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ESSAYS ON TAXES AND WAGE FORMATION

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ABSTRACT: The essays in this collection approach taxation as a group of policy instruments, and study how their use is connected to and affected by wage formation in the economy. In the first essay taxation acts as an automatic stabiliser in the face of different shocks. The study combines two theoretical models from previous literature. It shows that earlier results, concerning both income tax indexation and the role of openness in deciding the optimal degree of wage indexation, do not hold under more general assumptions. In the second essay the aim of policy is to cure an acute inflation problem. The instrument is a conditional threat to increase taxes. The study develops a one-shot game describing the determination of the threat and the decisions of the unions. The article gives one possible rationalisation for the use and success of tax threat policies, and discusses reasons why this instrument is not used more often. In the third essay the aim of policies is to increase efficiency and welfare in the economy. Households and firms have fully adjusted their behaviour to the tax and transfer structure. Wages are set by majority-voting in a centralised monopoly union. The essay extends a well-known general equilibrium simulation model to include a trade union. Taxes and transfers are shown to affect the economy through dynamic channels and in a way depending significantly on wage formation.

Key words: taxation, wage formation, trade unions

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TIIVISTELMÄ: Kokoelman esseissä verotusta tarkastellaan talouspolitiikan instrumenttina ja tutkitaan sen käytön vuorovaikutusta talouden palkanmuodostuksen kanssa. Ensimmäisessä esseessä verotus toimii automaattisena vakauttajana shokkien kohdatessa taloutta. Tutkimuksessa yhdistetään kaksi aiemmassa kirjallisuudessa esitettyä teoreettista mallia. Essee osoittaa, että aiemmat tuloverotuksen indeksointia ja talouden avoimuuden vaikutusta optimaaliseen palkkaindeksointiin koskevat tulokset eivät päde yleisempien oletusten vallitessa. Toisessa artikkelissa verojen korotusuhkauksella pyritään torjumaan palkkainflaatiota. Tilanne kuvataan kertaluontoisena pelinä, jossa sekä verouhkaus että ammattiliittojen päätökset muotoutuvat. Tutkimus esittää yhden mahdollisen perustelun verouhkauksen menestykselliselle käytölle, ja siinä pohditaan myös syitä tämän keinon käytön harvinaisuuteen. Kolmannessa esseessä tutkitaan verotuksen ja tulonsiirtojen vaikutusta talouden tehokkuuteen ja hyvinvointiin. Kotitalouksien ja yritysten oletetaan täysin sopeuttaneen toimintansa verorakenteeseen. Palkat määräytyvät jäsenten enemmistöpäätöksillä työmarkkinat kattavassa monopoliliitossa. Tutkimus laajentaa tunnettua yleisen tasapainon numeerista simulointimallia ammattiliitolla. Tulosten mukaan verotuksen ja tulonsiirtojen vaikutukset talouteen riippuvat merkittävästi palkkojen määräytymistavasta.

Asiasanat: verotus, palkat, ammattiliitot

PREFACE

This study consists of a summary and the following three articles on the interplay of taxation and wage formation:

Income Tax Indexation in an Open Economy. Journal of Money, Credit, and Banking Vol. 27 (2), May 1995, 389-403.

Tax Threats and Wage Formation. Labour Economics (5) 2 (1998), pp. 167-183

Wage Formation by Majority Voting and the Incentive Effects of Pensions and Taxation. Finnish Economic Papers (forthcoming).

I wish to express my gratitude to all who have helped me in this study. I am particularly grateful to Professor Erkki Koskela for his thorough supervision. Based on his vast knowledge of the relevant literature he has made invaluable suggestions about what to concentrate on. Especially the first article is a direct haul from the fresh trails he led me to. He and my other official examiner, Professor Mikko Puhakka, have also substantially influenced the third article.

I am deeply grateful to Dr Pentti Vartia for genuine continuous encouragement and support during the many years this process took. His characterisations and analyses of economic policies in Finland have greatly influenced my thinking. I have also had many inspiring discussions about Finnish incomes policies with Dr Kari Alho. Of my other colleagues and coworkers at ETLA I want to mention Dr Juha Ahtola from the early phases of the project, and Dr Tarmo Valkonen and Ms Eija Kauppi whose skillful work with the FOG model enabled the third article.

A large number of people have commented on the various drafts and conference and seminar presentations of the articles. I wish to mention Mikael Forss, M. B. Gregory, Vesa Kanniainen, Ryuta Kato, Heikki Palm, Matti Pohjola, Thomas Renström, Asbjørn Rødseth, Jouko Vilmunen and Jouko Ylä-Liedenpohja. Besides commenting on substances, Anthony de Carvalho, Catherine Reilly, and John Rogers have also checked the language. I wish to thank also the staff of ETLA for assisting the work in many ways.

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Helsinki, October 2000

Jukka Lassila

CONTENTS

JUKKA LASSILA

Income Tax Indexation in an Open Economy

THE VAST LITERATURE ON WAGE INDEXATION seems to have one rather robust result: compared to complete nominal wage rigidity, wage indexation stabilizes output with respect to demand shocks but makes it more sensitive to supply shocks. On the other hand, wage indexation destabilizes prices with respect to both kinds of shocks.

Bruce (1981) studies income tax indexation in connection with wage indexation in a closed economy. Bruce defines income taxes as indexed if the real amount of income taxes depends only on real income and not on the price level. Bruce claims that income tax indexation makes output more sensitive to demand shocks but less sensitive to supply shocks, which are precisely the opposite results as under wage indexation. Prices, however, become more sensitive to both kinds of shocks.

Unfortunately, in his analysis Bruce allows only one transmission mechanism for taxation: through disposable income to expenditure. This restriction makes it possible to achieve unambiguous results. But even in a very aggregated model, taxation may have other important effects. We explore here the view put forth by Holmes and Smyth (1972) and Mankiw and Summers (1986) that disposable income affects the demand for money.

If taxation directly affects the demand for money, then tax cuts may not be expansionary. Mankiw and Summers (1986) argue that consumer demand affects the demand for money more than other components of demand do. This makes the sign of the effect of a tax reduction on output unclear a priori. If the interest elasticity of the

The author thanks Erkki Koskela for suggesting this topic and for detailed and valuable advice. Helpful comments were provided by Vesa Kanniainen, John Rogers, Jouko Vilmunen, Jouko Ylä-Liedenpohja, and two anonymous referees. A grant from the Yrjö Jahnsson Foundation is gratefully acknowledged.

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demand for money is sufficiently low, tax reductions are contractionary. Mankiw and Summers present empirical evidence concerning the U.S. economy that supports the view that both consumer demand and disposable income are better explanatory variables in a money demand equation than total GNP.

This article studies the effects of income tax indexation in an open economy context. Our starting point is Aizenman (1985), who studied how optimal wage indexation is affected by the degree to which domestic output is exposed to the prices of internationally traded goods. His main finding is that, under flexible exchange rates, the more open the economy is, the higher is the optimal degree of wage indexation. We show that this result does not necessarily hold when taxation is progressive.

1. THE MODEL

Modeling Income Tax Indexation

With constant real income, price level changes may change the share of taxes relative to total income basically for two reasons. First, marginal tax rates are often graduated with respect to nominal income, not real income. Second, deduction limits are usually fixed in nominal terms.

We follow Bruce (1981) in the modeling of income tax indexation and define T as the ratio of gross to net real income. We assume that $\ln T$, the logarithm of T, is given by

$$\ln T = \tau + uy + (1 - r)up \tag{1}$$

where $\tau > 0$, $0 < u < 1$ and y and p are the logarithmic deviations of real output Y and the price level P from their equilibrium values, respectively. Various shocks cause output and prices to deviate from their equilibrium values, thus changing the share of taxes relative to income. Without any shocks the share of taxes would be $1 - e^{-\tau}$, where τ is a constant. The degree of tax progressivity is described by u, assumed to be positive.[1] It is also assumed to be less than one; otherwise, the marginal tax rate from real income would exceed unity. The parameter r measures the degree of tax indexation. If taxes are fully indexed, r is equal to one and the average tax rate depends only on the real income. If taxation is completely unindexed, r is equal to zero and the average tax rate depends only on the nominal income.

The Goods and Money Markets

We take Aizenman (1985) as a starting point for our model. As in his model, we consider a two-sector economy where the country is small in the traded goods sector and large in the nontraded sector. Thus the relative price of traded to nontraded

1. Income tax structure is progressive, proportional, or regressive depending on whether u is positive, zero, or negative.

goods is determined endogenously. The more open the economy is, that is, the larger the share of traded goods, the smaller is the relevance of this endogenous relative price. This affects also optimal wage and tax indexation.

Our model differs from Aizenman's model primarily with regard to taxation, which affects the demand in the goods market and may affect also the demand for money. Aizenman's model may be described as one with proportional taxation, which makes all tax parameters vanish from the analysis. Our model has progressive taxation.

Real output Y in period t is defined as

$$Y_t = (N_t P_{n,t} + Z_t P_{z,t})/P_t \tag{2}$$

where N and Z represent the output of nontraded and traded goods. P_n and P_z are prices of nontraded and traded goods, respectively, and P is a price index:

$$P_t = (P_{n,t})^{\theta_n}(P_{z,t})^{\theta_z} \tag{3}$$

where $\theta_n + \theta_z = 1$ The θ terms represent the shares of nontraded and traded goods.

For traded goods, the law of one price is assumed to hold:

$$P_{z,t} = S_t P^*_{z,t} \tag{4}$$

where S is the exchange rate and P^*_z is the international price of traded goods. The exchange rate is flexible.

The supply side of the economy is described by the labor supply and production functions. The labor supply is given by

$$L^s_{j,t} = Q_j(W_t/P_t)^\delta \tag{5}$$

where j denotes the sector (either n or z). W is the money wage. Labor is the only mobile factor of production. The elasticity of labor supply with respect to the real wage is assumed to be the same in both sectors.

Output is given by

$$N_t = Q_n(L_{n,t})^\alpha \exp(v_t) \tag{6}$$

and

$$Z_t = Q_z(L_{z,t})^\alpha \exp(v_t)\ . \tag{7}$$

The term v_t is a multiplicative productivity shock.

Wages are thought to be set to equate the expected labor supply and expected labor demand. The contracts are made before any shocks occur, and the prices prevailing during the contract period t are therefore not known. Nominal wages are, however, partially indexed to prices. The actual real wage differs from the expected level because of shocks, and actual employment is assumed to be demand determined.

We could have formulated the supply of labor to depend on the net real wage instead of the gross real wage, following Blinder (1973). It would not make any qualitative difference in the short run, as employment is demand-determined. Whether the full equilibrium is affected is considered in a footnote in section 3.

Thus far the model has been identical to Aizenman's (1985) model. The demand side, however, is different. Instead of real output Y as a scalar, we use real disposable income defined as follows.

$$Y_t^d = Y_t/T_t \, . \tag{8}$$

The demand for the nontraded goods is given by

$$(P_{z,t}/P_{n,t})^a Y_t^d \exp(\bar{c} - c(i_t - \pi_t)) \tag{9}$$

where a is the compensated demand elasticity, i is the money interest rate, and π is the expected inflation:

$$\pi_t = (E_t P_{t+1} - P_t)/P_t \, . \tag{10}$$

The term E_t denotes a conditional expectation operator, conditional on information available at time t.

The demand for money is specified in two different ways. First is the conventional one:

$$M_t^d = Y_t P_t \exp(-ki_t) \, . \tag{11}$$

The second specification approximates the claim of Mankiw and Summers (1986) that consumer spending is the best scalar in a money demand function; thus Y is replaced by Y^d:

$$M_t^d = Y_t^d P_t \exp(-ki_t) \, . \tag{12}$$

The next equation is the arbitrage condition that connects domestic and foreign interest rates under perfect capital mobility:

$$i_t - i_t^* = (E_t S_{t+1} - S_t)/S_t \, . \tag{13}$$

The supply of money, M^s, is formed from three components. First, there is a basic exogenous supply $\bar{M}$. Second, there is an exponential random shock m_t. Third, there is a negative element that comes from unindexed income taxation via the government budget constraint. Public spending is assumed fixed and does not appear in the analysis. Increased real taxes that result from unexpected inflation draw money off from the market. We assume that all this money is returned to the supply through open market operations.

$$M_t^s = \bar{M}\exp(m_t)\ . \tag{14}$$

2. SHORT-RUN EQUILIBRIUM

The Role of Expectations

The four disturbance terms m, v, i^*, and p_z^* are assumed to be uncorrelated and generated by white noise processes, for example,

$$m \sim N(0, \sigma_m^2) \tag{15}$$

and analogously for v, i^*, and p_z^*. The variances are henceforth denoted by V_m, V_v, V_{i^*}, and $V_{p_z^*}$, respectively.

It is convenient to express variables as percentage deviations around their nonstochastic equilibrium values. This nonstochastic equilibrium is achieved through solving the model under the assumption that $m_t = v_t = i_t^* = p_{z,t}^* = 0$. To get percentage deviations, we use log-linear approximations. We denote these transformed variables with lowercase letters.

Equating the marginal products of labor to the real product wages, we get the sectoral supplies:

$$n = \bar{h}(p_n - w) + hv\ ; \tag{16}$$

$$z = \bar{h}(p_z - w) + hv \tag{17}$$

where $\bar{h} = \alpha/(1 - \alpha)$; $h = 1/(1 - \alpha)$.

The determination of the short-run equilibrium can now be expressed with seven equations:

$$y = \bar{h}(p - w) + hv\ ; \tag{18}$$

$$\bar{h}(p_n - w) + hv = -a(p_n - p_z) + (1 - u)y - (1 - r)up - c(i + p)\ ; \tag{19}$$

$$m - p = (1 - gu)y - g(1 - r)up - ki\ ; \tag{20}$$

$$w = bp\ ; \tag{21}$$

$$i = i^* - s\ ; \tag{22}$$

$$p_z = p_z^* + s\ ; \tag{23}$$

$$p = \theta_n p_n + \theta_z p_z\ . \tag{24}$$

Aggregate supply y in equation (18) is obtained from sectoral supplies, using also equations (2) and (3). Equation (19) is obtained by equating the supply of and demand for nontraded goods. The parameter g in the money demand equation (20) varies with the specification. The conventional specification is achieved with $g = 0$, and the Mankiw-Summers specification with $g = 1$. Equation (21) is the wage indexing rule. The coefficient b shows the amount of wage indexation, with $b = 0$ referring to no indexation and $b = 1$ to full indexation.

The short-run equilibrium can be thought as the first-period solution of a two-period model. The second period affects the first only through expectations concerning inflation and the exchange rate. Since p is the unexpected deviation of the price level from its no-shock equilibrium value, it creates an expectation of inflation of the amount $-p$ in the next period; thus the real interest rate is $i + p$, as in equation (19). Similarly, if the shocks cause the exchange rate to devalue, that creates a revaluation expectation as in (22).

Expectations are crucial for the resulting equilibrium. If there were no expectations, the model solution would be extremely simple. The price level would be determined solely in the money market. It would be affected only by domestic monetary or productivity shocks and by foreign interest shocks. The openness of the economy would not affect prices. The exchange rate would balance the supply and demand of the nontraded goods by making the relative price of nontraded to traded goods appropriate.

The solution becomes much more complicated with exchange rate expectations. A change in the exchange rate creates an expectation of a reversal in the future, and this immediately affects the demand for money. Prices and the exchange rate are now simultaneously determined.

Solving the model given by equations (18)–(24) yields the following expression for p:

$$p = D^{-1}\{m - h[1 + u(k/\phi - g)]v + (i^* + p_z^*)k(1 - c/\phi)\} \tag{25}$$

where

$$D = [1 + u(k/\phi - g)]\bar{h}(1 - b) + k + 1 + u(k/\phi - g)(1 - r)$$

and

$$\phi = c + [a + \theta_z \bar{h}]/\theta_n \,.$$

We could now solve y from (18), and then the other variables. This is not, however, essential for the questions we wish to consider here.

Openness can be measured in this model by the share of traded goods θ_z, or by the substitutability in consumption a or in production $\bar{h}$. An increase in any of these will increase ϕ.

Taxation and the Demand for Money

Taxation brings three different ingredients into the analysis. One is progressivity, which means that also real income surprises affect real disposable income even though taxation may be indexed. The second is tax indexation or its absence, which determines whether price surprises have direct effects on real disposable income. The third effect is the option of modeling money demand to depend on real disposable income, instead of conventional total real income. Our primary interest lies with the second of these three points, but its inclusion requires nonproportional taxation and the results turn out to be sensitive with respect to the third option.

We notice that if taxation were proportional, that is $u = 0$, all tax parameters would vanish from (25). With $u \neq 0$, the degree of tax indexation r and the specification of money demand g have both separate and combined effects.

To show that the demand for money specification really makes a difference here, let us have a closer look at (25). Let us denote

$$\mu = k/\phi - g \,. \tag{26}$$

The term μ is clearly greater than -1, and thus $1 + u\mu$ is positive. But μ is not necessarily positive. This term is in the coefficients of all the tax terms in (25). Clearly, μ can only be negative if we assume that money demand depends on real disposable income, so that the parameter g is set equal to 1. Then the sign of μ depends on the magnitudes of k and ϕ.

Progressive taxation is often thought to be an "automatic stabilizer." But this depends on the specification of money demand, as Holmes and Smyth (1972) and Mankiw and Summers (1986) have noted. In our model this is straightforwardly seen from (25), assuming taxes to be fully indexed. If μ is negative, progressive taxation decreases the denominator and thus increases the effects of demand shocks (monetary and foreign shocks) and thus becomes an "automatic destabilizer." It may, however, stabilize supply shock effects. With positive μ these stabilatory properties are reversed. We may thus define the automatic stabilization condition as follows.

AUTOMATIC STABILIZATION CONDITION: Income taxation is an automatic stabilizer if and only if

$$k > g\phi \,. \tag{27}$$

To interpret this condition, let us consider a positive monetary shock. It creates excess supply in the money market. To return to balance, either the interest rate must fall or prices rise, or both. The interest rate falls if the currency depreciates, creating expectations of a future appreciation. It is crucial that for any given change in the interest rate, a larger movement of prices is needed when tax effects are present than when they are not, in order to balance the money market. The reason is that a rise in prices reduces the real money stock, but this is partly offset by the tax effect that reduces the demand for money. The situation is different in the nontraded goods market. A fall in the interest rate and the depreciation of the currency has led to excess demand. To balance this market, an increase in the price of the nontraded good is needed. This of course leads to an increase in the overall price level. Here it is crucial that for any given change in the interest rate (and the exchange rate change behind this), a smaller change in prices is needed when tax effects are present than when they are not, in order to balance the market for nontraded goods. This is so because with the tax effects, a price rise not only increases supply, it also reduces demand. So, with the tax effects, larger price movements are needed in the money market and smaller in the nontraded goods market. The relative importance of these conflicting factors depends on the exchange rate elasticities of the two markets. Condition (27) tells when the money market effects of taxation are weaker than the opposing effects in the goods market.

