy, it's value is the same as that of a renter not matched with a potential seller. Hence, its surplus from trade, denoted by $S^b(a,p)$, can be written as

$$S^b(a,p) = v^o(a-(1+\tau)p,2) - v^r(a,2). \tag{11}$$

In the same way, if a potential seller with financial wealth $\widetilde{a}$ sells its house with price p, it becomes a happy renter with financial wealth equal to $\widetilde{a}+p$. If it does not sell, its value is the same as that of an owner not matched with a potential buyer. Therefore, its surplus from trade is

$$S^s(\widetilde{a},p) = v^r(\widetilde{a}+p,1) - v^o(\widetilde{a},1). \tag{12}$$

Each party is only willing to trade if the surplus from trade is positive. If there exists no price p such that both $S^b(a,p)\geq 0$ and $S^s(\widetilde{a},p)\geq 0$, there is no trade. If such a price exists, the match leads to trade and the equilibrium price is

$$\arg\max_p \left\{S^b(a,p)\,S^s(\widetilde{a},p)\right\}. \tag{13}$$

We denote the equilibrium price by $p(a,\widetilde{a})$, where the first argument is the buyer's financial wealth and the second argument is the seller's financial wealth. Appendix A shows that if trade takes place, the Nash bargaining price is uniquely determined.

Let $Tr(a,\widetilde{a})$ be an indicator function that equals one if trade takes place and zero otherwise, when the buyer's and seller's financial wealth positions are a and $\widetilde{a}$, respectively. Hence, it is defined as

$$Tr\left(a,\widetilde{a}\right)=\begin{cases} 1 \text{ if } \exists\ p \text{ s.t. } S^{b}\left(a,p\right)\geq 0 \text{ and } S^{s}\left(\widetilde{a},p\right)\geq 0 \\ 0\text{, otherwise}\end{cases} \tag{14}$$

We can now define $W^b(a,\widetilde{a})$ and $W^s(a,\widetilde{a})$ as

$$W^{b}(a,\widetilde{a})=\begin{cases} v^{o}\left(a-\left(1+\tau\right)p\left(a,\widetilde{a}\right),2\right) \text{ if } Tr\left(a,\widetilde{a}\right)=1 \\ v^{r}\left(a,2\right) \text{ if } Tr\left(a,\widetilde{a}\right)=0\end{cases} \tag{15}$$

$$W^{s}(a,\widetilde{a})=\begin{cases} v^{r}\left(\widetilde{a}+p\left(a,\widetilde{a}\right),1\right) \text{ if } Tr\left(a,\widetilde{a}\right)=1 \\ v^{o}\left(\widetilde{a},1\right) \text{ if } Tr\left(a,\widetilde{a}\right)=0\end{cases}. \tag{16}$$

2.3 Matching

We follow the related literature in assuming that trading frictions can be represented by a matching function, which specifies the number of trading opportunities in a given period. Empirical evidence suggests that in labor-market applications matching technology features constant returns to scale (see for instance Petrongolo and Pissarides, 2001).[8] To our knowledge, there are no studies testing the constant returns to scale hypothesis in the housing market.[9]

We use the simplest possible specification. We have two cases depending on the relative sizes of the mass of potential buyers, m^b, and the mass of potential sellers, m^s. If $m^s \leq m^b$, the probability of being matched with a potential seller, ϕ^s, and the probability of being matched with a potential buyer, ϕ^b, are

$$\phi^{b}=\chi \text{ and } \phi^{s}=\chi\frac{m^{s}}{m^{b}} \tag{17}$$

otherwise

$$\phi^{b}=\chi\frac{m^{b}}{m^{s}} \text{ and } \phi^{s}=\chi, \tag{18}$$

[8]If matching does not feature constant returns scale, the probability of a match depends not only on the composition of those in the market (sellers/buyers), but also on the amount of sellers and buyers active in the market.

[9]For a thorough discussion on trading frictions in asset markets, see e.g. Rocheteau and Weill (2011) and Caplin and Leahy (2011).

where $\chi \in (0,1]$ is a matching efficiency parameter. Given the masses of potential buyers and potential sellers, the higher is χ, the more matches there are every period. If $\chi = 1$, traders in the short side of the market meet a potential trading partner with probability one.

2.4 Stationary equilibrium

We consider a stationary equilibrium where the distribution of households over their asset, tenure preference, and occupancy states is constant over time. The interest, wage and rental rates as well as the shares of owner and rental households are exogenously given.

Definition 1 *The stationary equilibrium consists of value functions* $\left\{V^d(a,z), v^d(a,z), W^b(a,\widetilde{a}), W^s(a,\widetilde{a})\right\}$, *household savings function* $s^d(a,z)$, *prices* $p(a,\widetilde{a})$, *indicator function* $Tr(a,\widetilde{a})$, *matching probabilities* ϕ^b *and* ϕ^s, *and distribution* $\mu^d(a,z)$ *(containing the information of* m^b *and* m^s*) which satisfy*

Matching:

Given $\mu^d(a,z)$, ϕ^b *and* ϕ^s *are determined by (17) or (18).*

Household optimization and bargaining:

a) Given $V^d(a,z)$, $s^d(a,z)$ *solves (6) with* $v^d(a,z)$ *as the resulting value function.*
b) Given $v^d(a,z)$, *surpluses* $S^b(a,p)$ *and* $S^s(a,p)$ *are determined by (11) and (12). Given the surpluses,* $Tr(a,\widetilde{a})$ *is determined from (14). For pairs* $\{a,\widetilde{a}\}$ *such that* $Tr(a,\widetilde{a}) = 1$, $p(a,\widetilde{a})$ *is determined by (13). Given* $Tr(a,\widetilde{a})$ *and* $p(a,\widetilde{a})$, $W^b(a,\widetilde{a})$ *and* $W^s(a,\widetilde{a})$ *are determined by (15) and (16).*
c) Given $v^d(a,z)$, $V^r(a,1)$ *and* $V^o(a,2)$ *are determined by (7) and (8). Given* $v^d(a,z)$, $W^b(a,\widetilde{a})$, $W^s(a,\widetilde{a})$, *and* $\mu^d(a,z)$, $V^r(a,2)$ *and* $V^o(a,1)$ *are determined by (9) and (10).*

Consistency:

$\mu^d(a,z)$ is the time invariant distribution that follows from the household savings policy, the outcome of the Nash bargaining, the probabilities $P(z',z)$ and φ_i for all $i = 1, 2, ..., n_\varepsilon$, and the exogenously determined masses of renter and owner households.

