

Essays on
INCOME DISTRIBUTION,
ECONOMIC WELFARE
and
PERSONAL TAXATION



Edited by Robert Hagfors and
Pentti Vartia



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ABSTRACT: The book is a collection of nine articles dealing with the distribution of income, economic welfare and personal taxation. The first four articles concentrate on the distribution of lifetime incomes, the effects of background variables on income distribution and the profitability of the investments on education. The approach is mainly empirical focussing on the case of Finland. In the next three articles the economic welfare approach is emphasized. The structure of low economic welfare households is studied and empirically determined equivalence scales are implemented. Also the effectiveness of social transfers is studied. The last two articles analyse the heaviness of personal taxation and the effects of deductions. Also other issues like reasons behind tax revenue changes, progression measures and neutrality concept are considered.

KEY WORDS: Income distribution, economic welfare, personal taxation

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TIIVISTELMÄ: Kirja koostuu yhdeksästä artikkelista, jotka käsittelevät tulonjakoa, taloudellista hyvinvointia sekä fyysisten henkilöiden verotusta. Neljässä ensimmäisessä artikkelissa keskitytään elinaikaisten tulojen jakautumiskysymyksiin, rakenteellisten taustamuuttujien tulonjakovaikutuksiin sekä henkiseen pääomaan suorilettujen investointien kannattavuuteen. Lähestymistapa on pääasiassa empiirinen ja tarkastelu koskee erityisesti Suomen oloja. Kolmessa seuraavassa artikkelissa kiinnitetään huomiota taloudelliseen hyvinvointiin. Huonosti toimeentulevien kotitalouksien rakennetta selvitetään ja käytetään empiirisesti laskettuja ekvivalenssiskaaloja. Viimeiset kaksi artikkelia käsittelevät välittömän verotuksen kiristymistä ja verovähennysten vaikutuksia. Myöskin verotulojen muutosten taustatekijöitä, progressiomittoja sekä verotuksen neutraalisuuskäsitettä valotetaan.

ASIASANAT: Tulonjako, taloudellinen hyvinvointi, välitön verotus

FOREWORD

In order to provide more empirical facts concerning the controversial subject of distribution of income and economic welfare, OKOBANK (The Central Bank of the Cooperative Banks of Finland) and The Central Association of the Finnish Cooperative Banks (OKL) decided to finance a research project in which the three major economic research institutes in Finland participated. This book represents the contributions of ETLA, the Research Institute of the Finnish Economy, to that project.

When the project was launched it was decided that ETLA's research topic would be distribution of income and economic welfare. The focus was then sharpened to concern the socio-economic and other so-called "natural" explanatory factors behind income distribution. In this book the original scope has somewhat expanded to cover, for instance, issues on personal taxation.

Many other persons in addition to that of the authors have contributed to the book. At the initial stages the project was coordinated on behalf of ETLA by Juhani Turkkila. Later the coordination work was passed to Timo Airaksinen and after him to Robert Hagfors. Along the way the authors have had a possibility to utilize the valuable comments of several persons: Yrjö Vartia in particular should be mentioned in this connection. Heikki Vajanne as the head of the data processing unit and as a creator of computation algorithms has made many of the research results possible. Kaija Hyvönen-Rajecki has contributed to the project as a statistical assistant. The diagrams were produced by Arja Selvinen and Arja Virtanen. The typing has been done by Tuula Ratapalo. The language was checked by John Rogers.

Helsinki, January 1989

Pentti Vartia

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INTRODUCTION

This book contains nine articles related to a project on income distribution which has been carried out at the Research Institute of the Finnish Economy in recent years and which has been sponsored by OKOBANK. The articles have been grouped into three broader categories: income distribution, economic welfare and personal taxation.

In the first four articles under the heading "Income distribution", like in most articles of the book, the empirical approach has been emphasized. This is clearly the case in the first article by Fredrik Nygård: "Relative Income Differences in Finland 1971-81".

The aim of Nygård is not only to describe the existing income distribution, but also to give an explanation of the observed distributions using some background variables. The income concept chosen is the available or disposable income and the income receiving unit is a household. The dynamic features are included by studying the distributions for different cross-section years and the focus is on the development of incomes in different kinds of households.

Nygård presents the distributions of disposable income per household and per household member for different cross-section years and different socioeconomic groups. Graphs for disposable nominal and real incomes of all households are presented, showing how they are distributed in the three survey years 1971, 1976 and 1981. For relative incomes the diagrams of pre-tax and disposable income distributions are presented.

They are followed by corresponding Lorenz curves.

Nygård then continues by trying to find out the effect of taxation on households' income distribution. This is done by the decomposition of the Gini coefficient to pre-tax income and paid transfers. According to the results there has been a growing tendency towards a more equal distribution due to the transfers paid from 1971 to 1976. During the period 1976-1981 the redistributive effect was somewhat diminished.

The rest of the article contains the pooling of the three household surveys and a cohort analysis. Using five-year cohorts the households are studied according to different socio-economic characteristics. Also the household cohort income profiles are presented for the period 1971-1981.

It is impossible to draw conclusions from the changes in inequality measures regarding changes in inequality in the normative sense when also the structure of the income receiving units is changing. For this, analysis of the effects of the background variables is required. Using the pooled data Nygård estimates a regression model where the relative income differences are explained with different characteristics of the households, like number of children, age of the head of the household, sex, socioeconomic group and education. The results seem to be satisfactory and the conclusions concerning part-time earners, inactive adults, children, age of the head of the household, socio-economic group, sex and education are given. It is concluded that the structural variables of the households explain a large part of the variation in incomes.

Nygård's second article "Lifetime Incomes in Finland - Desk Calculations Based on Civil Servant Salaries" represents another kind of approach to research on income distribution. The purpose is to highlight the importance of the time period for which the incomes are registered in the determination of the amount of inequality in the observed income distribution. This aim is approached by choosing some representative income receivers and studying their lifetime income streams. Income profiles are defined for representative civil servants and the corresponding lifetime discounted incomes are calculated. Here time used for education is considered as an investment which affects the future income streams. The differences between the discounted incomes chosen are sensitive to the given discount rate. The tax progression also affects the profitability of an investment project in human capital.

Nygård is also analysing the effect of the length of the accounting period on the income distribution measures using overlapping profiles of income receivers identical in the lifetime sense. The results indicate that the amount of inequality decreases when the time period for which incomes are registered is increased. Even though all lifetime income profiles are identical, the fact that individuals are at different stages of the life cycle at different times leads to considerable differences in yearly incomes.

While Nygård's desk calculations are based on hypothetical income earners, the third article in the first part of the book considers the time dimension in an empirical framework. Reija Lilja's "Accounting Period and Income Inequality - Empirical Evidence from U.S. Panel Data" is based on the sample from the Michigan panel data of income dynamics for the years 1967-81.

Lilja studies the dynamics of the income distribution during the 15-year period in question applying three different income definitions, the real labour income of the head of the household, real taxable income of the head and wife and real total income of the family. She then calculates the Gini coefficient and the square of the coefficient of variation for accounting periods of different length.

Lilja's results indicate, that the longer the time period, the lower the corresponding measure of inequality. The square of the coefficient of variation is more sensitive to the length of the accounting period than the Gini coefficient. She also calculates the Shorrocks index R for each accounting period and for the two inequality measures in question. Again the results indicate that the Gini index reveals a more modest reduction in inequality as the accounting period is becoming longer than does the squared coefficient of variation.

Part one is completed by Frank Cowell's article "On the Distribution of Lifetime Income in a Lognormal Model". The dynamic character of the income distribution is focussed from the theoretical point of view. A simple definition of "lifetime income" is applied within a stochastic model of income generation. The paper examines the relationship between the inequality of the current income and the distribution of the lifetime income. This is done in two ways: as it would appear to an outside observer, and as it would appear to an individual in the income distribution who knows his current income, but not his future prospects. The distributive impact of income taxation is also examined using inequality measures defined for a family of lognormal distributions.

In part two of this book a more concrete effort is made to look at the distribution of "economic welfare". In this connection the definitions of income and income receiving units appear to be important. This point comes out clearly in Risto Sullström's article "Characteristics of the Lowest Income Quintile in Finland in 1981".

Sullström studies in his paper "low economic welfare" households in the household survey data for the year 1981. He uses four different definitions for the income unit and sees which units are included in the lowest quintile according to various definitions. The individuals in separate cases differ greatly. Special attention is given also to cases, where there are handicapped and/or chronically ill members in a household.

Sullström calculates distribution indexes for several income definitions. He also calculates poverty indexes, where the head count ratio and the deviation of the incomes from the threshold level are taken into consideration. The most interesting results of Sullström are the descriptions of the household groups who stay at the lowest end of the income distribution no matter which income definition is used.

Consumer unit scales or equivalence scales, which are applied also in Sullström's article, are taken into closer consideration in Hagfors' article "Household Equivalence Scales in Finland for the Years 1976 and 1981". In his paper Hagfors aims at the empirical determination of the equivalence scales at two time points. The data used are household surveys from 1976 and 1981.

After a short introduction to the subject Hagfors presents empirical calculations on commodity specific and general equivalence scales. Several

interesting points are made regarding the scale numbers of different household types for different price and income relations. The limitations of the method used and some suggestions concerning future research are also given.

The third article of the welfare part of the book is by Hagfors and Sullström. The purpose is to put the equivalence scales defined in the previous article into practice, this time applying them to social transfers. The relevance of transfers is also investigated for various smaller demographic groups. Several distribution indexes are presented as well as concentration curves.

It is concluded that even though income distribution has not especially changed during the time period under consideration, the distribution of economic welfare has become more equal. It is also concluded that there exist some social transfers which, from the efficiency point of view, do not work satisfactorily. By efficiency in this case it is meant that the support is directed toward low welfare households. Finally, the fact that the results depend on the chosen definitions is emphasized.

The theme of the third part of the book, personal taxation, is by itself worth its own book. The two articles in this book can thus address only a few aspects of the subject. In the paper by Juhani Turkkila "Real Changes in Physical Persons' Income Taxation in Finland" the focus is on the heaviness of income taxation and how it could be best described.

Theoretical questions concerning real income changes are discussed, and the method for construction of an index which gives satisfactory description of the relative changes in income tax scales is outlined. The other central question addressed is how the deductions have affected the income taxation in the years 1949-1982.

In his article "The Effects of Taxation on Income Distribution" Christian Edgren concentrates on the direct effects of taxation. The units in the study are individuals. Edgren uses certain tax instruments, individual tax functions, the income distribution and the theory of consistent aggregation in order to construct measures of the progression.

Edgren provides sensitivity calculations related to the tax situation in the year 1987 considering an increase of one percent in total gross income. With four different alternative forms of change the values for relevant tax parameters are calculated, as well as some concentration indexes. These indexes are decomposed into the income and the income distribution effects. In his conclusion Edgren considers the reasons behind the changes in the tax revenue, suggestions for progression measures, the neutrality concept and the neutrality of indexation rules.

PART I

INCOME DISTRIBUTION

RELATIVE INCOME DIFFERENCES IN FINLAND 1971-1981

by Fredrik Nygård

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1. INTRODUCTION

The main idea of this paper is to delve 'behind' the observed Finnish income distribution and to adopt a fundamentally dynamic approach to illustrate how a number of background variates have influenced the income position of a household.

Before spelling out the empirical framework in any greater detail, it should be noted that the discussion will rely heavily on the following concepts:

(A) The main reason for attaching significance to the income distribution can be traced back to its welfare implications (welfare depends on consumption possibilities and these in turn on income). When discussing income distribution matters, the distribution of available income, closely related to the notion of consumption possibilities, should consequently be placed in the forefront.

(B) From a welfare point of view the appropriate income recipient unit seems to be the family/household, as members of a family may share their consumption goods and pay for the purchases from their joint income. Hence, even if incomes are earned largely by individuals, the 'consumption possibility' approach focuses on the pooled incomes that families/households control.

(C) It would be highly preferable if we could follow the development of annual incomes from a dynamic perspective, instead of confining the discussion to a set of annual 'snapshots'. A genuine dynamic analysis of trends in the income distribution would require panel data following the same families/household and their incomes over a number of years.

However, panel data on incomes in Finland is currently not available. On the other hand, the possibility of deriving average income profiles from cross-section income data covering several years (by disaggregating the income recipients according to age) should not be overlooked. A similar approach will be chosen below.

(D) When comparing the income positions between households, we will mainly confine the discussion to their relative position in the income distribution. Hence, if y_{it} denotes the income of the i^{th} household during year t and μ_t is the corresponding annual mean income in the whole household population, we will derive the distribution of incomes relative to the mean, y_{it}/μ_t , or log differences of household incomes from the mean, $\log(y_{it}/\mu_t)$, from the original y_{it} -distribution.

(E) The empirical analysis of income differences below will be based on survey data. As a consequence, the results must be interpreted as estimates involving sampling errors. However, the adopted data base lacks sufficient information for a rigorous treatment of the induced sampling errors and the analysis will mainly be carried out without any reference to the precision of the estimates.

2. THE DATA

Analyses of the personal income distribution in Finland during the last few decades can be based on either of the two following data sources:

- (a) the taxation records compiled by the National Board of Taxation (Verohallitus),

(b) the household and income distribution surveys conducted by the Central Statistical Office (Tilastokeskus).

At first sight the taxation records, covering the time period from 1920 onwards (with only a few exceptions), may appear as an appropriate base for studying trends in the income distribution. However, the records depend heavily on current fiscal legislation (exemption limits and tax-exempt income components, joint/separate taxation of married couples, etc.) and many shifts in reported incomes can be traced back to changes in the taxation rules. Moreover, as the taxation record uses the individual as the income recipient, records have to be matched if we wish to study the income distribution among families/households.

The income distribution surveys have been conducted annually since 1977, with forerunners in the form of household budget surveys for 1966, 1971, and 1976, and are based on samples consisting of some 10000 households. Besides the influence of sampling errors, the comparability between these surveys are to some extent restricted by some changes in the adopted income concepts and sampling procedures. Yet, the income distribution surveys (at least the later editions) must be considered as the most reliable Finnish data source for a study of income differences between households.

Table 2.1a summarizes the development of average available household income, transformed to real 1984 standards by the cost-of-living index, derived from the 'official' estimates published by the Central Statistical Office (CSO). The table, based on results from the household budget surveys of 1966, 1971, 1976, and 1981, and the income distribution surveys of 1977-80, adopts the 'traditional' CDO-classification of households into socio-economic groups.

Table 2.1a Finland 1966-1981: Real (1984 standards) available income per household; 1000 FIM.

	1966	1971	1976	1977	1978	1979	1980	1981
All								
households	57.5	64.2	71.7	70.2	71.6	72.8	72.8	75.8
Farmers	50.6	65.6	79.1	82.3	84.9	85.4	94.6	89.2
Own-account	77.6	80.6	86.5	84.6	93.9	93.3	92.0	95.6
White collar	84.4	84.7	86.8	87.4	89.3	93.5	91.8	93.0
Blue collar	54.5	64.3	77.8	76.7	77.4	79.7	78.9	82.4
Inactive	30.5	37.8	43.1	38.3	40.2	37.4	38.2	43.2

Table 2.1.b Finland 1966-1981: Real (1984 standards) available household income per capita; 1000 FIM.

	1966	1971	1976	1977	1978	1979	1980	1981
All								
households	17.2	21.6	25.8	26.4	27.3	28.3	28.8	29.2
Farmers	11.0	15.9	20.0	21.4	22.0	22.6	24.9	24.0
Own-account	20.5	21.9	24.4	25.1	28.3	28.6	27.9	29.1
White collar	25.7	28.9	30.6	31.7	32.0	33.5	33.7	34.6
Blue collar	16.2	20.0	25.0	25.3	26.3	27.2	27.8	29.0
Inactive	16.0	19.4	24.4	23.8	24.4	23.5	24.4	26.3

Sources: Household Budget Surveys for 1966
Household Survey 1971, 1976, 1981
Income Distribution Statistics 1977-1980.

As can be seen from the table, the real household income has exhibited a tendency to rise over the years. This trend is further accentuated when examining the available household income per capita in Table 2.1b, to be explained by the simple fact that the households have on average grown smaller over the period 1966-1981. We return to this point in Section 5.

In Figure 2.1a and 2.1b the level of real available income within the socio-economic groups are represented graphically for the three years 1971, 1976, and 1981.

Figure 2.1a Available (1984 standards) household income.

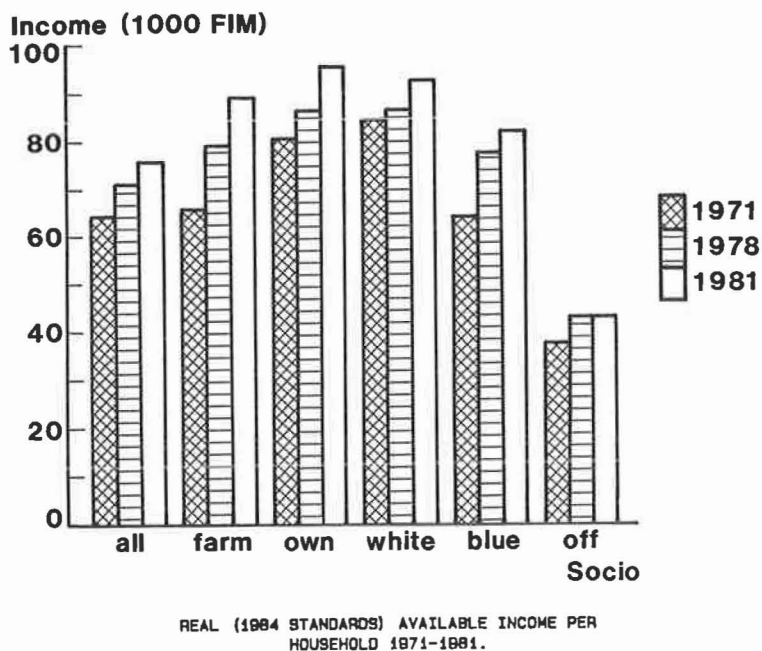
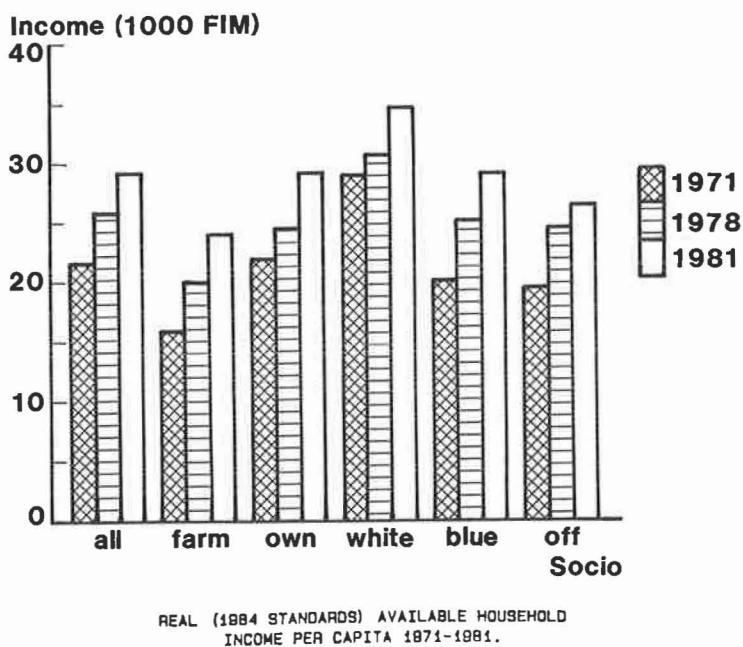


Figure 2.1b Available (1984 standards) household income per capita.



The data base to be adopted in this paper consists of micro-data from the samples used by the CSO in the household budget surveys (kotitaloustiedustelut) for 1971, 1976, and 1981, with effective sample sizes of 8816, 7971, and 7368 households, respectively. The survey designs were based on traditional probability sampling, without any panel-type links between the three samples, so that the 1971, 1976, and 1981 samples all include different households (possibly with a few exceptions due to pure chance).

The survey households were defined in a multigenerational setting as consisting of all persons who live together and jointly spend their income. Hence, in addition to the nuclear family a household may include other persons (presumably grandparents).

In co-operation with the CSO, the original 1971 and 1976 sample data were reworked (unifying a number of classification rules and including/excluding some income components) in order to improve the internal comparability between 1971 and 1976 microdata. However, the 1981 sample is included without any corresponding modifications. As a consequence, the data from 1971 and 1976 are not strictly comparable to the 1981 records, the major difference being a slightly narrower income coverage in the 1981 data. Even if the transformation to relative incomes, frequently used in the sequel, may be expected to improve the comparability (cf. Parkkinen (1985), who applies a similar argument), the differences in coverage should be kept in mind when interpreting the results of the following sections.

Figure 2.2a gives a first outline of the distribution of available household income in nominal amounts according to our data base. Note

that the 1981 distribution, due to its slightly different income coverage, probably should be shifted somewhat upwards.

Since the changes in the shape of the distribution (growth of the average nominal income accompanied by increasing dispersion) may be linked to the inflationary developments over the decade 1971-81, Figure 2.2b presents the three distributions with the horizontal axis transformed to real available household income. The transformation is based on the cost-of-living index, with 1984 as a reference standard, implying that 1971 incomes should be multiplied by a factor of 3.911, the 1976 incomes by a factor 2.065, and the 1981 incomes by 1.271. As can be seen, there are considerably less differences between the distributions according to this representation.

3. RELATIVE INCOME DIFFERENCES - GRAPHICAL DISPLAYS

As noted in the introduction, this paper will primarily deal with relative income differences. Now, if y_{it} denotes the annual income of a household in year t , its relative income is given by $u_{it} = y_{it}/\mu_t$, where μ_t is the average income in the household population during the same year, and most measures of income inequality (e.g. the Gini coefficient) may be interpreted as a weighted sum of the u_{it} 's. However, any monotone transformation of the relative income u_{it} could as well serve as an indicator of the relative income position of the household. In this paper we will frequently use the transformation

$$z_{it} = 100\log(u_{it}) = 100\log(y_{it}/\mu_t) = 100 \log(y_{it}) - \log(\mu_t)$$

Figure 2.2a The distribution of nominal available household income in Finland 1971, 1976, and 1981.

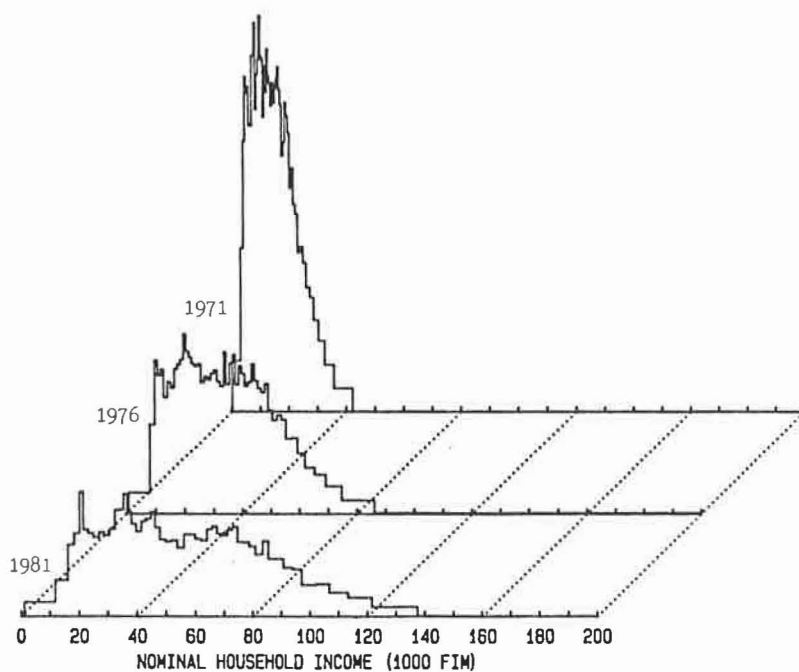
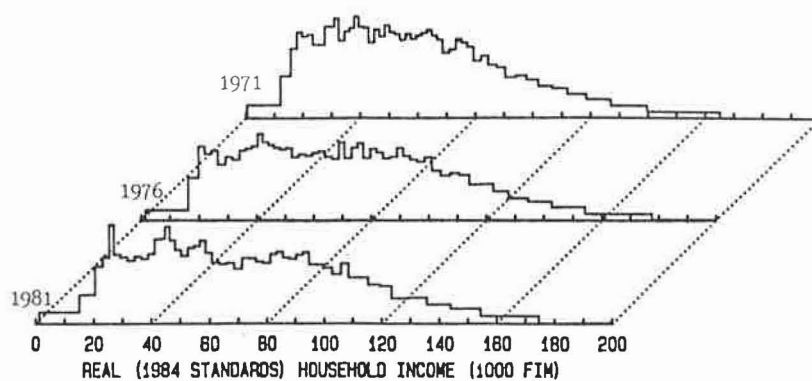


Figure 2.2b The distribution of real available household income (1984 standards) in Finland 1971, 1976, and 1981.



for descriptive purposes. The z_{it} 's, i.e. the log differences between household incomes and the mean income will be referred to as the relative incomes in log-scale, and their use is mainly motivated by the fact that a log-scale in many cases facilitates a graphical interpretation of income distribution data. In this context it should be noted that z_{it} is close to zero for households with an income close to the annual average, and that the magnitude of z_{it} roughly may be interpreted as the difference, expressed as a percentage, between the household income and the mean income.

Figures 3.1a and 3.1b give a first illustration of the relative income differences in log-scale. In Figure 3.1a the distributions of pre-tax household income are outlined, and Figure 3.1b represents the corresponding distributions of available household income.

The figures show three points clearly. First, the main difference between the distribution of pre-tax income and that of available income is that the latter is more peaked. Second, the income distributions from 1971, 1976, and 1981 are, on the whole, remarkably similar. Third, from 1971 to 1976 the distribution seems to have become more concentrated in incomes slightly above the mean income, at the expense of high incomes particularly, without any corresponding trend being evident between 1976 and 1981, and this applies to both the distribution of pre-tax and available income.

Figures 3.2a and 3.2b, in turn, illustrate the income distributions in the traditional form of Lorenz diagrams.

Figure 3.1a Pre-tax household income: relative income differences.

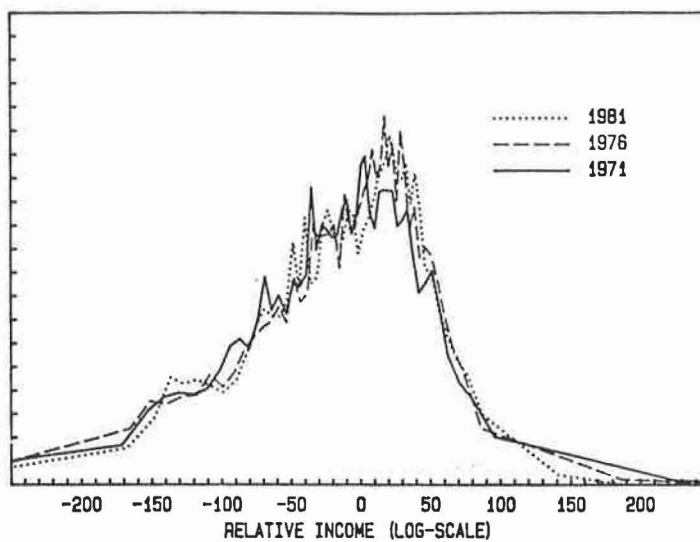


Figure 3.1b Available household income: relative income differences.

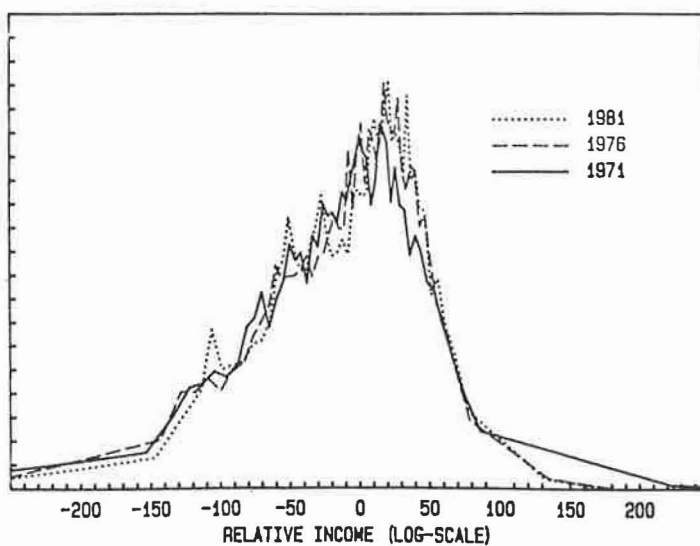


Figure 3.2a Pre-tax household income: Lorenz curves.

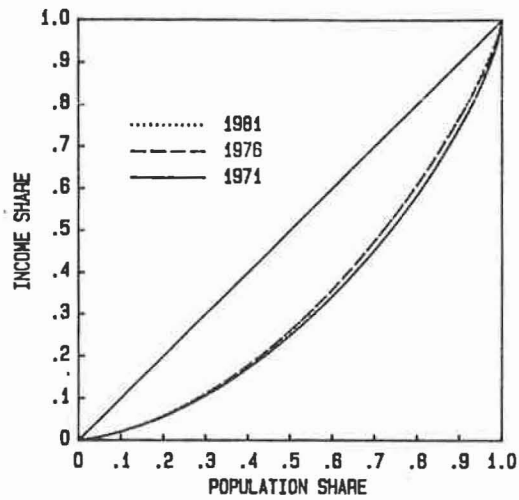
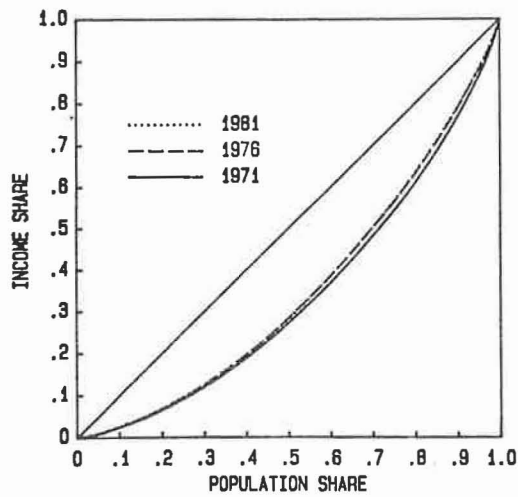


Figure 3.2b Available household income: Lorenz curve.



As implied by the Lorenz curves for both the distribution of pre-tax income, Figure 3.2a, and the distribution of available income, Figure 3.2b, from 1971 to 1976 the inequality across household in incomes has tended to decrease. Again, no similar pattern can be found to prevail between 1976 and 1981. In fact, the curves for 1976 and 1981 practically coincide in the Lorenz diagrams.

Similar conclusions regarding the trend of income inequality in the household population can be drawn by comparing Gini coefficients: The Gini coefficient of pre-tax income drops from its 1971 value of .361 to .337 in 1976, a value still holding in 1981, whereas the coefficient of available income develops according to the pattern of .326, .302 and .301 over the three years.

Figure 3.3 Available household income: Pen's parades.

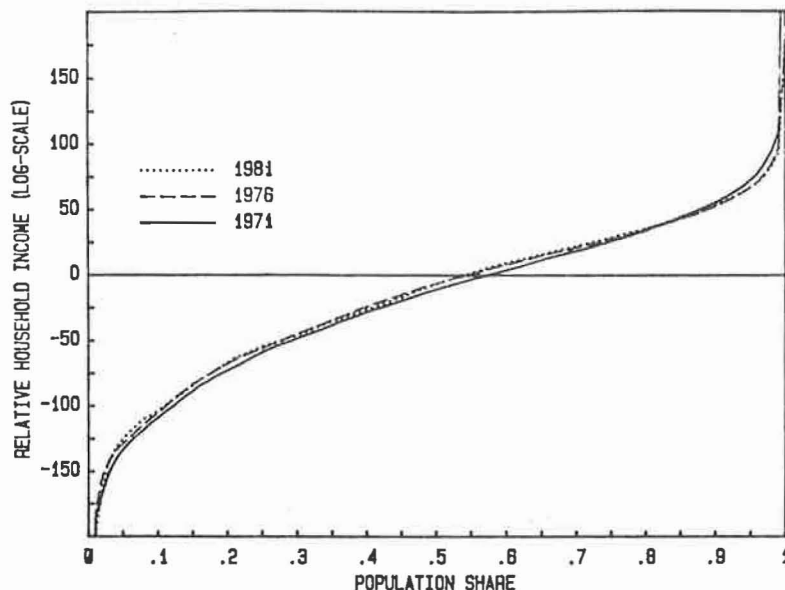


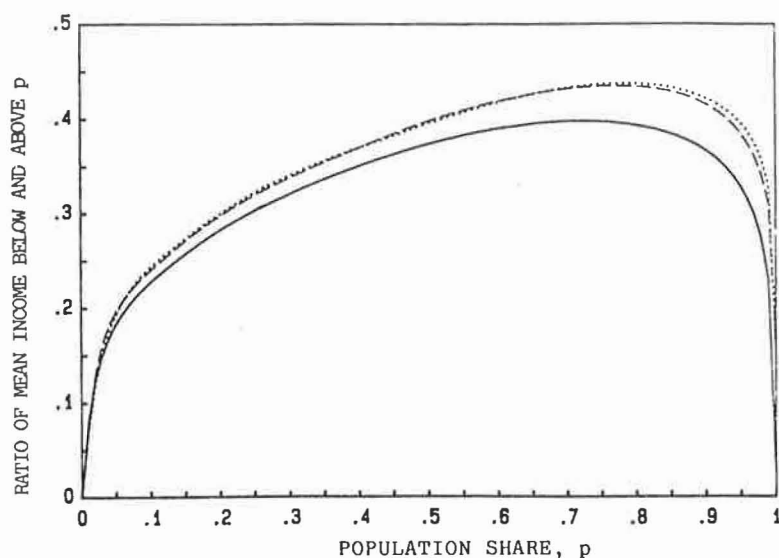
Figure 3.3 outlines the distribution of available income in a diagram similar to Pen's parade (with the relative income, originally suggested by Pen (1971), on the vertical axis replaced by its log-transformation).