Mankiw and Summers argued that for the U.S. economy the interest elasticity of money demand was sufficiently low compared to the interest elasticity of goods demand so that the analogy of condition (27) was probably not met. In our model the interest rate effects come from movements in the exchange rate, which create expectations about future reverse movements. In addition, the exchange rate changes the relative price of goods. Both these channels are captured in the term ϕ. The more open the economy the more likely it is that condition (27) does not hold. Whether or not it holds, the term D in (25) is positive.

It is noticeable that taxation has effects also on the relative price adjustment. The equation of the nontraded goods market can be expressed as

$$p_n - p_z = -c(i^* + p_z^*)/\theta_n\phi - u[1 - r + \bar{h}(1 - b)]p/\theta_n\phi - uhv/\theta_n\phi \,. \tag{28}$$

A transitory foreign price or interest rate increase leads to a higher real interest rate, which reduces the demand for both goods. The relative price of nontraded goods must fall to balance the supply and demand of nontraded goods. On the other hand, higher inflation that follows the shocks, according to (25), reduces disposable income through taxation and thus further reduces demand. Therefore, the needed adjustment of the relative price is larger the more progressive taxation is and the less it is indexed.

With progressive taxation, also domestic shocks change the relative price. This is evident from (28), where productivity shocks are explicitly present and monetary shocks have effects through changes in the price level.

3. OPTIMAL INCOME TAX INDEXATION

In the foregoing analysis, employment was determined by labor demand. The labor market did not clear, but rather the wage level was set by the wage indexing rule

(21). The resulting welfare loss from the labor market disequilibrium is proportional to the expected squared discrepancy of output from its equilibrium level, obtained with full market clearing [see, for example, Aizenman and Frenkel (1985) and Aizenman (1985)]. Thus we use the loss function adopted by, for example, Gray (1976) and Flood and Marion (1982):

$$H = E_0(y - \tilde{y})^2 \tag{29}$$

where a tilde (~) above a variable refers to the value of a variable in a fully flexible economy. In a flexible economy it follows from the labor market clearing[2] that

$$\tilde{w} = \bar{p} + \frac{h}{\delta + h} v\,; \tag{30}$$

$$\tilde{y} = hv - \frac{\bar{h}h}{\delta + h} v\,. \tag{31}$$

From equations (18) and (31) we get that

$$H = (\bar{h})^2 E_0 \left[(1 - b)p + \frac{h}{\delta + h} v \right]^2 . \tag{32}$$

Replacing (25) for p in (32), squaring, taking the expectation and differentiating with respect to r, we get the optimal value for tax indexation:

$$r^* = b + \delta(b - 1) + \frac{k + 1 - (1 - b)[\Phi/(1 + u\mu) + 1 + \delta]}{u\mu}$$

where

$$\Phi = \frac{\delta + h}{h^2} \left[\frac{V_m}{V_v} + k^2(1 - c/\phi)^2 \frac{V_{i^*} + V_{p_z^*}}{V_v} \right]$$

and $b \neq 1$.

It is evident from (33) that the automatic stabilization condition is indeed critical here. We proceed by assuming conventionally that μ is positive and condition (27) is met. If μ is negative, all the results below change signs.

2. If the supply of labor depends on the net real wage the following results may change. Assuming that wage taxation is similarly progressive as total income taxation, all depends on how we allow progression to affect the share of taxes relative to income. Assuming that the share of taxes in the full equilibrium is always $1 - e^{-\tau}$ would keep the qualitative results similar. If instead the share of taxes if $1 - e^{-\tau - u\tilde{y}}$ in the new equilibrium, the optimal income tax indexation (and also wage indexation) would depend on progressivity in a much more complex way. The complexity would further increase if the share of taxes were $1 - e^{-\tau - uy - u(1-r)\bar{p}}$.

The less flexible the real wage is, that is, the more wages are indexed, the higher is the optimal degree of income tax indexation.[3]

The relative importance of shocks plays a role here. The higher the ratio of productivity shock variance to the variances of other shocks, the higher is the optimal tax indexation.

The optimal rate of income tax indexation depends negatively on the real wage elasticity of the supply of labor.

The interest elasticity of the demand for money and the progressivity of taxation have ambiguous effects on the optimal degree of tax indexation. This is hardly surprising. And whatever the signs are, they change if the automatic stabilization condition is not met.

The openness of the economy has an ambiguous effect on the optimal tax indexation, except that the more open the economy, the more likely it is that (27) does not hold and the results above change sign.

Bruce (1981) claimed that both wage and tax indexation make prices more sensitive to shocks. This holds also here for wage indexation but not necessarily for tax indexation. This can be seen from (25). Both indexation parameters are in the denominator. But $(1 - r)$'s coefficient $u(k/\phi - g)$ is not necessarily positive. It is negative if the automatic stabilization condition is not met. In that case tax indexation would reduce the sensitivity of the price level to all kinds of shocks considered here.

Tax indexation affects also output variance. Using equations (18) and (25) we get

$$y = D^{-1}\{\bar{h}(1 - b)[m + k(1 - c/\phi)(i^* + p^*_{-})] + [k + 1 + u\mu(1 - r)]hv\} . \tag{34}$$

In the conventional money demand case, income tax indexation makes output less sensitive to productivity shocks and more sensitive to all other shocks. This result is similar to Bruce's (1981, p. 274). But the automatic stabilization condition is crucial here, too, for it changes the signs of the effects on output sensitivity with respect to all shocks.

Nothing guarantees here that optimal tax indexation yields a value for r that is between zero and one or in the limit. The necessary inequalities can be obtained from (33), but the results are too unwieldy to be fruitful.

4. TAXATION AND OPTIMAL WAGE INDEXATION

Replacing (25) for p in (32), squaring, taking the expectation and differentiating (32) with respect to b, we get the optimal value for wage indexation. It is interesting to note that b and r are determined only in relation to each other. Namely, differen-

3. With fully indexed wages, optimal tax indexation would, however, be indeterminate. This is straightforwardly seen from equation (32): with b equal to one, the loss function would depend only on the variance of productivity shocks, and the tax indexation parameter is not included in that relation.

tiating with respect to either b or r yields the same conditions for the optimal values. For a given r we get

$$b^* = 1 - [k + 1 + (1 - r)u\mu]/[\Phi(1 + u\mu)^{-1} + (1 + u\mu)(1 + \delta)] . \tag{35}$$

Equations (35) and (33) are the same. This means that in this model, forgetting for the moment that b and r should be either zero or unity or in between, wage indexation and income tax indexation are equally efficient methods for minimizing the loss function. The mathematics are looked upon more closely in the Appendix. The economic explanation might be that both instruments affect the transmission of shock effects only from price movements onward to other variables. Notice that this redundancy of one of these two instruments does not follow from the fact that we have two instruments for one target. We could get a better social optimum by optimizing the rate of progressivity u also. This progressivity parameter affects the transmission of shock effects also from output movements onward to other variables.

We could, of course, set an additional target, for example, the stability of the exchange rate, and optimize either b or r with respect to that target, without sacrificing anything from the original target.

In Bruce's (1981, p. 274) model, the higher the degree of income tax indexation is, the higher is the optimal degree of wage indexation. In our model this result holds if the automatic stabilization condition is met, but the sign of the dependence changes if the condition does not hold.

Aizenman's (1985) two main results were that an increase in openness will increase optimal wage indexation, and that optimal wage indexation decreases in accordance with the ratio of productivity variance to the variance of the other shocks. These results can be seen in a rather straightforward manner from (35), assuming that $u = 0$. The productivity variance case is obvious, and for openness, all three measures used by Aizenman lead to a similar conclusion:

$$b^*_a > 0,\ b^*_{\theta_t} > 0,\ b^*_{\bar{h}} > 0$$

where the partial derivatives are taken with respect to the share of traded goods (θ_t), the substitutability in consumption (a) and in production ($\bar{h}$).

Do these results hold when taxes are not proportional? For openness, not necessarily, because progressive taxation has made the case more complex. In Aizenman's article openness only showed in the term ϕ and thus in Φ. Here ϕ shows up also in μ, and it has effects in both directions. The results are ambiguous, and they may turn around for some parameter combinations, especially with high progressivity. Consider, for example, the case where the variance of productivity shocks is huge compared to the variances of other shocks. As $V_v \rightarrow \infty$, $\Phi \rightarrow 0$. Now, as openness increases, μ gets smaller in (35) and the optimal b also gets smaller. We also notice that with progressive taxation, openness affects optimal wage indexation even if there are no foreign shocks.

The productivity variance case is stronger. The results are the same qualitatively with both the conventional money demand specification and the Mankiw-Summers specification.

5. TEMPORARY SHOCKS VERSUS PERMANENT SHIFTS

The role of expectations is crucial in the determination of the short-run equilibrium. The shocks are assumed to be temporary, and the shocks of today are assumed to contain no information about the shocks of tomorrow. Thus it is rational to expect that whatever deviations from the no-shock equilibrium occur this period, these deviations are reversed in the next period because the best forecast of the next period is simply the no-shock equilibrium:

$$(E_t P_{t+1} - P_t)/P_t \approx -p \, , \; (E_t S_{t+1} - S_t)/S_t \approx -s \, . \tag{36}$$

These reversed-deviations expectations affect the present period through inflation expectations via the real interest rate and through exchange rate expectations via the interest parity condition.

Since the expectations are so crucial, it is useful to check how sensitive the results are to the assumption that shocks are strictly temporary. We forsake the assumption of temporariness in this section, and assume that all shocks are permanent shifts. All agents perceive that the shifts are permanent, and make rational forecasts for the next period.

Indexation here reflects rigidities both in wage formation and in taxation that are due to contract length. New contracts are made before the next period. Wages are set so that the labor market is in equilibrium, with the new shifted parameters. Taxes are assumed to be set so that the ratio of gross to net real income is e^{τ} [see equation (1)]. This is consistent with the idea that progression is not allowed to increase the share of taxes in time in a growing economy.

We assume that the agents are able to calculate the second-period full-equilibrium values of the relevant variables. This means either that the shocks are directly observable or that their magnitudes can be inferred from observable variables, following the line of reasoning of Karni (1983). Here this latter route can be followed, assuming that output, domestic prices, the domestic interest rate and the exchange rate are observable.

In the full equilibrium the labor market also clears, so that we replace the wage indexation rule by the following:

$$\tilde{w} = \tilde{p} + \frac{h}{\delta + h} v \, . \tag{37}$$

As there are no expected changes in the full equilibrium, the model solution is easily obtained. The price level comes from the money market equation:

$$\tilde{p} = m - \frac{1 + \delta}{h + \delta} v + ki^* . \tag{38}$$

The exchange rate is determined through the goods market:

$$\tilde{s} = \tilde{p} + \frac{\theta_n c(i^*)}{h\theta_z + a} - p_z^* . \tag{39}$$

These would be the required changes in the prices and exchange rate to achieve the full equilibrium. The actual changes in the first period are different, however, and this creates the rational expectation that there are further changes to come. These expectations are

$$(E_t P_{t+1} - P_t)/P_t \approx \tilde{p} - p , \ (E_t S_{t+1} - S_t)/S_t \approx \tilde{s} - s . \tag{40}$$

Comparing (36) and (40) we notice that in both cases inflation this period lowers the expected inflation next period, although in the permanent shift case from a higher initial expectation level, and analogously for the exchange rate expectations.

Solving the model with these modified expectations yields the following expression for the price level adjustment.

$$p = D^{-1} \left\{ (k + 1)m - \left[1 + u\mu - \frac{1 + \delta}{h + \delta} \right] hv + k^2 i^* \right\} \tag{41}$$

where D is the same as in equation (25).

Although these expectations make the short-run equilibrium somewhat different, the qualitative change is not very significant. The biggest difference is that, with permanent shifts, the foreign price of traded goods has no effects on the aggregate price level. The reason is that the immediate reaction in s is $-p_z^*$, which is also the full equilibrium change, so no further adjustment is expected. We also notice that the effects of monetary disturbances have grown while productivity effects have diminished.

6. CONCLUDING REMARKS

Income tax indexation may provide an alternative or a complement to wage indexation for stabilizing an economy subject to various kinds of shocks. It is an alternative because it turned out to be equally efficient in minimizing the loss function studied here as wage indexation. It can nevertheless also be a complement because the optimal values for wage and tax indexation are not necessarily in the range between zero and one, but there may be a combination of optimal values that fulfills this condition.

It seems, however, that there is much more theoretical ambiguity concerning income tax indexation and its effects than there is concerning wage indexation. Especially, the demand for money specification is crucial for tax indexation results but not for wage indexation results. If we accept the specification where disposable income rather than total income affects the demand for money, then it is possible that we get results that are exactly the opposite to those of Bruce (1981). The more open the economy is, the more likely this reversal of results is.

Moreover, the presence of progressive taxation makes qualitative results concerning the effects of wage indexation more complex. If the variance of productivity shocks is large, compared to other shock variances, it is possible that the optimal degree of wage indexation is a decreasing function of the openness of the economy, not increasing as in Aizenman (1985).

APPENDIX

1. Derivation of Equations (25) and (28)

To solve the model given by equations (18)–(24), we reduce it to two equations in p and s. We start with the equilibrium equation (19) of the nontraded goods market. Replacing y from (18), w from (21), and $i = i^* + p_z^* - p_z$ from (22) and (23), we get.

$$\bar{h}(p_n - p) = -a(p_n - p_z) - c(i^* + p_z^* + p - p_z)$$
$$-u[1 - r + \bar{h}(1 - b)]p - uhv . \qquad (42)$$

From (24) we notice that $p_n - p = \theta_z(p_n - p_z)$ and $p - p_z = \theta_n(p_n - p_z)$. Inserting these into (42) yields

$$(\bar{h}\theta_z + a + c\theta_n)(p_n - p_z) = -c(i^* + p_z^*) - u[1 - r + \bar{h}(1 - b)]p - uhv \qquad (43)$$

from which we get equation (28). On the other hand, $p_n - p_z = (p - p_z)/\theta_n$ from (24) and $p_z = p_z^* + s$ from (23). Inserting these into (43) we get the first of the two equations in p and s:

$$p = -c(i^* + p_z^*)/\phi - u[1 - r + \bar{h}(1 - b)]p/\phi - uhv/\phi + p_z^* + s . \qquad (44)$$

The second equation in p and s is obtained from the money demand equation (20). Replacing y from (18), w from (21), and i from (22), we get

$$ks = m - (1 - gu)hv + ki^* - [1 - (1 - gu)\bar{h}(1 - b) + gu(1 - r)]p . \qquad (45)$$

Equation (25) is now obtained from (44) and (45). Equation (41) is derived in an analogous manner.

2. *Optimizing the Tax Parameters*

The loss function (32) can be expressed as follows, after squaring and taking expectations:

$$H = C_1 z^2 + 2C_2 z + C_3 \tag{46}$$

where

$$z = (1 - b)/[(1 + u\mu)\bar{h}(1 - b) + k + 1 + (1 - r)u\mu] ;$$

$$C_1 = V_m + (1 + u\mu)^2 h^2 V_v + (V_{i^*} + V_{p_z})k^2(1 - c/\phi) ;$$

$$C_2 = -\frac{h^2}{h + \delta}(1 + u\mu)V_v ;$$

$$C_3 = \frac{h^2}{(h + \delta)^2} V_v .$$

The C_i terms are constants that do not depend on b or r. So we can minimize (46) with respect to z, which yields equations (33) and (35).

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Tax threats and wage formation

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Abstract

The government, concerned about employment, can threaten the trade unions: if wages exceed a certain limit, income taxes will be increased. A credibility problem arises because executing the tax increase, if the wage proposal is rejected, will deteriorate employment. We analyze these kinds of threats as an inverted Stackelberg game between the government and many trade unions. The credibility of the threat comes from costs related to cheating, and this limits the size of the threatened tax increase. The success of a tax threat policy requires that the conditions under which the threatened measures will be implemented are tailored to fit certain unions. For several reasons, the use of threats is likely to be restricted to temporary policy packages. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Tax threats have been both a research subject and a practical policy tool in the Nordic countries. The idea of using tax threats to make trade unions accept lower wage levels was first presented by Hansen (1958). [1] The idea was later developed using game theoretic methods in Johansen (1977). In practise, the idea was tried in Norway in the mid-70s and in Finland in 1989. There have also been other more informal attempts to affect wages through tax threats and promises. The anglo-

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[1] Appeared in Swedish in 1955.

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saxon counterpart is ‘TIP’ (tax-based incomes policies), where the idea is to influence employers instead of trade unions (see, e.g., Jackman and Layard, 1990).

The Finnish case was the starting point of this study. The centralized wage negotiation process that took off in autumn 1989 led to the so-called Kallio agreement. A tax threat was a part of the process: if a vast majority of unions did not accept Kallio’s proposal, income taxes would have been increased. Credibility of the threat was increased by printing the extra tax percentages into personal income tax withholding cards in advance. Some unions forsook the proposal. A majority accepted, however, and the tax threat was not executed.

The objective of this study is to set out a framework which rationalizes the threat policy from the government’s point of view, and allows the effects of the policy to wage formation be analyzed.

The credibility of the tax threat is a key issue. The government tries to achieve a lower level of wages using a threat which, if executed, will lead to higher wages and lower employment than would have resulted without the threat in the first place. Here, credibility is obtained by introducing costs of cheating. This makes credibility depend on the size of the threatened tax increase. By limiting the size of the threat, the distributional effects of the policy and the necessity to tailor the threat become apparent.

Game theory methods have rarely been applied to tax–wage interdependencies in a multi-union context. One reason may be that in a multi-union setting, taxation has been regarded as a poor policy instrument. Calmfors (1989) (p. 101) claims that unions look not only at the incomes of their own members but also at the incomes of members of other unions. As tax reductions cannot be tailored differently for different unions, tax threats may be ignored because of the ‘keeping up with the Jones’ effect, even though this leads to lower net income development. The main result of this study, however, is that although taxes cannot or must not be tailored, the tax threat can and must be tailored with some specific unions in mind.

Section 2 contains basic considerations about the nature of threat policies in general and tax threats in particular. It paves the way to the government preference function introduced in Section 3, after the wage formation and wage–price equilibrium of the economy in normal times, without threat policy measures, are described. The key-section is Section 4: it describes the tax threat game and its effects in detail. Reasons why threat policies are rarely used, even though they seem to be effective, are discussed in Section 5. The conclusions are drawn in Section 6.

2. Tax threats as a policy instrument

Hansen (1958) (p. 358) expressed clearly the basic idea in using taxes to influence wages. He suggested a method where ‘‘the State makes a declaration of its plans concerning future fiscal and monetary policy for the realization of full employment and a stable value of money with alternative future money wage rates. This declaration will include a promise that fiscal and monetary policy will be constructed in such a way that at one certain money wage rate, namely the one that the State considers suitable, wage earners will achieve the highest real disposable income, whereas at both higher and lower money wage rates, their real disposable incomes will be lower''.