2.5 Solving the model

The key computational challenge is related to the fact that households need to take into account the distribution. For instance, a potential seller wants to consider the distribution of asset holdings for all potential buyers. This is because its surplus from a match depends on the asset position of the potential buyer. Therefore, the value of not selling today and staying in the market depends on the distribution.

When solving the model, we thus need to find a distribution which is consistent with households' information about the distribution and the resulting household behavior. In practice, we iterate over the distribution. We first make an initial guess for the distribution. We use that distribution to determine the matching probabilities and also plug it in (9) and (10). We then solve recursively for all the value and policy functions. Finally, we simulate to find the associated stationary distribution. The resulting distribution provides us a new guess.

In our experience, this iteration converges quite nicely. We have also experimented with very different initial guesses for the distribution. The equilibrium we found was always independent of the initial guess. We discuss computational issues in more detail in Appendix B.

3 Numerical analysis

In this section, we analyze the model numerically. We begin by explaining how we calibrate the model and then present and discuss our results.

3.1 Calibration

We set the model period to be 3 months. Having a shorter time period might be useful, for instance, in order to describe the distribution of time-on-the-market for houses on the market. On the other hand, shortening the model period would further increase the computational costs.

We base our calibration on the Wealth Survey that was conducted by Statistics Finland in 2004. The survey contains register data about the asset holdings and incomes of a representative sample of Finnish households. Register data is supplemented by survey information. Importantly for our purposes, households were asked, among other things, to give an estimate of the current market value of their house and to report the length of stay in their current dwelling.

We consider only households where the age of the household head is between 30 and 60, in order to focus on the working age population. We also exclude social housing residents. We construct two variables for the analysis: 'house value' and 'financial wealth'. House value is the value of primary residence as estimated by the household. Financial wealth is the sum of all financial assets, quarterly after-tax return to financial assets, quarterly after-tax non-capital income, less all debt and quarterly interest payments on debt.

We set the interest rate parameter at $R = 1.01$ implying an annual interest rate of about 4%.

We consider the following utility function

$$u(c, z, d) = \frac{c^{1-\sigma}}{1-\sigma} - I(z, d) f,$$

where

$$I(z, d) = \begin{cases} 1 \text{ if } z = 2 \text{ and } d = r \\ 0 \text{ otherwise} \end{cases}.$$

Parameter $\sigma > 0$ measures risk-aversion (as usual $\sigma = 1$ would correspond to log-utility) and f is the disutility cost of living in rental housing while having a tenure preference for owner housing. We set $\sigma = 2$, which is a relatively conventional value.

In the sample, the average length of stay in current dwelling is 10 years. Based on this, we set the probability of a tenure preference shock so that in the model economy households face the need to move on average every 10 years. This implies

$$P(z', z) = \begin{bmatrix} 0.975 & 0.025 \\ 0.025 & 0.975 \end{bmatrix}.$$

For simplicity, we assume that the mass of owner houses is 0.5. In other words, half of the households are owners and half of them are renters. Recall that we also assumed that all unhappy renters and all unhappy owners participate in the housing market while other households are not active in the housing market. Together with the symmetric transition probility matrix P, these assumptions imply that the masses of potential buyers and potential sellers are always equal. Equation (17) shows that the matching probabilities, ϕ^b and ϕ^s, are then both equal to χ.

We assume that the income shock can take two values, that is $\varepsilon \in \{\varepsilon_1, \varepsilon_2\}$. We interpret the first shock as unemployment. The unemployed households receive an unemployment compensation. In Finland, the tax-funded unemployment insurance provides a fixed benefit that is about 25% of the average earnings. This implies

$$\varepsilon_1 = 0.25 \text{ and } \varepsilon_2 = 1.$$

We choose the probabilities of the income shocks so that the unemployment rate is 8%, which implies

$$\varphi_1 = 0.08 \text{ and } \varphi_2 = 0.92.$$

We further normalize the wage rate so that the average non-capital income equals 1. This results in $w = 1.064$. In what follows, we refer to after-tax non-capital income as simply 'income'.

In the data, the average rent-to-income ratio equals 0.22. Hence we set the rent at $\upsilon = 0.22$. We set the transaction tax at $\tau = 0.016$, which is the current transaction tax rate on apartments. We assume that households can only borrow against owner housing. Therefore, the borrowing constraint for renters is $\underline{s}^r = 0$.

We are then left with five parameters: owners' borrowing limit, $\underline{s}^o$, maintenance cost, κ, matching efficiency, χ, discount factor, β, and utility cost, f. We determine these parameters so that the model matches certain empirical targets. First of all, we want the model to feature a realistic average house price-to-average income ratio. In the data, the ratio of average house value to average quarterly income among owners is 14.6. Given that we normalized the average income to one, the average house price should thus be 14.6. We also want a realistic average financial wealth-to-average income ratio. However, the model does not feature enough parameters to match renters' and owners' average asset positions separately. If, on the other hand, we were to match the average financial asset position for all households, renters would be far too wealthy in the model. In order to capture the importance of borrowing constraints for potential buyers, we aim to match the ratio of average financial wealth to average income for renters only. For renters in the data, this figure is 2.7. Finally, we want the model to feature a realistic average time-on-the-market. Eerola and Lyytikäinen (2012) report an average time-on-the-market in the Finnish housing market of 55 days, which corresponds to 0.61 model periods.[10]

We choose the borrowing limit for owners, $\underline{s}^o$, so that it reflects a realistic down payment requirement for mortgages. In 2010, about half of the housing loans for first time buyers exceed 90% of the house value (Financial Supervisory Authority, 2011). We assume that owners can borrow up to 95% of the average house price. In the survey data, the average annual maintenance cost is about 3% of the average house value. We use this information to pin down the maintenance cost κ.