When the population 'marches by', starting from the household with the lowest relative income and ending with the highest, we observe that the 1971 parade falls short of both the 1976 and the 1981 parade up to a point when slightly more than 80 per cent of the population have marched by. After this point the 1971 parade exceeds the 1976/81 parades. Obviously, this is again an indication of the reduced inequality between 1971 and the two later years.

As has been seen above, the traditional graphical methods for displaying income distribution data do not succeed in differentiating between the 1976 and 1981 distributions. It is hard to get a summary interpretation of the differences between 1976 and 1981 from the frequency distributions of Figure 3.1b; the Lorenz diagram 3.2b is so 'smooth' that the difference between the 1976 and the 1981 curves disappears visually (this would still be true even if we made the diagram several times larger); and although there is a slight difference between the 1976 and 1981 parades in Figure 3.3, it may be hard to interpret.

To get a visual interpretation of the difference between the 1976 and the 1981 distributions, a graphical device suggested by Aaberge (1982) is presented in Figure 3.4.

Figure 3.4 Available household income: Aaberge's inequality curves.



Similar to the Lorenz diagram and Pen's parade, the horizontal axis denotes population shares, p , ranked according to available income. The vertical axis, in turn, represents the ratio between the mean incomes among households falling to the left and to the right, respectively, of a specific p -value. As can be seen from the figure, the 1976 and 1981 'inequality' curves are situated above the 1971 curve, again implying a reduction in income inequality. Moreover, the 1981 curve is slightly above the 1976 curve for population shares exceeding .80. Hence, the mean income among, for example, the poorest 80 per cent of the households as compared to the mean income among the richest 20 per cent of the households has increased from 1976 to 1981 and in this sense inequality has slightly been reduced.

4. THE EFFECT OF TAXATION

As was seen above, the distribution of household income tended to shift towards higher equality between 1971 and 1976: Measured by the Gini coefficient the inequality of available income was reduced by 7.4 per cent (from .326 to .302), and the inequality of pre-tax income by 6.6 per cent (from .361 to .337). Since the available income of a household is obtained from its pre-tax income by subtracting taxes (and other paid transfers), the drop in the inequality of available income depends on changes in the distribution of pre-tax income and on shifts in effective taxation schemes. Now, the slightly smaller drop in pre-tax income inequality, as compared to available income inequality, suggests that the effective taxation tended to promote equality more in 1976 than in 1971.

This tentative conclusion is supported by a decomposition of the Gini coefficient of the available income according to pre-tax income and paid transfers. The decomposition result is presented in Table 4.1 (decomposition rules are discussed in Nygård and Sandström (1981), and Lerman and Yitzhaki (1985)).

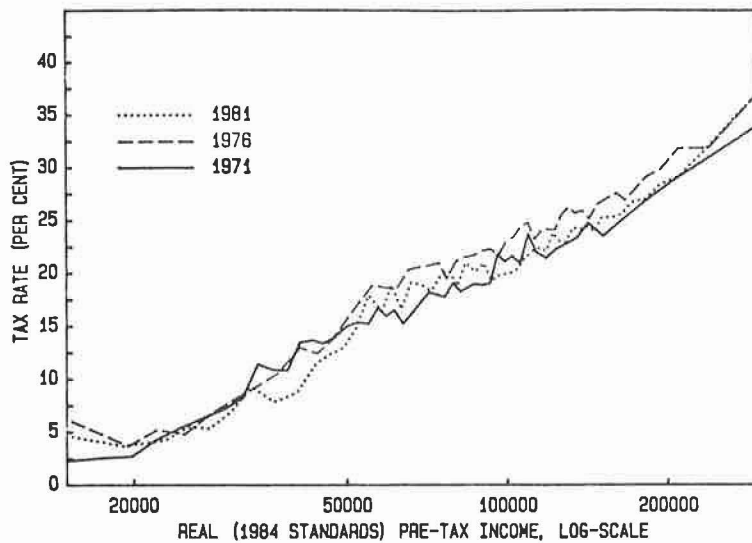
Table 4.1 Decomposition of the Gini coefficient of available income (relative contributions within brackets).

	Gini coefficient	Contribution from pre-tax income paid transfers	
1971	.326 (100.0)	.449 (137.8)	-.123 (-37.8)
1976	.302 (100.0)	.440 (145.9)	-.138 (-45.9)
1981	.301 (100.0)	.431 (143.4)	-.130 (-43.4)

As can be seen from the table, the relative inequality-reducing effect of paid transfers has increased from 37.8 per cent in 1971 to 45.9 per cent in 1976, followed by a slight drop to 43.4 per cent in 1981.

Figure 4.1 outlines the actual tax rates 1971, 1976, and 1981 as a function of real (1984 standards) pre-tax income.

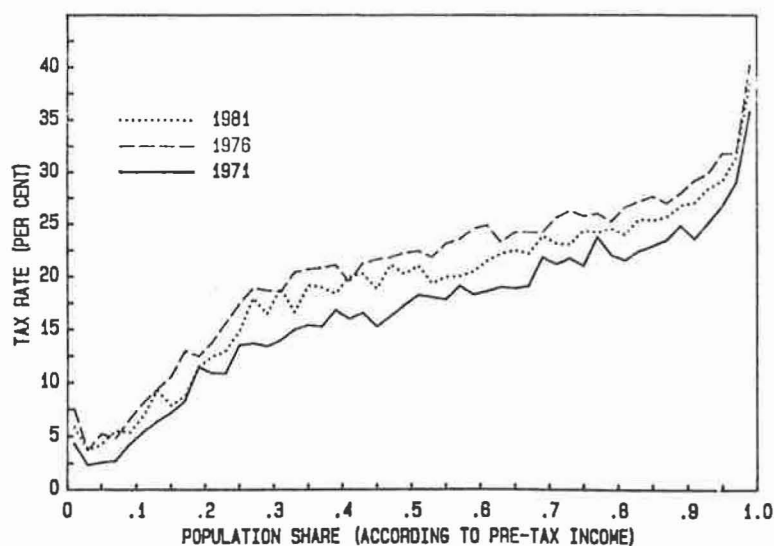
Figure 4.1 Tax rates at different levels of real pre-tax income (1984 standards).



Even if the tax rate profiles appear somewhat 'erratic', partly owing to the influence of sampling errors and partly the fact that the pre-tax incomes on the horizontal axis refer to different households, the tax progressivity has clearly increased from 1971 to 1976: For real pre-tax incomes above 50000 FIM the 1976 tax rate exceeds the 1971 rate by some 3 per cent on the average. On the other hand, from 1976 to 1981 the tax rates showed a tendency to decline, the 1981 situation being

rather close to the one in 1971. Figure 4.2 gives another visual interpretation of the tax rate changes from 1971 to 1981. In this figure the horizontal axis represents population shares, the households being ranked according to pre-tax income.

Figure 4.2 Tax rates for different population quintiles.



According to this representation, the difference between the 1971 and the 1976 situations becomes extremely clear: Due to the increasing real pre-tax income, a growing part of the household population is subject to relatively high tax rates, the average tax rate having increased by some 5 per cent between 1971 and 1976. From 1976 to 1981 taxation had again been alleviated, the 1981 tax profile lying roughly half-way between the 1971 and 1976 profiles.

5. POOLING THE DATA

5.1. Changes in the household population 1971-81

The data base includes, in addition to income amounts, information about some 'background' characteristics of each household. On one hand, there are variables referring to the household as a whole: number of persons belonging to the household, number of children, and number of earners working at least half-time. On the other hand, there is a set of variables characterizing the head of the household: the age, socio-economic group, sex, and education.

Having this information for the years 1971, 1976, and 1981, we may construct household 'cohorts' by a disaggregation of the population according to the age of the household head and pooling the data.

Table 5.1 Some household characteristics 1971, 1976 and 1981 according to age (head of the household).

Age group	Average number of (A) Persons			(B) Half-time earners			(C) Children		
	1971	1976	1981	1971	1976	1981	1971	1976	1981
20-24	2.28	2.08	1.75	1.39	1.20	.96	.52	.40	.22
25-29	3.02	2.72	2.44	1.52	1.45	1.46	1.10	.83	.63
30-34	3.67	3.24	3.03	1.60	1.55	1.53	1.68	1.32	1.18
35-39	4.00	3.69	3.38	1.63	1.64	1.64	1.99	1.68	1.46
40-44	4.04	3.77	3.40	1.80	1.71	1.76	1.83	1.51	1.24
45-49	3.68	3.53	3.10	1.86	1.82	1.77	1.27	1.00	.78
50-54	3.07	2.98	2.75	1.70	1.60	1.66	.77	.56	.39
55-59	2.69	2.54	2.31	1.50	1.38	1.32	.49	.31	.20
60-64	2.21	2.09	1.95	1.01	.88	.83	.19	.14	.07
65-69	1.89	1.72	1.66	.46	.26	.22	.08	.04	.03
70-74	1.79	1.57	1.57	.31	.15	.08	.11	.04	.02
75-79	1.73	1.44	1.52	.30	.09	.06	.05	.01	.02
20-79	3.00	2.80	2.58	1.38	1.29	1.27	.96	.77	.64

Table 5.1 presents one of the first results of the disaggregation. In the table the average number of persons, half-time earners, and children are given for households belonging to different age groups. Relying on the 'representativity' of the CSO-samples (this is indeed a heavy assumption, the average sample size within an age group being some 600-700 households for each year), we may derive household 'cohort' profiles over the years.

Table 5.2 The households 1971, 1976 and 1981 according to age and socio-economic group (head of the household).

Age group	Percentage (A) Farmers			(B) Own-account			(C) White collar		
	1971	1976	1981	1971	1976	1981	1971	1976	1981
20-24	2.0	1.4	1.4	.8	.6	1.1	37.9	34.2	24.7
25-29	4.7	2.7	3.7	2.4	2.9	1.1	39.7	43.2	41.9
30-34	8.7	5.2	5.1	4.8	4.2	4.6	38.6	46.0	46.4
35-39	10.9	7.4	6.5	8.5	4.3	4.6	32.7	44.8	44.2
40-44	16.8	9.5	8.3	9.2	5.2	7.1	25.1	35.8	42.7
45-49	19.2	13.2	10.4	6.2	6.1	6.2	23.7	30.3	38.2
50-54	19.1	13.8	12.6	6.0	6.8	5.6	21.2	25.2	34.5
55-59	20.0	16.0	12.1	5.9	5.3	4.2	18.2	20.6	23.5
60-64	15.8	12.2	8.9	4.0	3.1	5.0	7.7	13.2	14.5
65-69	8.8	5.4	5.0	1.7	1.6	1.2	1.1	1.6	2.2
70-74	6.1	3.8	2.0	1.5	.8	1.0	.4	1.4	.3
75-79	3.0	2.0	.9	.0	.0	.4	.9	.0	.0
20-79	12.3	8.2	6.7	4.8	3.9	3.9	23.0	28.5	30.9

Age group	Percentage (D) Blue collar			(E) Inactive		
	1971	1976	1981	1971	1976	1981
20-24	54.2	52.5	44.2	5.1	11.2	28.6
25-29	50.1	47.7	46.8	3.1	3.5	6.5
30-34	46.1	42.8	39.5	1.8	1.8	4.4
35-39	46.3	41.9	42.5	1.6	1.6	2.3
40-44	44.3	44.1	38.2	4.6	5.3	3.8
45-49	43.1	43.7	39.3	7.8	6.8	5.8
50-54	39.7	40.2	35.2	13.9	14.1	12.0
55-59	33.6	33.1	34.7	22.3	24.9	25.4
60-64	18.6	18.9	15.6	53.8	52.6	55.8
65-69	2.8	3.0	.6	85.5	88.4	91.0
70-74	1.5	.0	.0	90.5	94.0	96.7
75-79	2.2	.0	.0	93.9	98.0	98.7

To illustrate, the households belonging to the age group 20-24 in 1971, will in 1976 be represented by the age group 25-29, and in 1981 by the group 30-34, and looking at the number of half-time earners for this 'cohort' we find a profile of 1.39, 1.45, and 1.53 half-time earners on the average. In Tables 5.2-5.4 corresponding information about socio-economic groups, sex, and education is given.

Some trends are readily observable from the tables: (a) The household size, table 5.1, has grown smaller during the decade of 1971-81, largely due to the smaller number of children.

Table 5.3 The households 1971, 1976 and 1981 according to age and sex (head of the household).

Age group	Percentage with (A) Male head of household			(B) Female head of household		
	1971	1976	1981	1971	1976	1981
20-24	70.7	59.2	54.4	29.3	40.8	45.6
25-29	82.9	79.2	76.5	17.1	20.8	23.5
30-34	84.7	81.7	76.3	15.3	18.3	23.7
35-39	85.6	84.0	78.8	14.4	16.0	21.2
40-44	82.0	77.8	77.3	18.0	22.2	22.7
45-49	81.5	77.9	72.2	18.5	22.1	27.8
50-54	69.4	68.3	68.2	30.6	31.7	31.8
55-59	68.3	64.8	64.2	31.7	35.2	35.8
60-64	65.5	57.2	57.7	34.5	42.8	42.3
65-69	61.4	53.1	51.9	38.6	46.9	48.1
70-74	54.7	45.3	54.3	45.3	54.7	45.7
75-79	46.5	35.5	44.4	53.5	64.5	55.6

(b) Even if earnings activity, Table 5.1, as measured by the number of at least half-time earners seems to have declined on the average, from 1.38 to 1.27 per household, the reverse is true for the younger 'cohorts'.

(c) Regarding the distribution over socio-economic groups, Table 5.2, the heavy reduction in the share of farmer households, and the

corresponding increase in the white collar share, should be noted.

Another significant feature is the growing part of inactive households among the youngest (below 34 years) and oldest (above 65 years) age groups, also reflected in the number of half-time earners within these groups, Table 5.1.

(d) The households headed by females, Table 5.3, has increased.

(e) The population shares with secondary or higher education, Table 5.4, has grown rapidly.

Table 5.4 The households 1971, 1976 and 1981 according to age and education (head of the household).

Age group	Percentage with (A) Primary education			(B) Secondary education			(C) Higher education		
	1971	1976	1981	1971	1976	1981	1971	1976	1981
20-24	48.2	37.3	15.9	46.7	56.9	79.8	5.1	5.8	4.3
25-29	48.3	43.1	18.6	39.3	42.3	63.9	12.5	14.6	17.4
30-34	58.3	45.1	28.9	29.8	39.0	55.3	11.8	15.9	15.8
35-39	64.2	52.2	36.8	25.7	33.5	47.9	10.0	14.3	15.3
40-44	74.2	63.8	46.3	17.1	24.8	37.9	8.7	11.4	15.8
45-49	73.0	72.0	60.3	19.5	20.2	29.9	7.5	7.9	9.8
50-54	78.5	79.3	64.7	15.6	15.8	25.7	5.9	4.8	9.5
55-59	80.4	77.2	74.8	14.9	16.2	19.1	4.7	6.6	6.4
60-64	81.7	82.6	76.0	11.7	13.3	18.3	6.5	4.1	5.6
65-69	82.6	84.4	78.9	11.6	11.2	16.8	5.8	4.3	4.4
70-74	85.1	82.4	78.3	8.7	11.5	17.2	6.2	6.2	4.4
75-79	82.9	89.2	79.4	9.6	7.9	14.1	7.5	2.9	6.5
20-79	70.1	64.8	50.3	22.0	26.0	38.8	7.9	9.3	10.8

Bearing these significant structural changes of the household characteristics in mind, the stability of the relative income differences (cf. Figures 3.1a and 3.1b) turns out as quite startling.

5.2. Household income profiles

Table 5.5 presents the pre-tax income, the available income, and the Gini coefficient of available income 1971, 1976, and 1981 according to age group.

Table 5.5 Pre-tax household income, available household income, and the Gini coefficient 1971, 1976, and 1981 according to age (head of the household).

Age group	(A) Average pre-tax income (1000 FIM)			(B) Average available income (1000 FIM)			(C) Gini coefficient available income		
	1971	1976	1981	1971	1976	1981	1971	1976	1981
20-24	17.1	35.5	46.4	13.7	27.6	37.5	.277	.265	.296
25-29	22.7	49.4	72.8	17.5	36.8	56.6	.248	.219	.225
30-34	25.2	59.1	87.4	19.2	42.6	66.6	.244	.215	.211
35-39	27.4	62.1	98.5	20.9	45.2	73.7	.273	.206	.210
40-44	26.4	62.8	106.1	20.3	46.2	78.9	.256	.228	.222
45-49	26.1	62.7	103.5	20.2	46.2	76.5	.287	.252	.255
50-54	23.3	55.8	96.1	18.1	41.6	71.6	.321	.284	.277
55-59	19.5	50.7	80.7	15.6	37.1	61.2	.325	.317	.304
60-64	16.2	37.7	63.9	13.3	29.3	49.6	.370	.333	.323
65-69	12.0	28.2	45.0	10.5	23.7	41.1	.354	.329	.289
70-74	10.3	23.7	41.1	9.3	19.9	35.4	.348	.313	.284
75-79	11.1	19.2	36.5	9.6	16.9	32.0	.376	.292	.291
20-79	21.0	49.4	79.0	16.6	36.9	60.5	.323	.298	.296

The incomes in the table are given in nominal amounts, to be multiplied by the factors given in Section 2 when transformed to real (1984 standards) income amounts. The resulting real available household income profiles from the three cross-sections are outlined in Figure 5.1.

The cross-section profiles are quite similar in shape, the major difference being a upward drift when passing from 1971 through 1976 to 1981. The similarity in shape is, once again, accentuated by transforming the income amounts to relative incomes in log-scale as in Figure 5.2. below.

Figure 5.1 Cross-section real available household income profiles.

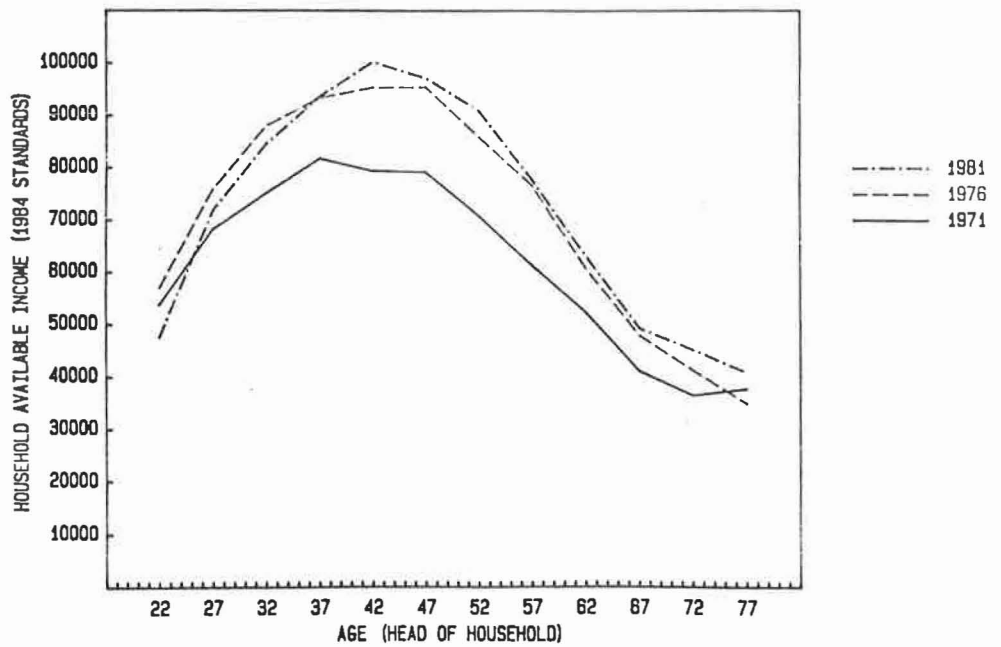
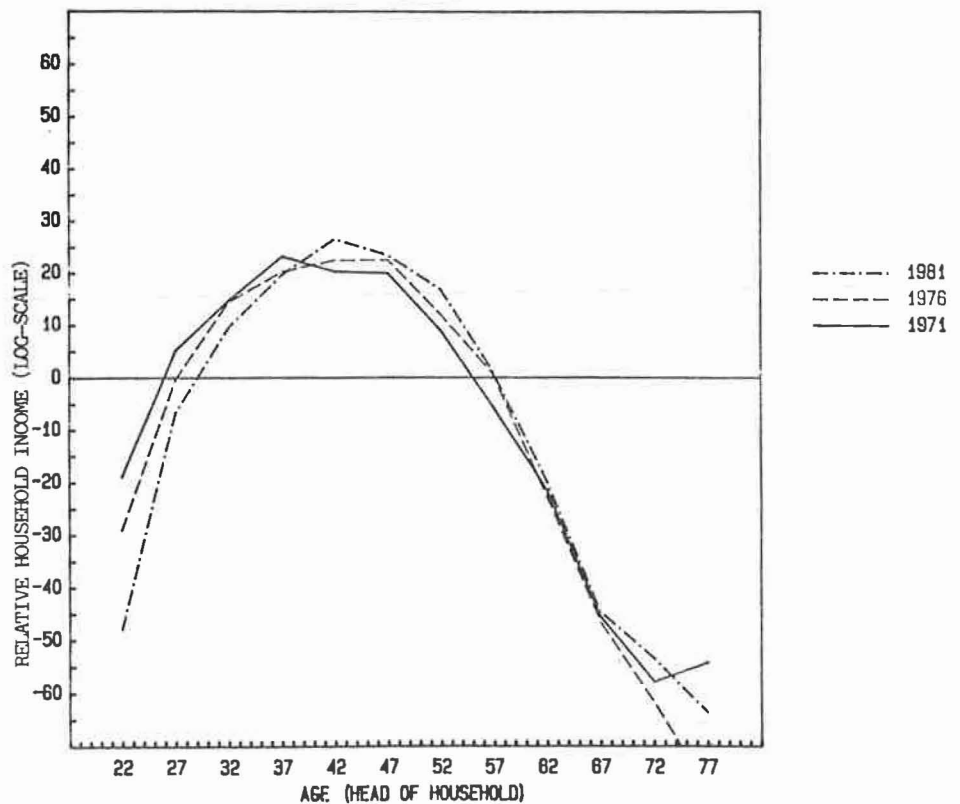


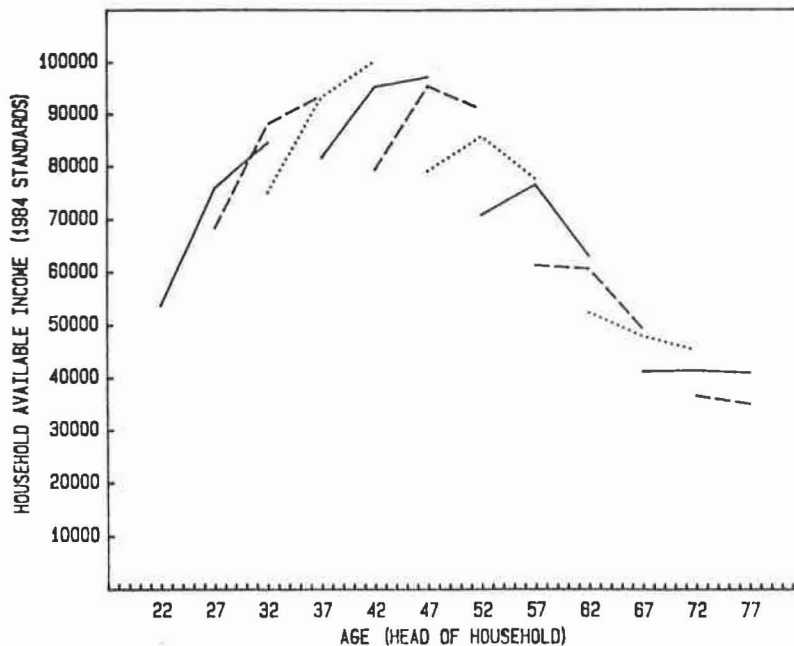
Figure 5.2 Cross-section relative income differences according to age group.



According to this representation the profiles for the years after 1971 start out from a lower relative income in the age group 20-24, this being compensated for by a shift of the peak towards higher ages.

Figure 5.3 outlines the average income profiles for eleven household 'cohorts' over the three years, with the profiles for the youngest 'cohorts' leftmost in the diagram.

Figure 5.3 Available household income (1984 standards): 'cohort' profiles.

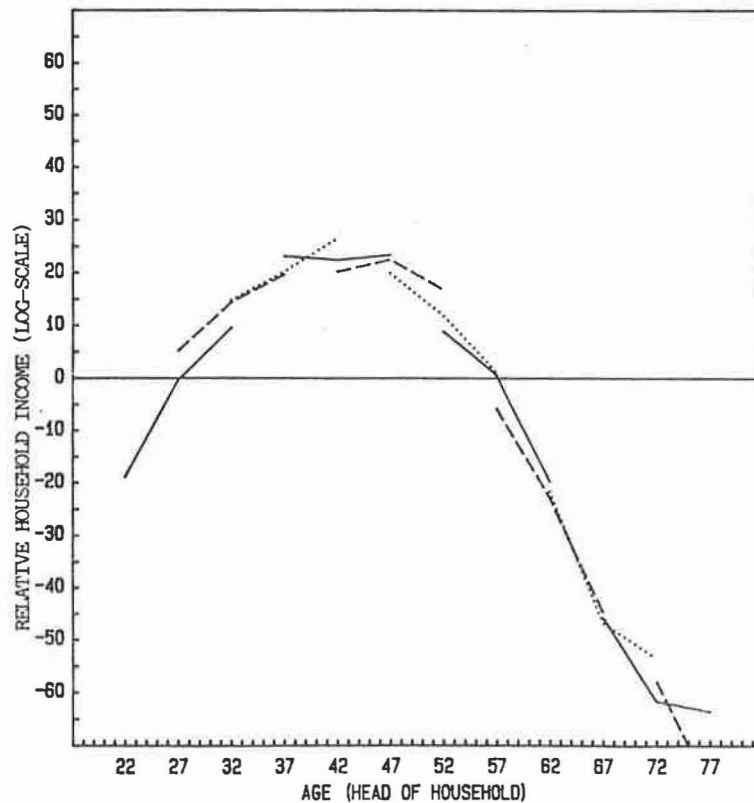


The four youngest 'cohorts', where the head is born between 1932 and 1951 (the age groups 20-24, 25-29, 30-34, and 35-39 in 1971), all display increasing available income profiles, whereas the fifth 'cohort' (the age group 40-44 in 1971) is the youngest showing a reduction in

available household income. The drop in available income between 1976 and 1981 for this 'cohort' may be contrasted with the information in Table 5.1, revealing that the reduced income is associated with a lower earning activity: In the 'cohort' the average number of half-time earners falls from 1.82 in 1976 to 1.66 in 1981.

Figure 5.4, finally, gives the eleven 'cohort' profiles transformed to relative incomes in log-scale. The profiles fall remarkably close to one another, with relative incomes appearing to be reasonably approximated by a quadratic function of age.

Figure 5.4 Relative income differences (log-scale): 'cohort' profiles.



5.3. A pooled cross-section regression

The discussion above indicates that there is a relationship between the 'age' of the household and its income. Similarly, the household income may be related to other 'background' characteristics of the household. Obviously, a descriptive study of these relationships could be based on calculations of average incomes conditional on the household characteristics to obtain a set of cross-tabulations. However, since a detailed analysis requires a large number of cross-tabulations, the results will tend to be 'messy' and awkward to interpret. As a consequence, we will drop detailed cross-tabulations in favour of a regressive type of analysis.

To spell out the adopted regression model in somewhat greater detail, we start with the case of data from one year only. The dependent variable in the analysis will be the relative income (in log-scale), and to this we relate the set of 'background' characteristics as independent variables. The model will be applied to economically active households only, as the socio-economic group 'inactive' may be anticipated to form a case of its own. From the data base we form five independent variables on a ratio scale:

- * Number of at least half-time earners as a proxy for earnings activity,
- * Number of children,
- * Number of non-earning adults, defined as the household size minus the number of half-time earners and children,
- * Age, and age squared, as proxies for seniority.

Moreover, we introduce three sets of dummy variables to take account of

- * Sex (two dummies),
- * Socio-economic group (four dummies), and
- * Education (three dummies).

In this way we obtain fourteen independent variables, and fifteen after introducing an intercept. However, a direct inclusion of the dummies into the regression will bring about singularity. Instead of using the mainstream method to ensure non-singularity, i.e. by forcing one regression coefficient within each dummy set to zero (being the same thing as excluding one dummy from each set), for ease of interpretation we rely on the method suggested by Klevmarken (1972) and restrict the problem by requiring the sum of regression coefficients within each dummy set to equal zero. For instance, we will include both a male and a female dummy under the restriction that the corresponding regression coefficients add up to zero.

Even if there are some indications of interactions between the independent variables, no interaction terms will be included in this analysis.

The effect of pooling the data over the three years could, in principle, be handled by defining a fourth set of dummy variables referring to the years. But, again for reasons of ease of interpretation, we prefer to introduce the effect of the three years in a way similar to the treatment of interventions in time series analysis. To be more specific, we define three 'time' variates, z_1 , z_2 , and z_3 , by

$$z_1 = 0 \text{ if year} < 1971,$$

$$1 \text{ if year} \geq 1971,$$

$$z_2 = 0 \text{ if year} < 1976,$$

$$1 \text{ if year} \geq 1976,$$

$$z_3 = 0 \text{ if year} < 1981,$$

$$1 \text{ if year} \geq 1981,$$

and replace each of the fifteen independent variables in the one-year model above with three variables, obtained by multiplying the 'original' independent variable with z_1 , z_2 , and z_3 , respectively. In this way we get a final regression model with 45 independent variables, in which each 'original' variable is associated with three regression coefficients. The first of these coefficients may be interpreted as a 'base' coefficient referring to the year 1971, the second represents an additional component which added to the 1971 base gives the 1976 regression coefficient, and the third a second additional component which added to the 1971 and 1976 terms gives the 1981 regression coefficient. Or, to put it in another way, the second of the three coefficients represents the change from 1971 to 1976, and the third the change from 1976 to 1981.

Table 5.6 gives the result of the 'descriptive' regression analysis, being based on a total of 18932 households, with the relative pre-tax income (log-scale) as the dependent variable and traditional OLS-estimation of parameters (using IMSL library subroutines).

Bearing the rather crude quality of the independent variables in mind, the model succeeds surprisingly well, as measured by R^2 , in explaining relative income differences. Regarding the interpretation of the results, it should be noted that the regression coefficients may roughly be thought of as the percentage contribution of each variable to the relative household income. A closer inspection of the results gives rise to the following tentative conclusions:

Half-time earners The influence of earnings activity on relative household income seems to have increased over time, the regression coefficient for the number of half-time earners growing from 36.07 in 1971 to 41.14 in 1981.

Table 5.6 Coefficient estimates of the pooled cross-section regression (t-values within brackets).