Hansen (1958) also wondered whether the State should give a detailed description of the policy measures that will be taken in various situations, or whether the State should merely announce the real disposable income levels that will result from alternative money wage rates, but leave open the exact measures by which these income levels will be achieved. Hansen concludes that from the point of view of the government, the latter way is better because it gives the government free hands to take whatever measures appear necessary to fulfil the declared intentions. A counter-argument is that this freedom may itself endanger the credibility of the proposal, because afterwards, it might be difficult to identify whether the government really tried to fulfil its intentions but failed because of some unexpected events, or whether the government did not try to do that. If the unions anticipate this difficulty, they are liable to dismiss the proposal.

Johansen (1977) (pp. 96–97) discusses the proposal of Hansen (1958) in a game-theoretic setting.[2] He mentions ''the serious problem of credibility'' as the reason why a ''policy using conditional statements is therefore not an easy and simple policy, but often something of a gamble''. The policy is successful only if the other players believe the government's declarations.

The threat game played in this article is an inverted Stackelberg game. The solution, as Pohjola (1985) notes, is subgame-perfect only under precommitment or credibility. Pohjola points out that reputation-building through repetitions of the game is a possible way to obtain credibility.

Whether a threat is credible or not, depends on the utility function of the maker of the threat, here the government. Johansen (1977) did not include anything directly connected with making threats in the government's utility function. Later, authors have not considered this, either. Yet there are reasons why threat-making should directly affect the government's utility.

The word 'threat' itself has strong negative connotations. Anyone who makes a threat without a justifiable reason is considered to be a trouble-maker. A govern-

[2] Although Hansen did not put forth his proposal in a game setting, he pondered about credibility, and his views seemed to precede the reputation building approach: ''The first condition for the success of a policy of this kind is obviously that the State announces its plans openly and then behaves in full accordance with them, neither concealing its intentions nor hesitating to carry out its declarations concerning the real incomes of the workers, even if the trade unions do not choose the money wage desired by the State. Obviously, this requires a tough and a consistent policy by the State, but this is nothing new in the field of wage-determination. Consistency in economic policy is indeed always necessary if the desired ends are to be realized'' (p. 360).

ment who in normal times systematically threatens a group in the society is seldom very popular, unless the motives of the government are generally thought important. In this respect, the case of preventing pollution is much stronger than the case of lowering wage inflation. It is important here to stress the word ‘normal’: in some situations, a threat for preventing wage increases may be generally accepted as a good thing, but not if used continuously without some specific reason.

Thus, making a threat has some direct costs attached to it. For the government, it costs popularity points. Even if a threat is totally credible after it is presented, this does not mean that it is a good instrument. Its presentation costs may be such that it is advantageous only on rare occasions. But in some cases, the cost may be negative: if the economic situation is bad or worsening rapidly, the government is expected to do something, expected to show character. A threat policy may be cheered by the public, eventhough in normal times, however, it would be received with appallment.

After a threat is made, and the possible popularity cost is paid, there may be another kind of cost. If the threat has not had the desired effect, the government has to decide whether to execute the threat or not. This is the credibility problem Johansen (1977) and Pohjola (1985, 1986) have stressed. The government clearly has a strong incentive to cheat. In the model we shall use, the tax increase would trigger more wage increases and make the situation worse than it would have been without the threat intervention.

But there is also an opposing factor. A cheater gets a bad reputation. In many areas of business, it is essential that a person’s word can be trusted. ‘‘My word is my bond’’, as the saying goes. In politics, things appear to be different. Promises seem to be given lightly and also broken lightly. But this appearance may be deceptive, because it usually concerns large abstract subjects which are neither clearly defined nor exactly measurable, and debates about cheating and breaking of promises have a different character. An explicit tax threat, however, can be compactly and exactly defined and measured. It is possible to follow whether one breaks one’s word.

If the cheater gets a bad reputation, then cheating has direct costs. These costs should be included in the utility calculation of the government. This is the line we follow here.

Including cheating costs in the utility function has the side effect that it makes minor threats more credible than major threats. Threats are executed if the cheating costs are greater than the costs from worsening unemployment. Doubling the tax rate would probably drive the economy into chaos, and such a threat would hardly ever be believed to be executed. A small increase in taxes, on the other hand, would not collapse the economy. Such a threat might be considered credible if the government is thought to value its own word. In the Norwegian and Finnish experiences, the threatened measures have been rather small.

A tax increase that is not put into force is similar to a tax relief that is put into force. Likewise, a tax threat is symmetric to a tax relief promise.[3] Assume that at the start of period one, the government announces the tax rate t_0, and threatens that with wages above a certain level $\overline{W}$ the tax rate will be increased to t_1. This is exactly the same with that of the beginning of the period when the government announces the tax rate t_1, and promises that with wages below a certain level $\overline{W}$ the tax rate will be lowered to t_0. This symmetry is consistent with the 'cost of making a threat and cost of cheating' formulation presented above. Rational agents notice that t_1 is artificially high, and deduct popularity points from the government. If the proposal is turned down by the unions and the government still lowers the tax rate, it has broken its word and is deemed 'wet'.

3. Wage formation, prices and government behavior

There are several ways to model the effects of one union's decisions on the well-being of other unions. The effects may come via the demand for labor and depend on whether the unions are in industries whose products are complements or substitutes (see, e.g., Wallerstein, 1990). The effects may also come in the form of varying alternative wages if labor can move easily between industries. A third alternative, via consumer prices, is chosen here. This choice is not crucial for the main results concerning the nature of tax threat policies.

We apply the so-called monopoly union model to the case of many trade unions. Each union is assumed to maximize its members' utility, which consists of the real net consumption wage and employment. The utility function of union i is:

$$U_i = U_i\big((1-t)W_i/P, L_i\big), \quad U_{i1}, U_{i2} > 0 \tag{1}$$

W_i and L_i are the wage rate and employment of union i, t is the proportional income tax rate and P denotes consumer prices. U_{i1} and U_{i2} are partial derivatives.

As we will compare the utility levels of different wage–consumer price combinations, it is useful to make an additional assumption: if a union is indifferent between two wage–price combinations, it will choose the one with lower price level. This assumption is similar to the 'epsilon truthfulness' assumption in Rasmusen (1989) (p. 161) and it makes the preferences lexicographic.

Employment is determined by the demand for labor, and is a decreasing function of the real product wage W_i/P_i, where P_i is the price of commodities

[3] The Norwegian experience was presented in relief form (Johansen, 1977, p. 96)—''The State Budget (was) formulated with a rather high tax level with room for later reductions in case of a wage settlement which conforms with the Government's intentions''.

produced by members of union i. We assume that P_i depends positively, but less than proportionally on W_i. Thus, we may write employment as:

$$L_i = L_i(W_i), \quad L_i' < 0. \tag{2}$$

The union maximizes Eq. (1) with respect to W_i, subject to Eq. (2). The condition for an interior solution is:

$$U_{i1}(1-t)/P + U_{i2} L_i' = 0. \tag{3}$$

The solution is a wage rate, $\tilde{W}_i$, that can be expressed as a function of the income tax rate and consumer price level. The second-order condition for maximum ensures that both partial derivatives are positive:

$$\tilde{W}_i = \tilde{W}_i(t,P). \tag{4}$$

We assume that there is a positive minimum wage below which the unions will not go. This rules out wage and price levels equal to zero in further parts of this analysis.

We will call $\tilde{W}_i$ the normal optimal wage rate. It is optimal for union i in normal times, when the government does not interfere with a tax threat and the union can take the tax rate as exogenous. In times when the government has made a threat the union cannot take the tax rate as given. Instead, it must take into account that if it sets its wage above some limit $\overline{W}$, that may cause a hike in the tax rate.

Inserting the normal optimal wage into the utility function, we get the indirect utility function of union i:

$$V_i(t,P) = U_i\Big((1-t)\tilde{W}_i/P, L_i\big(\tilde{W}_i\big)\Big). \tag{5}$$

By direct calculation, using Eq. (3), or by the envelope theorem, it could be shown that the optimal utility is decreasing both in taxes and in consumer prices.

Although unions above did not take into account the effect of their own wages on consumer prices when setting the normal optimal wage, they are aware that consumer prices depend positively on all producer prices. As producer prices depend on wages, we may write:

$$P = P(W_1, \dots, W_n). \tag{6}$$

The unions are assumed to act non-cooperatively, each according to Eq. (4). The equilibrium of the economy is found when the consumer price level Eq. (6) is consistent with unions' wage levels. The equilibrium level of consumer prices, resulting from unions' optimal wage levels, can be derived by inserting Eq. (4) to Eq. (6) and solving $\tilde{P}$ from:

$$\tilde{P} = P\Big(\tilde{W}_1(t,\tilde{P}), \dots, \tilde{W}_n(t,\tilde{P})\Big). \tag{7}$$

$\tilde{P}$ is an increasing function of t.

$$\tilde{P}=\tilde{P}(t), \quad \tilde{P}'>0. \tag{8}$$

In normal times, the outcome is the non-cooperative Nash equilibrium described by the equations above. But the wage set by each union has spillover effects to other unions through consumer prices. Tax threat is an instrument the government can use to make the unions internalize these effects.

We formulate the government's utility function so that it contains only one economic variable, aggregate employment L. As aggregate employment is a sum of employed union members, it can be expressed as a function of union wages, using Eq. (2).

$$L=L(W_1,\dots,W_n) \tag{9}$$

From Eqs. (4), (7) and (8), we notice that at the non-cooperative Nash equilibrium, the aggregate employment is a function of the tax rate only. We denote this solution by $\tilde{L}$:

$$\tilde{L}=\tilde{L}(t), \quad \tilde{L}'<0. \tag{10}$$

Two other variables are the cost of making a threat, M, and the cost of cheating, C. These are both private costs: they do not affect the social welfare, only the popularity of the government. The cost of making a threat is assumed to vary with the overall economic situation. It gets a value of 0 if a threat is not made, and a value of $\overline{M}_s$ if a threat is made in period s. The cost of cheating is thought to be a constant $\overline{C}$ if the threat is not executed although the unions have rejected the government's offer. If the threat is carried out, or if no threat was made in the first place, the variable gets the value 0. It is conceivable that both M and C depend also on other factors, for instance the details of the tax threat itself. That would, however, complicate the analysis without giving much further insight into the problem, so we have chosen the admittedly simple specification.

The government's utility function is thus: [4]

$$U_G=L-M-C. \tag{11}$$

Let us further assume that if the government is indifferent between two alternatives, it chooses the one with no cheating. If cheating is not involved, it chooses the one with higher employment. In what follows, we assume that all unions know the government's preferences.

[4] The tax rate could also be included directly into the government's utility function. That would not, however, change the nature of the results.

4. The tax threat game

4.1. The order of play

There is a specific order of play in a tax threat game. It consists of four phases.

1. The government calculates the wage and price outcome that is to come if no intervention is made. It also calculates the optimal threat, in a way to be shown later, and decides whether to use the threat or not.
2. The unions make their wage decisions, taking the possible threat into account. If no threat was made, the game ends.
3. If a threat was made, the government audits the outcome. If the conditions of the proposal are met, taxes are not increased and the wage decisions of the unions are realized, and the game ends. If the conditions are not met, the threatened tax increase is put to force.
4. If the tax increase is executed, all the unions make new wage decisions, taking the increased tax rate into account.

The sensitivity of the results of these types of games to the exact structure of information is demonstrated by Pohjola (1985). Here, it is essential that although the government presents the threat first (step 1), it acts after the unions have made their choices and knows the choices (step 3). [5] Step 4 is important because it hedges the unions against a very bad outcome that would otherwise be possible: a union accepting the government's proposal (in step 2) would be stuck with a low wage, even though the tax rate would still be increased because too many of the other unions rejected the proposal. Here, this situation is ruled out. In the Finnish 1989 experiment there was a similar clause.

All players are assumed to have certain, symmetric and complete information (see Rasmusen, 1989, p. 51). Information is not perfect, though: in step 2, the unions do not know the decisions of other unions.

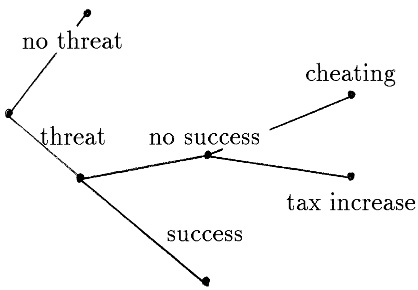
Table 1 summarizes the various outcome and utility possibilities. For each outcome, the utility of the government is indicated above and the utility of union, *i* is below in the far right column.

4.2. Desired properties of a tax threat

The tax threat is a triplet $(t_1, \overline{W}, F)$, where t_1 is the tax rate that will be put into force if a bigger share than F of the trade unions forsake the proposed ceiling, $\overline{W}$, to wage levels. $\overline{W}$ is a single number common to all unions. The share, F, means the number of employees belonging to unions that forsake the proposal,

[5] This is why the game is called an inverted Stackelberg game; in an ordinary Stackelberg game, the leader (the government) would move first.

Table 1
Possible outcomes of the tax threat game



Outcome:	Utilities:
$\tilde{L}(t_0)$ t_0	$\tilde{L}(t_0)$ $V_i(t_0, \tilde{P}(t_0))$
$\tilde{L}(t_0)$ t_0	$\tilde{L}(t_0) - \bar{M}_s - \bar{C}$ $V_i(t_0, \tilde{P}(t_0))$
$\tilde{L}(t_1)$ t_1	$\tilde{L}(t_1) - \bar{M}_s$ $V_i(t_1, \tilde{P}(t_1))$
$> \tilde{L}(t_0)$ t_0	$> \tilde{L}(t_0)$ see Section 4.5

compared to the total labor force. The threat is made whenever the gain from the threat, through higher employment, is greater than the cost of making the threat.

We will call a threat an *optimal tax threat* if it satisfies the following three conditions.:

1. The threat is credible;
2. At most, the share F of rational unions will reject it; and
3. From the threats satisfying the two conditions above, it leads to the highest value of the government's utility, that is, to the highest level of employment. 'Optimality' is, thus, defined from the government's point of view.

4.3. Credible tax threats

The threat is credible whenever the cost of cheating is greater than the loss from executing the threat, which results in unions setting even higher wages than they originally would have done. Using the government's utility Eq. (11) and the expression Eq. (10) for L, we find t_1 so that:

$$\tilde{L}(t_0) - \tilde{L}(t_1) \leq \overline{C}. \tag{12}$$

It is clear that the t_1 to be chosen is the one that satisfies Eq. (12) as an equality when we search for an optimal threat.

We note that the credibility of a threat does not depend on the proposed wage level or on the allowed rejection share or on the cost of making the threat. Credibility is a question of what happens if the proposal is rejected, and then $\overline{W}$, F and M play no role anymore.

Formulating credibility to depend on the size of the threat has important effects on the outcome of the game. If credibility is taken as given, as in, e.g., Salman and Cruz (1981), the government can set the threat so high that it achieves its target completely. In this model, it could press the wage levels arbitrarily low. In our formulation, this is not possible.

4.4. How to set the wage limit

After the threatened tax increase is determined, the government has to set a common wage limit for all unions and a statement of how many unions or how large a share of the unionized labor force is allowed to reject the proposal without triggering the tax increase. [6]

We classify the unions into three groups. Group R consists of those unions which reject a certain proposal (threat). Group A is the set of unions which accepts the offer and are restricted by the wage limit. Group B consists of unions which accept the offer but are not restricted by the wage limit because their optimal wage is below the limit. The government must do some calculations for all possible divisions of unions into groups A and other groups. During these calculations, the division of unions not in A into R and B will be made. From n unions, group A can be formed in 2^n different ways.

For a given A, the government's problem can be expressed as a mathematical programming problem:

$$\max_{\overline{W}} L = L(W_1, \ldots, W_n), \tag{13}$$

subject to:

$$W_i = \overline{W}, \quad \forall i \in A. \tag{14}$$

$$V_i\big(t_1, \tilde{P}(t_1)\big) \le U_i\big(\overline{W}, t_0, P, L_i(\overline{W})\big), \quad \forall i \in A. \tag{15}$$

$$W_i = \tilde{W}_i(t_0, P), \quad \forall i \notin A. \tag{16}$$

$$P = P(W_1, \ldots, W_n). \tag{17}$$

The government maximizes its utility by maximizing Eq. (13) with respect to $\overline{W}$, subject to Eqs. (14)–(17). Eq. (14) tells that the wages of unions in A are set to $\overline{W}$. Eq. (15) expresses the fact that the utility of any union in group A must be at least as high as it would be if the threatened tax increase were put into effect. Otherwise, the union would reject the proposal and thus belong to group R. Eq. (16) gives the wage setting of unions that are not in A.

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[6] It is clear that in this model, union-wise limits would be better than a common limit. The reason why common limit (for wage increases, not levels) policies are used in practice is probably the lack of sufficient information concerning the situation and preferences of different unions.

There are points that satisfy Eqs. (14)–(17), the no-threat equilibrium being one of them. The free minimum of Eq. (13), driving wages to minus infinity, is not possible as it violates Eq. (15). The solution, therefore, must be on the boundary. At least one union must be restricted exactly to the alternative utility level, otherwise the threat would not be optimal because a lower limit could be set without triggering any more rejections of the proposal. So, for some union $k \in A$, it must be that:

$$V_k\left(t_1,\tilde{P}(t_1)\right) = U_k\left(\overline{W},t_0,P,L_k(\overline{W})\right). \tag{18}$$

The government solves the above programming problem for all possible divisions of unions into A and other groups, and chooses the solution that gives the highest employment.[7] The optimal wage limit $\overline{W}*$ is thus found.

Next, the normal optimal wages for unions not in group A are calculated. If the wage is above $\overline{W}*$, the union is classified into group R. Otherwise, the union belongs to group B. The accepted share of rejections, $F*$, is simply the total size of unions in R in the optimal solution. F cannot be set to a lower value because all the unions in group R would reject the proposal anyway, and then the whole proposal would be deemed rejected. Setting a higher value for F could, on the other hand, create a possibility for free-riding for unions in group A; the incentive clearly exists.

The government must next check that the threat is worth making. From the government's utility function Eq. (11), the government gains if:

$$L - \tilde{L}(t_0) > \overline{M}. \tag{19}$$

If Eq. (19) does not hold, the government will not make the threat.

4.5. Distributional effects of tax threat policies

Provided that the parameters of the threat are set properly, the government achieves higher employment than without the threat. These aggregate effects are clearly favourable. But the policies also affect the wage distribution between the unions.

Some unions, the group R, reject the offer and resort to their normal wage-setting. Because some other unions lower their wage levels, prices will be lower, and also normal optimal wages will be lower. That means higher labor demand and greater utility for unions in R.

