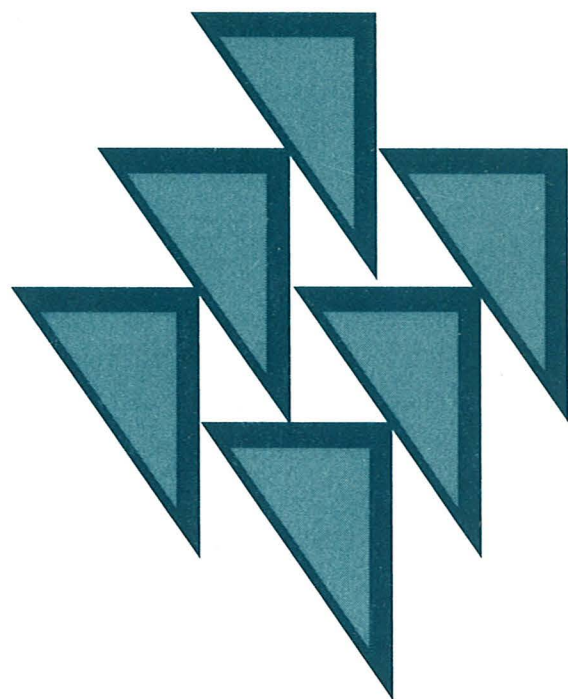


Juha Kettunen

Re-employment of
Finnish Unemployed Workers



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FINNISH UNEMPLOYED WORKERS

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ABSTRACT: The aim of this study has been to investigate the process of re-employment of Finnish unemployed persons using both search theoretical and microeconomic approaches. For the econometric analysis a sample of 2077 unemployed workers was drawn from the register of the Ministry of Labour. According to the results a higher re-employment probability can be achieved by paying stingy benefits during the spell of unemployment, but on the other hand the loss in the welfare of the workers can be offset by paying generous benefits to those finding jobs. The use of a waiting period does not substantially increase the re-employment probability. On the other hand, the incentive towards re-employment can be effectively increased by removing the protective rules of regional and occupational mobility and reducing benefits after a permitted period of higher benefits. The effects of an unemployed person's education on the duration of unemployment are not straightforward. It was found that the level of education is positively related to the re-employment probability for relatively low levels of education, but in the higher levels the relationship turns negative.

KEY WORDS: Unemployment duration, re-employment, unemployment insurance, education.

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TIIVISTELMÄ: Tässä tutkimuksessa tarkastellaan työttömien työllistymistä sekä etsintäteoreettista että ekonometristä analyysiä käyttäen. Ekonometrista tutkimusta varten muodostettiin työministeriön työnhakijarekisteristä 2077 työttömän otos. Tulosten mukaan työllistymistä voidaan edistää maksamalla niukempia työttömyyden aikaisia etuuksia, mutta toisaalta työttömän tulonmuodostusta ja työllistymistä voidaan edistää maksamalla korkeampia etuuksia työllistyneille henkilöille. Työttömyyspäivärahojen omavastuuajalla on vain hyvin pieni positiivinen vaikutus työllistymiseen, mutta alueellista ja ammatillista liikkuvuutta koskevan suojan poistaminen ja päivärahojen alenemien käyttö edistävät tehokkaammin työllistymistä. Peruskoulutuksen vaikutus työllistymiseen ei ole suoraviivainen. Koulutustason noustessa työllistymisen todennäköisyys nousee, mutta korkeimmin koulutetuilla on työllistymisongelmia.

AVAINSANAT: Työttömyyden kesto, työllistyminen, työttömyysturva, koulutus.

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The basis of this work was, however, laid down in 1986 - 1988 while I was completing my licentiates degree at the University of Helsinki. I am grateful to Professor Erkki Koskela and Dr. Heikki Loikkanen from the University of Helsinki for introducing me to this area of interest.

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The results of this thesis have been presented at several economic conferences including among others: The Conference on Longitudinal Data Analysis, Jyväskylä; The Symposium on Mass Unemployment in Finland, Joensuu; International Workshop on Labour Economics, Sannäs; Fourth Conference on Panel Data, Budapest; The Seventh Annual Congress of European Economic Association, Dublin; and The Fifth Annual Conference of European Association of Labour Economists, Maastricht. I am grateful to the participants of the conferences. Furthermore I have benefited from stimulating discussions with Kari Alho, Seija and Pekka Ilmakunnas, Reija Lilja, Matti Pohjola and Pentti Vartia.

I have also benefited from inspiring debate in 1991 - 1992 while working as an advisor for the committee of unemployment insurance set by the Ministry of Labour. Thanks are also due to John Rogers who carefully checked the language of the study.

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Juha Kettunen

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Chapter I

INTRODUCTION

This is a study of the re-employment of Finnish unemployed workers based on search theoretical and microeconomic foundations. The search approach of labour economics provides a productive interaction between economic theory and applied econometric work, because it explicitly incorporates uncertainty about the economic environment. Often economic theories and econometric work consider the phenomenon of re-employment from two different points of view, but in this case the economic theory and empirical research blend quite smoothly.

Search theories are based on the assumption of incomplete information, which spurs economic agents to initiate a search. The first papers in the area of search theory were the two seminal papers by Stigler (1961, 1962) with applications to buyer's optimal choice behaviour. However, search theory was mostly developed in the job search context, which is shown by the book of Phelps (1970) and surveys of Rothschild (1973) and Lippman and McCall (1976a,b). A number of contributions on search theory can be found, for instance, in Lippman and McCall (1979) and the latest contributions in Mortensen (1986) and Kiefer and Neumann (1989).

A considerable number of microeconomic studies concerning re-employment have been done after the

pioneering studies by Fowler (1968) and Kaitz (1970).

Perhaps the most notable of these studies are the ones by Lancaster (1979) and Nickell (1979a,b), which are followed by Kooreman and Ridder (1983), Narendranathan, Nickell and Stern (1985), Ham and Rea (1987), Engström and Löfgren (1987) among others. Nowadays there is a wide range of textbooks concerning this area, for example Gross and Clark (1975), Kalbfleisch and Prentice (1980), Lee (1980), Miller, Gong and Muños (1981), Lawless (1982), Cox and Oakes (1984) and Lancaster (1990).

In Finland there have been few studies based on individual data using adequate microeconomic methods. Sääski (1981) studied the duration of unemployment using ordinary least squares. The method is imprecise in this case, since the models can give negative fitted values for the durations of unemployment. However, the results indicate a positive relationship between the duration of unemployment and the receipt of unemployment benefits using dummy variables. Solttila (1983) studied factors affecting unemployment using logit models. However, the data did not include unemployment benefits. Eriksson (1985) studied models of unemployment duration using complete spells of unemployment using more than 500 individuals in the district of Turku. Eriksson found a statistically significant and positive relationship between benefit dummies and unemployment duration. The elasticities of unemployment duration with respect to the receipt of the earnings-related allowance were between 0.60 - 0.75 and with respect to the basic unemployment allowance between 0.18 - 0.33 in the estimated models. The studies of

Kettunen (1989, 1990a) are the first attempts in Finland to analyze the effects of unemployment benefit on the functioning of the labour market using the levels of unemployment benefits. Recently Lilja (1992) has studied the unemployment duration in Finland using semi-parametric discrete-time hazards. She used data from the Labour Force Surveys compiled by the Central Statistical Office combined with the data from the tax register. However, the number of observations is rather low bearing in mind the low efficiency of the semi-parametric procedure and wide intervals of the grouped data on unemployment durations.

The aim and outline of this study is as follows. Chapter II gives a brief description of the Finnish unemployment insurance (UI) system and microeconomic data collected for the econometric analysis.¹⁾ The Finnish system is studied after the reform at the beginning of 1985, when the level of unemployment benefits increased and the benefits became taxable. Unemployed persons can differ greatly from each other with respect to the level of unemployment benefits and durations of unemployment. The interest of this study focuses on the effects of the UI benefits on the re-employment and labour mobility. A microeconomic data set was collected for the study. The data of 2077 unemployed persons are fairly rich on individual and labour market specific characteristics. The sample has been taken from the persons becoming unemployed in 1985. Every hundredth individual was picked from the flow into unemployment. The individuals were then followed until the end of 1986. For the initial view of the data,

descriptive statistics and life table methods were used to describe the process of becoming employed again.

Chapter III describes the search theoretical background of the econometric work in the subsequent chapters.²⁾ A search theoretical model allowing for regional and occupational mobility of unemployed workers is presented and its properties are analyzed. A well-known result of search models is that the UI benefits have a disincentive effect on leaving unemployment. However, it is pointed out that giving conditional benefits to persons who become employed increases the re-employment probability. From the policy point of view it is partly a matter of whether the government wants to subsidize unemployment or employment. However, the issue is not so dichotomous, since a government of a welfare society wants to secure the basic income of its members. Three features of the UI system are analyzed. It is shown that the waiting period of UI benefits has only a slightly positive effect on the re-employment probability. On the other hand, it is shown that removing the protective rules regarding labour mobility and reducing benefits of the unemployed persons with long durations of unemployment substantially increase the re-employment probability.

Chapter IV deals with the econometric models of unemployment duration based on individual data.³⁾ Particular attention is given to the matter of unobserved explanatory variables, since in econometric models all the relevant variables may not be included or their importance can not even be suspected. Neglected heterogeneity may bias the parameter estimates. Therefore methods of correcting

the models are studied and methods for examining and testing the model specification are developed. A model of unemployment duration with a discrete mixing distribution is found to be well defined. The effects of explanatory variables on the duration of unemployment are illustrated using the estimated model. It is shown that the UI benefits have a large negative effect on the duration of unemployment.

A programme for estimating nonlinear maximum likelihood models with an application to a Weibull model allowing for gamma heterogeneity is reported. It is a modification of the programme used by Chesher (1986) and rewritten by Kettunen (1991d). It provides also a framework for developing specification tests. The modifications of the programme have been used for the maximum likelihood estimations of this thesis.

Chapter V studies the time-dependent effects of unemployment benefits.⁴⁾ The circumstances of unemployed persons do not usually stay constant over the unemployment spell. In the Finnish system persons who are eligible for the benefits risk losing them after the first three months. Another reason is that the earnings-related unemployment allowances decrease 20 per cent after the first 100 days unemployment. The data set includes the time series of the UI benefits during the unemployment spells for the unemployed persons. Therefore the interest is in testing and estimating the time-dependent effects of time-dependent benefits on the re-employment probability. It turns out that the effect of UI benefits is not constant during the unemployment spell. The replacement ratio has a negative

effect on the re-employment probability during the first three months. After that period the effect vanishes, since the benefits decrease and eligibility rules become stricter.

Chapter VI studies the effects of education on the duration of unemployment⁵⁾. The effects of education are explained by a search theoretical model. In the model education increases the arrival rate of job offers and shifts the offer distribution so that the more educated persons will get better offers. On the other hand, education will increase costs of re-employment. The costs of re-employment increase the reservation utility of the persons implying that there are fewer acceptable offers available. Hence the effect of education turns negative. Using Finnish data on unemployment durations it is noticed that an increasing level of education implies an increasing re-employment probability. However, the relationship turns negative on the highest levels of education. Unemployed persons who have about 13 - 14 years of education have the highest re-employment probability.

Chapter VII includes the inference based on the semi-parametric models of unemployment duration.⁶⁾ Cox's models with duration and calendar-dependent covariates are estimated. To model the macroeconomic seasonal effects the duration of unemployment is replaced by calendar time. Baseline hazard functions are used to illustrate the effects of the UI system and the seasonal effects on the hazard function. It is shown that the risk of losing benefits after the first three months and the reductions of

earnings-related unemployment benefits substantially increase the re-employment probability.

Chapter VIII concludes the study.⁷⁾ The results of the search theoretical and econometric work with respect to the outlines for the changes in the UI system are evaluated. The incentive towards re-employment can be increased by increasing the benefits of re-employment. The welfare of all the unemployed persons can be increased by removing the waiting period for benefits. On the other hand, removing the protective rules regarding labour mobility and returning to the old system, which included the reductions of benefits, will increase the re-employment probability. It can be concluded that the functioning of labour market can be increased by subsidizing re-employment instead of unemployment and by allowing higher benefit replacement ratios for the persons with short durations of unemployment.

Footnotes

1. This chapter is based on an article (Kettunen, 1991b). The detailed description of the data can be found in Kettunen (1991i).
2. An earlier version of this chapter has been published in Kettunen (1991e) and an article based on this study is published in Finnish Economic Papers (Kettunen, 1992a).
3. This chapter is partly based on a working paper (Kettunen, 1991c) and an article (Kettunen, 1991h) and a comment in Finnish Economic Papers (1993a).
4. This chapter is based on a working paper (Kettunen, 1991f) and an article (Kettunen, 1991g).
5. This chapter is based on a working paper (Kettunen, 1991a).
6. This chapter is based on a working paper (Kettunen, 1992b) and three articles (Kettunen, 1992c, 1993a,b). One of them (1993b) has been published in Finnish Economic Papers.
7. The results of this study have been discussed in two articles (Kettunen, 1990b and 1992d).

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Chapter II

THE EFFECTS OF UNEMPLOYMENT INSURANCE:**DESCRIPTION OF THE FINNISH SYSTEM AND DATA**

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Abstract

This chapter gives a brief description of the Finnish unemployment insurance system and microeconomic data collected for the econometric analysis of this study. The sample has been taken from the persons becoming unemployed in 1985 after the reform of the system at the beginning that year. The individuals were followed until the end of their unemployment spells but at most until the end of 1986. The data are described using descriptive statistics and life table methods.

1. Introduction

The purpose of this chapter is to describe the Finnish unemployment insurance (UI) system and data used for the econometric study of unemployment spells and the regional and occupational mobility. The Finnish system is studied after the reform at the beginning of 1985, when the level of unemployment benefits increased and the benefits became taxable. A detailed survey on the development of the Finnish unemployment insurance system can be found in Kettunen (1990).

Many studies of unemployment durations have been performed during the 1970's and 1980's. The focus of these studies has been the effects of explanatory variables on the re-employment probability using parametric models and maximum likelihood methods. The aim of this chapter is to

follow up on these studies but to use nonparametric methods. However, the interest is not only on the length of unemployment spells but on the means of becoming employed, i.e. regional and occupational mobility. The latter problem has not before got any notable interest. Attention is also paid to the persons who do not become employed.

The data set is collected from various registers and it is more reliable than the data sets based on interviews. In order to guarantee that the sample would be randomly generated and seasonally representative, every hundredth individual was sampled from the flow into unemployment during the year 1985. The sample was taken from the unemployment register of the Ministry of Labour. The individuals were then followed until the end of their unemployment periods, but at most until the end of 1986. The income and wealth information was compiled from the tax register into the data set. The information on unemployment benefits was compiled from the registers of the bank Postipankki and the Social Insurance Institution.

The remainder of this chapter is set out as follows. In the next section the main features of the Finnish UI system are discussed. Section 3 deals with the data, concepts of duration models and presents the life table estimates. Finally, section 4 concludes the study.

2. The Finnish Unemployment Insurance System

This section discusses first the macroeconomic context. It is worthwhile to take a look at the general state of the Finnish economy before, during and after the survey in order to put the overall picture of the Finnish labour market in context. After that the main features of the Finnish unemployment insurance system are discussed during the years of the study 1985 - 1986.

For an overall picture of the Finnish labour market the unemployment rate was decomposed into two parts. The level of unemployment can be expressed as a product of the inflow and duration in a stationary environment (see e.g. Leighton and Mincer, 1982). However, it is well known that all the assumptions of this identity are not completely satisfied (see Eriksson, 1985). Figure 1 illustrates the unemployment rate, weekly inflow and duration of unemployment. It can be seen that the both the inflow rate and the duration of unemployment vary with the unemployment rate. The variation of the duration of unemployment is, however, larger than the variation of the inflow. A simple variance decomposition of the unemployment rate using the method by Groshen (1991) confirms that the duration of unemployment is a more important component, because it explains 43 per cent of the variance of the unemployment rate. The inflow to unemployment explains 29 per cent of the variance and the interaction of these two variables explains 14 per cent.

The Finnish unemployment insurance system is a product of increasing corporatism. The degree of unionization in

Finland has risen rapidly as in the other Nordic countries, too. The breakthrough of the unionization occurred from the middle of the 1960's to the end of 1970's. During the 1980's the rate of unionization in Finland was around 85 per cent. The wage settlements in Finland during the last few decades have been most often one or two-year central agreements. The central agreements have often included the most notable improvements of the unemployment insurance. The measures favourable to unemployed persons have included, for example, the enlargement of the number of recipients of benefits, increases in benefits and the payment period.

There have been two notable reforms of the unemployment insurance since the 1960's. At the turn of the 1970's the number of recipients of the basic unemployment allowance was extended. The reform affected mostly females and young persons. The other reform took place in 1985 when the levels of benefits were increased.

Figure 2 illustrates the replacement ratios of unemployment benefits and the supply of and demand for labour. Two time-series of replacement ratios have been calculated. The first replacement ratio is calculated for the recipients of benefits using the aggregate data by the Ministry of Labour published in Finnish Labour Review. The second replacement ratio is calculated for all the unemployed persons including the non-recipients of benefits. The latter time-series measures both the level and coverage of unemployment benefits. An average wage is used in calculating the replacement ratios. These replacement ratios give, however, only a rough and

presumably a biased estimate of the true figures. One obvious bias is that the levels of the replacement ratios are too low, because the unemployed persons have typically wages that are lower than the average. The true time-series of wage levels of the unemployed persons are not available.

The reforms at the turn of 1970's did not increase the unemployment rate at once. The main reason is that the demand for labour was increasing during the years 1969 - 1974. A notable feature of the unemployment rate is that it almost doubled starting in 1975. The demand for labour decreased and the inflow to unemployment increased sharply. The rise of the unemployment rate can be explained by the increasing inflow. The unemployment rate remained high despite the recovery beginning in 1979. The reason is that the duration of unemployment spells stayed on a higher level.

History repeated itself when the second extensive reform of the UI system was carried out in the beginning of 1985. The level of the basic unemployment allowance was increased slightly, but the earnings-related unemployment allowance was increased substantially. The proportion of workers covered by the UI system was increased. One of the new features was that the benefits obtained from the UI funds depended on the previous earnings starting in 1985 (see Vähätalo, 1988). The high demand for labour kept the unemployment rate low, but in the late 1980's the decreasing demand for labour starting in late 1990 increased remarkably both the inflow and the duration of unemployment.

Figure 1. Unemployment rate, inflow and duration of unemployment

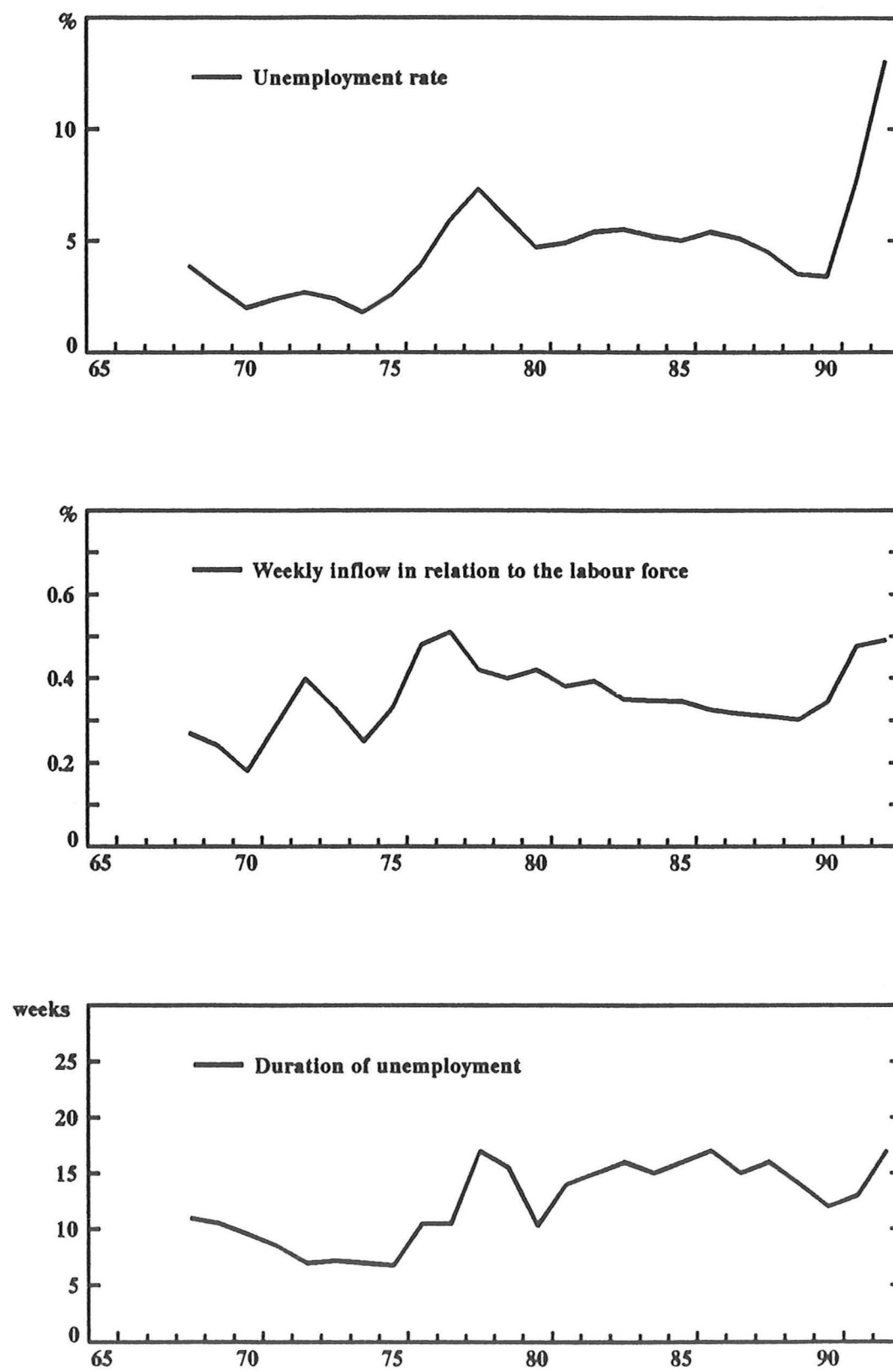
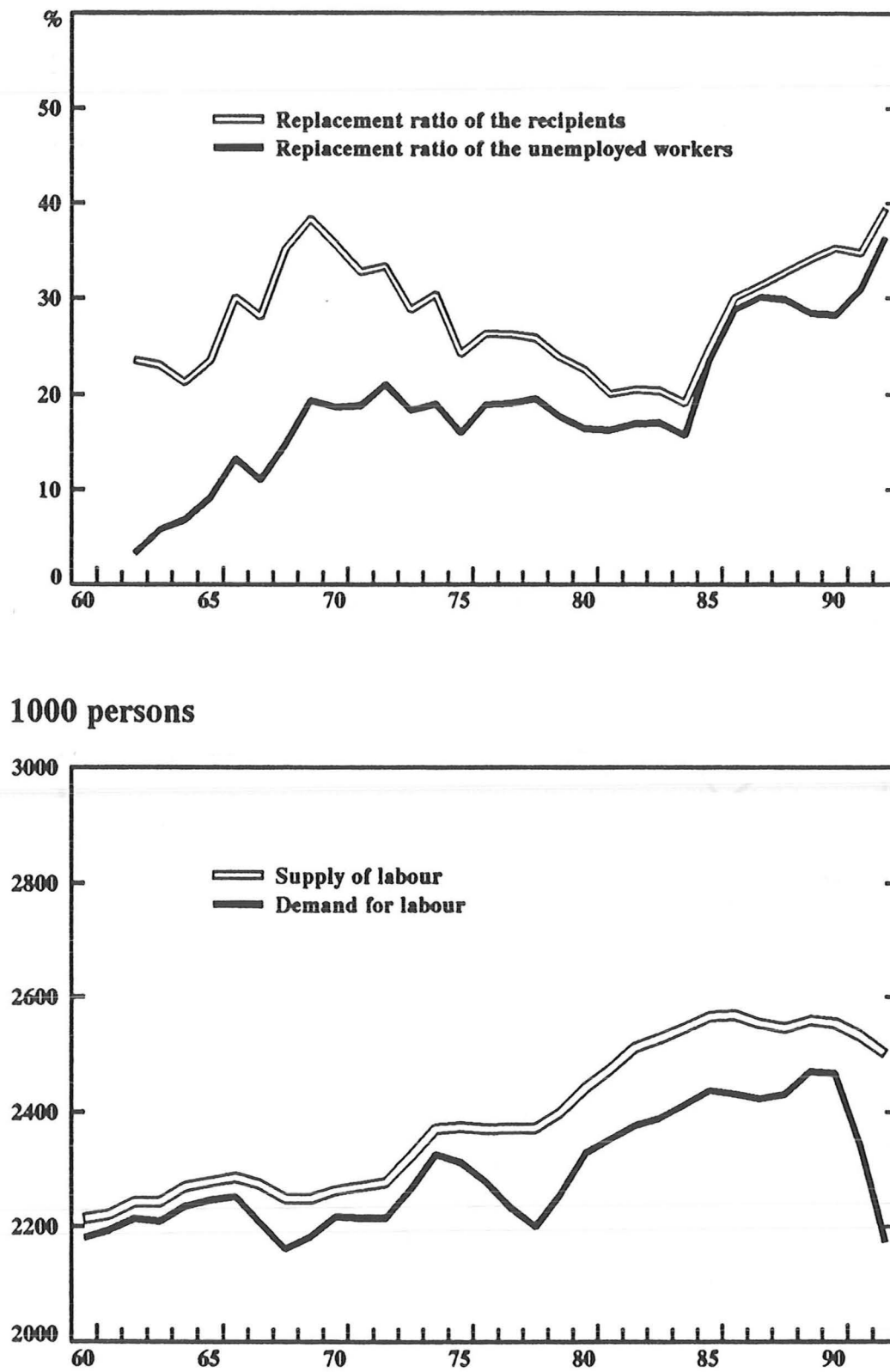


Figure 2. The replacement ratios of unemployment benefits and the supply of and demand for labour



The aim of the Finnish UI system is to achieve an efficient search pattern by means of subsidizing job search and by achieving efficient job acceptance by means of withholding benefits from reluctant workers. Unemployed persons aged 17 - 64 who register with the employment office qualify for benefits. Unemployment benefits can be paid normally after a 5-day waiting period. Persons who enter the labour market for the first time qualify for the basic unemployment allowance after a 6-week waiting period. This restriction is not applied to those who have just finished school or who have been self-employed. Workers who quit are eligible for benefits after 6 weeks.

There are two systems and therefore two kinds of unemployment benefits in use: the basic unemployment allowance and the earnings-related unemployment allowance. The basic unemployment allowance is financed wholly by the state and paid by the Social Insurance Institution. It is means tested. Persons who are in need of financial assistance are eligible for the allowance. There is no maximum unemployment period for the basic unemployment allowance. In 1985 the basic unemployment allowance was FIM 70 per day for an unemployed person whose family income during the spell of unemployment was at most FIM 3500 per month. The figures are before tax. For a single person the level was FIM 2340. If the family income was over FIM 3500, but less than FIM 5410, an unemployed person who has children was eligible for less than the maximum assistance. The child increases were FIM 15, 22 and 28 for one, two and three or more children, respectively. The dependence of the

daily basic unemployment allowance on the monthly income has been plotted in Figure 3.

The earnings-related unemployment allowance is paid by 75 unemployment insurance funds (in 1985), which are run by labour unions. It is financed by the government (48 per cent), employers (47 per cent) and employees (5 per cent). Members of the labour unions are normally also members of the unemployment insurance funds. The earnings-related unemployment allowance is paid to the unemployed persons who have been members of labour unions for at least 6 months and who have been working during that time. The earnings-related unemployment allowance was FIM 70 (the basic part of the allowance) plus 45 per cent of the difference between previous daily salary and FIM 70 (the earnings-related part of the allowance). If the monthly salary were more than FIM 6300, the corresponding per cent was 20 from the salary over FIM 6300. In Finland the average salary was about FIM 6000 per month. The earnings-related unemployment allowance, which is not means tested, can be at most 90 per cent of the salary. The child increases are as large as in the basic unemployment allowance. After the first 100 days of unemployment the allowance decreases 20 per cent, but it is in any case at least as high as the level of the basic unemployment allowance. After an unemployment period of 500 days the earnings-related unemployment allowance decreases to the basic unemployment allowance, which is means tested. The dependence of the daily earnings-related unemployment allowance on the monthly income has been plotted in Figure 4.

If there are no suitable jobs in the unemployed person's area of residence after the first three months of unemployment, the person must accept an offer outside his area of residence. Otherwise he may not be eligible for the benefits. This is the main principle in the Unemployment Insurance Act, but there are some minor exceptions to this basic rule. In practice, however, the unemployed persons do not very often get offers outside their area of residence.

Unemployed persons do not have to accept a job offer within the first three months of the unemployment period if the job is not suitable to him with respect to his education or previous work experience. This rule concerns persons with professional or vocational education and at least one year of job experience or alternatively persons without any higher education and at least two years experience in their job. A person who after being unemployed for the first three months does not accept an offer may lose his benefits. After the first three months the unemployment office tries to mediate a job from the previous occupation as far as this is possible.

Figure 5 illustrates the replacement ratios of the different types of benefits according to the Finnish Unemployment Insurance Act in 1985. The replacement ratios have been calculated before tax and they do not include child increases. For a typical monthly income of FIM 6000 the replacement ratios with the full child increase vary between 0.35 - 0.69.

Figure 3. The basic unemployment allowance for an unemployed single person and for a family with two children

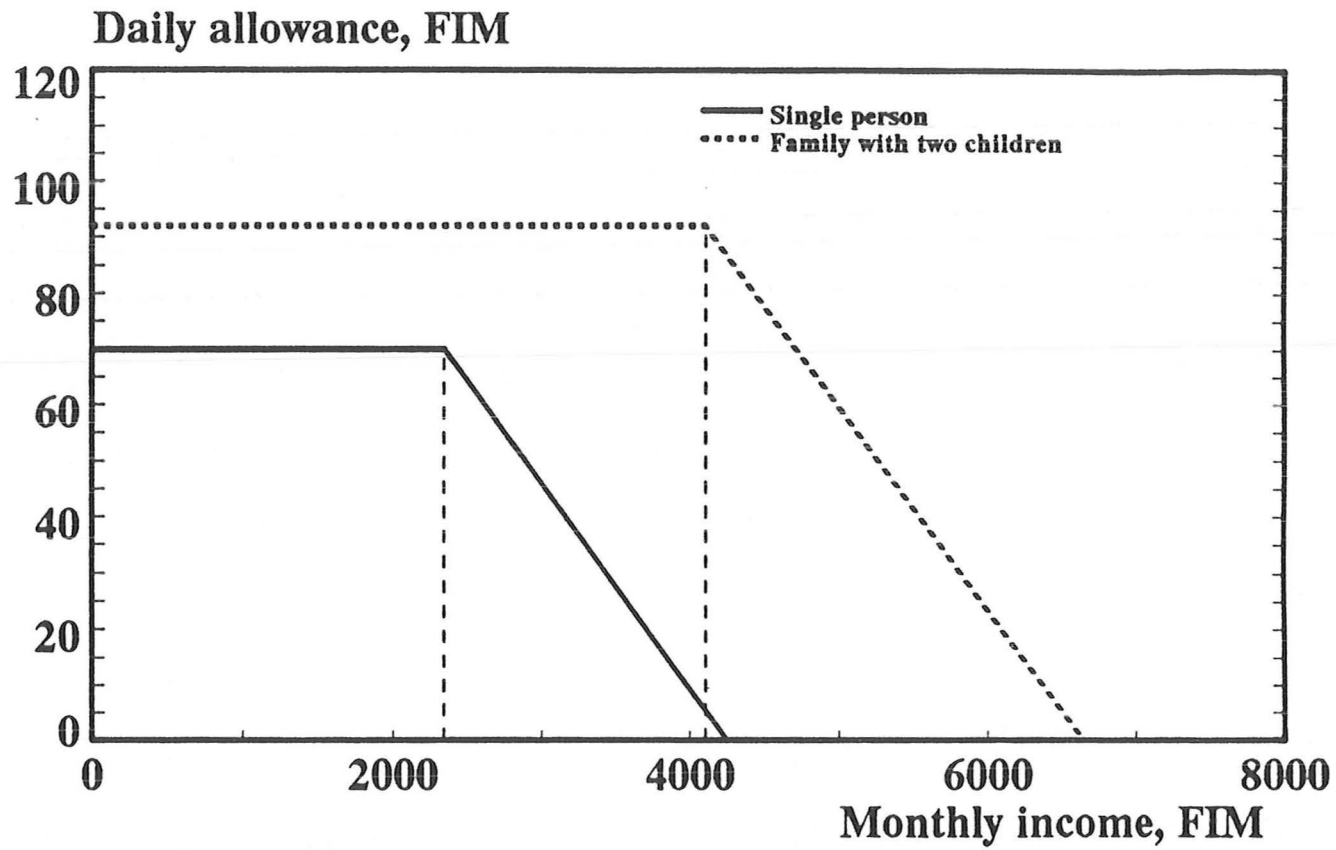


Figure 4. The earnings related unemployment allowance for an unemployed single person and for a family with two children

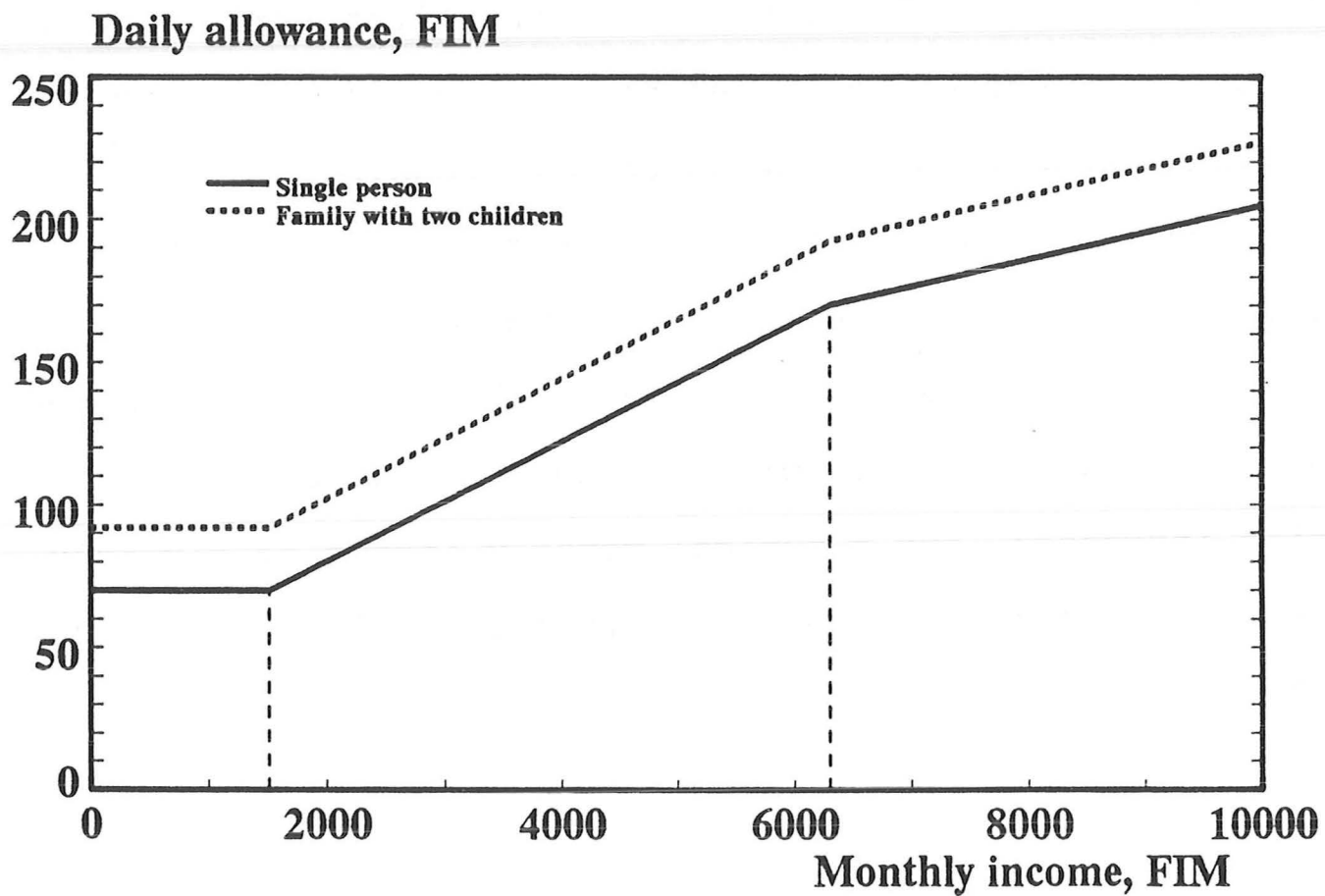
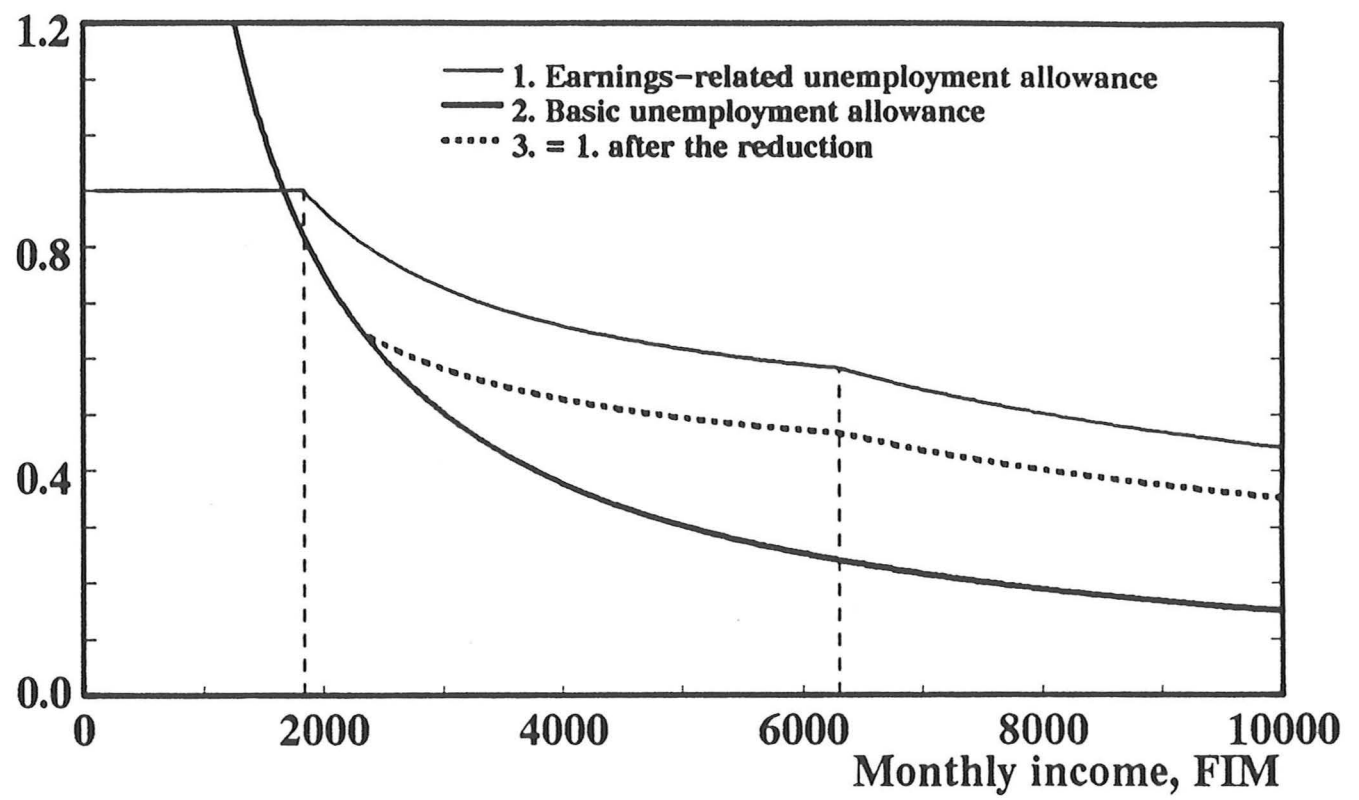


Figure 5. Replacement ratios of unemployment benefits



3. Unemployment Spells in View of the Data

3.1. The Sources and Structure of the Data

The sample was taken from the unemployment register of the Ministry of Labour. There are a number of ways in which data regarding the duration of unemployment can arise. The sampling can be made from the stock or from flows, which lead to different kind of models as shown by Chesher and Lancaster (1983). In this case the sampling has been made from the flow into unemployment. In order to guarantee that the sample would be randomly generated and seasonally representative, every hundredth individual was picked from the flow during 1985. The individuals were then followed until the end of their unemployment spells but at most until the end of 1986. So the longest lengths of unemployment spells of the data are nearly two years. The information of unemployed persons' and their spouses' annual income and asset figures was compiled from the tax register for the data set.

The government is responsible for the basic unemployment benefit system. The benefits paid under this system are called the basic unemployment allowance (sometimes called unemployment assistance). The Social Insurance Institution takes care of the payment of the basic unemployment allowance. The information on the basic unemployment allowance during the unemployment period was compiled from the unemployment allowance register of the Social Insurance Institution.

The earnings-related unemployment allowance (sometimes called UI benefits) is paid by the unemployment insurance funds. Members of the labour unions are normally also members of the unemployment insurance funds. Although the funds are formally associated with the labour unions, it is the government that has determined the most important regulations, including rules for benefit levels and criteria for receiving benefits. Nearly all the unemployment insurance funds pay the allowances through the bank Postipankki, which gave the information for the research on the earnings-related unemployment allowances during the unemployment periods. The Central Statistical Office of Finland helped with the collection of the data.