N = 18932 households

R² = .539

	1971	1976	1981
	Base	First addition	Second addition
Intercept	-178.42 (-35.95)	5.17 (.70)	-46.59 (-5.20)
Household characteristics			
Number of half-time earners	36.07 (54.74)	2.42 (2.41)	2.65 (2.40)
Number of other adults	17.54 (25.10)	.92 (.91)	-.51 (-.48)
Number of children	3.10 (8.23)	1.13 (1.87)	1.45 (2.05)
Head of household characteristics			
Seniority: Age	4.51 (17.80)	-.78 (-1.93)	1.72 (3.74)
Age ²	-.051 (-16.88)	.011 (2.28)	-.016 (-2.95)
Sex: Male	14.75 (22.78)	-3.02 (-3.30)	-1.66 (-1.78)
Female	-14.75 (-22.78)	3.02 (3.30)	1.66 (1.78)
Education: Primary	-24.71 (-30.50)	8.21 (6.94)	2.00 (1.60)
Secondary	-10.59 (-12.97)	4.04 (3.50)	2.84 (2.53)
Higher	35.50 (30.34)	-12.25 (-7.45)	-4.84 (-3.01)
Socio-economic group: Farmer	-37.40 (-34.91)	3.16 (1.85)	7.76 (3.97)
Own-account	5.28 (3.80)	1.97 (.91)	-3.28 (-1.38)
White collar	26.37 (27.56)	-4.91 (-3.46)	-3.83 (-2.55)
Blue collar	5.76 (7.37)	-.21 (-.18)	-.65 (-.50)

Inactive adults No significant trend over time.Children The number of children seems slightly to have increased its effect on relative household income.

Seniority The changes in the regression coefficients for age and age squared are all significant, implying a shift of the relative income peak towards higher ages. The estimated income peak in 1971 corresponds to an age of 44.2 years, in 1976 to 46.6 years, and in 1981 to 48.7 years (cf. Figure 5.4).

Socio-economic group In the base situation of 1971 the farmer households have the lowest relative income, and the white collar households the highest. Towards 1981 the farmers improve their relative income position at the 'expense' of white collar households.

Sex The relative income differences between households headed by females and males have significantly decreased over time.

Education Both the primary and the secondary education group have improved their relative income position at the 'expense' of higher education.

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LIFETIME INCOMES IN FINLAND - DESK CALCULATIONS BASED ON CIVIL SERVANT SALARIES 1985

by Fredrik Nygård

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1. INTRODUCTION

Any discussion of the income distribution among persons is crucially dependent on three definitional issues, i.e.

- (a) the definition of the 'income' concept itself, or which types of compensation/remuneration in cash and/or in kind should be regarded as 'income'?
- (b) the selection of an appropriate income recipient unit, and
- (c) the time period over which incomes are recorded.

Even if these issues are closely interrelated (cf. Nygård and Sandström [1981]), this paper seeks to illustrate the importance of point (c) above, or how the income distribution responds to changes in the time horizon.

In this context it should first, as a preliminary remark, be noted that the mainstream approach to both the theoretical measurement and empirical studies of income distributions tends to consider annual income as an appropriate income magnitude. Although this approach may seem reasonable to the practitioner - incomes are usually recorded on an annual basis due to taxation practice - it is open to objections from a more fundamental point of view, taking life-cycle considerations into account and distinguishing between transitory and permanent incomes.

Life-cycle aspects have been stressed by e.g. Paglin [1975], Lillard and Willis [1978], Weizsacker [1978], and Rosen [1984], and it is easily realized that, due to intertemporal variations in the income flow, the distribution of current incomes may differ substantially from the distribution of lifetime incomes.

Similarly, measures of income inequality operating on an annual basis may give misleading indications of the 'true' inequality, particularly in cases when the major source of interpersonal income variation is highly transitory. In fact, for a large group of income inequality measures it has been established (cf. Shorrocks [1978]) that the income inequality calculated on a 'lifetime' basis can never exceed a weighted average of the constituent 'annual' measures of inequality.

To place life-cycle aspects and the notion of income mobility on an empirical footing involves some difficulties. In broad terms, we may rely on one of the following three approaches. First, when genuine panel data on incomes is available, empirical estimates of lifetime incomes and/or income mobility patterns (quite similar to Markov transition matrices) may be calculated. Yet, the resulting figures relate to past history, and their significance and implications in the current situation may be elusive.

Second, it may be possible to derive life-cycle patterns from a set of cross-section data using some 'correction' technique, e.g. concentration curves and/or regression methods. This approach suffers from the evident drawback that the true underlying life-cycle patterns may be confounded by ill-conditioned 'corrections', a possibility hard to guard against in practice.

Third, the implications of income mobility may be illustrated by looking at 'representative' income recipients and their lifetime income streams. In the present paper, this approach will be applied to Finnish data.

Even if the adopted framework is rather rigid and over-simplified, it may still serve to illustrate the different implications of the 'annual' and the 'lifetime' approaches to income differences.

2. DESK CALCULATION SETUP

As noted above, the discussion in this paper will be based on the idea of 'representative' income recipients. To spell out the framework in somewhat greater detail, we will treat lifetime incomes from the perspective of a young man (Homo Economicus), 18 years old, who after completing secondary school considers whether he should (a) join the labor force, or (b) invest in further training for several years to qualify for a job with a higher salary.

For ease of exposition we further suppose that the young man's decision set is restricted to appointments as a Finnish civil servant - a brief review of the present salary agreements is given in Appendix A1 - and that only three alternative income careers are at issue. The first career requires no further training and pays at salary grade A5 (corresponding to an appointment as e.g. a messenger or caretaker). Qualifying for the second appointment, paying at grade A15 (as e.g. assistant accountant) requires three more years of education, while the third job, paying at grade A20 (as e.g. accountant or lawyer), would require seven additional years of schooling.

Moreover, your young man treats his choice as definitive, so that once he has reached a career choice he will hold this appointment until retirement at age 63, anticipating that he will die shortly before his

72nd birthday. Taking the situation of 1985 as the point of departure, with effective salary agreements and tax amounts as given in Appendix A, we may now try to calculate the lifetime payoffs of the optional career choices. This will be done in the following section.

3. LIFETIME INCOMES

3.1. Income streams

Obviously, the calculations of lifetime incomes must rest on assumptions about the future salary level paid at different grades and other possible changes in the salary agreements. Furthermore, if we wish to distinguish between gross and net income, this requires knowledge of future taxation schemes.

For simplicity we suppose that the young man expects the 1985 conditions to remain in effect throughout his lifetime, implying that he assumes away inflation and ignores all future changes in salary agreements and taxation schemes. From a real income point of view, this amounts to the same thing as imposing full inflationary corrections to salary levels and taxation schemes.

Under these conditions, the A5 career initially pays an annual gross salary of 48 730 FIM (net 35 990 FIM), the A15 salary at age 21 amounts to 62 690 FIM (43 380 FIM), while the A20 career starts off at age 25 with a salary of 80 070 FIM (52 380 FIM). The corresponding final salaries prior to retirement at age 63 are 64 830 FIM (44 480 FIM), 83 400 FIM (54 000 FIM), and 106 510 FIM (64 850 FIM), respectively. A broad outline of the yearly gross and net incomes applying to the three

Figure 3.1. Gross and net annual income according to age: Salary grades A5, A15, and A20.

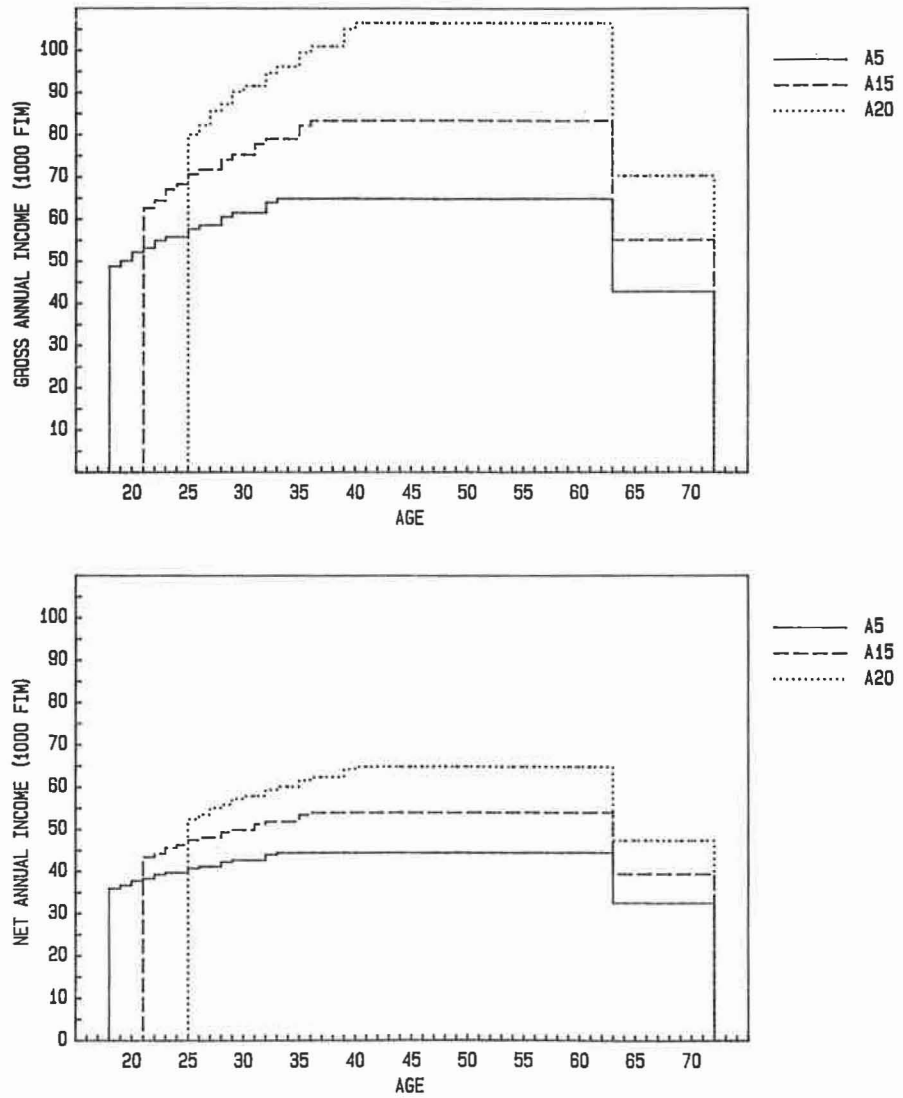
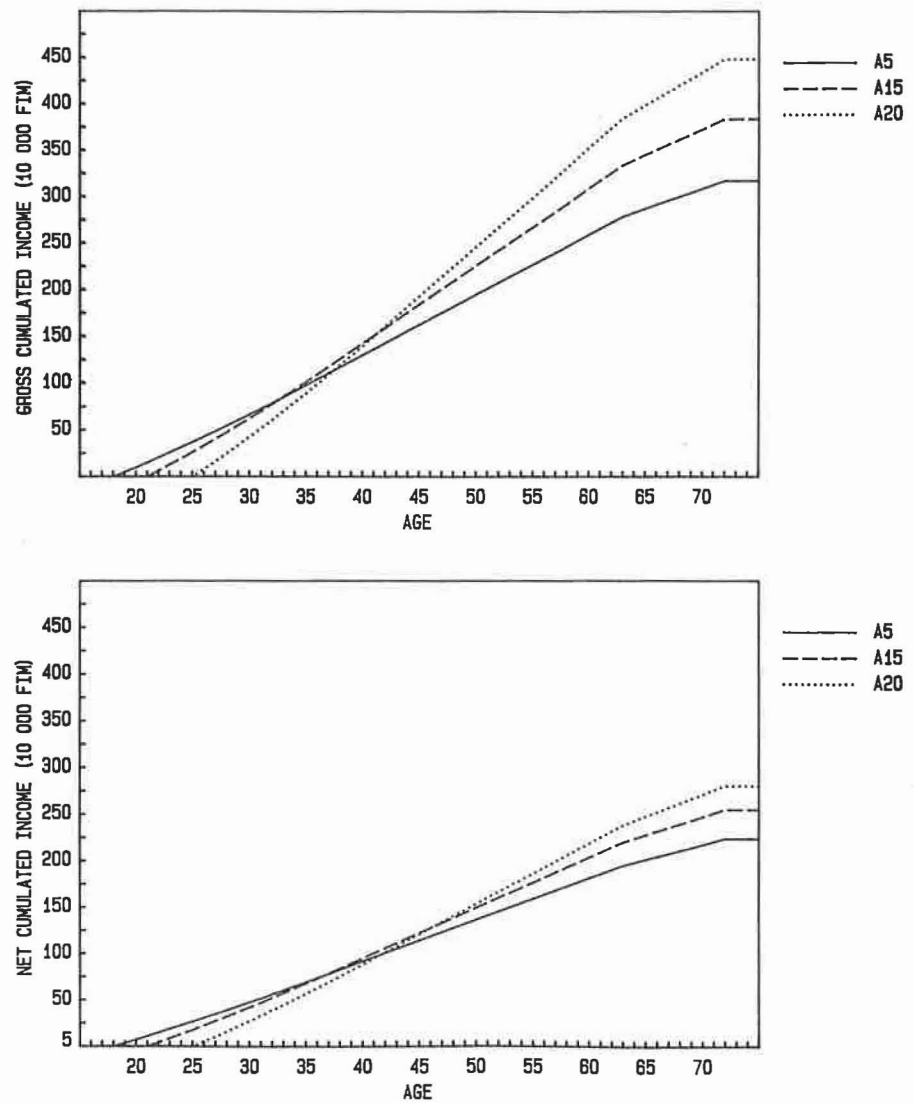


Figure 3.2. Cumulated gross and net incomes: Salary grades A5, A15 and A20.



careers at different ages is given in Figure 3.1. The consequences of tax progressivity should be noted: The differences in annual income between the careers are reduced by roughly one half when passing from gross to net income.

Figure 3.2, in turn, presents the result of cumulating annual incomes over the lifetime. The civil servant becomes a gross income millionaire at the age of 34-36 (depending on his salary grade), and a net income millionaire when he is 40-41 years old. The accumulated net income profiles intersect at age 36 (when the A15 servant catches up with the A5), at 41 years (the A20 career catching up with the A5), and at age 46 (the A20 servant having for the first time accumulated net earnings in excess of the A15).

According to the A5 pattern, the gross lifetime income amounts to 3.18 million, the A15 lifetime income to 3.85 million, and the A20 career results in a gross income of 4.49 million. The corresponding net pay-offs are 2.23, 2.54, and 2.80 million, respectively. According to this result, a rational young man should choose the A20 career, requiring seven more years of education, in order to maximize his lifetime income. Yet, the consequences of taxation should again be noted: The A20 gross lifetime income exceeds the A5 income by some 41 per cent, whereas the corresponding net incomes differ by only about 25 per cent.

Some supplementary taxation aspects are presented in Table 3.1, giving actual minimum, maximum, and average tax rates for the three careers. The minimum rate corresponds to the annual starting salary of each career, the maximum rate to the final salary prior to retirement, and the average rate, in turn, is derived as the ratio of lifetime taxes to lifetime gross income.

As can be seen from the table, preferring the A15 career to the A5, or the A20 to the A15, results in an increase by some 4 per cent of the actual tax rates.

Table 3.1. Actual and hypothetical tax rates for the careers A5, A15, and A20.

CAREER	TAX RATES (per cent)				
	Actual			Hypothetical	
	Min	Max	Life-time	Option 1	Option 2
A5	26.1	31.4	29.9	29.9	29.7
A15	30.8	35.3	33.9	33.7	32.9
A20	34.6	39.1	37.6	36.8	35.2

In addition, Table 3.1 includes lifetime tax rates corresponding to two hypothetical revisions of the Finnish taxation scheme.

Option 1 is related to a recent discussion in Finland about possible alleviations of the marginal tax rate for medium and high incomes. To be more precise, the lifetime tax rates under this option are derived by splitting the present state taxation scheme (cf. Appendix A2) into two parts: for annual incomes below 47 000 FIM no changes of present taxation are assumed, whereas the taxation of higher incomes (exceeding 47 000 FIM) is revised by setting the marginal state tax rate at 29 per cent. As can be seen from the table, the effect of such a revision is quite moderate. To the A5 servant, who falls below the 'critical' annual taxation level of 47 000 FIM, the revision would have no consequences at all, whereas the A20 servant would experience a modest decrease in his lifetime tax rate from 37.6 to 36.8 per cent (corresponding to a net income increase of some 34 000 FIM).

Option 2, in turn, illustrates the fact that annual tax progressivity may be accentuated during the lifetime as the higher paid careers are associated with no-income years during schooling. The calculations under this option are derived by shifting the tax base from annual to lifetime incomes, using the accumulated income during the active ages of 18-71 as an appropriate lifetime tax base and adopting a corresponding 54-fold adjustment of the 1985 taxation scheme for lifetime tax calculations.

As can be seen from the table, shifts of the tax base from annual towards lifetime incomes (option 2) may well have more marked consequences than simple revisions of the annual tax rates (option 1).

3.2. Discounted lifetime incomes

In the calculations of lifetime incomes in Section 3.1 the separate annual incomes are all treated as equally important, implying that the choice of career does not depend on how a given income is distributed over the life span. Several objections may be raised against this procedure, stressing the fact that it is preferable to obtain income now rather than to wait a few years for the same income amount. A frequently used method to take this type of consideration into account is to discount the annual earnings to their present value, or in our case to their value at age 18.

In Figure 3.3 and Figure 3.4 the gross and net annual salaries, discounted at 2 and 4 per cent, respectively, are depicted. To the young man the discounted annual incomes represent the amount of prospective

Figure 3.3. Gross and net annual incomes, discounted at 2 per cent:
Salary grades A5, A15, and A20.

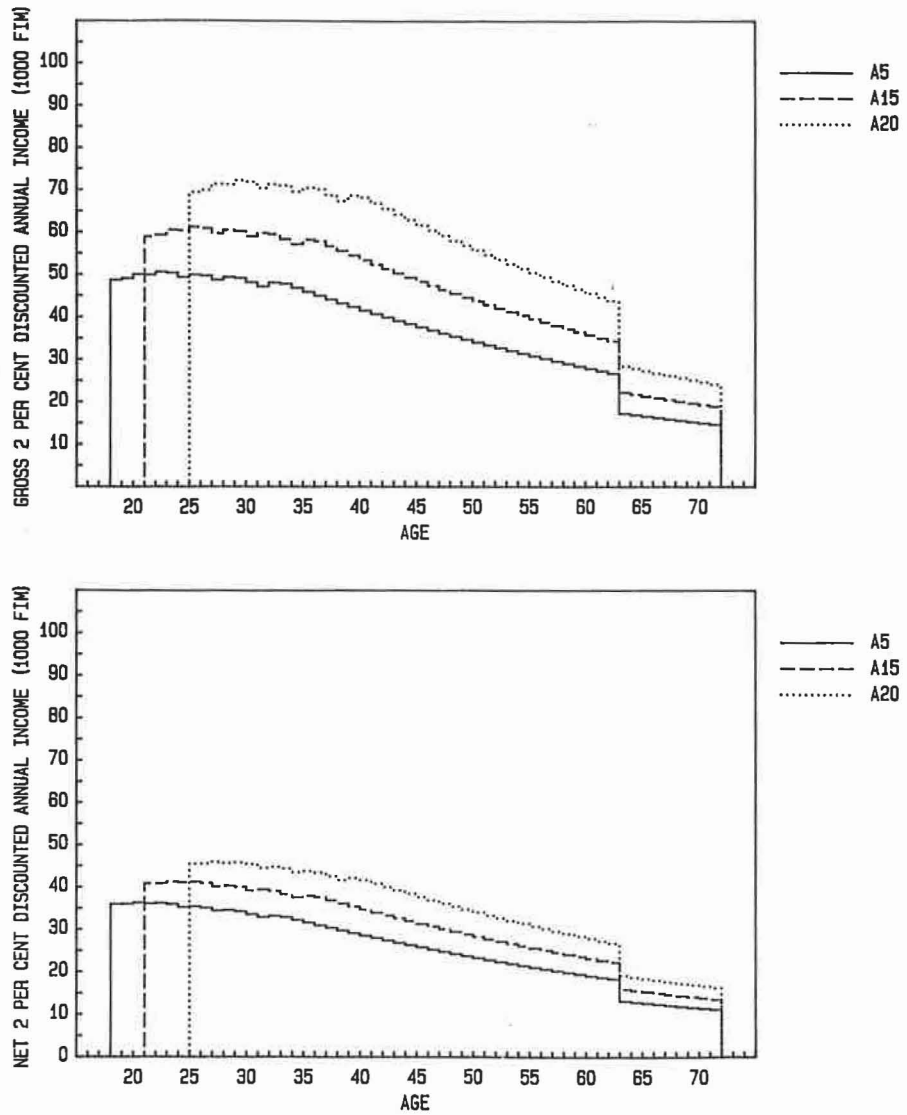


Figure 3.4. Gross and net annual incomes, discounted at 4 per cent:
Salary grades A5, A15, and A20.

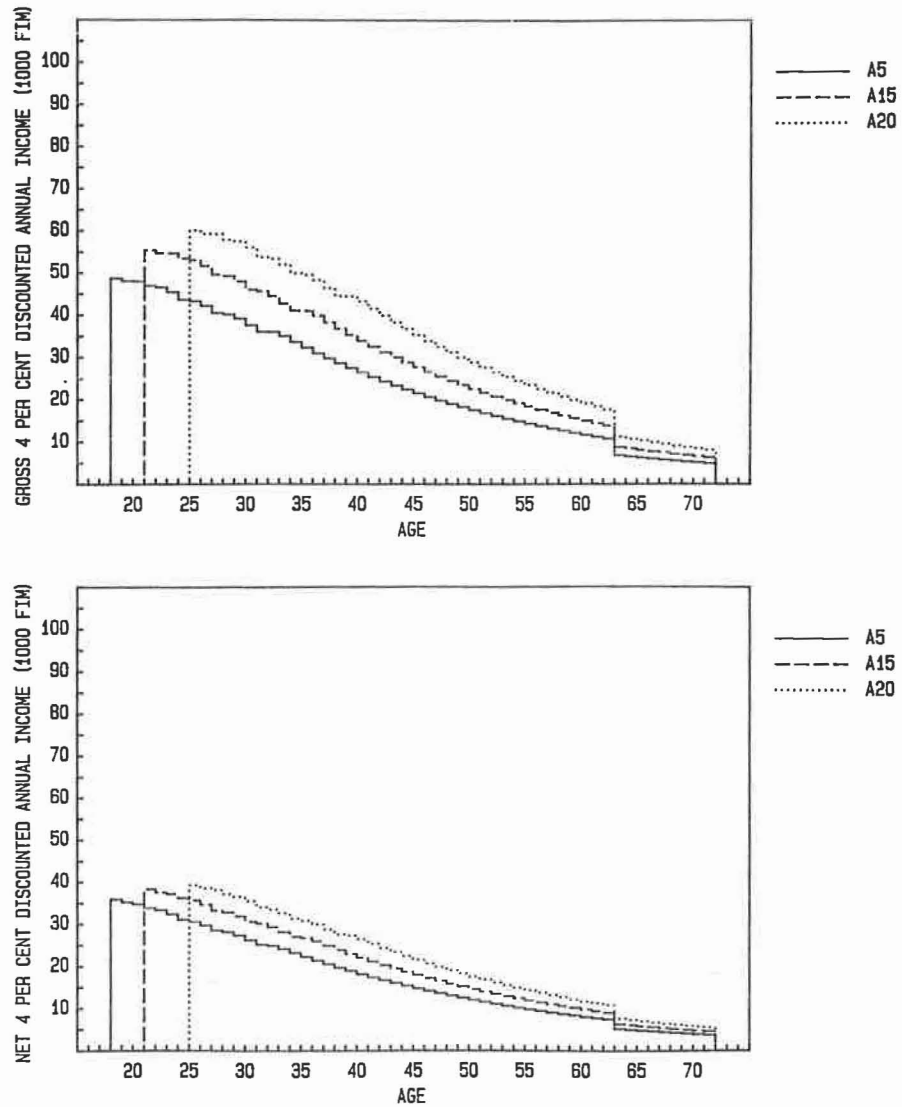


Figure 3.5. Cumulated gross and net incomes, discounted at 2 per cent: Salary grades A5, A15, and A20.

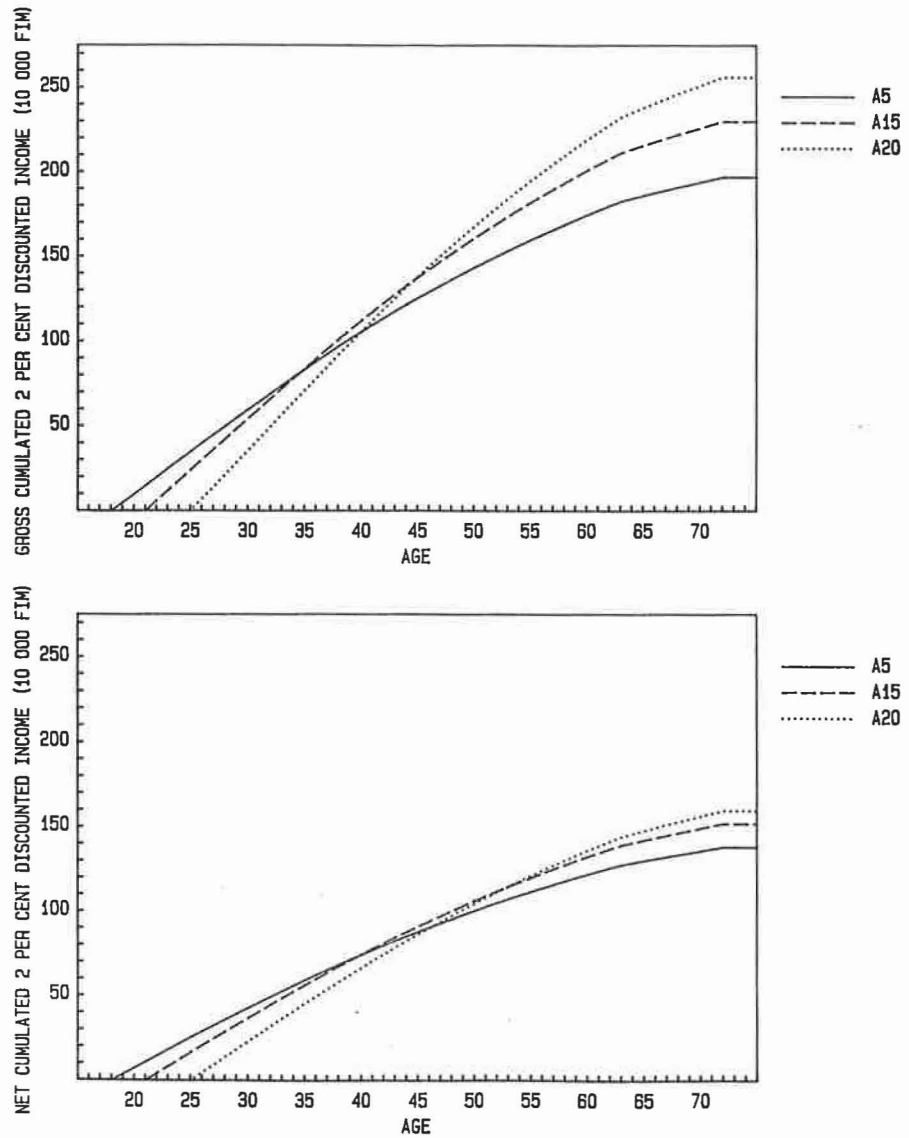
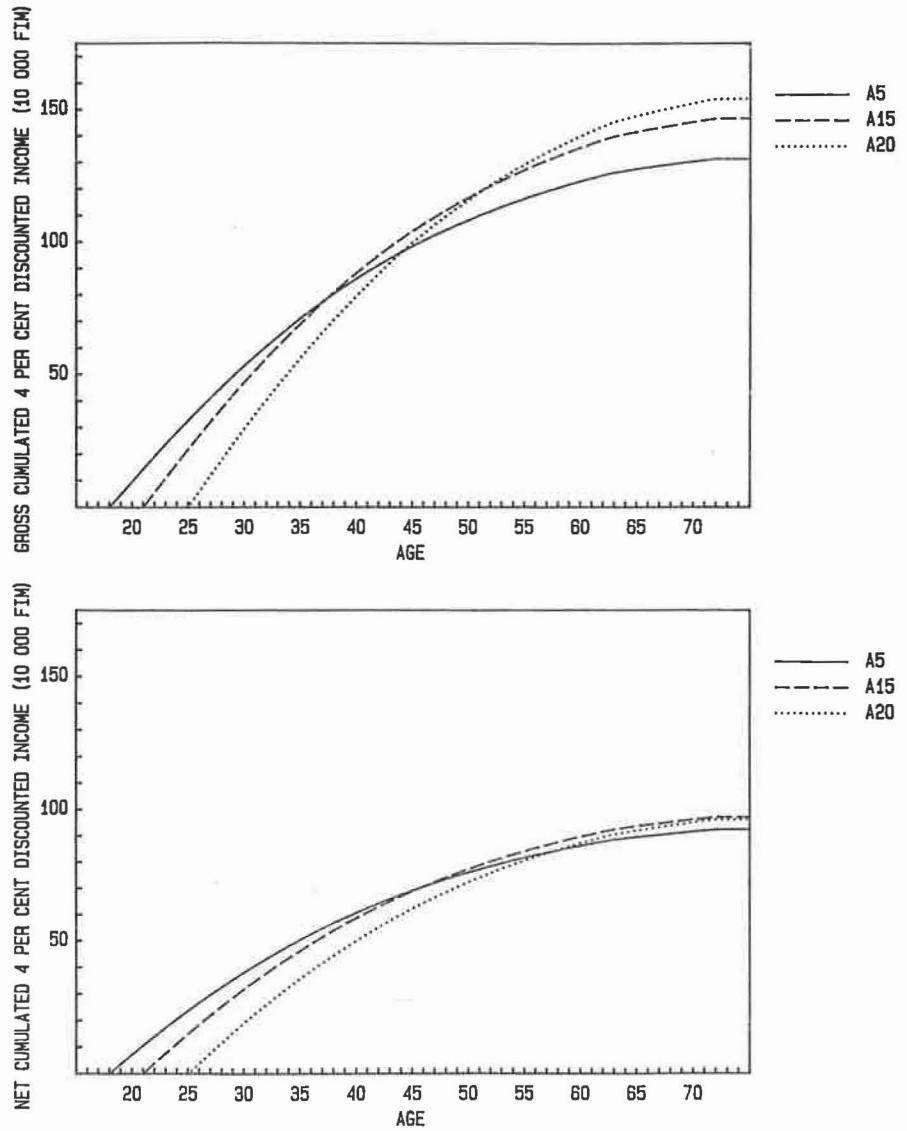


Figure 3.6. Cumulated gross and net incomes, discounted at 4 per cent:
Salary grades A5, A15, and A20.



salaries that he would be in control of at age 18 if he were able to find a creditor willing to borrow money at a real interest rate of 2 or 4 per cent with the servant's future earnings as security.

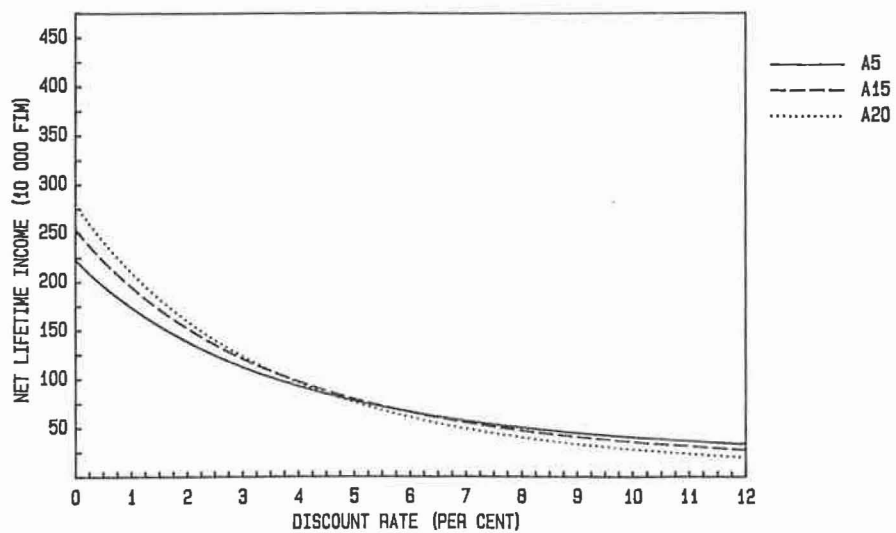
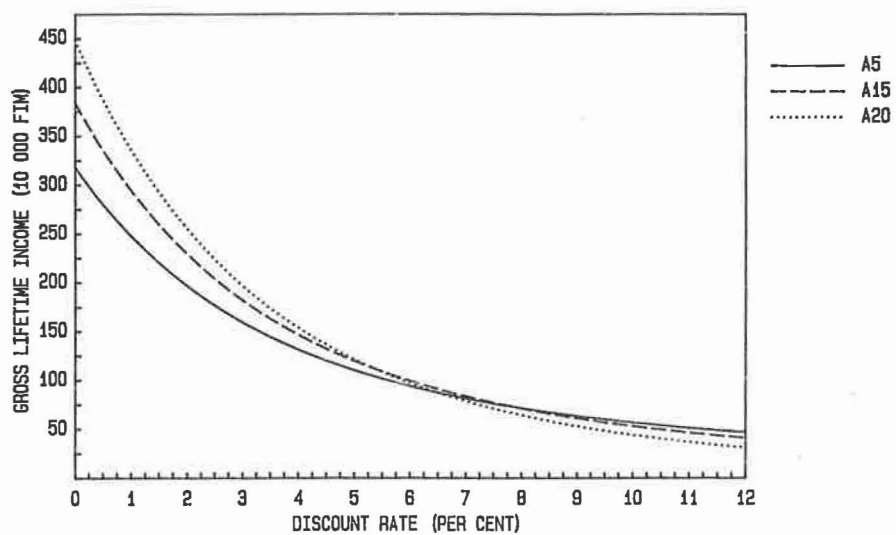
In Figures 3.5 and 3.6 the corresponding cumulated discounted incomes are presented, and in Table 3.2 a summary of the gross and net lifetime incomes at different discount rates is given. As can be seen from the figures and the table, the differences in lifetime income between the careers may largely be reduced by discounting. For instance, when comparing the A5 and A20 careers, the 41 per cent advantage of the A20 nominal gross lifetime income is reduced to a mere 4 per cent advantage when looking at the net lifetime income, discounted at 4 per cent. And, perhaps more remarkable, at a discount rate of 4 per cent, the A15 career beats the A20 when it comes to net lifetime earnings.