Then there are unions in the group A which accept the proposal and whose wage levels are restricted. Whether their utility is lowered or increased is not clear,

[7] For a given A, the solution may have the drawback that the normal optimal wage for some union in A might be below $\overline{W}$. That cannot, however, be true in the solution that gives the highest employment.

because for some unions, the increase in employment might offset the decrease in the real net wage. But if the government's policy is optimal, then there is at least one union k, whose utility falls to the acceptance limit $V_k(t_1, \tilde{P}(t_1))$.

The last group of unions is B: they accept the proposal but set their wages in a normal way, because the result is below the proposed wage ceiling. These unions benefit from the policy by the same reasoning as group R.

We can make one statement concerning the division of unions into these groups in the optimal policies. There is at least one union in group A. Groups R and B may be empty. All depend on the preferences of the unions and other parameters of the model.

Other things being equal, unions that have higher effect on the consumer price level are more likely to be in group A than unions whose wages have smaller effects on consumer prices.

4.6. An illustration

Let us consider an economy with two trade unions. Their wage-setting is illustrated in Fig. 1. The height of the bars denote the wage levels that would have been set in the absence of tax threats. The width of the bars denotes the sizes of the unions.

The government decides to make a threat. The threatened tax increase is found from the credibility constraint Eq. (12). The proposed wage level is calculated according to the procedure of the previous subsection. With two unions, there are four possibilities for group A. It may consist of union 1, union 2, both unions, or be empty. The last alternative does not require consideration.

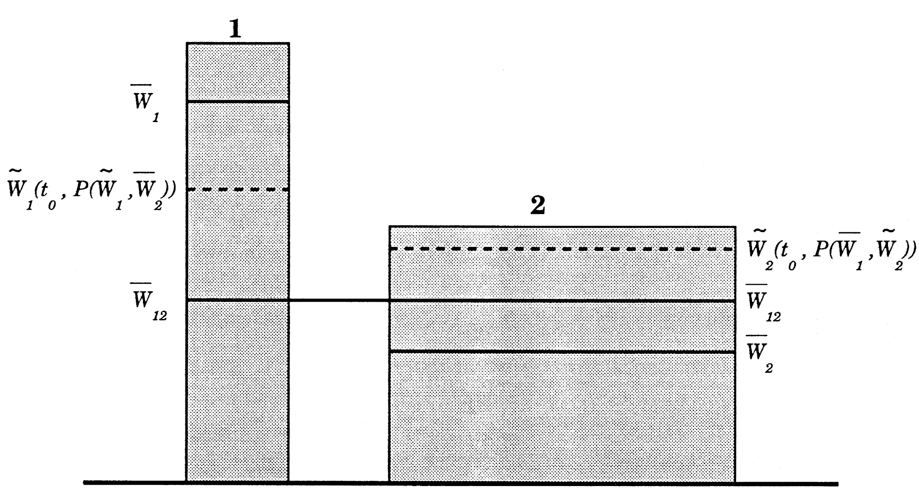

Fig. 1. A two-union example.

With the threat to increase taxes from t_0 to t_1, the government can achieve the wage level $\overline{W}_1$ for union 1, if it restricts only that union. Union 2 would then set its wage optimally to $\tilde{W}_2$; it is below the wage level that would have been set without the threat, because the wage of union 1 is lower, and thus also the price level is lower. If the government restricts only union 2, the resulting wage levels would be $\overline{W}_2$ for union 2 and $\tilde{W}_1$ for union 1. If both unions are restricted, the outcome would be $\overline{W}_{12}$ for both unions. Which of these is chosen?

We need to calculate the resulting employment level in each alternative. The employment levels are $L(\overline{W}_1, \tilde{W}_2)$ if union 1 is restricted, $L(\overline{W}_{12}, \overline{W}_{12}\})$ if both unions are restricted, and $L(\tilde{W}_1, \overline{W}_2)$ if union 2 is restricted. If we assume that the last is the highest, then $\overline{W}$ is set equal to $\overline{W}_2$ and F is set equal to the share of union 1 of the total employment. Before making the threat, the government checks that $L(\tilde{W}_1, \overline{W}_2) - \tilde{L}(t_0) > \overline{M}$. We assume that is the case. When the government makes this threat, union 2 realizes that union 1 will reject the proposal, and that if union 2 would also reject it, taxes would rise. Union 2 consults its preferences, behaves rationally and accepts the proposed wage level. It thus belongs to group A, and suffers from the policy. Union 1 belongs to group R, and benefits from the policy. Group B is empty.

4.7. Remarks on other types of tax threats

Tax threats can also be used to achieve outcomes Pareto superior to the Nash equilibrium that would result without any threats. We outline a case where the government chooses the best outcome from its own point of view, with the restriction that no union is worse off than it would be at the Nash equilibrium with no tax threats.

We assume that the threat satisfies conditions 1, 2 and 3 in Section 4.2 with the added restriction that no union is worse off than it would have been without threats. [8] For this restricted threat, any threatened tax increase would be sufficient; we use t_1 also in this case.

The procedure for finding this optimal restricted threat is otherwise similar than in Section 4.4 except that condition Eq. (15) is replaced by Eq. (20):

$$V_i\left(t_0, \tilde{P}(t_0)\right) \leq U_i\left(\overline{W}, t_0, P, L_i(\overline{W})\right), \quad \forall i \in A. \tag{20}$$

Again, the no-threat Nash equilibrium satisfies the constraint. It is evident that other possible solutions also exist. Consider, e.g., what happens if, starting from the Nash equilibrium, one union would lower its wage. To a first degree approximation, the utility loss from the wage would be exactly offset by the

[8] This brings the game very close to a team model. See Rasmusen (1989) (p. 170).

increase in employment. As other unions would also lower their wages because of a fall in the price level, all unions would be better off. Reducing the wage further, however, would eventually lead to a point where the utility from increased employment is small and wage reduction diminishes utility. This would certainly happen when the economy approaches full employment. So, there is a point where the utility would be the same as in the original equilibrium. Similar possibilities arise if several unions start to lower their wages together. This kind of behavior is not optimal for the unions if there are no threats, and thus cannot lead to an equilibrium. But it may become optimal if the government sets the threat properly.

If the government could set union-wise limits $\overline{W}_i$, it would achieve higher employment than when it sets a common wage limit to all unions. When the government is trying to find a value for the wage limit for union, i, it searches from the points where utility of union, i, is equal to what it would get if taxes were increased to t_1. In addition, the wage limits must yield the consumer price level that is used in calculating the utilities. There may be many solutions to this system of equations, but at least one can be assumed. From the solutions, the government then chooses the one with highest employment. The optimality of the obtained threat from the government's point of view is self-evident. Rational unions will set their wages lower than they otherwise would have done. An optimal policy leaves all unions with lower utilities, lower real net wages and higher employment.

5. Why are tax threats so rarely used?

The framework we have presented gives a theoretical basis for successful tax threat policies. The two practical applications mentioned, in Norway and in Finland, have also been successful, at least in the sense that the governments' proposals have been accepted by the unions. Why, then, are these policies not used more often?

A simple explanation might be that the cost of making the threat is usually very high. Interference, especially in the form of a threat, may make the government so unpopular that it cannot offset that with the gain from higher employment. Only under exceptional circumstances may the cost be low enough to warrant the threat. Such a situation might be an occurrence of an external shock. Consider, e.g., a major rise, expected to be temporary, in the prices of imported consumer goods. Trade unions may want to raise nominal wages and take less employment to offset the fall in real consumption wages. The government may want to prevent this with a threat. There is a temporary conflict of interests.

Is there room for threat policies in normal times? Assume that there is a genuine and permanent difference between the unions' and the government's preferences. Unions want more money and less employment than the government thinks optimal. This is the setting in many studies about the interplay of the government and the trade union. Tax threats were explicit in Johansen (1977), but the case is similar with wages and public expenditure and wages and the exchange rate, see, e.g., Gylfason and Lindbeck (1986, 1987) and Horn and Persson (1985). What happens if threats are used systematically, from one period to another?

Jackman and Layard (1990) mention three reasons why attempts to impose wage norms fail: ''First, of necessity they involve interferences with (or even suspension of) free collective bargaining between individual employers and their workers. Neither individual firms nor local union leaders like this. Second, they rigidify the wage structure, which can lead to labour shortages that are undesirable and generate huge pressures on wages. Third, the policies are typically too crude to contain earnings drift, through regrading of staff, bonuses and other evasive tactics''. All three reasons also discourage using tax threats.

Systematic threat policies by the government may create counter-threats. The situation can be compared to a devaluation policy. Hersoug (1985), e.g., says that a systematic devaluation policy would be anticipated by the union, and the union may hedge against it by demanding formal wage indexation.

The tailoring feature may be one reason why tax threats are so rarely used. Any threat of the common limit type we have considered can be interpreted as a threat tailored towards some unions, irrespective of whether that was the aim of the government or not. This may arouse aggression among those unions, and when agitated, behavior is not always rational.

The second point of Jackman and Layard (1990) concerning the wage structure applies directly also to tax threats. The third point is also relevant: tax threats policies may in reality have been ineffective because the wage limits have been dodged in various ways. Union-wise wage contracts are rarely transparent to anyone except, perhaps, the negotiators themselves, and in addition, wage drift is an important part of wage formation.

In repeated tax threat exercises, the credibility problem might warrant the reputation building approach. A more realistic and much more difficult way would be to consider reputation in a series of games where the actors are the same but the game situations differ. One period the game could be about wages and exchange rates, and the next period about wages and public expenditure. To further cite Johansen (1977) (p. 97), ''. . . the game in which economic planning and policy take place does not usually fall into a pre-determined class of games with a reasonably transparent structure. Precisely the influencing of the nature of the game is one of the important aspects of economic planning and policy''. Tax threats would probably have only a minor role in these repetitions.

6. Concluding remarks

We have analyzed tax threats as an inverted Stackelberg game between the government and many trade unions. A credibility problem is evident because executing the tax increase if the wage proposal is rejected will deteriorate employment. With costs of cheating, however, there is an upper limit for a credible tax threat. We sought optimal values of the proposed wage limits and of the share of rejections that is allowed without triggering taxes to be raised to that limit. With rational union behavior, the policies work. Employment is higher and the price level lower than they would have been without the threat. The threatened tax increase will not be put into force.

Tailoring is the essence of a successful tax threat policy. Earlier, we cited Calmfors (1989) as saying that tax relief is perhaps not a good instrument in a many-union economy because tax cuts cannot be tailored differently for different unions (in practise). Tax threats are entirely different in this respect. Although the threat was expressed as an increase in taxation that is the same for all unions, the setting of the threat was explicitly done by tailoring it to those unions which, when accepting the proposal, would lower their wage demands so that the effects were maximal. As remarked earlier, there is an exact correspondence between tax threats and tax relief promises. A conditional tax reduction executed is similar to a conditional tax increase not executed, so if a threat can be tailored, so can the reduction. The key word is ‘conditional’. Tailoring concerns the conditions under which the threat might be executed, not the tax rate that would then result.

A testable hypothesis can be derived from the fact that the threat treats unions differently. In a period with an optimally set and successful tax threat, wages of at least one union which accepts the government’s proposal are smaller than wages that would have been expected, considering the consumer price development and other factors. This could be tested, e.g., with a dummy variable in an empirical model of union-wise wage formation.

The main objective of the paper has been to present a framework for analyzing tax threat policy as a one-shot game. This is justified by the fact that in practise, tax threats, although at least seemingly successful, have been very rarely used. Several possible reasons exist, and are sketched in the paper, why threat policies are best suited for temporary policy packages.

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WAGE FORMATION BY MAJORITY VOTING AND THE INCENTIVE EFFECTS OF PENSIONS AND TAXATION*

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We study median voter wage-setting and its dependence on pensions and taxation in a centralized monopoly union framework using a dynamic computable general equilibrium model with overlapping generations structure. We show that the higher is the earnings-related PAYG pension benefit level, the lower is the wage the voter chooses, for two reasons. Firstly, if the voter claims high current wages his lifetime wage income falls, which will lead to lower pensions, and the advantages of lower pension contributions go to future working generations. Secondly, the median voter has to pay higher contributions both because the current wage bill falls and because current pensions may increase due to indexation. Both these generational effects lead the median voter to choose lower wages, which leads to higher employment. When we compare median voter wage setting with labour markets where wages adjust to equate supply and demand, the difference is bigger when the incentives to work are stronger in the market equilibrium, and gets smaller when the incentives are weaker. When e.g. the pension benefits and the corresponding payroll tax are increased, the voting equilibrium wage level approaches the market equilibrium wage. Similar results are obtained with respect to labour and consumption taxes. (JEL: J51, H55, D58)

1. Introduction

The effects of pensions on the economy are usually studied in an overlapping-generations framework and assuming that labour markets clear. Both pension benefits and pension contributions are included in these analyses, which often deal with the incentive effects of benefit rules and possible pre-funding on labour supply and saving. Important contributions are Samuelson (1958) and Aaron (1986). Research has increasingly used simulation methods such as dynamic computable general equilibrium (CGE) models since the pioneering work of Auerbach and Kotlikoff (1987), see, e.g., Feldstein (1998) and Siebert (1998).

The overlapping-generations framework has also been widely used in tax analysis. Labour income taxes, consumption taxes and capital income taxes automatically have different dy-

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namic roles. Theoretical contributions include Atkinson and Stiglitz (1976) and Atkinson and Sandmo (1980), and CGE applications are, e.g., Auerbach and Kotlikoff (1987) and Broer and Westerhout (1993).

Another widely used approach in tax analysis is based on trade union behaviour and bargaining. This research has not provided many robust insights on tax issues, partly because there is a wide range of models derived from the trade union or bargaining approach, and partly because taxation in its various forms and channels of influence is an extremely difficult subject to study. Pension contributions are present as part of payroll taxes, but pension benefits are absent.

Usually analytical studies of tax effects in bargaining models yield ambiguous results, unless specific restrictions are put on e.g. the union's objectives or parameter values. The results may also depend on the details of the tax system, as, e.g., Koskela and Schöb (1999) show concerning the composition of wage and payroll taxes. One result is theoretically well-established: tax progression is good for employment if wages are set by bargaining, but bad for employment if wages adjust to clear the supply of and demand for labour (see, e.g., Koskela and Vilmunen, 1996), but even this might not hold if hours of work are included. Empirical studies based on the bargaining approach, such as Tyrväinen (1995), must weigh up the different and conflicting tax effects.

Wage bargaining models have seldom incorporated an overlapping-generations structure. Huizinga (1990) considers efficient bargaining, where both wages and employment are agreed upon. In his model the union utility is the sum of the utilities of the members. Hawtrey (1990) combines trade unions and capital formation in an overlapping generations framework, modifying the Solow-Swan growth model. Workers live and work for two periods, and positive population growth ensures that the median voter belongs to the younger generation. There is no retirement and thus no pension system. Capital is owned by a separate group of people, capitalists. Hawtrey emphasizes the dynamic interaction between the union's decisions and capital formation.

Intertemporal CGE models usually assume that labour markets clear: wages adjust to equate the supply of and demand for labour. A notable exception is Jensen et al. (1996). Their model includes a Blanchard – Kiyotaki (1987) type labour market, and a household sector where the probability of death is independent of age.

The median voter approach in trade unions is not new. Blair and Crawford (1984) give a critical assessment of some applications in the 1970s. They stress that the conditions for the existence of a voting equilibrium are extremely stringent in models with choice spaces of dimensionality two or larger. Layard, Nickell and Jackman (1991, p. 86) state as one of their stylized conclusions that 'Union democracy means that unions maximize the welfare of the median member'. Applications are scarce, though. Renström and Roszbach (1995) study employee share ownership in an economy with one monopoly union for each firm. In their analysis union members differ in exogenous stock endowments. Booth (1995) gives a short overview of the median-voter approach to wage bargaining. She has used the approach herself to explain the individual decision to join a union. Outside trade union models, there are several applications of majority voting to taxation, from the 1970s onwards, and to pensions, see e.g. Lassila and Valkonen (1995) and Azariadis and Galasso (1996).

This study aims to analyse the effects of pensions and taxation so that both the overlapping-generations dynamics and trade union decision making are included. A novel feature in this paper is to incorporate a trade union with median-voter wage setting into an overlapping-generations framework of the Auerbach-Kotlikoff (1987) type. This model type has become popular in policy analysis, as it can be adapted rather easily to different institutional structures and other country-specific features (see, e.g., the articles in Broer and Lassila, 1997) while ensuring that the analysis is explicitly based on optimising behaviour by economic agents.

The median voter approach is almost directly applicable to a numerical OLG model. Workers differ in age, and their gains and losses from higher wages differ also, because the length of remaining working years, wealth, accumulated pension rights etc. differ. The median voter approach has the advantage of exactly defining the target of the trade union: no additional assumptions are needed as all relevant information is included in the households' utility functions. The approach also defines the dynamic aspects of the trade union's decision: the horizon is that of the median voter. Only one feature needs to be added to the model: a mechanism or rule stating how employment is divided among households. There are at least two obvious candidates. One is to divide employment equally among all households, the other is to leave some households entirely unemployed, and divide total employment among a smaller group. We shall use the first option because of its simplicity.

We show that there is an equilibrium wage level in our model which the majority of workers wants neither to increase nor decrease. This wage is higher than the market-clearing equilibrium wage, and correspondingly employment is lower. How big this difference is depends on many features of the economy, such as the substitutability of labour and capital in production, price elasticities between foreign and domestic goods in consumption, investment, intermediate use or in the export markets, and individual preferences between leisure and consumption. The difference is smaller when taxation and the pension system discourage individual work incentives. When, e.g., the pension benefits and the corresponding payroll tax are increased, the voting equilibrium wage level approaches the market equilibrium wage. Similar results are obtained with respect to labour and consumption taxes.

Our analysis stresses the importance of intergenerational aspects. For example, the higher is the earnings-related PAYG pension benefit level, the lower is the wage the median voter chooses, for two reasons. Firstly, if the voter claims high current wages his lifetime wage income falls, which will lead to lower pensions, and the advantage of lower pension contributions goes to future working generations. Secondly, the median voter has to pay higher contributions both because the current wage bill falls and because current pensions may increase due to indexation. Both these generational transfer effects lead the median voter to choose lower wages, which leads to higher employment.

The paper is organized as follows. Section 2 provides simple analytical examples of median-voter wage setting, to provide intuition of the basic features before proceeding to the complex simulation environment. In Section 3 the behaviour of individuals, both as members of trade unions and as households, is described and the median voter approach is specified. Section 4 discusses the features of the voting equilibrium. The incentive effects of pay-as-you-go (PAYG) pensions in the voting equilibrium are compared to those with labour-market clearing in Section 5. Incentives and taxation are discussed in Section 6, and concluding remarks are presented in Section 7.