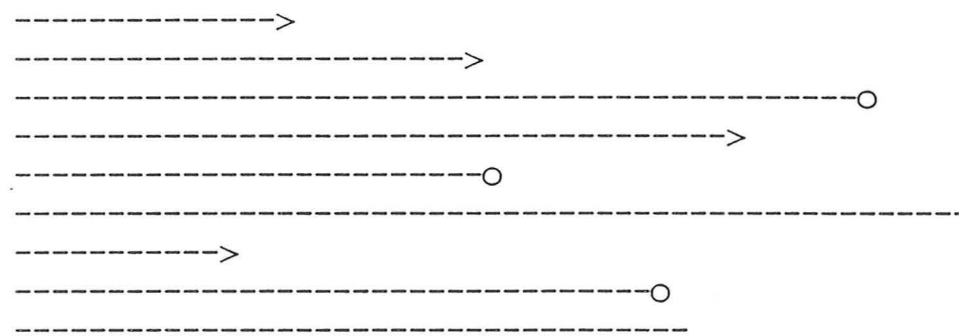
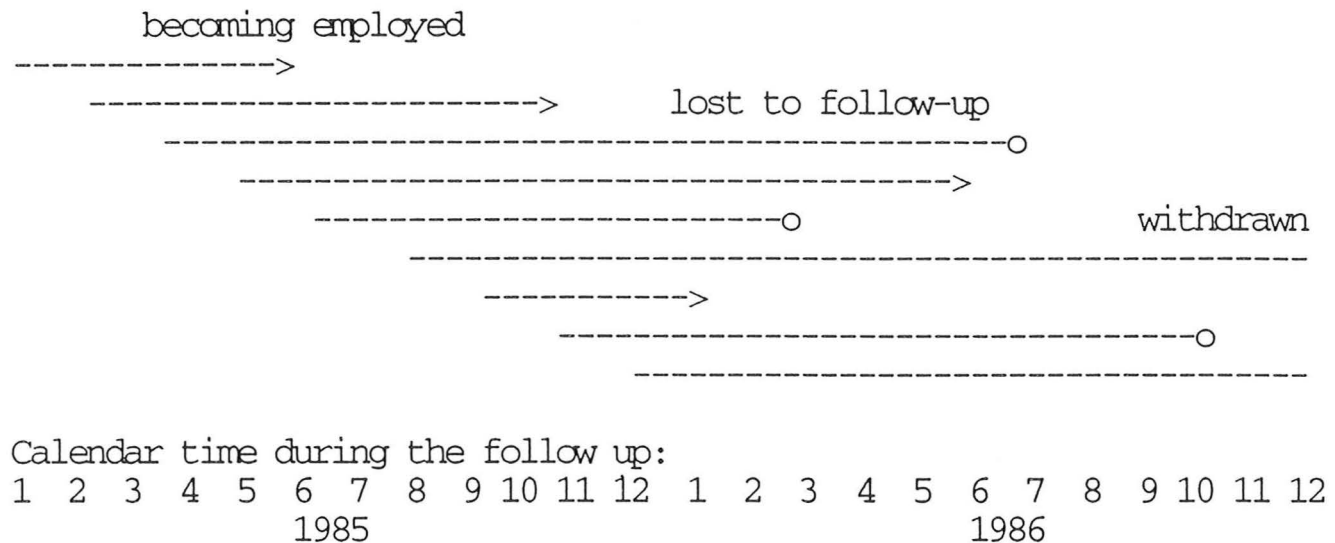
The individuals classified as unemployed have been taken into the sample from the original data set. Therefore individuals who are working part-time or receive an unemployment pension have been rejected. The search activity of laid-off persons is lower than others (see Lippman and McCall, 1979). It may be optimal for laid-off persons merely to wait for recalls. The unemployment duration depends on the recalls, which are determined by the demand for the firm's products. Therefore laid-off persons were rejected. About 34 per cent of the recipients of the earnings-related unemployment allowance had to be rejected, since some of the UI funds did not permit use of their data in the research (see Kettunen, 1989). About 13 per cent of the recipients got their benefits through other banks than Postipankki. These observations were lost. One may ask whether the data are representative with respect to all of the recipients. On the one hand, it can be argued

that the data are representative in econometric models, since the reasons why some of the observations were lost are exogenous. There is no reason to assume that some of the funds did not permit use of their data since they are misusing the UI system. Of course, on the other hand, some of the statistics, for instance means and variances of the replacement ratios of the whole sample, represent only the sample in use. In that case it is more appropriate to look at the means and variances of the replacement ratios of the recipients of the different benefits. In calculating the replacement ratio about one per cent of the observations had to be rejected because of missing income figures. Also some observations were rejected because of missing dates, incorrect social security codes or other invalid data. The final sample size is 2077 observations.

Persons can leave the ranks of the unemployed in different ways. Figure 6 illustrates some of the typical cases. The duration is calculated as a difference between the date of entry into unemployment and the date of exit into employment. These kinds of observations are complete spells of unemployment. Persons are no longer unemployed when they get an acceptable offer and start working. Unfortunately, a portion of the individuals was lost in the follow-up, because the individuals either can not be found or it can not be determined that they have found employment. An unemployed person can also be withdrawn from study, because the follow-up time ended. When some individuals may not be observed for the full time, it is said that the observations are right-censored. The censored observations include also transitions into the non-

participation. The censoring has to be taken into account in econometric work keeping these observations in the data.

Figure 6. Unemployed persons, entering and leaving the study



The length of unemployment (months):

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
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The data are cross-sectional and fairly rich on individual characteristics and labour market specific variables. Also, the data includes longitudinal (duration dependent) information on the actual benefit receipts during the job search period. So the data set is a combination of cross-sectional and time series data. The reference for details regarding the data should be made to Kettunen (1989, 1990 and 1991).

Usually in the studies of unemployment duration the replacement ratio of unemployment benefits is obtained as a ratio of after-tax benefits or unemployment income and the after-tax income while the individual is working. Clearly this ratio is important when studying the incentives of becoming employed. However, there are many ways of calculating this ratio. In a British study by Lancaster (1979) the numerator was obtained from the answer to the question of how much they had coming in from all sources, for instance, unemployment benefits, supplementary benefits and family income supplements during the main period of their unemployment. The denominator was the answer to the question of how much did they earn after deductions in their last job. No persons without any income were interviewed.

In the British study by Nickell (1979a) the unemployment benefits were imputed using the rules current at the appropriate date. The benefits included unemployment benefits, supplementary benefits, rate rebates, family allowances, free school meals and family income supplements. This unearned income was added to the spouse's income. It may, however, be difficult to interpret the results of the estimations, since the numerator of the replacement ratio includes other income than unemployment benefits (see Atkinson, 1980). Furthermore, the General Household Survey used by Nickell contains relatively little information on the actual unemployment benefits received, and the calculations by Nickell are based on the assumption that all the unemployed persons received full benefits. Clearly, this is a dubious assumption. A sizeable fraction

of those entitled to the benefits do not claim their entitlement. Supplementary Benefit Commission (1978) estimated that in 1979 the estimated take-up rate for the supplementary benefit was 78 per cent. In 1987 the take-up rate was 81 per cent (Atkinson and Micklewright, 1988). The income was estimated using the occupation specific earnings functions. An argument against the use of earnings functions is that the models usually explain only a low portion of the wage differentials. Another argument is that the unemployed individuals have in general lower potential earnings than those working. In a later study Nickell (1979b) calculated the replacement ratio by using actual current benefits and actual past earnings. The disadvantage of the data is that the observations with short durations had to be omitted due to missing data and the full pattern of benefits over the spell of unemployment is not observed.

In the Dutch study by Gorter et al. (1991) the unemployment benefits were determined by taking together the revenues directly related to the state of unemployment. Other sources of income such as labour market income of the partner, rent, interest, income from activities in the 'black market', etc. were added up in the variable additional income. The unemployed persons were asked their expected income. The persons were asked to compare themselves with other colleagues or friends with a similar education. For unemployed people who did not answer this question they used the result of a regression of the expected wage level. The wage level was estimated using personal characteristics, e.g. age, education and gender. The calculation of the replacement ratio may introduce a

selection bias in the way that the unemployed persons are too optimistic with respect to their value on the labour market. Secondly, there may be a selection bias if the group who answered the question about the expected income is different from the group who did not. Thirdly the benefits plus other income during the spell of unemployment do not represent the UI system well.

Access to administrative data recording the sequence of actual benefit payments throughout a spell of unemployment is a substantial advantage. However, there may be also difficulties. Moffitt (1985) points out that the disadvantage of the U.S. administrative data is that only the persons who have begun receiving UI benefits were available. The same problem appears in the U.S. studies by Solon (1985), Meyer (1990) and Katz and Meyer (1990) and a Canadian study by Ham and Rea (1987). The truncation of the data implies that the distribution of unemployment spells beyond the truncation cannot be analyzed. Also Atkinson and Micklewright (1991) point out that the absence from such data of those unemployed who are not claiming benefits must be balanced against this.

The unobserved wage offers are relevant considering the incentives for re-employment. However, in practice it is necessary to replace the offers with their expected value, which in this study is calculated for each individual using the actual pre- and post-unemployment income from the tax register during 1985. Clearly most income is pre-unemployment income. The weekly income was obtained by dividing the yearly income by the weeks during which the persons were working.

One might argue that there is potential endogeneity bias from using the actual level of income. As Narendranathan et al. (1985) point out the persons who are most likely to be selective about accepting jobs may well have had higher-than-average earnings in their previous job. However, with our data set there is no reason for that concern, because the actual earnings used do not have a statistically significant effect on the probability of becoming employed (see Chapter IV). Furthermore it can be argued that it is not only the income which is relevant in accepting an job offer but also the other characteristics of a job. That is a reason why the search model should be written in terms of utilities. In practice jobs include many characteristics.

The data provides plenty of variation in the replacement ratio to get accurate estimates of the impact of the level of UI benefits. When administrative data is used such things like the take-up rate, means testing, waiting period and risk of losing benefits and the other rules of the eligibility of benefits affect on whether benefits are actually paid to the unemployed workers. The reductions of benefits after a fixed number of unemployment days, non-linearities in the benefit schedules and reduced benefits due to the other income during the spell of unemployment are sources of the variation in benefit levels. In addition during the sample period there was a 7 per cent increase in the basic unemployment allowance (1st July, 1986). Many of these sources of variation can not be observed if benefits are imputed using the rules of the UI systems. For example, in Canada the variation in the

replacement ratio is small, since the benefits are a constant fraction of insurable earnings equalling the previous weekly wage up to a maximum (Ham and Rea, 1987). In the U.S. there is also variation between the states.

The descriptive statistics of the data are in Tables 1 - 3. Means and variances of the variables used in the study have been calculated separately for all duration lengths and durations longer than three and twelve months. An initial look at the data shows that some individual characteristics and labour market variables may be connected with longer durations of unemployment. Persons with long unemployment periods are more often men, are married and have children. They are older, have a low level of education and live in an area where the demand for labour is low. School graduates seem to have usually short spells of unemployment. On the other hand, the persons who have come from housework (usually housewives) seem to have longer spells of unemployment. The descriptive statistics of full samples and complete durations seem to be relatively close each other. The variables used in the study are described in Appendix 1.

Table 1. Descriptive statistics of the whole data

	Full sample		Complete spells	
	N=2077		N=1250	
	Mean	Std.dev.	Mean	Std.dev.
Unemployment spell in weeks	15.06	18.05	10.64	12.10
Number of children	0.23	0.62	0.25	0.66
Married (1=yes)	0.37	0.48	0.38	0.49
Sex (1=male)	0.53	0.50	0.54	0.50
Age (years)	31.14	11.94	29.63	10.23
Level of education	0.45	0.50	0.51	0.50
Training for employment (1=yes)	0.15	0.36	0.17	0.37
Member of UI fund (1=yes)	0.42	0.49	0.46	0.50
Came from schooling (1=yes)	0.14	0.34	0.16	0.37
Came from housework (1=yes)	0.07	0.26	0.05	0.21
Regional demand	0.11	0.13	0.10	0.12
Occupational demand	0.12	0.05	0.12	0.05
Taxable assets (mill.FIM)	0.011	0.03	0.010	0.03
Replacement ratio	0.16	0.21	0.16	0.21
Replacement ratio, $0 < t \leq 3$ months ¹	0.14	0.19	0.14	0.19

¹ The average figures for the first three months

Table 2. Descriptive statistics of the data for durations longer than three months

	Full sample		Complete spells	
	N=701		N=335	
	Mean	Std.dev.	Mean	Std.dev.
Unemployment spell in weeks	34.40	19.43	26.73	12.59
Number of children	0.36	0.77	0.43	0.84
Married (1=yes)	0.44	0.49	0.45	0.50
Sex (1=male)	0.57	0.49	0.59	0.49
Age (years)	36.59	13.24	33.50	10.76
Level of education	0.38	0.49	0.46	0.50
Training for employment (1=yes)	0.14	0.35	0.18	0.38
Member of UI fund (1=yes)	0.50	0.50	0.53	0.50
Came from schooling (1=yes)	0.08	0.26	0.09	0.29
Came from housework (1=yes)	0.10	0.30	0.06	0.23
Regional demand	0.09	0.10	0.07	0.08
Occupational demand	0.12	0.05	0.12	0.05
Taxable assets (mill.FIM)	0.016	0.03	0.013	0.03
Replacement ratio	0.26	0.23	0.27	0.22
Replacement ratio, $0 < t \leq 3$ months ¹	0.20	0.20	0.20	0.20
Replacement ratio, $3 < t \leq 12$ months ¹	0.33	0.32	0.37	0.34

¹ The average figures for these intervals

Table 3. Descriptive statistics of the data for durations longer than twelve months

	Full sample N=129		Complete spells N=23	
	Mean	Std.dev.	Mean	Std.dev.
Unemployment spell in weeks	66.60	14.04	60.48	7.23
Number of children	0.33	0.72	0.35	0.83
Married (1=yes)	0.48	0.50	0.57	0.51
Sex (1=male)	0.57	0.50	0.48	0.51
Age (years)	42.28	13.07	36.22	8.60
Level of education	0.20	0.40	0.26	0.45
Training for employment (1=yes)	0.15	0.36	0.35	0.49
Member of UI fund (1=yes)	0.53	0.50	0.70	0.47
Came from schooling (1=yes)	0.03	0.17	0.04	0.21
Came from housework (1=yes)	0.19	0.39	0.04	0.21
Regional demand	0.08	0.09	0.08	0.11
Occupational demand	0.12	0.05	0.12	0.06
Taxable assets (mill.FIM)	0.014	0.03	0.006	0.03
Replacement ratio	0.23	0.21	0.20	0.16
Replacement ratio, $0 < t \leq 3$ months ¹	0.15	0.17	0.13	0.13
Replacement ratio, $3 < t \leq 12$ months ¹	0.25	0.22	0.22	0.18
Replacement ratio, $12 < t \leq 24$ months ¹	0.21	0.47	0.27	0.25

¹ The average figures for these intervals

3.2. Functions Describing Unemployment Duration

Many of the theoretical concepts of the econometric models of unemployment duration are borrowed from the biostatistical literature. Survivor and hazard functions are the most obvious examples. The density, survivor and hazard functions define uniquely any specific duration distribution. However, each of them provides the researcher with a different view of the data. The density function is $f(t)$. The probability that an individual leaves unemployment during the time interval $t < \tau < t+dt$ is $f(t)dt$. It is also called the unconditional failure rate.

For the density function $f(t) \geq 0$ the integral from zero to infinity is one.

The survivor function $S(t)$ is the probability that an individual is unemployed at least until a fixed duration t . If T is a random variable that represents the duration, the survivor function can be defined using the cumulative distribution function of unemployment length $F(t)$ as follows

$$(1) \quad S(t) = 1 - F(t) = \int_t^{\infty} f(\tau) d\tau = \Pr(t < T).$$

The conditional instantaneous probability of leaving a state is expressed using the hazard function $h(t)$. The probability that an unemployed person becomes employed in the time interval $t < \tau < t+dt$ is $h(t)dt$, given he is still unemployed at time t . The hazard function is often termed the failure rate or the conditional instantaneous probability of leaving a state. The value of the hazard function is zero or positive for all t . By the definition of conditional probability the hazard function can be written

$$(2) \quad h(t) = f(t)/S(t),$$

where the density function can be written $f(t) = -\partial S(t)/\partial t$. Then the hazard function can be written as follows

$$(3) \quad h(t) = - \frac{\partial \log S(t)}{\partial t}.$$

Hence the well-known connecting relationship among the survivor and hazard functions can be written

$$(4) \quad S(t) = \exp\left[- \int_0^t h(\tau) d\tau\right].$$

The density, survivor and hazard functions are the main statistical concepts in the econometric study of unemployment spells. In the next section they are applied in describing the data using the life table method.

3.3. Life Table Analysis of Unemployment Duration

A nonparametric actuarial method applied to the durations is often a convenient way to get a touch on the data. According to Gross and Clark (1975) there are three types of life tables in common use - the population life table, the cohort life table and the clinical life table. There are also other non-parametric methods. One of them is the kernel-based method proposed, for instance, by Liu and van Ryzin (1985). Population and cohort life tables describe the actual survival experience of a population or cohort of individuals who were born at about the same time. A form of the clinical life table is applied here. The actuarial life

table analysis is based on the method by Cutler and Ederer (1958). Definitions and the notation are as follows:

1. Lengths of unemployment spells and times of loss or withdrawal are distributed into half-open time intervals $[t_{i-1}, t_i)$, $i = 1, 2, \dots, s+1$. The last interval is infinite in length. The length of unemployment is measured in weeks as a difference between the date of becoming unemployed and the date of becoming employed or the date of censoring. If both the date of entry into unemployment and the date of exit into employment are observed the observation is called a complete spell.
2. The midpoint of the i th interval is t_{mi} , $i = 1, 2, \dots, s$. The midpoints are used to plot the hazard and density functions.
3. The width of the i th interval is $h_i' = t_i - t_{i-1}$, $i = 1, 2, \dots, s$. The widths are required to calculate the hazard and death density functions.
4. The total number of individuals who are lost to follow-up during the i th interval is l_i .
5. The total number of individuals who are withdrawn from the study unemployed during the i th interval is w_i . Individuals who are lost to follow-up or withdrawn become censored observations.

6. The number of individuals who leave unemployment in the i th interval is d_i .

7. The total number of individuals who enter the i th interval is n_i' , $i = 1, 2, \dots, s+1$. Thus the total sample size for the study is n_1' . Clearly $n_i' = n_{i-1}' - l_{i-1} - w_{i-1} - d_{i-1}$.

8. The number of individuals who are searching for a job during the i th interval is $n_i = n_i' - (l_i + w_i')/2$. It is also called the risk set. It is assumed that times to loss or withdrawal are uniformly distributed.

9. The conditional proportion of becoming employed in the i th interval is $\hat{q}_i = d_i/n_i$, $i = 1, 2, \dots, s$. Clearly $\hat{q}_{s+1} = 1$ if there are no censored observations.

10. The conditional proportion of surviving the i th interval is $\hat{p}_i = (1 - \hat{q}_i)$, $i = 1, 2, \dots, s$.

11. The cumulative proportion of surviving is $\hat{S}_i = \hat{p}_{i-1}\hat{S}_{i-1}$, $i = 1, 2, \dots, s$. It is often referred to as the cumulative survival rate. Clearly $\hat{S}_0 = 1$.

The probability of becoming employed in the i th interval per unit width is a natural estimate for the density function

$$(5) \quad \hat{f}(t_{mi}) = (\hat{S}_i - \hat{S}_{i+1})/h_i' = \hat{S}_i \hat{q}_i / h_i', \quad i = 1, 2, \dots, s.$$

The estimate of the survivor function, i.e. the probability of being still unemployed at t_{mi} , is written

$$(6) \quad \hat{S}(t_{mi}) = (\hat{S}_i + \hat{S}_{i+1})/2 = \hat{S}_i(1 + \hat{p}_i)/2.$$

The estimate of the hazard rate is obtained as $f(t_{mi})/S(t_{mi}) = 2\hat{q}_i/h_i'(1 + \hat{p}_i)$. To get a more convenient form for the estimate of the hazard rate, the estimates of \hat{p}_i and \hat{q}_i are substituted into the estimate of the hazard function, which gives

$$(7) \quad \hat{h}_i(t_{mi}) = d_i/[h_i'(n_i - d_i/2)].$$

It can be seen that the estimate of the hazard function depends on the number of persons searching for a job and becoming employed, and it also depends on the width of the interval. It should be pointed out that the estimates and the clarity of the life tables depends on how the intervals are distributed over the unemployment period.

The estimates of the functions describing duration are used in life tables, which are used here to describe the duration of unemployment and the regional and occupational mobility. In the life table of regional mobility the interest is on the duration from the date of entry into unemployment until the date of exit to employment by moving. Becoming employed by moving implies a completed spell. Otherwise the observation is censored. Similarly in the life table of occupational mobility the completed spell

is an observation where the individual leaves unemployment by changing occupation.

Approximate variances for the functions describing survival are calculated. Instead of obtaining variances of the functions directly, a Taylor series expansion is made of the function and then the variance of the linear terms is found. The properties of variances of linear functions are well-known; thus by approximating an arbitrary function by a linear one, its variance can be approximated. The approximations are discussed in Bennett and Franklin (1954) and Kendall and Stuart (1963) even though the variances of the life table functions have already presented by Greenwood (1926). It should be pointed out that for small samples the approximated variances are not good approximations of the true variance. Kuzma (1967) found that the variance formula of Greenwood considerably underestimated the variance when the withdrawal rate was high. If there are only a few observations in the later intervals of a life table, the computation of variances is not necessarily worthwhile for these later stages.

The life table of unemployment spells is reported in Table 4. Nearly 40 per cent of the unemployed persons left unemployment during the ten first weeks. The density and hazard functions are decreasing except they are increasing slightly at about one year's unemployment and the hazard function is increasing slightly after the first three months of unemployment. However, the standard errors of the functions are too high to draw strong conclusions. More active and less selective persons leave the cohort sooner than others. This explains the basic decreasing nature of

the density and hazard functions. The survivor function is decreasing by definition. If there were no UI system the functions would probably be different. One hypothetical reason for the shape of the hazard function is that the earnings-related unemployment allowance decreases after the first 100 days of unemployment by 20 per cent. This remains to be shown. These allowances are paid until the 500th day. After that the basic unemployment allowance is paid.

About 40 per cent of the observations are censored, i.e. re-employment was not observed. The number of those who were lost track of in the follow-up increases after one year of unemployment. Employment offices have regulations that they have to offer any job to an unemployed person who have been unemployed at least one year. The aim is to stem long-term unemployment. It may be that some individuals do not accept the offer and stop searching. Another regulation which could affect the hazard function is that one year of unemployment is needed to get an unemployment pension.

The life table of regional mobility is reported in Table 5. More than 2 per cent of the unemployed persons became employed by moving to another region during the ten first weeks. The region is defined by the UI Act as an area of residence and other regions, where the persons normally go to work. Becoming employed by moving is a rather rare phenomenon in the labour market. About 98 per cent of the observations were censored, i.e. moving was not observed. The density and hazard functions are decreasing except that they increase strongly after the first three months. According to the Finnish Unemployment Insurance Act unemployed persons do not have to search outside their area

of residence within the first three months of unemployment. After that they may lose their benefits if they do not accept an offer outside their area of residence. It seems that the Unemployment Insurance Act has a positive effect on the probability of becoming employed by moving after the first three months. However, one can not draw strong conclusions about these estimates, because there is a limited number of complete spells available in the data. Therefore the standard errors of the estimates are rather high for the long durations.

The life table of occupational mobility is reported in Table 6. About 6 per cent of the unemployed persons left unemployment by changing occupations during the ten first weeks. About 90 per cent of the observations were censored, i.e. changing occupation was not observed. The occupation is calculated on a 5-digit level of the Nordic Occupational Classification. There are 1320 occupations on the most accurate 5-digit level. It is an empirical question on which level the occupational mobility is examined. The density and hazard functions are decreasing in the life tables except that they are increasing after the 30-weeks unemployment period. The three-month rule of labour mobility is applied to the choice of occupation. During the first months the unemployed persons do not have to accept an offer made by the employment office if they are not qualified by schooling or experience for the job. In practice the employment offices use rather narrow classifications of the occupations. Therefore the 5-digit level is the most appropriate.

An initial question is whether the UI system has a positive effect on the probability of changing occupations after the first three months of unemployment. Because the intervals of unemployment spells may not be the most appropriate and when the standard errors of the estimates of the hazard function are high we can not draw any strong conclusions based on these preliminary simple estimates.

Figure 7 illustrates the hazard functions of unemployment durations for the non-members and members of labour unions and labour mobility for the whole sample using slightly different intervals of the unemployment periods. It can be seen that these simple nonparametric hazard functions can be used to detect the effects of the UI system. The effects of the risk of losing benefits can be seen as an increasing hazard after the first three months of unemployment (about 13 weeks). The effect of reductions of the earnings-related unemployment allowances can be seen as a positive effect on the hazard function of the members of labour unions just after the first insured 100 days of unemployment (20 weeks). According to the rules of the system one week includes five insured days. Therefore 100 days of unemployment is 20 weeks. On the contrary no effect is found for the non-members who do not have the reductions.

Table 4. The life table of unemployment spells

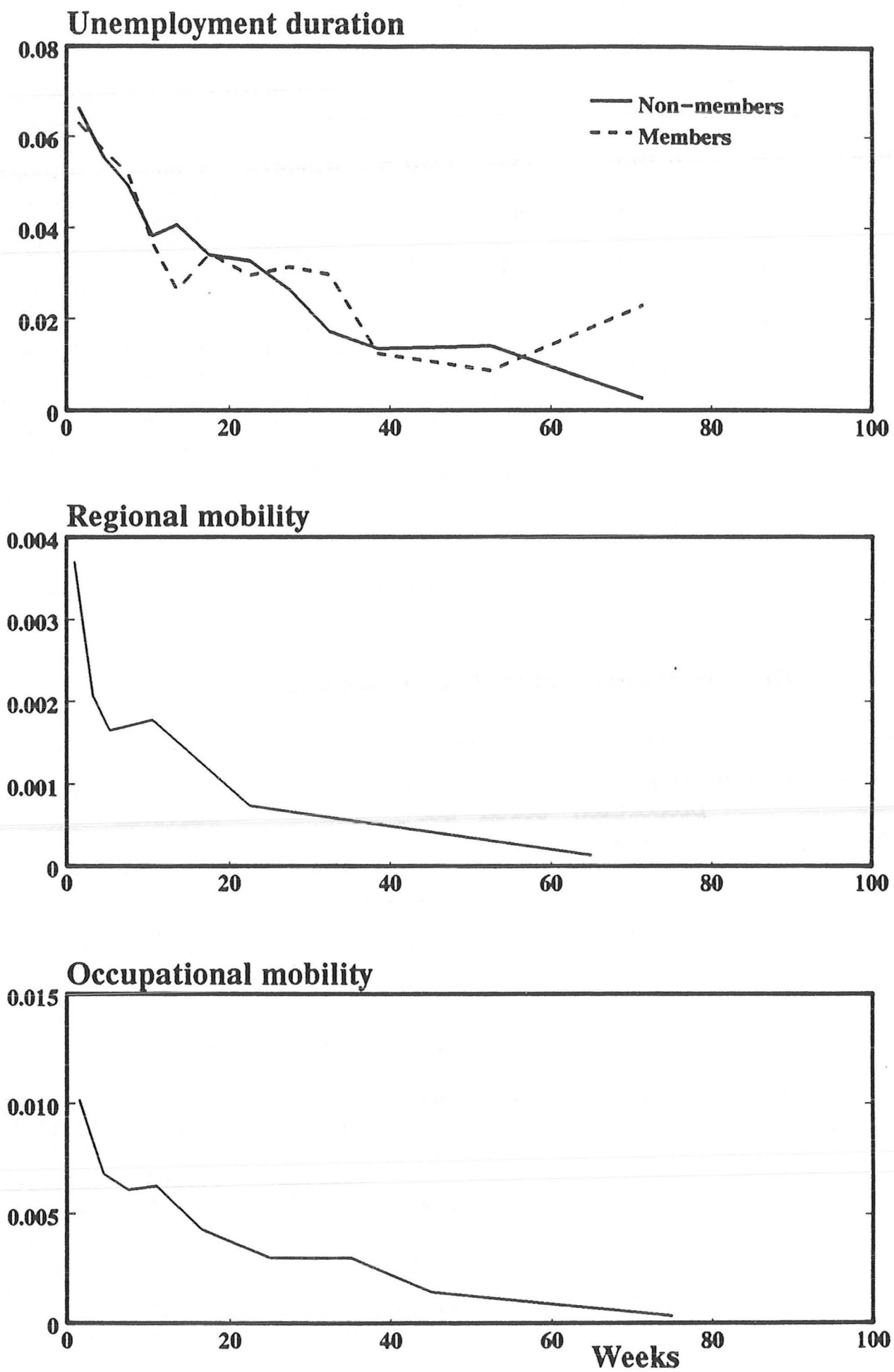
Interval in weeks	Exiting	Conditional proportion exiting	cen- sored	Risk set	Density	Cum. survival	Hazard
					Std.errors in parentheses		
0 -	548	0.278	208	1973.0	0.056 (0.002)	1.000 (0.000)	0.065 (0.003)
5 -	275	0.224	181	1230.5	0.032 (0.002)	0.722 (0.010)	0.050 (0.003)
10 -	122	0.149	88	821.0	0.017 (0.001)	0.561 (0.012)	0.032 (0.003)
15 -	94	0.151	65	622.5	0.014 (0.001)	0.478 (0.012)	0.033 (0.003)
20 -	67	0.140	37	477.5	0.011 (0.001)	0.405 (0.012)	0.030 (0.004)
25 -	52	0.138	30	377.0	0.010 (0.001)	0.349 (0.012)	0.030 (0.004)
30 -	31	0.103	20	300.0	0.006 (0.001)	0.300 (0.012)	0.022 (0.004)
35 -	16	0.064	16	251.0	0.003 (0.001)	0.269 (0.012)	0.013 (0.003)
40 -	11	0.050	15	219.5	0.003 (0.001)	0.252 (0.012)	0.010 (0.003)
45 -	9	0.047	21	190.5	0.002 (0.001)	0.240 (0.012)	0.010 (0.003)
50 -	8	0.057	61	140.5	0.003 (0.001)	0.228 (0.012)	0.012 (0.004)
55 -	7	0.076	20	92.0	0.003 (0.001)	0.215 (0.012)	0.016 (0.006)
60 -	5	0.075	17	66.5	0.003 (0.001)	0.199 (0.013)	0.016 (0.007)
65 -	2	0.043	13	46.5	0.002 (0.001)	0.184 (0.014)	0.009 (0.006)
70 -	3	0.133	31	22.5	0.001 (0.000)	0.176 (0.014)	0.005 (0.003)
100 -	0	0.000	4	2.0	. .	0.153 (0.018)	. .

Table 5. The life table of regional mobility

Interval in weeks	Exiting	Conditional proportion exiting	Cen- sored	Risk set	Density	Cum. survival	Hazard
					Std.errors in parentheses		
0 -	23	0.013	733	1710.5	0.0027 (0.0006)	1.0000 (0.0000)	0.0027 (0.0006)
5 -	14	0.013	442	1100.0	0.0025 (0.0007)	0.9866 (0.0028)	0.0026 (0.0007)
10 -	6	0.008	204	763.0	0.0015 (0.0006)	0.9740 (0.0043)	0.0016 (0.0006)
15 -	2	0.003	157	576.5	0.0007 (0.0005)	0.9663 (0.0053)	0.0007 (0.0005)
20 -	3	0.007	101	445.5	0.0013 (0.0007)	0.9630 (0.0058)	0.0014 (0.0008)
25 -	1	0.003	81	351.5	0.0005 (0.0005)	0.9565 (0.0069)	0.0006 (0.0006)
30 -	2	0.013	304	158.0	0.0002 (0.0001)	0.9538 (0.0074)	0.0002 (0.0001)
100 -	0	0.000	4	2.0	. .	0.9417 (0.0112)	. .

Table 6. The life table of occupational mobility

Interval in weeks	Exiting	Conditional proportion exiting	Cen- sored	Risk set	Density	Cum. survival	Hazard
					Std.errors in parentheses		
0 -	95	0.054	661	1746.5	0.0109 (0.0011)	1.0000 (0.0000)	0.0112 (0.0011)
5 -	39	0.035	417	1112.5	0.0066 (0.0010)	0.9456 (0.0054)	0.0071 (0.0011)
10 -	20	0.026	190	770.0	0.0047 (0.0011)	0.9125 (0.0074)	0.0053 (0.0012)
15 -	15	0.026	144	583.0	0.0046 (0.0012)	0.8888 (0.0089)	0.0052 (0.0013)
20 -	10	0.022	94	449.0	0.0039 (0.0012)	0.8659 (0.0104)	0.0045 (0.0014)
25 -	6	0.017	76	354.0	0.0029 (0.0012)	0.8466 (0.0119)	0.0034 (0.0014)
30 -	6	0.021	45	287.5	0.0035 (0.0014)	0.8323 (0.0130)	0.0042 (0.0017)
35 -	5	0.020	27	245.5	0.0033 (0.0015)	0.8149 (0.0146)	0.0041 (0.0018)
40 -	6	0.051	217	118.5	0.0007 (0.0003)	0.7983 (0.0160)	0.0009 (0.0004)
100 -	0	0.000	4	2.0	. .	0.7579 (0.0221)	. .

Figure 7. Nonparametric hazard functions

4. Conclusions

At the beginning of the year 1985 there was a notable reform in the Finnish unemployment insurance system. The main features of the reform are that the level of unemployment benefits was increased and the benefits became taxable. In this study the new UI system was examined, the collection of the data set for the econometric study was reported and a preliminary description of the data was made.

The descriptive statistics of the data show that the unemployment seems to be more often a problem of men. Moreover, the persons with long unemployment spells are often old and they live in an area where the demand for labour is low. The level of education seems to be an important factor helping people to get a job. School graduates seem to have usually short spells of unemployment, but those who have come from housework seem to have long spells of unemployment.

The life table analysis was applied to the duration of unemployment spells and the regional and occupational mobility. The probability of becoming employed, i.e. the hazard function, is decreasing, except after the first three months of unemployment it is slightly increasing and after one year of unemployment it is rather strongly increasing. Without the UI system the hazard function could be decreasing all the time. However, this can not be demonstrated using these simple nonparametric methods. One reason for the form of the hazard function may be that the reductions of the earnings-related unemployment allowances

after the 100th day of unemployment seem to increase the re-employment probability. Another reason which may affect the re-employment probability is the rule regarding labour mobility stipulated in the Unemployment Insurance Act. The hazard function is increasing around the unemployment of a year. One reason is that unemployment offices have to offer jobs to the persons who have been unemployed at least one year. Another reason is that one year's unemployment is required in order to be eligible for an unemployment pension. However, the main interest of this study is not in these matters.

The hazard function of regional mobility is decreasing except that it is increasing strongly after 20 weeks of unemployment. After the first three months of unemployment the persons may lose their benefits if they do not accept an offer outside their area of residence. It seems that the three-month rule of labour mobility may have a positive effect on the probability of becoming employed by moving after the first months.

The hazard function of occupational mobility is decreasing, except that it is increasing after the 30-week (7 months) unemployment period. The three-month rule of labour mobility is also applied to the choice of occupation. During the first few months unemployed persons do not have to accept an other occupation but after that period they may lose their benefits if they refuse to accept an offer. Using a slightly different partition of the duration it can be found that the hazard function is slightly increasing at the first three months of

unemployment. One could also claim that the inference is quite sensitive to the partitions chosen.

Looking at the levels of the hazard functions it seems that the most acceptable alternative is to find a job in their area of residence and in their occupation. If such vacancies are not available, they try to change occupations and stay in their area of residence. Persons can also leave unemployment by taking a training course for further employment, which usually takes some months. Finally, the least acceptable alternative is to move to a new location.

As a final conclusion some precautionary words concerning the discussion and interpretations have to be presented. The results of this chapter have been presented in order to get a touch on the data. There is no control for characteristics of individuals in this chapter. Therefore the preliminary non-parametric results of this chapter serve as a basis for the more sophisticated analysis. In addition, at this point it can be argued that the heterogeneity of the individuals is partly the reason for decreasing hazards, because the persons with the highest probabilities of becoming employed leave the cohort first. So far the interpretations and arguments serve as hypotheses, which remain to be tested in the search theoretical and econometric studies of the next chapters.

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Appendix 1. Variables of the Data

Duration of unemployment is calculated in weeks and it is the difference between the date of entry into unemployment and the date of becoming employed.

Number of children is the number of an unemployed person's children who are younger than 18 years.

Married is a dummy variable, 1 = yes.

Sex is a dummy variable, 1 = male.

Age is measured in years.

Level of education is a dummy variable, 1 = at least 12 years of education. The level of education is based on the education code of the Central Statistical Office of Finland.

Training for employment is a dummy variable, 1 = The person has received training for further employment. The training courses have been organized by the government. Training for employment is course participation, that has occurred before the unemployment, but not necessarily immediately before it.

Member of UI fund is a dummy variable, 1 = yes.

Came from schooling is a dummy variable, 1 = The person has come from schooling or from the military service.

Came from housework is a dummy variable, 1 = The person has come from housework or elsewhere outside the labour force (hospitals, prison, etc.).

Regional demand describes the regional rate of jobs available. It is the number of vacancies divided by the number of job seekers in the area.

Occupational demand describes the occupational rate of jobs available in the whole country. It is the number of vacancies divided by the number of job seekers in the occupation groups 0-9.

Taxable assets has been compiled from the tax register and it is measured in millions of Finnmark.

Replacement ratio is the unemployed persons' average replacement ratio for unemployment benefits during the unemployment period after tax. Weekly unemployment benefits after tax have been divided by the weekly income after tax. In the sample taken from the flow into unemployment 29 per cent of the individuals received the basic unemployment allowance, 29 per cent received the earnings-related unemployment allowance and 42 per cent did not receive any form of benefits. The means of the replacement ratios of the intervals (0, 3], (3, 12] and (12, 24] months are 0.29, 0.55 and 0.39 for the persons receiving earnings-related unemployment allowance and 0.26, 0.41 and 0.39, respectively, for the persons receiving the basic unemployment allowance. The average replacement ratios are lower during the first three months, since no benefits are paid during the qualifying waiting period of benefits. Also reductions and disqualifications of benefits decrease the average replacement ratios. Some persons do not even apply for the benefits. The average replacement ratios of benefits are higher when the data has been sampled from the stock of unemployed persons (length biased sampling). The flow sample includes more short durations with no benefits. Therefore the average replacement ratios of the unemployment spells calculated over all the individuals are rather low.

Chapter III

**A SEARCH THEORETICAL ANALYSIS OF THE FINNISH UNEMPLOYMENT
INSURANCE SYSTEM**

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Abstract

This chapter studies the effects of the Finnish unemployment insurance on the re-employment probability of unemployed workers using a search theoretical framework. It is well known that the unemployment benefits have a negative effect on the re-employment. In this chapter it is shown that the re-employment probability can be increased by lowering the costs of re-employment. Furthermore, it is shown that the qualifying waiting period has only a slightly positive effect on the hazard function, but removal of the mobility rules and reduction of benefits after a fixed period of unemployment substantially increase the re-employment probability.

1. Introduction

In the search theoretical literature [e.g. Lippman and McCall (1976a,b, 1979), Mortensen (1986) and Kiefer and Neumann (1989)] it has been generally considered that unemployment insurance (UI) has a disincentive effect on employment. Mortensen (1977) pointed out that the search behaviour of new entrants who are not currently eligible for UI benefits but who will be eligible after being employed is different. An increase in UI benefits or extension of the maximum benefit period will increase their re-employment probability, since unemployed workers must have been employed before they qualify for UI benefits.

This feature of the UI system has been well referenced and studied [e.g. Topel and Welch (1980) and Usategui (1988)]. However, there are many other features of the UI systems, which need more attention. This chapter analyses three features of the Finnish UI system using search models. Applications concerning the effects of the waiting period, mobility rules and reductions of benefits are presented. Their effects on the reservation utility, search intensity and re-employment probability are studied. The nonstationary features of search models in the context of housing demand have been studied by Loikkanen (1982). Van den Berg (1990) studied the effects of decreasing exogenous variables in search models with an empirical application to a structural model of unemployment duration.

Unemployed persons are not eligible for UI benefits at the beginning of their unemployment period. The insurance aspects of the waiting period have been earlier interpreted by Stafford (1977) using the economics of risk and insurance. In this chapter it is shown using a search model that during the qualifying waiting period the reservation utility is increasing and the search intensity is decreasing. Hence the re-employment probability is decreasing due to a fact that the unemployed persons are not yet eligible for benefits. However the effect is rather small.

Reluctant movers may lose their UI benefits after the first three months of unemployment. It is shown that the threat of removal of benefits decreases the reservation utility and increases the search intensity and re-employment probability. Furthermore, it is shown that the

reservation utility is slightly decreasing and the search intensity and re-employment probability are slightly increasing during the first three months.

Unemployed workers who are eligible for earnings-related unemployment allowances face a reduction of their benefits after the 100th day of unemployment. It is shown that the reductions decrease the reservation utility and increase the search intensity and re-employment probability. Hence the reservation utility is decreasing and the search intensity and re-employment probability are increasing before the reductions.

The remainder of this chapter is set out as follows. In section 2 the basic search theoretical model is presented and its properties are analyzed. In section 3 the main features of the UI system are analyzed: the qualifying waiting period, the threat of removing benefits from reluctant movers and the reduction of benefits. Their nonstationary effects on the reservation utility, search intensity and hazard function are analyzed. Section 4 concludes the chapter.

2. The Basic Model

In this section the basic search model of unemployment is presented and its comparative static properties are analyzed. Assume that an unemployed person gets utility from consumption C and leisure L and that there is no saving. The utility function is assumed to be a time separable function of these arguments. The utility of an unemployed person is $u_0(C, L)$, where C consists of UI benefits b minus the costs of search. Leisure is the time not spent in job search during the spell of unemployment, so that $L = 1 - s(t)$, where $s(t)$ is the search intensity, i.e. a fraction of time spent on search at time t . It is assumed that

$$(1) \quad u_C > 0, u_L > 0, u_{CC} \leq 0, u_{LL} \leq 0 \text{ and } u_{CL} = u_{LC} > 0,$$

where the subscripts denote derivatives.

If an individual is unable to find a job within the local labour market area, a suitable job may be found elsewhere, or if he is unable to find a job within his occupation, he may change his occupation. The arrival rate of job offers from area i and occupation j is assumed to follow a Poisson process with intensity $a_{ij}(s(t))$, which is a function of time spent on search. It is assumed that $a_{ij}(0) = 0$, $\partial a_{ij} / \partial s > 0$ and $\partial^2 a_{ij} / \partial s \partial s \leq 0$. The arrival rate of all the job offers $\Sigma \Sigma a_{ij} = \Sigma_i \Sigma_j a_{ij}(s(t))$ is convex as a sum of convex functions.

Moving from an area of declining industries and high unemployment to a region with growing employment or

changing occupations will also involve costs. These costs are measured in utility terms. It is assumed that in the model there are the searching costs c , the visiting costs c_i , the permanent cost of becoming employed c_j and moving costs c_i^m . The cost c is deterministic whereas c_i , c_j and c_i^m are probabilistic. The costs are of the flow type apart from c_i^m , which is of the lump-sum type. The effects of c_i^m have been studied, for instance, by Hey and McKenna (1979), Loikkanen (1982) and Burgess (1988), but the definition of c_j is new. It is a permanent loss in utility of a person who changes his occupation. For example, white collar workers may feel that they lose something if they accept any other occupation even at the same wage rate. Alternatively c_j could be assumed to depend on the area or both the occupation and area. For example, daily commuting costs between home and work are permanent costs of becoming employed.

Workers maximize the expected present value of the utility. During a short interval dt active search is undertaken and the unemployed person's utility evaluated at $t+dt$ is

$$\begin{aligned}
 (2) \quad V(t+dt) = & u_0(b - c - \sum \sum a_{ij} c_i, 1 - s(t)) B(dt) \\
 & + \sum \sum a_{ij} dt \int_{u_{ij}(t)}^{\bar{u}} [(u - c_j) B(t) - c_i^m] dF(u) D(dt) \\
 & + \{1 - \sum \sum a_{ij} dt [1 - F(u_{ij}(t))]\} V(t) D(dt) + o(dt).
 \end{aligned}$$

The first term of the value function $V(t+dt)$ on its right-hand side describes the discounted instantaneous utility during the search period dt . The second term is the expected discounted utility related to an acceptable offer. The third term is the expected discounted utility related to an unsuccessful search and $o(dt)$ is a remainder term. The expectation is taken with respect to the distribution function of utility $F(u)$. The maximum attainable utility is denoted by \bar{u} and $u_{ij}(t)$ is the reservation utility of an occupation j in an area i at time t . The offers that are at least $u_{ij}(t)$ are acceptable. The person may search for a job in one or more occupations in one or more areas. Also, it may not be optimal to search at all. This feature of search models has been studied by Loikkanen (1982).

$B(dt)$, $B(t)$ and $D(dt)$ are discount factors for dt , $t > 0$. It is assumed that $B(dt) = \int_0^{dt} e^{-r\tau} d\tau = [1 - \exp(-rdt)]/r$, where r is the subjective rate of time preference. By expansion it can be written as $B(dt) = dt + o(dt)$. The instantaneous utility of being unemployed is proportional to the length of the interval dt . In an infinite horizon case $B(t) = 1/r$, which discounts the utility of an acceptable offer. The simple infinite horizon case implies that the job separation rate is zero. The discount factor $D(dt) = \exp(-rdt)$ discounts the expected value of a search apart from the instantaneous utility from t to $t+dt$. By expansion $D(dt) = 1 - rdt + o(dt)$.