Table 3.2. Gross and net lifetime incomes, discounted at 0, 2, and 4 per cent (the numerals in brackets relate the income levels to the corresponding A5 value).

Career	Discount rate	Lifetime income, 100000 FIM	
		Gross	Net
A5	0 %	31.84 (100)	22.32 (100)
	2 %	19.71 (100)	13.80 (100)
	4 %	13.16 (100)	9.23 (100)
A15	0 %	38.46 (121)	25.45 (114)
	2 %	22.98 (117)	15.19 (110)
	4 %	14.67 (111)	9.70 (105)
A20	0 %	44.86 (141)	28.00 (125)
	2 %	25.59 (130)	15.95 (116)
	4 %	15.41 (117)	9.61 (104)

Obviously, the crucial element within this context is the discount rate: If the annual incomes are discounted at an increasing rate, the

Figure 3.7. Gross and net lifetime incomes at different discount rates:
Salary grades A5, A15, and A20.



A5 career - having an initial lead of 3 (7) years compared to the A15 (A20) - will eventually turn out as the one having the highest payoff. In Figure 3.7 this fact is illustrated, by plotting discounted gross and net lifetime incomes against discount rates in the range of 0 to 12 per cent.

Looking at the net income profiles in the lower half of the figure, it may be seen that if our young man bases his career choice on discounted net incomes, then the A20 career should be preferred at discount rates below 3.7 per cent, the A15 at rates between 3.8 and 5.8 per cent, and, finally, the A5 at rates above 5.9 per cent.

4. IMPLICATIONS FOR INCOME INEQUALITY

As noted in the introduction the notion of income inequality is crucially dependent on the accounting period. In this section this fact is illustrated by applying the well-known Gini coefficient as a measure of inequality to the income figures from Section 3.

To simplify the ideas we start out with a model society in which the individuals belong to one of three subsets, each with an equal number of persons and with a uniform age distribution. The members of the first subset are assumed to be predestined for the A5 career, the members of the second for the A15 career, while the third subset will choose the A20 career.

Hence, at any given point of time the income distribution in the three-subset society will be a simple mixture of the income levels

illustrated in Figure 3.1. Treating individuals presently investing in further education (the A15 age group 18-20 and the A20 age group 18-24) as having zero incomes, any annual 'snap-shot' will imply a Gini coefficient of .192 (gross income) or .161 (net income) for the age groups of 18-71. In Table 4.1 the annual Gini coefficients within the A5, A15 and A20 subsets are given. The table also includes the annual Gini coefficient within the merged society, plus coefficients calculated on the basis of lifetime incomes, discounted at 0, 2, and 4 per cent.

The annual income inequality within the isolated population subsets reveals an intrinsic feature: The inequality within the A20 subset exceeds the income inequality among the A15 servants, which in turn exceeds the Gini coefficient within the A5 group. Yet, when individual incomes are accumulated over more than one year, the corresponding 'within subset' Gini coefficients will tend to decrease and at the limit, accumulating over the life span, the individuals reach their lifetime income and the Gini coefficient within each subset reduces to zero.

Table 4.1 Gini coefficients for annual and lifetime incomes in the three-subset society

INCOME CONCEPT	GINI COEFFICIENT	
	Gross income	Net income
Annual		
A5 subset	.070	.054
A15 subset	.124	.109
A20 subset	.196	.181
Merged	.192	.161
Lifetime		
0 % discount rate	.075	.050
2 % discount rate	.057	.032
4 % discount rate	.035	.011

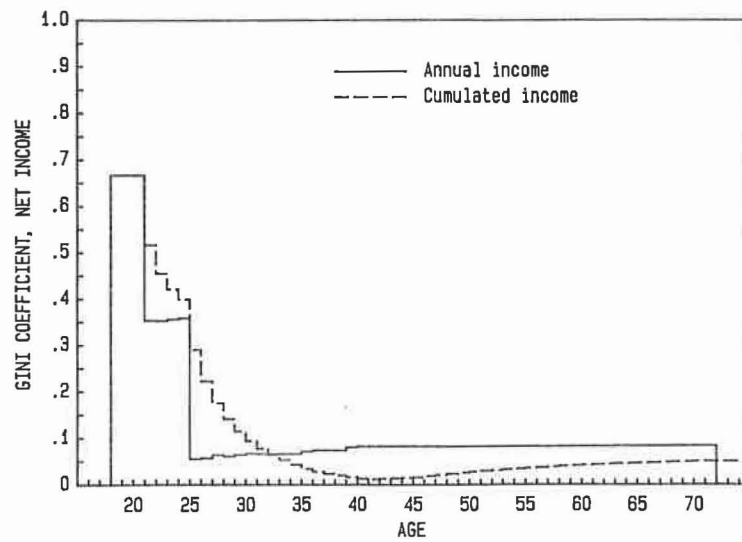
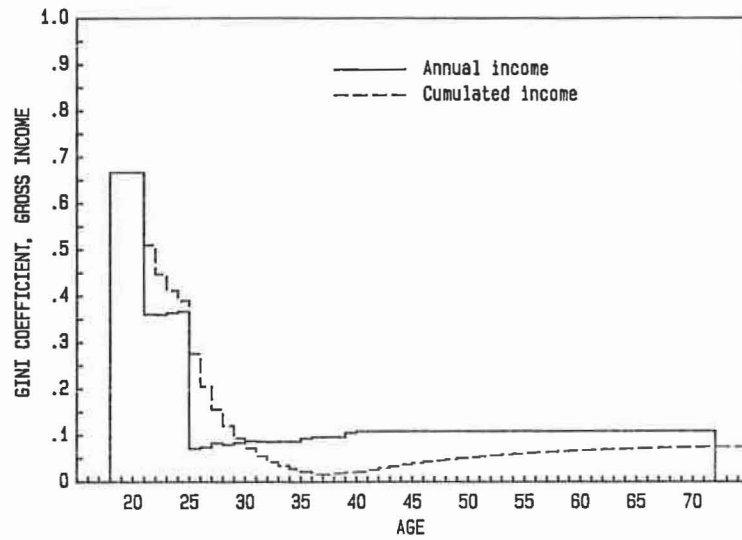
In our model society, a similar pattern emerges when taking the perspective of an age cohort, presently 18 years old, and examining the distribution of its prospective earnings. The fact that one-third (the A5 part) of the cohort enters the labor market straight away, while the remaining two-thirds choose further education for 3 or 7 years, is directly reflected by the annual Gini coefficient.

In Figure 4.1 the cohort's inequality profile is sketched in the case of yearly and cumulated gross and net earnings. The inequality of annual incomes shows two remarkable drops: At age 21, when the A15 group has completed its schooling, the initial gross (net) income Gini coefficient of .667 falls to .361 (.354), followed by a second drop to .072 (.055) at age 25, when the A20 part of the cohort joins the labor force. The accumulated income inequality develops in a quite similar fashion: Starting from a level of .667 the Gini coefficient of gross (net) cumulated income decreases to a minimum of .016 (.011) at age 37 (41), and then slowly increases to reach the final lifetime value of .075 (.050), given in Table 4.1. Accumulating discounted incomes would lead to even larger inequality reductions over the lifetime, as shown in Table 4.1.

5. CONCLUSIONS AND FINAL REMARKS

The simple lesson to be learned from the desk calculations is that although focusing only on annual incomes may be the proper procedure for comparing incomes in the short run, it may badly distort comparisons of economic welfare when the time horizon is placed further off. This fact is perhaps best illustrated by the calcula-

Figure 4.1 Gini coefficients at different ages in the model society:
Gross and net income



tions in Section 4, where e.g. the A20 subgroup in the model society 'suffers' from an annual net income Gini coefficient of .181. Yet, by assumption, the members of the A20 group are, in the long run, equally well off. Looking at the calculated lifetime incomes, issues associated with the choice between further education and straightaway labor market entrance should not be overlooked: Although schooling is expected to pay off by generating higher annual salaries, the lifetime payoff crucially depends on the spell of further education, with the associated low (zero) incomes, and on the anticipated discount rate.

Obviously, several objections may be raised against the over-simplified framework adopted in this paper. Objections concerning details of our basic setup (Sections 2 and 3) could well be taken into account without fundamentally changing the approach. For instance, treating individuals investing in further education as having zero incomes seems to be at odds with their behaviour in the real world. All the same, introducing a modest non-zero income amount during education would not overload the framework.

In addition, we could readily abandon the rigid assumption of a definitive choice of salary grade to allow for movements from one grade to another during the career, relying on real 'representative' career developments (as in Zettermark [1983]) or using probability transition matrixes. Similarly, we may relax the assumption of fixed 1985 conditions during the lifetime by specifying a probability distribution for possible changes in the salary agreements, taxation schemes, and inflation rates. With these modifications, the annual real income would be a random variate, and the lifetime payoffs of optional careers could be evaluated by calculating their expected values (and variances if we

wish to take the possibility of risk evasion into account). However, besides making the analysis more complex, this would hardly add anything essentially new to the picture.

On the other hand, it must also be realized that the adopted approach, placing its focus on purely economic (income) matters, may be doubted for more fundamental reasons. Apparently, there are many other aspects associated with career choices and the resulting lifetime earnings patterns: opportunity versus choice-related issues, social status aspects, and labor/leisure considerations, to mention only a few. The influence of these aspects on lifetime income can not be taken into account without drastic changes in the approach.

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APPENDIX A. SALARIES AND TAXES IN FINLAND 1985

A1. Civil servant salaries

Appointees to the Finnish civil service are mainly paid according to a scheme with salary grades ranging from A1, representing the lowest grade, to A32, representing the highest. The monthly salary depends not only on the salary grade but also on the location of the appointment (there is a regional division into two location groups according to cost of living). For simplicity, the calculations in this paper are based on appointments situated in Helsinki (representing location group 1).

Starting and final monthly salaries for some salary grades, according to the salary agreements running from March 1, 1985, are presented in Table A.1.

Table A.1 Starting and final gross monthly salaries 1985

Salary grade	Starting salary	Final salary
A1	3 481	4 732
A5	3 758	5 108
A15	4 834	6 570
A20	6 174	8 391
A32	16 900	22 099

A civil servant moves from the starting salary towards the final one according to seniority: The salary rises after the first (by 6 %), third (6 %), fifth (5 %), eighth (5 %), eleventh (4 or 5 %), and fifteenth (1.5 or 4.5 %) year of service. Seniority also affects the length of the yearly vacation. Initially, the servant is entitled to two days of leave per month of service, but after the first full year

of service the vacation is extended to five weeks, and after the fifteenth to six weeks. This aspect of seniority is indirectly reflected in the annual earnings through additional vacation pay, to which the civil servant is entitled, corresponding to one half of the ordinary salary during the vacation.

After retirement at age 63, the annual income reduces to 66 per cent of the income prior to retirement. Closer details of present salary agreements are found in e.g. Virkamieskalenteri (1984).

A2. Taxation

Incomes in Finland are principally subject to two types of taxes: a proportional municipal income tax, imposed by the local municipality, and a state income tax, acting according to a progressive scheme. Both taxes are determined annually on the basis of taxable income, derived from gross income by subtracting a set of deductions.

The calculations in this paper are based on the - not fully realistic - assumption that no deductions can be made from gross income in the municipal taxation. Similarly, regarding state taxation, minimal deductions are assumed, implying that only income acquisition, wage, and salary deductions are subtracted from the gross income to determine the taxable income. In Table A.2 a summary of the assumed determinants of the taxable income and tax rates is given.

Finally, Figure A.1 outlines the municipal, state, and aggregate tax rates as a function of taxable income.

Table A.2 Taxable income and tax rates in municipal and state taxation
1985

MUNICIPAL TAXATION:

Taxable income = gross income

Tax rate = 19.5 per cent (approximate Finnish average)

STATE TAXATION:

Taxable income = $Y - D1 - D2 - D3$

Y = Gross income

$D1$ = Income acquisition deduction =

$= 350 + 0.04Y$, if $Y < 26250$ FIM
1400, if $Y > 26250$ FIM

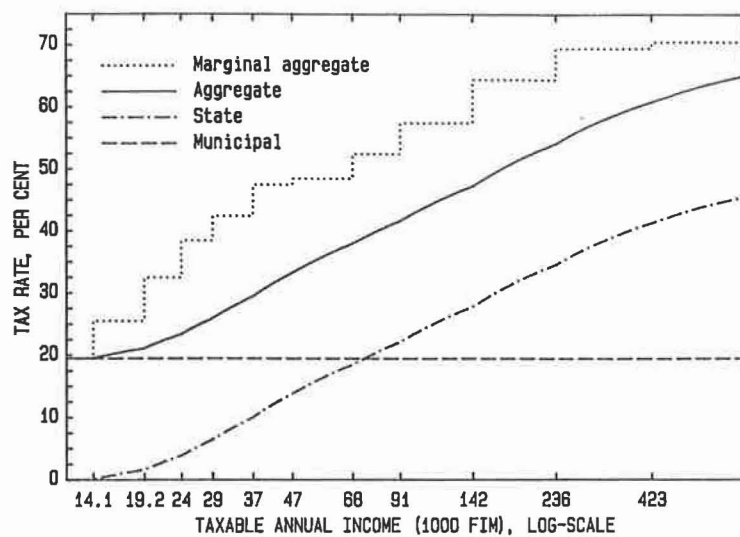
$D2$ = Wage deduction = $0.25(Y - D1)$, if $Y - D1 < 43200$ FIM
10800, if $Y - D1 > 43200$ FIM

$D3$ = Salary deduction = $0.01(Y - D1)$, if $Y - D1 < 80000$ FIM
800, if $Y - D1 > 80000$ FIM

Income tax according to the following table:

Taxable income FIM	Fixed tax amount at the lower bound	Tax rate (per cent) for income exceeding the lower bound
14100- 19200	10	6
19200- 24000	316	13
24000- 29000	940	19
29000- 37000	1890	23
37000- 47000	3730	28
47000- 68000	6530	29
68000- 91000	12620	33
91000-142000	20210	38
142000-236000	39590	45
236000-423000	81890	50
423000-	175390	51

Figure A.1 Municipal, state, and aggregate tax rates in Finland 1985.



ACCOUNTING PERIOD AND INCOME INEQUALITY - EMPIRICAL EVIDENCE FROM U.S. PANEL DATA

by Reija Lilja

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1. INTRODUCTION AND SUMMARY

The importance of time in income comparisons has been widely recognized in the literature on income inequality. One issue that has been frequently discussed is how to take account of short-run variations in incomes in connection with measured inequality. Temporary income fluctuations have been regarded as somewhat problematic in income comparisons since they tend to increase income dispersion and thus exaggerate the true degree of inequality. It has therefore been suggested that the appropriate income concept in welfare comparisons should be that of lifetime income.¹⁾ This idea, however, is to some extent restricted. Namely, lifetime income is a superior measure of economic welfare only in a world in which (among other things) there are no imperfections in labour or capital markets. If economic agents face binding constraints in their decision-making, lifetime income does not necessarily have clear welfare implications.²⁾

Nevertheless, the comparison between long-run and short-run incomes is in other respects important. For example, it is possible to examine income dynamics in a society by studying how the accounting period affects income inequality. In particular, the degree to which incomes of individuals (households) change over time, i.e. income mobility (stability), can be studied empirically if panel data on incomes are available. If there is a tendency for incomes to be equalized over time, measured inequality decreases as the accounting period used for calculating incomes is lengthened. To what extent this happens is an empirical

1) For example, Friedman (1962), Blinder (1974), Paglin (1975), among others have supported the life-cycle view.

2) See e.g. Holtzman (1984) for a more detailed analysis.

question.¹⁾ In this paper we shall examine the relationship between the inequality of incomes and the length of the accounting period for a group of U.S. households. Our sample is drawn from the Michigan panel data of income dynamics and consists of the period 1967-81. We also study income stability by the index R suggested in Shorrocks (1978). The index R measures the extent to which individual incomes fluctuate over time. It obtains a value of 1 if relative incomes are unchanged through time (complete immobility) and a value of 0 if complete equalization of incomes occurs. Our calculations have been made for two inequality measures and three income concepts for comparison.

The two inequality measures were the Gini coefficient (Gini) and the square of the coefficient of variation (SCV).

According to our findings, the Gini indicated much higher stability. The 15-year Ginis were 9-10 per cent below the average values for the annual Ginis, while the corresponding figures for the SCVs were 23-31 per cent. This indicates about three times greater mobility for the SCV, which is similar to the results reported in Shorrocks (1981). The differences between these inequality measures stems from the way in which observations in the tails of the income distribution are weighted in calculating the measures. The shapes of the stability profiles of incomes (i.e. plots of the index R against different accounting periods) suggest that the observed tendency of incomes to equalize over time in our sample was permanent by its nature.

1) Kravis (1962), Benus and Morgan (1975), Kohen, Parnes and Shea (1975) and Shorrocks (1981) are examples of earlier empirical studies on this question. In Finland Nygård (1985) has done some simulation experiments regarding this matter.

Moreover, our findings suggest that in income comparisons the chosen concept of income is at least as important a factor as income dynamics. For example, the average annual Gini coefficients differed as follows: the labour income of head of household indicated about a 6 per cent higher inequality than the taxable income of head and wife together and about a 22 per cent higher inequality than the total family income.¹⁾ These different income concepts, however, had similar stability profiles over the sample period.

This paper proceeds as follows: In the next section we discuss the problem of lengthening the accounting period in panel data and report inequality measures for different accounting periods. In the third section we study the income stability in the sample group and draw stability profiles suggested by the two inequality measures.

2. INCOME INEQUALITY

The relationship between income inequality and the accounting period is a matter for empirical research. In the investigations of this paper we have used a sample of 1,227 U.S. households from the Michigan panel data of income dynamics comprising 15 consecutive years (1967-81). The original sample consisted of 6,742 households and our sample was selected by eliminating the following categories of families:

1) Labour income of head refers to the earnings of the head. Taxable income of head and wife includes the earnings and taxable nonlabour income of the spouses. Total family income includes the earnings of all family members, unearned income, business profits and transfer payments.

1) Families that belonged to the non-random low-income sample from the Survey of Economic Opportunity at the beginning of the panel study.

Sample selection problems would arise if this sample were not eliminated, since in the remaining cross-section sample, which represents all U.S. dwellings, low-income families are included in the appropriate proportions (3,069 cases).

2) Families which reported a change either of the head of the household or of the spouse during the panel period. This exclusion was necessary for us to be able to follow the incomes of the same heads of households and married couples throughout the panel period¹⁾ (2,446 cases).

In our sample of families the heads of the households can be either men or women. However, if the family consisted of a married couple, the survey has always treated the husband as the head of the household. All the female heads are thus unmarried women. In our income comparisons this may cause a bit of confusion. We have used the following three income concepts in empirical calculations:

1) Labour income of head

This concept refers to the earnings of the head of the family.

2) Taxable income of head and wife

This concept is a sum of

- a) the taxable earnings and nonlabour income of the head of the household irrespective of the marital status or the sex of the head and
- b) the taxable earnings and nonlabour income of the spouse (by definition, the wife) if the head was married.²⁾

3) Total income of family

This concept includes the earnings of all family members, nonlabour income, business profits and transfer payments.

1) For a discussion about the sampling problems involved in panel data, see e.g. Lilja (1986).

2) Thus, if a female head got married during the panel period, her husband became the head of the household and the family reported a change in the head of the family. These families were excluded from our sample.

In our calculations all incomes are deflated by the consumer price index and expressed in 1967 dollars. Mean incomes of the sample for the whole panel period are reported in Appendix 1. Even though the above income concepts are not perfectly comparable, we hope that the comparison between the labour income of head and the taxable income of head and wife gives some idea about the effect of marriage on income inequality in the U.S. Moreover, the comparison between the total income of family and the other two income concepts should be able to shed some light on the effect of transfer payments on measured inequality. Because, in general, the purpose of transfer payments is to equalize the income distribution, one would expect the total income of a family to indicate lower inequality than the other income measures.

In our calculations two inequality measures have been used: the Gini coefficient (Gini) and the square of the coefficient of variation (SCV).¹⁾ Both these measures are convex functions of relative incomes and are thus 'mean independent'. The measures differ in their response to income transfers. Gini gives little weight to income transfers occurring in either tail of the income distribution, whereas SVC is very sensitive to the distribution of top incomes. This means, for example, that if income fluctuations within a period are to a large extent due to temporarily high incomes, SCV should decrease more than Gini when the accounting period of incomes is lengthened.

In Appendix 2 we have reported the Gini coefficients for annual incomes and incomes cumulated forwards in time. In Appendix 3 the same calculations have been reported for SCV. At first sight the numbers in these

1) See e.g. Shorrocks (1980) for definitions.

appendices may look peculiar. First, the inequality measures for annual incomes are usually increasing with time and secondly, the lengthening of the accounting period does not seem to have equalizing effects. Kravis (1962) was one of the first to be puzzled by this phenomenon and the one to explain it. It is a common feature in most income data that the relative inequality tends to increase as cohorts of individuals age. In other words, the dispersion of incomes within age-groups tends to increase over time. This is what we notice in our appendices: In Appendix 2 the annual Gini coefficients in 1981 were 19-23 per cent higher than in 1967. The corresponding figures for SCVs in Appendix 3 were 67-77 per cent. These results suggest that the dispersion of incomes within age groups in our sample is relatively strong in the top end of the income distribution.

It also appears from Appendices 2 and 3 that the increasing income dispersion within age-groups in the sample has offset the equalizing effect which the cumulation of incomes normally has. If incomes had been cumulated backwards (starting from the year 1981), the inequality measures would have decreased according to expectations. The problem of how to study pure accounting period effects on inequality can be solved by comparing the long-run inequality values with the means of the short-run values, and this is precisely what we have done in this paper. In Table 1 the means of Gini coefficients for different accounting periods are reported.

In Table 1 the n -period Gini refers to an arithmetic mean of the Ginis calculated for all consecutive n -period incomes ($n=1, \dots, 15$). For example, to obtain the value for the 2-period Gini we calculated an arithmetic mean of 14 Ginis for all consecutive 2-period incomes. This

Table 1. Means of Gini coefficients for different accounting periods

Accounting period, years	Real labour income of head	Real taxable income of head and wife	Real total income of family
1	0.472	0.445	0.368
2	0.463	0.437	0.360
3	0.457	0.431	0.354
4	0.453	0.427	0.350
5	0.449	0.423	0.347
6	0.445	0.420	0.345
7	0.442	0.417	0.343
8	0.439	0.415	0.341
9	0.437	0.413	0.339
10	0.434	0.411	0.338
11	0.432	0.410	0.336
12	0.430	0.409	0.335
13	0.428	0.407	0.335
14	0.427	0.407	0.334
15	0.425	0.405	0.334

procedure provides us with pure accounting period effects on income inequality. According to Table 1, the 15-period Ginis are about 9-10 per cent below their average annual values. The accounting period seems to have a similar effect on the inequality for all concepts of income. However, the way in which incomes are measured has a significant effect on inequality within each accounting period. Table 1 suggests that the additional income of the head's spouse as well as transfer incomes have equalizing effects on income distribution. The labour income of head of household indicates about a 6 per cent higher inequality than the taxable income of head and wife together and about a 22 per cent higher inequality than the total family income. A similar phenomenon appears in Table 2, in which means of the square of the coefficient of variation for different accounting periods are reported.

According to Table 2, the 15-period SCVs are about 24-30 per cent below their average annual values. Thus, the SCV seems to be about three times more sensitive to the lengthening of the accounting period than the Gini.

Table 2. Means of the square of the coefficient of variation for different accounting periods

Accounting period, years	Real labour income of head	Real taxable income of head and wife	Real total income of family
1	0.895	0.981	0.650
2	0.850	0.922	0.592
3	0.822	0.864	0.548
4	0.802	0.820	0.514
5	0.786	0.774	0.494
6	0.771	0.746	0.481
7	0.758	0.727	0.472
8	0.745	0.711	0.464
9	0.733	0.699	0.459
10	0.722	0.690	0.452
11	0.712	0.684	0.448
12	0.703	0.685	0.446
13	0.695	0.685	0.448
14	0.687	0.689	0.453
15	0.681	0.689	0.454

This result indicates that a considerable proportion of the observed within period inequality is due to temporarily high incomes in the top end of the income distribution. Furthermore, within period measured inequality according to Table 2 differs from that in Table 1. Table 2 suggests that the additional income of the head's spouse does not have any equalizing effect on income distribution. On the other hand, transfer incomes seem to contribute a large drop of about 27 per cent in inequality compared to that for the labour income of head. This result suggests that a change from one income concept to another has meant income transfers in the top end of the income distribution.

3. INCOME STABILITY

To examine income stability we follow a procedure suggested in Shorrocks (1981). For each accounting period we calculate an index R, which measures

the extent to which equalization takes place as the accounting period is lengthened. The index is defined as follows:

$$R = \frac{I(Y)}{\sum_1 w_1 I(Y_1)}$$

where $I()$ represents any inequality measure that is a convex function of relative incomes. $I(Y)$ represents the long-run inequality value (for n -period income) and $I(Y_1)$ refers to a single period inequality value. $I(Y_1)$ is weighted by w_1 , which represents the proportion of n -period income received in period 1. Thus the long-run inequality value is expressed as a proportion of the weighted average of short-run inequality values. R obtains values between 0 and 1. If relative incomes are unchanged through time, R equals 1 and the society is completely immobile. If R is zero, total equalization has happened. In a mobile society income changes are frequent and pronounced. This means high inequality values $I(Y_1)$ in subperiods and thus low values of index R . Thus a mobile income structure suggests low values of R , which can be regarded as a measure of income stability.

We have calculated index R for both of our inequality measures and for all accounting periods ($n=1, \dots, 15$). In Table 3 results for the Gini coefficients are reported.

It appears from Table 3 that the 15-year Gini coefficients are 9-10 per cent below the weighted average of annual Gini coefficients. This result confirms those reported in Table 1. Thus, according to Table 3 mobility in American income distribution has been rather modest during the years 1967-81. This conclusion is, however, challenged by Table 4, where index R for the square of the coefficient of variation is reported.

Table 3. Index R for Gini coefficients

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	1.00	1.00	1.00
1967-68	0.99	0.99	0.99
1967-69	0.97	0.98	0.97
1967-70	0.96	0.96	0.96
1967-71	0.96	0.95	0.95
1967-72	0.95	0.95	0.94
1967-73	0.94	0.94	0.94
1967-74	0.94	0.94	0.93
1967-75	0.93	0.93	0.93
1967-76	0.93	0.93	0.92
1967-77	0.93	0.93	0.92
1967-78	0.92	0.92	0.91
1967-79	0.92	0.92	0.91
1967-80	0.91	0.91	0.91
1967-81	0.91	0.91	0.90

Table 4. Index R for the square of the coefficient of variation

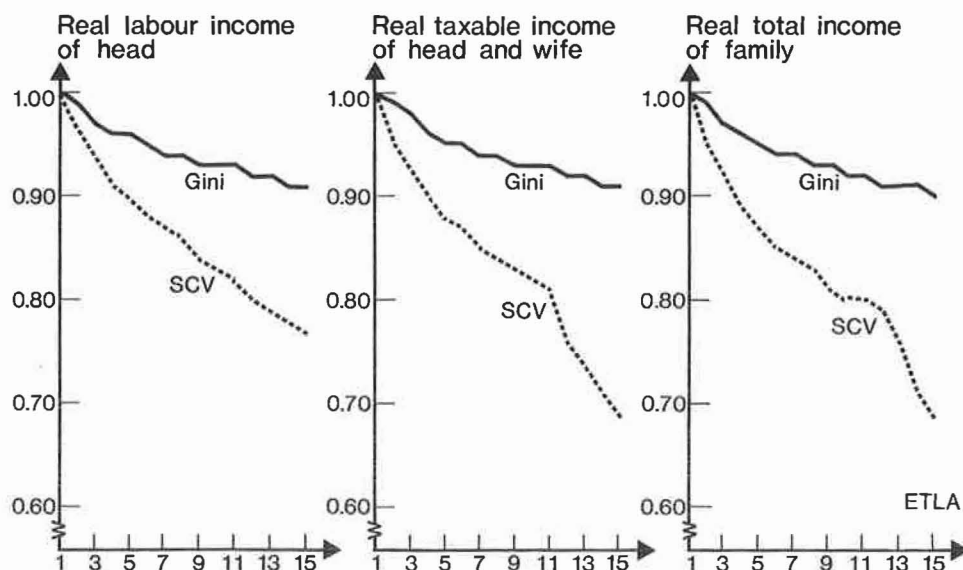
Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	1.00	1.00	1.00
1967-68	0.96	0.95	0.95
1967-69	0.94	0.93	0.92
1967-70	0.91	0.90	0.89
1967-71	0.90	0.88	0.87
1967-72	0.88	0.87	0.85
1967-73	0.87	0.85	0.84
1967-74	0.86	0.84	0.83
1967-75	0.84	0.83	0.81
1967-76	0.83	0.82	0.80
1967-77	0.82	0.81	0.80
1967-78	0.80	0.76	0.79
1967-79	0.79	0.74	0.76
1967-80	0.78	0.71	0.71
1967-81	0.77	0.69	0.69

According to Table 4 the 15-year SCV is about 23-31 per cent below the weighted average of annual SCVs. This result is similar to that obtained in Table 2. This indicates much higher mobility than the Gini due to reasons already known to us. Additional information that Tables

3 and 4 reveal comes from the evolution of R when the accounting period is lengthened. This information can be summarized in stability profiles. In Figure 1 we have drawn these profiles for each income concept and both inequality measures.

The shapes of the stability curves in Figure 1 suggest that in the group of our U.S. households there seems to be a continuous egalitarian trend as the time-horizon is stretched. This is because the curves for both the Gini and the SCV show a steady decline throughout the panel period. If the trend is to continue as such, differences between longer-run incomes are expected to be much smaller than those presented for the 15-period inequality measures.

Figure 1. Stability profiles



The results in this paper clearly indicate that in income comparisons the accounting period matters. When short-run fluctuations in incomes

can be smoothed out, the picture on overall inequality changes. What implications this has for economic welfare is a question that will not be considered here. When discussing welfare, such matters as market imperfections over the lifetime become important. These considerations are beyond the scope of this paper, and we leave them for future research.