To summarize, we choose β, f, χ, $\underline{s}^o$, and κ so as to match the following targets: i)

[10]We compute the average time-on-the-market by following households that have just become unhappy owners. In the model, transactions occur in the begining of each period. Therefore, an unhappy renter that buys a house in a given period, avoids paying the disutility cost associated with rental housing for the whole period. Accordingly, if a household sells its house in the same period it entered the housing market, the time-on-the-market is recorded as zero. If it sells in the next period, the time-on-the-market is recorded as 1 period, or 90 days, and so on. Some households withdraw their house from the market before they sell because they are hit by a tenure preference shock. Consistently with the empirical measure, the time-on-the-market is not recorded in that case.

Average house price equal to 14.6, ii) average financial wealth-to-average income ratio for renters equal to 2.7, iii) average time-on-the-market equal to 0.61 model periods, iv) owners can borrow up to 95% of the average house price, v) average annual maintenance cost is 3% of the average house price.

Given the targeted average house price, the last two targets directly imply $\underline{s}^o = 0.95 * 14.6 = 13.87$ and $\kappa = (0.03 * 14.6)/4 = 0.11$. The remaining three targets depend on all three remaining parameters. With parameter values $\beta = 0.98$, $f = 0.17$, and $\chi = 0.8$, the model closely matches also targets i)-iii).

3.2 Market inefficiency

There are several frictions in the model: a matching friction, a borrowing constraint, and a transaction tax. We use two measures of market inefficiency, namely average time-on-the-market and coefficient of variation of house prices, to study how these frictions influence the housing market.

Time-on-the-market is a commonly used measure of housing market conditions. As explained above, the calibrated model features a realistic average time-on-the-market. This means that households in the model economy face a realistic trade-off between trading today relative to staying in the market and waiting for a better match.

Time-on-the-market also indirectly measures the welfare cost related to the misallocation of housing units because it reflects the share of households that pay the disutility cost associated with rental housing. The disutility cost is borne by unhappy renters each period they are unable to move to owner housing. In the absence of both matching and financial frictions, no one would pay this disutility cost. In the benchmark calibration, the share of households that pay the disutility cost is 0.77%. This share changes almost one-to-one with the average time-on-the-market.

In the model, there are two reasons why unhappy renters do not trade immediately. First, since $\chi < 1$, some potential buyers are not matched with a potential seller so they don't even have a chance to buy a house. Second, some matches do not result in trade. As we explain below, the main reason for this is the borrowing constraint. In the benchmark calibration, 24% of the matches do not result in trade.

The coefficient of variation of house prices is a scale-neutral measure of price dispersion. Since houses are identical in the model, it is clear that in the absence of matching frictions, all houses would sell at the same price. In other words, some matching frictions are needed to generate price dispersion. However, matching frictions alone are not able to generate price dispersion, if all matches are identical. In the model, any house price dispersion stems from matching frictions together with wealth heterogeneity. In the benchmark calibration, the coefficient of variation for realized house prices is 0.21%. Hence, the model displays very little dispersion in house prices. However, as we show below, the price distribution is highly skewed.

3.3 Household policies and price determination

Figure 1 describes the optimal savings policy for all households. Current financial wealth is on the horizontal axis.[11] The vertixal axis shows the difference between the expected next period financial wealth (that is, $Rs^d(a, z) + E\varepsilon w$) and current financial wealth. If this difference is positive, the household is expected to be wealthier next period. Otherwise the household is expected to be poorer. The left hand panel shows the savings behavior of renters and the right hand panel that of owners. The figure shows the savings policy separately for those on the market (unhappy owners and unhappy renters) and those not on the market (happy renters and happy owners).

[11] The lowest financial wealth levels in the figures correspond to the maximum and minimum prices defined in (4) and (5). See Appendix B for details.

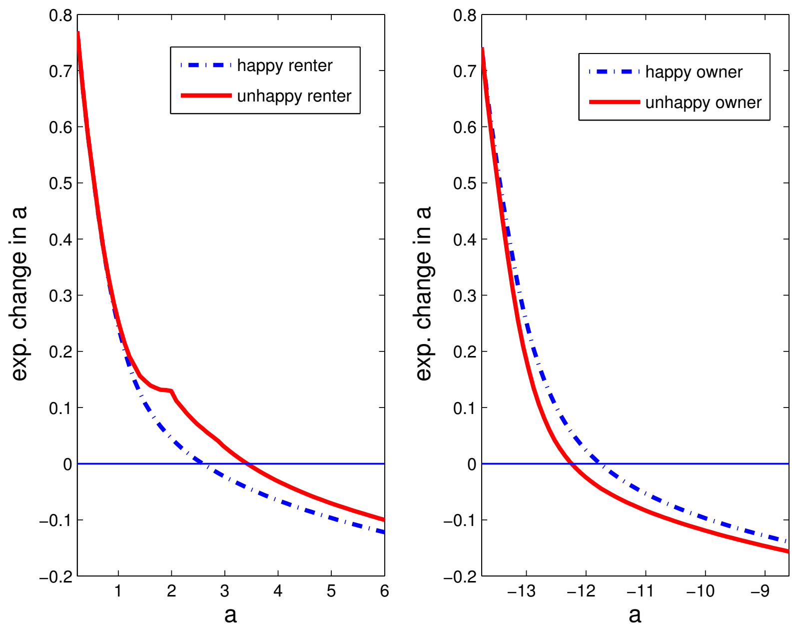

Figure 1: Savings policies.

Three features of the figure are worth discussing. First, for very low asset holdings, the curve is a straight line. At these asset levels, any increase in the current financial wealth is entirely spent on non-housing consumption in the current period. Therefore, an increase in current financial wealth is associated with a one-to-one reduction in the difference between expected future financial wealth and current financial wealth. This happens as long as households are borrowing constrained. Borrowing constrained owners borrow up to $-\underline{s}^o$ and borrowing constrained renters choose to save nothing ($\underline{s}^r = 0$). Their expected financial wealth is nevertheless higher than their current financial wealth because financial wealth also includes wage income and the unemployment benefit. For instance, the expected next period financial wealth for a renter that saves nothing is $E\varepsilon w = 1$.