Substituting the discount factors, rearranging terms, forming the difference quotient $[V(t+dt) - V(t)]/dt$ and taking the limits as dt approaches zero gives the

differential equation of expected utility stream with respect to time

$$(3) \quad \dot{V}(t) = u_0(b - c - \sum \sum a_{ij} c_i, 1 - s(t)) - rV(t) \\ + \sum \sum a_{ij} \int_{u_{ij}(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V(t)] dF(u).$$

It is assumed that the remainder term $o(dt)$ approaches zero with dt . It can be seen that $V(t)$ is constant over time, i.e. $\dot{V}(t) = 0$ in a model with an infinite horizon. The value function can now be written as

$$(4) \quad V(t) = \{u_0(b - c - \sum \sum a_{ij} c_i, 1 - s(t)) \\ + \sum \sum a_{ij} \int_{u_{ij}(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V(t)] dF(u)\} / r.$$

The necessary condition for the optimal $u_{ij}(t)$ can then be solved by setting $\partial V / \partial u_{ij} = 0$, which gives

$$(5) \quad u_{ij}(t) = c_j + r[c_i^m + V(t)].$$

The value function can be written $V(t) = [u_{ij}(t) - c_j]/r - c_i^m$. This means that the expected value of continuing the search, the value function, is equal to the utility of an acceptable offer minus the permanent cost discounted over the search horizon net of the moving cost. The reservation utility is chosen to equate the value of the worst acceptable offer with the expected value of unemployment.

Next the comparative static properties of the model are studied, i.e. the effects of exogenous variables on the optimal reservation utility relative to a given optimal search intensity. These effects are solved by differentiation in Appendix 1.

Summarizing the comparative static properties of the reservation utility the following results are obvious. The reservation utility is

a) a decreasing function of the searching cost c and visiting cost c_i ,

b) an increasing function of the UI benefits b , arrival rate of job offers a_{ij} , permanent cost of re-employment c_j and moving cost c_i^m , improvement of offer distribution and uncertainty of job offers. The effect of the subjective rate of time preference r is generally ambiguous, but the reservation utility is nearly always a decreasing function of r .

Another decision variable of the model is the search intensity. An unemployed person's objective is to maximize the expected discounted utility by choosing search intensity relative to the acceptance rule of job offers. The necessary condition for the optimal search intensity is obtained by differentiating $V(t)$ with respect to the search intensity s

$$(6) \quad V_s(t) = \left\{ - \sum \sum \frac{\partial u_0}{\partial c} \frac{\partial a_{ij}}{\partial s} c_i - \frac{\partial u_0}{\partial L} \right. \\ \left. + \sum \sum \frac{\partial a_{ij}}{\partial s} \int_{u_{ij}(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V(t)] dF(u) \right\} / r = 0.$$

It can be seen that the marginal utility of leisure and visiting costs is equated to the expected marginal utility gain from the search.

The derivation of the comparative static results is complicated by the fact that the necessary conditions involve not only endogenous and exogenous variables but also the value function. The endogenous variables are affected by exogenous variables directly and indirectly via the change in the value function. The results are solved by implicit differentiation in Appendix 1. The following results are obvious. The search intensity is

a) a decreasing function of the UI benefits b , permanent cost of re-employment c_j , moving cost c_i^m and the subjective rate of time preference r ,

b) an increasing function of searching cost c , arrival rate of job offers a_{ij} , improvement of offer distribution and uncertainty of job offers. The effect of the visiting cost c_i is generally ambiguous.

The hazard function is a product of the arrival rate and probability that an offer is acceptable

$$(7) \quad h(t) = \sum \sum a_{ij}(s(t)) [1 - F(u_{ij}(t))].$$

The hazard of moving is obtained by assuming that the moving cost c_i^m is positive. Correspondingly if the cost of changing occupations c_j is positive, $h(t)$ defines the hazard of changing occupations. The connection between search models and econometric models of unemployment duration is obtained by the well-known density function of duration models

$$(8) \quad f(t) = h(t) \exp\left(-\int_0^t h(\tau) d\tau\right),$$

and the connection with the expected value of an unemployment spell can be written as

$$(9) \quad E(T) = \int_0^{\infty} \exp\left(-\int_0^t h(\tau) d\tau\right) dt.$$

The hazard function is affected by two endogenous variables; the reservation utility and search intensity. Both of them have to be taken into account when examining the effects of exogenous variables on the hazard function. The UI benefits b and costs c_j and c_i^m increase the reservation utility and decrease the search intensity. Hence their effect on the hazard function is negative. The searching costs c decrease the reservation utility and increase the search intensity. Hence their effect on the hazard function is positive. The effect of the arrival rate of job offers on the hazard function has an ambiguous sign, since the direct effect is positive, but the indirect effect via the reservation utility is negative. The improvement of the offer distribution increases the reservation utility but by an amount which is less than the increase in the mean. In addition the offer distribution increases the search intensity. Therefore the effect is positive. The improvement of the uncertainty of job offers increases the reservation utility and search intensity. Hence the effect on the hazard is ambiguous. The effect of

the subjective rate of time preference on the hazard function is ambiguous, since it decreases the reservation utility and search intensity.

Summarizing the effects of exogenous variables on the hazard function, the following results are obvious. The hazard function is

- a) a decreasing function of the UI benefits b , permanent cost of re-employment c_j and moving cost c_i^m ,
- b) an increasing function of the searching cost c and improvement of job offers. The effects of the arrival rate of job offers a_{ij} , visiting cost c_i , subjective rate of time preference r and uncertainty of job offers on the hazard function are generally ambiguous.

The rest of this section is devoted to a finite horizon case. In that case the discounting factor of the expected income is $B(t) = [1 - \exp(-rt)]/r$. Figure 1 illustrates the effects of exogenous variables on the nonstationary paths of the reservation utility, search intensity and hazard function using numerical examples. The limited search horizon is the only cause of nonstationarity in these examples. The search horizon is assumed to be 40 years. Hence t measures the remaining time in the labour force. For simplicity it is assumed that the offers are uniformly distributed between 5000 and 15000 units of utility in a month.¹⁾ The value of time spent on search is assumed to be specified as $\delta s(t)^2$, where $\delta = 10000$ is a scaling factor and s is the search intensity. The arrival rate of job offers is specified as $\sum \sum a_{ij}(s) = 0.15s$. The remaining parameter values used in the numerical example are as follows: $b = 5000$, $r = 0.15/12$, $c_i = 1000$, $c_j = 2000$ and

$$c_i^m = 20000.^{2)}$$

It can be seen that the reservation utility is at first decreasing due to the limited search horizon, but during the last years it is increasing because of the lump sum type of moving cost. The search intensity is monotonously decreasing and during the last years it is not optimal to search at all. Finally it can be seen that the hazard function can be increasing or decreasing depending on the parameter values of the model. For the econometric specification of the hazard function it can be concluded that there is no monotonic form of the shape of the hazard function given by the search theory if a limited search horizon is assumed.

Figure 1. The effects of exogenous variables on the reservation utility, search intensity and hazard function

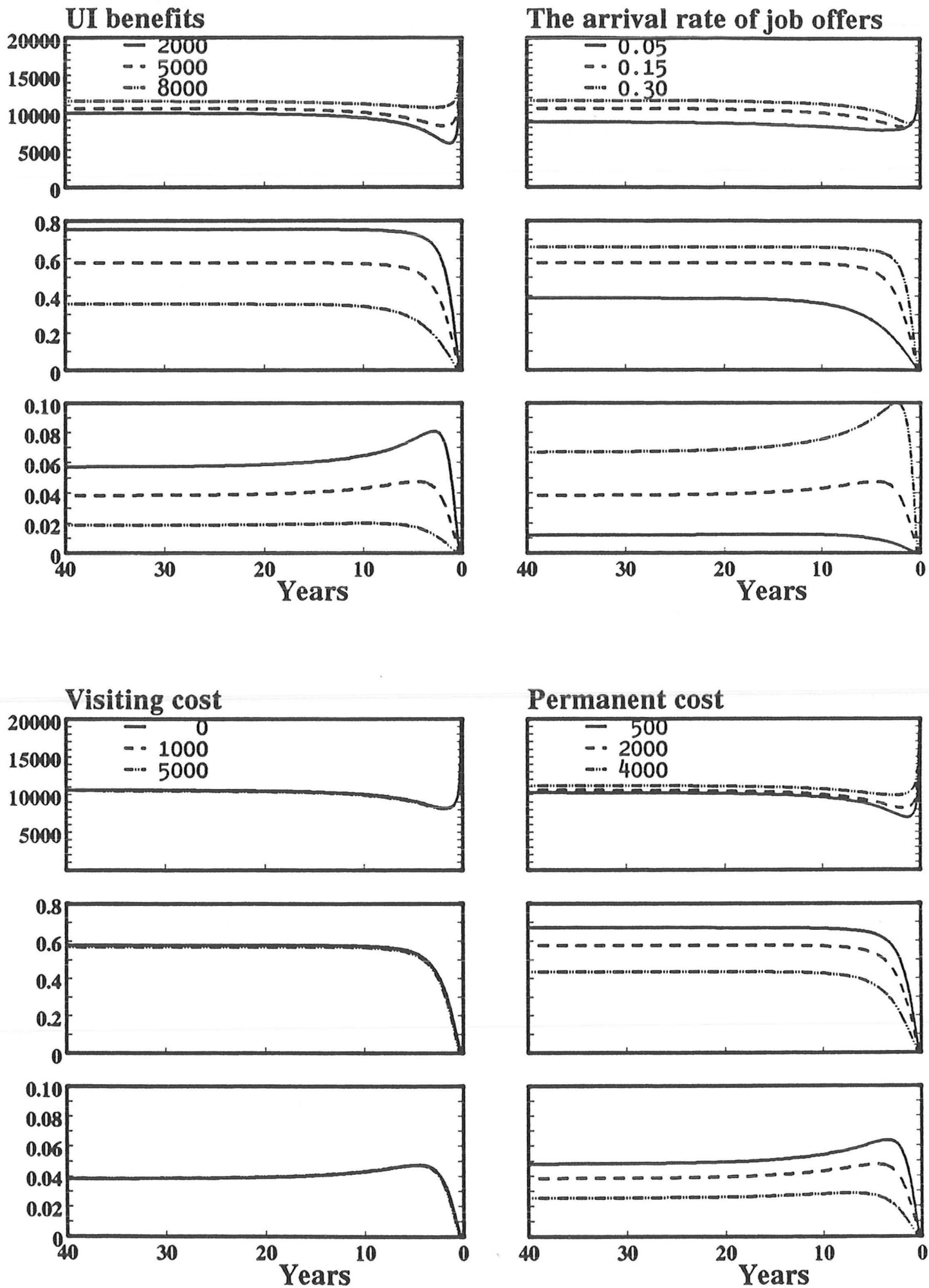
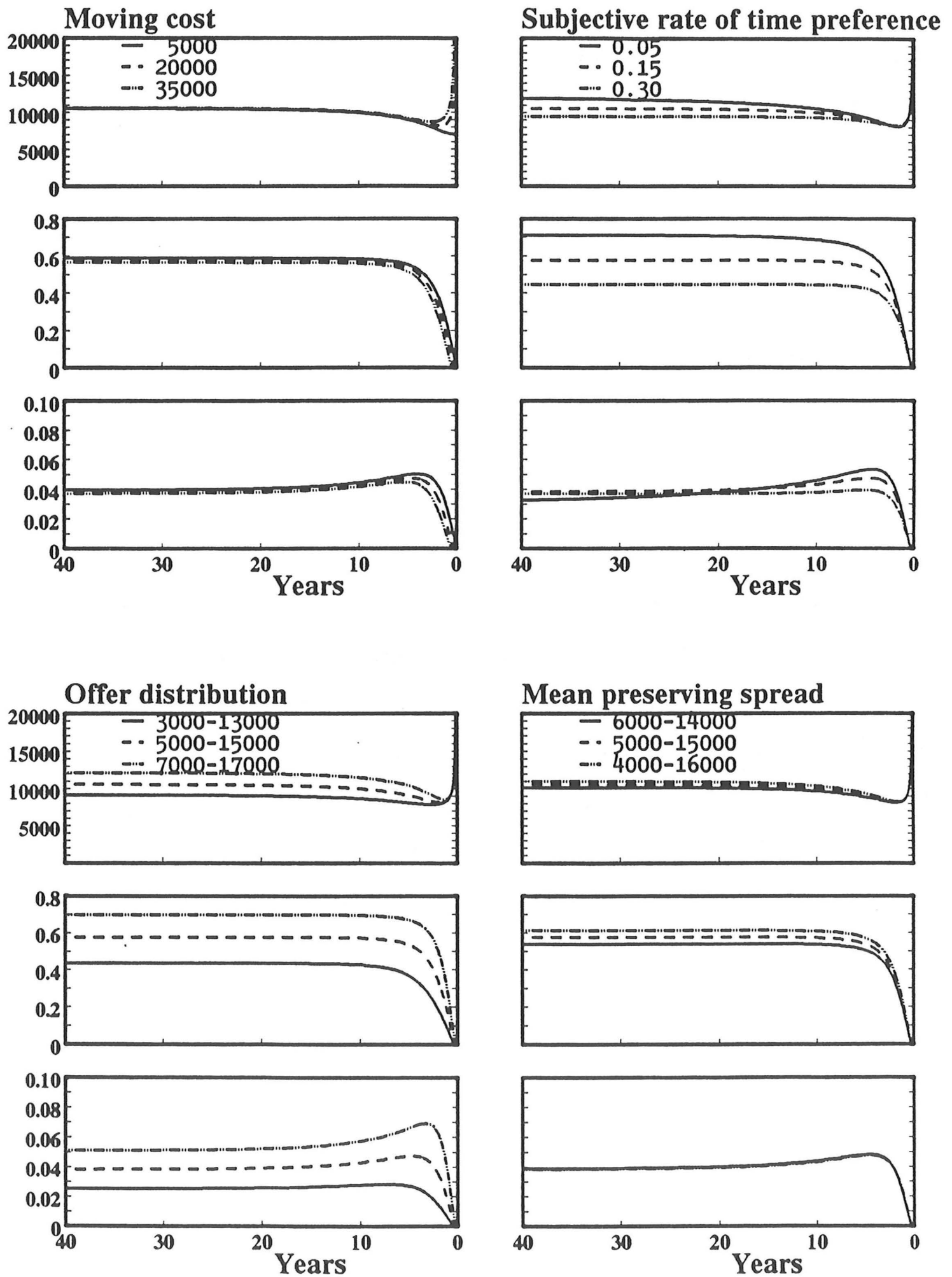


Figure 1. continued



3. The Effects of the UI System

3.1. The Waiting Period

According to the Finnish Unemployment Insurance Act benefits can be paid after a qualifying waiting period. It is normally one week or alternatively six weeks if the person has just entered the labour force or if he has quit his previous job. However, the waiting period of six weeks is not applied to a worker who has just finished school or who has been self-employed. In this section it is shown that the waiting period has a rather small effect on the re-employment probability and during the waiting period the hazard function is decreasing due to a fact that benefits are not yet paid.

The time concept in the applications to the UI system is such that at the outset of an unemployment period $t > 0$ and at the end of the waiting period $t = 0$. During the waiting period the instantaneous utility is $u_0(bD(t) - c - \sum \sum a_{ij}c_i, 1 - s^*(t))$, where $D(t) = \exp(-rt)$ and the asterisk is used refer to the functions affected by the feature of the UI system that is considered. If the person has not left unemployment, his instantaneous utility will be $u_0(b - c - \sum \sum a_{ij}c_i, 1 - s(t))$ after the waiting period once he has got his benefits.

The value of the search evaluated at $t+dt$ can be written as

$$\begin{aligned}
(10) \quad V^*(t+dt) = & u_0(bD(t) - c - \sum \sum a_{ij} c_i, 1 - s^*(t))B(dt) \\
& + \sum \sum a_{ij} dt \int_{u_{ij}^*(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V^*(t)] dF(u) D(dt) \\
& + V^*(t)D(dt) + o(dt).
\end{aligned}$$

It is obvious that $\lim_{t \rightarrow \infty} V^*(t) = V(t; b=0)$ and $\lim_{t \rightarrow 0} V^*(t) = V(t)$, i.e. $\dot{V}^*(t) < 0$, since $D(t) = \exp(-rt)$. If $t \leq 0$ then $V^*(t) = V(t)$. The reservation utility does not have a stationary solution during the waiting period, since the value function depends on how long the worker has been unemployed.

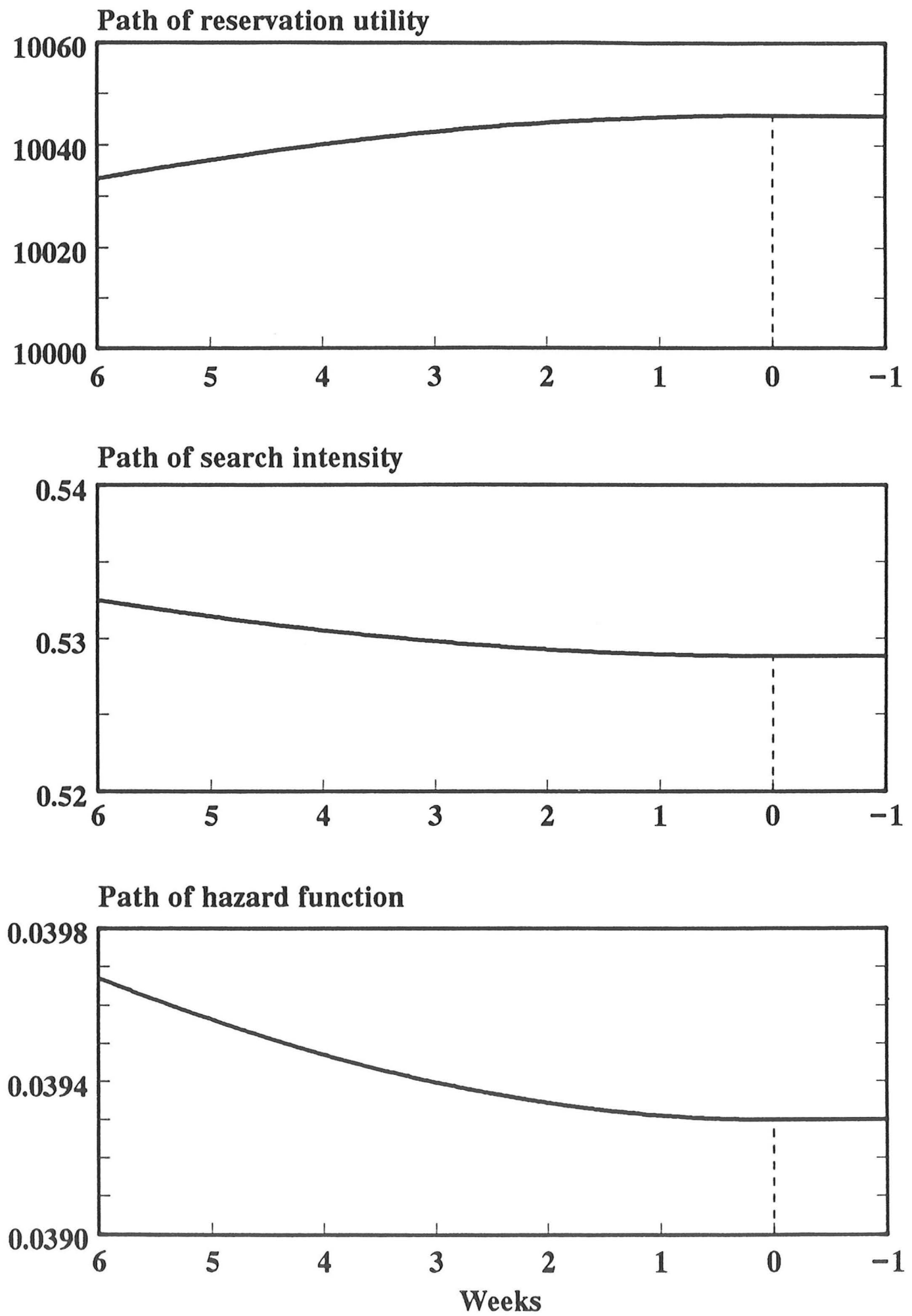
Solving the optimal reservation utility during the waiting period gives $u_{ij}^*(t) = c_j + r[c_i^m + V^*(t)]$. It is obvious that during the waiting period $u_{ij}^*(t) < u_{ij}(t)$, $s^*(t) > s(t)$ and $h^*(t) > h(t)$. Clearly $\partial u_{ij}^*(t)/\partial t < 0$, $\partial s^*(t)/\partial t > 0$ and $\partial h^*(t)/\partial t > 0$ during the waiting period, i.e. when the eligibility for UI benefits comes closer the reservation utility is increasing, and the search intensity and hazard function are decreasing.

A series of numerical examples are presented in this and the following sections to illustrate the nonstationary functions. It is assumed that the UI benefits $b = 5000$ if $t \leq 0$ and $b = 0$ during the waiting period. Furthermore, it is assumed that the offers are uniformly distributed between 5000 and 15000 units of utility in a month. This distribution is used, for instance, by Loikkanen and Pursiheimo (1979) and van den Berg (1987). Monthly figures are chosen since this is the common convention in Finland.

The value of time spent on searching is assumed to be specified as $\delta s(t)^2$, where $\delta = 10000$ is a scaling factor and s is the search intensity. The arrival rate of job offers is specified as $\sum \sum a_{ij}(s) = 0.15s$. The remaining parameter values used in the numerical example are as follows: $r = 0.15/12$, $c = 4000$, $c_i = 1000$, $c_j = 2000$ and $c_i^m = 20000$.

The effects of the qualifying waiting period have been illustrated in Figure 2. It can be seen that the changes of the reservation utility, search intensity and hazard function are small during the waiting period even though the subjective rate of time preference is rather high, and during the last week the functions are near the constant values. If r were lower, the changes in the functions would be smaller. The result is that the effects of the waiting period are very low. This finding leads to a conclusion that one way of improving the welfare of an unemployed person is to remove the waiting period, since it does not have much effect on the re-employment probability.

Figure 2. The effects of the waiting period



3.2. The Rule of Labour Mobility

The main rule in the Finnish Unemployment Insurance Act concerning labour mobility is that an unemployed person does not have to move outside his working area or change his occupation within the unemployment of the first three months. After that period he may no longer be eligible for UI benefits if he does not accept an offer obtained from the employment office. In this section it is shown that the threat of removal of benefits from a reluctant mover leads to a lower reservation utility and higher search intensity and hazard function. Furthermore, it is shown that the reservation utility is slightly decreasing, and the search intensity and hazard function are slightly increasing during the unemployment of the first three months. This approach is based on the argument by van den Berg (1990). He suggests for future research a model where, instead of perfect foresight with respect to the benefits, the individuals are aware of some additional elements of uncertainty and derive their optimal strategies given some probabilities that such changes occur.

The value of searching for a job can be written as

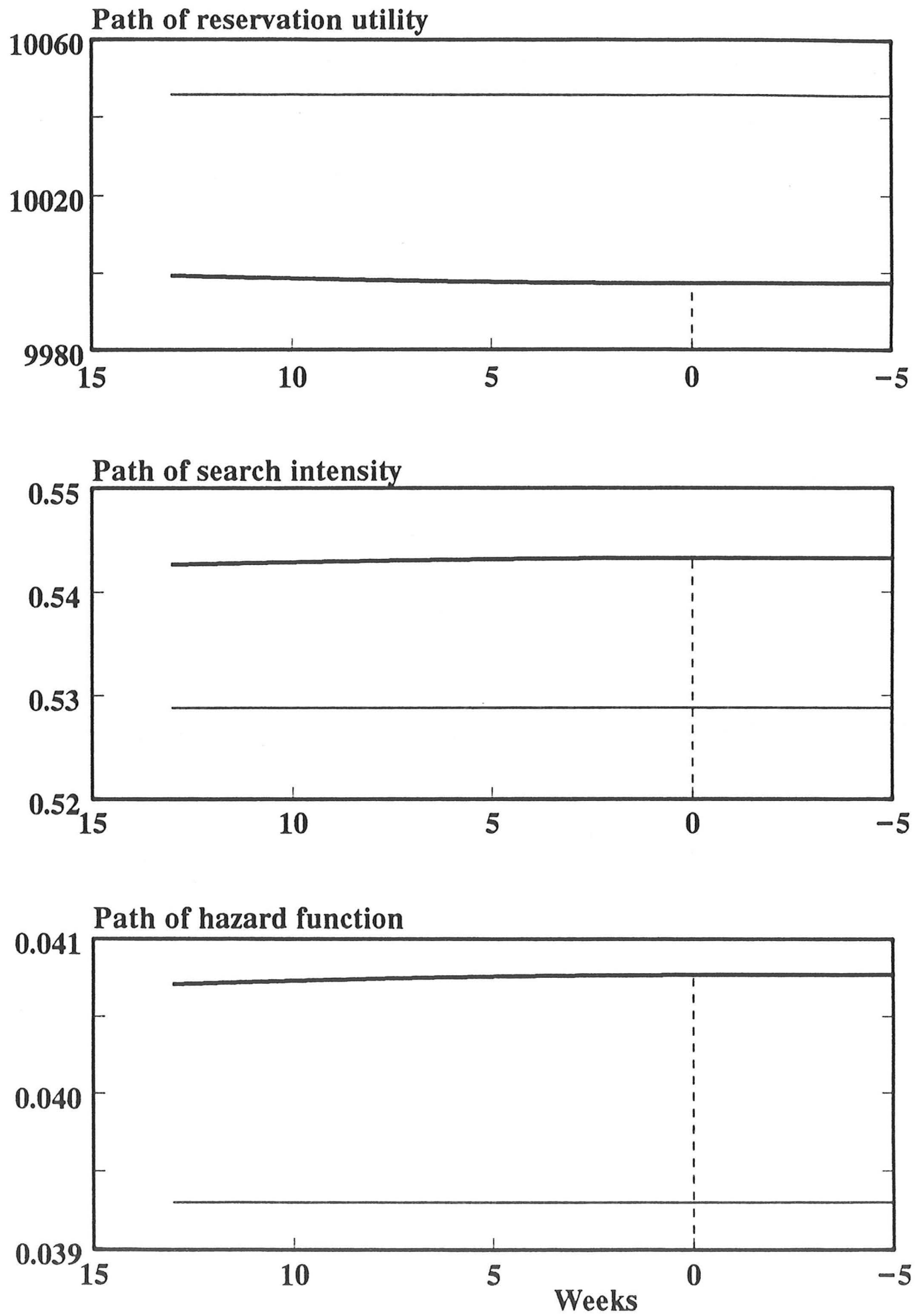
$$\begin{aligned}
 (11) \quad V^*(t+dt) = & u_0([1 - \sum \sum a_{ij} F(u_{ij}^*(t_0)) D(t)] b \\
 & - c - \sum \sum a_{ij} c_i, 1 - s^*(t)) B(dt) \\
 & + \sum \sum a_{ij} dt \int_{u_{ij}^*(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V^*(t)] dF(u) D(dt) \\
 & + V^*(t) D(dt) + o(dt),
 \end{aligned}$$

where $t_0 \leq 0$. The risk of losing UI benefits decreases the value of searching for a job. With a probability $\sum \sum a_{ij} F(u_{ij}^*(t_0))$ an unemployed person gets an offer, which is less than the reservation utility and loses his benefits. If an offer is accepted during the first three months, the person does not face such a risk. If he is unemployed and searching for a job, the associated instantaneous utility may change starting at $t = 0$. It is obvious that $\dot{V}^*(t) > 0$ before the risk period and $\lim_{t \rightarrow \infty} V^*(t) = V(t)$, since $D(t) = \exp(-rt)$. If the threat of removal of benefits is postponed, the threat of losing benefits matters less. If $\sum \sum a_{ij} = 0$ or the offers are at least $u_{ij}^*(t)$, then $V^*(t) = V(t)$ and the rule of labour mobility has no effects.

The optimal reservation utility during the first three months is $u_{ij}^*(t) = c_j + r[c_i^m + V^*(t)]$. It is obvious that $u_{ij}^*(t) < u_{ij}(t)$, $s^*(t) > s(t)$ and $h^*(t) > h(t)$. The risk of losing benefits after the first three months decreases the reservation utility and increases the search intensity and hazard function. Clearly $\partial u_{ij}^*(t)/\partial t > 0$, $\partial s^*(t)/\partial t < 0$ and $\partial h^*(t)/\partial t < 0$ during the first months, i.e. the path of the reservation utility is decreasing, and the paths of the search intensity and hazard function are increasing. Furthermore, it can be shown that the effects of UI benefits are decreasing over the spell of unemployment. The decreasing effect of UI benefits has been studied by Usategui (1988) in the case of a benefit period of finite duration.

The effects of the rules of labour mobility have been illustrated in Figure 3. The reservation utility is

decreasing, and the search intensity and hazard function are increasing during the first three months, and after the unemployment of three months the functions are constant. If there were no risk of losing benefits, the reservation utility would be higher and the search intensity and hazard function would be lower, which have been denoted by the straight horizontal lines. Compared to the waiting period it can be concluded that the rule of labour mobility has substantially larger effects. The effect of the risk of losing benefits has in these examples about four times as large an effect as the waiting period at six weeks until the eligibility of benefits. With a shorter waiting time the difference is even larger.

Figure 3. The effects of mobility rules

3.3. Reduction of UI Benefits

In this section a case where an unemployed person faces a relative reduction of UI benefits is considered. The earnings-related unemployment allowances decreased by 20 per cent after 100 days of unemployment in 1985 - 1987. It is shown that the path of the reservation utility is decreasing, and the paths of the search intensity and hazard function are increasing before the reduction. If the person faces a $k \cdot 100$ per cent reduction in his benefits, the instantaneous utility is $u_0((1-kD(t))b - c - \sum \sum a_{ij}c_i, 1 - s^*(t))$ at the beginning of the search. If the person has not left unemployment, his instantaneous utility is lower $u_0((1-k)b - c - \sum \sum a_{ij}c_i, 1 - s^*(t))$ once the reduction of benefits has happened.

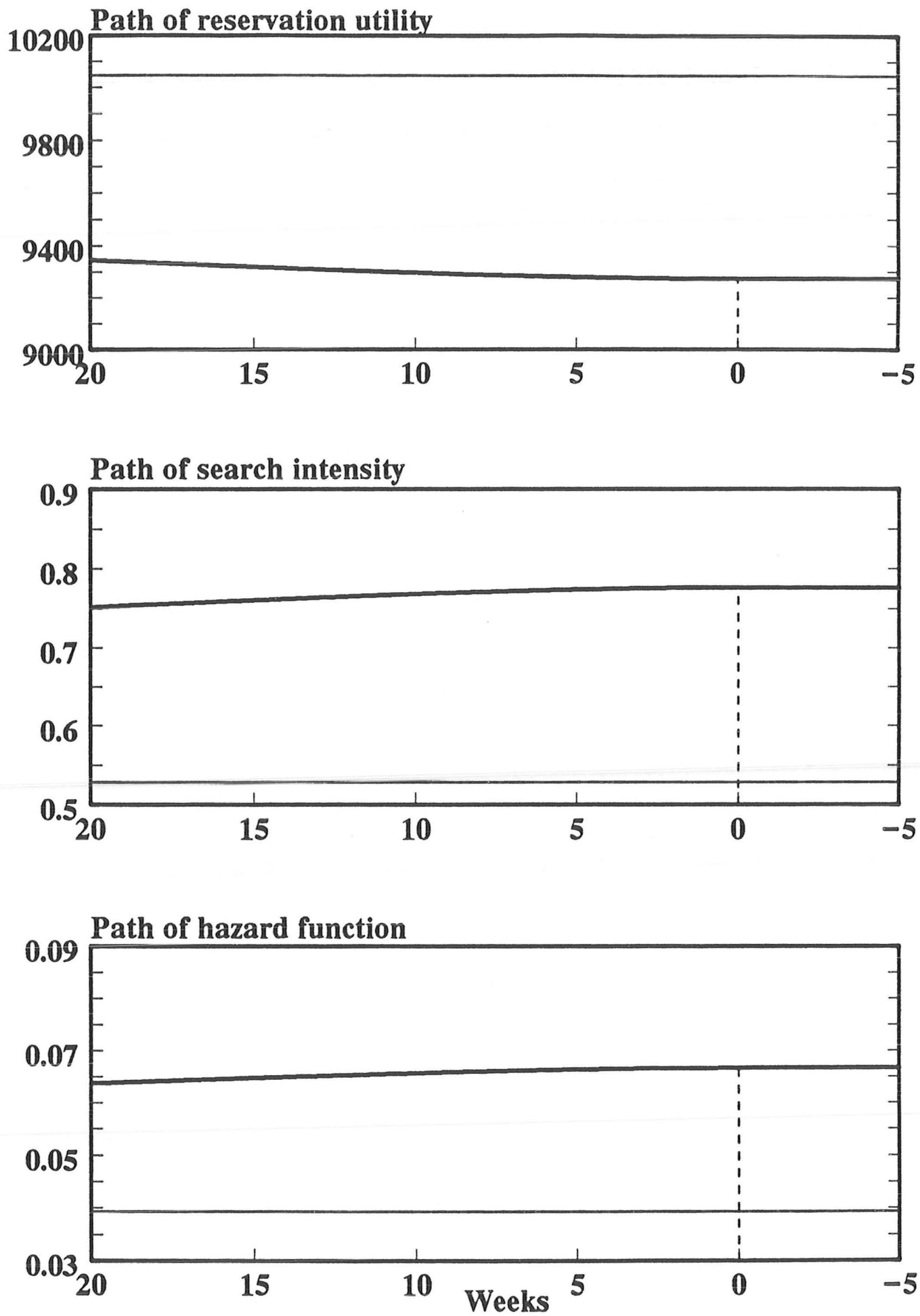
The value function can be written as

$$\begin{aligned}
 (12) \quad V^*(t+dt) = & u_0((1-kD(t))b - c - \sum \sum a_{ij}c_i, 1 - s^*(t))B(dt) \\
 & + \sum \sum a_{ij}dt \int_{u_{ij}^*(t)}^{\bar{u}} [(u - c_j)/r - c_i^m - V^*(t)]dF(u)D(dt) \\
 & + V^*(t)D(dt) + o(dt).
 \end{aligned}$$

The reductions decrease the expected value of utility. It is obvious that $\lim_{t \rightarrow \infty} V^*(t) = V(t)$, which is the value function with no reduction of UI benefits. If the reduction of benefits is postponed far into the future, the reduction does not matter. Clearly the value function is decreasing, i.e. $\dot{V}^*(t) > 0$.

The optimality condition for the reservation utility during the waiting period is found to be $u_{ij}^*(t) = c_j + r[c_i^m + V^*(t)]$. It is obvious that $u_{ij}(t) > u_{ij}^*(t)$, $s(t) < s^*(t)$ and $h(t) < h^*(t)$. Clearly $\partial u_{ij}^*(t)/\partial t > 0$, $\partial s^*(t)/\partial t < 0$ and $\partial h^*(t)/\partial t < 0$ before the reduction of benefits, i.e. as the cutoff date for the reduction comes closer the reservation utility decreases and the search intensity and hazard function increase.

The effects of the reduction of benefits have been illustrated in Figure 4. In the numerical example it has been assumed that the UI benefits have been reduced from 5000 to 1000 units of utility. The reservation utility is decreasing before the reduction, and the search intensity and hazard function are increasing. After the reduction the functions are constant. If there were no reductions, the reservation utility would be higher, and the search intensity and hazard function would be lower. These stationary functions have been described by the constant horizontal lines. It can be concluded that the reduction of benefits provides a substantial incentive to leave unemployment.

Figure 4. The effects of reductions of UI benefits

4. Conclusions

According to the comparative static results the UI benefits have a negative effect on the re-employment probability. This is a well-known result, but from the economic policy point of view it is interesting to know that the costs of re-employment have negative effects on the re-employment probability. Hence the conditional benefits can be used in order to reduce the re-employment costs and increase the probability of becoming employed. These findings support the "stick" and "carrot" approach of economic policy that the benefits paid during the unemployment should be stingy, but the benefits related to becoming employed can be generous.

Using search models it was shown that the hazard function is decreasing during the qualifying waiting period due to a fact that the benefits are not yet paid. Concerning the waiting period of UI benefits it can be concluded that it has only a slight effect on the re-employment probability. The improvement of the welfare of an unemployed person by removing the waiting period has a rather small negative effect on the re-employment probability.

Reluctant movers may lose their benefits if they do not accept an offer from other working areas or occupations after the first three months of unemployment. During the first three months of unemployment the hazard function is increasing for a person who gets benefits. The threat of removing benefits may substantially increase the re-employment probability if there are non-acceptable offers.

Persons who get earnings-related unemployment allowances face reductions of their benefits. The hazard function is increasing before the reduction. It was shown that the awareness of the reduction effectively increases the re-employment probability.

Footnotes

1. If the effects are uniformly distributed between \underline{u} and \bar{u} , the expected utility of an acceptable offer can be written in a closed form

$$E(u) = \int_{u_{ij}}^{\bar{u}} \tau / (\bar{u} - u_{ij}) d\tau = 0.5(\bar{u}^2 - u_{ij}^2) / (\bar{u} - u_{ij}),$$

where u_{ij} is the reservation utility.

2. The search intensity can be written as

$$s(t) = (0.5a/\delta) \{ [E(u)B(t)/0.5 - (1 - F(u_{ij}))(c_j B(t) + c_i^m + V)] D(dt) dt / B(dt) - c_i \},$$

which is in a closed form. Thus the numerical examples can be presented easily without numerical integration.

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Appendix 1. Comparative Static Results

Reservation utility

The fundamental equation for the reservation utility is solved from (4) by inserting $V = (u_{ij} - c_j)/r - c_i^m$, which gives

$$(13) \quad u_{ij} = u_0(b - c - \sum \sum a_{ij} c_i, 1 - s(t)) + c_j + r c_i^m \\ + \sum \sum a_{ij} \int_{u_{ij}}^{\bar{u}} (u - u_{ij}) dF(u) / r,$$

where the comparative static results can be solved:

$$(14) \quad \frac{\partial u_{ij}}{\partial b} = \frac{\partial u_0}{\partial C} > 0$$

$$(15) \quad \frac{\partial u_{ij}}{\partial c} = - \frac{\partial u_0}{\partial C} < 0$$

$$(16) \quad \frac{\partial u_{ij}}{\partial a_{ij}} = - \frac{\partial u_0}{\partial C} c_i + \int_{u_{ij}}^{\bar{u}} (u - u_{ij}) dF(u) / r > 0$$

$$(17) \quad \frac{\partial u_{ij}}{\partial c_i} = - \frac{\partial u_0}{\partial C} \sum a_{ij} < 0$$

$$(18) \quad \frac{\partial u_{ij}}{\partial c_j} = 1 > 0$$

$$(19) \quad \frac{\partial u_{ij}}{\partial c_i^m} = r > 0$$

$$(20) \quad \frac{\partial u_{ij}}{\partial r} = c_i^m - \sum \sum a_{ij} \int_{u_{ij}}^{\bar{u}} (u - u_{ij}) dF(u) / r^2.$$

The subjective rate of time preference r increases the reservation utility via the lump-sum type of moving cost, but on the other hand it decreases it via the expected utility. The total effect is usually negative. However, for example in an infinite horizon case the effect is negative only during the last few weeks. In the standard search model r has a negative effect on u_{ij} , since c_i^m does not exist.

To solve the effects of the offer distribution a translation of F to the right is made so that $F(u) = G(u + \mu)$, for all u and $\mu > 0$. This method was used by Mortensen (1986). The translation is said to first order stochastically dominate $F(u)$. Substituting the following useful transformation

$$(21) \quad \int_{u_{ij}}^{\bar{u}} (u - u_{ij}) dF(u) = E_F(u) - u_{ij} + \int_0^{u_{ij}} F(u) du$$

and $F(u) = G(u + \mu)$ for (13) and noting that $E_G(u) = \mu + E_F(u)$ gives

$$(22) \quad u_{ij} = u_0 + c_j + rc_i^m + \sum \sum a_{ij} [\mu + E_F(u) - u_{ij} + \int_0^{u_{ij}} F(u - \mu) du] / r,$$

where the effect of offer distribution on the reservation utility is solved as

$$(23) \quad \frac{\partial u_{ij}}{\partial \mu} = h / (r + h) > 0 \text{ and } < 1.$$

Next the effects of uncertainty of job offers are considered. Rothschild and Stiglitz (1970) have introduced the uncertainty to economics under the name 'mean preserving spread'. The distribution H is a mean preserving spread of F given that they have the same mean if and only if

$$(24) \quad \int_0^{u_1} H(u) du \geq \int_0^{u_1} F(u) du, \quad \text{for all } u_1 > 0.$$

Substituting (21) and $F(u) = H(u, \sigma)$ for (13) gives

$$(25) \quad u_{ij} = u_0 + c_j + rc_i^m + \Sigma \Sigma a_{ij} [E_F(u) - u_{ij} + \int_0^{u_{ij}} H(u, \sigma) du] / r,$$

where σ is the parameter of relative dispersion. The effect of uncertainty on the reservation utility is then

$$(26) \quad \frac{\partial u_{ij}}{\partial \sigma} = \Sigma \Sigma a_{ij} \int_0^{u_{ij}} H_\sigma du / (r + h) > 0.$$

Search intensity

The technique of solving the effects on the search intensity is presented, for instance, by Albrecht, Holmlund and Lang (1986). By the implicit function rule of differentiation the effect of the UI benefits is solved from (6)

$$(27) \quad \frac{\partial s}{\partial b} = - \frac{\partial V_s}{\partial b} / \frac{\partial V_s}{\partial s},$$

where $\partial V_s / \partial s < 0$ by the second order condition of the optimal search intensity. Therefore it is necessary to consider the sign of $\partial V_s / \partial b$, which is easily shown to be negative. The needed derivatives are

$$(28) \quad \frac{\partial V_s}{\partial b} = (- \Sigma \Sigma \frac{\partial^2 u_0}{\partial C \partial C} \frac{\partial a_{ij}}{\partial s} c_i - \frac{\partial^2 u_0}{\partial L \partial C}) / r < 0$$

$$(29) \quad \frac{\partial V_s}{\partial c} = - \frac{\partial V_s}{\partial b} > 0$$

$$(30) \quad \frac{\partial V_s}{\partial c_i} = \left[\Sigma \frac{\partial^2 u_0}{\partial C \partial C} \frac{\partial a_{ij}}{\partial s} c_i a_{ij} - \Sigma \Sigma \frac{\partial u_0}{\partial C \partial s} \frac{\partial a_{ij}}{\partial s} + \Sigma \frac{\partial^2 u_0}{\partial L \partial C} a_{ij} \right] / r$$

$$(31) \quad \frac{\partial V_s}{\partial c_j} = - \Sigma \Sigma \frac{\partial a_{ij}}{\partial s} [1 - F(u_{ij})] / r^2 < 0$$

$$(32) \quad \frac{\partial V_s}{\partial c_i^m} = - \sum \sum \frac{\partial a_{ij}}{\partial s} [1 - F(u_{ij})] / r < 0$$

$$(33) \quad \frac{\partial V_s}{\partial r} = - \sum \sum \frac{\partial a_{ij}}{\partial s} \int_{u_{ij}}^{\bar{u}} (u - c_j) dF(u) / r^2 - V_s / r < 0.$$

The sign of $\partial V_s / \partial c_i$ can not generally be determined, since the utility function is not known. In the numerical example the sign is clearly negative. Substituting the transformation (21) and $F(u) = G(u + \mu)$ for (6) and noting that $E_G(u) = \mu + E_F(u)$ gives

$$(34) \quad V_s = \left\{ - \sum \sum \frac{\partial u_0}{\partial c} \frac{\partial a_{ij}}{\partial s} c_i - \frac{\partial u_0}{\partial L} + \sum \sum \frac{\partial a_{ij}}{\partial s} [\mu + E_F(u) - u_{ij} + \int_0^{u_{ij}} F(u - \mu) du] / r \right\} / r.$$

Differentiation gives

$$(35) \quad \frac{\partial V_s}{\partial \mu} = \sum \sum \frac{\partial a_{ij}}{\partial s} [1 - F(u_{ij} - \mu)] / r^2 > 0.$$

Substituting (21) and $F(u) = H(u, \sigma)$ for (6) gives

$$(36) \quad V_s = \left\{ - \sum \sum \frac{\partial u_0}{\partial c} \frac{\partial a_{ij}}{\partial s} c_i - \frac{\partial u_0}{\partial L} + \sum \sum \frac{\partial a_{ij}}{\partial s} [E_F(u) - u_{ij} + \int_0^{u_{ij}} H(u, \sigma) du] / r \right\} / r.$$

Differentiation gives

$$(37) \quad \frac{\partial V_s}{\partial \sigma} = \sum \sum \frac{\partial a_{ij}}{\partial s} H(u_{ij}, \sigma) / r^2 > 0.$$

Chapter IV

**THE DURATION OF UNEMPLOYMENT ALLOWING FOR UNOBSERVED
HETEROGENEITY**

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Abstract

This chapter is concerned with the specification of parametric duration models and the effects of omitted explanatory variables, which may bias parameter estimates. Weibull models with gamma and mass point heterogeneity are estimated using Finnish unemployment duration data. Graphical examination of residuals derived for the heterogeneity models and numerical tests show that the discrete mass point mixing distribution is better than the continuous gamma distribution to rectify the model misspecification. Furthermore, a programme for estimating duration models by the maximum likelihood method is presented.