2. *Basic features of median-voter wage setting*

Like most union or bargaining models, the median voter approach also focuses on wages and employment. Employment comes indirectly as leisure; the median voter is an individual who values leisure and works only to facilitate consumption. From this angle, the trade union is an institution to whom the households have delegated their decisions concerning the amount of leisure, while keeping consumption decisions to themselves. When the trade union makes the leisure decisions, by setting wages which determine employment, it also considers the resulting consumption possibilities. The leisure – consumption outcome is different from what it would be if the households were to make both decisions directly. The choices are affected by taxes and the pension system, again differently when there are trade unions and when there are not.

This section tries to give intuition about the main features distinguishing the median voter approach from either assuming balancing atomistic labour markets or using the trade union approach without the median voter. Besides the leisure – consumption choice mentioned above, we highlight some crucial dynamic issues.

Firstly, the median voter is a different individual in each period, and each median voter must form expectations concerning future median voters' decisions and how they depend on current decisions. Secondly, the current wage affects capital formation and thus future wage and employment possibilities. Thirdly, pensions are inherently dynamic and create links both between periods in each generation's life cycle and between different generations.

Dynamic wage and employment effects

Consider an economy where all households care about consumption *c* and leisure *l*. Assume that each household lives two periods and has a lifetime utility function *U*. To simplify, assume that the household cannot freely borrow and lend, so there are two budget constraints.

(1) $U = u(c_1, l_1) + \beta u(c_2, l_2)$

(2) $(1 - l_1)w_1 - c_1 = 0$

(3) $(1 - l_2)w_2 - c_2 = 0$

where *w* is the wage rate. The price of the consumption good is set to unity, so *w* also represents the relative price of leisure and consumption. The total amount of time the household can allocate for work and leisure is also set to unity. The household's discount factor β is defined as $\beta = (1+\delta)^{-1}$, where δ is the positive rate of time preference. If the labour market equates the households' supply of and firms' demand for labour, the household decides both *c* and *l* optimally, taking *w* as given. This yields a familiar condition for the consumption – leisure choice (partial derivatives are denoted by subindices).

(4) $u_{l_i}/u_{c_i} = w_i \qquad i = 1, 2$

Let us now assume that the wage rate is set by a trade union. The firms' demand for labour then determines total employment, of which the share of each household is $e = 1 – l$. Households can now only decide consumption optimally, given *l* and *w*. The optimal solution is to spend total wage income on consumption. There is one household, however, who is the median voter in the trade union. Assuming that it belongs to the young generation, its wage-setting problem is

(5) $\max_{w_1} U$

subject to (2) and (3) and

(6) $1 - l = e = e(w), \;\; e_w < 0$

As the median voter the household decides the wage, taking into account the employment effect. It doesn't consider consumption here, but assumes that as a household it, as other households, will make an optimal consumption decision. Inserting the budget constraints into the utility function yields the following first-order condition.

(7) $$dU/dw_1 = u_c(e + e_w w_1) - u_l e_w + \beta(u_c(e + e_w w_2) - u_l e_w)\frac{dw_2}{dw_1} = 0$$

Besides the current consumption and leisure effects, the solution depends on the expected effect of w_1 on the second period's median voter's decision; an issue which will be discussed at length in Section 4. Disregarding the possibility that by chance $\beta\frac{dw_2}{dw_1} = -1$, which is outside the median voter's realm of influence, the only stationary solution[1], where $w_1 = w_2 = w$, is clearly

(8) $u_c(e + e_w w) - u_l e_w = 0$

With slight manipulation of (8) we have the following leisure – consumption relation, where μ is the elasticity of employment w.r.t. the wage rate.

(9) $\mu = e_w(w)w/e(w)$

(10) $u_l/u_c = w(1 + 1/\mu)$

[1] *We assume overlapping generations: in each period there is both a young cohort and an old cohort in the economy. This facilitates a stationary equilibrium.*

Since μ is negative, the marginal utility of leisure is lower in relation to the marginal utility of consumption than in the atomistic labour market case (4). With decreasing marginal utilities this implies that there is now more leisure in relation to consumption than there would be if the median voter household could freely choose both l and c, taking w as given. But the median voter cannot freely choose l (by choosing w), he internalizes the employment effect of his decision. The other households cannot choose l at all, it is given to them. The households would like to work more with the wage rate w, but are restricted by the firms' demand for labour.

Notice that, for an interior solution, the employment elasticity μ in (10) must be less than minus one. Otherwise it would always pay for the union to increase the wage somewhat more, as households could have both more leisure and more consumption. If the elasticity is a decreasing function of the wage rate, with values between zero and minus one for lower wages and values below minus one for higher wages, we know that the leisure-valueing monopoly union described above would set a higher wage than a wage-bill maximising trade union. Maximising the wage bill, with a wage rate where $\mu = -1$, would maximise consumption, but the trade union here is willing to trade some of that consumption for more leisure.

The direct effect of the current wage decision on the future wage decision is not the only way future employment is affected. One of the most important channels is that wage decisions can affect the capital stock, which affects future wage decisions, and so on (for an empirical monopoly-union application see Holm, Honkapohja and Koskela, 1994). To illustrate this, assume that employment depends also on the capital stock, which in turn adjusts slowly and depends on the wage prevailing in the previous period.

(11) $$e = e(K, w)$$

(12) $$K = K(w_{-1}), \qquad K_{w-1} < 0$$

Assuming for simplicity that now the wage in the second period is unaffected by the current period wage decision, condition (7) becomes

(13) $$dU/dw_1 = u_c(e + e_w w_1) - u_l e_w + \beta(u_c e_K K_{w-1} w_2 - u_l e_K K_{w-1}) = 0$$

Evaluating (13) in a stationary situation, the analogue of condition (10) becomes more complicated:

(14) $$u_l/u_c = w(1 + \tfrac{1}{\mu+\beta\kappa})$$

where $\kappa = e_K K_{w-1} w/e$ is the partial elasticity of employment w.r.t. the wage rate via the capital stock.

The right-hand side of (14) also includes the dynamic wage effect on capital stock and thus on future employment, in addition to the direct employment effect of the wage. Had we included the expected effect on the second period wage decision, it would have intertwined with the capital stock effect and further complicated the result. The examples demonstrate that dynamic employment effects are important for the median voter's decision-making.

The effects of taxes

Let us now assume that the government collects three types of taxes: a proportional wage tax, at a rate τ^w, a value-added tax τ^c on consumption, and a payroll tax τ^z. These taxes change the relative price of consumption and leisure, and the change is different from the households' and median voter's points of view. Now the household's budget constraint is

(15) $$(1-l)w_i(1-\tau^w) - (1+\tau^c)c_i = 0 \quad , \quad i = 1,2$$

and employment depends on the price of labour

(16) $$1 - l_i = e_i = e((1+\tau^z)w_i) \quad , \qquad i = 1,2$$

Instead of equation (10), the median voter's optimal decision now leads to the following leisure – consumption relation.

(17) $$u_l/u_c = \tfrac{(1-\tau^w)}{(1+\tau^c)} w(1 + \tfrac{1}{\mu})$$

As previously, the first term on the right side expresses the price ratio of leisure and consumption from the household's point of view. Wage and consumption taxes affect this ratio. The second term gives the employment elasticity effect, which now includes the payroll tax: $\mu = e_w((1+\tau^z)w)(1+\tau^z)w/e(w)$. The payroll tax thus affects the median voter's decision through the internalized employment effect. If households were to choose both consumption and leisure, they would neglect the payroll tax, but it would still affect the outcome because the market-clearing wage would depend on it.

Pensions

Future wage decisions may be important for the median voter even if he will then be retired. Earnings-related pensions connect the working periods of the lifecycle to the retirement periods. Working creates a right to receive pension benefits in the future. Contributions are paid from wage income, and used to pay for the pensions of current pensioners. Thus there are effects within and between lifecycles.

Consider again the decision setting of the period 1 median voter in a two-period lifecycle context. Now we assume that he will be retired during the second period ($l_2 = 1$). The constrained maximisation problem is as follows.

$$(18) \quad \max_{w_1} U = u(c_1, l_1) + \beta u(c_2, 1)$$

subject to the budget constraint and the demand for labour:

$$(19) \quad (1 - l_1)w_1 + z_2/(1+r) - c_1 - c_2/(1+r) = 0$$

$$(20) \quad 1 - l_1 = e_1 = e_1((1+\tau^z)w_1)$$

As the saving effects of pension systems are an important issue, we have now allowed saving. The sign of saving in period one and in period two is not constrained, so there is only one dynamic budget constraint (19).

By setting the wage rate the median voter determines his employment and thus leisure in period 1. The amount of leisure in period 2 is fixed: the current median voter will be retired and thus all his time is leisure. But he may still have to consider the second-period median voter's decision, because his pension z may depend on it. The benefit formula is

$$(21) \quad z_2 = \theta(1 - l_1)w_1\left(\frac{w_2}{w_1}\right)^{\lambda}$$

where θ is the pension replacement rate, relating the pension benefit to beneficiary's wage income. The higher θ is, the less households need to save for the retirement period, and less saving affects the wage and employment possibilities. λ is the indexation parameter with possible values of 0 (no indexation of pension benefits to current wages) and 1 (full indexation to current wages).

With no indexation, the budget constraint is

$$(22) \quad (1 - l_1)w_1 + \theta(1 - l_1)w_1/(1+r) - c_1 - c_2/(1+r) = 0$$

If the median voter is also an average household and there is no population growth, we can express the pension institute's budget constraint as (23). The equation says that, because it is a PAYG system, the contributions collected during period 1 are paid to current pensioners, whose pension benefits are determined by their earnings in period 0.

$$(23) \quad \tau^z(1 - l_1)w_1 = \theta(1 - l_0)w_0$$

When making his decision, the current median voter need not think about the wage during period 2; it doesn't affect his welfare since, with $\lambda = 0$, neither his wage income in period 1 nor his pension income in period 2 depend on the wage in period 2.

If the pension benefits are fully indexed to current wages, the household's and the pension institute's budget constraints change significantly:

$$(24) \quad (1 - l_1)w_1 + \theta(1 - l_1)w_2/(1+r) - c_1 - c_2/(1+r) = 0$$

$$(25) \quad \tau^z(1 - l_1)w_1 = \theta(1 - l_0)w_1$$

Now the current median voter's pension will depend on the next median voter's wage decision, so the possible effect of this period's wage on next period's wage must be taken into account. Another link now is that the current wage affects current pensions. The median voter knows that if he sets a higher wage, the current payroll tax rate must be increased: current pensions grow proportionately to the wage but the wage bill grows less. The change in the payroll tax rate will in turn also affect employment.

Combining median-voter wage setting in trade unions and overlapping-generations dynamics in analysing the effects of pensions and taxation brings about several issues. The examples above have illustrated three: expected future wage decisions, dynamic employment effects via the capital stock, and the dynamic nature of pensions.

An analytical approach would prove extremely difficult if not practically impossible unless some of the issues are dropped out. We have chosen another approach: we start from an existing overlapping-generations macro model and include the trade union in it. The dynamics of the capital stock are well known, and the pension system with its effects on saving are analysed extensively in earlier studies. Households' lifecycle contains several periods both in working-years and in retirement, facilitating a more realistic description of the time spans which in wage agreements is only one or several years but in pension systems several decades. The price to pay for this approach is that the results cannot be easily interpreted.

3. The simulation model

The model we use in simulations is the FOG model[2], modified to include a trade union with majority-voting wage setting. FOG is a dynamic general equilibrium simulation model with an overlapping-generations structure. It is an open economy version of Auerbach-Kotlikoff type of models, and has been used to analyse pension policies (Lassila et al., 1997a, Forss et al., 1998), other social transfers (Lassila and Valkonen, 1998), taxation (Valkonen, 1997 and 1999), globalisation (Lassila and Valkonen, 1999a) and ageing (Lassila and Valkonen, 1999b). In what follows we explain the basic structure of the model. For a detailed description of the model, see Lassila et al. (1997b).

3.1. Households

The economy has overlapping generations of households, each maximising lifetime utility U with respect to consumption. The maximisation problem is

$$(26)\quad \max_c U = \sum_{t=1}^{T} \frac{1}{1-\frac{1}{\gamma}} \frac{u_t^{1-\frac{1}{\gamma}}}{(1+\delta)^{t-1}}$$

where the periodic utility function is

$$(27)\quad u_t = (c_t^{1-\frac{1}{\rho}} + \alpha_0 l_t^{1-\frac{1}{\rho}})$$

and maximisation is subject to the budget constraint (28) stating that discounted lifetime wage income, pensions z and transfers E equal discounted consumption expenditure:

$$\begin{aligned}(28)\quad & \sum_{t=1}^{T_w}(1-l_t)w_t(1-\tau_t^w)(1+r)^{-(t-1)} \\ & + \sum_{t=T_w+1}^{T} z_t(1-\tau_t^w)(1+r)^{-(t-1)} \\ & + \sum_{t=1}^{T} E_t(1+r)^{-(t-1)} \\ & - \sum_{t=1}^{T} c_t P_t^C(1+\tau_t^c)(1+r)^{-(t-1)} = 0\end{aligned}$$

where c is consumption, l is leisure, and of the constant parameters γ is the elasticity of intertemporal substitution of the composite commodity u, δ is the rate of time preference and ρ is the elasticity of substitution between consumption and leisure. The household lives for T periods and works the first T^W periods. Labour incomes are taxed at a rate τ^w and the VAT rate is τ^c. P^C denotes consumer prices (see Equation A 21 in Appendix 2). Incomes and expenditures are discounted with an exogenous interest rate r.

Leisure l is determined by total employment L^T (see Appendix 2), which is divided equally among all working-age households, whose number is N^w, according to Equation (29). Utility maximization is also subject to the rules of the pension system, described in Section 5.1. The transfers are taken as given by the households.

[2] The name is abbreviated from Finnish Overlapping-Generations Model.

(29) $1 - l_t = L_t^T / N_t^w$

As a reference, we shall also look at competitive labour markets without trade unions. There the supply of and demand for labour balance, and households do not take leisure as given but instead maximise U with respect to both consumption c and leisure l.

3.2 The trade union

We assume that there is one nationwide trade union. It is a monopoly union, i.e. it sets wages independently and employers have no say in the outcome. Employment is determined by firms. Trade union membership is compulsory. This is to avoid problems of whether or not it is advantageous for all workers to be members.

In each period the members decide the wage of that period. The voting procedure is not specified, we simply assume that the outcome is such that the majority of members do not want to change it.

Each member bases his/her voting on lifetime utility considerations. When comparing the wage alternatives for the current period, they calculate their combinations of leisure and optimal consumption, and the resulting period utilities, for all the periods of their remaining lifetime, subject to their budget constraint during their remaining lifetime. Then they aggregate period utilities in their lifetime utility function and choose the wage that yields the highest lifetime utility. In these calculations they take into account all the general equilibrium effects, of which they have perfect foresight.

Formally, the median voter maximises the utility of the rest of his life, with respect to the wage in the period in which he is the median voter. Without loss of generality we denote both the age group and the period by m in the following formulas. The problem is

(30) $$\max_{w_m} U = \Sigma_{t=m}^{T} \frac{1}{1-\frac{1}{\gamma}} \frac{u_t^{1-\frac{1}{\gamma}}}{(1+\delta)^{t-1}}$$

subject to the budget constraint

(31) $$W_{m-1} + \sum_{m}^{T_w} (1-l_t) w_t (1-\tau_t^w)(1+r)^{t-m} + \sum_{T_w+1}^{T} z_t (1-\tau_t^w)(1+r)^{t-m} + \sum_{m}^{T} E_t (1+r)^{t-m} - \sum_{m}^{T} c_t P_t^C (1+\tau_t^c)(1+r)^{t-m} = 0$$

where W_t is financial wealth at the end of period t, subject to the rules of the pension system which determine z (see Equations 33–35), and subject to the general equilibrium effects that come from the total model (see Section 3.3. and Appendix 2).

We may write the overall condition for optimal wage setting as follows.

(32) $$\begin{aligned} \frac{dU}{dw_m} = (&\Sigma_m^T \frac{u_t^{-\gamma^{-1}}}{(1+\delta)^{t-1}} u_{l_t} \\ &- \Omega \, \Sigma_m^{T_w} w_t (1-\tau^w)(1+r)^{t-m}) \frac{dl_t}{dw_m} + \Omega \frac{dW_{m-1}}{dw_m} \\ &+ \Omega \, \Sigma_m^{T_w} (1-l_t)(1-\tau^w)(1+r)^{t-m} \frac{dw_t}{dw_m} \\ &+ \Omega \, \Sigma_{T_w}^{T} (1-\tau^w)(1+r)^{t-m} \frac{dz_t}{dw_m} \\ &- \Omega \, \Sigma_{t=m}^{T} c_t (1+\tau^c)(1+r)^{t-m} \frac{dP_t^C}{dw_m} \\ &- \Omega (\Sigma_{t=m}^{T} c_t P_t^C (1+r)^{t-m} \frac{d\tau_t^c}{dw_m} \\ &+ \Sigma_{t=m}^{T} (1+r)^{t-m} \frac{dE_t}{dw_m}) = 0 \end{aligned}$$

The median voter considers what will happen if he increases the wage marginally. He feels the effects through six channels[3]. Rather than a rigorous tool of analysis, we use (32) as a list of terms, helping to describe the median voter's decision making on a general level.

The first term, divided into two rows, describes employment, and thus leisure, effects (the term Ω is the Lagrangian multiplier of the budget constraint). The assumption that firms determine employment after wages are set

[3] *The seventh channel is consumption, which changes in every period of the median voter's remaining lifetime. This is the result of his reoptimization as a consumer, and by the envelope theorem we know that the magnitude of this effect is zero.*

means that employees are not on their notional labour supply curve, and that wages and labour input are negatively correlated.[4] Unemployment is only indirectly present in our model: households would like to work more at the wage rate determined by the trade union, but are restricted by the demand for labour by firms. There are no unemployed people in the model, however, as employment is divided equally among working-age households. The critique by Lucas (1987, section 5) applies: representative agent macro models can tell us something about employment and wages, but have very little to say about unemployment, which is a concept dealing with disruptions in, or difficulties in forming, employer – employee relationships.

Second, the value of the median voter's financial wealth changes, as the stock market reevaluates the future streams of dividends. When wages are increased, this comes as a surprise to the stock market and share prices immediately fall. The effects depend on the distribution of ownership of these shares. A part of the shares is owned by foreign citizens; the bigger this part is the less the median voter has to worry about the wealth effect. The domestically owned shares are assumed to be distributed equally among all households. Besides shares, households' financial wealth consists of firm and government bonds.

Third, wages change. The change in the initial period is decided by the median voter, but he must also consider what median voters in future periods will do. This is discussed in Section 4.

Fourth, the pension benefits that he will get in the future will change. The pension effects are dealt with in Section 5.

Fifth, consumption prices change. The price of the domestically produced good changes in relation to the price of the imported good, which is the numeráire in the model, to equalise supply and demand. Consumption goods are composites made of domestically produced and imported goods.