Second, the savings behavior of unhappy renters is quite different from other households' savings behavior. In particular, for unhappy renters the curve is almost flat over a certain financial wealth range. Below that range, when given a little bit more financial wealth, unhappy renters prefer to use most of it to finance current consumption simply because their marginal utility of consumption is very high. These renters do not expect to be able to finance a house in the near future. In contrast, in the range where the curve is flat, households' savings policy reflects mainly the need to be able to finance a house, if given the chance to buy one. Unhappy renters in this financial wealth range know that while they might be able to buy a house, the borrowing constraint is an issue for them. Hence, when given a little bit more financial wealth, they prefer to save most of it in order to make sure that they are able to finance a house without sacrificing too much current non-housing consumption.

Also relatively wealthy unhappy renters save more than happy renters. This is because owners spend more for housing than renters (when both maintenance and capital costs are taken into account). So unhappy renters, who are likely to buy a house soon, expect to spend more for housing than happy renters. For the same reason, unhappy owners, who expect to move soon to rental housing, save less than happy owners.

Third, all the curves cross the zero line only once. Those that have relatively high current financial wealth are expected to have a lower financial wealth in the future and those with low current financial wealth are expected to become wealthier. This suggests that at least given the savings policies, the stationary distribution is unique.

Figure 2 illustrates how the outcome of the Nash bargaining depends on potential buyer's and seller's asset positions. The left hand panel plots the Nash bargaining price as a function of seller's financial wealth and the right hand panel the price as a function of buyer's financial wealth. Both panels show two different cases: one where the potential trading partner is relatively poor in terms of financial wealth and another where it is relatively wealthy. For some combinations of seller's and buyer's financial wealth, a match does not result in trade. Obviously, in those cases there is no price to be plotted.

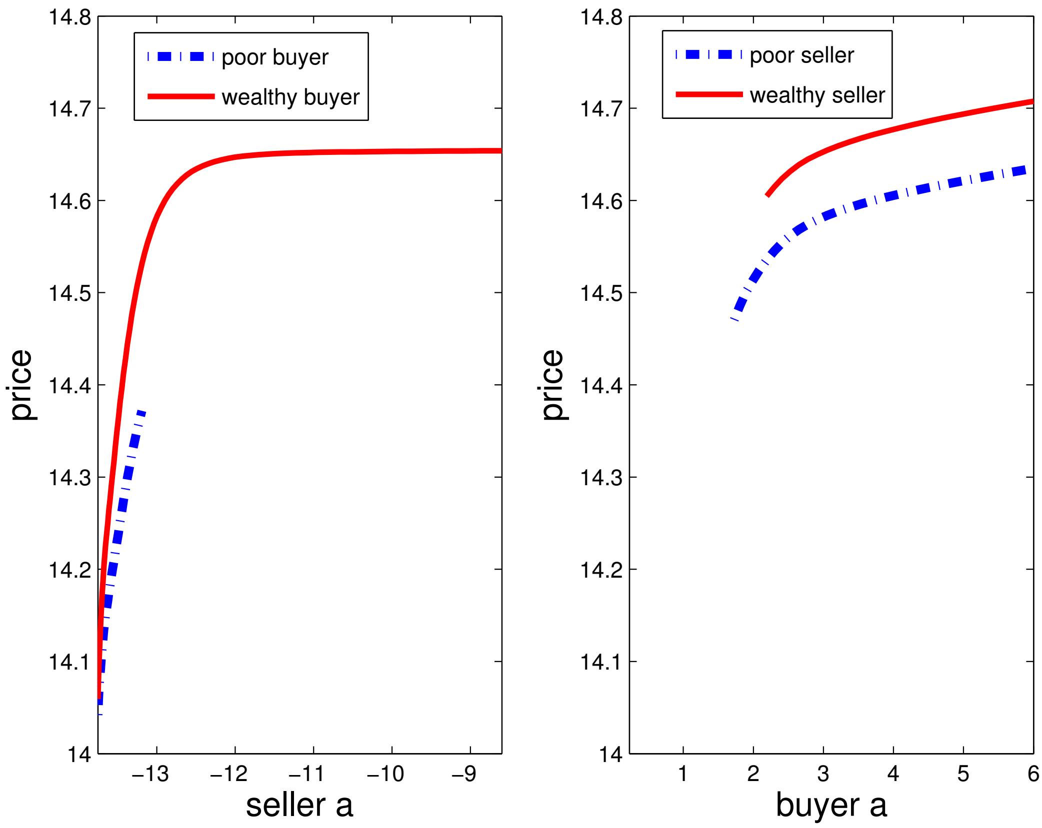

Figure 2: Price as a function of seller's (left) and buyer's (right) asset position.

Consider first the left hand side of the figure and the case of a relatively wealthy buyer. When the seller is very poor and likely to be borrowing constrained, the Nash bargaining price is relatively low. Selling the house allows a highly indebted owner to smooth consumption over time. For a given price, these households benefit much more from trade than wealthier sellers. Therefore, the Nash product is maximized with a relatively low price. However, the need to sell for liquidity reasons diminishes quickly as current financial wealth increases. This means that the outside option of the seller increases rapidly. Hence, for there to be trade, the price must increase rapidly as well. Further away from the borrowing constraint, consumption smoothing is not an issue anymore. Therefore, the price as a function of seller's financial wealth becomes almost flat.

When looking at the case of a relatively poor buyer, one observes that trade only occurs if the seller is also relatively poor. Because of the borrowing constraint, a poor buyer is only able to trade if the price is relatively low. However, when faced with such a buyer, a wealthier seller, who does not need to sell for liquidity reasons, prefers to wait for a better match.

The right hand side panel shows, first of all, that the price is relatively insensitive to changes in buyer's asset position. However, when the buyer is very poor, there is no trade. Because of the borrowing constraint, a buyer with very little savings is unable to pay a price that would satisfy the seller. On the other hand, as long as trade occurs, the price is always increasing in buyer's financial wealth. Again, because of the desire for consumption smoothing, the price increases somewhat more rapidly with buyer's financial wealth at low asset levels than at high asset levels. However, even for high asset levels, the price function does not flatten out as much as in the left hand panel. A wealthier buyer is always willing to pay more than a poorer one in order to avoid the disutility cost associated with rental housing already today rather than later.