1. Introduction

In this chapter reduced form models of unemployment duration are estimated. The structural job search models act as a guide for the estimation. The main results of the search models are that the unemployment benefits decrease the probability of becoming employed, but on the other hand the members of UI funds have more elements of incentives for becoming employed. For example, in the line with the well known results by Mortensen (1977) it can be argued that the members of the UI funds have higher incentives for re-employment than the non-members, because the search behaviour of the persons who will be eligible for the

earnings-related part of benefits in the future after being employed at least 6 months is affected by the UI scheme. The earnings-related unemployment allowance make intermittent employment more attractive than it would otherwise be, because the value of re-employment is higher for the members [see also Hamermesh (1979) and Burdett (1979)].

In addition, the functional form of the distribution of unemployment duration has to be flexible, since the rules of the UI system do not stay constant during the long spells of unemployment. In order to take the imperfect nature of econometric models into account two methods for incorporating unobserved individual heterogeneity into Weibull duration models of unemployment spells are considered using Finnish unemployment data. It is well known that omitted variables cause bias to parameter estimates if duration models are not controlled for omitted variables [see Lancaster (1979), Nickell (1979a,b) Lancaster and Nickell (1980)]. Especially the shape of the hazard function of finding a job during a spell of unemployment is considered in this study. A Weibull model applied to the data produces a decreasing hazard function, but controlling for heterogeneity implies an increasing hazard function, which is in concordance with the standard search theory [see Kiefer and Neumann (1989)]. Hence the correction for heterogeneity and model specification tests are particularly important.

The improvement of model specification, when introducing heterogeneity, is shown using a graphical procedure based on examination of residuals derived for the

heterogeneity models. The discrete mass point mixing distribution is shown to provide a better pattern of heterogeneity than the continuous gamma heterogeneity.

The chapter is organized as follows. A Weibull model of unemployment duration is estimated in section 2 with the allowance for gamma heterogeneity and a discrete mass point heterogeneity. Section 3 deals with the model specification. The residuals of the estimated models are derived, and a graphical examination based on the residuals is presented. Furthermore, numerical tests are presented. The information matrix test is used to confirm the conclusions from the graphs, which may be subject to incorrect interpretations. Section 5 concludes the study. The programme for estimating nonlinear maximum likelihood models with an application to duration models of unemployment is described and presented in Appendix 1.

2. The Duration of Unemployment Allowing for Unobserved Heterogeneity

2.1. The Specification of Unemployment Benefits

In this section the specification of unemployment benefits is studied using a Weibull model of unemployment duration. Let us consider independent pairs of independent random variables T and Z , where T is the duration variable of primary interest and Z is a censoring variable.¹⁾ A duration or a censoring time is observed, $t = \min(T, Z)$, with the indicator for complete spells c . If $T < Z$, then $c = 1$ and otherwise $c = 0$.

Econometric models of duration are specified in terms of the hazard function $h(t)$, which is the conditional probability that an unemployed person leaves unemployment at time t given that he still is unemployed. The probability of being still unemployed until the duration t is given by the survivor function. The survivor function for T is equal to one minus the distribution function of the duration variable and it can be written

$$(1) \quad S(t) = e^{-I(t)},$$

where $I(t)$ is the integrated hazard

$$(2) \quad I(t) = \int_0^t h(\tau) d\tau.$$

Using the rule of conditional probabilities, the unconditional probability, i.e. density function, that an individual becomes employed at time t is a product of the hazard and survivor functions

$$(3) \quad f(t) = h(t)e^{-I(t)}$$

for $t \geq 0$. The likelihood contribution of an individual can be written in view of the above definitions as

$$(4) \quad \ell = h(t)^c e^{-I(t)},$$

which is equal to $f(t)$ if $c = 1$ and $S(t)$ if $c = 0$. The distribution of unemployment spells needs to be parametrized, and maximizing the likelihood function ℓ over the unknown parameters ϕ may be accomplished by maximizing a concave functional $L(\phi) = \sum \log \ell(\phi)$.

The Weibull model is a versatile family of duration distributions in view of its interpretation and its flexibility for empirical fit, and it has been widely used in applications of duration models to unemployment spells. The hazard function can be written as

$$(5) \quad h(t) = \alpha t^{\alpha-1} e^{x\beta},$$

where x is a vector of explanatory variables for an individual, β is a vector of structural parameters and α is the shape parameter. If $\alpha > 1$, the hazard function is increasing in duration and it is said that there is

positive duration dependence. If $\alpha = 1$, the hazard function is constant and the distribution of unemployment spells is exponential. If $\alpha < 1$, the hazard function is decreasing in time and it is said that there is *negative duration dependence*. The explanatory variables are introduced into the model in a log-linear form. An advantage of this form is that it renders positive estimates. The integrated hazard is written as $I(t) = \int_0^t h(\tau) d\tau + C$. The constant C is chosen such that $I(0) = 0$. Then the integrated hazard can simply be written as

$$(6) \quad I(t) = t^{\alpha} e^{x\beta}.$$

Consequently, the survivor, density and hazard functions of the Weibull distribution can be written as

$$(7) \quad S(t) = e^{-t^{\alpha} e^{x\beta}}$$

$$(8) \quad f(t) = \alpha t^{\alpha-1} e^{x\beta} - t^{\alpha} e^{x\beta}$$

$$(9) \quad h(t) = \alpha t^{\alpha-1} e^{x\beta}.$$

To estimate the unknown parameters, the hazard function (9) and the integrated hazard (6) are substituted into the likelihood contribution (4).

The first econometric attempt using parametric models is to study the specification of the replacement ratio of UI benefits. Table 1 includes the results concerning the

effects of unemployment benefits, monthly earnings and replacement ratios. The models include twelve other explanatory variables, but to save space their parameter estimates have been left out from the table. In the models (A), (B) and (C) continuous explanatory variables are used. The benefits and replacement ratio decrease the probability of becoming employed, but the effect of monthly earnings is statistically insignificant. The model (C) with the replacement ratio is superior, because it leads to the highest value of the log likelihood function.

Table 1. The effects of benefits, earnings and compensation ratios on the probability of becoming employed

	(A)	(B)	(C)
	Standard errors in parentheses		
Benefits	-0.395 (0.050)		
Earnings		-0.019 (0.012)	
Replacement ratio			-1.223 (0.150)
Log likelihood	-4964.2	-4994.7	-4962.5

Benefits, earnings and replacement ratio are thousands of Finnmark after tax in a month. Other explanatory variables are number of children, married, sex, age, level of education, training for employment, member of a labour union, came from schooling, came from housework, regional demand, occupational demand and taxable assets.

Lilja (1992) has emphasized that other features of the UI system than just the benefit level may involve effects on the probability of becoming employed. Using Finnish data

she estimated hazard models for the probability of becoming employed separately for the persons who received the basic and earnings-related unemployment allowances and for the persons who did not receive any benefits. The approach is comparable to the studies for the U.S., where the disadvantage of the administrative data is that figures for only the persons who have begun receiving UI benefits are available [see e.g. Moffitt (1985)]. The problem related to the truncated data has been pointed out also by Atkinson and Micklewright (1991). In the light of these studies separate models for the recipients and non-recipients of these two benefits were estimated.

Table 2 presents the results of the models, which are estimated separately for the non-recipients and recipients of the different benefits. There are some notable differences in the parameter estimates. The level of education has a negative effect for the recipients of earnings-related benefits, but a positive effect for the persons receiving the basic benefit. The persons who entered the labour force from housework and obtained basic benefits have more problems in finding acceptable jobs. The occupational demand and the taxable assets of labour increase strongly the re-employment probability of the recipients of the earnings-related unemployment allowance. The estimated coefficients of replacement ratios are both negative, but their absolute values are lower than in the model where the non-recipients of benefits are included. The coefficient of the replacement ratio for the recipients of the earnings-related unemployment allowance does not statistically differ from zero. This result shows that care

is needed in interpreting the results based on data sets where the non-recipients of benefits are truncated.

The replacement ratios have a different effect on the re-employment probability depending whether the non-recipients of benefits are included in the sample. This result leads us to consider another kind of specification of the model. Using Finnish data Eriksson (1985) estimated models of unemployment duration with indicators for the recipients of the benefits of the two different benefit schemes.

The first model of Table 3 corresponds to the specification used by Eriksson. The results of these two studies are rather similar. The receipt of the basic unemployment allowance has a negative effect on the re-employment probability. It is nearly twice as high as the effect for the recipients of the earnings-related unemployment allowance. In the second model only the recipients of the benefits are included in the sample. The effects are estimated separately for the replacement ratios in both of the benefit schemes. In the third model the whole sample including the non-recipients of benefits is used for the estimation of the effects of the corresponding replacement ratios. It turns out that the total effect of the replacement ratio can be decomposed into the receipt of the benefits and the level of the replacement ratio of the persons receiving benefits.

Similar models were estimated without making the distinction between the two benefit schemes. The parameter of the receipt of benefits took a statistically significant value of -0.582. The parameter of the level of the benefits

took a significant value of -0.602. The sum of these two effects is approximately the parameter estimate of the replacement ratio, which took a value of -1.223. These results show that the truncated benefit data without non-recipients of benefits tells only a part of the effects of the replacement ratios.

**Table 2. Weibull models of unemployment duration with
duration-dependent replacement ratios**

	(A)	(B)	(C)
	Std.errors in parentheses		
(A) recipients of the basic allowance			
(B) recipients of the earnings-related allowance			
(C) non-recipients			
Shape	1.115 (0.046)	1.064 (0.075)	0.742 (0.027)
Constant	-2.248 (0.284)	-1.801 (0.573)	-1.406 (0.190)
Number of children	0.002 (0.069)	0.090 (0.095)	-0.068 (0.850)
Married	0.074 (0.114)	0.304 (0.177)	0.128 (0.096)
Sex	-0.168 (0.106)	-0.065 (0.148)	0.138 (0.084)
Age	-0.044 (0.006)	-0.069 (0.008)	-0.029 (0.005)
Level of education	0.339 (0.108)	-0.372 (0.161)	0.076 (0.082)
Training for employment	0.036 (0.135)	0.101 (0.188)	0.298 (0.102)
Member of UI fund	0.188 (0.148)	0.162 (0.432)	0.098 (0.089)
Came from schooling	0.187 (0.120)	0.136 (0.294)	0.436 (0.117)
Came from housework	-1.026 (0.193)	-0.449 (0.226)	-0.665 (0.222)
Regional demand	-0.527 (0.601)	-0.009 (0.899)	0.165 (0.286)
Occupational demand	0.194 (1.156)	3.824 (1.362)	0.050 (0.905)
Taxable assets	0.171 (2.538)	3.959 (1.825)	-1.874 (1.739)
Replacement ratio	-0.995 (0.236)	-0.124 (0.387)	
Log likelihood	-1823.8	-925.7	-2156.0
Number of observations	720	337	1025

**Table 3. Weibull models of unemployment duration with
duration-dependent replacement ratios**

(A) indicators for the receipt of benefits
(B) replacement ratios for the recipients
(C) replacement ratios for the whole sample

	(A) Std.errors	(B) in parentheses	(C) Std.errors
Shape	0.868 (0.020)	1.086 (0.038)	0.866 (0.020)
Constant	-1.385 (0.133)	-2.031 (0.227)	-1.423 (0.136)
Number of children	0.032 (0.052)	0.035 (0.052)	-0.007 (0.051)
Married	0.133 (0.065)	0.155 (0.089)	0.167 (0.065)
Sex	0.029 (0.057)	-0.135 (0.079)	0.017 (0.056)
Age	-0.041 (0.003)	-0.053 (0.004)	-0.043 (0.003)
Level of education	0.112 (0.057)	0.181 (0.085)	0.073 (0.057)
Training for employment	0.190 (0.071)	0.075 (0.105)	0.193 (0.072)
Member of UI fund	0.062 (0.068)	-0.055 (0.114)	0.100 (0.066)
Came from schooling	0.309 (0.079)	0.178 (0.106)	0.286 (0.078)
Came from housework	-0.672 (0.125)	-0.788 (0.145)	-0.684 (0.126)
Regional demand	-0.035 (0.240)	-0.347 (0.504)	0.115 (0.239)
Occupational demand	0.835 (0.584)	1.391 (0.829)	0.639 (0.599)
Taxable assets	0.848 (1.066)	2.261 (1.322)	0.860 (1.086)
Recipient of ERUA	-0.399 (0.089)		
Recipient of BUA	-0.683 (0.071)		
Replacement ratio of ERUA		-0.108 (0.257)	-0.706 (0.212)
Replacement ratio of BUA		-1.054 (0.222)	-1.625 (0.186)
Log likelihood	-4953.0	-2156.0	-4956.5
Number of observations	2077	1052	2077

ERUA = earnings-related unemployment allowance
BUA = basic unemployment allowance

2.2. The Gamma Mixing Distribution

In this section an approach to the incorporating of gamma heterogeneity into duration models is described and the integrated hazard for graphical examination of residuals is derived. It is inevitable that in an econometric analysis relevant variables will be omitted either because they are unmeasurable or because their importance is unsuspected. Even if the omitted variables are uncorrelated with those which are included in the model the parameters will be biased towards zero [Nickell (1979b)]. The usual method for incorporating heterogeneity is to assume a parametric functional form for the pattern of the heterogeneity. The gamma mixing distribution has been chosen, because it is analytically simple to use and it provides quite a flexible model for the distribution of the heterogeneity component.

Lancaster (1979) found that the estimated falling hazard function represents, at least in part, merely the effect of unrecognized heterogeneity of the sample individuals, i.e. omitted variables. He introduced regressors into the model one at a time and each time found that the parameter estimates increased. Rather than being an estimate of a behavioral parameter, α is, at least in part, merely an index of specification error. The more significant regressors are included, the larger it becomes. It may be shown under fairly general conditions that the coefficients of explanatory variables are then biased towards zero [Lancaster and Nickell (1980)]. Therefore, we

may expect the parameters of the model to increase in absolute value when the effects of omitted variables are taken into account.

The method of correcting for gamma heterogeneity has been widely used during the 1980's in duration models. [e.g. Kooreman and Ridder (1983), Newman and McCulloch (1984), Narendranathan, Nickell and Stern (1985) and Engström and Löfgren (1987)]. The model specification has not, however, been examined in these studies.

Suppose the individuals of the sample differ to some certain degree with respect to some unobservable variable, say, motivation v . Each individual has his own v and hence his own hazard function $h(t)$. Lancaster using data from the stock of unemployed persons assumed that these hazard functions have a gamma distribution. The conditional hazard function in a Weibull model allowing for gamma heterogeneity is

$$(10a) \quad h(t|v) = v\alpha t^{\alpha-1} e^{-\beta v t^\alpha},$$

where v has a gamma density

$$(10b) \quad g(v) = \frac{\psi^\mu}{\Gamma(\mu)} v^{\mu-1} e^{-\psi v} \quad \text{with} \quad \Gamma(\mu) = \int_0^\infty w^{\mu-1} e^{-w} dw.$$

The expected value of the heterogeneity component $E(v) = \mu/\psi$ is normalized to one by setting $\psi = \mu$ and its variance, i.e. $\sigma^2 = 1/\mu$, is estimated. Integrating the survivor function over the assumed mixing distribution gives a

closed form for the survivor function with gamma heterogeneity. Differentiation gives the corresponding density function. The marginal hazard function, not conditional on v , is obtained as a ratio of the density and survivor functions. The hazard function allowing for gamma mixing distribution can then be written as

$$(11) \quad h(t) = \alpha t^{\alpha-1} e^{x\beta} [1 + \sigma^2 t^\alpha e^{x\beta}]^{-1}.$$

Integrating (11) from zero to t gives the needed integrated hazard

$$(12) \quad I(t) = 1/\sigma^2 \log[1 + \sigma^2 t^\alpha e^{x\beta}].$$

$I(t)$ has a unit exponential distribution, as will be seen in section 3. The hazard function (11) and the integrated hazard (12) are substituted into the likelihood contribution (4) to estimate the unknown parameters.

The data of 2077 Finnish unemployed persons were used to estimate the econometric models. For estimating duration models and developing statistical tests it was found useful to write the needed programmes using the SAS/IML (1985) programming language. The programme is reported in Appendix 1. The Berndt, Hall, Hall and Hausman (1974) algorithm was used to estimate the unknown parameters. It requires the analytic first derivatives of the log likelihood function with respect to the parameters to be estimated.

The results of the estimations assuming a Weibull distribution are in the first column of Table 4.

Exponential, lognormal, loglogistic and gamma distributions were also tried to estimate different hazard models, but it turned out that the parameter estimates of the structural parameters were only slightly different from the corresponding estimates using the Weibull distribution. The constant of the model, where the effect of omitted variables is captured, decreases and the absolute values of the statistically significant parameter estimates increase in most cases when gamma heterogeneity is introduced into the Weibull model, as is to be expected. The basic Weibull model produces a decreasing hazard function, but the shape parameter of the Weibull model with gamma heterogeneity takes a value larger than one indicating increasing hazard functions for the individuals. The sample hazard function with a gamma mixing distribution is increasing at the beginning of unemployment, but later on it turns into a decreasing function.

Table 4. Gamma heterogeneity in a Weibull model

Dependent variable:		
The length of the spell of unemployment		
	(A)	(B)
(A) A Weibull model	Std.errors	
(B) A Weibull model allowing for gamma heterogeneity	in parentheses	
Shape parameter	0.861 (0.020)	1.201 (0.058)
Variance of heterogeneity		1.045 (0.162)
Constant	-1.478 (0.136)	-1.157 (0.210)
Number of children	-0.004 (0.050)	-0.020 (0.080)
Married	0.170 (0.065)	0.135 (0.101)
Sex	-0.007 (0.056)	-0.066 (0.090)
Age	-0.042 (0.003)	-0.057 (0.005)
Level of education	0.064 (0.058)	0.035 (0.095)
Training for employment	0.176 (0.072)	0.321 (0.119)
Member of UI fund	0.213 (0.060)	0.364 (0.096)
Came from schooling	0.291 (0.078)	0.375 (0.130)
Came from housework	-0.711 (0.124)	-0.892 (0.176)
Regional demand	0.168 (0.238)	0.353 (0.338)
Occupational demand	0.641 (0.600)	-0.098 (0.963)
Taxable assets	1.021 (1.080)	0.822 (1.379)
Replacement ratio	-1.223 (0.150)	-2.243 (0.261)
Log likelihood	-4962.5	-4920.6

2.3. The Discrete Mixing Distribution

In this section a mass point approach to the incorporation of unobserved heterogeneity into duration models is described. A method to estimate a discrete mixing distribution is described and integrated hazards for graphical examination of residuals are derived. The main method for incorporating unobserved heterogeneity has been to assume a parametric functional form for the pattern of heterogeneity. Heckman and Singer (1984a,b), who propose a discrete pattern of heterogeneity, have shown that estimates of the structural parameters may be sensitive with respect to the parametric forms assumed for heterogeneity. Furthermore, there are a limited number of tractable forms for mixing distributions available.

The approach dispensing with the need to specify a parametric distribution for the heterogeneity component has its origins in the work of Kiefer and Wolfowitz (1956), who showed that a nonparametric characterization of the heterogeneity distribution ensures consistent estimation of simultaneously estimated structural parameters. Further work on the properties of mass point mixing distributions has been carried out by Simar (1976), Laird (1978), Lindsay (1983a,b) and Heckman and Singer (1984a,b). Applications of the mass point approach in the context of discrete choice models have been presented by Davies and Crouchley (1984), Dunn, Reader and Wrigley (1987), Davies (1987) and Card and Sullivan (1988). Applications to duration models have been

presented by Brännäs (1986a,b), Trussell and Richards (1987) and Ham and Rea (1987).

To illustrate the discrete heterogeneity problem with discrete variables, suppose for simplicity that there are in the sample two groups, which have different constant hazard functions $h_1(t) > h_2(t)$ for all $t \geq 0$ and which are not controlled by explanatory variables in the data. At $t = 0$ the estimated hazard is the average of the hazards of these groups. The proportion of the low hazard group increases over time and the estimation gives an indication that the hazard function of the individuals is falling when it is in fact constant. The average hazard of the sample is converging asymptotically to the hazard function $h_2(t)$. In the sequel of this section it can be seen that this example happens to come true with the data in the case of two mass points.

Define the function $f_Q = \int_0^\infty f_u(t) dQ(u)$ to be the mixture density corresponding to a mixing distribution Q . The densities f_u are atomic densities for each value of u . A convex combination of m elements of f_u can be written as $\sum p_i f_{u_i}$ with the restriction $\sum p_i = 1$. It is assumed that the density of unobserved heterogeneity has a particular functional form, namely the likelihood function has been specified so that there are m types of individuals in the sample not controlled by explanatory variables. The probabilities p_i are the shares of these groups, but it is not possible to distinguish between m types of individuals.

In the case of parametric duration models the mixing likelihood contribution can be written as

$$(13) \quad f_Q = \sum_{i=1}^m p_i h_i(t) e^{-I_i(t)},$$

where $h_i(t) = \alpha t^{\alpha-1} e^{u_i+x\beta}$ and $I_i(t) = t^\alpha e^{u_i+x\beta}$ are the atomic hazard functions and integrated hazards respectively. The objective is to estimate the discrete mixing distribution consistently with the atomic densities, a maximizer of the mixture likelihood function $\mathcal{L}(Q) = \pi f_Q$. Maximizing the likelihood function $\mathcal{L}(Q)$ over Q may be accomplished by maximizing the concave function $L(f) = \sum \log f_Q$. The problem is equivalent to the maximization of a concave function subject to finitely many linear constraints.

To ensure that the probabilities $p_i \in (0, 1)$ and that $\sum p_i = 1$, the probabilities associated with each location have been defined using a multinomial logit type of formula

$$(14) \quad p_i = \frac{e^{g_i}}{1 + \sum_{k=1}^{m-1} e^{g_k}}, \quad i = 1, \dots, m-1,$$

where g_k , $k = 1, \dots, m-1$ are parameters to be estimated. The probability of the last mass point $p_m = 1 - p_1 - p_2 - \dots - p_{m-1}$. By definition $p_1 = 1$, when $m = 1$. The parameters g_k work only as a device. They do not have an interesting economic interpretation in this context.

The standard errors of the probabilities p_i can be approximated by the well-known delta method. The first order Taylor series expansion gives

$$(15) \quad \hat{p}_i(\hat{g}) = p_i(g) + (\hat{g} - g)' \frac{\partial p_i}{\partial g},$$

where $g = (g_1 \dots g_{m-1})$. The variance can then be approximated by

$$(16) \quad \text{Var}[\hat{p}_i(\hat{g})] = \frac{\partial \hat{p}_i}{\partial \hat{g}}' \text{Var}(\hat{g}) \frac{\partial \hat{p}_i}{\partial \hat{g}}.$$

Locations of mass points are defined as $\exp(u_i)$. The vector of ones has been left out from the explanatory variables to avoid singularity. The idea of mass point models can be expressed so that the constant parameter β_0 of the basic model is partitioned in m location parameters u_i and each of the location parameters is given a probability p_i . In the case where $m = 1$, when there is one location parameter, the parameter u_1 is equal to the constant of the basic Weibull model β_0 . Consequently, the likelihood function of mass point models reduces to the likelihood function of the basic Weibull model, and the model with one mass point and the basic Weibull model coincide.

Following Lindsay (1983a) it can be seen that the log likelihood function $L(f) = \sum \log f_Q$ is differentiable with the directional derivative of L at L_{Q_0} towards L_{Q_1} being

$$(17) \quad D(u; Q) = \lim_{p \rightarrow 0} \{L[(1-p)f_{Q_0} + pf_{Q_1}] - L(f_{Q_0})\}/p$$

$$= \sum [(f_{Q_1} - f_{Q_0})/f_{Q_0}]$$

$$= \sum f_{Q_1}/f_{Q_0} - n,$$

where it will be understood that the summing is over observations. The procedure of estimating a discrete mixing distribution is to increase the number of points of support until $D(u; Q) \leq 0$. Then the procedure is stopped and the semi-parametric ML estimator is obtained. This procedure is suggested also by Brännäs and Rosenqvist (1988). Maximum likelihood algorithms are directly applicable to the constrained problem of maximization over discrete mixtures Q with a fixed number of support points. A simple first order check for a global maximum is to verify that the second derivative $D''(u^*; Q) \leq 0$ at the support points of measure Q . The Berndt, Hall, Hall and Hausman (1974) algorithm is used to estimate the unknown parameters.

It should be noted that only the consistency of the estimates has been established (Kiefer and Wolfowitz, 1956). A formal inferential framework beyond their proof has not yet been established for mass point methods. The standard errors of the estimated parameters are obtained from the estimated information matrix. Therefore these have no rigorous justification even though this procedure has been used in practice by, for example, Heckman and Singer (1984b) and Davies and Crouchley (1984).

The integrated hazard of the mass point models needs to be derived. The density and survivor functions are obtained from the mixing likelihood contribution (13) by setting $c = 1$ and $c = 0$ respectively. The hazard function $h(t)$ is the ratio of these two functions, i.e. $h(t) = f(t)/S(t)$.

$$(18a) \quad f(t) = \sum_{i=1}^m p_i h_i(t) e^{-I_i(t)}$$

$$(18b) \quad S(t) = \sum_{i=1}^m p_i e^{-I_i(t)}$$

$$(18c) \quad h(t) = \sum_{i=1}^m p_i h_i(t) e^{-I_i(t)} / \sum_{i=1}^m p_i e^{-I_i(t)}.$$

Integrating the hazard function gives a rather simple expression for the integrated hazard

$$(19) \quad I(t) = \log\left\{\left[\sum_{i=1}^m p_i e^{-I_i(t)}\right]^{-1}\right\},$$

which is needed in the graphical examination of residuals. It is based on the fact that $I(t)$ has a unit exponential distribution in the absence of censoring, as will be seen in the next section. Note that if $m = 1$ the integrated hazard (19) reduces to the integrated hazard of the basic Weibull model (6).

The results of estimations of the mass point models are presented in Table 5. The values of function D of the models with 2, 3, 4 and 5 mass points are 0.86, 5.97, 1.13 and -3.45 respectively, showing that five points of support are enough to rectify the effect of omitted variables with this data. The model with two mass points produces constant hazard functions for the two groups, which are not controlled for explanatory variables. Models with three or more mass points produce increasing hazard functions. An

increasing hazard function is in concordance with standard search theories with a limited search horizon. The absolute values of statistically significant parameter estimates increase in most cases when more mass points are introduced into the model, as is to be expected.

Many of the explanatory variables have significant effects on the re-employment probability. Age is a very significant factor like in many other countries [see Lancaster (1979), Nickell (1979a,b), Heckman and Borjas (1980), Kooreman and Ridder (1983), Atkinson, Gomulka, Micklewright and Rau (1984), Narendranathan, Nickell and Stern (1985) and Folmer and van Dijk (1986)]. Older people are more likely to have problems in finding jobs. Training for further employment has a significant and positive effect on the re-employment probability. Members of the UI funds, i.e. members of the labour unions in the Finnish system, become employed earlier than the non-members, as expected by the search models. Similar results have been obtained for Finland by Lilja (1992) using the data of the Labour Force Surveys. On the other hand, Narendranathan, Nickell and Stern (1985) found for the U.K. that the members of labour unions had lower re-employment probabilities. This result is most likely due, however, to the different systems of unemployment insurance. The persons leaving school or the military service usually have no great problems. They leave unemployment clearly earlier than the others. The persons who have come from housework find it very difficult to find a job. The effects of unemployment benefits are measured in this study using the benefit replacement ratio. This variable is not always

defined in the same way. There is a unanimity, however, in the qualitative impacts of how the replacement ratio affects the probability of becoming employed. Similar results have been obtained by Lancaster (1979), Nickell (1979a,b), Atkinson, Gomulka, Micklewright and Rau (1984) and Narendranathan, Nickell and Stern (1985). The benefits decrease significantly the re-employment probability, as is expected by the search theoretical models. The number of children, marriage, gender, level of education, demand variables and taxable assets do not have statistically significant effects on the re-employment probability in these models.

In Figure 1 the probabilities of mass points p_i are plotted against the locations $\exp(u_i)$. The mixing distribution does not seem to be very far from a gamma distribution. There seems to be a pattern in the way new mass points are located. When the number of mass points is increased, each location in the previous model seems to get new locations on both its sides in the next model. Furthermore, they seem to take less mass than the neighbour mass points in the previous model.

The sample hazard functions (11) and (18c) have been illustrated in Figure 2 for a person with average characteristics in the sample. Even though the hazard functions of the different groups of the mass point models are constant or increasing, the hazard function for the sample does not need to be monotonous. The sample hazard function of the final mass point model is decreasing except for the first few weeks.

Table 5. Mass point heterogeneity in a Weibull model

Dependent variable: The length of the spell of unemployment

	Number of mass points			
	m=2	m=3	m=4	m=5
	Std.errors in parentheses			
Shape parameter	0.998 (0.034)	1.245 (0.063)	1.457 (0.114)	1.671 (0.182)
Number of children	-0.004 (0.059)	-0.035 (0.080)	-0.020 (0.095)	-0.034 (0.108)
Married	0.126 (0.080)	0.119 (0.099)	0.144 (0.117)	0.157 (0.132)
Sex	-0.058 (0.070)	-0.050 (0.088)	-0.044 (0.105)	-0.088 (0.119)
Age	-0.049 (0.004)	-0.060 (0.005)	-0.070 (0.007)	-0.082 (0.011)
Level of education	0.045 (0.074)	0.063 (0.095)	0.059 (0.112)	0.074 (0.125)
Training for employment	0.257 (0.091)	0.276 (0.117)	0.375 (0.140)	0.367 (0.157)
Member of UI fund	0.260 (0.074)	0.333 (0.094)	0.407 (0.113)	0.473 (0.136)
Came from schooling	0.261 (0.099)	0.384 (0.128)	0.450 (0.155)	0.399 (0.171)
Came from housework	-0.765 (0.143)	-0.950 (0.184)	-1.029 (0.218)	-1.232 (0.271)
Regional demand	0.221 (0.274)	0.396 (0.348)	0.542 (0.408)	0.432 (0.464)
Occupational demand	0.233 (0.736)	0.038 (0.932)	-0.529 (1.129)	0.020 (1.268)
Taxable assets	0.781 (1.176)	2.166 (1.546)	2.124 (1.930)	0.553 (1.877)
Replacement ratio	-1.689 (0.189)	-2.339 (0.267)	-2.761 (0.362)	-3.032 (0.451)
u ₁	-1.154 (0.168)	-0.162 (0.241)	1.096 (0.391)	2.717 (0.570)
u ₂	-3.362 (0.389)	-2.102 (0.250)	-0.848 (0.332)	0.441 (0.373)
u ₃		-5.336 (0.781)	-2.711 (0.376)	-1.533 (0.414)
u ₄			-6.094 (0.808)	-3.396 (0.550)
u ₅				-7.291 (1.142)
p ₁	0.834 (0.046)	0.303 (0.017)	0.106 (0.015)	0.038 (0.010)
p ₂	0.166 (0.046)	0.605 (0.007)	0.342 (0.010)	0.174 (0.006)
p ₃		0.092 (0.024)	0.461 (0.001)	0.354 (0.003)
p ₄			0.091 (0.024)	0.354 (0.002)
p ₅				0.080 (0.022)
Log likelihood	-4929.0	-4916.5	-4913.9	-4912.7

Figure 1. Mass point probabilities in a Weibull model

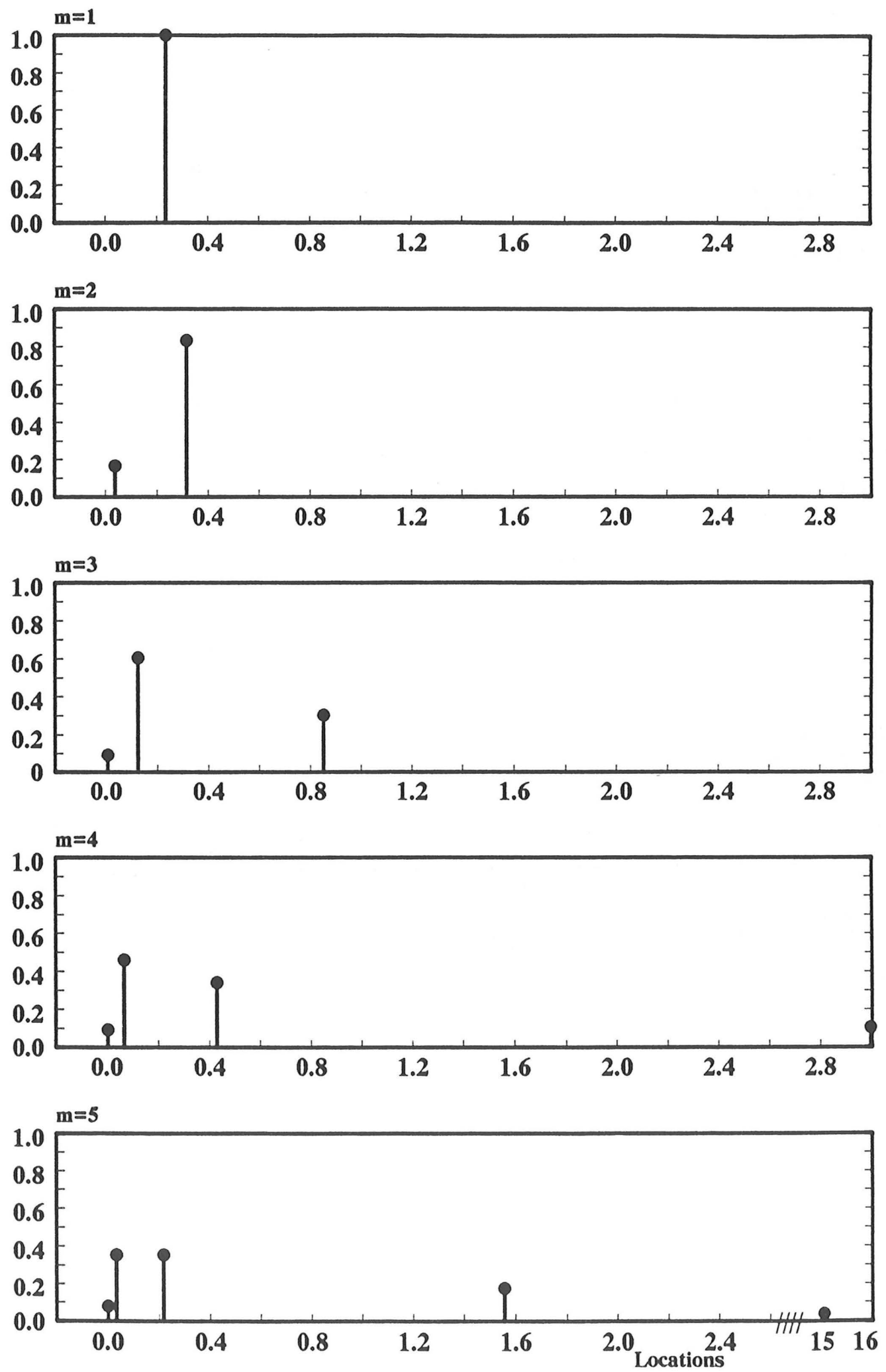
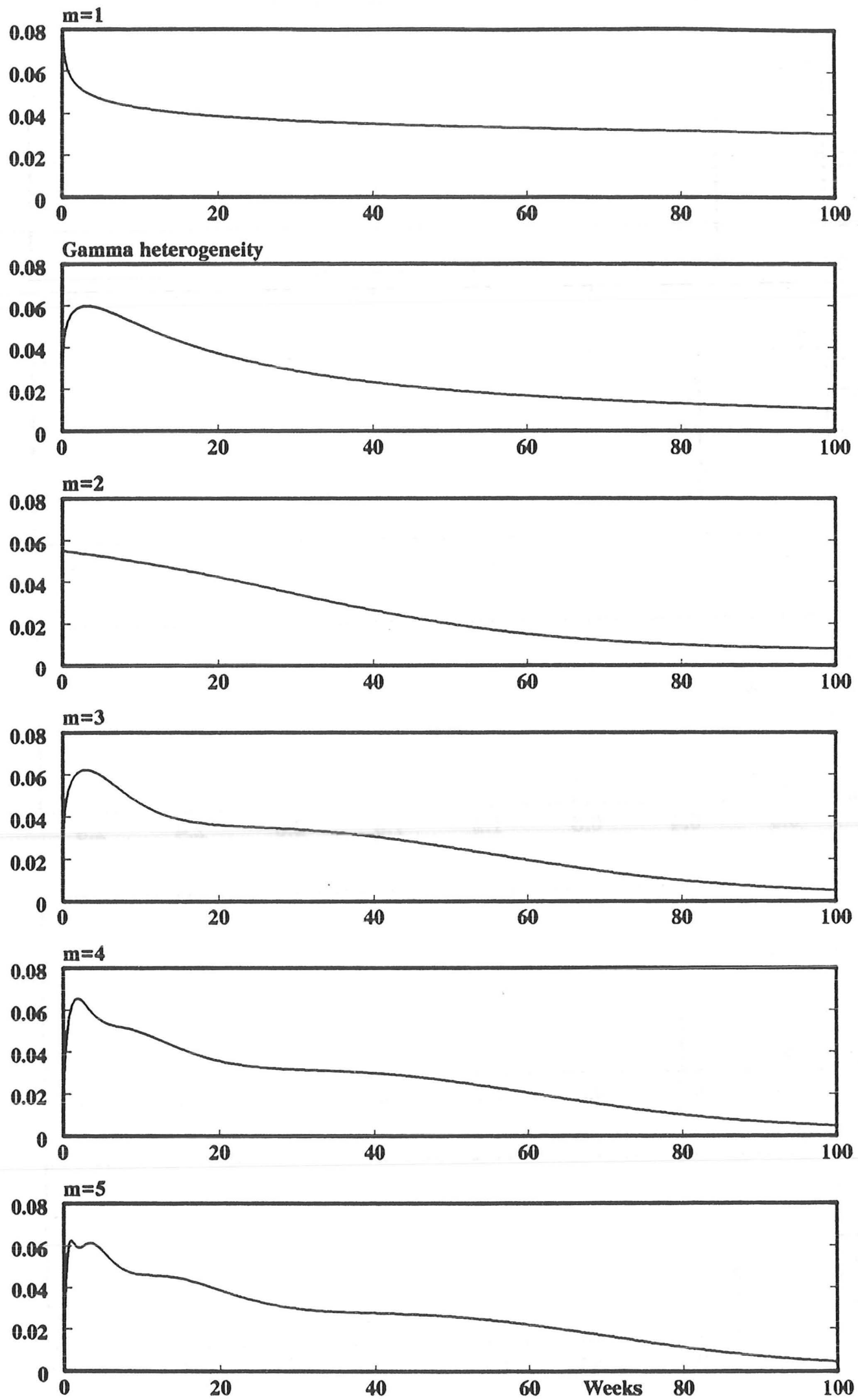


Figure 2. Sample hazard functions of Weibull models

At any time t_0 the remaining spell of unemployment is obtained by integrating the survivor function from t_0 to infinity. Thus the expected value of the unemployment spell in the case of a Weibull model allowing for discrete mass point heterogeneity can be written as

$$\begin{aligned}
 (20) \quad E(T) &= \int_0^{\infty} \sum_{i=1}^m p_i \exp(-t e^{\alpha u_i + x\beta}) dt \\
 &= \sum_{i=1}^m p_i (1/\alpha) e^{-(u_i + x\beta)/\alpha} \Gamma(1/\alpha),
 \end{aligned}$$

where Γ is the gamma function and the integration is done by a change of variables letting $I_i = t e^{\alpha u_i + x\beta}$.

The Weibull model with mass points of support is illustrated by way of example. Let the fictive person be a single 30-year-old woman who has no children. She has less than 10 years of education and no training for further employment. She has left employment, but has not been a member of a labour union. She faces an average regional and occupational demand of labour and she has no taxable assets and does not get unemployment benefits. The expected unemployment spell of the person is 37.8 weeks. The effects of the changes in the characteristics of the person on the duration of unemployment are illustrated in Table 6. It can be seen that most important factors affecting the duration of unemployment is the age, work experience (not come from housework) and the high level of unemployment benefits.

Schooling and skill (member of the labour union) seem to have positive effects on the re-employment.

Table 6. The effects of explanatory variables on the expected duration of unemployment for a person

The change of the explanatory variable	The change of the expected duration of unemployment	
Number of children: 0 → 1	0.8	
Married: not → yes	-3.4	
Gender: female → male	2.0	
Age: 30 → 40 years	23.8	*
At least 10 years schooling: no → yes	-1.6	
Training for further employment: no → yes	-7.4	*
Member of UI fund: no → yes	-9.3	*
School graduate: no → yes	-8.0	*
Came from housework: no → yes	41.2	*
Regional demand: 0.1 → 0.5	-3.7	
Occupational demand: 0.1 → 0.5	-0.2	
Taxable assets: 0 → 0.2 millions of Finnmark	-2.4	
Replacement ratio: 0 → 0.1	7.5	*
0 → 0.2	16.5	*
0 → 0.3	27.3	*

* Statistically significant effect on the 5 per cent level

3. Misspecification Analysis

3.1. Graphical Examination of Residuals

A graphical method to examine model misspecification is described and illustrated in this section. The integrated hazards, i.e. generalized residuals of fitted models, derived in the previous sections are examined. Exponential and Weibull models in the absence of censoring have been studied by Lancaster (1983, 1985). Lancaster and Chesher (1985a,b) have described the construction of residuals for right-censored duration data. In this study their procedure based on product-limit estimates [Kaplan and Meier (1958)] has been applied to residual definitions (12) and (19) to examine model specification when gamma and mass point heterogeneity have been introduced into the model. Furthermore, critical regions for the residual plots are derived.