[4] *Oswald and Walker (1993) provide empirical evidence that this seems to hold for unionized workers, whereas for non-union workers the correlation seems to be positive, implying that they may be on their labour supply curve.*

Sixth, consumption taxes or transfers change to balance the budget. The government's budget must be balanced in the present value sense. If a large share of public outlays is salaries, and the budget is balanced each period by e.g. transfers or VAT, then the voting outcome may be very near, or even below, the market wage. This is so because an increase in wages increases public outlays so much that the following cut in transfers leaves workers worse off than initially without the wage increase. In our simulations the government's budget is balanced by increased borrowing in the first period and by transfers thereafter.

Only the labour demand effects are usually included in the literature dealing with local bargaining. Of the effects in this study, the pension effects should also be taken into account under local bargaining, as wages and pensions are connected also at the individual level, and not only at the economy-wide level. Wealth effects could be ignored by local bargainers, unless their holdings are in the firms they work. The effects on the behaviour of future local bargainers, however, should be taken into account, especially if labour is immobile between sectors. Each local bargainer probably thinks that the effects of his own decision on consumer prices and taxes are too small to be taken into account. These general equilibrium effects are more relevant with highly centralized wage bargaining.

3.3 The rest of the economy

The rest of the economy in the FOG model is described in more detail in Appendix 2, but its main features are as follows

- a forward-looking value-maximizing firm sector, which chooses the optimal path of investment, use of labour and intermediate goods and produces the domestic good which can be exported and which is an imperfect substitute for the imported good
- a government sector, which collects taxes and produces public services which are provided free of charge (and, for simplicity, are not taken into account in individual utility considerations)

– a pension institution, which pays the pensions and collects contributions from the employers. The employers' contribution rate is endogenous and balances the fund's budget each period (see Section 5.1)
– the rest of the world, with which goods can be traded and capital interchanged. The domestic interest rate is equal to the world interest rate.

4. *Voting equilibrium*

4.1 Market-wage equilibria and fixed-wage equilibria

When the model described above is used to study e.g. the effects of tax policies, it is usually assumed that there is full equilibrium, i.e. goods, financial and labour markets clear in each period, with all expectations fulfilled (see, e.g., Auerbach and Kotlikoff, 1987 and Broer and Lassila, 1997).

Here we try to illustrate how the outcome of the economy changes when wages no longer clear the labour market, but are instead set by the trade union, employment is determined by firms' labour demand, and total employment is distributed equally among all households whose members are of working age. We call this equilibrium a fixed-wage equilibrium, to separate it from the full equilibrium, later referred to as the market-wage equilibrium. Notice that there is a stationary fixed-wage equilibrium for any wage rate the trade union cares to set, irrespective of whether it is set by majority voting or in some other way. The voting equilibrium is just one special case of fixed-wage equilibria.

The numeraire in the FOG model is the imported good: its price is unity. Compared to the numeraire, the higher the wage level is in the fixed-wage model the less labour will be used as a production factor, and output is smaller. Domestically produced goods are more expensive relative to foreign goods. This induces substitution away from domestically produced goods, so the import content of consumer goods, investment goods and intermediate goods increases. The increase in the terms of trade decreases exports. Imports decline also, because the level of output, investment and consumption declines as higher wages imply lower employment. The decline in imports is smaller, however, than in exports. This is only consistent in equilibrium if foreign debt service costs are smaller, so net foreign debt is lower the higher the wage rate is. Although the capital-labour ratio is higher the higher wages are, the capital stock also declines with the level of output when wages are set at a higher level. It is not clear what happens to aggregate household wealth as both the capital stock and net foreign debt decline, although usually it declines as well.

4.2 Voting equilibrium

When the workers of some period decide the wage of that period, they try to take into account what future workers will decide and how future wage decisions depend on the current wage decision. This forms an infinite forward linkage, as the future decision makers will also consider the effects of their wage decisions on the decisions of the worker generations to come still further in time, and so on. The current median voter thus faces an enormous mental task.

The question is: Are there any circumstances under which the median voter can confidently predict the consequences on future wages of a particular choice for the current wage? Assume that there is: what kind of situation could that be? The median worker may think as follows: “I must make a decision, and I may as well start from some assumption concerning the path of future wages. I’ll make an optimal decision of the current wage, conditional on that path. Will the median voter of the next period decide the exact wage I have assumed? He will do that only under four conditions. One, he must reason like I do. He probably does, because he is just as rational as I am. Two, he must have the same assumption concerning wages from his time onwards. Three, the wage level I assumed he would set must be optimal for him, conditional on the assumed wage path during his future. Four, he must think that next median voter also fulfills these conditions. If these conditions are met for all the median voters to come, all that remains is to correctly assume the path of future wages.” Whether the path is found also depends on the history of the economy, but the thinking process itself leads us to consider the concept of 'voting equilibrium'. That is what we define in this section and use throughout the rest of the paper.

Let $w_t^V(w_{t,t+1}^e, w_{t,t+2}^e, \ldots)$ be the wage set by majority voting in period t, given the expected future path of wages $w_{t,t+1}^e, w_{t,t+2}^e, \ldots$, expected by the voters in period t. By definition, the change in the (remaining) lifetime utility resulting from an increase in wages from w_t^V to $w_t^V + \varepsilon$ is not positive for the majority of working-age persons. Also, the utility from decreasing the wage to $w_t^V - \varepsilon$ is non-positive for the majority, for all ε.

Definition 1. A voting equilibrium path of wages is a sequence $\{w_t\}$, where (i) $w_t = w_t^V(w_{t,t+1}^e, w_{t,t+2}^e, \ldots)$ for all t, and (ii) $w_{j,t}^e = w_t$ for all t and for all $j < t$.

The first part in Definition 1 states that each wage on a voting equilibrium path is such that the majority of workers wants neither to increase it nor to decrease it, given the anticipated future wages. The second part states that voters in each period anticipate all future wages correctly.

Considering all time points simultaneously, the voting equilibrium path is analogous to a Nash equilibrium in noncooperative games: workers of any one period do not want to change their decision, if workers of some other period do not change their decision. If one takes the time structure into account, each median voter acts as a Stackelberg leader vis-à-vis the future median voters.

In what follows we concentrate on stationary equilibria, where the wage level, and all other things, remain constant in time.

Definition 2. A steady-state voting equilibrium is a voting equilibrium path where, in addition to the two requirements of Definition 1, also $w_t = \overline{w}$ holds for all t.

The procedure of finding the steady-state voting equilibrium in the simulation model is described in Appendix 1.

4.3 Multiple equilibria?

Voting models may have multiple equilibria. This is a standard problem in models where the level of pensions is determined by voting. Azariadis and Galasso (1996, p.65) describe the problem as follows: "Today's decisions depend on the expectations of how tomorrow's policymakers will react to situations they expect to prevail the day after tomorrow, and so on forever. Policy choice is indeterminate because there is no way to pin down the behaviour of the policymaker at + ∞." In their model, however, and in many similar models, changes in policy can be made without cost. Future policies are the main thing, in fact the only thing, the median voter has to take into account.

That is not the case in this study. Voting is now about wages, not pensions. As in all trade union models, wages affect the demand for labour. Thus changes in wage policy cannot be made without costs. Changes in wages affect current employment directly as well as future employment and labour income because the capital stock changes. Costs come also in the form of various general equilibrium effects.

Trade union models with overlapping generations have not paid attention to multiple equilibria. In her two-period model, Hawtrey (1990) lets the trade union be myopic in the sense that the first-period optimum is the choice in each period. She notes that in the real world the span between generations is in the vicinity of twenty to forty years. Hawtrey does not specifically mention any possibility of multiple equilibria due to expectations concerning future wage decisions or any other matter. Huizinga (1990, p. 84 footnote) deals with future wages briefly: "The important aspect is that the implicit agreement between the firm and the union is stationary and expected to be everlasting." In his model either party could always force the return to the setup in period 0, so the only believable contract is the one that duplicates the conditions of period 0 forever. This is in contrast to our model, where the adjustment of the capital stock is costly and slow. The trade union literature with majority voting also neglects multiple equilibria. Booth (1995) does not mention multiple equilibria in her median voter overview. Rehnström and Roszbach (1995) consider the changes dynamization would bring to their static analysis, but do not mention the possible multiple equilibria problem.

Studies using voting models usually cannot provide convincing evidence about uniqueness or its absence. Beauchemin (1998), studying fiscal policy in an intergenerational framework, uses numerical methods to verify the uniqueness of his model solutions. Our approach has been similar. The simulations we have carried out provide no hints of the existence of multiple equilibria. We cannot rule out their possibility in our model, but there are strong factors that limit the role of expected future wage decisions.

4.4 Leisure – consumption choice in the simulation model

As noted in section 2, the leisure-consumption trade-off is important in the median voter's decision-making. Here we describe this trade-off in our simulation model.

If in any steady state fixed-wage equilibrium we increase the wage for one period, the dynamic simulation shows the following pattern for the median worker. In the first period leisure increases as the demand for labour decreases, and this increases the utility of the worker. It is the dominant utility effect during the worker's remaining lifetime. During the following working periods leisure is also greater than it would have otherwise been, because the capital stock was adjusted downwards during the first period, and is not immediately raised to the original level because of adjustment costs, although wages have returned to the original level. These leisure effects after the first period are small compared to the first period effect. The total effect of a temporary wage increase and a longer-lasting decline in employment is a fall in discounted wage income during the remaining lifetime. The counterpart of the positive leisure effects on utility is the decline in consumption, which follows from the reduction in lifetime wage income. This consumption decline is smoothed over all periods of the remaining lifetime. The total effect on lifetime utility is the weighted sum of the leisure and consumption effects.

Basically, thus, the trade union trades consumption for leisure when it increases the wage level above the initial fixed-wage equilibrium level. More leisure is paid for in the form of reduced consumption. The crucial issue then is how these two items affect the utility of the household. The normal assumption is that both goods have declining marginal utility. That is also the case in our model. Additional leisure is more appreciated the less leisure there is, in relation to consumption, to begin with. If there is quite a lot of leisure to begin with, the utility of the household is not much increased by having more leisure.

4.5 Who is the median voter?

Union members are identical in all respects, except that they are in different phases of their life cycles. Among other things, that means that they have different numbers of periods ahead, and they have accumulated different amounts of financial wealth. In our simulations both these features cause the young cohorts to prefer higher wages to the older cohorts. Thus the endogenously determined median voter is the median-age worker.

A reassuring feature is that, qualitatively, the utility changes from a temporary wage increase are similar in all cohorts although quantitatively they differ. This means that the qualitative properties of the voting equilibrium in comparison to market-wage equilibrium are probably robust to the age of the median voter, although quantitative results could be more sensitive. If, however, the median voter changes as a result of a change in an exogenous or policy variable, the effects may become volatile. This does not happen in the simulations of this paper.

4.6 Sensitivity analysis

Steady-state sensitivity analysis concerning the relations between market-wage and voting-wage equilibria, with respect to households' behavioural parameters, firms' production function parameters and other parameters describing the economy, does not point towards any great sensitivity (see Table 1). In all cases considered, the voting-wage equilibrium had high-

Table 1. Market-wage and voting equilibria: sensitivity calculations.

Parameter	w	L	C	K	F	P^d	A^f	W	CV
basic	102.16	85.59	86.73	86.07	85.88	101.57	1.03	3.36	–3.73
$\gamma = 0.4$	102.20	85.43	86.67	85.92	85.72	101.60	1.84	4.21	–3.80
$\gamma = 0.75$	102.07	85.94	86.81	86.40	86.22	101.50	–0.96	1.27	–3.57
$\rho = 0.375$	100.35	97.39	97.51	97.48	97.45	100.25	–0.61	–0.24	–0.57
$\rho = 0.9$	103.61	77.29	79.04	78.01	77.72	102.61	2.07	6.00	–6.25
$\sigma = 0.6$	102.52	84.74	86.20	85.39	85.14	101.69	0.99	3.47	–3.92
$\sigma = 1.4$	101.85	86.52	87.39	86.86	86.72	101.44	1.06	3.22	–3.45
$\beta = 0.6$	105.51	70.74	73.19	71.77	71.47	103.46	–1.29	4.74	–14.72
$\beta = 0.9$	100.73	94.32	94.73	94.50	94.40	100.60	1.32	1.97	–0.48
$\sigma^E = -5$	105.18	82.91	85.24	84.00	83.56	103.75	–0.17	5.53	–4.03
$\sigma^E = -20$	101.05	86.03	86.70	86.27	86.18	100.76	1.52	2.63	–4.01
$s^F = 0.5$	103.16	79.70	81.21	80.35	80.09	102.29	1.18	4.58	–7.34
Budget rule:									
E	101.32	90.91	91.66	91.22	91.10	100.96	0.78	2.20	–1.40
τ^c	101.32	90.90	91.65	91.21	91.09	100.96	0.78	2.20	–1.41
B^G, τ^c	102.16	85.58	86.72	86.06	85.87	101.57	1.03	3.36	–3.73

Notes: The basic parameter values are: $\gamma = 0.5$, $\rho = 0.75$, $\sigma^E = -10$, $s^F = 0.333$ (see Equations 1, 2, A1 and A14. s^F is foreign owners' share of stocks). The figures express ratios of voting equilibrium values to corresponding market-wage equilibrium values. Net foreign assets and total household wealth are expressed in relation to private production, and the figures are percentage point differences between voting and market-wage equilibria. The budget rule tells how the public sector is balanced in the dynamic simulations describing the median voter's calculations. At "*E*" the budget is balanced by transfers and at τ^c by value-added taxes in all periods, at B^G, τ^c the first period balance is achieved by taking on debt, and from then on VAT is used. Compensated variation *CV* expresses the compensation needed to achieve the same lifetime utility in *VW* that prevails in corresponding *MW*. It is expressed as a percentage of discounted lifetime consumption expenditure, and a negative sign implies a welfare loss.

w = wage rate
C = total private consumption
F = production
A^f = net foreign assets
L = employment
K = capital stock
P^d = terms of trade (price of domestic good)
W = total household wealth

er wages, lower labour input, lower consumption levels, a lower capital stock, lower output and higher export prices than the market-wage equilibrium. It is possible, however, that there exist parameter values where the voting wage would be below the market wage[5], and most of the relations stated above would reverse. High values of the capital – labour substitution elasticity would probably produce that situation. In such a case the trade union would have to force people to work more than they would like to at the going wage and prices.

The trade-off of the union is, briefly, more leisure and higher wages today and more leisure tomorrow versus less consumption now and in the future. The crucial parameters are those that affect this trade-off strongly. The elasticity of substitution between capital and labour is obvious: the higher the elasticity, the easier it is for the firm to reduce labour demand after a temporary wage increase, and thus workers lose more income and consumption. The price elasticity of exports is important: with a high elasticity, the wage increase would hit exports badly and thus reduce output and labour demand. The price elasticity of imports works in a similar fashion: a high elasticity implies that domestic production is greatly reduced after the wage increase affects prices, as the imported good is used more in the consumption, investment and intermediate good. The reduc-

[5] *Hawtrey (1990, p. 90) concludes that in her model this situation is the prevailing one, as it is often wise for the union not to use its power to increase wages. The model is different from ours but the trade union faces an analogous trade-off between higher wages today and an increased capital stock tomorrow.*

tion in production means less employment and less labour income. The ownership of firms' shares is also important: the more foreigners own the shares, the less the median voter gives weight to the negative share value effects of wage increases. On the household side, the intratemporal elasticity of substitution is important. If the elasticity is high, households are more willing to sacrifice consumption to get more leisure, and the median voter, having the same preferences, drives the wage level higher.

5. *The effects of pensions on median voter's wage decision*

5.1 PAYG pension system

By replacing the labour-market equilibrium assumption with majority-voting wage setting behaviour, we could in principle use the simulation model to analyse, e.g., pension policies in a similar fashion as in, for example, Broer and Lassila (1997). The dynamic calculations are, however, very difficult with the median voter approach. We can gain insight of the way the wage formation assumptions affect the incentive effects of earnings-related pensions by doing some exercises with the steady-state market-wage and voting-wage equilibria.

When we compare voting equilibrium to market equilibrium, we do not want the employment division rule to affect the comparisons. Thus we adjust the model so that in the market equilibrium steady states all workers wish to have an equal amount of leisure, irrespective of their age. To achieve this, two conditions must be met. First, the price of leisure must be constant, so that there is no intertemporal substitution concerning leisure. Second, the real net interest rate faced by the households must equal the rate of time preference, so that there is no intertemporal substitution concerning consumption, because that would also be reflected in leisure. The latter is straightforward, we set $r = \delta$. Condition one is slightly more complicated, because the price of leisure includes, besides the net wage rate, also the present value of the future pension right that comes from working. What is required is that the PAYG pension right accrues steadily in time and yields interest, so that the present value of future pensions accruing from each period, discounted to that period, is constant. With these features, the market-wage equilibrium is a special case of fixed-wage equilibria.