More generally, figure 2 shows that the existence of a borrowing costraint is important regarding both price dispersion and whether or not trade takes place. If all households are far away from the borrowing constraint, trade always occurs and the price is relatively insensitive to changes in seller's or buyer's asset positions.

3.4 Asset and house price distributions

Figure 3 displays the stationary distributions of renters' and owners' financial wealth holdings. The financial wealth distributions are very concentrated. This is not surprising for two reasons. First, the only source of income uncertainty is the i.i.d. unemployment shock. Second, the discount factor, which was chosen so as to match renters' average financial wealth holdings in the data, is quite low relative to the interest rate. Related to this, all owners are highly leveraged. That is, of course, unrealistic. However, as discussed above, the Nash bargaining outcome is sensitive to changes in the seller's asset position only when the seller is close to the borrowing constraint.

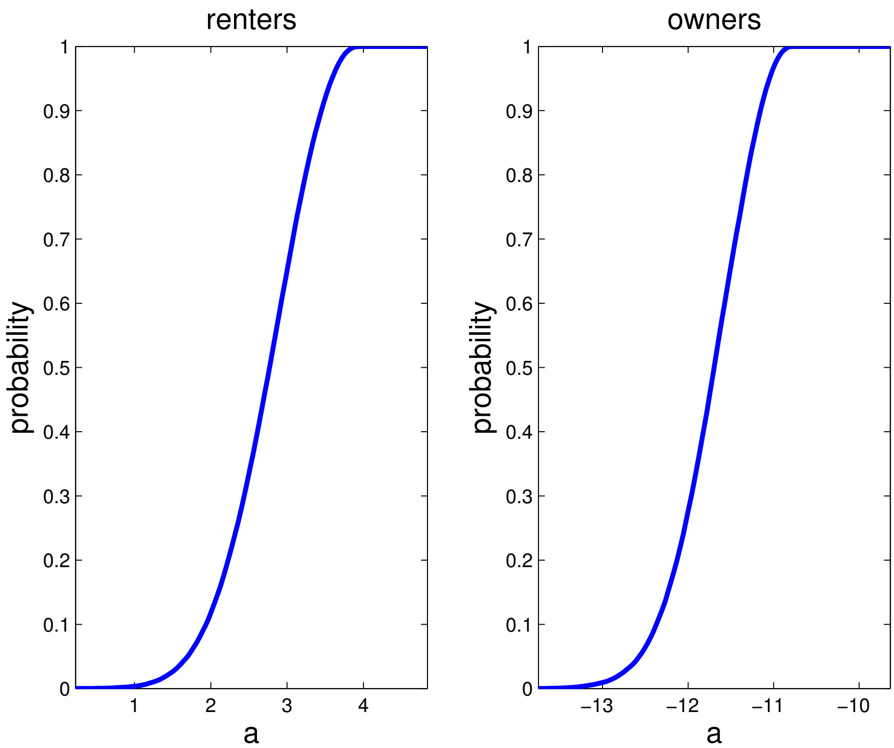

Figure 3: Financial asset distributions (cdf).

Figure 4 displays the distribution of realized house prices. The distribution in the figure is cut so that all prices less than 14.45 are collected to the first bar. For 0.4% of the trades, the price is less than 14.45 and the lowest price is about 14.2. In other words, the price distribution is very skewed and features a long and very thin left hand tail. The thin tail is explained by the borrowing constraint: Trade occuring with very low prices requires that *both* parties are very close to the borrowing constraint. Such matches are unlikely. When only one of the trading partners is very close to the borrowing constraint, the other partner prefers to wait for a better match and there is no trade.

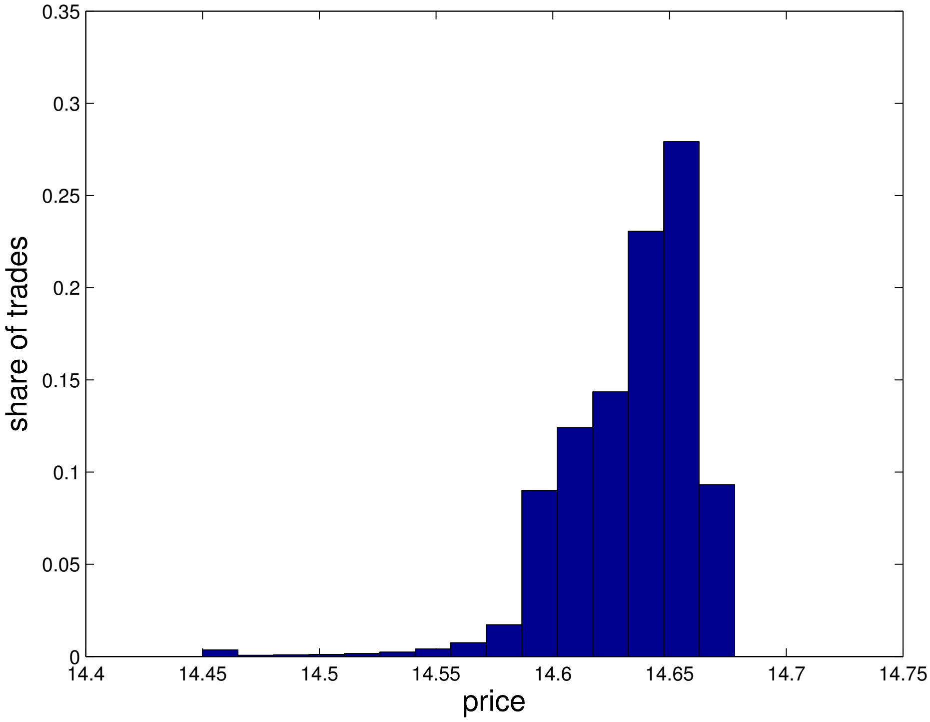

Figure 4: House price distribution.