Consider any duration distribution with a hazard function $h(\tau; \phi)$ depending upon a parameter vector ϕ . Then the random variable

$$(21) \quad I(T) = \int_0^T h(\tau; \phi) d\tau$$

has a unit exponential distribution since at any time point $t > 0$ the survivor function is

$$(22) \quad e^{-I(t)} = P(T > t) \\ = P[I(T) > I(t)].$$

Thus for every $I(t) \geq 0$, the survivor function $P[I(T) > I(t)] = \exp[-I(t)]$, which is the survivor function of unit exponential distribution. The moments of $I(T)$ are $E[I(T)^q] = q!$, $q = 1, 2, \dots$. The definitions of the generalized residuals $\hat{I}(T_j)$ in the absence of censoring is given by Cox and Snell (1968) and in the Weibull case the residuals are

$$(23) \quad \hat{I}(T_j) = T_j^{\hat{\alpha}} e^{x_j \hat{\beta}}, \quad j = 1, \dots, n,$$

where the $\hat{\cdot}$ indicates maximum likelihood estimates and n is the size of the sample. Hence, if the negative of the logarithm of the residual survival function is plotted against the ordered sequence of the residuals, it should give approximately a straight plot on a 45° line through the origin. For graphical plots when a Weibull model with gamma heterogeneity is fitted to the data the residuals can be written as

$$(24) \quad \hat{I}(T_j) = 1/\hat{\sigma}^2 \log[1 + \hat{\sigma}^2 T_j^{\hat{\alpha}} e^{x_j \hat{\beta}}]$$

and the residuals for the mass point models can be written as

$$(25) \quad \hat{I}(T_j) = \log\left\{\left[\sum_{i=1}^m \hat{p}_i e^{-T_j^{\hat{\alpha}} e^{\hat{u}_i + x_j \hat{\beta}}}\right]^{-1}\right\}.$$

With censoring $t = \min(T, Z)$, where Z is a censoring time. If the model is correct, the residuals approximate a

censored random sample from the unit exponential distribution, where the approximation is due to use of the estimated values instead of the true ones. Now the residuals have not got a unit exponential distribution, because its distribution depends on that of the censoring time. However, it is possible to define a set of residuals which do have simple properties under correct specification.

In the case of right-censored observations a procedure based on product-limit estimates suggested by Lancaster and Chesher (1985a,b) can be used to estimate the survivor function of residuals and this is a distributed unit exponential when the model is correct. Consider the ordered sequence of the residuals. The hazard function of the residuals can be calculated for each residual corresponding to an uncensored observation as the ratio of the number of residuals with value equal to the particular residual and the number of residuals greater than or equal to it. Let this ratio for the s th ordered uncensored residual be $\hat{h}(\hat{I}_s)$. Then the product-limit estimate of the residual survivor function is

$$(26) \quad \hat{S}(\hat{I}_j) = \prod_{s=0}^{j-1} [1 - \hat{h}(\hat{I}_s)],$$

and minus the logarithm of the residual survivor function is given by

$$(27) \quad -\log \hat{S}(\hat{I}_j) = -\sum_{s=0}^{j-1} \log[1 - \hat{h}(\hat{I}_s)].$$

The plot of the negative of the logarithm of the residual survivor function (27) against the residuals should give approximately a 45° line through the origin for large samples when the model is correct.

To evaluate the model specification using residual plots it is useful to calculate critical regions for minus log residual survivor functions. The product-limit method gives a maximum likelihood estimate for the survivor function. If there are d_j persons becoming employed among the r_j individuals in the risk set at t_j , the contribution to the likelihood function can be written as

$$(28) \quad \ell = h_j^{d_j} (1 - h_j)^{r_j - d_j},$$

where h_j is the hazard function. The log likelihood contribution is then

$$(29) \quad \log \ell = d_j \log h_j + (r_j - d_j) \log(1 - h_j).$$

The maximizing $\hat{h}_j = d_j/r_j$ is the solution of

$$(30) \quad \partial \log \ell / \partial h_j = d_j / \hat{h}_j - (r_j - d_j) / (1 - \hat{h}_j) = 0.$$

The sample information matrix at \hat{h} is

$$(31) \quad -\partial^2 \log \ell / \partial h \partial h = r_j / [\hat{h}_j (1 - \hat{h}_j)],$$

which is obtained by substituting $d_j = \hat{h}_j r_j$. In order to estimate the variance of $-\log \hat{S}(\hat{t}_j)$, consider the logarithm of the product-limit survivor function

$$(32) \quad \log \hat{S}(\hat{I}_j) = \sum_{l=0}^{j-1} \log \hat{p}_l,$$

where $\hat{p}_1 = 1 - \hat{h}_1$. The delta method implies

$$(33) \quad \begin{aligned} \text{Var}(\log \hat{p}_j) &\approx \text{Var}(\hat{p}_j) (\partial \log p_j / \partial p_j)^2 \\ &\approx p_j(1 - p_j) / r_j p_j^2, \end{aligned}$$

where $\text{Var}(\hat{p}_j) = \text{Var}(\hat{h}_j)$ is estimated by the inverse of the observed information matrix. Assuming that $\log \hat{p}_1, 1 = 1, 2, \dots$, are independent

$$(34) \quad \begin{aligned} \text{Var}[-\log \hat{S}(\hat{I}_j)] &= \hat{\sigma}_j^2 = \sum_{l=0}^{j-1} (1 - \hat{p}_l) / r_l \hat{p}_l \\ &= \sum_{l=0}^{j-1} d_l / [r_l(r_l - d_l)], \end{aligned}$$

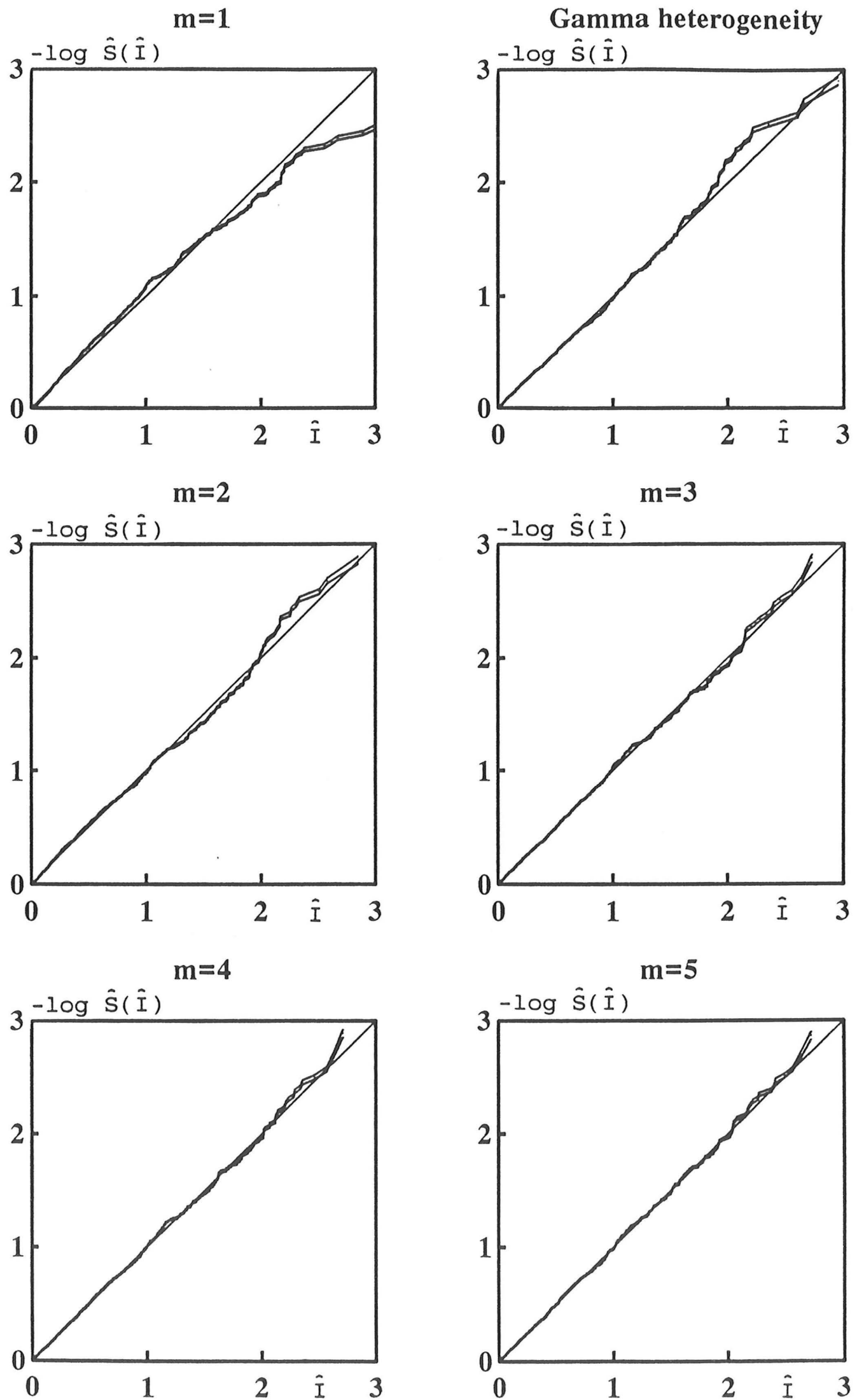
since $\hat{p}_1 = 1 - d_1/r_1$. Another way of calculating (34) is to assume that the distribution of $r_l p_l$ is binomial. Greenwood (1926) followed this approach in calculating his famous formula for the variance of the survivor function. The confidence limits for the estimated $-\log \hat{S}(\hat{I})$ can be computed using the estimates of standard errors $\hat{\sigma}_j$ as follows

$$(35) \quad -\log \hat{S}(\hat{I}_j) \pm C_{\alpha/2} \hat{\sigma}_j,$$

where $C_{\alpha/2}$ is the critical value for the normal distribution. That is, $\Phi(-C_{\alpha/2}) = \alpha/2$, where Φ is the distribution function of the normal distribution. Under the null hypothesis $\Pr[-\log \hat{S}(\hat{I}) > \hat{I}] = \alpha/2$ and $\Pr[-\log \hat{S}(\hat{I}) < \hat{I}] = \alpha/2$ for all values of \hat{I} . Then $\alpha = 0.1$ requests the 90 per cent critical region for the $-\log \hat{S}(\hat{I})$. It should be pointed out that for small values of r_1 equation (34) is not a good approximation of the true variance.

Figure 3 illustrates the residual plots with the confidence limits for the models estimated in section 2. The limits are rather narrow, not allowing very much departure from the 45° diagonal. It should be pointed out that the departure from the 45° line is larger for high values of the residuals. Thus the graphical method reveals best the right tail behaviour of the duration distribution. A plot above (below) the 45° line indicates that the estimated hazard function is too low (high). The behaviour of residuals seems to be slightly better after allowing for gamma heterogeneity. In the mass point models no specific assumption about the distribution is required for the unobserved heterogeneity. Thus the risk of misspecification is reduced. The departure from the 45° line decreases when the number of mass points has been increased. In the last graph the plot is fairly precisely on the 45° line except for the last few observations.

Figure 3. Residual plots of the models of unemployment duration



3.2. The Information Matrix Test

In this section the Information Matrix (IM) test is used to specify the pattern of heterogeneity. The test is used to test the Weibull model against gamma heterogeneity and the Weibull mass point models against more mass points. To focus on the particular parameters of interest the IM test introduced by White (1982) is used to reveal the constancy of the shape parameters, constants and location parameters of the models. The interpretation of the test was first given by Chesher (1984). To avoid the computation of the analytic second derivatives of the IM test a numerical procedure to ease the computation is presented.

The likelihood ratio test cannot be used when the hypothesis of interest is on the boundary of the parameter space. The test statistic can have non-standard distributions under null hypothesis. This happens if the MLE of the variance has to be constrained to be non-negative and if it can be zero with non-negligible probability. The problem does not affect score tests. Hence the focus is on the IM test, which is a score test as shown by Chesher (1984).

Let $L = \sum \log[\ell(x, \phi)]$ be the log likelihood function and let p be the number of parameters in the model. Write the derivatives of L with respect to the parameters as

$$(36) \quad \partial L / \partial \phi_i = L_i$$

$$(37) \quad \partial^2 L / \partial \phi_i \partial \phi_j = L_{ij},$$

where L_i is a $(p \times 1)$ vector and L_{ij} a $(p \times p)$ matrix. The IM test compares the familiar IM identity in maximum likelihood theory to zero

$$(38) \quad E(L_{ij} + L_i L_j) = 0, \quad i, j = 1, \dots, p.$$

It examines whether alternative forms of the information matrix, the Hessian L_{ij} and the outer product $-L_i L_j$, are approximately equivalent in the sample. This essentially means that when the model is correctly specified, the information matrix can be expressed in either Hessian form or outer product form.

The test will be based on the indicators \hat{D}_A . For an observation, \hat{D}_A is a vector with one element corresponding to each index pair in the interesting set of distinct index pairs A

$$(39a) \quad \hat{D}_A = n^{-1} \sum \hat{d}_{ij}, \quad i, j \in A, \text{ where}$$

$$(39b) \quad \hat{d}_{ij} = \hat{L}_{ij} + \hat{L}_i \hat{L}_j.$$

The summation is over the n individuals of the sample and the $\hat{}$ indicates that the parameters are replaced by their MLE. Under regularity conditions given by White (1982) the joint distribution of $n^{1/2} \hat{D}_A$ is asymptotically normal with mean zero and covariance matrix V_A , which depends on the index pairs selected.

The IM test presented by White requires the analytic third derivatives of log likelihood function. Lancaster

(1984) showed that the test can be calculated using the analytic second derivatives. In this study it is shown that the second derivatives can be calculated numerically using the analytic first derivatives. The asymptotic covariance matrix of $n^{1/2}\hat{D}_A$ is obtained as shown by Lancaster by applying the first order Taylor series expansion, using the IM identity and rewriting the terms as

$$(40) \quad V_A = E(d'd) - E(d'L_\phi) [E(L_\phi'L_\phi)]^{-1} E(L_\phi'd),$$

where d is a vector of IM identities and L_ϕ includes the first derivatives. V_A is consistently estimated by replacing E by $n^{-1}\Sigma$ and ϕ by $\hat{\phi}$. The generic form of the IM statistic is then

$$(41) \quad \hat{T}_A = n\hat{D}_A'\hat{V}_A^{-1}\hat{D}_A.$$

When the likelihood function is correctly specified the IM test statistic displays an asymptotic chi-squared distribution with as many degrees of freedom as there are indicators, i.e. $\text{rank}(V_A)$. There will be at most $p(p+1)/2$ indicators and test statistics, but it is in many cases inappropriate to base the test on all the indicators. If the interest is on single parameters, then it is necessary to consider the IM identities d_{ii} . If the computed test statistic exceeds the critical value, the null hypothesis that the model has been correctly specified can be rejected.

Chesher (1983) showed that $T_A = nR^2$, equivalently as the explained sum of squares, from the least squares

pseudo-regression $\mathbf{1} = [\hat{\mathbf{d}} : \hat{\mathbf{L}}_\phi] \boldsymbol{\beta} + \boldsymbol{\varepsilon}$ in which the dependent variable $\mathbf{1}$ is an n element vector of ones and the regressors are the selected IM identities \mathbf{d} and the derivatives \mathbf{L}_ϕ for all the parameters in the model. The identities and derivatives are evaluated at MLE of parameters and at each element of \mathbf{t} , \mathbf{c} and \mathbf{x} . To show the result the sum of squared residuals of the estimation of $\boldsymbol{\beta}$ is written

$$(42) \quad \hat{\boldsymbol{\varepsilon}}' \hat{\boldsymbol{\varepsilon}} = \mathbf{1}' (\mathbf{I} - \mathbf{R}(\mathbf{R}'\mathbf{R})^{-1}\mathbf{R}') \mathbf{1}$$

$$= n - \mathbf{1}' \mathbf{R}(\mathbf{R}'\mathbf{R})^{-1} \mathbf{R}' \mathbf{1},$$

where \mathbf{I} is a identity matrix and $\mathbf{R} = [\hat{\mathbf{d}} : \hat{\mathbf{L}}_\phi]$. Since $\mathbf{1}' \hat{\mathbf{F}}_\phi = 0$ by the first order condition of MLE, it is needed to consider the top left block of $(\mathbf{R}'\mathbf{R})^{-1}$ so that

$$(43) \quad \hat{\boldsymbol{\varepsilon}}' \hat{\boldsymbol{\varepsilon}} = n - \mathbf{1}' \hat{\mathbf{d}} (\hat{\mathbf{d}}' \hat{\mathbf{d}} - \hat{\mathbf{d}}' \hat{\mathbf{L}}_\phi (\hat{\mathbf{L}}_\phi' \hat{\mathbf{L}}_\phi)^{-1} \hat{\mathbf{L}}_\phi' \hat{\mathbf{d}})^{-1} \hat{\mathbf{d}}' \mathbf{1}$$

$$= n - T_A.$$

Defining R^2 in the pseudo-regression as $1 - \hat{\boldsymbol{\varepsilon}}' \hat{\boldsymbol{\varepsilon}} / \mathbf{1}' \mathbf{1}$, it can be seen that T_A is precisely nR^2 from the pseudo-regression.

The second derivatives may be difficult and time consuming to derive analytically. Analytic second derivatives are not necessarily needed, however, to compute the IM test. Numerical approximations based on the analytic first derivatives can be used to compute the second

derivatives. The difference quotient $[F_i(\hat{\phi}+\Delta) - F_i(\hat{\phi})]/\Delta$, when starting from $\hat{\phi} + \Delta$ and taking one iteration step towards the maximum of the log likelihood function, approximates the magnitude of second derivatives. The diagonal IM identities are then of the form

$$(44) \quad d_{ii} = F_i(\hat{\phi})F_i(\hat{\phi}) - [F_i(\hat{\phi}+\Delta) - F_i(\hat{\phi})]/\Delta,$$

where the small number $\Delta = 0.0000001$. Using numerical examples it is straightforward to show that this procedure gives fairly accurate estimates of the analytic second derivatives.

The results of calculations of IM test statistics are presented in Table 7. The shape parameter and the constant terms, which are the location parameters in mass point models, have been tested. The calculated test statistics for the shape parameters are less than the critical value with gamma-heterogeneity and with two or more mass points. This gives support to the conclusion that the correction for heterogeneity rectifies the shape of the hazard function, which shows that the groups of the sample may consist of individuals with non-decreasing hazard functions even though the sample hazard function may be decreasing.

The IM test statistics for the constants and location parameters are lower after introducing heterogeneity to the model, as was expected, because the influence of omitted variables is captured in the constant and location parameters. Gamma heterogeneity improves substantially the model specification but not enough to pass the IM test. The mass point method provides an excellent pattern of

heterogeneity. According to the IM test three mass points are enough to rectify the model specification with respect to the tested parameters.

Table 7. Information Matrix test statistics

Models	Parameters	Test statistics	
Weibull, m=1	α	28.30	
	β_0	44.24	
Weibull, gamma heterogeneity	α	2.48	*
	β_0	8.63	
m=2	α	4.33	*
	u_1	22.29	
m=3	u_2	8.61	
	α	0.06	*
	u_1	4.49	*
	u_2	0.26	*
m=4	u_3	0.71	*
	α	0.40	*
	u_1	0.46	*
	u_2	1.49	*
	u_3	0.34	*
m=5	u_4	0.46	*
	α	0.01	*
	u_1	0.13	*
	u_2	5.07	*
	u_3	0.12	*
	u_4	0.11	*
	u_5	0.31	*

* Significant at 1 per cent level ($\chi^2_{1,0.99} = 6.63$)

Numerical tests confirmed the conclusions of the graphs, which may be subject to incorrect interpretations. The conclusion of the IM test is that correction of heterogeneity of duration models is of great importance even with fairly rich and reliable data. The basic Weibull model did not fit very well, but after introducing gamma heterogeneity the model was better specified. Allowing for

mass point heterogeneity, however, made the specification even better. According to the IM test three mass points were enough to rectify the effects of omitted variables in the models with a fairly rich set of explanatory variables.

4. Conclusions

According to the results of this chapter the unemployment benefits have a negative effect and the labour union membership indicator has a positive effect on the probability of becoming employed, as is expected by the search models. It is important to note that the rules of the UI system are not similar for the members and non-members of the labour unions. Under the full information about the rules of the UI system an unemployed person takes the changes of the system into account in advance. There is reason to assume that at least some of the unemployed persons know of the reductions in advance. Since the reductions of benefits only apply to the earnings-related unemployment allowances, they increase the hazard function of only the recipients of those benefits during the whole spell of unemployment. Therefore it is quite possible that even though the benefits have a negative effect on the probability of becoming employed, the recipients of the lower basic unemployment allowance have lower hazard rates.

It can be argued using search models that the recipients of the basic unemployment allowance do not have as many elements of incentives for re-employment during their spells of unemployment. A proportional decrease in UI benefits has larger positive effects on the re-employment probability for the persons having higher benefits, because it has a higher economic importance. This result has been shown by Usategui (1988). Econometric models take this feature into account, since the elasticity of an

explanatory variable with respect to the hazard function is equal to its value multiplied by the parameter estimate.

In addition, it can be argued that the members of labour unions find employment more attractive when jobs have uncertain durations. According to well-known results [see Mortensen (1977), Hamermesh (1979) and Burdett (1979)] the members of labour unions have higher incentives for re-employment than the non-members, because the higher earnings-related unemployment benefits create a closer attachment to the labour force via the higher value of search. These elements of the UI system are studied more carefully in Chapter V allowing for time-dependent effects of replacement ratios.

The higher earnings-related unemployment allowance creates also incentives for joining a trade union. Actually when these benefits have become more generous in Finland since the 1960's the degree of unionization has risen distinctively (see Tyrväinen, 1989). Nevertheless the trade unions do not attract all the workers (students, self-employed, temporary workers, etc.). The reasons for joining a trade union are, however, beyond the focus of this study.

The models of unemployment duration allowing for unobserved heterogeneity were studied in this chapter. Weibull models with gamma and mass point heterogeneity were estimated using Finnish microeconomic data. In the basic Weibull model the estimated value of the shape parameter was less than one indicating negative duration dependence. However, the parameter estimates of the basic Weibull model were biased. The absolute value of the estimate of the shape parameter increased substantially after allowing for

unobserved heterogeneity. In the Weibull model with gamma heterogeneity and in mass point models with three or more points of support the parameter estimate was larger than one indicating an increasing hazard function for an individual. These results are in concordance with standard search theories. Also the parameter estimates of the structural parameters increased substantially when unobserved heterogeneity was taken into account. These results show that there is unobserved heterogeneity in the data and it is important not to neglect it.

The residuals of estimated heterogeneity models were derived and examined by a graphical method. It seems that the model with gamma heterogeneity is slightly better than the basic Weibull model and the mass point models with three or more mass points were rather well specified. Numerical tests confirmed the conclusions of the graphs, which may be subject to incorrect interpretations. Five points of support were enough to rectify the effects of omitted variables according to Lindsay's rule. Furthermore the information matrix test was used to specify the pattern of heterogeneity. A numerical method to ease the computation of the IM test was developed. According to the test three mass points were enough to rectify the effect of unobserved heterogeneity. The conclusion drawn from the specification tests is that correction of heterogeneity of duration models is of great importance even with fairly rich and reliable data.

The estimation of the microeconomic models with a large number of parameters and likelihood function specified by the researcher may be difficult in practice using micro-

computers or standard statistical packages. The needed statistical tool with an application is presented. It is written using the SAS/IML matrix language, which is close to matrix algebra notation. The use of this programme is not limited to this particular application, but with slight modifications it is a useful tool to estimate a wide class of maximum likelihood models. Furthermore, the programme is a basis for further development and a framework for developing specification tests according to the needs of the users.

Footnote

1. Recently a number of Dutch studies using panel data have paid special attention to the problem of attrition bias [e.g. Ridder (1990), Gorter et al. (1991) and van den Berg et al. (1991)]. The use of panel data may lead to biased estimates if the stochastic processes underlying labour market behaviour depend on the behaviour concerning participation in a panel survey or on the omitted variables correlated with the endogenous variable. If the persons who have a relatively high probability of finding a job also have a high probability of censoring, the empirical hazard rate underestimates the rate at which individuals become employed. No grounds based on economic theory are presented for the attrition bias. Gorter et al. (1991) conclude that there are no indications of an attrition bias in their study. Van den Berg et al. (1991) found that the unobserved explanatory variables for the duration of panel survey participation are not related to unobserved variables for the duration of unemployment. In that study the attrition could be treated as censored observations. The attrition bias could in this study using flow data be a smaller problem, since the durations are shorter, but we can not be sure about this.

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Appendix 1. A Programme for Estimating Nonlinear Maximum Likelihood Models

General Features of the Programme

An estimation of a user specified maximum likelihood model is often difficult or impossible using standard statistical software. Statistical packages often have restrictions regarding the parametrization of the likelihood function and the development of statistical tests. This section provides the needed statistical tool which enables estimation of the unknown parameters of the user specified structural nonlinear model with an application to duration models. A programme for estimating a Weibull model allowing for gamma heterogeneity is provided. The programme is nevertheless a useful tool for a wider class of maximum likelihood models and a basis for further development depending on the needs of the researcher. The programme is written using the SAS/IML (1985) matrix language, which is close to matrix algebra notation. The programme can be translated with a little effort into any other matrix language.

The origin of the programme should be made clear. It is based on Andrew Chesher's programme, which was used to estimate censored Weibull models for time to pavement cracking. Subsequently it was used to estimate multiple spell models of female fertility (Chesher, 1986). The programme was converted from the SAS/MATRIX to SAS/IML language and the necessary changes were made to do the various estimations.

The practical problem of estimating the models with a large number of parameters and likelihood function specified by the researcher is solved using the programme. The data handling and estimation of this kind of model using micro computers is painstakingly slow. Mainframe computers can speed the computation, but the estimation of user specified models using standard statistical packages may be difficult or even impossible. Writing programmes using low-level languages like Fortran and using libraries

of subroutines may be tedious, because programming every detail is time consuming and the subroutines may not allow for changes by the user. Especially this work is motivated by the estimation and test development of duration models, where the low-level programmes may be very long as can be seen in Kalbfleisch and Prentice (1980) or Lee (1980). In that area there is clearly a need for a flexible and powerful high-level programme which allows the user to specify the likelihood function and which provides an environment for the development of specification tests.

The version of the programme presented in this Appendix estimates the unknown parameters of a Weibull model allowing for gamma heterogeneity. It reads an ASCII data file including the duration t , indicator for complete spells c and matrix of explanatory variables x and saves it to a SAS data set. The user can control the programme using a set of requirements for the iteration and solution at the beginning of the SAS/IML programme. The set of explanatory variables can be changed using an indicator. The limits for the maximum number of iterations and linear search can be given as well as the criteria for the convergence. The requirement of the precision of solution can be controlled by the user using the accuracy requirements and the proportion of the step length.

The estimates of structural parameters of a Weibull model can be used as starting values for those parameters of the corresponding model with gamma heterogeneity and the starting value for σ^2 can be randomly allocated. A safeguard against the possibility of convergence to a local maximum that is not a global maximum is to choose several initial values of the parameters. If the iterations do not converge to the same solution, the shape of the log likelihood function should be investigated with care until the global maximum is located.

During the iterations the iteration monitor prints the number of iteration, values of the likelihood function, parameters and gradients. In the case of a linear search the number of the searches, the value of the likelihood function, the step adjustment and the parameters are provided. It is suggested, however, to use the options to

suppress the printings of the iteration monitor and linear search during the iterations if they are not needed. If the accuracy requirement is achieved, the programme prints to a listing file including, for instance, the starting values of the parameters, the log likelihood function, the parameter values and their standard errors at solution and the number of iterations. More printings can be easily added.

There are two links, which are written at the end of the programme, but which are called and executed during the iterations. The likelihood function can be changed by the user. It is written in LINK LIKEF. The algorithm requires the evaluation of first partial derivatives of the log likelihood function with respect to the parameters to be estimated. They are written in LINK LIKED. The Berndt, Hall, Hall and Hausman (BHHH) (1974) algorithm is used to estimate the unknown parameters, but it can be easily replaced by another algorithm.

The Algorithm

The convergence of the BHHH algorithm is guaranteed by the theory unlike the method of scoring and some other statistical maximisation procedures. The ideal is to reach the values of parameters $\phi = (\sigma^2, \alpha, \beta)$ such that the gradient $L_\phi = 0$. The likelihood equations to be solved in a case of the Weibull model allowing for gamma heterogeneity can be written

$$(45) \quad L_\alpha = \sum [c(1/\alpha + \log t) - (c + 1/\sigma^2)(N - 1)\log t/N] = 0$$

$$(46) \quad L_{\sigma^2} = \sum [\log N/\sigma^4 - (c + 1/\sigma^2)(N - 1)/\sigma^2 N] = 0$$

$$(47) \quad L_\beta = \sum \{[c - (c + 1/\sigma^2)(N - 1)/N] \# x\} = 0,$$

where $N = 1 + \sigma^2 t^\alpha e^{x\beta}$ and # indicates elementwise multiplication.

Iterations will move uphill along the likelihood function. Each iteration consists of computing the log likelihood function $L(\phi)$ and the gradient L_ϕ , which is used to derive a direction of increase of $L(\phi)$. According to the classical Gradient Theorem, which is proved, for instance, in Jacoby, Kowalik and Pizzo (1972), any vector \mathbf{d} with $L_\phi \mathbf{d} > 0$ is a direction of increase of $L(\phi)$ in the sense that $L(\phi + \lambda \mathbf{d})$ is an increasing function of the step length λ for small enough values of λ . The directions \mathbf{d} can be derived from the gradient by multiplying it by a positive definite matrix \mathbf{Q} . The convergence is speeded by a choice of \mathbf{Q} such that it is the inverse of the Hessian matrix of second derivatives of $L(\phi)$. The use of the information matrix identity - $E(L_{\phi\phi}) = E(L_\phi)E(L_\phi)'$ avoids the need of analytical or numerical calculation of second derivatives. The updated estimates of parameters calculated during the i th iteration can be written as

$$(48) \quad \phi^{i+1} = \phi^i + \lambda^i L_{\phi}^{i'} (L_{\phi}^i L_{\phi}^{i'})^{-1}.$$

If λ is too large leading to a decreasing value of $L(\phi)$, the linear search subiteration is used. There are plenty of methods of linear search, as noted by Quandt (1986). As a simple method to avoid computation during the iterations, it is suggested that the step length λ is halved as many times as needed to find an increasing value of $L(\phi)$ and then new values of ϕ^{i+1} are calculated.

There are many criteria for the stopping rule of iterations [see e.g. Quandt (1986)]. In the neighbourhood of a maximum the algorithm takes small steps in the sense that $|L(\phi)^{i+1} - L(\phi)^i|$ is small. The ideal of reaching values of ϕ such that $L_\phi = 0$ is not attainable in practice. In the neighbourhood of a maximum $(\sum L_\phi^2)^{1/2}$ is likely to be small. Both of these conditions are used as stopping criteria.

*A SAS/IML programme for estimating a Weibull duration model
allowing for gamma heterogeneity*

```

*A SAS/IML PROGRAMME FOR ESTIMATING A WEIBULL DURATION MODEL
ALLOWING FOR GAMMA HETEROGENEITY IN THE VMS OPERATING SYSTEM;
OPTIONS LS=80 PS=500;
LIBNAME SASLIBR '[JKETTUNEN.SASFILES]';
FILENAME RAWDATA 'GL.DAT';
DATA SASLIBR.ADATA;
  INFILE RAWDATA;
  INPUT T 1-7 .3 C 9 CONST 11 KIDS 13 MARRIED 15 SEX 17 AGE 19-20
        EDU 22 UEDU 24 MEMBER 26 CAME1 28 CAME2 30 REGDEM 32-35 .3
        PROFDEM 37-40 .3 ASSETS 42-46 .4 BENEFITS 48-54 .6;
PROC IML;
START;
*POS OF T DATA;      IND1=1;
*POS OF C DATA;      IND2=2;
*POS OF X DATA;      IND3={3 4 5 6 7 8 9 10 11 12 13 14 15 16};
*LINE SEARCH LIMIT;  LLS =100;
*ITERATION LIMIT;    LIT =300;
*ACCY REL FUN;        ACC1=0.001;
*ACCY GRAD;           ACC2=0.01;
*STEP ADJUSTMNT;     LAMDA=1.0;
*SUPPRESS LINE SRCH; SUPRESLS=1;
*SUPPRESS ITERATION; SUPRESIT=1;
*2ND DERIV STEP;      DIFR=0.0000001;
*START POINT;         B={1.201  1.045
                        -1.157 -0.002  0.135 -0.066
                        -0.057  0.035  0.321  0.364
                        0.375 -0.892  0.353 -0.098
                        0.822 -2.243};
PRINT 'MAXIMUM LIKELIHOOD ESTIMATION: BHHH MODIFIED NEWTON RAPHSON
METHOD';
USE SASLIBR.ADATA;
READ ALL INTO A;
T=A(1,IND1); C=A(1,IND2); X=A(1,IND3);
NAMES1={'SEARCH NO' 'OLD L' 'NEW L' 'LAMDA'};
NAMES2={'OLD B' 'NEW B'};
NAMES3={'ITER NO' L};
NAMES4={PARAMETR GRADIENT};
NAMES5={PARAMETR 'S.ERROR' 'T STAT' GRADIENT};
NAMES9={ACC1 ACC2 LAMDA 'NO OBS' 'NO PARS'};
NAMES13={T C CONST KIDS MARRIED SEX AGE EDU UEDU
        MEMBER CAME1 CAME2 REGDEM PROFDEM ASSETS BENEFITS};
FREE ADATA A;
OBS=NROW(T);
PAR=NCOL(X);
PAR2=PAR+2;
LOGT=LOG(T);
NAMEB1='SHAPE'; NAMEBD='SIGMA';
VARNAME=NAMEB1||NAMEBD||NAMES13(1,IND3);
PRINT 'ACCURACY REQUIREMENT:STEP LENGTH PROPORTION:NO OF OBS &
PARAMETERS';
PRINT '*****';
PRT=ACC1||ACC2||LAMDA||OBS||PAR2;
PRINT PRT (1 COLNAME=NAMES9 1);
FREE PRT;
PRINT 'DEFAULT START POINT PROVIDED';
PRINT '*****';
PRINT B;
J1=J(OBS,1,1);

```

```

NIT=1;
BOLD=B;
LINK LIKEF;
LOLD=LL;
MAR4:  LOLD=LL; BOLD=B;
        LINK LIKED;
        IF SUPRESIT=0 THEN DO;
            LOGLIK =NIT||LL;
            PARAMS =B' ||DL;
            IF NIT=1 THEN DO;
                PRINT '*****';
                PRINT 'ITERATION MONITOR';
                PRINT '*****';
            END;
            PRINT LOGLIK (| COLNAME=NAME3 |);
            PRINT PARAMS (| COLNAME=NAME4 |);
            FREE LOGLIK  PARAMS;
            END;
        NLS=1;
        FAC=DL'*INV(CL);
MAR3:  B=BOLD-LAMDA#FAC;
        IF NLS>1 & SUPRESLS=0 THEN DO;
            NLS1=NLS-1;
            IF NLS=2 THEN DO;
                PRINT 'LINE SEARCH SUBITERATION';
                PRINT '*****';
            END;
            LOGLIK =NLS1||LOLD||LL||LAMDA;
            PARAMS =BOLD//B;
            PRINT LOGLIK (| COLNAME=NAME1 |);
            PRINT 'PARAMETER VALUES';
            PRINT PARAMS (| ROWNAME=NAME2 |);
            FREE LOGLIK PARAMS ;
            END;
        LINK LIKEF;
        IF LL > LOLD THEN GOTO MAR1;
        NLS=NLS+1;
        IF NLS < LLS THEN GOTO MAR2;
        PRINT '*****';
        PRINT '*SORRY - LINE SEARCH FAILURE*';
        PRINT '*****';
        PRINT BOLD B DL CL FAC LOLD LL NLS NIT;
        GOTO MAR7;
MAR2:  LAMDA=LAMDA/2;
        GOTO MAR3;
MAR1:  NIT=NIT+1;
        IF ABS((LL-LOLD)/LOLD) < ACC1*LAMDA & SQRT(DL(|##,|))
            <= ACC2 THEN GOTO LEN;
        IF NIT>LIT THEN GOTO MAR5;
        GOTO MAR4;
MAR5:  PRINT '*****';
        PRINT '*ITERATION LIMIT EXCEEDED*';
        PRINT '*****';
        PRINT BOLD B DL CL FAC LOLD LL NLS NIT;
        GOTO MAR7;
LEN:   PRINT '*****';
        PRINT '*CONGRATULATIONS ACCURACY REQUIREMENT ACHIEVED*';
        PRINT '*****';
MAR7:  LINK LIKED;
        PRINT 'LOG LIKELIHOOD FUNCTION AT SOLUTION';
        PRINT '*****';
        PRINT LL;
        VAR=INV(-CL);
        SERR=SQRT(VECDIAG(VAR));
        TSTAT=B'/SERR;

```

```

SOLN =B' ||SERR||TSTAT||DL;
PRINT 'PARAMETER VALUES AT SOLUTION';
PRINT '*****';
PRINT SOLN (| COLNAME=NAME5 ROWNAME=VARNAME |);
FREE SOLN;
XBETA=X*B2';
MEANXB= XBETA(|+,|)/OBS; PRINT MEANXB;
*THE FILE FOR THE GRAPHICAL EXAMINATION OF RESIDUALS;
CENSOR=1-C; KL=T||CENSOR||IH;
GRAFCOLS={T CENSOR IH};
CREATE SASLIBR.KL FROM KL (| COLNAME=GRAFCOLS |);
APPEND FROM KL;
CLOSE SASLIBR.KL;
*THE FILE FOR THE IM-TEST;
BOLD=B;
VOLD=V;
B=BOLD; B(|1,1|)=BOLD(|1,1|)+DIFR; LINK LIKEF; LINK LIKED;
IND=VOLD(|1,|)'#VOLD(|1,|)'-(VOLD(|1,|)'-V(|1,|)')/DIFR;
B=BOLD; B(|1,3|)=BOLD(|1,3|)+DIFR; LINK LIKEF; LINK LIKED;
IND=IND|| (VOLD(|3,|)'#VOLD(|3,|)'-(VOLD(|3,|)'-V(|3,|)')/DIFR);
B=BOLD; B(|1,16|)=BOLD(|1,16|)+DIFR; LINK LIKEF; LINK LIKED;
IND=IND|| (VOLD(|16,|)'#VOLD(|16,|)'-(VOLD(|16,|)'-V(|16,|)')/DIFR);
STF=J1||VOLD' ||IND;
OLSCOLS={ONE FA FSIGMA FCONST FKIDS FMARRIED FSEX01
          FAGE FEDU38 FUEDU01 FMEMBER0 FCAME1 FCAME2
          FREGDEM FPROFDEM FASSU FRUN INDA INDC INDR};
CREATE SASLIBR.L FROM STF (| COLNAME=OLSCOLS |);
APPEND FROM STF;
CLOSE SASLIBR.L;
STOP;
LIKEF: *THE LIKELIHOOD FUNCTION;
B1=B(|1,1|); BD=B(|1,2|); B2=B(|1,3:PAR2|);
M=X*B2';
N=1+BD#(T##B1)#EXP(M);
H=B1#(T##(B1-1))#EXP(M)/N;
IH=1/BD#LOG(N);
L=C#LOG(H)-IH;
LL=L(|+,|);
RETURN;
LIKED: *THE FIRST DERIVATIVES;
V1=C#(1/B1+LOGT)-(C+1/BD)#(N-1)/N#LOGT;
V2=1/BD/BD#LOG(N)-(C+1/BD)#(N-1)/N/BD;
V3=(C-(C+1/BD)#(N-1)/N)#X;
V=V1'//V2'//V3';
DL=V(|,+,|);
CL=-V*V';
RETURN;
FINISH;
RUN;

```

Chapter V

TIME-DEPENDENT EFFECTS OF UNEMPLOYMENT BENEFITS

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Abstract

This chapter presents methods of estimating the effects of time-dependent covariates in parametric duration models. Using Finnish microeconomic data it is shown that unemployment insurance benefits have a negative effect on the probability of becoming employed during the first few months, but later on the effect vanishes. One reason is that in the Finnish system persons who are eligible for the benefits have a risk of losing them after the first three months. Another reason is that the earnings-related unemployment allowances decrease 20 per cent after the first 100 days unemployment. These results remain after correcting for omitted variables assuming gamma and mass point heterogeneity across unemployed persons.

1. Introduction

In this chapter the effects of unemployment insurance on spells of unemployment are examined. The circumstances of unemployed persons do not usually stay constant over the duration of unemployment. The purpose of this study is to estimate the time-dependent effects of time-dependent benefits on the re-employment probability during the unemployment duration. A technique for estimating these effects is presented using a Weibull model and Finnish microeconomic data.

Duration models based on the proportional hazards (PH) assumption imply constant effects of explanatory variables over time. A score test for testing the PH assumption is presented and a method for estimating the time-dependent coefficients in a Weibull model is developed. Often it may be preferable to avoid estimating an alternative nonproportional hazards model and therefore the focus is on a score test. A computationally convenient form of the test statistic and the appropriate connection with the pseudo-regression based on ordinary least squares is presented.

It is inevitable that econometric models do not include all the necessary explanatory variables either because they are unmeasurable or because their importance is unsuspected. Neglected heterogeneity may bias the parameter estimates towards zero. To correct for unobservable variables gamma and mass point heterogeneity across individuals is introduced into the model.

The chapter is organized as follows. Section 2 introduces the parametric duration models and time-dependent effects. Furthermore, it provides a score test for the time-dependent effects. Section 3 analyzes the effects of omitted variables and introduces gamma and mass point heterogeneity into the model. The results of the estimations are presented in section 4 and section 5 concludes the study.

2. Time-Dependent Effects of UI Benefits

2.1. Time-Dependent Effects

In the Finnish UI system the circumstances of an unemployed person are different during the first three months of unemployment than later. If no suitable jobs are found in the unemployed person's area of residence within the first three unemployment months, the person does not have to accept an offer outside his area of residence. Also during the first months the unemployed person does not have to accept an offer if the job is not suitable for him with respect to his education or previous work experience. This rule concerns persons with education and at least one year of job experience or alternatively persons without proper education and at least two years experience in their jobs. A person who after being unemployed for three months does not accept an offer may lose his benefits. If the effect of the time-dependent change is handled in a flexible manner, it should account for the higher hazard just after the first three months. This is allowed for letting the unemployment benefits and their parameters vary over time.¹⁾

Severance pay may have some effects on the re-employment probability. It is paid by the fund for severance pay to the persons who have been dismissed because of reasons which are related to the decline in the production or the demand for the products of the firm. The severance pay is financed by employers within the context of the payment of unemployment insurance. One of the

prerequisites is also that the recipients of the severance pay must be at least 40 years old and they must have a sufficient amount of experience in their work. The severance pay consists of two parts. The A-part of the severance pay can be paid to any of the persons who fulfil the above mentioned requirements. In 1985 - 1986 the B-part of the severance pay could be paid to the persons who had difficulties in finding acceptable jobs and who had been unemployed three months.

There were 1036 persons (that is about 50 per cent) in the data who were unemployed at least three months. There is no data on how many of these received severance pay, but it can be estimated using aggregate data that about 5-7 per cent of them were eligible to the B-part of the pay. If these persons really have problems in finding acceptable jobs, as is supposed by the rules of the system, they contribute to the likelihood function as censored observations providing relatively small amount of information. Therefore the system of severance pay may probably have a slight positive effect on the hazard function. As a whole the total effect is most likely small compared to the effects of the UI system, because rather few persons are eligible for the severance pay.