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Appendix 1. Mean incomes in 1967 dollars

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	7 214	8 915	9 736
1968	7 523	9 486	10 501
1969	7 726	9 699	10 769
1970	7 936	9 934	11 156
1971	7 658	9 716	11 047
1972	7 844	9 983	11 411
1973	7 962	10 228	11 765
1974	7 539	9 794	11 429
1975	7 170	9 465	11 126
1976	7 171	9 650	11 456
1977	7 221	9 695	11 636
1978	7 158	10 253	11 916
1979	6 706	9 623	11 890
1980	6 192	9 416	11 673
1981	5 744	8 919	11 213

Appendix 2. Gini coefficients for annual and cumulated incomes

Table 2A. Gini coefficients for annual incomes

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	0.394	0.379	0.337
1968	0.403	0.390	0.344
1969	0.401	0.383	0.337
1970	0.415	0.394	0.345
1971	0.427	0.406	0.351
1972	0.434	0.413	0.348
1973	0.439	0.416	0.351
1974	0.461	0.438	0.362
1975	0.484	0.451	0.370
1976	0.499	0.458	0.372
1977	0.509	0.464	0.372
1978	0.528	0.507	0.384
1979	0.541	0.505	0.401
1980	0.565	0.538	0.428
1981	0.582	0.534	0.415

Table 2B. Gini coefficients for incomes cumulated forwards in time

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	0.394	0.379	0.337
1967-68	0.391	0.377	0.332
1967-69	0.388	0.372	0.328
1967-70	0.389	0.372	0.326
1967-71	0.391	0.373	0.326
1967-72	0.392	0.374	0.324
1967-73	0.394	0.375	0.323
1967-74	0.397	0.378	0.324
1967-75	0.401	0.381	0.324
1967-76	0.405	0.384	0.325
1967-77	0.409	0.387	0.325
1967-78	0.413	0.392	0.326
1967-79	0.417	0.397	0.329
1967-80	0.421	0.402	0.332
1967-81	0.425	0.405	0.334

Appendix 3. Square of the coefficient of variation for annual and cumulated incomes

Table 3A. Square of the coefficient of variation for annual incomes

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	0.627	0.570	0.460
1968	0.674	0.671	0.530
1969	0.663	0.598	0.467
1970	0.711	0.627	0.480
1971	0.740	0.671	0.503
1972	0.719	0.674	0.489
1973	0.755	0.692	0.501
1974	0.872	0.766	0.536
1975	0.960	0.819	0.569
1976	0.982	0.796	0.534
1977	1.000	0.801	0.520
1978	1.082	1.690	0.542
1979	1.103	1.513	0.968
1980	1.232	2.190	1.416
1981	1.300	1.988	1.232

Table 3B. Square of the coefficient of variation for incomes cumulated forwards in time

Period	Real labour income of head	Real taxable income of head and wife	Real total income of family
1967	0.627	0.570	0.460
1967-68	0.627	0.590	0.469
1967-69	0.616	0.569	0.448
1967-70	0.612	0.555	0.432
1967-71	0.613	0.555	0.426
1967-72	0.608	0.551	0.417
1967-73	0.608	0.549	0.411
1967-74	0.616	0.554	0.410
1967-75	0.623	0.560	0.410
1967-76	0.634	0.564	0.408
1967-77	0.643	0.566	0.406
1967-78	0.651	0.596	0.403
1967-79	0.661	0.621	0.416
1967-80	0.671	0.661	0.438
1967-81	0.681	0.689	0.454

ON THE DISTRIBUTION OF LIFETIME INCOME IN A LOGNORMAL MODEL

by F.A. Cowell

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1. INTRODUCTION

An analysis of the distribution of lifetime income runs into two obvious problems (1) What is lifetime income? (2) How may its interpersonal distribution be specified? Problem 1 involves specification of a simple aggregate which captures the level of well-being of a person over his whole lifetime. Should this be lifetime earnings plus initial assets? Or should we try to find some appropriate concept of permanent income? The answers to these questions depend on assumptions made about capital market constraints and uncertainty of incomes and of lifetimes, but suitable general definitions will be provided below.¹⁾ However, it is essential to take some suitable special case of this general definition in order to deal with Problem 2. One simplified solution is to look at the interpersonal distribution of lifetime earnings discounted at the market rate of interest.²⁾ This, however, is only one of several versions of the lifetime income variable that may be appropriate under different model specifications. The disadvantages are that it does not allow for the personal welfare effects of uncertainty and capital market imperfections, and that analytically it is extremely difficult to handle even in the simplest of dynamic income distribution models.

In this paper we pursue a special case of the general lifetime income concept that combines tractability readily with interpretability. As a special case, of course, it can be used as no more than a suggestion of the more general behavioural characteristics, but, nevertheless, it does demonstrate a number of interesting things. This income concept we use in conjunction with a simple model of the dynamics of actual current income. This dynamic model should not be taken as an "explanation" of

the personal income distribution, but rather as a fairly rich framework for the complex of forces which govern income dispersion and variability. Given these two methodological components we can examine the structure of inequality and the effect of redistribution policy in current and lifetime terms.

2. INDIVIDUAL LIFETIME INCOME

For ease of exposition I shall keep the notation and terminology in line with Cowell (1979). Consider a person currently of age θ who will die at age $\bar{\theta}$. He has a noninterest income stream $\{y(t) | t \in [\theta, \bar{\theta}]\}^3$, and has assets $S(t)$ at any age t , with his "inheritance from the past" $S(\theta)$ given exogenously. We shall ignore the problem of bequests, and assume that the person is constrained to die with zero assets. There are a number of ways in which the time path of his assets may be determined. For example, if he can borrow or lend as much as he likes at a rate r , subject only to the terminal capital condition, then the appropriate condition is

$$(1) \quad \frac{d}{dt} S(t) = rS(t) + y(t) - c(t), \quad t \in [\theta, \bar{\theta}]$$

In general, however, if there are restrictions on borrowing, the motion of $S(t)$ will be more complicated than the simplified differential equation (1), although we do not need to pursue this here. We assume the person derives utility from his lifetime consumption stream $\{c(t) | t \in [\theta, \bar{\theta}]\}$ in a manner given by the additively separable function $W(\theta, \bar{\theta}) = \int_{\theta}^{\bar{\theta}} V(c(t), t) dt$ and we shall define the following

$$(2) \quad \left[\begin{array}{l} W^*(\theta, \bar{\theta}) = \max_{\{c(t)\}} \int_{\theta}^{\bar{\theta}} V(c(t), t) dt, \\ \text{S.T.} \quad \left[\begin{array}{l} S(\theta) = S_{\theta}, \\ \{y(t)\} \\ \text{market constraints.} \end{array} \right] \end{array} \right]$$

Now let us derive an index that summarises the "economic position" attainable from the stream $\{y(t)\}$. Suppose the individual were to forfeit this stream, but be compensated by an increase in his initial assets, $S(\bar{\theta})$. In particular, consider a quantity $V(\theta, \bar{\theta})$ such that:

$$(3) \quad \left[\begin{array}{l} W^{**}(\theta, \bar{\theta}) = \max_{\{c(t)\}} \int_{\theta}^{\bar{\theta}} V(c(t), t) dt, \\ \text{S.T.} \quad \left[\begin{array}{l} S(\theta) = V(\theta, \bar{\theta}), \\ \{y(t)\} = \{0\}, \\ \text{market constraints.} \end{array} \right] \end{array} \right]$$

Then we can make the following definition.

Definition 1. $V(\theta, \bar{\theta})$ is the lifetime welfare equivalent capital sum of the income stream $\{y(t) \mid t \in [\theta, \bar{\theta}]\}$ in the given market environment if $W^{**}(\theta, \bar{\theta}) = W^*(\theta, \bar{\theta})$ in (2), (3) above.

A related concept that is perhaps easier to define can now be introduced. Here we suppose that $V(c(t), t)$ can be written $e^{\rho[\theta-t]} u(c(t)) / \Omega(\rho)$, where ρ is the (constant) rate of pure time preference, and $\Omega(\rho) = [1 - e^{\rho[\theta-\bar{\theta}]}] / \rho$, a normalising factor.

Definition 2. The lifetime-welfare-equivalent current income for the income stream $\{y(t) \mid t \in [\theta, \bar{\theta}]\}$ and initial asset holding $S(\theta)$ in the given market environment is

$$y(\theta, \bar{\theta}) = u^{-1}(W^*(\theta, \bar{\theta})),$$

where $W^*(\theta, \bar{\theta})$ is defined in (2).

The quantity in Definition 1 has been termed "Wergild" (in Cowell (1979)) and equals discounted lifetime noninterest income if capital markets are perfect. If capital markets are imperfect, then $y(\theta, \bar{\theta})$ will depend on the shape of the utility function. In the opposite polar case, where consumption is continuously equal to current income, $y(\theta, \bar{\theta})$ is a function of discounted lifetime utility of income, where ρ , the subjective discount rate, rather than r , the market rate of interest, is used. It is this polar case on which we shall focus for the simple exercises in this paper. There are a number of reasons why this is a suitable alternative approximation. Firstly, it is appropriate if all persons face total restrictions on borrowing (for example, because of the lack of life insurance markets) and have a high rate of pure time preference, ρ . Secondly, it is appropriate in the case of extreme income uncertainty which may lead to the nonexistence of capital markets. Thirdly, even if capital markets do exist, income uncertainty will cause people's life cycle consumption profiles to follow their current income profiles.⁴⁾ Also, this particular case will be extremely convenient when we "assemble" the population in the next section.⁵⁾ In the case of imperfect capital markets there are technical reasons why the income concept $y(\theta, \bar{\theta})$ may be more useful - see Cowell (1979).

Finally, let us note that in the case of an uncertain date of death the discount factor $e^{\rho[\theta-t]}/\Omega(\rho)$ must be modified to allow for the probability of survival. If we assume a uniform "force of mortality" γ_1 for the interval $[\theta, \bar{\theta}]$ where $\bar{\theta}$ is now the maximum possible death date and may be infinite, then we may modify the utility function so

that we now write $\rho = \rho_1 + \gamma_1$ where ρ_1 is the pure rate of time preference.

3. A SIMPLE MODEL OF INCOME DYNAMICS

If enough data are specified, then it is clear that we may in principle define a concept such as "wergild" for an isolated individual and use this as a measure of his lifetime well-being. If we translate this to the problem of working out what the "distribution of lifetime income" looks like, then a number of new problems arise. Firstly, is it a valid procedure to incorporate $V(\theta, \bar{\theta})$ or $y(\theta, \bar{\theta})$ as a variable in the size distribution of income? As has been noted elsewhere⁶⁾ any such variable must satisfy criteria of measurability and comparability. By ignoring family formation and differential needs we have virtually assumed away the problem of comparability within a given generation. However, the problem of measurability will remain unless we deal with a subset of the model that guarantees that $V(\theta, \bar{\theta})$ is proportional to $y(\theta, \bar{\theta})$. Then we would have a situation comparable to that under perfect capital markets where total real wealth is directly proportional to real income at a given moment via the interest rate, and for any mean-independent inequality measure it would be irrelevant which distribution were to be used. Except under special circumstances, however, $V(\theta, \bar{\theta})$ and $y(\theta, \bar{\theta})$ are not proportional. Either has a good claim to be a cardinal individual welfare measure, and so the distribution of each perhaps needs to be treated separately.

Secondly, even if we agree on the measure of personal welfare or lifetime income, there is an ambiguity of meaning as regards its inter-

personal distribution for any generation. One meaning of "the distribution of lifetime income" is the distribution as it appears to an observer, and is in principle an "objective" entity, rather like the conventionally-defined distribution of income or of wealth at any instant of time. Let us call this the observed distribution. The other meaning that we need to consider is the distribution as it appears to the individual. He, of course, has information that is not available to the observer concerning his own income and income profile - his education, training, health, mental ability, relationship with his boss and so on. However, even though he has more information, we must still accept that his income profile is stochastic. The distribution as it appears to him will be called the self-perceived or subjective distribution.

It is evident that the self-perceived distribution changes with the age of the person concerned. This suggests that it may be helpful to consider both this and the observed distribution on an age-specific basis. Then, cohort by cohort, we may consider the distributional effects of parametric shifts in the static and dynamic determinants of individual incomes. We shall return to the problem of aggregation over the cohorts at a later stage.

Let $y_{\theta}(t)$ be the income of an individual in the θ -cohort (the cohort currently of age θ) at some future age t . We shall assume that for all $t \in [\theta, \bar{\theta}]$ this income is determined by a simple Markovian stochastic process:

$$(4) \quad \frac{d}{dt} (\log y_{\theta}(t)) = \frac{d}{dt} \mu_{\theta}(t) + u(t)$$

Obviously much turns on the specification of the behaviour of the random sequence $\{u(t)\}$. For the present we shall make the simplest assumptions possible, namely that $u(t)$ is uncorrelated with $u(t')$ for any $t' \neq t$, that $u(t)$ is uncorrelated with $y_\theta(t')$ for any $t' \leq t$, and that the sequence is normal, homoscedastic and of zero mean:

$$(5) \quad u(t) \sim N(0, v_\theta^2)$$

We shall also assume that the current incomes of the θ -cohort are distributed as

$$(6) \quad y_\theta(\theta) \sim \Lambda(\mu_\theta(\theta), \sigma_\theta^2(\theta))$$

These assumptions⁷⁾ lead us to write immediately

$$(7) \quad y_\theta(t) \sim \Lambda(\mu_\theta(t), \sigma_\theta^2(t))$$

$$\sigma_\theta^2(t) = \sigma_\theta^2(\theta) + [t-\theta]v_\theta^2$$

$$(8) \quad \theta \leq t \leq \bar{\theta}$$

If we make the further assumption of stationarity⁸⁾ of the process then the stochastic experience of each cohort is identical. This enables us to drop the θ -subscript in all the above expressions and the entire process is specified for any cohort, given v^2 , $\mu(\theta)$ and $\sigma^2(\theta)$.

4. INEQUALITY OF LIFETIME INCOME

We may now proceed to compute the lifetime aggregates and averages for the θ -cohort for the special case in which

$$(9) \quad u(y(t)) = \alpha_0 + \alpha_1 \log(y(t)), \alpha_1 > 0.$$

We find that for the definitions on page three, we have

$$y(\theta, \bar{\theta}) = \exp \left[\int_{\theta}^{\bar{\theta}} e^{\rho[\theta-t]} x(t) dt \right]$$

where $x(t) \equiv \log(y(t))$. For this case, however, it is probably more convenient to look at lifetime average income (using the "wergild" concept) which comes to

$$(10) \quad y(\theta, \bar{\theta}) = \exp \left(\int_{\theta}^{\bar{\theta}} e^{\rho[\theta-t]} x(t) dt / \Omega(\rho) \right).$$

Let $x(\theta, \bar{\theta}) \equiv \log y(\theta, \bar{\theta})$. Then, we find that (10) yields

$$(11) \quad x(\theta, \bar{\theta}) = M(\theta, \bar{\theta}) + x(\theta) + \int_{\theta}^{\bar{\theta}} u(t) [e^{\rho[\theta-t]} - e^{\rho[\theta-\bar{\theta}]}] dt / [\rho \Omega(\rho)],$$

where $M(\theta, \bar{\theta}) = \int_{\theta}^{\bar{\theta}} e^{\rho[\theta-t]} \mu(t) dt / \Omega(\rho) - \mu(\theta)$. Hence the observed value of expected average lifetime logincome of the cohort currently of age θ is

$$Ex(\theta, \bar{\theta}) = M(\theta, \bar{\theta}) + \mu(\theta);$$

and the self-perceived value of expected average lifetime logincome of an individual currently at age θ is

$$Ex(\theta, \bar{\theta}) | x(\theta) = M(\theta, \bar{\theta}) + x(\theta).$$

The variance of each of these two entities is given respectively by

$$\text{var } x(\theta, \bar{\theta}) = \hat{\sigma}^2(\theta, \bar{\theta}) = \sigma^2(\theta) + \hat{\sigma}^2(\theta, \bar{\theta}).$$

$$\text{var } x(\theta, \bar{\theta}) | x(\theta) = \hat{\sigma}^2(\theta, \bar{\theta}),$$

where

$$\begin{aligned}\hat{\sigma}^2(\theta, \bar{\theta}) &= v^2 \int_{\theta}^{\bar{\theta}} [e^{2\rho[\theta-t]} - 2e^{\rho[2\theta-t-\bar{\theta}]} + e^{2\rho[\theta-\theta]}] dt / [\rho\Omega(\rho)]^2 \\ (12) \quad &= v^2 [2\Omega(\rho) - 3\Omega(2\rho) + [\bar{\theta}-\theta][1-2\rho\Omega(2\rho)] / [\rho\Omega(\rho)]^2.\end{aligned}$$

This expression enables us to establish the following.

Lemma 1 In the simple logarithmic utility model, the variance of the logarithm of self-perceived lifetime-welfare-equivalent income increases monotonically with the unexpired portion of the lifetime. For infinite programmes this converges to an upper limit of $v^2/2\rho$.

Proof Let $\tau = \bar{\theta} - t$, the unexpired portion of the lifetime. Notice that

$$\frac{\partial \hat{\sigma}^2(\theta, \bar{\theta})}{\partial \tau} = 2\zeta v^2 \frac{\Omega(2\rho) - \tau\zeta}{\rho^2 \Omega^3(\rho)}$$

where $\zeta = \partial\Omega(\rho)/\partial\tau > 0$. Expanding the numerator in $\rho\tau$ we find this positive. Hence $\hat{\sigma}^2(\theta, \bar{\theta})$ increases with τ . The second part of the Lemma follows directly from (12) and noting that $\Omega(\rho) \rightarrow 1/\rho$ as $\bar{\theta} \rightarrow \infty$.

Evidently as $\bar{\theta} \rightarrow \infty$, $[\theta - \bar{\theta}][1 - 2\rho\Omega(2\rho)] \rightarrow 0$, and so

$$\hat{\sigma}^2(\theta, \bar{\theta}) \rightarrow v^2 [2/\rho - 3/2\rho] / [1] = v^2/2\rho. \quad \text{Q.E.D.}$$

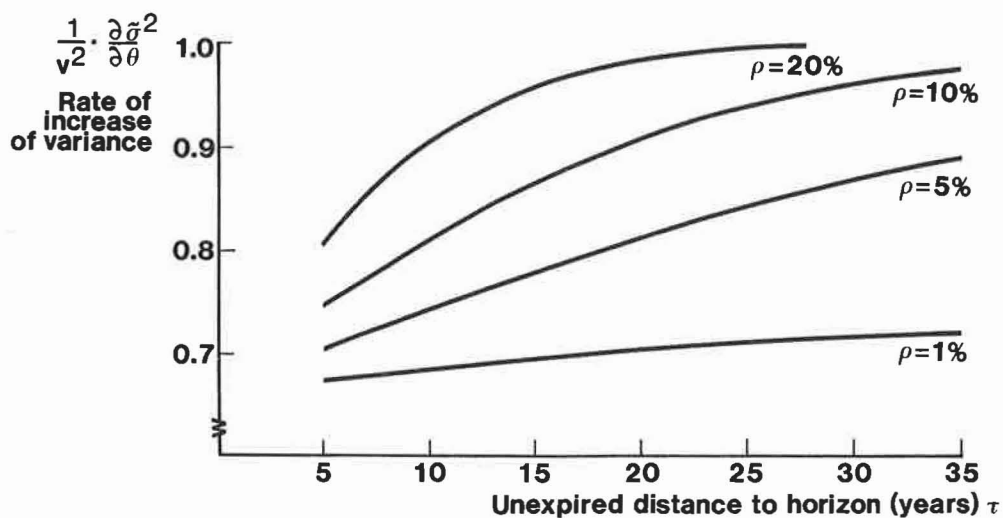
Thus for very long lifetimes it is convenient to assume that the self-perceived variance remains constant as the individual ages, and hence, from (12) the observed variance increases linearly. However, for finite $\bar{\theta}$, this conclusion may not be drawn, since $\hat{\sigma}^2(\theta)$ increases with θ , while $\hat{\sigma}^2(\theta, \bar{\theta})$ falls. This is illustrated in Figure 1. Along the horizontal axis the unexpired portion of the lifetime τ is plotted. On the vertical axis, the rate of increase of observed variance $\tilde{\sigma}^2(\theta, \bar{\theta})$, normalised by v^2 , is plotted. The functional relationship is graphed for a selection of values of ρ , and for each curve, the value of τ at which $\tilde{\sigma}^2$ starts to increase is marked in. Note that each curve is

asymptotic to $\frac{1}{2} \frac{\partial \tilde{\sigma}^2}{\partial \tau} = 1$; i.e. the "standard" case of linearly increasing variance. Note also that convergence is very slow for ρ as low as 1 % although for lifetimes greater than five years to run, the change in $\tilde{\sigma}^2$ has attained at least 65 % of its asymptotic value. The behaviour of each function of $\tau < 5$ years suggests that in empirical applications care is taken over the older age groups.

Hence it is convenient to assume that $\bar{\theta}$ is effectively infinite, in which case

$$\tilde{\sigma}^2(\theta, \infty) = \sigma_0^2 + v^2[\theta + 1/2\rho] .$$

Figure 1. Finite Horizon and the Behaviour of $\tilde{\sigma}^2(\theta, \bar{\theta})$



Let us consider the relationship between the observed dispersion of lifetime - equivalent income, and the observed dispersion of current incomes in the population. One difficulty that we shall encounter here

relates to the "drift" of $\mu(\theta)$, the mean of log-income in cohort θ . In the process of aggregating the random sequence $\{y(t)\}$ over the lifetime and then computing the variances $\hat{\sigma}^2(\theta, \bar{\theta})$ and $\tilde{\sigma}^2(\theta, \bar{\theta})$, it is clearly important that this drift is netted out. Hence it is useful to introduce the following.

Definition 3. The Adjusted Aggregate Income Distribution (AAID) is the distribution over the entire population of the quantity $y(\theta)e^{-\mu(\theta)}$. In effect the AAID is the Observed Aggregate Income Distribution (OAID) where the income of each person has been normalised by dividing by the median income of the cohort to which he belongs. We shall proceed by examining first the relationship between inequality in the distribution of lifetime income and inequality in the AAID. Let us consider for a moment what this AAID looks like.

As we have seen, the Age-Specific Income Distribution (ASID) for any cohort θ will have its income $y(\theta)$ distributed according to $\Lambda(\mu(\theta), \sigma^2(\theta))$ where $\sigma^2(\theta) = \sigma_0^2 + v^2\theta$, so that the normalised distribution is obviously $\Lambda(0, \sigma_0^2(\theta))$. The AAID, however, is not log-normal. If the age distribution is negative exponential ($\theta \sim \gamma e^{-\gamma\theta}$) then for the AAID we have $y \sim \text{LG}(0, \sigma_0^2, v^2/\gamma)$ where LG is the so-called log-Gram-Charlier distribution described in Cowell (1975) and Rutherford (1955).⁹⁾ This distribution may be approximated by the distribution $\Lambda(0, \sigma_0^2 + v^2/\gamma)$. To see the nature of this approximation, consider the bias involved by taking the moments of Λ rather than of LG. The ratio of the k^{th} moment about zero for the Λ -distribution to the equivalent for the LG-distribution is given by¹⁰⁾

$$\delta_k = \frac{1 + \alpha + \alpha^2/2! + \alpha^3/3! + \dots}{1 + \alpha + \alpha^2 + \alpha^3 + \dots} = [1 - \alpha]e^\alpha$$

where $\alpha = v^2 k^2 / 2\gamma$. Since the mean of the age distribution is $1/\gamma$, we see immediately that as long as mean age is not too high, and mobility is fairly low, the bias in the lower-order moments will be negligible. As an illustration, consider $v^2 = .005$ and $1/\gamma = 0$. Then $\alpha = .01 k^2$, and the ratio of the variances is $\delta_2 = 0.999950$. The bias will obviously be more important for higher moments, and in order to ensure convergence of the expression defining the LG-moments we must have $\alpha < 1$. In fact v^2/γ must not be too large if the LG distribution (which is the transformation of an infinite sum of Hermite polynomials) is to be well-defined anyway. Restriction of v^2/γ within the range that is theoretically convenient and empirically plausible means that δ_k is likely to be close to unity for low positive integer values of k .

In view of this, we may not only take the distribution of current income, of self-perceived lifetime income and of observed lifetime income as lognormal, but the AAID as well, at least approximately. If so, this is a tremendous advantage in the analysis of the structure of inequality, since all issues involving ordinal comparisons of inequality in the various distributions can be settled merely by examination of the variance of logincome. All other mean-independent inequality measures may be derived as functions of this statistic for comparisons within this family distributions. This attractive property must be treated with caution, however, for the following reasons. Firstly, we have assumed lognormality of $x(\theta)$ throughout $[0, \bar{\theta}]$. If $x(o)$ is not lognormal the convergence to a limiting lognormal distribution may be slow, and thus the lognormality assumption unsatisfactory and the variance of logarithms unsuitable as a sole criterion of inequality.

Secondly, any such convergence will depend on some version of the law of proportionate effect holding. This may not be borne out in practice. Thirdly, for other definitions of lifetime equivalent income (appropriate to different market conditions) or other utility functions, the distribution of this quantity will not be strictly lognormal, though lognormality may be a good approximation. Fourthly, even if there is no problem of approximation and bias here, there certainly is such a problem for the AAID. What this means is that estimates of those inequality measures that are effectively functions of high-order moments of the distribution may be substantially biased if they are computed on the assumption that the distribution is Λ rather than LG.¹¹⁾ Fifthly, although all mean-independent inequality measures are (trivially) ordinally equivalent¹²⁾ to the variance of logarithms on the hypothesis they are certainly not cardinally equivalent. As we shall see in section 6, the cardinal properties are also important.

Bearing in mind these warnings, let us then look at the ordinal structure of inequality. First of all, from page 8 it is obvious that inequality in the observed distribution of lifetime income at age must exceed that in the subjective distribution of lifetime income or that of current income at that age. Next let us use the following two simple lemmas.

Lemma 2. For a sufficiently long horizon there must exist some θ such that $\sigma^2(\theta) > \hat{\sigma}^2(\theta, \bar{\theta})$. Where it exists, the critical age $\hat{\theta}$ at which $\sigma^2(\theta) = \hat{\sigma}^2(\theta, \bar{\theta})$ is given in the infinite horizon case by $\hat{\theta} = 1/2\rho - \sigma_0^2/v^2$.

The proof of this is trivial given lemma 1 and the fact that $\sigma^2(\theta) = \sigma_0^2 + v^2\theta$.

Lemma 3. Given an infinite horizon, for at least some values of θ , $\hat{\sigma}^2(\theta, \infty) > \sigma_W^2$, the variance of logincome in the AAID for every value of ρ, γ . Where it exists, the critical age $\tilde{\theta}$ at which $\tilde{\sigma}^2(\tilde{\theta}, \infty) = \sigma_W^2$ is given by $\theta = 1/\gamma - 1/2\rho$.

Proof. We know (from Cowell (1975)) that the variance of logincome in the AAID is $\sigma_W^2 = \sigma_0^2 + v^2/\gamma$. Hence, using equation (13):

$$(14) \quad \tilde{\sigma}^2(\theta, \infty) - \sigma_W^2 = v^2[\theta + \frac{1}{2\rho} - \frac{1}{\gamma}]$$

which must eventually be positive as θ gets large. Now if $\gamma > 2\rho$ there will exist no θ such that the RHS of expression (14) is zero. However, for $\gamma \leq 2\rho$, this is obviously given by $\theta = 1/\gamma - 1/2\rho$, for then $\tilde{\sigma}^2(\theta, \infty) = \sigma_0^2 + v^2/\gamma$. Q.E.D.

Hence, for any cohort older than some age $\hat{\theta}$ observed current income inequality must exceed subjective lifetime income inequality; $\hat{\theta}$ obviously is greater the greater is income mobility, and the lower the rate of pure time preference. Inequality of observed lifetime income will exceed inequality in the AAID for all cohorts at least as old as $\tilde{\theta}$. This result is particularly interesting if we consider the initial age group $\theta = 0$. Recall that $\rho = \rho_1 + \gamma_1$, the pure rate of time preference plus the survival probability parameter. We also have $\gamma = \gamma_1 + \gamma_2$, where γ_2 is the exponential rate of growth of new entrants to the population. Hence Lemma 3 implies that $\tilde{\sigma}^2(0, \infty) > \sigma_W^2$ as $\frac{1}{2}[\gamma_2 - \gamma_1] > \rho_1$.

For a large rate of growth of new entrants and/or small force of mortality and/or small rate of time preference, observed inequality of lifetime income of a new-born cohort exceeds observed inequality of (standardised) current income in the whole population. But if pure time preference is non-negative and the rate of growth of new entrants falls short of the force of mortality, observed inequality of lifetime

income (viewed at age zero) must be less than observed inequality of current income in the whole population.

The last assertion slipped in one further point, since it will be noticed that the word "standardised" was omitted. The reason for this is that $\sigma_W^2 \leq \sigma_T^2$, the variance of the logarithms in the OAID¹³).

The mean and variance of logincome in the OAID are in fact given by

$$(15) \quad \mu_T = \int_0^{\infty} \mu(\theta) g(\theta) d\theta$$

$$(16) \quad \sigma_T^2 = \int_0^{\infty} \sigma^2(\theta) g(\theta) d\theta + \int_0^{\infty} [\mu(\theta) - \mu_T]^2 g(\theta) d\theta$$

where $g(\theta)$ is the frequency distribution of ages of the population over $[0, \infty)$, taken to be $\frac{1}{\gamma} e^{-\gamma\theta}$ in our earlier discussion. The second term in the RHS of (16) may be interpreted as a "variance between groups" component of total variance of logincome, σ_B^2 let us say, while the first (within groups) term is the variance of logs of the AAID. So (16) can also be written

$$(17) \quad \sigma_T^2 = \sigma_W^2 + \sigma_B^2$$

The use of (17) in conjunction with our earlier results using σ_W^2 as a basis of comparison leads us to some obvious further conclusions about the relative magnitudes of inequality in lifetime income and in current income, though it should be borne in mind that there is no reason to suppose that the OAID is also approximately lognormal, since this will depend crucially on the behaviour of the cross-section profile $\mu(\theta)$.

5. GENERALISATIONS OF THE DISTRIBUTION PROCESS

So far the structure of lifetime inequality has been carried out using the restrictive assumptions about the shock sequence $u(t)$ for any projection of individual incomes: (i) homoscedasticity; (ii) absence of serial correlation; (iii) absence of correlation with incomes. In this section we shall examine the implications of $\sigma^2(\theta)$, $\hat{\sigma}^2(\theta, \bar{\theta})$ and $\tilde{\sigma}^2(\theta, \bar{\theta})$ of relaxing each one of these assumptions.

Consider first heteroscedastic, non-autocorrelated, and non-income-correlated disturbances. We shall continue to assume stationarity of the process in historical time so the only modification is to rewrite (5) as

$$(18) \quad u(t) \sim N(0, v^2(t))$$

It is evident that the integrated solution to (4) for the stationary case is

$$(19) \quad x(t) - x(\theta) = \mu(t) - \mu(\theta) + \int_{\theta}^t u(\tau) d\tau$$

where $x(t) \equiv \log y(t)$. From the assumptions about the absence of correlation on $u(t)$ one may then immediately see that

$$\sigma^2(t) = E[x(t) - \mu(t)]^2 = \sigma^2(\theta) + \int_{\theta}^t v^2(\tau) d\tau$$

This obviously involves only a trivial extension of the basic model. However if, as seems plausible, the variance of the shock process declines with age, so that chance has a smaller role to play in income determination in latter life, then obviously profiles of the variance of observed logincome will be concave rather than linear.

The modifications required for $\hat{\sigma}^2(\theta, \bar{\theta})$ and $\tilde{\sigma}^2(\theta, \bar{\theta})$ are immediate.

We have

$$(20) \quad \hat{\sigma}^2(\theta, \bar{\theta}) = \int_{\theta}^{\bar{\theta}} v^2(t) [e^{2\rho[\theta-t]} - 2e^{\rho[2\theta-t-\bar{\theta}]} + e^{2\rho[\theta-\bar{\theta}]}] dt / [\rho\Omega(\rho)]^2$$

If we differentiate the equation (20) with respect to θ we find that $\partial \hat{\sigma}^2(\theta, \bar{\theta}) / \partial \theta = v^2(\theta) + 2\hat{\sigma}(\theta, \bar{\theta}) / \Omega(\rho)$ so that whatever the profile of the variance of the disturbance term $u(t)$ in this model, the first part of Lemma 1 still holds.