3.5 Experimenting with frictions

In this section, we analyze how different types of frictions influence housing market outcomes. We consider the following three frictions: borrowing constraint, matching friction, and transaction tax. We vary one friction at a time and study how the changes influence the measures of market inefficiency discussed in section 3.2. We also report changes in the average house price, in the average asset positions for renters and owners, as well as in the share of matches that result in trade.

The benchmark values of the parameters describing the frictions are $\underline{s}^o = -13.87$ (borrowing constraint), $\chi = 0.8$ (matching probability), and $\tau = 0.016$ (transaction tax). We first tighten the borrowing constraint for owners by setting $\underline{s}^o = -12.41$ and $\underline{s}^o = -10.95$. These figures correspond to mortgages of approximately 85% and 75% of the average house price in the benchmark economy. Recall that the benchmark borrowing constraint corresponds to a mortgage of 95% of the average house price. For the matching probability χ, we consider values $\chi = 1.0$ and $\chi = 0.6$. Finally, we eliminate the transaction tax altogether, and double it to 3.2%.

Table 1 displays the results as percentage changes relative to the benchmark economy. The first three columns report the relative changes in the average financial wealth of owners ($\overline{a}^o$) and renters ($\overline{a}^r$) and the average house price ($\overline{p}$). (Owners' average financial wealth is negative. We report the change relative to the absolute value of average financial wealth.) The last three columns report the relative changes in the average time-on-the-market (tom), the coefficient of variation of house prices ($cv(p)$), and the share of matches that result in trade (tr).

	$\overline{a}^o$	$\overline{a}^r$	$\overline{p}$	tom	$cv(p)$	tr
Matching probability						
$\chi = 1.0$	0	0	-0	-42	-17	-4
$\chi = 0.6$	0	-0	0	66	23	5
Borrowing constraint						
$\underline{s}^o = -12.41$	12	12	-6	77	78	-24
$\underline{s}^o = -10.95$	24	29	-11	199	183	-45
Transaction tax						
$\tau = 0$	0	1	1	-21	11	9
$\tau = 0.032$	0	-1	-1	34	-10	-12

Table 1: Percentage changes in selected statistics relative to the benchmark economy.

Consider first the matching friction. Changes in the matching probability χ have virtually no effect on households' average financial asset holdings or the average house price. Naturally, however, they do affect our measures of market inefficiency. For instance, increasing the matching probability from 0.8 to 1.0, decreases the average time-on-the-market by 42% and the coefficient of variation of house prices by 17%. The effects on the time-on-the-market are somewhat mitigated by the fact that when the matching probability is lower, a larger share of matches result in trade.

Consider then the borrowing constraint. Not surprisingly, tightening the borrowing constraint increases households' average financial asset positions and decreases the average house price. The effects are relatively large, reflecting the fact that households are relatively poor in terms of financial wealth. For instance, decreasing the maximum mortgage by 10%, increases both renters' and owners' average financial wealth by 12% and decreases the average house price by 6%. Decreasing the maximum mortgage by 20%, roughly doubles these effects.

Interestingly, tightening the borrowing constraint also works to substantially increase the measures of market inefficiency. Decreasing the maximum mortgage by 10%, increases the average time-on-the-market and the coefficient of variation of house prices by 77% and 79%, respectively. Decreasing the maximum mortgage by 20%, almost triples these measures relative to the benchmark. The increase in the average time-on-the-market reflects the fact that a smaller share of matches result in trade. For instance, decreasing the maximum mortgage by 20%, decreases the share of successful matches by 45%, or from 0.76 to 0.42.

In other words, the borrowing constraint magnifies the effects of matching frictions. For instance, while some matching frictions are needed to create any price dispersion between identical houses, tightening the borrowing constraint increases price dispersion substantially. In fact, Table 1 reveals that decreasing the maximum mortgage by just 10% increases the coefficient of variation of house prices much more than decreasing the probability of a match from 0.8 to 0.6 (79% vs. 23%).

The intuition behind both the smaller share of successful matches and the increased price dispersion relates to figure 2. As the figure shows, the main link between the Nash bargaining price and traders' asset positions is the borrowing constraint. As the borrowing constraint is tightened, it becomes relevant to a larger share of households, even though households' average financial wealth positions increase. Related to this, a larger share of potential buyers are so close to the borrowing constraint that with some sellers there is no scope for trade.

Finally, consider the transaction tax. Eliminating the transaction tax leads to an approximately 1% increase in the average price by while doubling it to 3.2% leads to a similar house price depreciation. This means that the transaction tax partly capitalizes into house prices.[12] The transaction tax has a substantial effect on the average time-on-the-market. According to these results, a transaction tax of 1.6% increases the average time-on-the-market by about 25% compared to a situation without a transaction tax. As a result, it also increases the share of unhappy renters by about as much. In other words, even a rather moderate transaction tax makes the housing market much less efficient.[13] However, the transaction tax works to decrease price dispersion. One reason is that it makes some matches where the potential buyer has little savings unsuccessful. In the absence of the tax, such matches would contribute positively to price dispersion, because the price would be relatively low.

4 Discussion

We have developed a model of the housing market that features both financial and matching frictions. In the model, both sides of the market are heteregeneous in their assets and the outcome of the bargaining process between a seller and a buyer depends on the seller's and the buyer's financial asset positions. As a result, some matches do not result in trade and identical houses may sell at different prices. These are all natural features of the market for owner housing.

Our results illustrate how a financial friction in the form of a borrowing constraint works to magnify the effects of matching frictions. For instance, while some type of a matching friction is needed for the model to generate any price dispersion, tightening the borrowing constraint increases price dispersion substantially. This is because the borrowing constraint makes the outcome of the bargaining process more sensitive to traders' asset positions. The borrowing constraint also explains why some matches do not result in trade. Therefore, tightening the borrowing constraint also increases the average time-on-the-market. Our results also suggest

[12] Dachis, Duranton and Turner (2012) find that an increase of 1.1% in transaction tax in Toronto caused a decline in houses prices about equal to the tax. Our setting is different in that transaction tax cannot be avoided by trading outside a certain geographical area.