The time trended variables may be replaced with their within spell average or using beginning-of-spell values [Heckman and Singer (1984)]. Usually in parametric models the variation in the explanatory variables across observations is used to take into account the time-dependent effects. The problem with these kinds of models is that the over time variation of the covariates may be

absorbed by the baseline specification. Prentice and Gloeckler (1978) specify a semi-parametric model and use the variation in the mean of the covariates, i.e. the variation in the covariates across observations, to estimate the baseline hazard and structural parameters. No assumptions are made about the baseline hazard. In that sense the Prentice and Gloeckler approach is similar to Cox's partial likelihood technique [Cox (1972, 1975)]. Their method has been proposed also by Han and Hausman (1986) and used by Moffitt (1985). Recently Meyer (1990) divided the duration of the unemployment into intervals of one week and extended the Prentice and Gloeckler model by using time-dependent covariates.

A commonly applied specification is the proportional hazards model, where the hazard function $h(t) = h_0(t)h_1(x)$ factors into the product of a function of duration t , the baseline hazard, and function of the explanatory variables x . The PH model assumes that the effect of an explanatory variable is constant during the duration of the unemployment. An alternative is to assume that the effect varies with the duration, remaining constant within predefined intervals. Such an alternative may be relevant in long-term studies, and in cases where the environment of an individual changes starting at a known point in time it may even be the more natural model to apply.

Many studies have shown that plotting the hazard function by duration indicates spikes around the moment of benefit exhaustion [see Marston (1975), Moffitt (1985), Ham and Rea (1987) and Katz and Meyer (1990)]. One may expect similar kind of spikes when the risk period of benefits

starts and when the benefits are reduced. This issue is also discussed in a survey by Atkinson and Micklewright (1991), who expect that the effect of benefits have the least effect in the countries where the administration of the unemployment compensation is very tight. Follman et al. (1990) specify their model such that at the moment of benefit exhaustion the effect of explanatory variables may change. However, they do not allow for unobserved heterogeneity, whereas in our study a discrete mixing distribution is used. It allows for a non-monotonous flexible baseline hazard, which can to some extent reflect the effect of spikes.

Consider q intervals of duration $(t_0, t_1], \dots, (t_{q-1}, t_q)$ with $t_0 = 0$ and $t_q = \infty$. The hazard function of the Weibull model with time-dependent effects can be written

$$(1) \quad h(t) = \alpha t^{\alpha-1} e^{x(\beta + \mu_j)}, \text{ for } t_{j-1} < t \leq t_j, j=1, \dots, q,$$

where $\beta = (\beta_1 \dots \beta_p)$ and $\mu_j = (\mu_{1j} \dots \mu_{pj})$ are $1+q$ vectors of p parameters. To avoid singularity it is defined that $\mu_j = 0$, as $j = 1$. One reason for this kind of specification of time-dependent effects is that the integrated hazard has a closed-form expression. The integrated hazard is obtained by integrating the hazard functions by intervals, which leads to the expression

$$(2) \quad I(t) = \sum_{s=1}^{j-1} [I_s(t_s) - I_s(t_{s-1})] + [I_j(t) - I_j(t_{j-1})].$$

In the Weibull case, for instance, in the third interval, it

would be $I(t) = t_1^\alpha e^{x\beta} + (t_2^\alpha - t_1^\alpha) e^{x(\beta+\mu_2)} + (t^\alpha - t_2^\alpha) e^{x(\beta+\mu_3)}$.

The likelihood contribution of an individual can be written

$$(3) \quad \ell = \frac{q}{\pi} \prod_{j=1}^c [h(t) e^{-I(t)}]^{d_j},$$

where c is an indicator for a complete spell of unemployment and d_j is an indicator for the interval, i.e. $d_j = 1$ if $t_{j-1} < t \leq t_j$ otherwise $d_j = 0$.

The explanatory variables may be time-dependent as well, i.e. the time-dependent variables may take different values in the intervals. In this study the interest concerning the PH assumption is in a single time-dependent explanatory variable, the benefit replacement ratio. Its effect is tested in two intervals $(t_0, t_1]$ and $(t_1, t_2]$, where $t_0 = 0$, $t_1 = 3$ and $t_2 = 24$ months. The longest spells of unemployment in the data are nearly two years. The economic reason for estimating the change of the hazard function is that after the first three months of unemployment the rules of the UI system are different.²⁾ Thus, in our case $x(\beta+\mu_j)$ is written as $x\beta + x_{rj}(\beta_r+\mu_j)$, where x_{rj} is the benefit replacement ratio in the intervals $j = 1, 2$, β_r is its parameter and μ_j is the additional parameter in the interval j . The rest of the explanatory variables x are constant over time.

The approach taken in this study has several advantages. The method is more efficient than the earlier models by Prentice and Gloeckler (1978) and Meyer (1990) in the sense that the duration is continuous. It is not partitioned into intervals. The loss of efficiency of the

procedure suggested by Prentice and Gloeckler is due to aggregating continuous data into grouped data (weekly, monthly, etc.). However, the loss in efficiency is not necessarily a serious problem with large data sets. The parameter estimates may be sensitive with respect to how the duration is classified into days or weeks. The approach avoids inconsistent estimation of covariate coefficients due to allowing for the time-dependent covariates and their parameters to vary over time. Furthermore, unobserved heterogeneity across observations will be taken into account.

Another approach is to allow the shape parameter of the Weibull model to vary in the predefined intervals. This line of argument was pursued in Kettunen (1989). The more flexible specification of the shape of the hazard function leads to a steeper decreasing hazard, but there are not any notable changes in a Weibull model with gamma heterogeneity.³⁾

2.2. Testing the Proportional Hazards Assumption

In this section a score test for the PH assumption is presented. Specification tests are particularly important for many econometric models estimated by maximum likelihood, such as parametric duration models, where few diagnostic tests are currently available. A chi-squared test for the PH assumption based on the difference between the number of failure times observed and its expected value in each category from a given partition of the time axis is

suggested by Schoenfeld (1980). A Wald type of test for the PH assumption in a two-step regression model has been suggested by Anderson and Senthilselvan (1982). Moreau, O'Quigley and Mesbach (1985) presented a score test for checking the assumption. The test was extended by O'Quigley and Pessione (1989). All the tests have been developed in the context of Cox's model and they are not directly applicable to parametric duration models.

The alternative nonproportional hazards model of this study assumes that the effect of a duration-dependent covariate varies as a step function. However, this model may be awkward to estimate. Therefore it may seem preferable at first to avoid such an estimation and develop a score test. The null hypothesis for the PH model is $H_0: \mu_2 = \mu_3 = \dots = \mu_q = 0$, which leads to the hazard function

$$(4) \quad h(t) = \alpha t^{\alpha-1} e^{x\beta + x_{rj}\beta_r}, \text{ for } t_{j-1} < t \leq t_j, j=1, \dots, q.$$

At any time t the hazard function depends on the duration-dependent value of the replacement ratio x_{rj} .

The score test is based on the statistic $S_j = n^{-1/2} L_{\mu_j}$, where $L_{\mu_j} = \partial L / \partial \mu_j$ is evaluated under the null hypothesis. In the Weibull case it becomes

$$(5) \quad S_j = \begin{cases} 0, & t \leq t_{j-1} \\ n^{-1/2} \{ \hat{C} - \hat{I}(t) [1 - (t_{j-1}/t)^\alpha] \} x_{rj}, & t_{j-1} < t \leq t_j \\ n^{-1/2} \{ -\hat{I}(t) [(t_j/t)^\alpha - (t_{j-1}/t)^\alpha] \} x_{rj}, & t_j < t, \end{cases}$$

which have been written using the generalized residuals

$\hat{I}(t) = t^{\hat{\alpha}} e^{\hat{\beta} + x_{rj} \hat{\beta}_r}$, i.e. the integrated hazard of the fitted Weibull model. The definitions of the residuals are found in Cox and Snell (1968).

The information matrix - $E[\partial^2 L / \partial \phi \partial \phi']$, $\phi = (\mu, \alpha, \beta)$, can be consistently estimated by

$$(6) \quad I = - n^{-1} \begin{bmatrix} L_{\mu\mu} & L_{\mu\psi} \\ L_{\psi\mu} & L_{\psi\psi} \end{bmatrix},$$

where $\psi = (\alpha, \beta)$. It can be inverted using the method outlined by Theil (1983, p. 13). The top left-hand block of the inverse of the information matrix is of relevance towards calculating the test statistic. It can be written

$$(7) \quad V = - n^{-1} (L_{\mu\mu} - L_{\mu\psi} L_{\psi\psi}^{-1} L_{\psi\mu}').$$

Under the null hypothesis \hat{S} is asymptotically $N_{p(q-1)}(0, V)$. The asymptotic null distribution of the score test is not affected if the required estimates of V are evaluated using any estimator, which is consistent under H_0 . The matrix V can be expressed consistently as the outer product form of the information matrix identity

$$(8) \quad V = n^{-1} (L_{\mu}' L_{\mu} - L_{\mu}' L_{\psi} (L_{\psi}' L_{\psi})^{-1} L_{\psi}' L_{\mu}),$$

which is a convenient form, because it requires neither an expression for the Hessian of the log likelihood function

nor analytic evaluation of the information matrix. The score test statistic is then of the form

$$(9) \quad T = \hat{S}' \hat{V}^{-1} \hat{S}.$$

The statistic is based upon the result that the quadratic form $S'V^{-1}S$ manifests an asymptotic chi-squared distribution when the null hypothesis is true. The test statistic can then be calculated as nR^2 from a pseudo-regression based on ordinary least squares, where a vector of ones is regressed on L_μ , L_α and L_β . The procedure based on the pseudo-regressions is described by Chesher (1983) and Lancaster (1984).

3. Mixing Distributions

The first approach in this section combines the correction for omitted variables based on gamma mixing distribution and a parametric duration model with time-dependent covariates. If unobserved characteristics are not adequately captured by explanatory variables, this may lead to biases in parameter estimates. It is known that the bias is towards zero [see Lancaster (1979), Lancaster and Nickell (1989)]. Therefore, the parameters of the model may be expected to increase in absolute value when omitted variables are taken into account.

The hazard function of a Weibull model allowing for time-dependent effects and gamma heterogeneity can be written as

$$(10) \quad h_g(t) = \alpha t^{\alpha-1} e^{x(\beta+\mu_j)} [1 + \sigma^2 I(t)]^{-1}$$

and the corresponding integrated hazard can be written as

$$(11) \quad I_g(t) = 1/\sigma^2 \log[1 + \sigma^2 I(t)],$$

where $I(t)$ is the integrated hazard of the original model (2). In our case $x\beta + x_{rj}(\beta_r + \mu_j)$ is substituted for $x(\beta + \mu_j)$, because we are interested in the time-dependent effects of a single time-dependent variable.

The second approach in this section combines the correction for omitted variables based on the mass point approach and the parametric duration model with time-

dependent covariates. Meyer (1990) found the computation of the discrete mixing distribution difficult. In this context the computation can be carried out. In the case of parametric duration models the mixing likelihood for an individual can be written as

$$(12) \quad f_Q = \prod_{j=1}^Q \pi_j \left[\sum_{i=1}^m p_i h_i(t) e^{-I_i(t)} \right]^{d_j},$$

where $h_i(t) = \alpha t^{\alpha-1} e^{u_i+x(\beta+\mu_j)}$ and $I_i(t) = t^\alpha e^{u_i+x(\beta+\mu_j)}$ are the hazard function and integrated hazard for the group i in the Weibull case. In a model with one time-dependent explanatory variable $x\beta + x_{rj}(\beta_r+\mu_j)$ is substituted for $x(\beta+\mu_j)$.

4. The Results

Data on 2077 Finnish unemployed persons were used in estimating the models. The results are presented in Table 1. The first model is the basic Weibull model with the hazard function $h(t) = \alpha t^{\alpha-1} e^{x\beta + x_r\beta_r}$, where an average replacement ratio over the unemployment period is used. A Weibull model with a hazard function $h(t) = \alpha t^{\alpha-1} e^{x\beta + x_{rj}\beta_r}$, including the time-dependent replacement ratios is estimated in the two intervals, $(t_0, t_1]$ and $(t_1, t_2]$, where $t_0 = 0$, $t_1 = 3$ and $t_2 = 24$ months. The second column of Table 1 presents this model. The negative effect of the replacement ratio β_r decreases substantially when time-dependent replacement ratios are introduced into the model.

A score test for the PH assumption is made. The test statistics calculated under H_0 takes a value of 8.84, which exceeds the critical value $\chi^2_{1,0.95} = 3.84$. The conclusion is that the PH assumption is rejected for the replacement ratio. The test suggests estimating a model with time-dependent parameters.

A model with the hazard $h(t) = \alpha t^{\alpha-1} e^{x\beta + x_{rj}(\beta_r + \mu_2)}$ including the time-dependent replacement ratios and their time-dependent effects is in the third column. The parameter estimate β_r takes a value -0.894, and after the first three months the additional parameter estimate μ_2 takes a positive value 0.871. Unemployed persons who are eligible for benefits face a risk of losing benefits after the first three months. The risk increases the re-employment probability and it is captured by the

parameter μ_2 .

The PH assumption is not valid either after allowing for gamma heterogeneity in the fourth model, since the corresponding parameter estimates take the values -1.506 and 1.475. It is interesting to note that Nickell (1979) using data from the U.K. and a different kind of model found similar effects of UI benefits. In both of the studies the effect is first negative and statistically significant but later on the effect vanishes.

**Table 1. Time-dependent effects of UI benefits allowing
for gamma heterogeneity**

	(1)	(2)	(3)	(4)
	Std.errors in parentheses			
Shape parameter	0.861 (0.020)	0.853 (0.021)	0.822 (0.022)	1.096 (0.054)
Variance of heterogeneity				0.955 (0.161)
Constant	-1.478 (0.136)	-1.576 (0.137)	-1.454 (0.139)	-1.183 (0.207)
Number of children	-0.004 (0.050)	-0.088 (0.050)	-0.082 (0.049)	-0.134 (0.074)
Married	0.170 (0.065)	0.207 (0.066)	0.203 (0.065)	0.198 (0.099)
Sex	-0.007 (0.056)	-0.040 (0.057)	-0.041 (0.057)	-0.074 (0.088)
Age	-0.042 (0.003)	-0.044 (0.003)	-0.044 (0.003)	-0.060 (0.005)
Level of education	0.064 (0.058)	0.053 (0.059)	0.066 (0.059)	0.029 (0.093)
Training for employment	0.176 (0.072)	0.187 (0.074)	0.181 (0.073)	0.331 (0.116)
Member of UI fund	0.213 (0.060)	0.243 (0.062)	0.237 (0.061)	0.356 (0.093)
Came from schooling	0.291 (0.078)	0.299 (0.079)	0.291 (0.078)	0.380 (0.128)
Came from housework	-0.711 (0.124)	-0.716 (0.125)	-0.731 (0.124)	-0.895 (0.171)
Regional demand	0.168 (0.238)	0.308 (0.240)	0.271 (0.240)	0.487 (0.330)
Occupational demand	0.641 (0.600)	0.743 (0.609)	0.715 (0.602)	0.457 (0.943)
Taxable assets	1.021 (1.080)	1.997 (1.102)	1.224 (1.073)	1.223 (1.393)
Replacement ratio, β_r	-1.223 (0.150)	-0.376 (0.120)	-0.894 (0.191)	-1.506 (0.264)
Replacement ratio, μ_2			0.871 (0.208)	1.475 (0.267)
Log likelihood	-4962.5	-4993.4	-4985.4	-4950.4

The results of estimations of mass point models are presented in Table 2. The model with two mass points produces approximately constant hazard functions for the two groups which are not controlled for explanatory variables. The models with three or four mass points produce increasing hazard functions. The absolute values of statistically significant parameter estimates increase in most cases when more mass points are introduced into the model, as is to be expected. Lindsay's (1983) criterion was used to determine the number of mass points. It turns out that four mass points are enough to rectify the effect of omitted variables using this specification and data set. It can be concluded that allowing for time-dependent replacement ratios reduces the unobserved heterogeneity, since five points of support were needed to correct for unobserved heterogeneity in the models with the average replacement ratios (cf. Chapter IV).

The parameter estimates of the explanatory variables differ only slightly from the estimates of the model with average replacement ratios over the duration of unemployment. The effects of children and marriage, however, became statistically significant. Unemployed persons who have children or persons who are not married have lower re-employment probabilities.

In the final model with four mass points the parameter estimate of the replacement ratio takes a value of -1.890 and the additional parameter μ_2 due to the risk of losing benefits takes a value of 1.752. The negative effect of the

replacement ratio vanishes after the first three months and the PH assumption is not valid.

Separate models were estimated for the non-members and members of the labour unions, because the rules of the system are not similar in these groups. A model with two intervals was estimated for the non-members. The parameter μ_2 captures the effect of increased risk of losing benefits. The estimation of the mass point models for the non-members of labour unions was problematic. The problems arose when estimating the model with two or more mass points. During the iterations the estimate for the parameter u_2 approached a large negative value of -32.3 leading to a non-singular Hessian matrix even with the double precision of the programme. Therefore the method failed in obtaining the maximum likelihood estimates, though the solution was almost reached.

A large value of u_2 means that in the sample there is a group which has virtually a zero probability of becoming employed. There is clearly a serious technical problem. It was solved by modifying the likelihood function. The new estimator is based on the assumption that there is indeed a group in the data consisting of the persons who will not become employed. The hazard function of these persons is equal to zero and the survivor function is equal to one. The share of these problematic persons in the sample is p_m , which remains to be estimated. The mixing likelihood contribution can then be written as

$$(13) \quad f_0 = \prod_{j=1}^g \left[\sum_{i=1}^{m-1} p_i h_i(t) e^{-I_i(t)} + p_m (1 - c) \right]^{d_j}.$$

Complete spells do not contribute to the likelihood function via the last term of the likelihood contribution. With censored values the contribution comes with probability p_m through the survivor function, which is set equal to one. The method was used successfully. One point worth noting is that the solution of this model is not far from the parameter estimates of the model, which was near the convergence. This is an indication that the result is reasonable.

The results of the estimations of the models for the non-members are presented in Table 3. Three points of support of the discrete mixing distribution were needed to rectify the effect of omitted variables. The results indicate that among the non-members there are about 11 per cent of the individuals who would never become employed. After the first three months there is a positive shift in the effect of the replacement ratio. The total effect of the replacement ratio remains statistically significantly negative, however, during the risky period of losing benefits.

A more detailed analysis is to divide the duration of unemployment into three intervals $(t_0, t_1]$, $(t_1, t_2]$ and (t_2, t_3) , where $t_0 = 0$, $t_1 = 3$, $t_2 = 4.6$ and $t_3 = 24$ months. This model is reasonable for the members of labour unions. The parameter μ_2 captures the effect of the risk period and μ_3 captures both the effect of risk and the reductions of benefits. In addition the severance pay may affect both of these parameters and the rules of the employment office may affect the estimate of μ_3 . One might argue that in order to get robust estimates the second interval from 3 to 4.6

months (100 days of unemployment) might be too short for the investigation of the rules regarding labour mobility using parametric models of unemployment duration. For example, Korpi (1991) has estimated the effects of benefit indicators only in two intervals. With short intervals the parameter estimates may not be reliable if there are not enough observations in each of the intervals. However, since there are 81 complete spells in the second interval, such a model was estimated.

The results of the estimations for the members of labour unions are presented in Table 4. It turns out that the replacement ratio has a small negative effect on the re-employment probability during the first three months. The negative effect is different from zero at the 9 per cent significance level. The negative effect of the replacement ratio is clearly higher for the non-members than for the members of labour unions. In the second risky period the effects vanishes. In the third interval the positive shift is larger. A similar jump of the hazard function was not observed for the non-members. Simple tests of the significance of the parameter estimates show that the effects of the replacement ratios of benefits do not statistically differ from zero in the second and third intervals. During an interval at any point in time the hazards of any two persons are proportional to each other. The result means that after the reduction of benefits the person with higher benefits has a higher probability of becoming employed. This result supports the search theoretical result presented by Usategui (1988) that the

proportional decrease of benefits has a larger incentive effect for the person having higher benefits.

Even though the hazard functions of the groups are monotonously increasing in the final model the sample hazard is not necessarily monotonous. The sample hazard is defined as follows

$$(13) \quad h(t) = \frac{\sum_{i=1}^m p_i h_i(t) e^{-I_i(t)}}{\sum_{i=1}^m p_i e^{-I_i(t)}}.$$

Figure 1 illustrates the sample hazard functions for an average person receiving benefits in the sample. It can be seen that the hazard functions are rather flexible. The first hazard function is plotted using the model with two intervals for the whole sample. The second hazard is the corresponding model for the non-members of labour unions. It can be seen that the hazard functions make prominent jumps after the first three months of unemployment, since the eligibility of benefits becomes stricter. For the non-members of labour unions there seems to be difficulties in obtaining acceptable job offers during the first few weeks of unemployment. The third hazard function is plotted for the members of labour unions and it is based on the estimation of the effects of replacement ratios in three intervals. There exist jumps in the hazard function after the first three months and the hundredth day of unemployment. Another feature which should be pointed out is that the hazard function of the long-term unemployed members of the labour unions is higher than the corresponding function of the non-members.

Table 2. Time-dependent effects of UI benefits and mass point heterogeneity

	Number of mass points		
	m=2	m=3	m=4
	Std.errors in parentheses		
Shape parameter	0.987 (0.040)	1.194 (0.064)	1.288 (0.091)
Number of children	-0.109 (0.065)	-0.175 (0.079)	-0.184 (0.089)
Married	0.201 (0.087)	0.216 (0.103)	0.199 (0.115)
Sex	-0.060 (0.077)	-0.088 (0.092)	0.085 (0.104)
Age	-0.055 (0.004)	-0.063 (0.006)	-0.069 (0.007)
Level of education	0.029 (0.080)	0.066 (0.098)	0.040 (0.109)
Training for employment	0.331 (0.102)	0.240 (0.118)	0.296 (0.132)
Member of UI fund	0.333 (0.083)	0.368 (0.097)	0.411 (0.110)
Came from schooling	0.341 (0.110)	0.408 (0.131)	0.441 (0.148)
Came from housework	-0.819 (0.153)	-1.011 (0.187)	-1.114 (0.206)
Regional demand	0.440 (0.297)	0.420 (0.350)	0.489 (0.388)
Occupational demand	0.609 (0.826)	0.451 (0.980)	0.512 (1.106)
Taxable assets	0.881 (1.255)	2.087 (1.594)	1.369 (1.621)
Replacement ratio, β_r	-1.276 (0.229)	-1.741 (0.303)	-1.890 (0.345)
Replacement ratio, μ_2	1.302 (0.244)	1.644 (0.297)	1.752 (0.327)
u_1	-0.980 (0.193)	-0.004 (0.247)	0.459 (0.366)
u_2	-2.738 (0.238)	-2.103 (0.259)	-1.360 (0.478)
u_3		-4.865 (0.816)	-2.792 (0.489)
u_4			-7.277 (7.803)
p_1	0.648 (0.060)	0.272 (0.014)	0.187 (0.009)
p_2	0.352 (0.060)	0.617 (0.023)	0.345 (0.052)
p_3		0.112 (0.037)	0.399 (0.016)
p_4			0.069 (0.027)
Log likelihood	-4956.5	-4945.9	-4943.6

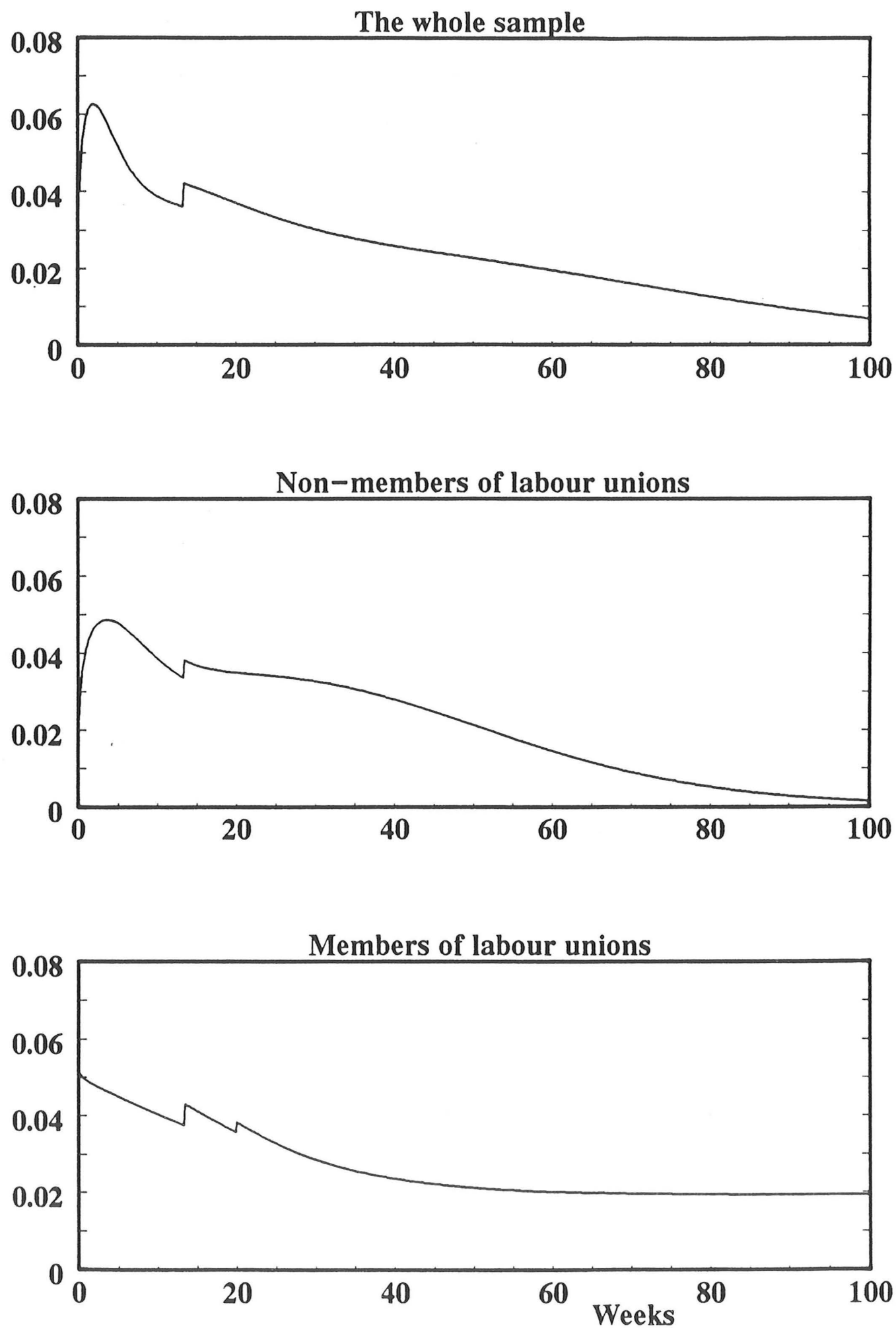
**Table 3. Time-dependent effects of the replacement ratios
of UI benefits for the non-members of labour
unions**

	Number of mass points		
	m=1	m=2	m=3
	Std.errors in parentheses		
Shape parameter	0.847 (0.028)	0.993 (0.041)	1.272 (0.082)
Number of children	-0.074 (0.069)	-0.093 (0.077)	-0.167 (0.112)
Married	0.089 (0.100)	0.008 (0.113)	0.012 (0.151)
Sex	-0.009 (0.074)	-0.067 (0.087)	-0.085 (0.117)
Age	-0.035 (0.005)	-0.042 (0.005)	-0.054 (0.008)
Level of education	0.196 (0.075)	0.185 (0.094)	0.240 (0.129)
Training for employment	0.146 (0.108)	0.171 (0.124)	0.245 (0.168)
Came from schooling	0.303 (0.087)	0.248 (0.106)	0.404 (0.146)
Came from housework	-0.805 (0.178)	-0.924 (0.186)	-1.150 (0.242)
Regional demand	0.098 (0.318)	0.105 (0.341)	0.335 (0.457)
Occupational demand	-0.150 (0.936)	-1.660 (1.022)	-2.103 (1.296)
Taxable assets	0.569 (2.103)	0.174 (2.229)	1.823 (2.889)
Replacement ratio, β_r	-1.523 (0.257)	-2.083 (0.281)	-2.784 (0.403)
Replacement ratio, μ_2	1.004 (0.295)	1.230 (0.312)	1.778 (0.404)
u_1	-1.582 (0.192)	-1.155 (0.216)	-0.025 (0.329)
u_2			-2.076 (0.339)
p_1		0.888 (0.020)	0.286 (0.016)
p_2		0.112 (0.020)	0.608 (0.007)
p_3			0.106 (0.023)
Log likelihood	-2671.5	-2640.9	-2632.9
Number of observations	1212	1212	1212

Table 4. Time-dependent effects of the replacement ratios
of UI benefits for the members of labour unions

	Number of mass points m=1 m=2 Std.errors in parentheses	
Shape parameter	0.810 (0.033)	0.988 (0.060)
Number of children	-0.120 (0.070)	-0.126 (0.103)
Married	0.311 (0.091)	0.364 (0.123)
Sex	-0.054 (0.091)	-0.035 (0.121)
Age	-0.054 (0.005)	-0.068 (0.007)
Level of education	-0.159 (0.103)	-0.396 (0.139)
Training for employment	0.184 (0.107)	0.329 (0.149)
Came from schooling	0.226 (0.187)	0.279 (0.268)
Came from housework	-0.587 (0.176)	-0.603 (0.243)
Regional demand	0.483 (0.374)	0.841 (0.489)
Occupational demand	1.515 (0.829)	1.855 (1.165)
Taxable assets	1.683 (1.296)	1.713 (1.590)
Replacement ratio, β_r	-0.293 (0.286)	-0.584 (0.344)
Replacement ratio, μ_2	0.480 (0.323)	0.657 (0.350)
Replacement ratio, μ_3	0.652 (0.304)	1.004 (0.355)
u_1	-0.987 (0.233)	-0.381 (0.342)
u_2		-2.063 (0.392)
p_1		0.612 (0.108)
p_2		0.388 (0.108)
Log likelihood	-2295.4	-2285.3
Number of observations	865	865

Figure 1. Sample hazard functions of Weibull mass point models



5. Conclusions

In this chapter the effects of unemployment benefits on the unemployment spells were examined. In the Finnish system the circumstances of an unemployed person change starting at a known point in time. After the first three months unemployed persons are encouraged to move and change their occupations to become employed. Furthermore the recipients of the earnings-related unemployment allowances have a 20 per cent reduction in their benefits after the first 100 days of unemployment. In this study the effect of these changes were estimated. The effects of unemployment benefits were allowed to vary with the duration of unemployment spell, remaining constant within predefined intervals. A technique for estimating the time-dependent effects of time-dependent explanatory variables on the re-employment probability was presented.

Often it may be preferable to avoid estimating an alternative nonproportional hazards model, because the estimations of these kinds of models may not be straightforward. Therefore the focus was at first on a score test. Tests for the PH assumption have been studied in the context of Cox's model by many authors. In this study a score test for the PH assumption was extended to parametric duration models. The test shows that the effect of benefits does not stay constant during the unemployment spell.

If the average replacement ratio during the unemployment spell is used, the effect of the replacement

ratio on the re-employment probability is negative. The microeconomic data collected from various registers include, however, the time-dependent replacement ratios. Alternative models with time-dependent effects of unemployment benefits were estimated. The replacement ratio has a negative effect on the re-employment probability during the first three months, but after that period the effect vanishes.

A more detailed analysis using separate models for the non-members and members of labour unions show that after the first three months there is a positive shift in the parameter estimate and the effect vanishes for the members of labour unions, but for the non-members the negative effect decreases appreciably. After the reductions of earnings-related unemployment allowances the effect of the replacement ratio for the members of labour unions turns positive. It supports the search theoretical result that the proportional decrease of benefits has a larger incentive effect for the persons having higher benefits.

Even though the data are rich in explanatory variables and more reliable than the data based on interviews, there is reason to assume that relevant variables have been omitted from the model. The influence of omitted variables was taken into account in estimation assuming that the effects have a gamma and discrete mass point distribution. When heterogeneity is introduced into the model, the absolute values of parameter estimates increase, but the correction for omitted variables does not eliminate the result that the effect of UI benefits vanishes after the first three months.

Footnotes

1. The search theoretical model suggests a very slowly increasing hazard function somewhat before the risky period and reductions of benefits. The effect of the risky period is higher for the recipients of the higher earnings-related unemployment allowance. Furthermore the reductions are only applied for these benefits. This effect comes through the membership dummy. However, it is based on full information on the rules of the UI system. If a person suddenly learns that his benefits are reduced, the hazard jumps up (see Kettunen, 1992a). Empirical evidence based on the baseline hazard of semiparametric models gives support to the limited information (see Kettunen, 1992b).

2. The elasticity of the hazard function with respect to the replacement ratio is in a logarithmic form $\partial \log h(t) / \partial \log x_r = x_r \beta_r$. Since the unemployed person loses the right to refuse a job offer, it is reasonable to let the elasticity be $x_{rj}(\beta_r + \mu_j)$ after these changes of the UI system. A similar change may occur when the benefits are reduced.

3. The likelihood contribution of a Weibull model with time-dependent shape parameters in three intervals $(t_0, t_1]$, $(t_1, t_2]$ and $(t_2, t_3]$, where t_0, t_1, t_2 and t_3 are 0, 3, 12 and 24 months, can be written as

$$\ell = \prod_{j=1}^3 \pi [h_j(t)^c e^{-I_j(t)}]^{d_j},$$

where the hazard functions in the intervals are

$$h_1(t) = \alpha_1 t^{\alpha_1 - 1} e^{x\beta}$$

$$h_2(t) = \alpha_1 t_1^{\alpha_1 - \alpha_2} t^{\alpha_2 - 1} e^{x\beta}$$

$$h_3(t) = \alpha_1 t_1^{\alpha_1 - \alpha_2} t_2^{\alpha_2 - \alpha_3} t^{\alpha_3 - 1} e^{x\beta}$$

and the corresponding integrated hazards are

$$I_1(t) = t^{\alpha_1} e^{x\beta}$$

$$I_2(t) = I_1(t_1) + (\alpha_1/\alpha_2) t_1^{\alpha_1 - \alpha_2} (t^{\alpha_2} - t_1^{\alpha_2}) e^{x\beta}$$

$$I_3(t) = I_2(t_2) + (\alpha_1/\alpha_3) t_1^{\alpha_1 - \alpha_2} t_2^{\alpha_2 - \alpha_3} (t^{\alpha_3} - t_2^{\alpha_3}) e^{x\beta}.$$

Note that $h_j(t_j) = h_{j+1}(t_j)$ and $I_j(t_j) = I_{j+1}(t_j)$ so that the hazard and integrated hazard are continuous. The model with the time-dependent shape parameters was estimated. The estimates of the structural parameters changed slightly. Allowance for the time-dependent shape parameters of the hazard did not lead to an increasing hazard function but instead to a steeper decreasing function.

After allowing for gamma heterogeneity the hazard and integrated hazard can be written as

$$h_{gj} = h_j(t) [1 + \sigma^2 I_j(t)]^{-1}$$

$$I_{gj} = 1/\sigma^2 \log[1 + \sigma^2 I_j(t)]^{-1}.$$

With gamma heterogeneity the estimates of the structural parameters and the variance of heterogeneity are very near the corresponding model without allowance for time-dependent shape. The shape of the hazard function after allowing for the time-dependent shape parameters is rather close to the shape of the basic Weibull model with gamma heterogeneity.

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Chapter VI

THE EFFECTS OF EDUCATION ON THE DURATION OF UNEMPLOYMENT

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Chapter VI

THE EFFECTS OF EDUCATION ON THE DURATION OF UNEMPLOYMENT

This chapter studies the relationship between the level of education and probability of re-employment. Using a search theoretical model it is shown that on the lowest levels additional education increases the probability of re-employment, but on the highest levels the relationship turns negative. Using Finnish microeconomic data on unemployed workers it is shown that the unemployed persons who have about 13 - 14 years of education have the highest re-employment probability.

1. Introduction

The role of education over the life-cycle has been seen as an investment in human capital. In the theories of human capital it is usually assumed that the optimum amount of education is chosen to maximize life-time earnings or utility [e.g. Blinder and Weiss (1976)]. Empirical applications have been presented, for instance, by Wolff and van Slijpe (1973), Willis and Rosen (1979), Garen (1984) and a replication of these studies by Oosterbeek (1990). Recently a paper by Groot and Oosterbeek (1990) studies the optimum amount of education and introduces a probability of becoming unemployed after school. There are

other theories which support the argument that overeducation can be a long-lasting problem with negative effects on productivity [e.g. Spence (1973), Hartog (1981, 1986), Ducan and Hoffman (1981), Rumberger (1981), Tsang and Levin (1985) and Hartog and Oosterbeek (1988)].

The purpose of this study is to analyze the effects of education on the life-time utility and re-employment in the light of search theories. The optimal behaviour of an unemployed person is examined assuming a given level of education. It is shown that the effect of education on the value of life-time utility and the probability of re-employment is not straightforward. Highly educated unemployed persons have problems in finding acceptable job offers.

Empirical evidence is presented using Finnish microeconomic data. About 37 per cent of the unemployed persons having at least the lowest university degree are seeking jobs in teaching and research. About 19 per cent of these unemployed persons' occupations are in the construction or technical occupations in factories. Furthermore at least 11 per cent of them can be classified into the sparsely located service or production jobs. Typically those jobs requiring a high level of education are in towns, building sites or factories. If these persons lose their jobs, they have to pay moving or commuting costs in order to get a new job. A change of occupation in their area of residence would involve costs in terms of lower wages.

Assuming a finite search horizon it is evident that the time-path of the reservation utility is decreasing, which

leads to an increasing hazard function. However, if lump-sum types of re-employment costs are assumed, the reservation utility is increasing and the hazard function is decreasing during the last few years. These theoretical findings are in accordance with the empirical evidence that the hazard function of an unemployed person is increasing and that elderly persons are apt to have serious problems in finding acceptable offers.

A higher level of education leads to a higher reservation utility, but the effect of education on the probability of re-employment is analytically ambiguous. Numerical examples show that to some extent the probability of becoming employed increases when the level of education increases. The function subsequently becomes decreasing, however, since the possibility of getting an acceptable offer decreases. Empirical evidence is in accordance with these theoretical results.

The remainder of this study is set out as follows. In section 2 the search theoretical model is presented. The numerical examples of the search models are presented in section 3. Section 4 presents the empirical evidence. A Weibull model of unemployment duration is estimated assuming that the effect of omitted variables can be taken into account using a discrete mass point distribution. Section 5 concludes the study.

2. The Effects of Education in a Search Model

In this section a search theoretical model is developed to analyze the effects of education on the time-path of the value function, reservation utility and hazard function during the unemployment period. Recent surveys in search theories can be found in Mortensen (1986) and Kiefer and Neumann (1989). It is assumed that the unemployed person evaluates job opportunities in terms of utility, which may include consumption and other characteristics. For simplicity the utility of an individual is assumed to be additively separable. It is assumed that the individual's remaining time horizon is limited. It may be interpreted as an unemployed person's remaining time in the labour force from the beginning of his or her unemployment period. The search is assumed to take place during a unit of time dt after which the remaining search horizon is denoted as t .

While an unemployed person is searching for a job, he is assumed to obtain instantaneous utility b . Often b is taken to be identical with the amount of unemployment benefits received plus other income net of searching costs.

The arrival rate of job offers $a(s)dt$ is assumed to be related to the length of search interval and to depend on the level of education s . It is assumed that the arrival rate of offers is increasing and a concave function of education. Job opportunities rise with the length of schooling, as one can accept a job below the educational level but can not elicit a job offer above the educational level.¹⁾

If an individual is unable to find employment from the local job market, an acceptable job may have to be sought elsewhere. It is assumed in the model that there are two kinds of costs of re-employment which depend on the level of education. Some of the costs of re-employment will remain permanent. The permanent cost of re-employment $c(s)$ is of a flow type. When an offer is accepted, the individual pays these costs daily. These costs may include travelling expenses between home and work and the loss of utility related to changing occupations. Some of the costs of re-employment, e.g. the moving cost $c_m(s)$, may be of a lump-sum type. The costs are probabilistic and they are measured in terms of utility. It is assumed that $c(s)$ and $c_m(s)$ are increasing and convex functions of the level of education.

According to the search model well-educated persons have higher reservation utilities, i.e. reservation wages. Therefore they have probably on average also higher wages. Since they spend all that they earn, their costs are also higher. It is also plausible that well-educated unemployed persons have less acceptable jobs near where they live. The same line of argument is followed also in the discussion by Holmlund (1984). These assumptions are confirmed by the data, which shows that these persons move more often than less educated persons. Holmlund presents also similar empirical evidence for Sweden. Unfortunately there are no data on the costs of becoming employed in our study.

Better educated persons usually have more consumer durables and expensive housing. To get a job persons often have to move and sell their homes. Therefore the fixed

costs increase over the level of education. Concerning the costs of re-employment it does not matter whether the costs are of the flow or fixed type, since the discounted flow-type costs can be seen as fixed costs. Therefore if the assumption of fixed costs are relaxed, the basic results remain unchanged.

Workers maximize the expected present value of the utility. The value of the search can be written

$$\begin{aligned}
 (1) \quad V(t+dt) = & bB(dt) \\
 & + a(s)dt \int_{u^*(t)}^{\bar{u}} [(u - c(s))B(t) - c_m(s)]dF(u, s)D(dt) \\
 & + \{1 - a(s)dt[1 - F(u^*(t))]\}V(t)D(dt),
 \end{aligned}$$

where F is the distribution function of job offers in terms of utility. It is assumed that the level of education shifts F to the right in such a way that F is first order stochastically dominated by the new distribution function. The first term of the value function $V(t+dt)$ on the right-hand side of (1) is the instantaneous utility during dt . The multiplicand $B(dt) = [1 - \exp(-rdt)]/r$ is the discount factor, where r is the subjective rate of time preference. The second term is the expected discounted utility related to an acceptable offer. The parameter \bar{u} is the maximum attainable utility and $u^*(t)$ is the reservation utility at time t . The utility $u^*(t)$ is the endogenous variable of this model. Offers which are at least $u^*(t)$ are acceptable. $B(t)$ discounts the expected utility related to an acceptable offer during the remaining search period

$[t, 0)$. The third term is the expected discounted utility related to an unsuccessful search, where $D(dt) = \exp(-r dt)$ discounts the expected value of search apart from the instantaneous utility from t to $t+dt$.

By expansion $B(dt) = dt + o(dt)$, where $o(dt)$ is the remainder term. The instantaneous utility of being unemployed is proportional to the length of the time interval dt . Correspondingly the discount factor of expected utilities $D(dt) = 1 - r dt + o(dt)$. Substituting the discount factors in $V(t+dt)$, forming the difference quotient $[V(t+dt) - V(t)]/dt$, taking the limits as dt approaches zero and rearranging the terms gives the differential equation of expected utility stream with respect to time

$$(2) \quad \dot{V}(t) = b - rV(t)$$

$$+ a(s) \int_{u^*(t)}^{\bar{u}} [(u - c(s))B(t) - c_m(s) - V(t)] dF(u, s).$$

After the active search period the search does not produce any expected utility. Solving from (1) the value function $V(t) = b/r$ during the passive search, which implies $\dot{V}(t) = b - rV(t) = 0$.

The optimal reservation utility is a solution to a dynamic optimal control problem. Differentiating $\dot{V}(t)$ with respect to the reservation utility $u^*(t)$ gives the necessary condition for the optimal reservation utility

$$(3) \quad u^*(t) = c(s) + [c_m(s) + V(t)]/B(t).$$

Rewriting (3) we see that the value function $V(t) = [u^*(t) - c(s)]B(t) - c_m(s)$. This means that the expected value of continuing a search, the value function, is equal to the utility of an acceptable offer minus the permanent loss of utility due to becoming employed discounted over the lifetime, net of the lump-sum moving costs.