Now take the case of simple autocorrelation. We may introduce this by supposing the disturbance term to behave as follows

$$(21) \quad \frac{d}{dt} u(t) = \xi u(t) + u'(t), \quad t \in [\theta, \bar{\theta}]$$

where $u'(t)$ is independently normally distributed $N(0, v^2)$ and $u(\theta)$ is normally distributed $N(0, v_{\theta}^2)$. From (21) we may immediately write

$$(21') \quad u(t) = [u(\theta) + \int_{\theta}^t e^{-\xi\tau} u'(\tau) \cdot d\tau] e^{\xi t},$$

which in turn gives us

$$E x(t) \mid x(\theta), u(\theta) = \mu(t) - \mu(\theta) + x(\theta) + u(\theta) e^{\xi t}$$

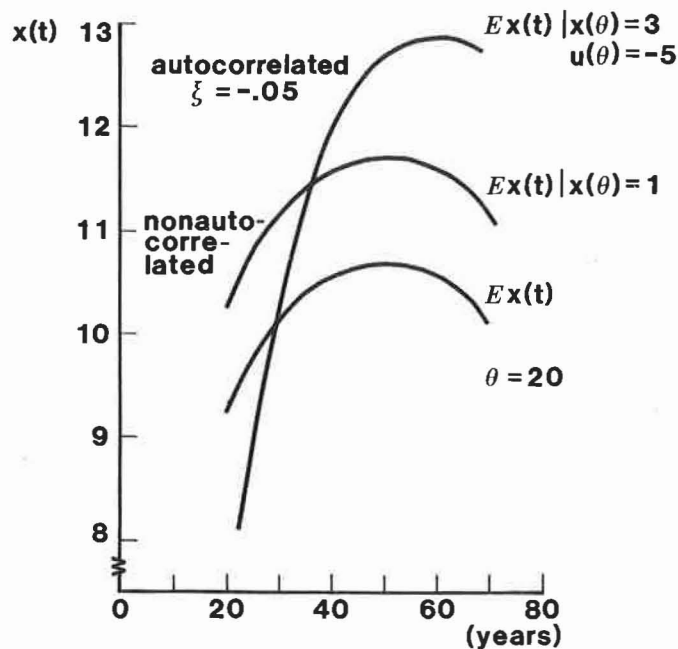
$$E \frac{dx(t)}{dt} \mid x(t), u(t) = \frac{d\mu(t)}{dt} + \xi u(t) e^{\xi t}$$

Now we see that the expected log-income profile facing an individual is generated by three parameters $\mu(\theta), x(\theta), u(\theta)$. The first is the log-median income in the cohort when he enters the population. The second is the particular income with which he starts out. The third characterises the rate of growth of his income. In the non-autocorrelated model

$u(\theta)$ will affect the size of the income he receives in the first instant of his life after age θ , but not subsequent expected growth rates. It is easy to see that, under those circumstances and with given $x(\theta)$, lifetime profiles of expected logincome corresponding to different initial shocks $u(\theta)$ will simply be uniform vertical displacements of each other in (θ, x) -space. In the autocorrelated model this is not so. Initial mobility affects future expected growth.

The interpretation of this is simple. We are now able not only to characterise random influences that operate on individual incomes as they develop over time, and an individual's "good fortune" in starting

Figure 2. Expected Profiles of Logincome

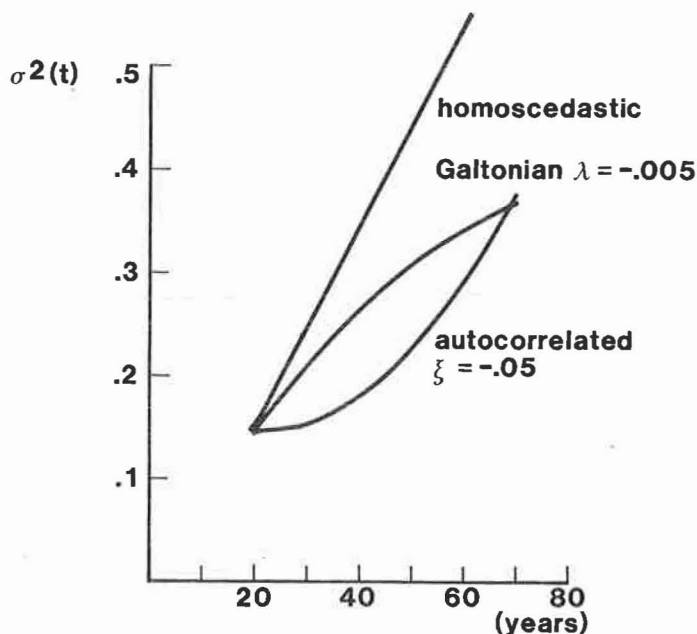


out in life (his initial logincome, $x(\theta)$) - but also systematic differences operating on his income. This is not only in the more obvious ways - success breeding success, mediocrity breeding mediocrity, the cumulative effects of illness and disabilities and so on. It also captures interoccupational differences in income and in income growth within a model that does not explicitly differentiate the population by occupational grouping. The differential growth in incomes for autocorrelated and non-autocorrelated system is shown in Figure 2.

We now form the expected logincome profiles to the dynamics of the autocorrelated model. From equation (21) we may derive

$$(22) \quad x(t) = \mu(t) + x(\theta) - \mu(\theta) + u(\theta)[e^{\xi[t-\theta]} - 1]/\xi + \int_{\theta}^t u'(\tau)[e^{\xi[t-\tau]} - 1]d\tau/\xi$$

Figure 3. Profiles of Variance of Logincome



Using (22) to calculate the variance of logincome at any future age t (taking $\theta = 0$ for convenience here) we find

$$\sigma^2(t) = \frac{v^2}{\xi^3} \left[\frac{1}{2} e^{2\xi t} - 2e^{\xi t} + 1 \right] + \frac{1}{2} + \xi t + \frac{v_0^2}{2\xi} [e^{2\xi t} - 1] + \sigma_0^2,$$

so that autocorrelation obviously introduces a nonlinearity into the variance profile. This is illustrated in Figure 3.

We also use (22) to derive

$$x(\theta, \infty) = M(\theta, \infty) + x(\theta) + [u(\theta) + \int_{\theta}^{\infty} e^{\rho[\theta-t]} u'(t) dt] / [\rho - \xi],$$

$$\hat{\sigma}^2(\theta, \infty) = \text{var } x(\theta, \infty) | x(\theta), u(\theta) = \frac{v^2/2\rho}{[\rho - \xi]^2},$$

and

$$\tilde{\sigma}^2(\theta, \infty) = \sigma^2(\theta) + v_0^2 e^{2\xi\theta} + v^2 [1/2\rho + e^{\frac{2\xi\theta-1}{2\xi}}] / [\xi - \rho]^2.$$

Note that the relationship between the observed and the self-perceived variance is almost exactly the same as that which obtained in the simple model - the discrepancy between the two is now larger since the individual has information about the realised value of $u(\theta)$ which now influences his future prospects of "success" or "failure". Notice that

if $v_0^2 = 0$ and $v^2 = v_1^2 \xi^2$ for $\xi < 0$, $\frac{\partial \tilde{\sigma}^2}{\partial \xi} < 0$, $\frac{\partial \hat{\sigma}^2}{\partial \rho} < 0$, but that $\frac{\partial \tilde{\sigma}^2}{\partial \xi} \Big|_{d\sigma^2(\theta)=0} = v^2 [\rho^2 - \xi\rho]^{-3} > 0$. Thus, whereas we expect an increase in the

"strength" of the autocorrelation effect (an increase in ξ , to increase $\tilde{\sigma}^2$ and $\hat{\sigma}^2$, given a particular profile of the variance of logincome, when we allow for the effect of the increase in ξ upon the variance of logincome itself, the overall result is a reduction in $\tilde{\sigma}^2$ and $\hat{\sigma}^2$.

The final extension of the basic model that we shall examine is that of correlation of $u(t)$ with $y(t)$. Specifically if $u(t)$ is negatively cor-

related with $x(t) - \mu(t)$ the model is one of Galtonian regression toward the mean in which equation (4) is modified to

$$(23) \quad \frac{d}{dt} x(t) = \lambda [x(t) - \mu(t)] + \frac{d}{dt} \mu(t) + u'(t)$$

where $\lambda < 0$, and $u'(t)$ is distributed as in (5). Integrating (23) we have

$$(24) \quad x(t) = \mu(t) + [x(\theta) - \mu(\theta)] e^{\lambda[t-\theta]} + \int_{\theta}^t e^{\lambda[t-\tau]} u'(\tau) d\tau$$

Remembering that in the case we are interested in, $\lambda < 0$, the interpretation of this model is evident from (23) or (24). There are built-in mechanisms such that the very rich are likely to receive less-than-average proportionate increases in income, and the poor are likely to receive greater-than-average increases in income. Put another way, the effect of an abnormally lucky (or unlucky) start in life is systematically damped away - and similarly for favourable and unfavourable economic shocks later in life. Evidently this process will result in a somewhat different profile for dispersion. Note that we can derive this profile from (24) directly. Suppose that $u'(\theta)$ is independently normally distributed with variance parameter v^2 - Cf. the specification of $u(\theta)$ in the simple, nonautocorrelated non-regressive model. Then we find:

$$(25) \quad \sigma^2(t) = \sigma^2(\theta) e^{2\lambda t} + [e^{2\lambda t} - e^{2\lambda \theta}] v^2 / 2\lambda$$

Note that as $\lambda \rightarrow 0$, equation (25) degenerates to (8). If λ is very large and negative, then σ^2 may be a decreasing function of θ . In the "normal" case, where $\sigma^2(\theta)$ is an increasing function $\sigma^2(\theta)$ is concave in θ . The profile defined by (25) is illustrated in Figure 3.

Now let us turn to lifetime income. Using (11) and (24) we find

$$x(\theta, \infty) = M(\theta, \infty) + [\rho x(\theta) - \lambda \mu(\theta) + \int_{\theta}^{\infty} \rho e^{\rho[\theta-t]} u(t) dt] / [\rho - \lambda].$$

Note that $x(\theta, \infty) = M(\theta, \infty) + \mu(\theta)$, as before, but that now

$$(26) \quad \hat{\sigma}^2(\theta, \infty) = [1 - \lambda/\rho]^{-2} v^2 / 2\rho,$$

$$(27) \quad \tilde{\sigma}^2(\theta, \infty) = [1 - \lambda/\rho]^{-2} [\sigma^2(\theta) + v^2 / 2\rho]$$

where $\sigma^2(\theta)$ is given by equation (25). Hence the broad conclusions about the observed and the self-perceived variance remain, although in this case $\sigma^2(\theta)$ is deflated by a factor that is larger, the larger is $-\lambda$, the Galtonian regression parameter. Observing that $\sigma^2(\theta)$ increases with λ , we find that the stronger is the effect of regression toward the mean, the less is the dispersion of self-perceived lifetime income, which is what we would expect. However, consider the size of observed variance cohort by cohort. By differentiating (27) with respect to θ we find

$$(28) \quad \frac{\partial}{\partial \theta} \tilde{\sigma}^2(\theta, \infty) = [2\lambda\sigma_0^2 + v^2] \frac{e^{2\lambda\theta}}{[1 - \lambda/\rho]^2}$$

Evidently if λ is sufficiently negative, inequality of lifetime income may actually decline with age, a result that is in sharp contrast with the other cases we have considered where observed inequality of lifetime income always rises the older the cohort considered.¹⁴⁾

6. REDISTRIBUTION

Let us now turn to the problem of measuring redistribution, for example by means of an income tax system. This involves measuring the change in inequality between two distributions - "gross of tax" and "net of tax". A preliminary question we should consider is whether in moving from an untaxed to a taxed state of the world we may allow for shifts in individuals' income profiles as a result of the incentive effects of taxation. Unfortunately, in a general model of the individual life cycle it is virtually impossible to predict even the direction of such shifts with the a priori information at our disposal. When it comes to an aggregative analysis over persons with heterogeneous preferences and endowments, this shift effect cannot be satisfactorily allowed for in practice. We shall therefore ignore the distinction between "income distribution gross of tax" and "income distribution without the tax". Straightaway we need the following definition.

Definition 4. Let $I(y)$ be an inequality measure mapping an income distribution $\{y, F(y)\}$ on to the real line, where $F(\cdot)$ is a distribution function. Let $F_G(y(\theta))$ and $F_N(y(\theta))$ be the group of tax and net of tax distributions of current income respectively in the θ -cohort. Then $i^\theta = I(F_N(y(\theta)), y(\theta)) / I(F_G(y(\theta)), y(\theta))$ is the index of redistribution, or the incidence index relative to the inequality measure I for cohort θ 's current income.

Note that the smaller is i^θ , the more effective is the redistribution and if estimated i^θ is less than true i^θ , redistribution has been over-estimated. This remark also applies to the next definition relating to redistribution of lifetime income.

Definition 5. Let I be an inequality measure, and $F_G(y(\theta, \bar{\theta}))$, $F_N(y(\theta, \bar{\theta}))$ be the distribution of $y(\theta, \bar{\theta})$ gross of tax and net of tax respectively. Then $\lambda(\theta, \bar{\theta})$, the index of incidence of the tax relative to the inequality measure I defined on lifetime average income for age group θ is $\lambda(\theta, \bar{\theta}) = I(F_N(y(\theta, \bar{\theta})), y(\theta, \bar{\theta})) / I(F_G(y(\theta, \bar{\theta})), y(\theta, \bar{\theta}))$. When $\bar{\theta} = \infty$, we shall abbreviate this to λ^θ .

In addition we may define in an obvious manner i^T and i^A - the index of incidence in the OAID and AAID respectively.

We now formalise the relationship between the gross and net distributions by specifying a tax function. This is taken to be the so-called Constant Residual Progression Tax Function defined by¹⁵⁾

$$(29) \quad T(y) = y - Ay^b$$

where $A > 0$ and $b \in [0, 1]$ are parameters. The index of progression is $1-b$. Apart from the fact that this functional form is a fairly good approximation to many progressive tax schedules in the real world, it has the added attractions that net of tax distributions with lower b values Lorenz-dominate distributions with higher b -values, and that if the gross-of-tax distribution is lognormal¹⁷⁾ so also is the net-of-tax distribution. Using the earlier notation, therefore, we have gross income and gross lifetime income distributed as

$$(30) \quad y(\theta) \sim \Lambda(\mu(\theta), \sigma^2(\theta))$$

$$(31) \quad y(\theta, \infty) \sim \Lambda(M(\theta, \infty) + \mu(\theta), \tilde{\sigma}^2(\theta, \infty))$$

and thus, writing $a \equiv \log A$, we have the net-of-tax distributions corresponding to (30) and (31) given respectively by

$$\Lambda(b\mu(\theta) + a, b^2\sigma^2(\theta)) \text{ and } \Lambda(bM(\theta, \infty) + b\mu(\theta) + a, b^2\sigma^2(\theta, \infty)).$$

Once again, given the deliberate oversimplification over our dynamic model, the underlying preference structure and the prevailing market conditions, we have the tremendous advantage of carrying out the analysis in terms of the parameters of the lognormal. More specifically it was shown in Cowell (1975) that all inequality measures in common use for any distribution $\Lambda(m, s^2)$ as $\chi(m)\phi(s^2)$, or in the mean-independent case as $I = \phi(s^2)$. Now the behaviour of the incidence index will obviously depend on the nature of the function $\phi(\cdot)$ and is sensitive to the cardinalisation of the inequality measure. In the absence of a satisfactory theory of the cardinal (as apposed to ordinal) properties of inequality measures, we shall use the standard cardinalisations in the literature. For the lognormal, these are summarized in Cowell (1975, 1977). The crucial property of the function $\phi(s^2)$ relates to its elasticity $\eta(\sigma^2) = [s^2/\phi(s^2)]\partial\phi(s^2)/\partial s^2$, and we may borrow results on this from the related problems in Cowell (1975). We now use the results of section 4 to establish the following simple results on incidence indices.

Theorem 1. Given an infinite horizon, a mean-independent inequality measure written as a function $I = \phi(s^2)$ in the case of the distribution $\Lambda(m, s^2)$ then;

(a) for every age group above $\hat{\theta} = 1/2\rho - \sigma_0^2/v^2$ the apparent reduction in dispersion inferred from the measured inequality change on current income i^θ is greater/less than the true reduction in self-perceived dispersion according as I is increasingly/decreasingly elastic in s^2 ;

(b) for any age group that may exist below $\hat{\theta}$, the inequality in part (a) is exactly reserved.

Proof. (a) From Lemma 2 we know that $\theta > \hat{\theta} \Rightarrow \sigma^2(\theta) > \hat{\sigma}^2(\theta, \infty)$. Now $i^\theta = \phi(b^2_{\sigma^2}(\theta))/\phi(\sigma^2(\theta))$ and the actual reduction in self perceived lifetime inequality is given by $\phi(b^2_{\hat{\sigma}^2}(\theta, \infty))/\phi(\hat{\sigma}^2(\theta, \infty))$. The proof then follows immediately from a comparison with the proof of Theorem 1* in Cowell (1975)

(b) where $\hat{\theta} > 0$, the case of $\theta < \hat{\theta}$ follows by symmetry from part (a). Q.E.D.

Theorem 2. Given an infinite horizon, a particular age group θ , a mean-independent inequality measure $I = \phi(s^2)$, for the distribution $A(m, s^2)$, and the incidence index defined on the AAID, i^A , then:

(a) i^A under/overestimates i^θ as I is increasingly/decreasingly elastic in s^2 if $\theta > \tilde{\theta} = 1/\gamma = 1/2\rho$.

(b) i^A under/overestimates i^θ as I is decreasingly/increasingly elastic in s^2 if $\theta \leq \tilde{\theta}$ where such values of θ exist.

(c) The absolute size of the bias increases with v^2 .

Proof (a) From Lemma 3 we observe that for $\tilde{\theta} > \tilde{\theta}, \tilde{\sigma}^2(\theta, \infty) > \sigma_w^2$. Since $i^A = \phi(b^2_{\sigma_w^2})/\phi(\sigma_w^2)$ and $i^\theta = \phi(b^2_{\tilde{\sigma}^2}(\theta, \infty))/\phi(\tilde{\sigma}^2(\theta, \infty))$, the proof follows immediately from a comparison with the proof of Theorem 1 in Cowell (1975).

(b) Follows by symmetry from part (a).

(c) Follows from the proof of Theorem 1* in Cowell (1975). Q.E.D.

Theorem 1 tells us what happens if we try to use the redistribution index on current income in some cohort as an estimate of the change in self-perceived inequality of lifetime income prospects as viewed by a member of that cohort. For example, if we take the Gini concentration index (decreasingly elastic) then for the middle aged, the net current income/group current income ratio of this statistic will always be less than the post-tax/pre-tax ratio relating to the Gini coefficient of the subjective dispersion of lifetime income as perceived by some one member of the middle-aged group.

Theorem 2 gives the following information. For any age group, as we have seen, we can construct an index of the redistributive effect of the income tax that is based on lifetime average income (using the wergild concept) rather than current income. To do this, however, we need to make some assumption about the value of such quantities as the subjective rate of time preference. Suppose we use as an estimate of this "true" incidence in each age group the incidence index computed from the AAID. Then Theorem 2 tells us for which age group i^A overestimates the redistributive effect of the tax, and for which age groups it provides an underestimate. For example, suppose we employ Atkinson's measure of inequality (decreasingly elastic in s^2) to compute incidence, and let $\gamma = 1/65$. Then if the discount rate is 2.5 %, i^A underestimates the redistributive effect of the income tax for all age groups younger than 45 years (and overestimates it for the over-45's) - if the discount rate increases to 5 %, the critical age increases to 55 years; if ρ is as high as 10 %, the critical age is 60 years... and so on.

Theorem 2 is of limited value for three reasons. (1) It deals only with the simplest model of income dynamics. (ii) It uses an arbitrary value

of θ for comparison using χ^θ . (iii) It uses the AAID rather than the OAID as a standard for comparison. We shall now deal with each of these shortcomings.

Firstly, let us consider the various extensions of the basic model discussed in section 5. The behaviour of $\tilde{\sigma}^2(\theta, \infty)$ is of key importance in each case, and for the autocorrelated and the regressive models we can immediately state the following results.

Theorem 3. Given an infinite horizon, a mean-independent inequality measure $I = \phi(s^2)$, and the associated incidence index defined on life-time average income for each age group θ, χ^θ , then (a) for all age groups, χ^θ increases/decreases with ξ according as ϕ is increasingly/decreasingly elastic; (b) for all age groups and for $\lambda < 0, \chi^\theta$ increases/decreases with λ according as ϕ is decreasingly/increasingly elastic; (c) for all age groups and for $\lambda = 0, \chi^\theta$ increases/decreases with ρ as ϕ is increasingly/decreasingly elastic; (d) for $\lambda < 0$, then there always exists an age group $\hat{\theta}$ such that for $\theta > \hat{\theta}$, the conclusion of (c) is exactly reversed.

Proof

(a) We use $\tilde{\sigma}^2$ as shorthand for $\tilde{\sigma}^2(\theta, \infty)$. It is sufficient to observe that $\partial \tilde{\sigma}^2 / \partial \xi < 0$. For we have

$$\frac{\tilde{\sigma}^2}{\chi^\theta} \frac{\partial \chi^\theta}{\partial \xi} = \left[\tilde{\sigma}^2 b^2 \frac{\phi'(b^2 \tilde{\sigma}^2)}{\phi(b^2 \tilde{\sigma}^2)} - \tilde{\sigma}^2 \frac{\phi'(\tilde{\sigma}^2)}{\phi(\tilde{\sigma}^2)} \right] \frac{\partial \tilde{\sigma}^2}{\partial \xi};$$

and the expression in square brackets is evidently positive/negative as ϕ is decreasingly/increasingly elastic.

(b) This follows immediately from the proof of part (a) once it is noted that $\partial \tilde{\sigma}^2 / \partial \lambda > 0$.

(c) From the discussion of the autocorrelated case it is found that $\partial \tilde{\sigma}^2 / \partial \rho = \frac{1}{2} v_1^2 [1/\rho \xi - \rho^{-2} + [e^{2\xi\theta} - 1]/2\xi^2]/[1-\rho/\xi]^3 < 0$. This holds also for the case $\xi = -\infty$, i.e. the nonautocorrelated case. Given this behaviour of $\tilde{\sigma}^2(\theta, \infty)$, the sign of $\partial \tilde{\sigma}^2 / \partial \rho$ may be determined in a manner analogous to that of parts (a) and (b).

(d) Observe that in the case of Galtonian regression, differentiating (27) we have

$$(32) \quad \partial \tilde{\sigma}^2 / \partial [1/\rho] = [1-\lambda/\rho]^{-3} [2\lambda e^{2\lambda\theta} \sigma_0^2 + v^2 [e^{2\lambda\theta} - \frac{1}{2} [1-\lambda/\rho]]]$$

Evidently, if $\lambda < 0$, it must be true that there exists $\hat{\theta} > 0$ such that $e^{2\lambda\hat{\theta}} = \frac{1}{2} [1-\lambda/\rho]$ and such that for $\theta > \hat{\theta}$, expression (32) is negative. Hence for $\theta > \hat{\theta}$, $\tilde{\sigma}^2$ increases with ρ . The remainder of the proof follows immediately in the manner of parts (b) and (c).

Q.E.D.

Let us consider the interpretation of these results. It is clear that for any age group if incidence is measured using the distribution of lifetime average income (based on the wergild concept) rather than actual current income, the size of the incidence index depends not only on objective data, such as the dispersion of incomes within the group, but also on subjective quantities such as the rate at which future incomes are discounted. Thus the effectiveness in redistribution of the income tax system depends on the discount rate and, as we have seen, on the strength of the autocorrelation and regression effects. If the regression effect is nil, then the relationship is simple - even in the presence of autocorrelation in the shock system; for indices based on decreasingly elastic inequality measures (such as Atkinson or Gini), the measured value of

incidence decreases with $\rho - 1$, i.e. the higher the discount rate, the more effectively redistributive is the tax; the opposite conclusion holds for measures such as the logarithmic variance or the coefficient of variation which are increasingly elastic. The regression effect complicates the picture: while the "non-regression" result described in the last sentence may hold over a number of age groups, we can be sure that eventually it is reversed. Finally we note that as the regression effect or the autocorrelation effect becomes stronger (as $-\lambda$ or ξ increases) then according to incidence indices based on decreasingly elastic inequality measures the tax system becomes ceteris paribus more effective in terms of redistribution.

Now let us deal with the objection (ii) on page 116. We have observed that the dispersion of lifetime average income, $\tilde{\sigma}^2(\theta, \infty)$ varies with θ and thus the redistributive effect of taxation depends on which age group we examine. Hence Theorem 2 tells us that the relative magnitudes of $\tilde{\sigma}^\theta$ and $\tilde{\sigma}^A$ depends on the arbitrary θ chosen. There is obviously some attraction in choosing $\theta = 0$, but a newly-introduced tax of the form (29) will affect all age groups' lifetime inequality simultaneously. There is, therefore, a certain advantage in selecting a "representative age group", within which redistribution can be compared with apparent redistribution of current income in the economy as a whole. Once again we use the inequality measure $I = \phi(s^2)$ defined for the distribution $\Lambda(m, s^2)$.

Definition 6. Given a system of weights $\{w(\theta)\}$, and a particular function $\phi(\cdot)$, the representative age group θ' is defined implicitly by

$$\phi(\tilde{\sigma}^2(\theta', \infty)) = \int_0^\infty \phi(\tilde{\sigma}^2(\theta, \infty)) w(\theta) d\theta$$

The structure of weights may take various forms, although a natural one to adopt is $w(\theta) = \frac{1}{\gamma} e^{-\gamma\theta}$, i.e. weight according to the population density by age. However, the following result, which is a corollary of Theorem 3 of Cowell (1975) does not depend on the particular functional form of the age structure. In addition, it is true for the more general models of the stochastic process.

Lemma 4. Let $i^W = \phi(b^2 \tilde{\sigma}^2(\theta', \infty)) / \phi(\tilde{\sigma}^2(\theta', \infty))$ and $i^{A'} = \phi(b^2 \sigma_{A'}^2) / \phi(\sigma_{A'}^2)$, where $\sigma_{A'}^2 = \int_0^\infty \tilde{\sigma}^2(\theta, \infty) g(\theta) d\theta$, and $g(\theta)$ is the age distribution. Then $i^W < i^{A'}$ if ϕ is concave and decreasingly elastic and $i^W > i^{A'}$ if ϕ is convex and increasingly elastic.

In this Lemma $\sigma_{A'}^2$ is the average income in the whole population if crude aggregation regardless of age were carried out. Lemma 4 shows the relationship between incidence defined using the distribution and the more suitable weighted index.

Theorem 4. In the case of no Galtonian regression, if ϕ is increasingly elastic and convex, then i^A , the incidence index based on the AAID underestimates the value of the ideal weighted wergild incidence index where the age structure is used as the system of weights.

Proof. If ϕ is convex, then $\phi(\sigma_W^2) = \phi(\int_0^\infty \sigma^2(\theta) \gamma e^{-\gamma\theta} d\theta) < \int_0^\infty \phi(\sigma^2(\theta)) \gamma e^{-\gamma\theta} d\theta$. Since ϕ is an increasing function, we have $\int_0^\infty (\sigma^2(\theta)) \gamma e^{-\gamma\theta} d\theta < \int_0^\infty \phi(\tilde{\sigma}^2(\theta, \infty)) \gamma e^{-\gamma\theta} d\theta = \phi(\tilde{\sigma}^2(\theta', \infty))$. Hence $\sigma_W^2 < \sigma^2(\theta', \infty)$. Recalling that $i^A = \phi(b^2 \sigma_W^2) / \phi(\sigma_W^2)$, and using the definition of i^W in Lemma 4 the increasing elasticity of ϕ implies $i^W < i^A$.

Q.E.D.

Hence, taking the ideal weighted wergild-based index as a datum, for inequality measures such as the logarithmic variance or Herfindahl's measure, the incidence index computed from the adjusted aggregate income distribution always underestimates the reduction in inequality actually achieved by the income tax. Unfortunately it is not possible in general to derive a symmetrical result for concave inequality measures for this particular theorem.

Now let us turn to the pre and post-tax OAID. Whereas the log median of net income in the OAID (given by $b\mu_T + a$; see equation (15)) closely resembles that in any one ASID, this is not true for the variance of logs of net income in the OAID. To get this we do not take a simple function of the pre-tax variance σ_T^2 . Instead the dispersion of net incomes in the OAID is given by $b^2\sigma_W^2 + b\sigma_B^2$ (see equations (16 and (17)). Now, as stated on page 104, there is no a priori reason for presuming the OAID to be approximately lognormal. However, some empirical studies suggest that nevertheless lognormality may be a reasonable assumption. If this is so, then the following theorem is relevant.

Theorem 5. Let θ^* be that value of θ such that $\bar{\sigma}^2(\theta, \infty) = \sigma_W^2 + \sigma_B^2/b$.

Let i^θ denote the incidence index for lifetime income in age group θ , and i^T denote the incidence index in the OAID, and let the incidence indices be defined relative to some mean-independent inequality measure $I = \phi(s^2)$ for the distribution $\Lambda(m, s^2)$

(a) if I is decreasingly elastic, i^T understates the redistributive effect i^θ in age group θ for all $\theta < \theta^*$

(b) if I is increasingly elastic, then i^T understates the redistributive effect i^θ in age group θ for all $\theta > \theta^*$

(c) this bias increases with v^2 .

Proof. (a) It is sufficient to observe that if $\delta, \Delta > 0$ the decreasing elasticity of ϕ implies that $[x+\delta x] \frac{\phi'(x+\delta x+\Delta)}{\phi(x+\delta x+\Delta)} < [x+\delta x+\Delta] \frac{\phi'(x+\delta x+\Delta)}{\phi(x+\delta x+\Delta)}$

$$< x \frac{\phi'(x)}{\phi(x)}$$

Comparison of this expression with that in the proof of Theorem 1* in Cowell (1975) shows that this condition is sufficient to ensure that

$$\frac{\phi(b^2\sigma_W^2 + b\sigma_B^2)}{\phi(\sigma_W^2 + \sigma_B^2)} > \frac{\phi(b^2\sigma^2(\theta, \infty))}{\phi(\tilde{\sigma}^2(\theta, \infty))}$$

for all $\theta < \theta^*$.

(b) This follows by symmetry from part (a)

(c) As for Theorem 1 in Cowell (1975). Q.E.D.

Notice that because of the presence of the "between-ages" component we do not have a symmetrical result for increasingly-elastic I in part (a) and decreasingly elastic I in part (b). In fact it is possible to show that for some age groups i^T understates the true amount of re-distribution i^θ for every age group in the population. As an example of this, we state the following theorem.

Theorem 6. For the inequality measure defined as the variance of the logarithms of incomes i^T underestimates the amount of redistribution implied by i^θ for every age group θ if $\sigma_B^2 > 0$.

Proof. $\forall \theta: i^\theta = b^2 \tilde{\sigma}^2(\theta, \infty) / \tilde{\sigma}^2(\theta, \infty) = b^2$

$$i^T = \frac{b^2\sigma_W^2 + b\sigma_B^2}{\sigma_W^2 + \sigma_B^2} = b^2 + \frac{[b-b^2]\sigma_B^2}{\sigma_W^2 + \sigma_B^2}$$

$$0 < b < 1 \text{ and } \sigma_B^2 > 0 \quad i^T > b^2 = i^\theta.$$

Q.E.D.

Notice that $[b-b^2]\sigma_B^2/[\sigma_W^2 + \sigma_B^2]$ provides a convenient measure of bias. This bias increases with σ_B^2 (i.e. with the variance of average log-income over the life cycle), is zero when $\sigma_B^2 = 0$, and is independent of θ . Observe, too, that Theorem 6 does not make the assumption that the OAID is lognormal. Finally we can easily see the following important corollary.

Corollary to Theorem 6. Consider an inequality index χ^w defined as in Lemma 4 where w is any weighting function. Then if I is taken as the variance of logincome, i^T always understates the amount of redistribution according to χ^w . So, whatever we consider to be the "right" combination of the χ^θ , i^T always indicates that there is less redistribution than there "really" is. Thus we can see that, loosely speaking, the presence of "drift" in the income generation process makes it more likely that i^T will understate the amount of redistribution vis à vis any ideal weighted incidence index.

Footnotes

- 1) The reasoning behind these definitions are given in Cowell (1979).
- 2) See, for example, Creedy (1977).
- 3) Where possible we will use $y(t)$ as a shorthand form for this.
- 4) See Nagatani (1972).
- 5) Weiss (1971) gives a further rationale for the use of discounted lifetime utility.
- 6) See, for example, Cowell (1977, p. 6).
- 7) We shall consider the implications of relaxing these later in the paper.
- 8) Secular income growth, represented by an upward drift in all the $\mu(\theta)$ s in historical time can readily be incorporated.
- 9) See the former reference for proofs of the assertions in this paper about the LG-distribution.
- 10) See Cowell (1975, pp. 360, 361).
- 11) For example, using Atkinson's Social Welfare Function inequality index with a high value of inequality aversion may lead to such a bias.
- 12) For a discussion of ordinal equivalence, and formulae for inequality measures in the case of the lognormal see Cowell (1977).
- 13) An equivalent inequality will hold for any inequality measure even if the OAID departs substantially from lognormality.
- 14) There is one unlikely exception. Differentiation of $\tilde{\sigma}^2(\theta, \infty)$ in the autocorrelated model reveals that this perverse result would be obtained if these were very strong negative serial correlation.
- 15) See Jakobsson (1976) and Cowell (1975).
- 16) See Cowell (1977, Chap. 4) and Aitchison and Brown for the following standard results on the lognormal.