[13] Empirical evidence indicates that transaction taxes also reduce residential mobility. See for instance Hilber and Lyytikäinen (2012), Dachis et al. (2012), and Van Ommeren and Van Leuvensteijn (2005).

that even a moderate transaction tax makes the housing market less efficient by substantially increasing the average-time-on-the market.

The results of the present paper can be extended and complemented in several ways. For instance, financial frictions might become even more important for housing market outcomes if we allow for endogenous housing market participation together with 'thick-market effects' (as in Ngai and Tenreyro 2012). In future work, it should also be possible to consider aggregate dynamics (as in, for instance, Díaz and Jerez 2012). One potentially very interesting question in this respect is the role of borrowing constraints for house price dynamics. Even a moderate fall in house prices may reduce the net worth of highly leveraged households drastically. Stein (1995) and Ortalo-Magné and Rady (2006) have described how such a reduction in households' net worth may feed back into house prices through household borrowing constraints and create a multiplier effect. However, in models where the spot market for housing works perfectly, borrowing constraints can influence house price dynamics substantially only if the share of borrowing constrained households is very large (see Eerola and Määttänen 2012). This is partly because non-constrained households take advantage of any future house price predictability that may be caused by borrowing constraints. With matching frictions, however, non-constrained households cannot immediately invest in housing, even if they anticipate large capital gains. In other words, matching frictions might make borrowing constraints more relevant for house price dynamics.

Appendix A

In this appendix, we show that the Nash bargaining price is unique. The value function of the household in occupancy state d and tenure preference state z with financial wealth a is

$$v^d(a,z) = \max_{s \geq \underline{s}^d} \left\{ u(c,z,d) + \beta \sum_{j=1}^{2} P(j,z) \sum_{i=1}^{n_\varepsilon} \varphi_i V^d(Rs + w\varepsilon_i, j) \right\}.$$

Denote the level of savings that solves the household problem by $s^d(a,z)$. The optimal level of savings is determined by the first-order condition

$$-\frac{\partial u(c,z,d)}{\partial c}+\beta R\sum_{j=1}^{2}\sum_{i=1}^{n_\varepsilon}P(j,z)\varphi_i\frac{\partial V^d(Rs+w\varepsilon_i,j)}{\partial a'}+\mu^d=0, \tag{A1}$$

where μ^d is the Kuhn-Tucker multiplier on the borrowing constraint $s\geq \underline{s}^d$.

Taking into account that households optimally choose savings after trade, we can write the surplus from trade for the potential buyer and the potential seller as

$$\begin{aligned}
S^b(a,p) &= u\left(c^{trade},2,o\right)+\beta\sum_{j=1}^{2}\sum_{i=1}^{n_\varepsilon}P(j,2)\varphi_i V^o\left(Rs^o(a-(1+\tau)p,2)+w\varepsilon_i,j\right)\\
&\quad -v^r(a,2)\\
S^s(\widetilde{a},p) &= u\left(\widetilde{c}^{trade},1,r\right)+\beta\sum_{j=1}^{2}\sum_{i=1}^{n_\varepsilon}P(j,1)\varphi_i V^r\left(Rs^r(\widetilde{a}+p,1)+w\varepsilon_i,j\right)\\
&\quad -v^o(\widetilde{a},1)
\end{aligned}$$

where

$$\begin{aligned}
c^{trade} &= a-\kappa-(1+\tau)p-s^o(a-(1+\tau)p,2)\\
&\quad \text{and}\\
\widetilde{c}^{trade} &= \widetilde{a}-\upsilon+p-s^r(\widetilde{a}+p,1)
\end{aligned}$$

Using the above expressions for the surpluses and taking into account condition (A1), the effect of price changes on the surpluses can be written as

$$\begin{aligned}
\frac{\partial S^b(a,p)}{\partial p} &= -(1+\tau)\frac{\partial u\left(c^{trade},2,o\right)}{\partial c}+(1+\tau)\frac{\partial s^o(a-(1+\tau)p,2)}{\partial a}\mu^o\\
&\quad \text{and}\\
\frac{\partial S^s(\widetilde{a},p)}{\partial p} &= \frac{\partial u\left(\widetilde{c}^{trade},1,r\right)}{\partial c}-\frac{\partial s^r(\widetilde{a}+p,1)}{\partial a}\mu^r
\end{aligned} \tag{A2}$$

The standard Kuhn-Tucker optimality conditions imply that $\mu^d>0$ if the borrowing constraint is binding. In this case, however, $\frac{\partial s^d(a,z)}{\partial a}=0$. If, in turn, the borrowing constraint is not binding, $\mu^d=0$. Therefore, the surplus from trade only depends on the price through its effect on current non-housing consumption.

If trade is mutually beneficial, price is determined by Nash bargaining. Assume that $S^b(a,p) > 0$ and $S^s(\widetilde{a},p) > 0$ so that trade is mutually beneficial.

The Nash bargaining aims to choose p so as to maximize

$$S(a,\widetilde{a},p) = S^b(a,p)\, S^s(\widetilde{a},p)\,.$$

The first order condition for the optimal price is given by

$$\frac{\partial S^b(a,p)}{\partial p} S^s(\widetilde{a},p) + \frac{\partial S^s(\widetilde{a},p)}{\partial p} S^b(a,p) = 0$$

By using (A2) this can be written as

$$-(1+\tau)\frac{\partial u\left(c^{trade},2,o\right)}{\partial c} S^s(\widetilde{a},p) + \frac{\partial u\left(\widetilde{c}^{trade},1,r\right)}{\partial c} S^b(a,p) = 0. \tag{A3}$$

If the second order condition is satisfied over the relevant range of prices, (A3) determines a unique equilibrium price. The second order condition is

$$\begin{aligned}
\frac{\partial^2 S(a,\widetilde{a},p)}{\partial p \partial p} \quad = \quad & \frac{\partial^2 u\left(\widetilde{c}^{trade},1,r\right)}{\partial c \partial c}\left(1 - \frac{\partial s^r(\widetilde{a}+p,1)}{\partial a}\right) S^b(a,p) \\
& -2(1+\tau)\frac{\partial u\left(\widetilde{c}^{trade},1,r\right)}{\partial c}\frac{\partial u\left(c^{trade},2,o\right)}{\partial c} \\
& +(1+\tau)^2 \frac{\partial^2 u\left(c^{trade},2,o\right)}{\partial c \partial c}\left(1 - \frac{\partial s^o(a-(1+\tau)p,2)}{\partial a}\right) S^s(\widetilde{a},p)
\end{aligned}$$

Together with $1 - \frac{\partial s^r(\widetilde{a}+p,1)}{\partial a} > 0$ and $1 - \frac{\partial s^o(a-(1+\tau)p,2)}{\partial a} > 0$, this implies that $\frac{\partial^2 S(a,\widetilde{a},p)}{\partial p \partial p} < 0$. Therefore, whenever trade is mutually beneficial, (A3) determines a unique equilibrium price.