For the person who started working in period 1 the pension z in period t is

$$(33)\quad z_t = \frac{1}{T_W}\sum_{s=1}^{T_W}\theta(1-l_s)w_s(1+r)^{t-s}\left(\frac{w_t}{w_s}\right)^{\lambda},\quad T_W < t \le T$$

The parameter θ expresses the pension replacement rate and λ is the indexation parameter of pensions to current wages. We consider only the limit cases when $\lambda = 0$, representing no indexation, and $\lambda = 1$, representing full indexation. From (33), the discounted pension right that accrues from marginal work during period s is the following:

$$(34)\quad \sum_{t=T_W+1}^{T}(1+r)^{-(t-s)}\frac{dz_t}{d(-l_s)} = \frac{1}{T_W}\sum_{t=T_W+1}^{T}(1+r)^{-(t-s)}\theta w_s(1+r)^{t-s}\left(\frac{w_t}{w_s}\right)^{\lambda} = \frac{1}{T_W}\sum_{t=T_W+1}^{T}\theta w_s\left(\frac{w_t}{w_s}\right)^{\lambda},\quad 1 \le s \le T_W$$

which is constant and independent from s in an equilibrium where $w_s = w_t = w$ for all s and t. The employers' contribution rate is determined by the pension institute's budget constraint. That is obtained by aggregating pensions and contributions over cohorts i.

$$(35)\quad \sum_{i=T_{W+1}}^{T} z_{i,t} - \tau_t^z\sum_{i=1}^{T_W}(1-l_{i,t})w_t = 0$$

5.2 PAYG pensions and the median voter's decision

Figure 1 shows the pattern between the market-equilibrium labour cost (wage rate + employers' pension contribution) and the voting-equilibrium labour cost, when the pension benefit level, as a percentage of wage income, is increased. The curves show that the higher the PAYG pensions, the lower are work incentives and wages. Actually, wages decline more rapidly than labour costs when the pension level

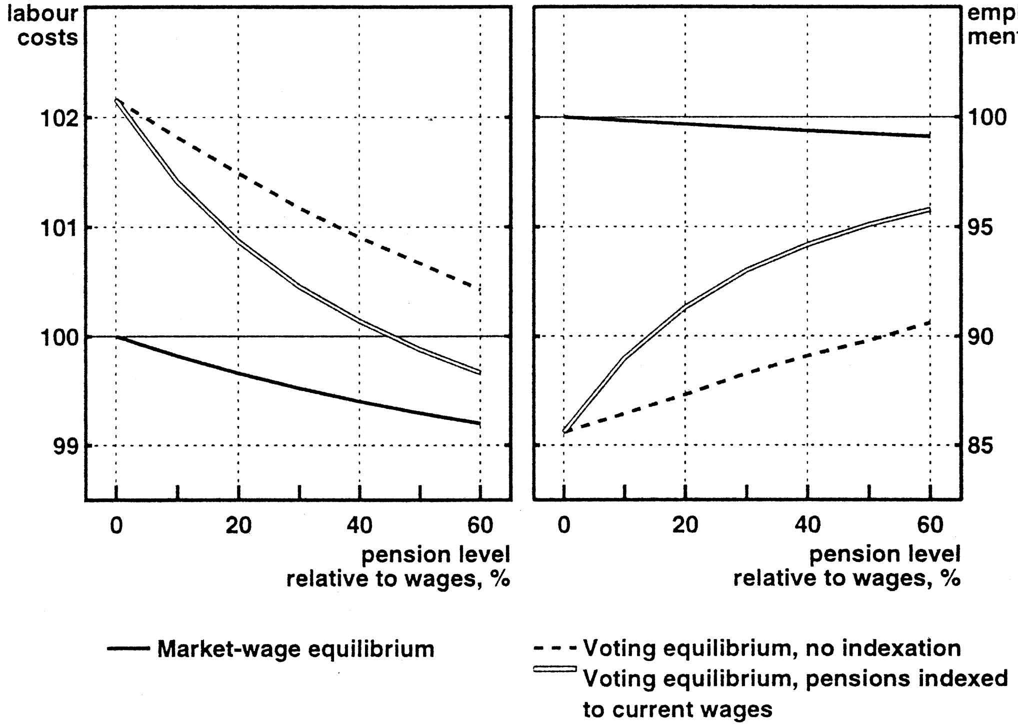

Figure 1. Effects of pension level on equilibrium labour costs and employment.

is increased, since the latter include pension contributions which increase. The voting-equilibrium wages approach the market-equilibrium wages when pension levels are higher. The second part of the figure shows the corresponding employment levels (see also Table 2).

In the market-wage equilibria, the increase in pension level decreases the work incentives of households. This is entirely a consequence of missing funding. The pension rule (Equation 33) itself has no distortive effect: rational households take the accruing pension rights into account, in the way shown in Equation (34), when they make labour supply decisions. If there were funding, exactly matching the accruing pension rights, and no indexing to current wages ($\lambda = 0$), the system would be actuarially fair, and the market-wage equilibria would be identical in all economically meaningful respects. Household wealth would be lower if pension rights were excluded, but the pension fund would match that exactly. Gross wages and the contribution rate would vary, but labour costs would be constant. Net labour incomes would differ in timing in households' lifecycles, but their discounted amount would be the same. But since no funds are collected, the interest income from funds is missing and a corresponding amount must be collected from current wages. This part of the employers' contribution rate is pure tax, and it distorts the labour supply decision.

The contribution rate effect of the missing fund is also present in the voting equilibria. But there are other effects also. The voter, thinking about the effects of a possible wage increase in the current period, balances in his mind the increase in leisure and the decrease in consumption, the latter being the consequence of a decline in discounted labour incomes. This decline is the sum of the increase in work income he receives in the first period and the decrease in each period during the rest of his working life. The aggregate effect of these on pensions is

Table 2. Market-wage and voting-wage outcomes with different replacement rates θ.

a) no indexation (λ = 0)

Parameter		*w*	*L*	*C*	*K*	*F*	P^d	A^f	*W*	τ^z	*CV*
θ = 0	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
	VW	1.022	0.856	0.867	0.861	0.859	1.016	1.030	3.355	0.000	–3.730
θ = 10	*MW*	0.939	0.998	0.982	0.998	0.998	0.999	–12.622	–12.818	6.260	–1.310
	VW	0.958	0.864	0.860	0.868	0.867	1.013	–11.899	–9.949	6.260	–5.004
θ = 20	*MW*	0.886	0.997	0.965	0.996	0.996	0.998	–23.809	–24.176	12.516	–2.494
	VW	0.902	0.873	0.855	0.877	0.875	1.011	–23.329	–21.727	12.516	–6.119
θ = 30	*MW*	0.838	0.995	0.951	0.994	0.994	0.997	–33.795	–34.310	18.774	–3.569
	VW	0.852	0.883	0.852	0.886	0.885	1.009	–33.504	–32.242	18.774	–7.068
θ = 40	*MW*	0.795	0.994	0.937	0.992	0.993	0.996	–42.764	–43.409	25.032	–4.551
	VW	0.807	0.891	0.848	0.893	0.892	1.007	–42.617	–41.644	25.032	–7.998
θ = 50	*MW*	0.756	0.992	0.925	0.991	0.991	0.995	–50.864	–51.624	31.290	–5.451
	VW	0.767	0.898	0.845	0.900	0.899	1.005	–50.828	–50.110	31.290	–8.877
θ = 60	*MW*	0.721	0.991	0.915	0.989	0.990	0.994	–58.215	–59.079	37.548	–6.278
	VW	0.730	0.906	0.843	0.907	0.907	1.003	–58.260	–57.801	37.548	–9.636

b) full indexation (λ = 1)

Parameter		*w*	*L*	*C*	*K*	*F*	P^d	A^f	*W*	τ^z	*CV*
θ = 0	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
	VW	1.022	0.856	0.867	0.861	0.859	1.016	1.030	–5.941	0.000	–3.730
θ = 10	*MW*	0.939	0.998	0.982	0.998	0.998	0.999	–12.622	28.270	6.260	–1.310
	VW	0.954	0.890	0.884	0.893	0.892	1.010	–11.964	25.004	6.260	–3.846
θ = 20	*MW*	0.886	0.997	0.965	0.996	0.996	0.998	–23.809	47.177	12.516	–2.494
	VW	0.896	0.913	0.891	0.915	0.914	1.006	–23.393	45.211	12.516	–4.516
θ = 30	*MW*	0.838	0.995	0.951	0.994	0.994	0.997	–33.795	60.711	18.774	–3.569
	VW	0.846	0.930	0.894	0.931	0.931	1.003	–33.538	59.458	18.774	–5.404
θ = 40	*MW*	0.795	0.994	0.937	0.992	0.993	0.996	–42.764	70.878	25.032	–4.551
	VW	0.801	0.942	0.893	0.942	0.942	1.001	–42.612	70.033	25.032	–6.388
θ = 50	*MW*	0.756	0.992	0.925	0.991	0.991	0.995	–50.864	78.795	31.290	–5.451
	VW	0.761	0.951	0.890	0.951	0.951	0.999	–50.782	78.213	31.290	–7.364
θ = 60	*MW*	0.721	0.991	0.915	0.989	0.990	0.994	–58.215	85.135	37.548	–6.278
	VW	0.725	0.958	0.887	0.957	0.957	0.998	–58.180	84.723	37.548	–8.315

Notes: The figures express ratios of market wage (*MW*) and voting equilibrium (*VW*) values to market-wage equilibrium base values (top row). Net foreign assets and household wealth are expressed in relation to private production, and the figures are percentage point differences between voting and market-wage equilibria and the base values. *CV* expresses the welfare loss compared to the base case. σ^z is the employer's pension contribution rate. For the other variables see Table 1.

negative. The advantage of this fall in pensions, in the form of lower contributions, does not come to the median voter but goes to future workers: there is a generational transfer. The higher the pension level, the higher is this transfer, and selfish workers react to this transfer by resorting to lower wage levels. There is also another generational transfer: a wage increase raises the pension contribution rate and this deteriorates the leisure – consumption trade-off of the median voter. The rise in the contribution rate follows from the fact that the labour income of the median voter falls. The rise is higher if current pensions are indexed to wages. Indexation also further reduces the median voter's own pension, as the current higher wage is replaced by the future wage which remains constant (see eq. 34). Both these generational effects work to the disadvantage of the median voter, and are greater the higher is the PAYG benefit rate and the higher is the degree of indexation of pensions to current wages.

Figure 1 shows that employment increases with the PAYG benefits, if wages are set by majority voting. Firms increase their demand for labour as labour costs decline. The market-wage level of employment also reflects the distortionary effect on households' labour supply, which dominates the labour demand increase by firms.

From the above reasoning, short aggregating periods of pension rights, at the end of individual's working life, may also contribute to higher employment than aggregating systems based on the whole working history of the worker. The loss of work income in future periods weighs more in the median voter's calculations if those future periods are crucial for the determination of the pension level.

The capital stock is also affected by the pension level. This is not due to saving effects: although household wealth declines and the net foreign asset position deteriorates with higher pension levels, interest rates do not rise because financial capital is assumed to be perfectly mobile internationally. The increase in capital stock in voting equilibria takes place because of output expansion: cheaper labour makes it profitable to produce more, and thus more capital is also needed. There are, of course, general equilibrium effects: part of the increased profitability goes abroad as the price of the domestically produced good slightly falls in relation to the foreign good, and this partly increases the cost of capital as the capital good price falls less than output price. There is some input substitution towards labour, but this substitution effect is smaller than the effect of the output increase.

Even though employment increases with PAYG, the utility of workers decreases with increasing pension benefits, as compensated variation results in Table 2 show. It is also noticeable that welfare is higher in every market-wage equilibrium than in the corresponding voting-wage equilibrium with the same pension level. Again, this applies to steady state comparisons. Further research should try to establish whether there is a voting-wage equilibrium path, described in Section 4.2., leading from the market-wage equilibrium to the steady-state voting-wage equilibrium. If there is, imagine that a nation-wide trade union is established in the market-wage equilibrium. The initial median voter would raise the wage and be better off, and probably would some of his successors. But gradually the capital stock would decline and other changes would occur, and future workers would be worse off than they would have been in the market-wage equilibrium.

6. *The effects of taxation on median voter's wage decision*

This section provides insight of the way the wage formation assumptions affect the incentive effects of taxation. As with the pension system in the previous section, this is done by describing simulations with the steady-state market-wage and voting-wage equilibria.

If the incentives in society are such that people work a lot, there is little leisure in the market equilibrium. In this case the trade union can increase the utility of its members by increasing wages, which leads to more leisure and less consumption. The wage level will be far higher than the market-equilibrium wage. But if the incentive system is such that not much work is done in the market equilibrium, the trade union cannot increase the utilities much by acquiring more leisure, and the wage level in the voting equilibrium will be closer to the market-equilibrium wage.

This simple explanation points to a general conclusion about the incentive effects of taxation and social security. Comparing the voting-equilibrium wage level and the market-equilibrium wage level, in the region where the former is higher than the latter, we thus expect to find that they are closer to each other

– the higher are labour income taxes used to finance larger transfers, and
– the higher are value added taxes used to finance larger transfers

The simulations (in Table 3) support these conclusions. Furthermore, sensitivity analysis shows that the taxation results are not very sensitive to those parameter values that showed the largest effects in Table 1. Still, as is typical with

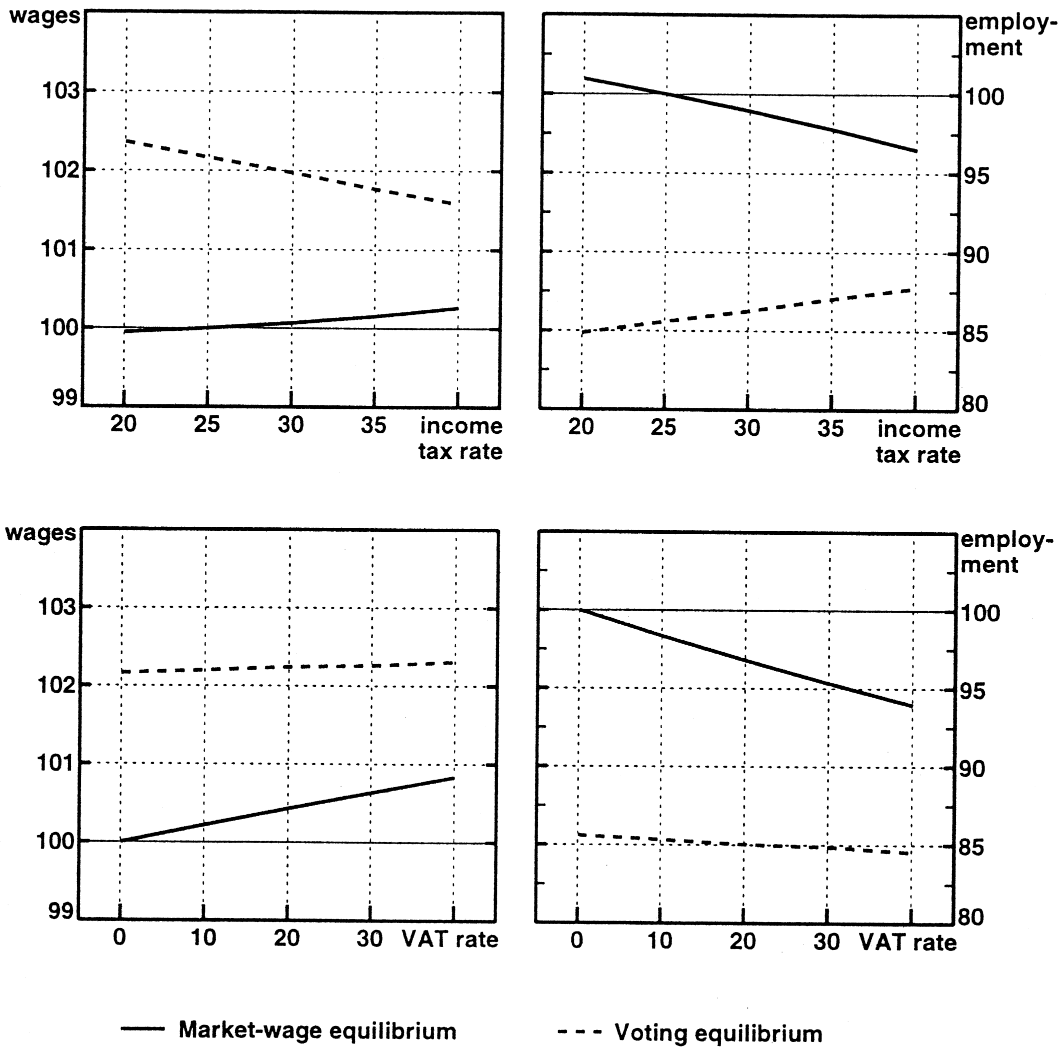

Figure 2. Effects on tax rates on equilibrium wages and employment.

a simulation approach, no generality can be claimed.

The effects of higher taxes, either income or consumption taxes, on the market-wage equilibrium outcomes are explained by the work incentives. The higher are taxes, the less rewarding is working: with higher income taxes the net wage is lower, with higher consumption taxes consumption prices are higher. In both cases the higher transfers compensate roughly for the income effects, so only the substitution effect, leisure becoming cheaper in relation to consumption, remains. More leisure is consumed, less work is done.

The voting equilibrium outcomes are not so straightforward. There is also a clear difference between labour taxes and consumption taxes: with the former, higher taxes mean lower wages and higher employment, with the latter slightly higher wages and correspondingly lower employment. To understand why, we must go through the median voter's choices in some detail.

Table 3. Market-wage and voting-wage outcomes with different taxes.

a) labour taxes (τ^w)

Parameter		*w*	*L*	*C*	*K*	*F*	P^d	A^f	*W*	*E*	*CV*
$\tau^w = 20$	*MW*	1.000	1.010	1.015	1.009	1.009	1.000	4.655	4.598	–6.080	0.420
	VW	1.024	0.849	0.866	0.854	0.852	1.017	5.940	8.480	–6.062	–3.480
$\tau^w = 25$	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
	VW	1.022	0.856	0.867	0.861	0.859	1.016	1.030	3.355	0.000	–3.729
$\tau^w = 30$	*MW*	1.001	0.989	0.984	0.990	0.990	1.000	–4.660	–4.586	6.150	–0.465
	VW	1.020	0.863	0.868	0.867	0.865	1.014	–3.864	–1.744	6.136	–4.019
$\tau^w = 35$	*MW*	1.002	0.978	0.967	0.978	0.978	1.001	–9.327	–9.160	12.372	–0.987
	VW	1.018	0.871	0.870	0.875	0.873	1.013	–8.746	–6.845	12.349	–4.283
$\tau^w = 40$	*MW*	1.003	0.965	0.949	0.965	0.965	1.002	–14.003	–13.720	18.666	–1.583
	VW	1.016	0.877	0.870	0.881	0.880	1.011	–13.611	–11.912	18.638	–4.611

b) VAT (τ^c)

Parameter		*w*	*L*	*C*	*K*	*F*	P^d	A^f	*W*	*E*	*CV*
$\tau^c = 0$	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
	VW	1.022	0.856	0.867	0.861	0.859	1.016	1.030	3.355	0.000	–3.729
$\tau^c = 10$	*MW*	1.002	0.984	0.984	0.984	0.984	1.002	–0.427	–0.191	10.000	–0.035
	VW	1.022	0.853	0.864	0.858	0.856	1.016	0.606	2.970	10.000	–3.923
$\tau^c = 20$	*MW*	1.004	0.968	0.969	0.969	0.969	1.003	–0.831	–0.367	20.000	–0.167
	VW	1.022	0.850	0.861	0.855	0.853	1.016	0.191	2.600	20.000	–4.143
$\tau^c = 30$	*MW*	1.006	0.953	0.955	0.955	0.954	1.005	–1.214	–0.531	30.000	–0.376
	VW	1.023	0.849	0.859	0.854	0.852	1.016	–0.228	2.202	30.000	–4.284
$\tau^c = 40$	*MW*	1.008	0.939	0.942	0.941	0.941	1.006	–1.580	–0.684	40.000	–0.645
	VW	1.023	0.845	0.855	0.850	0.848	1.017	–0.629	1.853	40.000	–4.524

Notes: The figures express ratios of market wage (*MW*) and voting equilibrium (*VW*) values to market-wage equilibrium base values (second row in (a), top row in (b)). Net foreign assets and total household wealth are expressed in relation to private production, and the figures are percentage point differences between voting and market-wage equilibria and the base values. Transfers *E* are expressed as a percentage of total consumption expenditure (excluding VAT). For the other variables see Table 1.

The median voter trades consumption for leisure, as noted in Section 4.4. The wage increase in the first period leads to lower employment during several periods. The higher the labour income tax rate is, the more unfavourable is this trade-off for the median voter: a small increase in the net wage produces a large reduction in employment, which is affected by the gross wage. Thus the reduction in lifetime gross incomes is bigger. Net incomes also fall substantially and consumption need be reduced accordingly. Thus the median voter tends to settle for lower gross wages if income taxes are higher. Because the higher income tax rate also means higher transfers, we should talk about "the effects of an increase in the marginal tax rate".