Next the comparative dynamics are studied. The focus is on the effects of education with respect to the reservation utility and hazard function. The hazard function is defined as

$$(4) \quad h(t) = a(s)[1 - F(u^*(t), s)].$$

It is a product of the arrival rate of job offers and the probability that an offer is acceptable. The effects of education on the arrival rate of job offers, offer distribution and re-employment costs are assumed to be positive. Then the effects of education can be examined via the arrival rate, offer distribution and costs. The details of the calculations are presented in Appendix 1.

The effect of education via the arrival rate of job offers has an ambiguous effect on the hazard function. The direct effect is positive, since the number of occasions on which one is able to leave unemployment increases. The indirect effect via the reservation utility is negative, because of the increased selectivity of the searchers. Recently, a number of papers have been written in which sufficient conditions are derived for the hazard function

to be non-negative. A short survey is given by van den Berg (1990). This issue is, however, beyond the scope of this study.

To solve the effects of education via the offer distribution, a translation of the distribution function F to the right is made so that $F(u, s) = G[u + \mu(s)]$, for all u and $\mu > 0$. The translation is said to first order stochastically dominate $F(u, s)$. This method has been used e.g. by Mortensen (1986). The result is that an increase in the offer distribution increases both the reservation utility and hazard function.

The effects of education via the re-employment costs $c(s)$ and $c_m(s)$ are straightforward. The costs increase the reservation utility and decrease the re-employment probability.

Summarizing the effects of education on the reservation utility, the following results are obvious. The reservation utility $u^*(t)$ is an increasing function of education via the arrival rate of job offers $a(s)$, offer distribution $F(u, s)$, permanent cost $c(s)$ and lump-sum costs of re-employment $c_m(s)$. The hazard function is an increasing function of education via the offer distribution and a decreasing function via the costs of re-employment. The effect via the arrival rate is analytically ambiguous. The connection between the search and unemployment duration models is defined by the hazard function. The density function of unemployment durations can be written as

$$(5) \quad f(t) = h(t) \exp\left[-\int_0^t h(\tau) d\tau\right],$$

which is the product of the hazard and survivor functions of unemployment durations.

3. Numerical Examples

In order to illustrate the time-paths of the value function, reservation utility and the hazard function numerical examples are given for the model with a limited search horizon. Furthermore, the effects of education on these functions are studied. The search horizon is assumed to be 40 years, but of course the person can enter or exit unemployment at any time during this period. For simplicity the offer distribution is assumed to have a uniform distribution between $[\underline{u}, \bar{u}] = [5, 15]$. The assumption has been used earlier, for instance, in the studies of Loikkanen and Pursiheimo (1979) and van den Berg (1987).

The time-paths have been calculated in reverse order using a fact that at the end of the search horizon $V(t) = b/r$. The arrival rate of job offers, offer distribution and re-employment costs are defined as follows

$$(6a) \quad a(s) = as$$

$$(6b) \quad \underline{u}(s) = 0.5s + \underline{u}, \quad \bar{u}(s) = 0.5s + \bar{u}$$

$$(6c) \quad c(s) = cs$$

$$(6d) \quad c_m(s) = c_ms.$$

The remaining parameter values used in the numerical example are as follows: $b = 2$, $a = 0.15$, $r = 0.05/12$, $c = 1$ and $c_m = 150$. In order to illustrate the time-paths of the

value function, reservation utility and hazard function the level of education was set as $s = 4$.

Figure 1 presents the time-paths. The value of search is a decreasing function over time. When there are 8.5 years left in the labour force, the active search period is over and the passive search with the constant value function starts. The reservation utility is a decreasing function, but during the last few years it is increasing because of the lump-sum types of moving costs. A high wage is needed to offset the moving cost. A decreasing function of reservation utility implies an increasing hazard. During the last few years the reservation utility is higher than the highest attainable utility, and the hazard function is zero, since there are no acceptable offers.

In Figure 2 the effects of education have been studied at $t = 480$ months (40 years left in the labour force). The time paths have been calculated for the different levels of education and the values of $V(t)$, $u^*(t)$ and $h(t)$ have been plotted. It can be seen that there is an optimal level of investment in education for an unemployed person. The reservation utility is increasing over the level of education, as expected. The effect of education on the hazard function is interesting. At first the hazard function is increasing, but later on it turns into a decreasing function. Using the uniform distribution it is straightforward to show that $\partial^2 h(t) / \partial s \partial s < 0$, i.e. the hazard function is concave.

Since the reservation utility increases with the level of education, there are fewer acceptable offers for the highly educated unemployed persons. Therefore the hazard

function begins to decrease. The result depends on the assumed effects of education. Greater education increases the arrival rate of offers and offer distribution, but the other crucial assumption is that it increases also the costs of becoming employed. The numerical examples show only the possibility of getting such results as determined by specific parameters in a dynamic search model. Depending on the values of the parameters of the model, Figures 1 and 2 can be different.

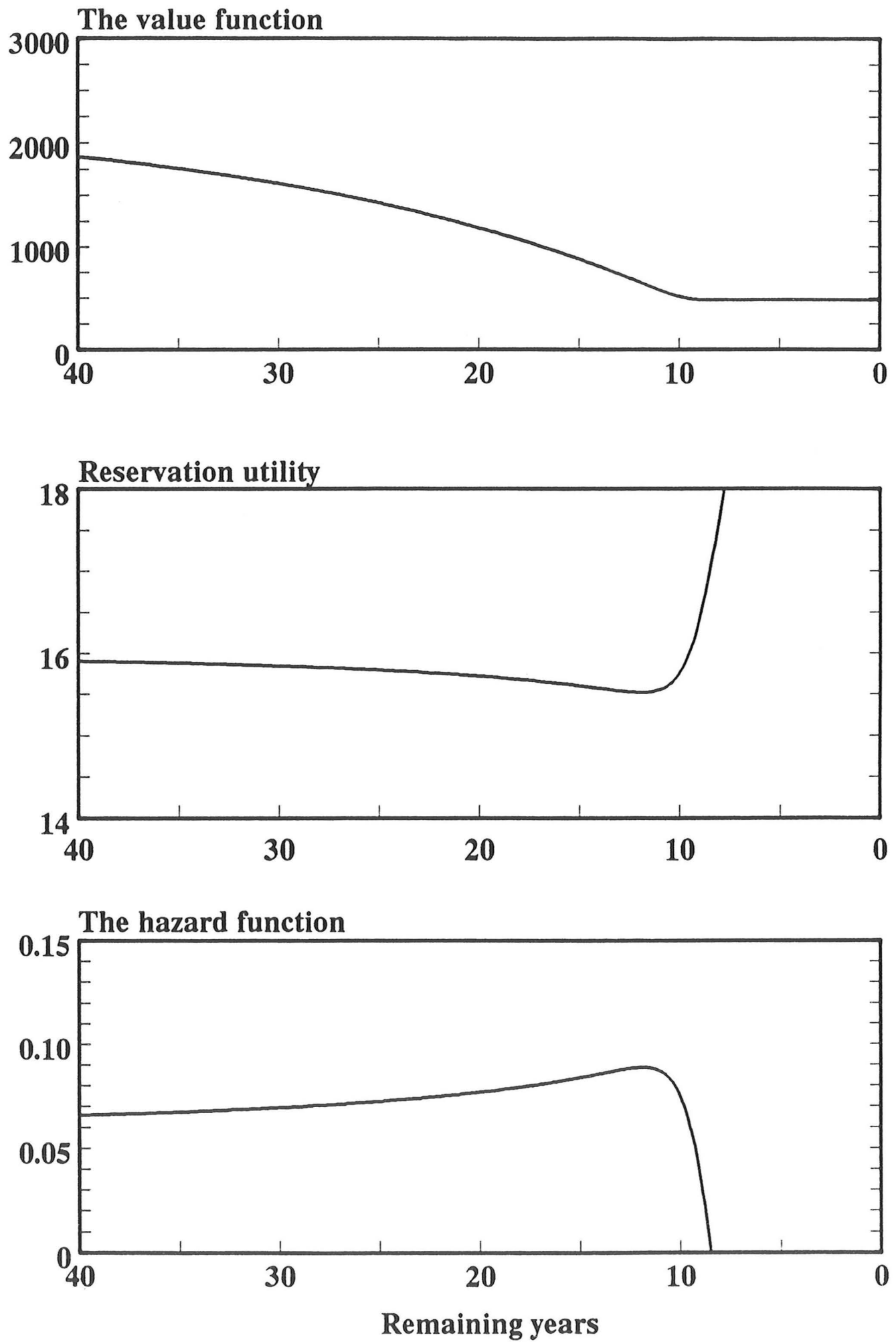
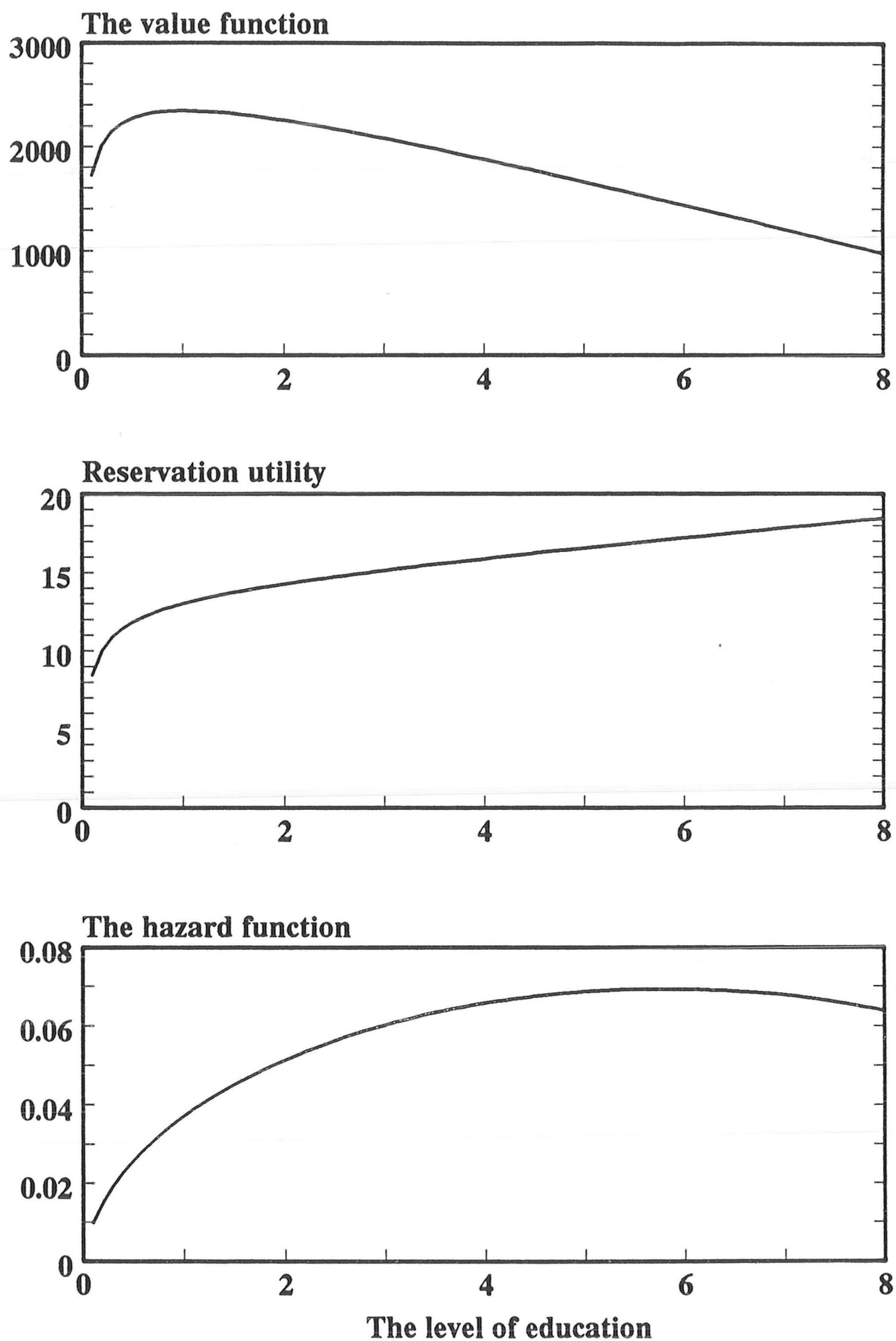
Figure 1. Time-paths of various functions

Figure 2. The effects of education

4. Empirical Evidence

In this section models of unemployment duration are estimated using Finnish microeconomic data. It is shown that the persons with less than 9 years of education and, on the other hand, the persons with master's, licentiate or doctor's degrees have the lowest re-employment probabilities. A Weibull model of unemployment duration is estimated and a mass point approach allowing for unobservable differences across persons is followed. Data on 2077 Finnish unemployed persons have been used in the estimations.

A graphical method is used to get an initial view on the effects of education on the duration of unemployment. Score plots are useful for detecting effects of omitted variables. One might hope that graphical analysis would aid in selecting an alternative specification of education variables. It is possible that the association between the scores and candidate omitted variable in a scatter plot might indicate ways of remedying misspecification by alerting us to the possibility of a nonlinear effect for an omitted regressor. Furthermore, graphical procedures may be valuable in indicating whether departures are of operational significance. Sometimes it is possible to find evidence from misspecification not detected by formal test procedures.

Chesher and Irish (1987) have examined graphical methods for detecting omission of regressors for grouped or censored data in the context of normal linear models.

Lancaster (1990) has derived the residuals of duration models using the scores of omitted variables. Graphical residual analysis can be informative about model misspecification, but some care is required in interpreting residual scatter plots derived from censored data. In this study a graphical procedure allowing for censoring is based on the scores of candidate regressors.

The likelihood contribution can be written using the hazard $h(t)$ and integrated hazard $I(t)$ as follows

$$(7) \quad \ell = [h(t)u]^c e^{-I(t)u},$$

where $u = \exp(x_s \beta_s)$ and c is the censoring indicator. The variable x_s (level of education) is deliberately excluded from the model. A way of testing whether β_s is not zero is to examine the variation in the log likelihood contribution $\log \ell$, when this parameter is allowed to depart from zero, in either direction. This suggests that a test for adding explanatory variables could be based on the scores $\partial \log \ell / \partial \beta_s$ at $\beta_s = 0$, which can be written

$$(8) \quad \partial \log \ell / \partial \beta_s = [c - I(t)]x_s.$$

A graphical examination of the effect of the explanatory variable x_s can be obtained by plotting it against the scores.

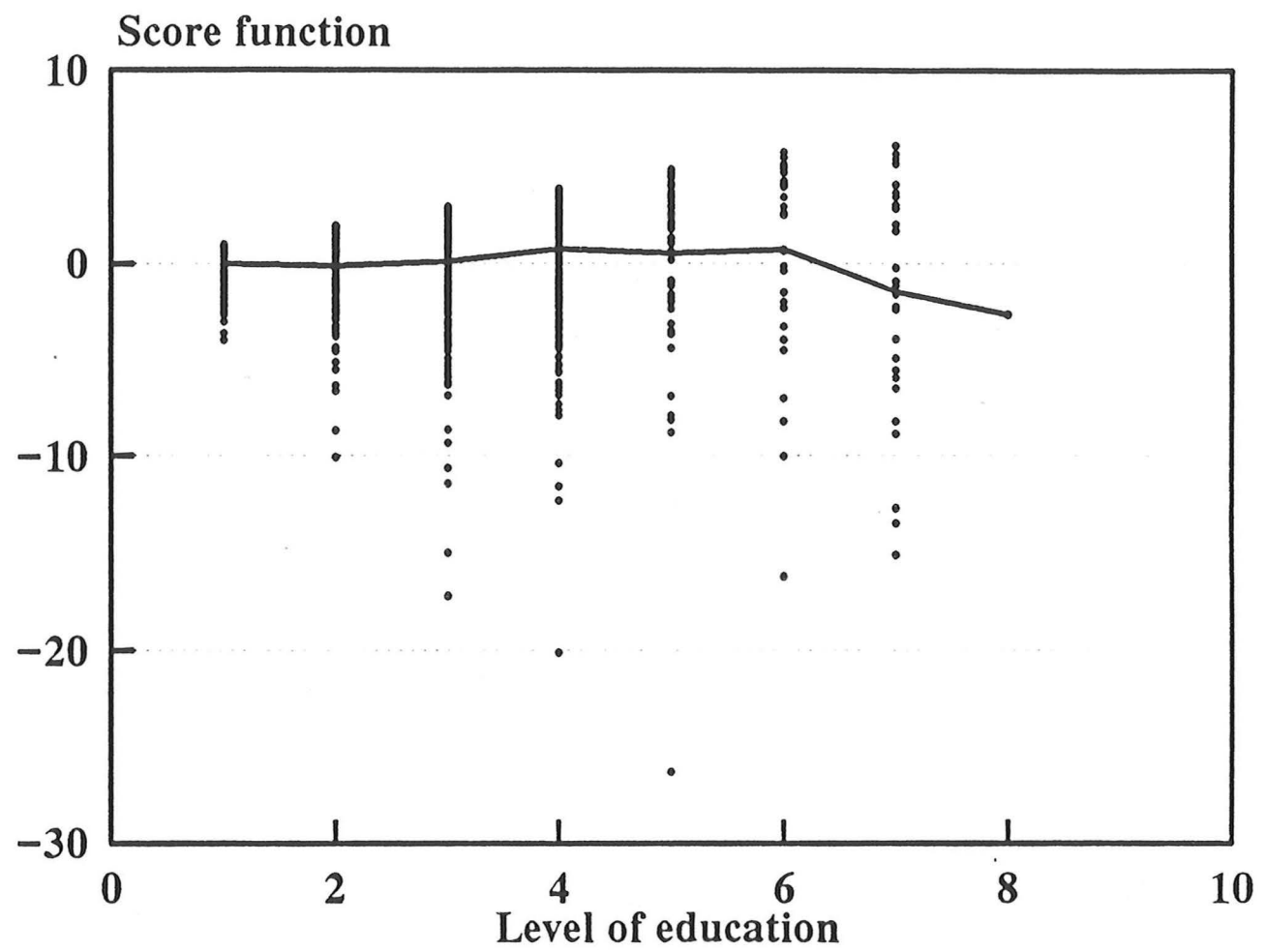
The models of unemployment duration are presented in Table 1. The level of education, ranging from 1 to 8, has no statistically significant effect on the re-employment probability. Figure 3 plots the score function against

education. Regressing the scores on the level of education yields a slope coefficient of 0.008 and intercept of -0.005, so that the regression is virtually a horizontal line. Joining up the average scores on each level of education gives the line drawn in Figure 3. The average scores suggest the possibility of a nonlinear effect for this variable. Low levels of education seem to be associated with relatively low re-employment probabilities. High levels of education are also associated with relatively low re-employment probabilities and long unemployment durations.

Table 1. Weibull models of unemployment duration

	Std.errors in parentheses	
Shape parameter	0.861	0.861
	(0.021)	(0.021)
Constant	-1.720	-1.695
	(0.132)	(0.126)
Number of children	-0.009	-0.011
	(0.050)	(0.050)
Married	0.153	0.154
	(0.065)	(0.065)
Sex	-0.017	-0.015
	(0.056)	(0.056)
Age	-0.040	-0.040
	(0.003)	(0.003)
Level of education (1-8)	0.014	
	(0.022)	
Training for employment	0.174	0.170
	(0.071)	(0.071)
Member of UI fund	0.199	0.198
	(0.060)	(0.060)
Came from schooling	0.298	0.303
	(0.080)	(0.080)
Came from housework	-0.717	-0.716
	(0.124)	(0.125)
Regional demand	0.117	0.123
	(0.236)	(0.235)
Occupational demand	0.752	0.943
	(0.643)	(0.585)
Taxable assets	0.927	0.927
	(1.078)	(1.076)
Replacement ratio	-1.240	-1.245
	(0.151)	(0.151)
Log likelihood	-4967.8	-4968.0

Figure 3. A score plot of education



An approach allowing for discrete unobservable differences across unemployed persons is followed. In the case of parametric duration models the discrete mixing likelihood contribution can be written as

$$(9) \quad f_0 = \sum_{i=1}^m p_i h_i(t)^c e^{-I_i(t)},$$

where $h_i(t) = \alpha t^{\alpha-1} e^{u_i+x\beta}$ and $I_i(t) = t^\alpha e^{u_i+x\beta}$ are the atomic hazard functions and integrated hazards in a Weibull case and c indicates complete spells of unemployment. If $c = 1$, then t is a complete spell, otherwise $c = 0$.

The results of estimations of mass point models are presented in Table 2. Models with two or more mass points produce increasing hazard functions. The absolute values of statistically significant parameter estimates increase in most cases when more mass points are introduced into the model, as is to be expected. Lindsay's (1983) D function is used to determine the optimal number of mass points. It turns out that five points of support are enough to rectify the effect of omitted variables with this data set.

The effects of education have been estimated using the lowest level of education, defined as less than 9 years of schooling, as the base for comparison. A higher level of education implies a higher hazard for the levels of education up to 13 - 14 years of education. The result is statistically significant. On the other hand, the re-employment probability begins to decrease for the persons with a bachelor's degree. The persons with a master's,

licentiate or doctor's degree have the lowest re-employment probabilities. The levels 6 - 8 do not statistically differ from the first level. These problematic levels of education include 3.3 per cent of the unemployed persons.

About 37 per cent of the occupations where the level of education is at least the lowest university degree are in teaching and research. About 19 per cent of these occupations are in the construction or technical occupations. At least 11 per cent of them can be classified into the production or service jobs with a fixed place. Typically the jobs with a requirement of a high level of educational are situated in towns or factories. If these persons lose their jobs they have to pay the moving or commuting costs in order to get a job. The change of the occupation in their area of residence would involve costs in means of lower wages.

Another variable which is interesting to education economists is the training for further employment arranged by the state. It includes participation in courses which have occurred before the onset of unemployment, but not necessarily immediately before it. Training for further employment has a significant and positive effect on the re-employment probability. It increases the probability of re-employment by about 60 per cent.

Table 2. Mass point heterogeneity in a Weibull model

Dependent variable: The length of the spell of unemployment					
	Number of mass points				
	m=1	m=2	m=3	m=4	m=5
	Std.errors in parentheses				
Shape parameter	0.843 (0.020)	1.079 (0.039)	1.250 (0.060)	1.415 (0.099)	1.751 (0.138)
Number of children	-0.036 (0.051)	-0.116 (0.069)	-0.101 (0.082)	-0.162 (0.097)	-0.145 (0.118)
Married	0.005 (0.065)	-0.007 (0.091)	-0.116 (0.101)	-0.125 (0.119)	-0.126 (0.144)
Sex	-0.086 (0.057)	-0.166 (0.080)	-0.116 (0.092)	-0.146 (0.108)	-0.163 (0.132)
Age, 56-65 years ²⁾	-2.051 (0.244)	-2.602 (0.255)	-2.931 (0.280)	-3.225 (0.342)	-3.921 (0.441)
Education: ³⁾ Level 2	0.275 (0.083)	0.293 (0.116)	0.334 (0.136)	0.341 (0.160)	0.458 (0.197)
Level 3	0.411 (0.072)	0.342 (0.101)	0.442 (0.117)	0.455 (0.137)	0.565 (0.169)
Level 4	0.357 (0.090)	0.365 (0.124)	0.377 (0.139)	0.410 (0.163)	0.497 (0.203)
Level 5	0.467 (0.136)	0.486 (0.203)	0.789 (0.271)	0.915 (0.311)	1.069 (0.393)
Level 6	0.340 (0.187)	0.471 (0.298)	0.492 (0.352)	0.522 (0.426)	0.695 (0.486)
Levels 7 and 8	-0.095 (0.267)	-0.379 (0.360)	-0.517 (0.445)	-0.638 (0.556)	-0.660 (0.620)
Training for employm.	0.184 (0.072)	0.288 (0.103)	0.340 (0.122)	0.422 (0.143)	0.479 (0.176)
Member of UI funds	0.090 (0.060)	0.222 (0.083)	0.224 (0.097)	0.261 (0.113)	0.332 (0.139)
Came from schooling	0.435 (0.078)	0.550 (0.114)	0.690 (0.136)	0.731 (0.164)	0.971 (0.205)
Came from housework	-0.740 (0.123)	-0.807 (0.155)	-0.905 (0.182)	-1.067 (0.211)	-1.319 (0.255)
Regional demand	0.140 (0.235)	0.339 (0.303)	0.514 (0.360)	0.674 (0.416)	1.030 (0.510)
Occupational demand	0.159 (0.653)	-0.720 (0.921)	-0.693 (1.048)	-0.897 (1.256)	-0.774 (1.546)
Taxable assets	-0.872 (1.009)	-1.834 (1.209)	0.595 (1.564)	-0.621 (1.719)	-2.414 (2.083)
Replacement ratio	-1.281 (0.149)	-1.947 (0.212)	-2.457 (0.266)	-2.763 (0.346)	-3.667 (0.453)
u ₁	-2.620 (0.128)	-2.382 (0.171)	-1.993 (0.214)	-1.332 (0.319)	-0.471 (0.358)
u ₂		-4.500 (0.243)	-3.870 (0.278)	-3.081 (0.380)	-2.652 (0.421)
u ₃			-7.303 (0.889)	-4.746 (0.459)	-4.448 (0.527)
u ₄				-8.399 (1.101)	-6.310 (0.628)
u ₅					-10.245 (1.175)
p ₁		0.683 (0.037)	0.352 (0.018)	0.159 (0.019)	0.080 (0.001)
p ₂		0.317 (0.037)	0.539 (0.006)	0.376 (0.016)	0.227 (0.002)
p ₃			0.109 (0.024)	0.366 (0.007)	0.330 (0.021)
p ₄				0.098 (0.028)	0.270 (0.009)
p ₅					0.093 (0.027)
Log likelihood	-4972.3	-4926.9	-4915.2	-4912.4	-4910.0

The survivor function of the model is obtained from the mixing likelihood contribution (9) by setting $c = 0$, which gives

$$(10) \quad S(t) = \sum_{i=1}^m p_i e^{-I_i(t)}.$$

Integrating the survivor function gives the expected value of unemployment spells $E(T; \phi)$, where $\phi = (\alpha, \beta, u_i, g_k)$, $i = 1, \dots, m$, $k = 1, \dots, m-1$. The expected value of the unemployment spell $E(T; \phi)$ can be written as a weighted average of the expected values of the m groups $E(T_i; \phi_i)$, $\phi_i = (\alpha, \beta, u_i)$, as follows

$$(11) \quad E(T; \phi) = \sum_{i=1}^m p_i E(T_i; \phi_i),$$

where

$$(12) \quad E(T_i; \phi_i) = (1/\alpha) e^{-(u_i + x\beta)/\alpha} \Gamma(1/\alpha).$$

The gamma function is denoted by Γ . The variance of $E(T; \phi)$ can be approximated by the delta method. The first order Taylor series expansion gives

$$(13) \quad E(T; \hat{\phi}) \approx E(T; \phi) + (\hat{\phi} - \phi)' \frac{\partial E(T; \phi)}{\partial \phi},$$

The approximative variance can then be written as

$$(14) \quad \text{Var}[E(T; \hat{\phi})] \approx \frac{\partial E(T; \hat{\phi})}{\partial \hat{\phi}}' \text{Var}(\hat{\phi}) \frac{\partial E(T; \hat{\phi})}{\partial \hat{\phi}}.$$

Table 3 includes the effects of education on the duration of unemployment calculated for an average person in the sample. It can be seen that the level of education has a strong effect on the duration of unemployment. The fifth level of education implies the shortest duration. Many persons having the a bachelor's degree have problems in finding acceptable job offers. The persons having at least the master's degree have even more difficulties.

Table 3. The effects of education on the duration of unemployment

Level of education	The effect of education in weeks*	Std.errors**
2	-14.8	6.6
3	-17.7	6.7
4	-15.9	6.6
5	-29.3	7.1
6	-21.0	9.1
7-8	+29.4	40.5

* The effects are compared to the first level of education

** $\sqrt{\text{Var } E(T; \phi, \text{ level 1}) + \text{Var } E(T; \phi, \text{ each level})}$

5. Conclusions

In this study the effects of education on the re-employment probability were analyzed using a search model. It was shown that the effect of education on the probability of re-employment is not straightforward. Using a model with a finite search horizon it was shown that the time-path of the reservation utility is decreasing, which leads to an increasing hazard function. However, during the latter years the reservation utility is increasing and the hazard function is decreasing due to the lump-sum type of cost of re-employment.

It was shown that there is an optimal level of education for an unemployed person maximizing life-time income. Furthermore, a higher level of education leads to a higher reservation utility. The re-employment probability increases over the lower levels of education. However, the possibility to get an acceptable offer decreases when the reservation utility increases with the level of education. Therefore the hazard function begins to decline toward the highest levels of education.

The models of unemployment duration allowing for a discrete pattern of unobserved heterogeneity across unemployed individuals were estimated. Weibull models with mass point heterogeneity were estimated using Finnish microeconomic data. In the basic Weibull model the estimated value of the shape parameter was less than one, indicating negative duration-dependence. The absolute values of the parameter estimates increased substantially,

however, after allowing for unobserved heterogeneity and the hazard function turned increasing. Five points of support were enough to rectify the effects of omitted variables. According to the model the re-employment probability is low for the persons who are near the age of retirement.

According to the models of unemployment duration the education has a positive effect on the re-employment probability up to about 13 - 14 years of education, but the unemployed persons with a master's, licentiate or doctor's degree have problems in finding acceptable offers. These results clearly support the possible outcome of the search model that the probability of becoming employed begins to decline toward the highest levels of education. According to the empirical evidence these effects have a substantial economic significance.

It is necessary to point out that this study tells only half of the story about the effect of education on unemployment. Even though the well-educated persons have problems in finding jobs they rather seldom become unemployed. Furthermore, the data represents only the unemployed persons who have been searching a job using the unemployment office. Using aggregate data it can be estimated that 12 per cent of the individuals on the first and second levels of education became unemployed in unemployment offices during 1985. On the third and fourth level of education 7 per cent became unemployed. On the highest levels only 4 per cent of the workers become unemployed. The transition intensity of well-educated

persons into unemployment is rather low, but on the other hand their unemployment can be a serious problem.

Footnotes

1. Another compelling explanation which could yield predictions that education decreases the re-employment probability is that the arrival rates may at first increase and then decrease with each higher level. The direct effect of the arrival rate on the hazard function is positive, but the indirect effect via the increasing reservation utility is negative. Therefore the effect of education via the arrival rate on the hazard function remains ambiguous. If the indirect effect based on the increasing selectivity is rather small, the argument may be relevant. The assumption of this chapter that job opportunities rise with the length of schooling has been used, for example, by Groot and Oosterbeek (1990).

2. Age, 56-65 years is a dummy variable and it is measured in years, 1 = yes. Mean = 0.05.

3. Level of education is a dummy variable, 1 = yes:

Level 1 = less than 9 years of education. Mean = 0.368.

Level 2 = 9 years of education. Mean = 0.174.

Level 3 = 10 - 11 years of education. Mean = 0.245.

Level 4 = 12 years of education. Mean = 0.152.

Level 5 = 13 - 14 years of education. Mean = 0.028.

Level 6 = 15 years of education. Bachelor's degree.
Mean = 0.017.

Level 7 = 16 years of education. Master's degree.
Mean = 0.015.

Level 8 = licentiate or doctor's degree. Mean = 0.0005. The level of education is based on the education code of the Central Statistical Office of Finland.

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Appendix 1. Comparative Static Results: The Effects of Education on the Reservation Utility and Hazard Function

For simplicity the comparative static results have been solved in an infinite horizon model, where $\dot{V}(t) = 0$ and $B(t) = 1/r$. Since $\partial a/\partial s$, $\partial \mu/\partial s$, $\partial c/\partial s$ and $\partial c_m/\partial s$ are positive, it is necessary to consider the effects of a , μ , c and c_m on the reservation utility and hazard function.

The effects of the arrival rate of job offers

Since $\dot{V}(t) = 0$, the reservation utility is solved from the differential equation (2) by inserting $V = (u^* - c)/r - c_m$, which gives

$$(15) \quad u^* = b + c + rc_m + a \int_{u^*}^{\bar{u}} (u - u^*) dF(u, s)/r,$$

where the effect of the arrival rate of offers can then be solved directly

$$(16) \quad \frac{\partial u^*}{\partial a} = \int_{u^*}^{\bar{u}} (u - u^*) dF(u, s)/r > 0.$$

The effect of a on the hazard function is

$$(17) \quad \frac{\partial h}{\partial a} = [1 - F(u^*, s)] - af(u^*, s) \frac{\partial u^*}{\partial a},$$

where $f(u^*, s)$ is the density function of offers. Clearly $\partial h/\partial a$ has an ambiguous sign.

The distribution of offers

Substituting the following useful transformation

$$(18) \quad \int_{u^*}^{\bar{u}} (u - u^*) dF(u) = E_F(u) - u^* + \int_0^{u^*} F(u) du$$

into (15) gives

$$(19) \quad u^* = b + c + rc_m + a[E_F(u) - u^* + \int_0^{u^*} F(u) du]/r.$$

Substituting $F(u) = G(u + \mu)$ and noting that $E_G(u) = \mu + E_F(u)$ gives

$$(20) \quad u^* = b + c + rc_m + a[\mu + E_F(u) - u^* + \int_0^{u^*} F(u - \mu) du]/r.$$

The effect of the offer distribution on the reservation utility can be solved as

$$(21) \quad \frac{\partial u^*}{\partial \mu} = h/(r + h) > 0,$$

where $h = a[1 - F(u^* - \mu)]$. The effect of an increase of the offer distribution on the hazard function is

$$(22) \quad \frac{\partial h}{\partial \mu} = af(u^* - \mu) \left(1 - \frac{\partial u^*}{\partial \mu}\right) > 0,$$

since $\partial u^*/\partial \mu < 1$.

The costs of re-employment

The effect of c and c_m on the reservation utility can be written as

$$(23) \quad \frac{\partial u^*}{\partial c} = 1 > 0$$

$$(23) \quad \frac{\partial u^*}{\partial c_m} = r > 0.$$

The effects of c and c_m on the hazard function are clearly negative.

Chapter VII

SEMI-PARAMETRIC INFERENCE

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Abstract

This chapter examines the duration and calendar-dependence in the context of Cox's semi-parametric proportional hazards model. Duration-dependent UI benefits are used to study duration-dependent features of the Finnish UI system and calendar-dependent covariates are used to study the seasonal effects on the re-employment probability. Calendar-dependent covariates may not always be adequate to model the macroeconomic changes. Therefore the roles of duration and calendar time are successfully changed in estimating the model. The underlying baseline hazards of the models based on duration and calendar time are presented.

1. Introduction

The proportional hazards model presented by Cox (1972) studies the effects of explanatory variables on the hazard rate without specifying the form of duration-dependence. The estimation of Cox's model leads to the partial conditional likelihood function, where the time-dependent part of the likelihood function is cancelled out, because it is identical for the individuals becoming employed and individuals in the risk set.

The environment of an unemployed person changes over time. The changes may be duration-dependent. The Finnish unemployment insurance system is such that the rules

concerning the eligibility of benefits vary over the duration of unemployment. On the other hand, the changes may be calendar-dependent. Probably the most important calendar-dependent changes in the short run are the seasonal factors, which have an important effect on the re-employment probability. The interest of this study is in the calendar and duration-dependent effects on the re-employment probability.

Concerning any period the individuals experience two events, the entry τ^0 and exit τ^1 , measured in calendar time. The calendar time is measured as the duration between the date in question and any fixed date before these two events. The duration of unemployment is then $t = \tau^1 - \tau^0$. The hazard function of the proportional hazards model presented by Cox (1972) can be written as

$$(1) \quad h(t, x) = h_0(t)h_1(x; \beta),$$

where the first factor $h_0(t)$ is the unknown baseline hazard. These kinds of models are called semi-parametric, since one does not have to define the baseline hazard. The second factor, which is known up to a finite dimensional parameter vector β , usually takes the log linear form $h_1(x; \beta) = \exp(x\beta)$.

Let t_1, t_2, \dots, t_n denote the ordered durations of n individuals. The partial likelihood contribution can be written as follows [Cox (1975)]

$$(2) \quad \ell_i(\beta) = \frac{\exp(x_i\beta)}{\sum_{j \in R(t_i)} \exp(x_j\beta)},$$

where $R(t_i)$ denotes the risk set, i.e. the observations with $t \geq t_i$. Multiplying the numerator and denominator by $h(t)dt$ it can be seen that the contribution of an observation i is just the probability that the duration ends in $[t_i, t_i+dt)$ given that some duration in the risk set ends in that interval.

Usually duration data includes censored observations. In estimation it is helpful to use an indicator to signify whether observations are complete durations or censored times. The censoring indicator and values of the duration variable T or censoring variable C are observed. The censoring indicator $c_i = 1$ if t_i is a complete spell, otherwise $c_i = 0$ and $t_i = \min(T_i, C_i)$. The risk set includes censored observations, which appear only in the numerator of the likelihood function. With a censoring indicator the partial likelihood function can be written as

$$(3) \quad \ell(\beta) = \prod_{i=1}^n \ell_i(\beta)^{c_i}.$$

If $c_i = 0$, no contribution is made to the likelihood function. A similar type of modelling censoring was used by Aalen (1978).

Usually duration data are to some degree grouped, i.e. there are spells which end during the same unit of time. In this data the duration of unemployment is measured using dates. Therefore the grouping is rather mild. One way of taking the grouping into account is to write the partial likelihood function following Breslow (1974) as

$$(4) \quad \ell(\beta) = \pi \prod_{i=1}^d \frac{\exp(\sum_{l \in T(t_i)} c_l x_l \beta)}{[\sum_{j \in R(t_i)} \exp(x_j \beta)]^{m_i}},$$

where d is the number of distinct exit times observed in the data and $T(t_i)$ denotes a set of m_i individuals who are observed to leave unemployment at t_i . Note that if $T(t_i)$ includes only censored observations then $m_i = 0$ and no contribution is made to the likelihood function.

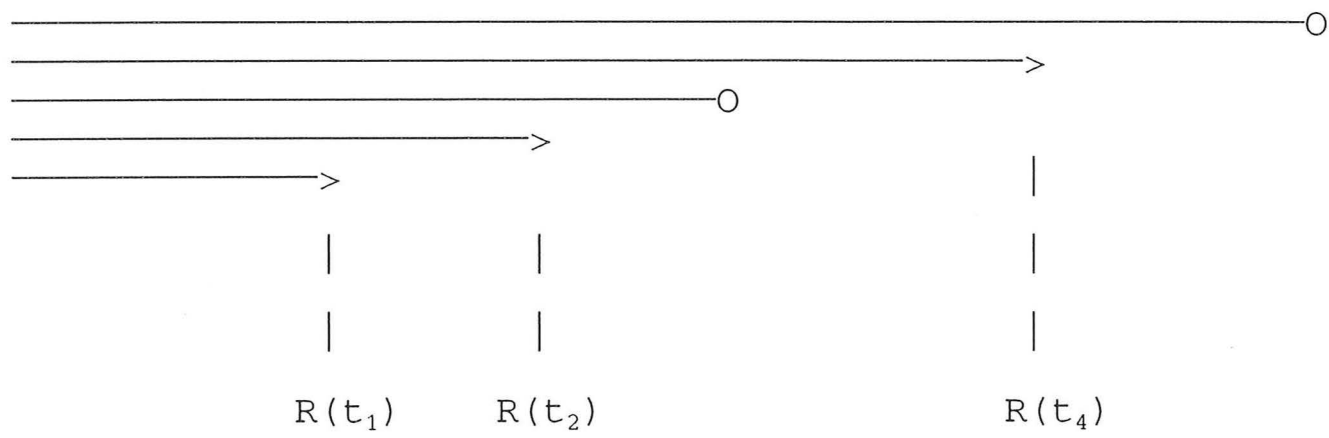
In estimation of the unknown parameters β it is helpful to rank the durations and censoring times in descending order for the calculation of the risk set as shown in Figure 1. The calculation of the risk set for each distinct duration can then be done by adding $\exp(x_j \beta)$ terms of the individuals starting from the first observation. The partial likelihood function can then be easily maximized with respect to the parameters.

The explanatory variables of the risk sets may vary even though the individuals are homogenous, since some of the explanatory variables may be calendar or duration-dependent. The effects of these variables are studied using Finnish data of unemployed workers.

Using parametric models the specification of the duration distribution and duration dependence may be difficult. The Finnish UI system is such that the rules concerning the eligibility of benefits vary over the duration of unemployment. Therefore the interest of this chapter is in the semi-parametric models and duration dependent effects of the system on the re-employment and regional and occupational mobility of unemployed workers.

Figure 1. Unemployment durations of five individuals

(0 = censored, > = complete spells)



2. Time-Dependent Effects

2.1. Duration-Dependence

The values of explanatory variables may change over the duration of unemployment for the individuals. This section considers the inclusion of such variables in the semi-parametric proportional hazards model. Models with duration-dependent replacement ratios of unemployment benefits are estimated using different kinds of model specifications. With duration-dependent covariates the hazard function of the proportional hazards model can be written as

$$(5) \quad h(t, x, z) = h_0(t) e^{x\beta + z(t)\beta_z},$$

where x includes the covariates, which are constant in time, and $z(t)$ includes the duration-dependent covariates.

Models with duration-dependent benefit replacement ratios are estimated. In the model (A) of Table 1 the benefit replacement ratio is fixed at an average value over the unemployment spell. In the model (B) the duration-dependent unemployment benefits were used in the two intervals $(t_0, t_1]$ and $(t_1, t_2]$, where $t_0 = 0$, $t_1 = 3$ and $t_2 = 24$ months. The results show that the effect of unemployment benefits is lower with duration-dependent replacement ratios. The third possibility is to assume that the effects of duration-dependent variables vary over time, remaining constant within predefined intervals [model (C)]

of Table 1]. The reason for this kind of specification is that the explanatory variables may have different effects on the re-employment probability with respect to different durations. With duration-dependent effects the hazard function in our case can be written for the interval $t_{j-1} < t \leq t_j$, $j = 1, 2$, as $h_j(t, x, z) = h_0(t) \exp[x\beta + z(t)\beta_z(t)]$, which can be rewritten as follows

$$(6) \quad h_j(t, x) = h_0(t) e^{x\beta + z(t)(\beta_z + \mu_j)}.$$

To avoid singularity it is defined that $\mu_1 = 0$. The partition of the time axis into predefined intervals allows the proportional hazards assumption to be tested. This approach has been followed by Moreau, O'Quigley and Mesbach (1985), Moreau, O'Quigley and Lellouch (1986) and O'Quigley and Pessione (1989).

The results of model (6) with duration-dependent replacement ratios are presented in the third column of Table 1. It can be seen that the unemployment benefits have a negative effect on the re-employment probability during the first three months, but after that period the effect turns positive. An obvious reason is that the eligibility rules of benefits become stricter. The unemployed persons have a risk of losing benefits after the first three months if they do not move to an other region or change their occupations. Furthermore, after the 100th day of unemployment the earnings-related unemployment allowances decrease by 20 per cent. Because of these rules the incentive for re-employment is higher for the persons with high benefits. These findings are confirmed by Table 2,

which shows that the negative effect of benefits is higher for the non-members of labour unions even though their benefits are lower.

Since the rules concerning the labour mobility change during the spell of unemployment, it is reasonable to model the probabilities of moving and changing occupations. The semi-parametric models of labour mobility are presented in Table 3. In the case of regional mobility the censoring indicator $c_i = 1$ if the person has moved to get a job, otherwise $c_i = 0$. In the case of occupational mobility the censoring indicator is defined in a similar way.