Appendix B

We use the following algorithm to solve the model: i) Guess distribution $\mu^d(a,z)$ and determine the matching probabilities ϕ^s and ϕ^b. ii) Solve for the value and policy functions using value function iteration. iii) Simulate to find the resulting stationary distribution. iv) Update the guess for distribution. v) Repeat i)-iv) until the distribution has converged.

In step ii), given a guess for $V^d(a,z)$, we first solve for $v^d(a,z)$ from (6). We then determine $Tr(a,\widetilde{a})$ and $p(a,\widetilde{a})$. It is clear that $S^b(a,p)$ is decreasing and $S^s(a,p)$ is increasing in price: other things equal, the buyer's surplus from trade is always smaller and the seller's larger the higher the price. We therefore begin by calculating prices $\overline{p}^b$ and $\underline{p}^s$ such that $S^b\left(a,\overline{p}^b\right) = 0$ and $S^s\left(a,\underline{p}^s\right) = 0$. If $\overline{p}^b < \underline{p}^s$, there is no price that would render trade mutually beneficial. If instead $\overline{p}^b \geq \underline{p}^s$, we know that trade takes place. In this case, we find $p\left(a,\widetilde{a}\right) \in [\underline{p}^s,\overline{p}^b]$ by solving (13), which is a one dimensional maximization problem. Given $Tr\left(a,\widetilde{a}\right)$ and $p\left(a,\widetilde{a}\right)$, we first determine $W^b\left(a,\widetilde{a}\right)$ and $W^s\left(a,\widetilde{a}\right)$ from (15) and (16). We then solve $V^d(a,z)$ for $d = r,o$ and $z = 1,2$ from (7), (8), (9), and (10).

Of course, we need to use discrete grids of possible financial wealth levels for both owners and renters. We use cubic splines to interpolate value functions $V^d\left(a,z\right)$ and $v^d\left(a,z\right)$ between grid points. Since the match values $W^b\left(a,\widetilde{a}\right)$ and $W^s\left(a,\widetilde{a}\right)$ feature kinks around asset levels where trade becomes mutually beneficial, we apply linear interpolation to them.

For a given match and a given price, we compute the surplus from trade using value functions $v^d\left(a,z\right)$ according to (11) and (12). The minimum financial wealth levels that we may need to consider correspond to the maximum and minimun prices defined in (4) and (5). The minimum financial wealth is $\underline{s}^r + \upsilon + c_{\min}$ for renters and $\underline{s}^o + \kappa + c_{\min}$ for owners. We assume $c_{\min} = 0.01$. In the benchmark calibration, these limits are 0.23 and -13.75, respectively. These limits provide the lower bounds for the financial wealth grids. We set the maximum financial wealth levels in the benchmark at 6 for renters and at -8.6 for owners. These bounds are not binding. The difference between them corresponds to the average house price.

We approximate the distribution by a discrete density function. The financial wealth of a household in occupancy state d is forced to belong to a set $A^d = \{a_1^d, a_2^d, ..., a_m^d\}$. As usual, we use a lottery to force next period financial wealth to be on the grid A^d (see algorithm 7.2.3 in Heer and Maussner, 2010).

In step iii), we first determine optimal savings for all unmatched households as well as the outcome of the bargaining process and the associated optimal savings for all possible matches. That is, we determine, among other things, $Tr\left(a_j^r, a_k^o\right)$ and $p\left(a_j^r, a_k^o\right)$ for all $j = 1, 2, .., m$ and $k = 1, 2, ..., m$. Since $Tr\left(a,\widetilde{a}\right)$ is a discrete function and the price is not defined everywhere, we do not interpolate these functions, but solve for the outcome of the bargaining process in the same way as in step ii).

We then determine three transition probability matrices. The first one determines transition probabilities from a given current state (a, d, z) to different next period states for unmatched households. The next period financial wealth is determined by the savings decision and the income shock. With two income shocks and two tenure preference states (and the lottery), a household in a given state that is not matched with a potential trading partner may generally move to 8 different states. The second matrix determines probabilities at which a potential buyer (unhappy renter) with a given financial wealth $a \in A^r$ that is matched with a potential seller with a given financial wealth $\widetilde{a} \in A^o$ moves to different next period states. Given the match, there are again generally 8 different states where the household can move. Similarly, the third matrix determines the probabilities at which a potential seller that is matched with a given potential buyer moves to different next period states. Given these transition probability matrices and an initial density function, we iterate over the density function to find the stationary distribution. At this stage, we need to take into account the probabilities of different matches which are in turn determined by the density function. For instance, of those unhappy renters in state $\left(a_j^r, r, 2\right)$ that are matched with a potential seller, fraction $\mu^o\left(a_k^o, 1\right) / \sum_{l=1}^{m} \mu^o\left(a_l^o, 1\right)$ are matched with a potential seller with financial wealth equal to a_k^o.

The results reported here have been computed using 75 non-linearly spaced gridpoints for financial wealth in the value function and 150 linearly spaced gridpoints for financial wealth in the density function. Before we simulate to find the stationary distribution in step iii), we need to determine the outcome of the bargaining process for 150^2 combinations of seller's and buyer's financial wealth. Further increasing the number of gridpoints had virtually no impact on the reported statistics of the benchmark calibration.

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