Higher consumption taxes do not affect the fall in lifetime incomes resulting from an increase in the wage in the first period. But this fall in income will result in a decline in consumption which is smaller in volume, because the price of consumption is higher with the higher VAT. This makes the overall trade-off between leisure and consumption slightly better for the median voter, and he claims higher wages. Remember that higher consumption tax revenues are paid back to the households in the form of transfers, so this is a "marginal VAT rate effect".

Taxation can, in principle, have similar effects on wage formation as pensions did in Section 5.2, as different taxes may be targeted at different phases of the life cycle. For instance, taxing pensions more heavily than wages may make the median voter assign less weight to future income and thus claim higher current wag-

Table 4. Sensitivity of pension and tax effects to key parameter values.

a) $\beta = 0.6$

Parameter		w	L	C	K	F	P^d	A^f	W	E
$\beta = 0.6$	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
	VW	1.055	0.707	0.732	0.718	0.715	1.035	–1.293	4.743	0.000
$\tau^w = 40$	*MW*	1.003	0.967	0.953	0.968	0.968	1.002	–10.278	–9.917	18.661
	VW	1.035	0.792	0.796	0.800	0.798	1.022	–11.198	–7.374	18.611
$\tau^c = 40$	*MW*	1.009	0.944	0.947	0.946	0.946	1.006	–1.440	–0.473	40.000
	VW	1.050	0.728	0.750	0.738	0.735	1.032	–1.944	3.569	40.000
$\theta = 60$,	*MW*	0.722	0.992	0.916	0.990	0.991	0.996	–41.963	–42.658	0.000
$\lambda = 0$	*VW*	0.749	0.788	0.745	0.795	0.793	1.019	–43.992	–40.730	0.000
$\theta = 60$,	*MW*	0.722	0.992	0.916	0.990	0.991	0.996	–41.963	–42.658	0.000
$\lambda = 1$	*VW*	0.729	0.933	0.867	0.934	0.933	1.002	–42.340	–41.982	0.000

b) $\rho = 0.375$

Parameter		w	L	C	K	F	P^d	A^f	W	E
$\rho = 0.375$	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
	VW	1.004	0.974	0.975	0.975	0.974	1.003	–0.607	–0.236	0.000
$\tau^w = 40$	*MW*	0.998	0.994	0.975	0.993	0.994	0.999	–14.426	–14.591	18.505
	VW	1.000	0.979	0.961	0.979	0.979	1.000	–14.773	–14.729	18.511
$\tau^c = 40$	*MW*	1.002	0.988	0.989	0.988	0.988	1.001	0.116	0.298	40.000
	VW	1.004	0.974	0.976	0.975	0.975	1.003	–0.250	0.126	40.000
$\theta = 60$,	*MW*	0.720	1.002	0.923	0.999	1.000	0.993	–58.010	–59.034	0.000
$\lambda = 0$	*VW*	0.721	0.988	0.911	0.986	0.987	0.994	–58.407	–59.241	0.000
$\theta = 60$,	*MW*	0.720	1.002	0.923	0.999	1.000	0.993	–58.010	–59.034	0.000
$\lambda = 1$	*VW*	0.720	0.999	0.921	0.996	0.997	0.993	–58.106	–59.087	0.000

c) $\sigma^E = -5$

Parameter		w	L	C	K	F	P^d	A^f	W	E
$\sigma^E = -5$	*MW*	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
	VW	1.052	1.026	0.852	0.840	0.836	1.038	–0.168	5.532	0.000
$\tau^w = 40$	*MW*	1.005	1.003	0.950	0.966	0.965	1.004	–14.523	–13.949	18.667
	VW	1.037	1.019	0.860	0.866	0.863	1.027	–15.097	–10.983	18.633
$\tau^c = 40$	*MW*	1.017	1.008	0.945	0.943	0.941	1.012	–2.063	–0.242	40.000
	VW	1.052	1.026	0.847	0.837	0.832	1.038	–1.874	3.894	40.000
$\theta = 60$,	*MW*	0.716	0.992	0.911	0.987	0.989	0.989	–58.875	–60.619	0.000
$\lambda = 0$	*VW*	0.740	1.009	0.819	0.879	0.877	1.013	–60.758	–58.784	0.000
$\theta = 60$,	*MW*	0.716	0.992	0.911	0.987	0.989	0.989	–58.875	–60.619	0.000
$\lambda = 0$	*VW*	0.721	0.996	0.890	0.962	0.963	0.994	–59.221	–60.143	0.000

Notes: The figures express ratios of market wage (*MW*) and voting equilibrium (*VW*) values to market-wage equilibrium base values (top row). Net foreign assets and total household wealth are expressed in relation to private production, and the figures are percentage point differences between voting and market-wage equilibria and the base values. Transfers *E* are expressed as a percentage of total consumption expenditure (excluding VAT). For the other variables see Table 1.

es. This remains an area of future research. So does progressivity: Simulations (not reported here) show that progressive taxation has similar effects for voting-equilibrium wages and employment in this model to those reported by Koskela and Vilmunen (1996) for most bargaining-type models of wage formation and employment. Increasing progressivity, while keeping the total amount collected by the tax constant, makes the trade union choose lower wages and higher employment. It is not directly possible to compare the effects of progressivity between market-wage equilibria and voting-wage equilibria, because the employment division rule blurs the picture. Progressivity has effects only if there is redistribution between persons, or between periods in a person's life-cycle, which requires differences in period earnings. In market-wage equilibria this leads to a varying supply of labour either between periods or between persons, and the equal division of labour rule that is employed in the voting-wage equilibria causes itself differences between these two regimes, which intertwine with the effects of progressivity.

7. Conclusions

We have studied majority-voting wage-setting in an overlapping generations economy, using a numerical simulation model. Wages are set by a nationwide labour union where all persons in working age are members. Employment is determined by firms, and is divided equally among all workers. Households take employment as given, and determine their consumption and saving by maximizing lifetime utility under perfect foresight. Although employment is given and thus leisure also, leisure is an argument in the utility function.

If there were no trade union, each household would make its own decisions concerning leisure and consumption. Here the trade union makes the leisure-consumption choice for the households. Furthermore, its decision affects the terms of the choice, the trade-off that there is between leisure and consumption in each period. The current trade-off is affected by the previous choices, and the current choice will affect the trade-off of the current and future periods. There is a dynamic element: by increasing the wage, the trade-off is better for the union today but worse tomorrow. The combined effect is what counts.

The economy has an equilibrium wage level, which the majority of workers wants neither to increase nor decrease. We compare this voting equilibrium outcome to the full market-clearing outcome where wages adjust to equate the supply of and demand for labour. The voting outcome has higher wages and lower employment than the market-clearing outcome. How big this difference is depends on many features of the economy. The production technology has effects, especially the substitutability between labour and capital. Individual preferences concerning the substitution between leisure and consumption are important; one advantage of the median voter approach is that household preferences directly affect the trade union's decisions. The openness of the economy also affects wage outcomes: the easier it is to replace domestic products with foreign goods in consumption, investment, intermediate use or in export markets, the less room the trade union has to operate. Even if the interest rate is not affected by the wage decisions, there is still an openness issue in financial markets: the more foreigners own domestic shares, the less the trade union cares about the adverse share-value effects of wage increases.

Pension policies and tax policies have both saving incentive and work incentive effects. Without trade unions these incentives affect household behaviour. With a trade union the work incentives affect its decisions, while saving incentives still operate through households. The central incentive result of this study is that when we compare trade union wage setting with labour markets where wages adjust to equate supply and demand, the difference is bigger when the tax rates are lower, and gets smaller when taxes increase. When e.g. the benefits of a PAYG-type pension system are increased, and the corresponding payroll tax is also increased, the voting equilibrium wage level approaches the market equilibrium wage. Similar results are obtained with respect to labour and consumption taxes.

If wages are set by voting, the resulting level of employment is higher, the higher is the earnings-related PAYG pension benefit level. The reason is that if the voter claims high current wages his lifetime wage income falls, which will lead to lower pensions, and the advantage of lower pension contributions goes to future working generations, not the current median voter. Also, the median voter has to pay higher contributions both because the current wage bill falls and because current pensions may increase due to indexation. Both these generational transfer effects lead the median voter to choose lower wages, which leads to higher employment. This example shows that incentive effects of e.g. pension policies can be drastically different in a unionised economy from the effects in an economy with non-union labour markets. In this study, institutions do matter.

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Appendix 1: How to find a steady-state voting equilibrium wage level

A steady-state voting equilibrium is found by trial and error. Starting from a stationary fixed-wage equilibrium, we increase the first-period wage slightly, find the new dynamic solution to the economy, calculate the compensated variations and see whether the majority gains or loses. If the majority gains, we calculate a new fixed-wage equilibrium with a higher wage, repeat the dynamic exercise and calculate the compensated variations. After some initial wage level the majority usually loses from a wage increase; then we have passed the voting equilibrium point. The exact point when the gains become losses also depends on the size of the first-period wage increase. Ideally we would like to make an infinitesimal change to the wage.

Figure 3 describes the outcome of a series of wage increase trials. Wages in the initial fixed-wage equilibrium are on the horizontal axis. They are related to the market-wage equilibrium outcome. The gains decline smoothly, and once wages are about three percent above the market-wage equilibrium level, the median voter suffers from a one-period wage increase.

A practical proof of the existence of a voting equilibrium is that it can be found in practice by the procedure described above. Uniqueness or its absence is probably very difficult to prove. Studies such as Azariadis and Galasso (1996) clearly show that multiple equilibria are possible. Nothing, however, has pointed towards multiple equilibria in the simulation experiments.

Notice that market-wage and fixed-wage equilibria can be found using only a steady-state model, but to find the voting equilibrium among the fixed-wage equilibria one needs a dynamic model.

Compensated variation as a percentage of discounted consumption expenditure during remaining lifetime

Figure 3. Relative gain from 0.5 percent one-period wage increase in age group 35–40

Appendix 2: The model

Firms

A representative small firm produces the domestic good using capital inherited from the previous period, intermediate goods and labour. Infinite horizon decisions of investment, employment and use of intermediate goods are made to maximise the firm's market value. The firm takes the prices, demand of production and supply of factors at given prices, and production technology and taxation as given. Intermediate and capital goods are costs minimising CES composites of domestic and imported goods. Investments are financed by retained earnings and debt.

The formulation of the production structure follows Keuschnigg and Kohler (1994). The structure applied in this study is essentially a one-sector version of a model intended for multisector use. The structure can be described as follows:

(A 1) $$F_t = A\left\{\varepsilon K_{t-1}^{(1-1/\beta)} + (1-\varepsilon)L_t^{(1-1/\beta)}\right\}^{\frac{\beta}{\beta-1}}$$

(A 2) $$G(I_t, K_{t-1}) = \xi\frac{I_t^2}{K_{t-1}}$$

(A 3) $$Y_t = \frac{F(K_{t-1}, L_t) - G(I_t, K_{t-1})}{1-\zeta}$$

The value-added production function F is a CES function of capital and labour. In the process of installing new capital some production is lost due to investment adjustment costs G. These installation costs depend positively on investments and negatively on the amount of capital. The use of the composite intermediate good is determined as a fixed proportion ζ of gross production Y.

Domestic households consider bonds and firms' shares as perfect substitutes in their portfolios. The arbitrage condition between (after-tax[6]) returns on bonds and shares is:

(A 4) $$r_{t-1}V_{t-1} = D_t + V_t - V_{t-1}$$

where the left-hand side describes the invested amount yielding the domestic interest rate. On the right-hand side, the first term is the dividend income and the second term the capital gain.

The arbitrage condition can be transformed to a form where the market value of the shares equals the present value of expected future dividends:

(A 5) $$V_t = \sum_{s=t+1}^{\infty} D_s \prod_{v=t+1}^{s} \frac{1}{1+r_v}$$

The dividends are a residual from the firm's cash flow identity:

(A 6) $$D_t = P_t^F(F_t - G_t) - (1+\tau_t^z)w_t L_t - r_{t-1}B_{t-1}^f - P_t^K I_t + B_t^f - B_{t-1}^f$$

where the dividend in period t is determined by profits before depreciation minus investment expenditure plus any increase in corporate debt. Corporate debt is preferred when financing investments, but its use is limited to a fixed ratio of the replacement value of corporate capital.

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[6] *All corporate and capital income tax rates in the model have been set to zero and removed from the equations. Valkonen (1995, 1997) has used the same model for tax reform analysis.*

The firm chooses the optimal path of investment, use of labour and intermediate goods to maximise the current period dividend and the firm's value at the end of the period. If there are no unexpected shocks, there is no need to revise the optimal plan and it will be followed forever. Capital depreciates at a constant annual rate of *d*. The constraints of the maximisation are the initial capital stock and an equation describing its dynamics:

(A 7) $$K_t = K_{t-1}(1-d) + I_t$$

Three of the four first order conditions of the constrained optimisation are used as model equations. The first implies that investments should be carried out until the marginal benefit from an additional unit of investment equals the marginal cost. The marginal cost includes the price of a unit of capital plus the installation cost. The condition can be transformed to a q-theory investment equation (A 8). The optimality condition of capital says that capital should be installed until the return of an additional unit is large enough to cover the expenses of carrying the capital to the next period. These expenses include interest, depreciation and the change in the replacement price of capital. This condition is transformed to an equation (A 9) describing the path of the shadow value of capital. In a steady state this marginal productivity condition of capital can be written as (A 10). The terms within the brackets are the depreciation rate *d* and the interest cost of the capital stock. The two terms to the right of the brackets are based on adjustment costs linked to replacement investments.

(A 8) $$I_t = \frac{(q_t - P_t^K)K_{t-1}}{2\xi p_t^F}$$

(A 9) $$q_t = (P_{t+1}^F(F_K - G_K) + q_{t+1}(1-d))\frac{1}{1+r_t}$$

(A 10) $$F_K - G_K = \frac{P^K}{P^F}[d+r] + r\xi d + \xi d^2$$

The third condition says that the marginal benefit of an extra unit of labour should cover wage costs plus the employer's pension contribution:

(A 11) $$P_t^F F_L = (1 + \tau_t^z)w_t$$

The fourth condition is a transversality condition ensuring that the discounted shadow value of capital goes to zero as time approaches infinity.

The market value of the firm is linked to the shadow value of capital in the leveraged firm as follows:

(A 12) $$V_t = K_t q_t - B_t^f$$

where B_t^f is the firm's debt. This link has been derived using the homogeneity of production and capital installation technologies. The value of the firm jumps whenever unexpected news about the firm's future profitability enters the market. Domestic households own a part $(1–s^F)$ of the firms. When the value of the firms jumps, and changes households' wealth, they reoptimise their life-cycle plans immediately.

The numerical values of the firm parameters applied in this study are the following: $\beta = 0.7$, $\varepsilon = 0.36$, $\xi = 6$, $\zeta = 0.1$, $d = 0.5$, $s^F = 0.333$. The household parameter values are: $\gamma = 0.5$, $\rho = 0.75$, $\delta = 0.01$, $\alpha_0 = 0.8$.

Government sector

The government collects income taxes from wages and from pensions Z, and consumption taxes, and uses the proceeds to pay interest on outstanding debt and to employ civil servants to produce public services. These services are provided free of charge and are not taken into account in individual utility considerations.

$$\text{(A 13)} \quad E_t + L_t^G w_t(1+\tau_t^z) + r_{t-1}B^G = (L_t w_t + Z_t)\tau_t^w + C_t P_t^C \tau^c$$

In the steady states, transfers are used to balance revenues and expenditures of the government every period. The share of public employment is 0.25 of the total employment in steady states. In dynamic simulations describing the median voter's calculations public employment is held constant throughout, the government holds all tax rates constant in the first period, and runs a deficit, and in the following periods balances the budget and freezes the public debt using transfers. Public debt in steady states equals zero.

Foreign sector

The model imitates a small open economy, where the export share of total demand is large. The amount exported depends on the price elasticity of foreign demand:

$$\text{(A 14)} \quad X_t = x\left(\frac{P_t^d}{P_t^M}\right)^{\sigma^E}$$

A large negative value for the elasticity implies that a small country has to adjust to the price level of international markets. The basic parameter values are: $x = 0.6$, $\sigma^E = -10$.

The imported good is used in consumption, investments and as an intermediate good in production. Its price is determined in international markets. It is an imperfect substitute for the domestically-produced good. The demand conditions are described by a CES structure.

Markets

The model includes four markets, which balance every period. The two labour market specifications have been described in the text. Total employment equals the sum of private employment and public employment (Equation A 15). In the domestic goods market, firms are the sole supplier. The product is used by other firms as a part of the composite intermediate and investment goods, by households as a part of the composite consumption good and by foreign agents. The demand of domestic agents is determined by a cost minimising CES structure. The equilibrium condition which determines the price of the domestic good is Equation A 16. Domestic demand for the fixed-price imported good is also determined by minimising costs of the composite goods (the price of the imported good serves as a numeraire in the model). The perfectly elastic supply adjusts to demand in these markets (Equation A 17). The fourth market is the capital market. In this market, savings and investment are balanced. The arbitrage condition of domestic households ensures that they are ex ante indifferent between investing their savings in bonds and in firms' shares. Total saving is the sum of domestic savings and foreign portfolio investments. Equation A 18 describes the parallel stock equilibrium.

$$\text{(A 15)} \quad L_t^T = L_t + L^G$$

(A 16) $Y_t = \zeta Y_t v_t^d + I_t i_t^d + C_t c_t^d + X_t$

(A 17) $M_t = \zeta Y_t v_t^M + I_t i_t^M + C_t c_t^M$

(A 18) $W_t = V_t + B_t^f + B_t^G + A_t^f$

where the unit demands are

(A 19) $v_t^M = \left(\frac{mP_t^C}{P_t^M}\right)^{\sigma^M} = i_t^M = c_t^M, \quad v_t^d = \left(\frac{(1-m)P_t^C}{P_t^d}\right)^{\sigma^M} = i_t^d = c_t^d$

The price of the domestic good P^d is endogenous and the price of the imported good serves as the numeraire in the model. Other prices are linked to them according to the following equations. The model facilitates the use of different share parameters m and price elasticities σ for consumption, investment and intermediate goods, but in this study we use the common values $m = 0.3$ and $\sigma^M = 0.99$ for all goods, which explains Equations (A19) and (A 22). P^H is the price of the intermediate good.

(A 20) $P_t^F = (P_t^d - \zeta P_t^H)/(1-\zeta)$

(A 21) $P_t^C = \left[m^{\sigma^M}(P_t^M)^{1-\sigma^M} + (1-m)^{\sigma^M}(P_t^d)^{1-\sigma^M}\right]^{1/(1-\sigma^M)}$

(A 22) $P_t^C = P_t^K = P_t^H$