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PART II

ECONOMIC WELFARE

CHARACTERISTICS OF THE LOWEST INCOME QUINTILE IN FINLAND IN 1981

by Risto Sullström

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1. INTRODUCTION

Questions regarding the well-being of the lowest income quintile are investigated in this study in order to shed some light on the situation of low income recipients for the discussions on redistribution of income in Finland. The aim of the study is twofold. First, consideration is given to several theoretical and data-related issues that are encountered in research on low income questions. Second, the study seeks to characterize the distinctive features of the households belonging to the lowest income quintile and compare them with higher income households, without taking a stand on the success or failure of the incomes distribution policy. The data used is from a 1981 household survey.

Unlike in many other countries, for example in England and in the USA, in Finland there are no official estimates made of an income level below which households are defined as being poor. This study also bypasses the problem of how to estimate a certain figure for the poverty line and other related controversial questions. Instead, low income households will be those categorized according to the lowest quintile point below which 20 % of the households remain. The calculations are then carried out using four different income definitions utilizing disposable income.

Because low incomes and the health situation are apparently intertwined with the well-being issue, the households are divided in the study according to whether they include members who are disabled or chronically ill. Other background variables used for the household were the age of the head of the household, the life cycle of the household, the socio-economic status of the household, and the form of the local community.

First, the central questions associated with poverty research and the basics of poverty indexes will be dealt with, and the data used will be described. Then the results of the indexes obtained will be assessed, and the characteristics of certain household groups with low incomes, medium incomes (2nd-4th quintiles) and high incomes (5th quintile) will be compared and contrasted. Special attention will be paid during the analysis to how the choice of the income definition affects the results.

The study presents only rough descriptions so that the results clearly do not enable drawing any conclusions of the like that "household X should receive aid of p markkaa so that its situation with respect to utility corresponds to some other household Y" or "that income compensation is a better way of dealing with problems of low economic welfare than price compensation". This type of analysis requires better knowledge of household preferences, several years worth of cross-sectional data as well as more detailed requirements for the contents of the data. But because structural and distributional changes occur relatively slowly, many of the features observed in the study can perhaps give an indication of the current trends or at least pose some questions for later investigations.

The main sources for the empirical calculations have been the investigations regarding social welfare undertaken by the research department of the Ministry of Social Affairs and Health for the year 1981. Of these, particular mention should be made of the research report specifying the incomes and expenses of disabled and chronically ill individuals (The Impact of Social Security in 1981, part 10, 1986).

2. THE DEFINITION OF "LOW ECONOMIC WELFARE"

2.1. The origins of poverty research

In recent years poverty research has focused especially on developing countries; in industrialized countries the center of attention has been the so-called "new poor". In poverty research at a very general level, without taking into consideration here whether poverty even exists, the evaluation can be looked at from the point of view of the standard of living of an individual or a household, which emphasizes the significance of consumption, or then it can be looked at using some other alternatively defined income earning unit which can be understood as being entitled to a certain minimum level of resources (cf. Atkinson 1985). In empirical studies the problems with these approaches include the lack of relevant data and the drawbacks that accompany use of surrogate variables.

Research on low economic welfare can thus seek to find answers to the following questions:

- 1) what is an appropriate definition for the income used as the basis for calculations,
- 2) how does one determine the income level below which households are defined as poor or low income,
- 3) how are different households to be compared,
- 4) what is a suitable measure for this comparison?

2.1.1. Choice of the income definition

Instead of just one type of transformation of disposable income, this study uses four different ones:

per household, per household member, per OECD consumer unit, and per Tasku (Economic Planning Centre) consumer unit. These are used in various Finnish distribution studies, though they have seldom been the subject of direct comparison.

Disposable income is not necessarily the best basis for evaluation. First of all, it can temporarily fluctuate a great deal in a household. Part of the income can be carried forward for later consumption. The household may also have other benefits which are not calculated as disposable income. Likewise the impact of property income is not included as an income variable. Furthermore, precise measures of the concept of utility in economic theory cannot as such be defined. Indeed, this study makes no attempt to evaluate political alternatives.

2.1.2. Poverty line

Table 2.1. compares different cut-off points for distributions (5 %, 10 %, 15 %, 20 % and 25 %). First, it is evident that for all the data groups each of the first income quintiles includes about 375,000 households. Average disposable income per person in 1981 ranges from 12025 to 17001 markkaa according to the income definition, which in 1987 prices corresponds to between 17352-24532 markkaa.¹⁾ The lowest figure was obtained for disposable income per household member, where each household member received a weight of 1, while the highest figure was for disposable income per household. The poverty line (20 % point) for

1) The value for the consumer price index for 1987 was 144.3 (1981="100").

income per household was 30518 mk (44037 mk in 1987 prices) and for income per household member 15632 mk (22557 mk).¹⁾ The calculations as per consumer unit fell between these two extremes. Income defined as per household member emphasized the large households most clearly.

Income variables defined according to the health status gave slightly different population shares. The relative share of low income households with an ill or disabled member was higher than the average for the whole population (41.4 %) except for the Tasku income definition. On the other hand, this group's average values for the disposable income of low income households were larger than the corresponding figures for all the households. Evidently this group's low income households receive some sort of support that the other low income households do not.

Furthermore, this group's variation coefficients and Gini coefficients (column b) were smaller in the lower end of the distribution than for the corresponding total household figures (column a), which indicates a more even distribution of income. This result depends to some extent on the income definition used.

1) For calculation purposes the seven negative income figures in the data were set equal to zero. The quintile point for each income definition was then calculated so that the incomes (per household, per household member, and per consumer units) were placed in rank order. The lowest quintile was then the lowest 20 % of the households. This solution emphasizes households as the unit of assessment. Another way would have been to have defined the cut-off point according to the unit used, for example 20 % of the household members or consumer units. In the resulting calculations this affected to some extent the number of households ranked in each quintile and also the internal structure of the quintiles. The effect on the index calculations, however, was slight.

Table 2.1. Some characteristics of the lowest parts of income distribution.

Income definition	Cut-off point	Number of households		Share of (b) %	Mean income FMK		Disposable income per person FMK		Size of household		Gini coefficient		Quadratic coefficient of variation		Value of income at cut-off points (a)
		(a)	(b)		(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	
Disposable income per household	5 %	93688	34217	36.5	12104	12979	10551	10977	1.15	1.18	0.194	0.158	0.125	0.090	17134
	10 %	187732	90948	48.4	15762	16982	13967	15067	1.13	1.13	0.164	0.121	0.093	0.056	21234
	15 %	280988	141814	50.5	18308	19287	15626	16826	1.17	1.15	0.160	0.126	0.087	0.055	25861
	20 %	375105	187846	50.1	20797	21449	17001	17510	1.22	1.22	0.171	0.142	0.094	0.066	30518
	25 %	468600	228646	48.8	23167	23447	17917	17828	1.29	1.22	0.181	0.159	0.103	0.079	34590
	100 %	1873139	775570	41.4	59648	56795	23455	23087	2.54	2.46	0.301	0.317	0.296	0.336	362362
Disposable income per household member	5 %	93678	33168	35.4	7832	8198	7832	8198	3.16	3.44	0.154	0.144	0.084	0.080	10509
	10 %	187545	69795	37.2	9738	9819	9738	9819	3.30	3.41	0.137	0.127	0.067	0.061	12656
	15 %	281247	114829	40.8	11020	11150	11020	11150	3.30	3.28	0.135	0.127	0.063	0.057	14384
	20 %	374656	160565	42.9	12025	12288	12025	12288	3.28	3.21	0.133	0.125	0.061	0.054	15632
	25 %	468594	200367	42.8	12855	13156	12855	13156	3.21	3.11	0.132	0.122	0.059	0.052	16674
	100 %	1873139	775570	41.4	23455	23087	23455	23087	2.54	2.46	0.230	0.221	0.192	0.172	164732
Disposable income per OECD unit	5 %	93871	33198	35.4	10766	11223	8272	8702	2.33	2.44	0.166	0.153	0.097	0.087	14652
	10 %	187329	81307	43.4	13492	13990	10327	10886	2.39	2.40	0.143	0.121	0.074	0.057	17554
	15 %	281057	132210	47.0	15163	15627	11620	12356	2.41	2.28	0.132	0.113	0.064	0.049	19400
	20 %	374847	184752	49.3	16444	16877	12637	13432	2.41	2.24	0.128	0.111	0.059	0.046	21033
	25 %	468661	234314	50.0	17510	17873	13430	14211	2.44	2.25	0.126	0.112	0.056	0.045	22482
	100 %	1873139	775570	41.4	30535	29388	23455	23087	2.54	2.46	0.217	0.214	0.166	0.157	164732
Disposable income per Tasku unit	5 %	93710	24662	26.3	12169	12563	8462	8141	2.23	2.55	0.179	0.172	0.109	0.107	16784
	10 %	187708	63601	33.9	15503	16235	10310	10282	2.56	2.77	0.152	0.130	0.082	0.066	20473
	15 %	280987	101928	36.3	17615	18178	11600	11572	2.66	2.68	0.142	0.124	0.072	0.057	23170
	20 %	374966	146090	39.0	19234	19878	13443	13557	2.70	2.66	0.138	0.120	0.067	0.053	24986
	25 %	468320	188363	40.2	20554	21190	13622	13725	2.71	2.64	0.134	0.117	0.063	0.050	26666
	100 %	1873139	775570	41.4	35219	35725	23455	23087	2.54	2.46	0.213	0.207	0.172	0.163	205915

(a) = All households

(b) = Handicapped and/or chronically ill members in a household

2.1.3. Consumer unit scales

The basic idea behind using consumer unit scales is to enable comparison of different households. We can justifiably ask whether there are scale effects associated with the size of a household, and if so, how can they best be taken into consideration. Solutions to this problem were presented by Engels already during the last century when he made the observation that the share of expenses for necessities falls as household size increases and that at the same income level consumption expenditures on necessities per household member is inversely dependent on the size. There has been extensive empirical research on this issue recently (cf. Deaton-Muellbauer 1980).

Table 2.2. OECD and Tasku consumer units according to household structure (1981).

Group	OECD consumer unit	Tasku consumer unit
1 person	1.00	0.83
2 adults	1.70	1.43
1 adult, 1 child	1.50	1.47
3 adults	2.40	1.96
2 adults, 1 child	2.20	2.07
1 adult, 2 children	2.00	1.88
2 adults, 2 children	2.70	2.49
3 adults, 1 child	2.90	2.52
2 adults, 3 children	3.20	2.79
3 adults, 2 children	3.40	2.95
4 adults, 1 child	3.60	3.00

A basic research study incorporating equivalence scales is unfortunately still lacking for Finland. Table 2.2. presents the consumer unit scales for the 1981 household survey grouped according to household structure. The table indicates that for the OECD figures the first adult gets a

weight of one while any additional adults have a weight of 0.7. A child's weight is always 0.5. The Tasku unit scale factors for the age structure are more complicated (Hagfors and Koljonen 1984). According to the Tasku index a household with an individual under the age of 45 gets a weight of one. As the head of the household gets older, the Tasku unit scale decreases.

One criticism of consumer units is that they take into consideration certain consumption behaviour features while neglecting others. They emphasize the significance of consuming goods, but they do not necessarily depict the ability to completely utilize the goods and services, and they bypass many considerations which goods consumption does not measure.

2.1.4. Poverty indexes

In this study the poverty line has been defined by choosing a 20 % cut-off point for low income households instead of a certain income level. In this respect the solution is arbitrary. On the other hand, as indicated above coming up with a certain markka figure for the poverty line is a particularly problematic question and requires thorough research. By relating the income levels given by the cut-off points to reality, everyone can decide for him- or herself what kind of preconditions there are for a full life below the line.

A slightly more developed method than the simple head count ratio, which is 1/5 in this investigation, is to look at how the incomes below the poverty line deviate from the poverty level itself, i.e. the gap relation.

Neither of these, however, fulfills the requirements for a proper index with respect to A. Sen (1976):

- (a) when there are decreases in the income below the poverty line, *ceteris paribus*, the inequality index increases,
- (b) when income moves from below to above the poverty line, *ceteris paribus*, the index increases.

A more formalized version of the head count ratio (P0) and the income deviating gap relation (P1) can be presented as follows (Kakwani 1980):

$$(1) \quad P0 = F(z) = h/n = 1/5 ,$$

where h = number of households below z

n = total number of households

z = the maximum level at poverty line for incomes ranked according to size.

$$(2) \quad P1 = F(z)(z - y^*)/y ,$$

where y^* denotes the average for incomes below the line and y is the average for all incomes. The measure P1 does not take into consideration inequalities in the distribution among low income households. Instead two other indexes P2 (Kakwani index) and PS (Sen index)

$$(3) \quad P2 = F(z)(z - y^*(1-G^*))/y$$

$$(4) \quad PS = F(z)(z - y^*(1-G^*))/z ,$$

where G^* denotes the Gini coefficient for the low income households' distribution, make it possible to also evaluate the distribution effects.

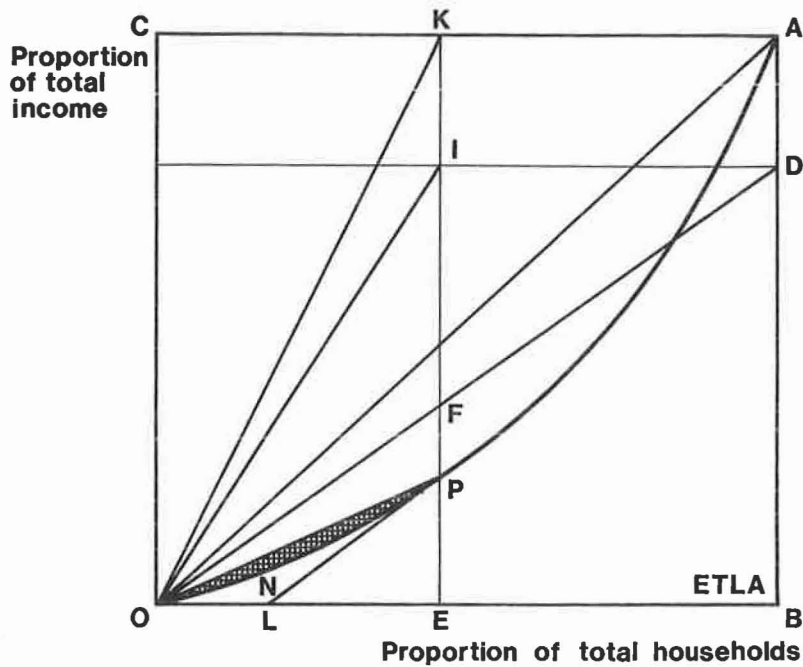
Sen derived his index (PS) axiomatically. Kakwani's index includes the idea that low income households are compensated by high income households until they reach the poverty line. The Kakwani index deviations are calculated with respect to the average of the whole income distribution, while for the Sen index this is done with respect to the poverty line. In this study the indexes are then used in calculations with respect to various background variables. This way more detailed information on the poverty structure can be obtained.

The main features of Sen's (PS) and Kakwani's (P2) poverty indices are presented grafically in figure 2.1. The figure shows the differences between the indexes and combines the problem with investigations into the inequality of the total income distribution.

The income distribution of all the households is given by the Lorenz curve ONPA in the Figure 2.1. The line OA corresponds to a perfectly even distribution. Point P depicts the level z below which households are defined as low income households. OE ($=h/n$) represents the share of low income earners and EP ($=hy^*/ny$) is the share of their income. The line LP is the tangent at point P on the Lorenz curve and its slope is equal to z/y ($=PE/LE=FE/OE$).

It can be seen that the poverty index P1 is the area of triangle OPF divided by the area OEK. Note that $EK=1$. By dividing the shaded area of ONP by OEP we get the Gini index G^* for low income households. The poverty index P2 corresponds to the area ONPF divided by the area OEK.

Figure 2.1. Graphical presentation of poverty indices P1, P2 and PS.



It is clear from the Figure 2.1 that $ONPF$ is greater than the area OPF or $P1 < P2$. Sen's measure of poverty PS can be shown to equal the area $ONPF$ divided by the area OEI . This explains also the differences between the estimates of $P1$, $P2$ and PS in the results later.

2.2. The data

The study is based on the household survey of 7368 households and their 22792 members made by the Central Statistical Office of Finland in 1981 (Household Survey 1981). The data consists of detailed information on, for instance, household consumption, the composition of income, and use of public services.

The household survey has the drawback that it does not include individuals living in institutions and this affects its suitability for studying the problem at hand. The sample of the survey emphasized certain interesting groups, however, from the research point of view.

The final non-response rate was about 27 per cent, about 30 % of the urban communities and about 18 % of rural communities. The non-response rate was proportionally smaller the more members belonging to the household. The largest non-response rate was in Helsinki (about 40 %) and the smallest was for the rural communities in central Finland.

As regards the socioeconomic position the non-response rate was greatest for retired persons, in the group "other professionals and employers". The non-response rate of students (about 14 %) and private entrepreneurs (about 19 %) was small. With respect to income the non-response rate

Table 2.3. Distribution % for households in various quintile groups by income definition.

	Households including ill or disabled member % quintile			Households not including ill or disabled member % quintile			Total %
	1.	2.-4.	5.	1.	2.-4.	5.	
Factor income/ household	14.0	21.0	6.4	6.0	39.0	13.6	100.0
Factor income/ person	14.3	21.8	5.3	5.7	38.2	14.7	100.0
Disposable income/ household	10.0	23.7	7.7	10.0	36.3	12.3	100.0
Disposable income/ person	8.6	25.7	7.1	11.4	34.3	12.9	100.0
Disposable income/ OECD consumer unit	9.9	24.3	7.2	10.1	35.7	12.8	100.0
Disposable income/ Tasku consumer unit	7.8	24.8	8.8	12.2	35.3	11.2	100.0

was largest for low income and high income households, which makes the distribution more even than in real life. The impact on the average figures is difficult to estimate. With respect to this particular study, it is troublesome that the non-response rate in the survey was higher than average for the low income households.

The background variables used in the study are presented in appendix 1. All four groupings, the age of the head of the household, the life cycle of the household, the socioeconomic status of the head of the household, and the form of the local community are specified according to the health status of the household because its significance is plain, as can be seen from table 2.2.. When ranked according to factor income excluding income transfer payments made and received, households with disabled or chronically ill members are usually in lower income quintiles. This indicates the importance of income transfers for this group. The formulation of disposable income per household gives similar results. This explains the large number of small households or retired persons in the lowest quintile.

3. RESULTS

3.1. The results by background variables

Information for the two extreme disposable income definitions - per household and per household member - are presented in tables 3.1.1.-3.1.4. In the first column is the percentage of the groups belonging to the total population. In the second column appear the groups' averages for the respective variables as regards the total population, and in the third column is the corresponding information for the low income households. The fourth column gives each group's percentage share of low income households with respect to the total for the group. Columns five, seven and nine are the poverty indices P1, P2 and PS respectively. Column six presents the weight of each background variable group in the overall index for the income variable. Column eight is the elasticity of the P2 index with respect to the Gini coefficient for the income distribution of the low income households. If the Gini index decreases by one percent, i.e. the income distribution for low income earners becomes more even, then the value of the P2 index drops in accordance with the elasticity estimate. The results are presented for the four background variables.

When the poverty criteria are judged with respect to the income definition broken down in terms of the age of the head of the household, group six (one ill or disabled member and the head of the household is over 64 years old) and group one (head of the household is under 25) stand out in the indexes. The results were significant in that they were independent of the income definition used. With respect to income equalization relation E2, the most sensitive were groups three, eight, and nine regardless of the income criteria. For example,

in group two (head of household 25-64 years old, no ill or disabled) a one percent decrease in the low income households' Gini coefficient, i.e. poverty distribution, would decrease the P2 index by 0.3 %.

In the calculations compiled according to the life cycle of the household, the significance of the one-person households, groups one and six, stands out particularly with respect to the income definitions calculated per household. The situation of single parents, groups two and seven, again stood out for the income definitions calculated per household member. In both cases the results were more pronounced for households with ill or disabled members. With respect to income distribution the most elastic were groups five and eight.

In the classifications according to the socioeconomic status of the head of the household, the average income for the low income households was lowest for group three, i.e. healthy households where the head of the household is in one of the "other socioeconomic groups" such as retired persons receiving social benefits or students living on student loans, and group four, which are self-employed heads of household with one ill or disabled member. The index figures drew special attention to group three again as well as group six, i.e. one ill or disabled member and the head of the household is in one of the "other groups", and group seven, i.e. self-employed heads of household with two or more ill or disabled members. The results for group seven were particularly clear for the income definitions calculated per household member. With respect to income distribution the most elastic were groups five, seven and nine.

As regards the breakdown by the form of the local community, the lowest income averages for low income households were for groups two and four, which were rural forms of communities. The only urban community average that was smaller than its corresponding rural average was when the income definition was calculated per household member and the households had one or more ill or disabled members, group five. The income per household figures for the Tasku consumer units stood out for households with one ill or disabled member: the largest elasticity value was for group five.

According to these results the health status of the household was a distinguishing factor for the low income households. The health status stood out most clearly for the income definitions calculated as per household. This is natural because there are more retired persons among the low income households when the income variables are defined this way. Retired persons' households consist mainly of elderly members, whose health is worse than average. The significance of the age structure in the Tasku consumer units comes out through the weightings.

These observations are but a few of the most central points that can be picked out from the results within tables 3.1.1.-3.1.4. They clearly indicate that in addition to the significance of the classification breakdown, the choice of the income definition is of particular importance when the poverty structure is investigated and when target groups are sought.¹⁾

1) The education of the head of the household and ranking in the lowest quintile have been found to be independent (The Impact of Social Security in 1981. Vol 10, 1986).

Table 3.1.1. Poverty index by the age of the head and the health status of the households.

Disposable incomes per household

Group	Share of the population	Mean income of all households FMK	Mean income of poor households FMK	Share of poor households %	Poverty index P1	Contribution by groups to the total poverty %	Poverty index P2	Elasticity E2	Poverty index PS
1	6.30	33749	17853	53.55	0.2010	21.94	0.2742	0.2670	0.3032
2	48.08	67819	21608	8.65	0.0114	19.03	0.0164	0.3056	0.0364
3	4.22	33181	20805	58.48	0.1712	12.31	0.2139	0.1999	0.2326
4	0.99	45957	19492	36.67	0.0880	2.05	0.1182	0.2554	0.1779
5	22.51	65230	21715	15.92	0.0215	16.20	0.0297	0.2771	0.0635
6	9.04	33336	20987	59.00	0.1687	26.12	0.2151	0.2160	0.2350
7	0.13	81656	0	0.00	0.0000	0.00	0.0000	0.0000	0.0000
8	5.36	70566	23060	4.55	0.0048	0.93	0.0071	0.3211	0.0164
9	3.38	43782	25059	14.93	0.0186	1.42	0.0276	0.3258	0.0396
Total	100.00	59648	20797	20.03	0.0326	100.00	0.0443	0.2680	0.0771

Disposable income per household member

Group	Share of the population	Mean income of all households FMK	Mean income of poor households FMK	Share of poor households %	Poverty index P1	Contribution by groups to the total poverty %	Poverty index P2	Elasticity E2	Poverty index PS
1	3.95	21129	11137	30.63	0.0652	7.55	0.0926	0.2961	0.1252
2	53.70	23876	11919	19.54	0.0304	54.01	0.0435	0.3012	0.0664
3	2.29	24064	12351	11.74	0.0160	1.22	0.0237	0.3243	0.0365
4	0.84	21124	11699	23.67	0.0441	1.09	0.0575	0.2334	0.0777
5	24.21	23843	11994	18.28	0.0279	22.32	0.0403	0.3080	0.0615
6	5.03	23568	12936	16.85	0.0193	3.17	0.0274	0.2953	0.0413
7	0.19	21593	13643	16.79	0.0155	0.09	0.0223	0.3057	0.0308
8	6.83	21775	12430	25.84	0.0380	7.84	0.0567	0.3298	0.0790
9	2.94	19755	13043	25.69	0.0337	2.71	0.0494	0.3185	0.0624
Total	100.00	23455	12025	20.00	0.0308	100.00	0.0443	0.3073	0.0663

Table 3.1.2. Poverty index by the life cycle and the health status of the households.

Disposable incomes per household

Group	Share of the population	Mean income of all households	Mean income of poor households	Share of poor households	Poverty index P1	Contribution by groups to the total poverty	Poverty index P2	Elasticity E2	Poverty index PS
	%	FMK	FMK	%		%			
1	16.82	30710	19828	51.61	0.1797	47.68	0.2469	0.2725	0.2485
2	2.43	47116	21146	11.16	0.0222	1.31	0.0284	0.2187	0.0439
3	9.82	65063	23515	4.71	0.0051	1.67	0.0073	0.3072	0.0156
4	22.31	77218	22204	1.75	0.0019	1.66	0.0028	0.3313	0.0071
5	7.21	86022	20696	2.65	0.0030	0.97	0.0043	0.3028	0.0123
6	11.88	27504	20837	68.17	0.2399	40.29	0.3091	0.2238	0.2786
7	0.73	47614	19230	15.09	0.0358	0.64	0.0462	0.2258	0.0721
8	11.23	51839	24614	12.43	0.0142	4.23	0.0214	0.3372	0.0363
9	7.61	77279	19227	2.09	0.0031	0.92	0.0046	0.3329	0.0116
10	9.96	82363	25772	2.64	0.0015	0.64	0.0023	0.3305	0.0061
Total	100.00	59648	20797	20.03	0.0326	100.00	0.0443	0.2680	0.0679

Disposable income per household member

Group	Share of the population	Mean income of all households	Mean income of poor households	Share of poor households	Poverty index P1	Contribution by groups to the total poverty	Poverty index P2	Elasticity E2	Poverty index PS
	%	FMK	FMK	%		%			
1	6.61	30710	10231	13.29	0.0234	6.58	0.0328	0.2880	0.0645
2	2.28	19789	11002	26.73	0.0625	3.90	0.0854	0.2675	0.1081
3	7.73	32504	11937	4.96	0.0056	1.96	0.0080	0.2983	0.0167
4	33.14	20441	11983	25.55	0.0456	42.82	0.0651	0.2992	0.0851
5	10.19	23950	12320	16.06	0.0222	7.51	0.0324	0.3152	0.0497
6	4.67	27504	12023	8.85	0.0116	2.07	0.0172	0.3256	0.0303
7	0.74	18569	12292	37.36	0.0672	1.27	0.0987	0.3192	0.1173
8	8.83	25920	12633	13.86	0.0160	5.09	0.0241	0.3334	0.0399
9	11.30	20473	12008	27.38	0.0485	15.53	0.0706	0.3138	0.0925
10	14.52	22206	12470	20.82	0.0296	13.25	0.0423	0.2996	0.0601
Total	100.00	23455	12025	20.00	0.0308	100.00	0.0443	0.3073	0.0647

Table 3.1.3. Poverty index by the socio-economic status of the head and the health status of the households.

Disposable incomes per household

Group	Share of the population	Mean income of all households FMK	Mean income of poor households FMK	Share of poor households %	Poverty index P1	Contribution by groups to the total poverty %	Poverty index P2	Elasticity E2	Poverty index PS
1	5.75	69639	20755	10.08	0.0141	2.91	0.0202	0.2990	0.0460
2	44.04	67129	23605	8.18	0.0084	12.80	0.0125	0.3274	0.0276
3	8.80	29107	17936	66.03	0.2854	37.57	0.3693	0.2272	0.3523
4	3.47	74668	19750	10.58	0.0153	2.03	0.0218	0.2986	0.0532
5	15.22	71889	25479	5.45	0.0038	2.15	0.0059	0.3492	0.0138
6	13.86	33366	20838	58.35	0.1693	40.20	0.2180	0.2236	0.2384
7	1.30	75919	21909	4.18	0.0047	0.24	0.0069	0.3107	0.0171
8	2.79	80162	25415	0.97	0.0006	0.07	0.0006	0.0017	0.0016
9	4.78	44856	24570	13.96	0.0185	2.04	0.0277	0.3306	0.0407
Total	100.00	59648	20797	20.03	0.0326	100.00	0.0440	0.2680	0.0721

Disposable income per household member

Group	Share of the population	Mean income of all households FMK	Mean income of poor households FMK	Share of poor households %	Poverty index P1	Contribution by groups to the total poverty %	Poverty index P2	Elasticity E2	Poverty index PS
1	7.69	20475	11386	36.60	0.0759	16.57	0.1083	0.2990	0.1418
2	47.08	24690	12372	15.63	0.0206	33.27	0.0294	0.2984	0.0465
3	5.17	19496	10434	34.75	0.0927	12.94	0.1287	0.2803	0.1606
4	4.97	20482	11396	33.54	0.0694	9.78	0.0982	0.2940	0.1287
5	16.86	25519	13050	11.96	0.0121	7.22	0.0176	0.3108	0.0287
6	8.26	21999	11776	21.67	0.0380	9.57	0.0537	0.2926	0.0756
7	2.09	18498	11947	40.71	0.0811	4.35	0.1196	0.3219	0.1415
8	3.57	24664	13359	14.35	0.0132	1.61	0.0195	0.3207	0.0307
9	4.30	19584	12798	27.64	0.0400	4.67	0.0594	0.3265	0.0744
Total	100.00	23455	12025	20.00	0.0308	100.01	0.0441	0.3073	0.0645

Table 3.1.4. Poverty index by the type of community and the health status of the households.

Disposable incomes per household

Group	Share of the population	Mean income of all households	Mean income of poor households	Share of poor households	Poverty index P1	Contribution by groups to the total poverty	Poverty index P2	Elasticity E2	Poverty index PS
	%	FMK	FMK	%		%			
1	38.71	61996	20977	16.42	0.0253	31.15	0.0355	0.2873	0.0720
2	19.89	61017	18688	18.31	0.0355	22.13	0.0478	0.2576	0.0956
3	19.85	56896	22039	25.58	0.0381	22.12	0.0505	0.2455	0.0942
4	12.69	54036	20208	33.12	0.0632	22.25	0.0828	0.2366	0.1466
5	4.65	62358	21331	4.99	0.0074	1.10	0.0108	0.3192	0.0221
6	4.21	58484	25792	12.25	0.0099	1.25	0.0142	0.3030	0.0272
Total	100.00	59648	20797	20.03	0.0326	100.00	0.0444	0.2680	0.0862

Disposable income per household member

Group	Share of the population	Mean income of all households	Mean income of poor households	Share of poor households	Poverty index P1	Contribution by groups to the total poverty	Poverty index P2	Elasticity E2	Poverty index PS
	%	FMK	FMK	%		%			
1	37.23	25346	12291	15.04	0.0198	25.94	0.0286	0.3066	0.0464
2	22.71	21007	11461	28.05	0.0557	36.84	0.0790	0.2948	0.1061
3	17.42	25493	12433	12.93	0.0162	9.99	0.0233	0.3045	0.0380
4	12.67	21283	11917	25.43	0.0444	16.59	0.0640	0.3063	0.0871
5	4.96	23023	11718	16.71	0.0284	4.50	0.0421	0.3258	0.0621
6	5.01	19346	13063	34.45	0.0457	6.14	0.0673	0.3205	0.0833
Total	100.00	23455	12025	20.00	0.0308	100.00	0.0443	0.3073	0.0653

3.2. Total effect in indexes

Although the results for the four income measures vary noticeably regarding the poverty structure, their calculated poverty index values deviate only slightly (table 3.2.1). Two of the structural distribution measures, the income definitions calculated per household and per member, were extremes between which the figures for the OECD and Tasku consumer units were situated (cf. the Impact of Social Security in 1981, Part 10, 1986). According to these results the structural distribution of poverty depends fundamentally on the criteria for the basis used in the evaluation.

According to indexes P1, P2 and PS, the total income transfer to low income households required to bring them up to the poverty line was smallest for index P1 because it does not take into consideration the unevenness of the associated distribution. The high value for the PS measure results from the way it is related to the deviation from the poverty line instead of the average income distribution, as is the case with the P2 index. For example, according to the Kakwani P2 index the income compensation required to bring the low income households up to the poverty line is about 4.5 % of total income. The alternative indexes' usefulness is supplemented by the extra information they offer.

Table 3.2.1. Total Index Values by Income Definition.

Index	Disp. inc./ house	Disp. inc./ OECD unit	Disp. inc./ Tasku	Disp. inc./ pers.
P1	.032	.030	.033	.031
P2	.044	.044	.047	.044
PS	.068	.060	.064	.065

4. MAIN FEATURES OF THE HOUSEHOLDS IN THE LOWEST INCOME QUINTILE

4.1. Income quintile averages

Some selected average information on the lowest, the middle three, and the highest income quintiles have been collected together and presented in tables 4.1-4.3. The comparison according to disposable income has been expanded by including quintile classifications using factor income and gross income. The former classification represents income before household transfer payments made and received while the latter is roughly income before direct taxes. In the last column of the tables there is, for the sake of comparison, the averages calculated for the total data.