Table 1. Semi-parametric models of unemployment spell with duration-dependent replacement ratios

	(A)	(B)	(C)
(A) Average replacement ratios			
(B) Time-dependent replacement ratios			
(C) Time-dependent replacement ratios and coefficients			
	(A)	(B)	(C)
	Std.errors in parentheses		
Number of children	-0.002 (0.048)	-0.097 (0.049)	-0.086 (0.049)
Married	0.143 (0.067)	0.171 (0.070)	0.159 (0.069)
Sex	-0.014 (0.063)	-0.054 (0.061)	-0.056 (0.061)
Age	-0.039 (0.004)	-0.037 (0.003)	-0.036 (0.003)
Level of education	0.045 (0.067)	0.081 (0.064)	0.102 (0.064)
Training for employment	0.183 (0.074)	0.206 (0.079)	0.202 (0.079)
Member of UI fund	0.209 (0.063)	0.216 (0.065)	0.205 (0.065)
Came from schooling	0.283 (0.090)	0.300 (0.083)	0.280 (0.084)
Came from housework	-0.648 (0.140)	-0.655 (0.137)	-0.671 (0.137)
Regional demand	0.114 (0.256)	0.248 (0.252)	0.131 (0.252)
Occupational demand	0.551 (0.586)	0.656 (0.622)	0.547 (0.590)
Taxable assets	0.783 (0.994)	0.770 (1.112)	0.682 (1.112)
Replacement ratio, β_z	-1.232 (0.132)	-0.325 (0.138)	-1.375 (0.205)
Replacement ratio, μ_2			2.127 (0.271)
Log likelihood	-8415.6	-8453.6	-8422.1

**Table 2. Semi-parametric models of unemployment duration
for the non-members and members of labour unions**

(A) Non-members (B) Members	(A) (B) Std. errors in parentheses	
Number of children	-0.019 (0.069)	-0.016 (0.073)
Married	0.048 (0.106)	0.235 (0.095)
Sex	-0.021 (0.083)	-0.047 (0.096)
Age	-0.032 (0.005)	-0.048 (0.005)
Level of education	0.154 (0.085)	-0.154 (0.103)
Training for employment	0.181 (0.119)	0.179 (0.108)
Came from schooling	0.300 (0.097)	0.200 (0.188)
Came from housework	-0.729 (0.191)	-0.548 (0.201)
Regional demand	-0.061 (0.327)	0.410 (0.411)
Occupational demand	-0.387 (0.925)	1.276 (0.886)
Taxable assets	0.120 (2.015)	1.494 (1.193)
Replacement ratio	-1.725 (0.219)	-0.851 (0.221)
Log likelihood	-4178.7	-3362.2
Number of observations	1212	865

Table 3. Semi-parametric duration models of labour mobility

(A) Duration model (B) Calendar model	(A) (B) Std. errors in parentheses	
Age	-0.053 (0.022)	-0.035 (0.010)
Level of education		0.305 (0.182)
Training for employment		0.340 (0.194)
Member of UI fund	-1.382 (0.467)	0.376 (0.156)
Came from schooling		-0.034 (0.248)
Came from housework		0.084 (0.254)
Regional demand	-2.036 (1.433)	1.223 (0.597)
Occupational demand	2.836 (3.179)	-3.544 (1.637)
Taxable assets		-3.918 (3.264)
Replacement ratio	-5.116 (1.005)	-0.446 (0.334)
Log likelihood	-321.8	-1364.3

2.2. Calendar-Dependence

This section studies the seasonal effects on the re-employment using semi-parametric models of unemployment duration. With calendar-dependent covariates the hazard function of Cox's model can be written as

$$(7) \quad h(t, x, z) = h_0(t) e^{x\beta + z(\tau)\beta_z},$$

where x includes the covariates which are constant in time, and $z(\tau)$ includes the calendar-dependent covariates. The seasonal variation and the effects of quarterly unemployment rates are examined in this study.

One way of introducing time-dependence is to use dummy variables to indicate periods in calendar time. Ridder (1987) allows for dependence on calendar-time by introducing dummies for two-year intervals. Examining the seasonal effects quarterly dummy variables are in this study introduced into the model. The variation across observations is used to estimate the effects of calendar-time, and the parameters are estimated for each dummy variable. The first column of Table 4 includes this model. The results show that the last quarter has the lowest re-employment probability, whereas the first two quarters have the highest probabilities. In great measure this result is due to the considerable variation in the Finnish climate.

It may be of interest to use a continuous calendar-dependent variable, for instance, the unemployment rate of

the whole economy, and estimate its effect. This is done in the second model of Table 4. The unemployment rate varies with calendar time, but in statistics it is calculated as an average value for intervals of calendar time, i.e. months, quarters and years. Quarterly data on the whole economy are used in this model. The variation across observations is used to take into account the calendar-time effects. The unemployment rate has a negative and statistically significant effect on the re-employment probability. A ten per cent increase of the unemployment rate decreases the probability of becoming employed by nearly 7 per cent.

**Table 4. Semi-parametric models of unemployment duration
using calendar-dependent covariates**

	Std. errors in parentheses	
Number of children	-0.017 (0.049)	-0.017 (0.049)
Married	0.090 (0.070)	0.139 (0.069)
Sex	-0.052 (0.061)	-0.019 (0.061)
Age	-0.037 (0.003)	-0.031 (0.003)
Level of education	0.085 (0.064)	0.079 (0.064)
Training for employment	0.148 (0.079)	0.214 (0.079)
Member of UI fund	0.193 (0.065)	0.200 (0.065)
Came from schooling	0.324 (0.084)	0.180 (0.084)
Came from housework	-0.609 (0.137)	-0.623 (0.137)
Regional demand	0.179 (0.252)	-0.001 (0.253)
Occupational demand	0.609 (0.620)	0.275 (0.619)
Taxable assets	0.530 (1.164)	0.345 (1.094)
Replacement ratio	-1.272 (0.156)	-1.129 (0.154)
Quarter 1	0.763 (0.095)	
Quarter 2	0.765 (0.089)	
Quarter 3	0.362 (0.093)	
Unemployment rate		-0.684 (0.068)
Log likelihood	-8368.3	-8364.1

2.3. Changing the Roles of Duration and Calendar Time

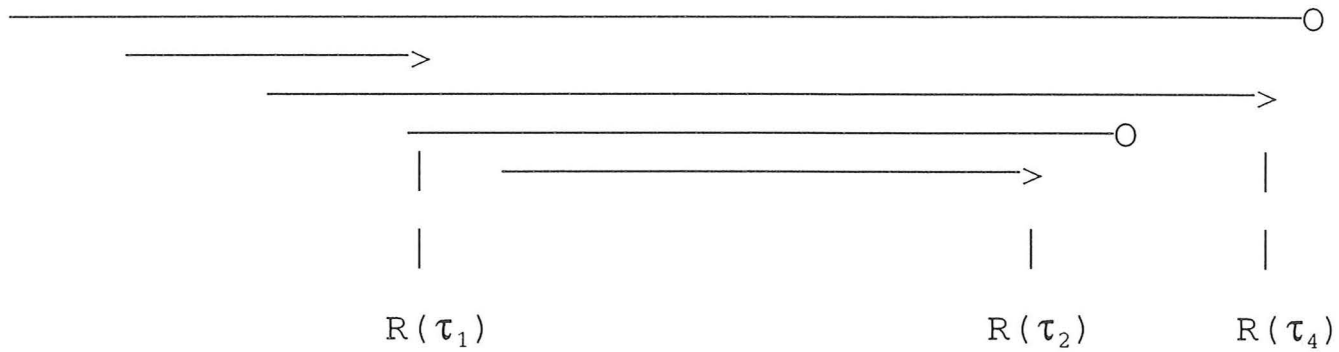
In this section the roles of the duration and calendar time are changed in order to study the seasonal effects and to estimate and test the duration-dependence. Turning back to the seasonal effects it is obvious that the specification of the calendar-dependent part may be difficult in practice, because the macroeconomic environment is changing continuously. If this process is not fully observed, one way to overcome the problem is to restrict the seasonal effects to a particular form. Using the models with calendar-dependent covariates the functional form of the calendar-dependent part has to be specified completely, but it may be difficult in practice. Here it is not preferred to specify the form of seasonal effects, but instead to ignore the specification of calendar-dependence by using calendar time as the basic time variable.

The idea of changing the roles of duration and calendar times is presented by Imbens (1990) and Ridder and Tunali (1990), even though they have not estimated this kind of model. Imbens suggests replacing the duration by the calendar time in order to eliminate common calendar-time related to macroeconomic shocks that affect all individuals in the same way. Ridder and Tunali study child mortality and discuss the properties of these two observation plans. After changing the roles of duration and calendar time the hazard function of the model can be written as

$$(8) \quad h(\tau, x) = h_0(\tau)h_1(x; \phi).$$

On any day of exit the risk set of a calendar time model consists of the individuals who are unemployed on that day. The estimation of the calendar time model is not possible using standard statistical packages. A scheme for estimating a calendar time model can nevertheless be formulated in the following manner. Consider calendar time-dependent unemployment spells as shown in Figure 2. It is helpful to recode the dates of entry and exit as a difference between these dates and any date earlier than the first date of entry in the sample. The calculation of the risk set can then be performed easily starting from the latest date and moving towards the next latest date and so on. In the case of exit or censoring the observation is added to the risk set and in the case of entry it is subtracted from it. This procedure is continued until all the dates of exit have been processed in order to calculate the needed log likelihood function and the derivatives. The needed programming is done using the SAS/IML (1985) matrix language. Appendix 1 includes a part of the programme including the log likelihood function and the first derivatives.

Figure 2. Unemployment durations of five individuals in a model based on calendar time (0 = censored, > = complete)



The results of estimations of the calendar time model are presented in Table 5. Compared to the corresponding duration model in the first column of Table 1 and the models of labour mobility in Table 3, it can be seen that the parameter estimates of the calendar models do not differ substantially from the corresponding duration models. It can be concluded that durations of unemployment can be replaced by the calendar time in this case. The loss of efficiency of the calendar model can be large, however, if there are a lot of durations that are not overlapping each other. Then some fraction of the persons does not contribute to the likelihood function.

The estimation of the calendar model is more time consuming than the model based on durations, since the calculation of the risk is based on comparisons of individual calendar times. However, the calendar model has an advantage. The baseline hazard can be expressed as a smoothly varying function of calendar time.

**Table 5. Semi-parametric models of unemployment duration
with calendar time**

	(A)	(B)	(C)
	Std.errors in parentheses		
(A) Unemployment duration			
(B) Regional mobility			
(C) Occupational mobility			
Number of children	-0.042 (0.049)		
Married	0.195 (0.070)		
Sex	0.018 (0.062)		
Age	-0.042 (0.004)	-0.050 (0.021)	-0.037 (0.010)
Level of education	0.037 (0.065)		0.309 (0.180)
Training for employment	0.214 (0.079)		0.381 (0.193)
Member of UI fund	0.257 (0.064)	-1.234 (0.466)	0.425 (0.156)
Came from schooling	0.237 (0.086)		-0.064 (0.251)
Came from housework	-0.695 (0.137)		0.027 (0.252)
Regional demand	0.149 (0.252)	-1.986 (1.463)	1.171 (0.589)
Occupational demand	0.503 (0.629)	2.561 (3.214)	-3.733 (1.644)
Taxable assets	0.756 (1.006)		-3.675 (3.266)
Replacement ratio	-1.157 (0.148)	-4.841 (0.965)	-0.452 (0.323)
Log likelihood	-7176.6	-272.8	-1167.9

3. Baseline Hazard Functions

The estimated values of the parameters can be used to construct an estimator for the integrated baseline hazard, which has been proposed for Cox's model by Breslow (1972, 1974). The integrated baseline hazard can be written as

$$(9) \quad \hat{I}_0(t_i) = \sum_{t_j \leq t_i} \frac{c_j}{\sum_{k \in R(t_j)} \exp(x_k \hat{\beta})},$$

where c_j is an indicator for a non-censored observation. The corresponding Breslow's estimate for the baseline hazard is based on the subdivisions of the time scale at those points where the event occurs

$$(10) \quad \hat{h}_0(t_i) = \frac{c_i}{(t_i - t_{i-1}) \sum_{k \in R(t_i)} \exp(x_k \hat{\beta})}.$$

In the calendar time model τ_i is substituted for t_i . For the graphical presentation of the baseline hazard it is more natural to assume that $h_0(t)$ is a slowly varying function of t . If the estimates of the integrated baseline hazards are available, an estimate for the baseline hazard for each distinct duration can be rewritten as a difference quotient of the integrated baseline hazards

$$(11) \quad \hat{h}_0(t_i) = \frac{\hat{I}_0(t_i) - \hat{I}_0(t_{i-v})}{t_i - t_{i-v}},$$

where $v = 1$. Then it can be seen that (10) is essentially equal to (11) for non-censored observations. It is suggested here that these smoother estimates are obtained by choosing v for each t_i such that $\min(t_i - t_{i-v})$ is larger than a predefined constant ϵ . In this application ϵ was set equal to 5 weeks and the estimates of the baseline hazard function were centred at the midpoint of the intervals (t_i, t_{i-v}) . An advantage of this kind of simple smoothing is that the baseline hazard can not obtain negative values, which is possible using the method suggested by Anderson and Senthilselvan (1980). The graphical procedure is valuable in indicating the operational significance of the changes in the baseline hazard.

The estimates of the baseline hazards of the duration and calendar time models are presented in Figure 3. In the first box the function is derived from the first model of Table 1. The baseline hazard functions resemble very much the corresponding life table hazard functions, which are presented in Chapter II. Reluctant movers risk losing benefits after the first three months. The first box of Figure 3 shows that the risk increases the re-employment probability. The elasticity of the hazard function with respect to the replacement ratio is the product of the replacement ratio and its parameter estimate. Therefore the effect of risk is larger for the members of labour unions, who are usually eligible for higher benefits.

Members of labour unions face a 20 per cent reduction in their benefits at the end of the 20th week of unemployment. The reduction has a very strong positive effect on the re-employment probability as the third box of

Figure 3 shows. The baseline hazard is approximately 100 per cent higher just after the reduction than it would otherwise be. These findings are confirmed by Table 2, which shows that the negative effect of benefits is higher for the non-members of labour unions.

During the period under investigation the employment office had to offer a job to a person who had been unemployed for a year. Therefore the baseline hazard functions are increasing at the one-year mark. The low estimates of the baseline hazard function for the durations just less than a year are rather low. This is affected by the rules and practices of the employment office.

The calendar time model of unemployment duration is presented in the first column of Table 5 and the seasonal variation is illustrated by the baseline hazard function in the last box of Figure 3. The seasonal variation of the baseline hazard is rather similar during 1985 and 1986 except that the baseline hazard is lower in 1986. Figure 3 shows that the re-employment probability is rather high in May and June, whereas it is low during the last quarter of the year.

The semi-parametric models of labour mobility are presented in Tables 3 and 5. The corresponding baseline hazard functions are presented in Figure 4. The unemployed persons seem to be prone to move at the beginning of their unemployment spell and just after the three months of unemployment. There also seems to be two moving peaks in calendar time. Unemployed persons move often in the beginning of June and August. However, one can not draw

very strong conclusions about the regional mobility, since it is a rather rare phenomenon.

Occupational mobility is measured on the most accurate 5-digit level, which includes 1320 occupations. Unemployed persons change their occupations most often at the beginning of their unemployment spells and just after the first three months of unemployment. There is a peak in occupational mobility at the beginning of September. People change their occupations quite often also in the beginning of May, June and July, but rather seldom at the end of the year.

Figure 3. Baseline hazard functions of unemployment duration

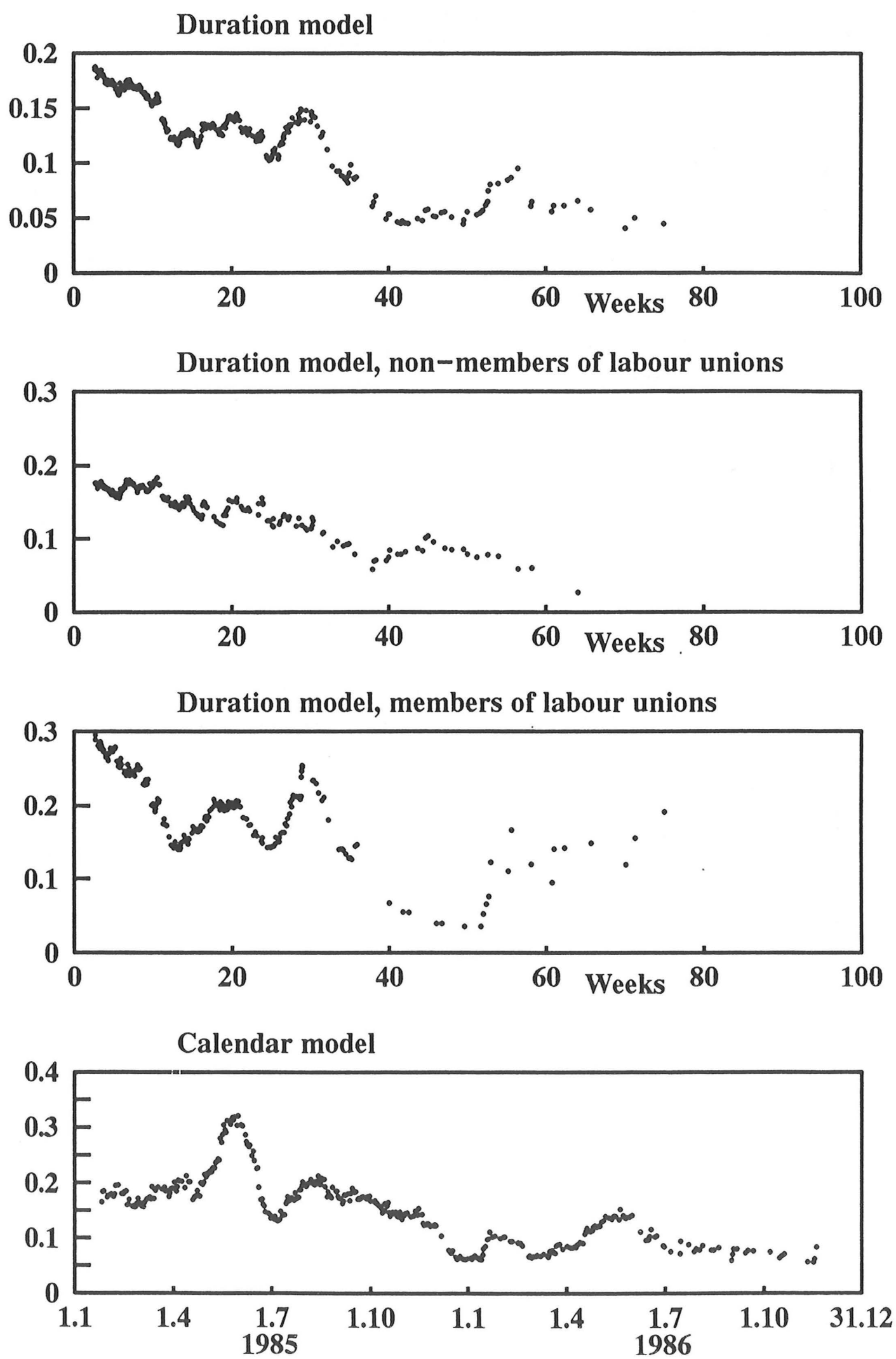
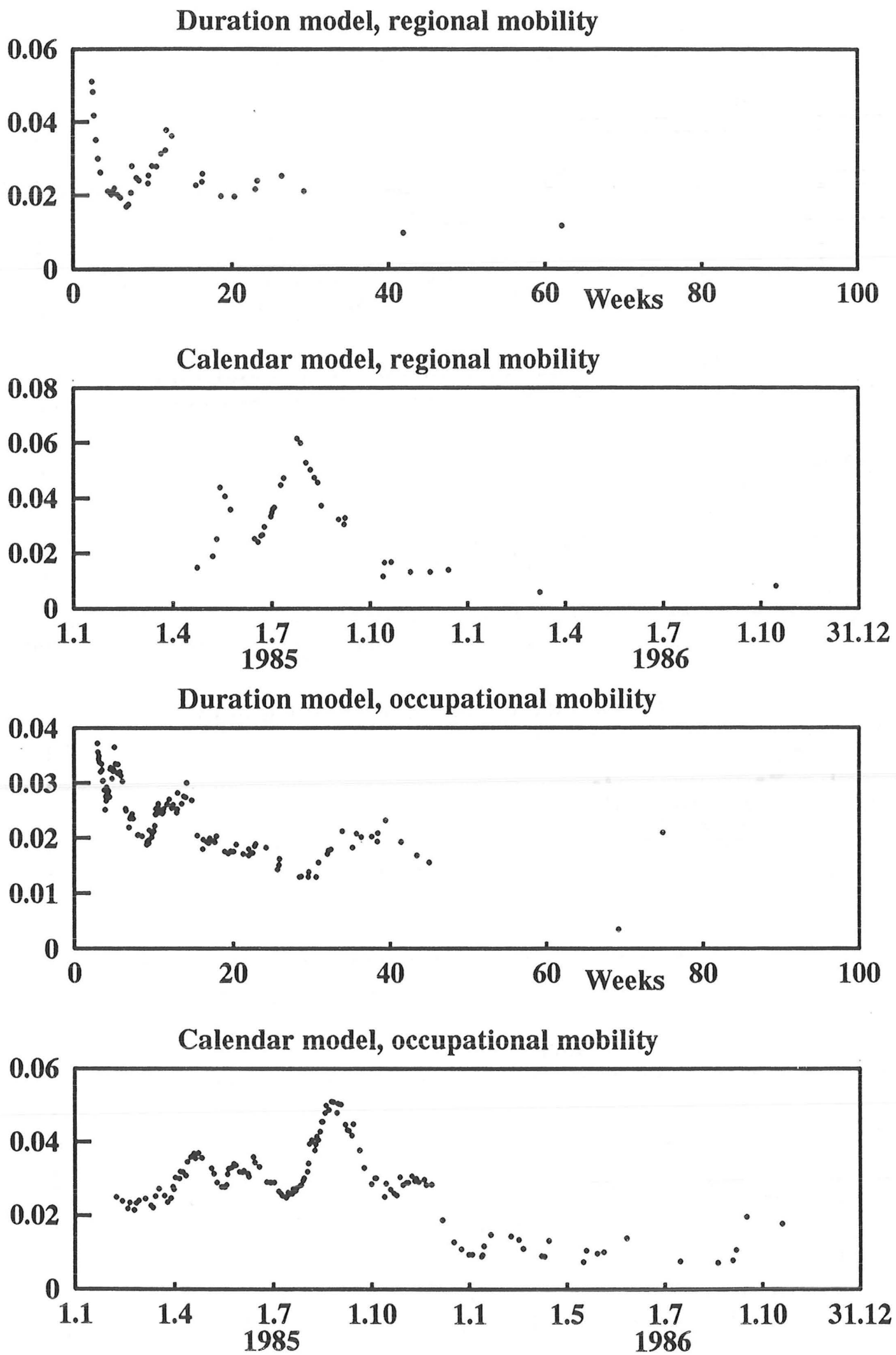


Figure 4. Baseline hazard functions of labour mobility



4. A Graphical Method for Assessing Goodness of Fit

This section considers a graphical method which can be used to detect lack of fit in the semi-parametric models of duration. The method for examining fitted models is based on the generalized residuals and it examines whether they have a unit exponential distribution. The generalized residuals of a fitted model are defined as

$$(16a) \quad \hat{I}(t_i) = \hat{I}_0(t_i) \exp(x_i \hat{\beta}),$$

where

$$(16b) \quad \hat{I}_0(t_i) = \sum_{t_j \leq t_i} \frac{c_j}{\sum_{t_j \leq t_k} \exp(x_k \hat{\beta})}, \quad i = 1, \dots, n.$$

The residuals should behave approximately as censored unit exponentials in large samples if the model is correctly specified. Note that $\hat{I}(t_i)$ estimates the expected value of the residuals rather than the residuals itself, which may cause their distribution to depart from that of a right-censored unit exponential.

A large number of tests has been proposed for the semi-parametric proportional hazards model. Arjas (1988) gives an extensive reference list. Kay (1977) suggested plotting the residuals against the expected order statistics, i.e. the straight line. To extend the method to the censored case Kay suggests plotting the residuals against the

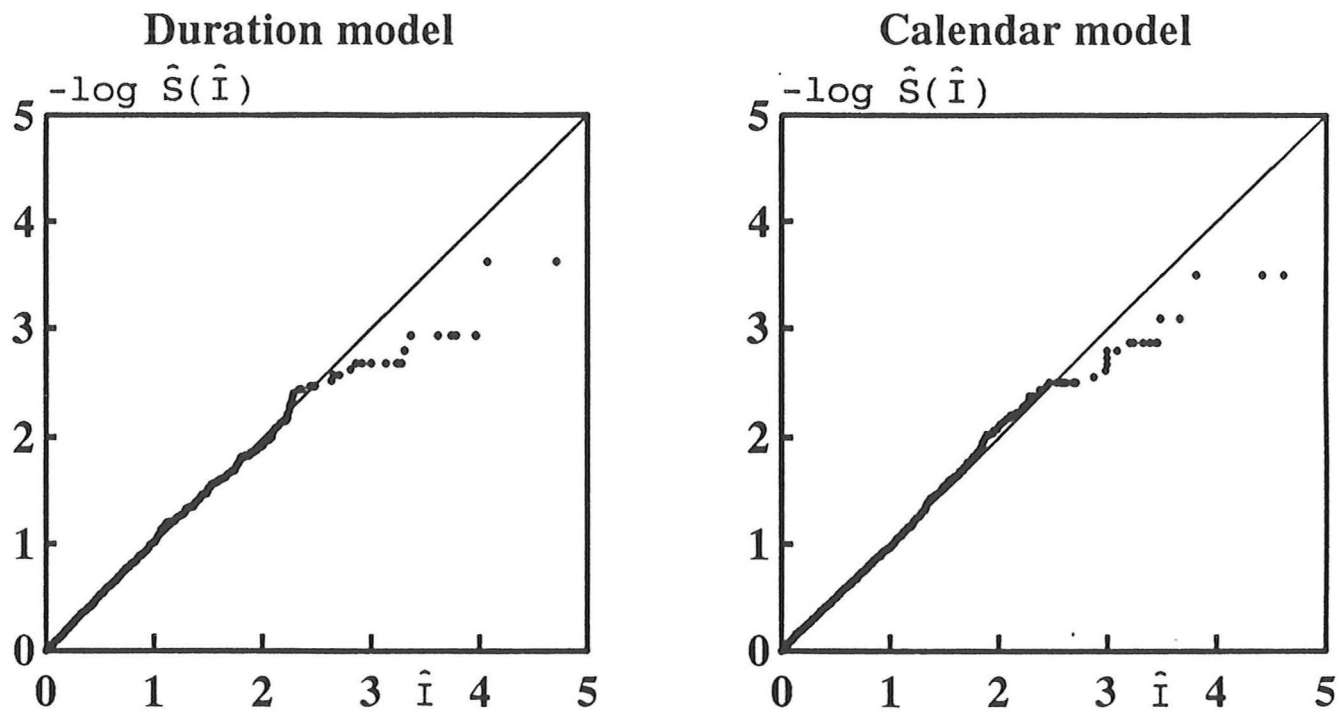
cumulative hazard function of the residuals. Lagakos (1980) noted that cumulative hazard function or log survival function of residuals versus a straight line could be quite misleading and should not be used. Crowley and Hu (1977) suggested adding $\log(2)$ (the conditional median additional duration for unit exponentials) or 1 (the conditional expected additional duration) to each censored residual. Crowley and Storer (1983) studied cross-plots of estimated generalized residuals either against a set of order statistics or against a covariate value from the unit exponential distribution. They found that a cross-plot of generalized residuals against a set of order statistics revealed very little. However, they noted that crossplots of residuals versus covariates may still be of some value in determining which covariates to include in a model.

The model specification is examined here using a graphical procedure suggested by Lancaster and Chesher (1985) in the context of parametric duration models. The product-limit procedure allowing for censored data is here applied to the generalized residuals from the fitted models in order to estimate the residual survivor functions $\hat{S}(\hat{I}(t_i))$. The residuals $\hat{I}(t_i)$ plotted against $-\log \hat{S}(\hat{I}(t_i))$ should give an approximately straight line with unit slope in large samples when the model is correctly specified.

Figure 5 illustrates the residual plots of proportional hazards models. The residuals of the calendar model are obtained from (16a) and (16b) by replacing the duration concept t by dates τ . Note that the expression $\tau_j \leq \tau_k$ then does not define the risk set, since the risk set includes the persons who are unemployed on that day. The plots of

the duration and calendar models are very similar. This is no wonder since the parameter estimates of these two models do not differ substantially.

Figure 5. Residual plots of semi-parametric models of unemployment duration



5. Conclusions

This chapter studied the time-dependence in semi-parametric proportional hazards models of unemployment duration. Models with duration-dependent replacement ratios of unemployment benefits were estimated. The model with a duration-dependent replacement ratio gives a substantially lower parameter estimate of the replacement ratio than the model where the benefit replacement ratio is fixed at an average value over the unemployment spell. Studying more carefully the duration-dependent effects, it was noted that the benefits have a negative effect on the re-employment probability during the unemployment of the first three months, but after that period the effect turns positive. One reason is that the unemployed persons may lose their benefits after the first three months if they do not move or change occupations. Another reason is that the reduction of benefits by 20 per cent after the 100th day of unemployment increases the re-employment probability by about 100 per cent. Using the graphs of the baseline hazard functions it was shown that these features of the UI system are of a great importance when looking at the probabilities of re-employment and regional and occupational mobility.

Specifications with calendar-dependent explanatory variables were studied and estimated using Finnish microeconomic data on unemployment spells. Statistically significant effects were found using calendar-dependent dummy variables and unemployment rates. The models were found useful in estimating the seasonal effects on the re-

employment probability. However, using these kinds of models the functional form of the calendar-dependence has to be specified completely, which may be difficult in practice.

In order to allow a flexible form of the calendar-dependence the roles of duration and calendar time were changed following the suggestions by Imbens (1990) and Ridder and Tunali (1990). The baseline hazard functions of the calendar models were used to illustrate the seasonal variation of the re-employment and labour mobility. The baseline hazard of re-employment is rather high in May and June. Regional mobility has peaks in the beginning of June and August. Occupational mobility has a peak at the beginning of September, but it is rather high also in the beginning of May, June and July. Labour mobility is rather low at the end of the year.

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Appendix 1. Log Likelihood Function and the Derivatives
of a SAS/IML Programme for Estimating a Calendar-
Dependent Model

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*****;
*      COMMENTS:
*      T0  = ENTRY TIME
*      T1  = EXIT TIME
*      RS  = RISK SET FOR EACH DURATION
*      DRS = THE DERIVATIVE OF RS
*****;
LIKEF:  XB=X*B'; TT0=T0; TT1=T1; RS=0; DRS=0; N=0; FREE RST;
A:      IF TT0(|<>|) < TT1(|<>|) THEN DO;
          I=TT1(|<:>,|); TT1(|I,1|)=0; D1= 1; N=N+1;
          END;
        ELSE DO;
          I=TT0(|<:>,|); TT0(|I,1|)=0; D1=-1;
          END;
        RS  = RS  + D1#EXP(XB(|I,|));
        DRS = DRS + D1#X(|I,|)#EXP(XB(|I,|));
        IF D1 = 1 THEN DO;
          L  = L/(C(|I,|)#(XB(|I,|)-LOG(RS)));
          V  = V/(C(|I,|)#( X(|I,|)-DRS/RS));
          RST = RST/RS;
          END;
        IF N < OBS THEN GOTO A;
        LL = L(|+,|); FREE N L RS DRS TT0 TT1;
RETURN;
LIKED:  V  = V';
        DL = V(|, +|);
        CL = -V*V'; FREE V;
RETURN;

```

Chapter VIII

SUMMARY AND CONCLUSIONS

The aim of this study has been to investigate the process of re-employment of Finnish unemployed persons using both the search theoretical and microeconomic approaches. In the following the course of the study with the main results are summarized.

For the econometric analysis of re-employment a sample of 2077 unemployed workers was drawn from the register of the Ministry of Labour. Every hundredth individual was picked for the sample from the flow into unemployment during 1985. The persons were followed until the end of their spells of unemployment, but at most to the end of 1986. Therefore the data set includes censored observations, which are rather common in econometric studies of unemployment duration. The information of unemployed persons' annual income and taxable assets was compiled from the tax register. The information on the basic unemployment allowance and the earnings-related unemployment allowance during the unemployment period was compiled from the unemployment allowance register of the Social Insurance Institution and the bank Postipankki, respectively. The data are fairly rich on individual characteristics and labour market specific variables. The interest of this study is on the effects of unemployment benefits on the duration of unemployment. This is the first

Finnish econometric study where the levels of unemployment benefits are available. In addition the effects of education on the duration of unemployment were studied.

A well-known result of search theoretical models is that the unemployment benefits have a disincentive effect on becoming employed. It is shown in Chapter III, however, that a higher re-employment probability can be achieved by paying stingy benefits during the spell of unemployment, but on the other hand the loss in the welfare of the workers can be offset by paying generous benefits to the persons who find jobs. This kind of system represents the "stick" and "carrot" approach, which decrease the selectivity and increase the search intensity of unemployed workers more effectively than just the cutting of benefits. Benefits related to the regional and occupational mobility and re-employment bonuses are means of increasing the re-employment probability and welfare of unemployed persons. According to the results in Chapter III the welfare of unemployed persons can be increased by removing the waiting period for benefits. It is shown that the waiting period does not substantially increase the re-employment probability. On the other hand, the incentive towards the re-employment can be effectively increased by removing the rules of regional and occupational mobility and reducing benefits after a permitted period of higher benefits. These kinds of changes in the UI system would increase the incentive towards short durations of unemployment.

In the econometric study of Chapters IV - VII some factors decreasing the probability of becoming employed were investigated. Aged persons are apt to incur problems

in finding acceptable offers. They are less prone to move and change occupations than the younger persons. The persons who came from housework have lower probabilities than the others. Training for further employment was found to help the re-employment. A reason for concern is that the replacement ratio of benefits has a negative effect on the probability of becoming employed. This is not a general result, however.

More detailed analysis shows that the receipt of benefits has a smaller negative effect for the earnings-related unemployment allowance than for the basic unemployment allowance. In addition it was found that the level of the replacement ratio for the recipients of the basic unemployment allowance had a negative effect on the probability of becoming employed, but for the recipients of the earnings-related allowance the effect was not statistically significant. One reason for this is that the elements of the incentives of the UI system are more efficient for the recipients of the higher earnings-related unemployment allowance. The risk of losing benefits matters more for the persons obtaining higher benefits. In addition the reductions of benefits are only applied to the earnings-related allowances.

The study of the time-dependent effects of unemployment benefits in Chapter V revealed that the effect of the replacement ratio was negative but statistically rather weak during the first three months. After that period the effect vanishes for the members of labour unions. For the non-members the negative effect decreases substantially. The reasons for the positive shift of the effect of the

replacement ratio can be found from the features of the UI system. The eligibility rules of benefits become stricter, so that the persons who are reluctant to move or change occupations may lose their benefits after the first three months. Furthermore the earnings-related benefits decreased 20 per cent after the hundredth day of unemployment. The reductions of earnings-related unemployment allowances increase the incentives for the re-employment after the hundredth day of unemployment. After that the effect of the replacement ratio obtains a positive shift, but the effect of the replacement ratio does not statistically differ from zero.

The effects of education on the duration of unemployment are not straightforward. It was found that the level of education is positively related to the re-employment probability within the relatively low levels of education, but in the higher levels the relationship turns negative. The level of education is positively related to the arrival rate of job offers and well-educated persons get better job offers. On the other hand, well-educated persons have higher costs of re-employment and therefore higher reservation utilities. Hence the persons with the highest levels of education have fewer acceptable offers available. The persons with 13 - 14 years of formal education have the highest re-employment probabilities.

The results of the parametric models are complemented by the semi-parametric models in Chapter VII. The unemployment benefits have a disincentive effect on becoming employed during the first months, but after that the stricter rules of eligibility increase the incentives

for getting a job. Alternative methods were developed in order to study the effects of the risk period and reductions of benefits on the duration dependent baseline hazard. After the first three months the baseline hazard turns increasing for the members of labour unions. For the non-members the effect is less distinctive. Upon the hundredth day of unemployment the earnings-related unemployment allowances decrease by 20 per cent. Just after the reduction the baseline hazard increases about 100 per cent.

The previous results concerning the effects of unemployment benefits are, however, not so straightforward. The system of earnings-related unemployment allowance has also other incentive effects on the re-employment. The rules of the unemployment insurance require that the recipients of these benefits must have been working before the unemployment. This requirement makes the intermittent employment more attractive for the members of labour unions than it would otherwise be, because the value of search related to re-employment is higher for them. This theoretical result is supported by the empirical finding that the members of labour unions have higher probabilities of becoming employed.

Tiivistelmä

Tutkimuksessa tarkastellaan työttömien työllistymiseen vaikuttavia tekijöitä sekä etsintäteoreettisen että yksilötason tutkimusaineistoon perustuvan ekonometrisen tutkimuksen avulla. Tutkimusaineistona on 2077 työttömän otos, joka koottiin tätä tutkimusta varten työvoimaministeriön työnhakijarekisteristä. Otokseen valittiin joka sadas vuonna 1985 työttömäksi tullut. Työttömyyttä seurattiin työttömyyskauden loppuun saakka, mutta enintään vuoden 1986 loppuun asti. Tutkimusaineistoon yhdistettiin verotusrekisterin tiedoja sekä päivärahatiedot Kansaneläkelaitoksen ja Postipankin rekistereistä. Erityisenä mielenkiinnon kohteena on miten työttömän taloudellinen asema sekä koulutus vaikuttavat työttömien työllistymiseen sekä alueelliseen ja ammatilliseen liikkuvuuteen.

Työttömyysturva parantaa työttömän taloudellista asemaa luomalla taloudellisia edellytyksiä sopivan työpaikan valintaan, mutta toisaalta se on ongelmallinen, koska korkea työttömyysturva heikentää työvoiman saatavuutta pidentämällä työttömyyden kestoa ja vähentämällä työvoiman alueellista ja ammatillista liikkuvuutta.

Eräänä tutkimuksen tavoitteena oli selvittää, mitkä työntekijäryhmät muodostuvat ongelmalliseksi työllistymisen kannalta ja herättämään kysymyksiä järjestelmän toimivuuden parantamiseksi. Ikääntyneiden henkilöiden työllistäminen on osoittautunut ongelmalliseksi mm. siitä syystä, että heidän muutto- ja ammatinvaihtotodennäköisyytensä on vähäinen. Myös kotityöstä ja muualta työvoiman ulkopuolelta tulevien

työllistyminen on vaikeaa. Korkeaa työttömyysturvaa saavien työllistäminen on ongelmallista. Joillekin henkilöille työttömyysturva ja vapaa-aika sinänsä muodostavat riittävän hyvinvointitason, joten työntekoa pienellä palkalla voidaan pitää huonompana vaihtoehtona kuin työttömyyttä.

Tutkimuksessa järjestelmää tarkastellaan työmarkkinoiden tehokkaan toimivuuden kannalta. Toimivilla työmarkkinoilla työttömillä on mahdollista löytää nopeasti heille sopivaa työtä ja vastaavasti työnantajilla löytää nopeasti sopivaa työvoimaa. Voidaankin kysyä miten työttömyysturvajärjestelmää voitaisiin muuttaa, että työmarkkinoiden toimivuus paranisi.

Tutkimuksen mukaan työllistymistä voitaisiin edistää siirtymällä niukempaan työttömyyden aikaiseen ja runsaampaan työllistymiseen liittyvään tukeen. Työllistymisen todennäköisyyttä edistävät työllistymistuet maksettaisiin työttömälle työllistymisen yhteydessä. Esimerkiksi muuttoavustusten korottaminen sekä erorahan ja osaksi työttömyyspäivärahan maksaminen työllistymisen yhteydessä olisivat tällaisia keinoja. Päivärahojen osittainen maksaminen vasta työllistymisen yhteydessä toimisi työllistymistä kannustavammin kuin vastaavansuuruisen päivärahan osan leikkaaminen.

Työttömyysturvajärjestelmällä voitaisiin kannustaa myös nopeaa työllistymistä. Työttömyysturva voisi olla suhteellisen hyvä työttömyyden alkuvaiheessa, mutta mikäli työtöntä halutaan kannustaa työllistymään, työttömyysturvan tulisi huonontua työttömyyden pitkittyessä. Kaikkien päivärahan oikeutettujen työttömien taloudellista asemaa voitaisiin parantaa poistamalla työttömyyspäivärahojen omavas-

tuu aika, koska omavastuuajalla ei ole juurikaan työllistymistä edistävää vaikutusta.

Työllistymistä voitaisiin merkittävästi edistää poistamalla työttömyysturvalaista päivärahojen alueellista ja ammatillista liikkuvuutta koskeva suoja ja ottamalla käyttöön ns. ansioturvan alenemat. Tutkimustulosten mukaan kolmen ensimmäisen työttömyyskuukauden aikana voimassa oleva liikkuvuutta koskeva suoja passivoi työllistymistä. Alenemat olivat tutkimusajanjaksolla 20 prosenttia sadannen työttömyyspäivän kohdalla. Tulosten mukaan työllistymisen todennäköisyys kasvoi alenemien toteutumisen jälkeen. Vuodesta 1987 vuoteen 1989 saakka päivärahat alenivat 200. työttömyyspäivän jälkeen 12.5 prosenttia. Työttömyyspäivärahojen alenemat poistettiin vuoden 1989 heinäkuun alusta alkaen. Alenemien käyttöönotto kannustaisi työttömiä nopeampaan työllistymiseen.

Työttömyyspäivärahajärjestelmä on kannustinvaikutusten osalta epäyhtenäinen perus- ja ansioturvaan oikeutetuilla henkilöillä, sillä ansioturvaan sisältyy perusturvaa enemmän työllistymistä kannustavia piirteitä. Vain ansioturvassa on alenemat. Samantyyppinen piirre on myös se, että ansioturva on ajallisesti rajoitettu. Lisäksi ansioturvassa on työssäoloehto, joka lisää henkilön kiinteämpiä yhteyksiä työmarkkinoihin. Se lisää työllistymismotivaatioita, sillä vain riittävän pitkään työssä olleet työttömät ovat oikeutettuja korkeampaan ansioturvaan. Tämän vuoksi ansioturvaan oikeutettujen työttömyyskassan jäsenten työllistymisen todennäköisyys on muita suurempi.

Koulutusta voidaan tarjota eräänä ratkaisuna työttömyysongelmaan, sillä aikuiskoulutuksella voidaan merkittä-

västi edistää työllistymistä. Kuitenkin työllisyyspolitiikassa olisi tarkoin ja yksilökohtaisesti harkittava, missä määrin ikääntynyttä henkilöä olisi koulutettava uusiin tehtäviin, ja missä määrin hänen toimeentulonsa olisi pyrittävä varmistamaan työttömyysturva- tai eläkejärjestelmien avulla.

Tässä tutkimuksessa tarkasteltiin kuitenkin keskeisimmin peruskoulutuksen merkitystä työllistymisessä. Työttömiin korkeampi peruskoulutus edistää merkittävästi työllistymistä tiettyyn rajaan saakka, mutta korkeimmat koulutusasteet muodostuvat usein esteeksi nopealle työllistymiselle. Nopeimmin työllistyvät ne, joilla on 13 - 14 vuoden peruskoulutus. Koulutus lisää työtarjousten saannin todennäköisyyttä ja parantaa työtarjouksia, mutta korkea koulutus merkitsee useimmiten myöskin korkeampia työllistymiskustannuksia. Ne saattavat liittyä joko uudelleen koulutukseen tai muuttoon uudelle työssäkäyntialueelle. Näistä tekijöistä johtuen koulutus lisää myöskin työntekijän vaatimustasoa, mikä johtaa siihen että korkeasti koulutetut saavat vähemmän hyväksyttävissä olevia työtarjouksia.