The general observations regarding the lowest quintile correspond to the results presented above, that the income definitions per household highlight more clearly than the other income criteria the elderly single-person households and small single-parent households. The share of transfer payments received within disposable income was greater and the share of social services offered free of charge was smaller than in other quintile groups. In the lowest quintile the gross income consisted almost exclusively of income transfers. The tax burden was greater on the indirect tax side.

The average propensity to consume in the lowest quintile was greater than unity for all income classifications. The share of food and housing expenses was above average, which corresponds to the above-mentioned results of Engel. The transportation expense share was lower than average. The average size of the households in the middle three quintiles corresponded fairly well to the average for the total data and was of the same

Table 4.1. Some typical figures for households in the lowest quintile.

Variable	Factor income		Gross income		Disposable income		
	/house-	/member	/house-	/member	/house-/OECD	/member	unit
	hold		hold		hold		
Household size	1.41	1.65	1.28	2.87	1.22	2.39	3.21
Age of Head of Households	64.11	63.96	55.66	43.13	53.39	50.37	44.36
Number of Children	0.06	0.17	0.07	0.97	0.07	0.67	1.20
Children under age 7	0.02	0.07	0.03	0.42	0.03	0.30	0.53
Number of Disabled	0.88	0.92	0.62	0.65	0.54	0.62	0.57
Number of Retired	1.22	1.27	0.80	0.76	0.68	0.75	0.60
Persons per room	0.70	0.74	0.72	0.95	0.70	0.88	0.99
Average household gross income (mk)	29628	32022	22894	41635	23564	34139	49522
Average household disposable income (mk)	26950	29026	21096	36542	20739	29658	41395
Household wage income/disposable income	0.02	0.03	0.24	0.49	0.37	0.44	0.63
Household entrepreneurial income/ - " -	0.02	0.03	0.06	0.18	0.06	0.17	0.19
Household property income/ - " -	0.07	0.06	0.07	0.04	0.06	0.05	0.03
Transfer payments received/ - " -	0.99	0.97	0.72	0.43	0.64	0.49	0.33
Transfer payments paid/ - " -	-0.10	-0.10	-0.08	-0.13	-0.13	-0.14	-0.19
Indirect taxes/ - " -	-0.18	-0.32	-0.23	-0.25	-0.24	-0.26	-0.26
Public services/ - " -	0.19	0.20	0.24	0.33	0.22	0.32	0.33
Consumption expenditure/ - " -	1.00	1.01	1.18	1.17	1.22	1.24	1.17
Food expenditure/consumption expenditure	0.31	0.32	0.29	0.29	0.27	0.29	0.28
Beverage and tobacco expenditure/ - " -	0.03	0.03	0.03	0.04	0.03	0.04	0.03
Clothing expenditure/ - " -	0.05	0.05	0.06	0.07	0.06	0.07	0.07
Housing expenditure/ - " -	0.18	0.17	0.18	0.15	0.19	0.16	0.15
Household accessories expenditure/ - " -	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Health services expenditure/ - " -	0.04	0.04	0.03	0.02	0.03	0.02	0.02
Transportation expenditure/ - " -	0.10	0.11	0.10	0.15	0.12	0.14	0.17
Recreation expenditure/ - " -	0.07	0.07	0.08	0.08	0.08	0.08	0.08
Other service expenditure/ - " -	0.12	0.12	0.14	0.12	0.15	0.12	0.12
Spouse's wage income/household wage income	0.19	0.45	0.23	0.26	0.20	0.27	0.26
Spouse's income/household income	0.44	0.42	0.45	0.38	0.39	0.41	0.36
Spouse's total retirement benefits/ household retirement benefits	0.42	0.37	0.45	0.31	0.36	0.30	0.29
Spouse's pension income/ household pension income	0.33	0.29	0.22	0.24	0.32	0.26	0.26
Average household interest expenditure (mk)	253	328	400	1060	451	847	1368
Average household loans (mk)	2660	3242	4730	13036	4967	10065	18251
Average community housing aid (mk)	87	105	68	720	51	507	772
Average KELA housing aid (mk)	678	665	569	251	518	300	124

Table 4.2. Some typical figures for households in the 2nd-4th quintiles.

Variable	Factor income		Gross income		Disposable income		
	/house-	/member	/house-	/member	/house-/OECD	/member	
	hold		hold		hold	unit	
Household size	2.56	3.05	2.58	2.68	2.56	2.69	2.59
Age of Head of Households	41.08	41.49	49.50	45.71	44.31	44.94	46.71
Number of Children	0.67	0.92	0.67	0.67	0.65	0.71	0.61
Children under age 7	0.31	0.37	0.30	0.26	0.29	0.28	0.23
Number of Disabled	0.43	0.44	0.49	0.51	0.51	0.50	0.53
Number of Retired	0.40	0.40	0.49	0.51	0.51	0.50	0.54
Persons per room	0.85	0.89	0.85	0.83	0.84	0.85	0.82
Average household gross income (mk)	70230	82546	71790	76742	72137	77025	78945
Average household disposable income (mk)	55282	64550	56595	62252	56331	60260	61239
Household wage income/disposable income	0.89	0.91	0.87	0.90	0.88	0.90	0.91
Household entrepreneurial income/ - " -	0.12	0.14	0.11	0.12	0.11	0.12	0.11
Household property income/ - " -	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Transfer payments received/ - " -	0.21	0.19	0.24	0.22	0.25	0.22	0.22
Transfer payments paid/ - " -	-0.27	-0.27	-0.26	-0.28	-0.28	-0.27	-0.28
Indirect taxes/ - " -	-0.21	-0.21	-0.20	-0.20	-0.20	-0.20	-0.20
Public services/ - " -	0.16	0.18	0.15	0.15	0.15	0.16	0.15
Consumption expenditure/ - " -	0.96	0.95	0.94	0.92	0.94	0.93	0.92
Food expenditure/consumption expenditure	0.23	0.23	0.23	0.23	0.23	0.23	0.23
Beverage and tobacco expenditure/ - " -	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Clothing expenditure/ - " -	0.07	0.07	0.07	0.07	0.07	0.07	0.07
Housing expenditure/ - " -	0.15	0.14	0.15	0.14	0.15	0.17	0.14
Household accessories expenditure/ - " -	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Health services expenditure/ - " -	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Transportation expenditure/ - " -	0.19	0.20	0.19	0.20	0.19	0.19	0.19
Recreation expenditure/ - " -	0.09	0.09	0.09	0.09	0.09	0.09	0.09
Other service expenditure/ - " -	0.16	0.15	0.16	0.15	0.16	0.15	0.15
Spouse's wage income/household wage income	0.33	0.34	0.33	0.40	0.33	0.38	0.41
Spouse's income/household income	0.39	0.38	0.38	0.41	0.38	0.41	0.42
Spouse's total retirement benefits/ household retirement benefits	0.40	0.33	0.33	0.27	0.33	0.29	0.28
Spouse's pension income/ household pension income	0.41	0.35	0.31	0.30	0.29	0.28	0.30
Average household interest expenditure (mk)	1476	1911	1467	1710	1481	1616	1657
Average household loans (mk)	21012	27606	21470	25086	21493	24057	24223
Average community housing aid (mk)	365	340	366	162	355	231	143
Average KELA housing aid (mk)	40	46	75	182	90	164	221

Table 4.3. Some typical figures for households in the highest quintile.

Variable	Factor income		Gross income		Disposable income		
	/house-/member	hold	/house-/member	hold	/house-/OECD	member	unit
Household size	3.62	1.91	3.68	1.81	3.82	2.25	1.72
Age of Head of Households	43.02	41.94	44.20	45.12	44.04	45.18	45.87
Number of Children	1.06	0.19	1.06	0.15	1.12	0.34	0.10
Children under age 7	0.30	0.05	0.31	0.04	0.33	0.10	0.02
Number of Disabled	0.39	0.30	0.44	0.36	0.48	0.41	0.40
Number of Retired	0.14	0.09	0.28	0.26	0.34	0.30	0.31
Persons per room	0.83	0.67	0.83	0.63	0.85	0.67	0.62
Average household gross income (mk)	150293	110996	152318	109723	150561	125362	104232
Average household disposable income (mk)	105427	75554	107326	74909	108442	87767	73094
Household wage income/disposable income	1.16	1.24	1.09	1.12	1.05	1.08	1.06
Household entrepreneurial income/ - " -	0.16	0.13	0.17	0.13	0.17	0.14	0.13
Household property income/ - " -	0.04	0.05	0.05	0.06	0.05	0.05	0.06
Transfer payments received/ - " -	0.06	0.04	0.11	0.15	0.11	0.15	0.17
Transfer payments paid/ - " -	-0.42	-0.46	-0.41	-0.46	-0.38	-0.42	-0.42
Indirect taxes/ - " -	-0.20	-0.19	-0.19	-0.18	-0.19	-0.18	-0.18
Public services/ - " -	0.13	0.05	0.13	0.05	0.13	0.07	0.05
Consumption expenditure/ - " -	0.88	0.87	0.88	0.86	0.87	0.84	0.84
Food expenditure/consumption expenditure	0.19	0.16	0.19	0.17	0.20	0.17	0.17
Beverage and tobacco expenditure/ - " -	0.03	0.04	0.03	0.04	0.03	0.04	0.04
Clothing expenditure/ - " -	0.05	0.05	0.06	0.07	0.07	0.07	0.07
Housing expenditure/ - " -	0.13	0.14	0.13	0.14	0.12	0.13	0.14
Household accessories expenditure/ - " -	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Health services expenditure/ - " -	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Transportation expenditure/ - " -	0.24	0.24	0.24	0.23	0.24	0.25	0.23
Recreation expenditure/ - " -	0.10	0.09	0.09	0.09	0.09	0.09	0.09
Other service expenditure/ - " -	0.18	0.24	0.17	0.22	0.17	0.20	0.22
Spouse's wage income/household wage income	0.32	0.45	0.31	0.45	0.31	0.40	0.48
Spouse's income/household income	0.34	0.47	0.33	0.47	0.33	0.41	0.48
Spouse's total retirement benefits/ household retirement benefits	0.24	0.33	0.23	0.36	0.24	0.31	0.37
Spouse's pension income/ household pension income	0.31	0.34	0.26	0.34	0.28	0.32	0.37
Average household interest expenditure (mk)	3628	2246	3505	2117	3412	2612	1967
Average household loans (mk)	51995	33794	50713	30985	50615	37842	29157
Average community housing aid (mk)	25	2	41	2	91	7	6
Average KELA housing aid (mk)	7	3	10	8	16	13	18

magnitude for all the income definitions. The same holds also for the propensity to consume. The proportion of transfer payments paid out of disposable income was greater than in the lowest quintile and the proportions of indirect taxes and public services correspondingly smaller. Wages had a marked significance in the composition of income.

In the highest quintile the transformation per household highlights the larger households more than the income definitions calculated per person. The average propensity to consume was lower than in the other quintiles and the transportation expenditures had the greatest portion in the household budget. While moving from the lowest to the highest quintile, the amount of loans compared to disposable income grew. Housing assistance was greatest in the lowest quintile. The share of property income within disposable income, however, was on the same magnitude in all the quintile groups.

4.2. Identification of the households

When it was seen how often the households were in a certain income quintile group according to all four disposable income formulations, the result was the following table 4.4. This table indicates that over half of the households were such that they switched groups according to at least one criteria. All in all there were 1,873,139 households in the basic population.

This result shows the sensitivity of household identification to different income criteria. There can be vast differences in the

Table 4.4. Households always in same income quintile.

	Households	Households members
1st quintile	125910	199965
2nd-4th quintiles	577333	1518386
5th quintile	110674	294393

Table 4.5. Averages for households always in same income quintile.

Variable	Quintile group			Disposable income/ household (all data)
	1.	2.-4.	5.	
Household size	1.64	2.63	2.66	2.54
Age of Head of Households	45.44	43.77	47.66	46.07
Number of Children	0.21	0.59	0.32	0.63
Children under age 7	0.10	0.26	0.08	0.25
Number of Disabled	0.54	0.52	0.42	0.51
Number of Retired	0.64	0.54	0.32	0.51
Persons per room	0.80	0.86	0.63	0.82
Average household gross income (mk)	19479	75951	175582	78114
Average household disposable income (mk)	16630	59791	119079	59649
Household wage income/disposable income	0.35	0.90	1.08	0.90
Household entrepreneurial income/ - " -	0.14	0.10	0.20	0.13
Household property income/ - " -	0.05	0.04	0.07	0.04
Transfer payments received/ - " -	0.63	0.23	0.12	0.23
Transfer payments paid/ - " -	-0.16	-0.27	-0.47	-0.30
Public services/ - " -	0.47	0.14	0.05	0.15
Consumption expenditure/ - " -	1.80	0.93	0.81	0.99
Food expenditure/consumption expenditure	0.26	0.23	0.16	0.22
Beverage and tobacco expenditure/ - " -	0.04	0.04	0.03	0.04
Clothing expenditure/ - " -	0.07	0.07	0.07	0.07
Housing expenditure/ - " -	0.17	0.15	0.13	0.15
Household accessories expenditure/ - " -	0.06	0.06	0.06	0.06
Health services expenditure/ - " -	0.02	0.02	0.02	0.02
Transportation expenditure/ - " -	0.14	0.19	0.25	0.20
Recreation expenditure/ - " -	0.08	0.09	0.09	0.09
Other service expenditure/ - " -	0.14	0.15	0.21	0.16
Spouse's wage income/household wage income	0.10	0.26	0.30	0.40
Spouse's income/household income	0.17	0.29	0.29	0.41
Spouse's total retirement benefits/ household retirement benefits	0.19	0.26	0.23	0.30
Spouse's pension income/household pension income	0.20	0.25	0.22	0.30
Average household interest expenditure (mk)	867	1589	3574	1661
Average household loans (mk)	8343	23788	56545	24015
Average community housing aid (mk)	150	220	8	241
Average KELA housing aid (mk)	124	108	4	161

household ranking in the income distribution depending on the income transformation.

Certain average figures for the households that belong to the same income group according to all four disposable income criteria have been selected and presented in table 4.5. Comparison with tables 4.1.-4.3. indicates that in the group of households remaining in the lowest quintile there were relatively many low income households of small families with children whose propensity to consume and the share of loans within disposable income was high. The household budget proportions were typical of low income households: the shares of expenditures on necessities were great. The shares of transfer payments received and public services as a part of disposable income were also clearly greater than for other quintile groups. On the other hand, the spouse's share of household wage income was lower.

The households consistently in the highest quintile were those with high incomes and a slightly older head of the household whose children were more seldom under school age. The shares of transfer payments received and public services within disposable income were lower than for the other groups. They lived in more spacious housing and their housing assistance was also less. The propensity to consume was lower than average and the significance of consumption on transportation and other services was greatest.

In the discussion above which takes into consideration only those households that are consistently in the same income group gives a relatively low estimate of the number of households that fulfill the conditions used in the study to determine who are low income households. Exactly

the opposite approach would have been to investigate all those households which fulfilled at least one of the criteria. This way the number of households to be studied would have been noticeably higher. The investigation indicates the importance of research on questions related to the poverty line.

5. FINAL REMARKS

When social security or taxation or the existence of poverty are being discussed, it is essential to be able to successfully identify the matters under consideration. Four different income definitions were dealt with in this study. The study made no attempt to define any certain poverty line. Instead, the interest was concentrated on the households belonging to the lowest income quintile and their associated characteristics.

The scope of the issues investigated by the study was limited due to the nature of the data available from the 1981 household survey. From a research point of view, numerous interesting aspects were not able to be included in the investigation because the relevant data was lacking.

The well-being problem of low income households was approached from the traditional poverty index point of view. Poverty indexes offer a very justifiable means for making empirical investigations. The indexes used in the calculations indicate that the distribution factors were significant for low income households. Nearly a third of the total effect of the indexes could be seen as resulting from the distribution factors. One important focal point regarding the assessment of the well-being of low income households was the composition of the household's income. This aspect was not focused upon very much in the study.

It is possible to get a picture of this, however, from tables 4.1-4.4. Numerous calculations regarding income composition for the 1981 household survey data have been made, for example, in the publications of the Ministry of Social Affairs and Health.

The health situation was an important factor when studying questions concerning the well-being of the lowest quintile. From the results it appears that those households with an ill or disabled member are more closely bound to the social welfare services available than others are. It appeared that the OECD consumer unit scales put more emphasis on the connection between health and ranking as a low income household than the Tasku consumer unit scales.

The study demonstrated the importance of poverty research and related isolated questions. Especially research regarding the comparability of households should be made more precise.

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Appendix 1. Group variables

In the variable list the term "disabled" is used to mean both the disabled and chronically ill

Household health status and age of the head of household

1.	no disabled and the household head is age	0-24
2.	no " " " " " " " "	25-64
3.	no " " " " " " " "	over age 64
4.	one " " " " " " " "	age 0-24
5.	one " " " " " " " "	age 25-64
6.	one " " " " " " " "	over age 64
7.	more than one " " " " " " " "	age 0-24
8.	more than one " " " " " " " "	age 25-64
9.	more than one " " " " " " " "	over age 64

Household health status and household life cycle

1.	no disabled, one person household	(life cycle group 1)
2.	no disabled, single parent household	(life cycle group 2)
3.	no disabled, childless couple	(life cycle group 3)
4.	no disabled, children in household	(life cycle groups 4-8)
5.	no disabled, so-called "other household"	(life cycle group 9)
6.	one or more disabled, one person household	(life cycle group 1)
7.	" " " " , single parent household	(life cycle group 2)
8.	" " " " , childless couple	(life cycle group 3)
9.	" " " " , children in household	(life cycle groups 4-8)
10.	" " " " , so-called "other household"	(life cycle group 9)

Appendix 1. Cont.

Household health status and socioeconomic status of the head of household

1.	no disabled and household head is self employed						(groups 0-20)
2.	"	"	"	"	"	"	an employee (groups 21-72)
3.	"	"	"	"	"	"	in another group(groups 73-92)
4.	one	"	"	"	"	"	self employed (groups 0-20)
5.	one	"	"	"	"	"	an employee (groups 21-72)
6.	one	"	"	"	"	"	in another group(groups 73-92)
7.	more than one	"	"	"	"	"	self employed (groups 0-20)
8.	"	"	"	"	"	"	an employee (groups 21-72)
9.	"	"	"	"	"	"	in another group(groups 73-92)

Household health status and form of local community

1.	no disabled and form is an urban community					
2.	"	"	"	"	"	a rural community
3.	one	"	"	"	"	an urban community
4.	one	"	"	"	"	a rural community
5.	more than one	"	"	"	"	an urban community
6.	more than one	"	"	"	"	a rural community

HOUSEHOLD EQUIVALENCE SCALES IN FINLAND FOR THE YEARS 1976 AND 1981

by Robert Hagfors*

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* I would like to thank Risto Sullström for parameter estimates in table A.2.

1. INTRODUCTION

In recent years there has been a lot of debate concerning the poverty in welfare states. This is at least partly due to the observations made in different countries that the share of the population living under the poverty threshold has not decreased despite the prevalence of large social welfare programs. This means that social transfers to households are inefficient and that in this respect they do not fit in well with the liberalization of the market forces or with the aims of the current tax reforms.

These elements are also present in the Finnish debate. However, some central questions have still not been satisfactorily answered:

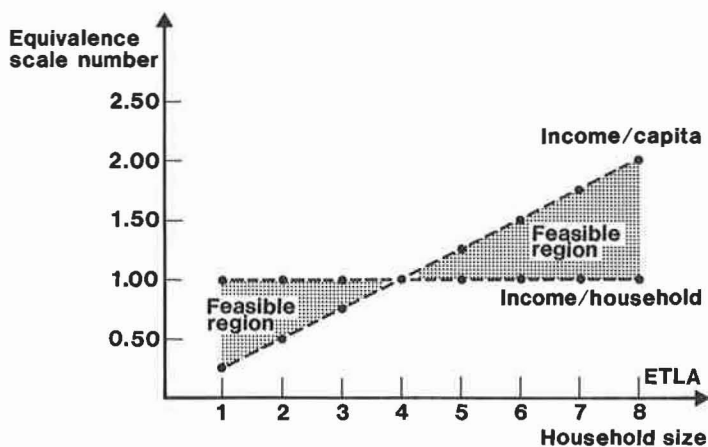
- how many households live under the poverty threshold,
- how poor are they and
- who are they?

In empirical research done lately in Finland the great variation of results can be explained with differences in the definitions which have been used. The main concepts to be defined in distribution studies are a) the relevant income unit and b) the relevant income receiving unit. (See Grootaert 1982, Hagfors 1987 and Uusitalo 1985, 1987, about the relevant income unit).

Usually, but not self-evidently, a household's disposable income after taxes and transfers has been chosen as an income concept. This choice means that it is actually the purchasing power and its distribution that is studied. This definition does not take into account the fact that households differ in size and other characteristics. In order to

adjust the disposable income by differences in household size, per capita measures has been used. This definition has the weakness that it counts adults and children alike. The third income definition used in research is the disposable income adjusted by using an equivalent number of household members. Here the so-called equivalence scales are used. By using equivalence scales the disposable income of the household can be seen as adjusted, in a way, on a needs corrected basis. The household as a unit and the number of household members are the two extremes as income deflators between which the equivalence scale settles. The idea is presented in figure 1.

Figure 1. The feasible region for equivalence scales when a four-member household has the scale number value of 1.00



As can be seen from figure 1. the equivalence scales shall have some curvature in the shape. The reason for this is that there exist returns to scale in consumption when the size of the household is growing. There are of course also other factors, such as the age structure, which may have an opposite effect. The effect of the returns to scale is, however, dominating (Hagfors 1988).

The other concept which has relevance from the point of view of income distribution studies is the income receiving unit. In principle there are three possibilities: a household, an individual or the number of equivalent members in the household. The last one should not be considered because in the sense of aggregation the total amount of equivalent members has no empirical meaning.

The choice of an individual or a household member has been justified on the grounds that only the individuals are consistent with a well defined social welfare function. (For references see Grootaert 1982 and van Ginneken 1981.) On the other hand, the choice of the household as an income receiving unit has been based on the argument that the household is the smallest unit which has its own budget to make decisions on. Individuals do not necessarily have one. As a result households are assumed to behave in the same way as individuals when maximizing a household social welfare function (Jorgenson and Slesnick 1987).

In order to be consistent in the aggregation of income units there are now three possibilities:

- The income unit is income per household and the income receiving unit is a household.
- The income unit is income per capita and a household member is an income receiving unit.
- The income unit is income per household equivalence scale and a household equivalent member is an income receiving unit.

When the distribution and level of economic welfare is studied, the last one of these is the best choice on the condition that household

equivalent members as income receiving units are replaced by individuals.

The problem that remains is to determine the equivalence scales for different types of households. There has been already some scales in use in Finland. These are the so-called "calory scales" and OECD-scales introduced by the Central Statistical Office of Finland (Household surveys 1976 and 1981) and TASKU-scales (Hagfors and Koljonen 1984). All of these suffer from serious shortcomings (Hagfors 1987).

In this paper an effort is made to construct household equivalence scales for Finland for the years 1976 and 1981. We make the calculations at two points of time in order to see if the changes in relative prices or other factors have any effect on equivalence scales.

The next section contains a short survey of different approaches applied in equivalence scale calculations.

The problem of the identification of equivalence scale numbers in econometric models is presented in section three. Here we make also the choice of the econometric demand system used in this paper.

In section four we present the data and some estimation results.

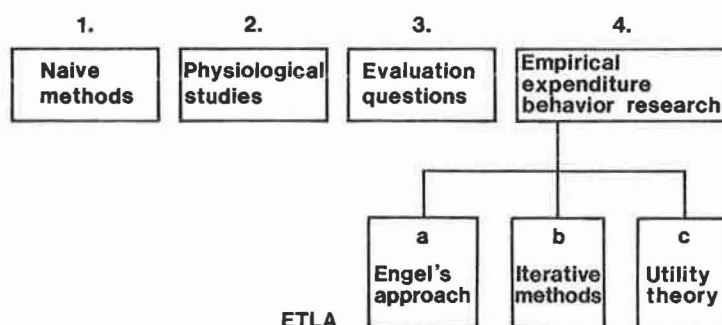
Commodity specific scales for different commodity groups in the years 1976 and 1981 are computed in section five, and in section six we present general household equivalence scales.

The last chapter contains some conclusions.

2. A BRIEF SURVEY OF APPROACHES

In this paper the equivalence scales are computed by using econometric techniques and data on households' empirical consumption behaviour. This, however, is not the only way to proceed. Some other principal approaches are presented in figure 2.

Figure 2. Some approaches to computing equivalence scales.



The first approach in figure 2 is the naive methods in box 1. These mean that the household as a unit or the number of household members are used as an income deflator. We have already mentioned the shortcomings of this first approach. In the second approach a scale is constructed on the basis of an individual's nutritional needs. This is estimated for individuals according to age and sex. The household scale is constructed by summing up the individual scale numbers. The above-mentioned calory scales are an example. (Other examples are the Amsterdam scale, Deaton and Muellbauer 1980.) These scales can not, however, react to returns to scale in consumption when the size of the household is growing. Secondly, the nutritional needs form too narrow a basis for economic welfare comparisons between households, at least in developed countries.

The third approach is based on an idea that the households themselves are the best experts on their relative positions among other households. The method has been applied mainly at the University of Leyden and at the University of Gothenburg (Kapteyn and van Praag 1976, van Praag, Hagenaars and van Weeren 1981, Gustafsson 1986).

The households are asked to tell the amount of money that they suppose corresponds to different levels of economic welfare in their household's case. From the answers the individual welfare functions can be estimated and equivalence scales for different household types constructed.

The last type of approach, and the one chosen in this paper, is to base the equivalence scale calculations on the empirical consumption behaviour of the households. The first of the three subgroups within this approach is the oldest one, originating from research done at the end of the last century. Engel's approach means that the share of food consumption of all consumption reveals the welfare level of the household. While this idea seems to work well in developing countries, where food forms the largest part of the consumption basket (Deaton 1981), this is not necessarily so in developed countries. Other commodity groups can be used instead of food, like for instance all necessities or luxuries. The problem here seems to be that the scale numbers depend on the commodity group chosen (Nicholson 1976 and empirically Hagfors 1988, which includes other references).

The second subgroup, iterative methods, is actually a generalization of Engel's approach, where all commodity groups are considered simultaneously. Originating from Prais and Houthakker (1955), the purpose was to estimate commodity specific weight coefficients for all commodity groups.

The income equivalence scales can then be calculated from commodity specific coefficients as a weighted average, where the weights are the expenditure shares of the commodity groups. (For empirical calculations see McClements 1977; some problems with the approach are presented in the next section.)

The third subgroup includes applications of utility theory. Barten (1964) was the first to put the equivalence scales within the context of a utility function. In this way he opened the way to the utilization of the theory of consumer choice. (For critical comments, see Pollak and Wales 1979.)

In this paper we have chosen the Barten approach. We are, however, deferring the further introduction of the method until the next section. (For empirical comparisons between the Prais and Houthakker model and the Barten model, see Muellbauer 1977.)

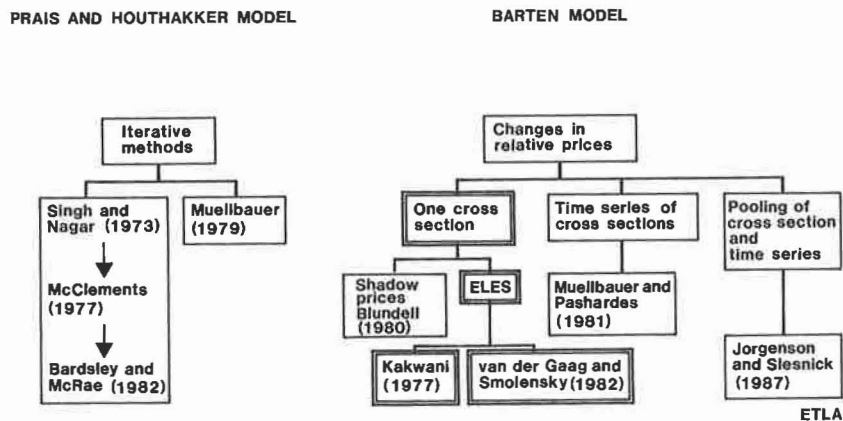
3. IDENTIFICATION AND CHOICE OF THE MODEL

When trying to estimate equivalence scales from demand systems, we face the problem of identification. The reason for this is the fact that we have to estimate n scale numbers from n demand equations or Engel curves. However, only $n-1$ of these are independent. The last one will be determined as a residual from the budget constraint. (See Muellbauer 1975.)

As a solution to the identification problem various methods have been used. Some of these are presented in figure 3, where the Prais and Houthakker model and the Barten model are considered.

In the studies referred to in figure 3 concerning the Prais and Houthakker model the results are pessimistic. The scales are reported to be dependent on the initial values given in iterations or ex ante restrictions or the iteration algorithm.

Figure 3. Different solutions to the identification problem in calculating equivalence scales from demand systems



In the Barten model, on the other hand, the information on changes in relative prices is sufficient for identification of the equivalence scale numbers. Now the central question is what kind of data is available.

If a time series of cross sections or a panel data is available, one can choose the method used by Muellbauer and Pashardes (1981) for England. They applied a demographic AIDS demand system to cross sections for 1968-1973.

Another method has been used by Jorgenson and Slesnick (1987). They have applied a method for pooling of cross-section and time series data. In this study they estimate an econometric exact aggregation model of

consumer behaviour. This model has a translog form. The cross-section year is 1973 and the time series cover the years 1947-1982. In Finland there have not yet been data available for application of either method mentioned above.

The model used in this paper is based on one cross section. As can be seen from figure 3 there are still some possibilities. In order to get the price variability into the model one can imply some shadow prices. Blundell (1980) used shadow prices for leisure time. This possibility is ruled out in our study, because in Finland the information concerning the time use of the households is available only in the household survey for 1971.

The other way is to apply the ELES (Extended Linear Expenditure System) model together with the Barten approach. The identification of the equivalence scales is derived through the variation of the saving behaviour among the households. This method was introduced by Kakwani (1977). In his study the households were characterized by the number of household members, and in this way only the returns to scale were explicitly estimated. We have repeated Kakwani's calculations concerning Australian households with Finnish data.

Van der Gaag and Smolensky (1982) made another application of the ELES model in the U.S.A. using a 1972-1973 household survey. The difference with Kakwani was that several household characteristics could be included in addition to the size of the household by using the dummy techniques. We have implemented the dummy variables also in this paper. The models we estimate for the years 1976 and 1981 are ELES (according to the size of the household) and ELES_D (using dummies for different types of households).

The ELES model has been widely used in the literature (see Lluch 1973, Lluch, Powell and Williams 1977 and Howe 1975) and we are skipping the details here. It is useful, however, to keep in mind that ELES demand equations are derived from the Stone and Geary utility function of the form:

$$(1) \quad u(q) = \sum_{k=1}^{n+1} b_k \log(q_k - c_k)$$

where q is a commodity vector, c_k is the subsistence consumption of commodity k and b_k is an allocation parameter. Commodity $n+1$ is savings and it is supposed to be zero at the subsistence level.

The demand equations for commodities can be derived in a usual way:

$$(2) \quad p_k q_k = p_k c_k + b_k \left(y - \sum_{k=1}^n p_k c_k \right), \quad k=1, \dots, n,$$

where p_k is the price of the commodity k and y is the household's income.

The interpretation of equation (2) is the familiar one, that the consumer first chooses commodities up to the subsistence level and then allocates the remaining expenditures according to the allocation parameters b_k .

When applied to one cross section, the prices are assumed to be constant and the same for all households.

For estimation purposes equation (2) can be transformed so that the estimation is possible, equation by equation, using the OLS method.

From the estimated regression parameters the parameters of ELES, b_k and c_k can be derived. (For more formal presentations see Lluch, Powell and Williams 1977 and Hagfors 1988.)

The general and commodity specific equivalence scales for households of different size are estimated in this paper by using the method presented by Kakwani (1977). Here a one-member household is selected as a reference household. We can present the reference household's utility function in the following form:

$$(3) \quad u\left(\frac{q_k}{m_k}\right) = \sum_{k=1}^n b_k \log \left(\frac{q_k}{m_k} - c_k \right),$$

where m_k is the commodity specific scale number of commodity k . It depends on the characteristics of the household. For the reference household all m_k numbers are equal to one.

The corresponding indirect utility function is:

$$(4) \quad v(y, m) = \log a + \log(y - \sum_k p_k m_k c_k) - \sum_k b_k \log m_k - \sum_k b_k \log p_k + \sum_k b_k \log b_k.$$

For a reference household the indirect utility function is:

$$(5) \quad v(y_0) = \log a + \log(y_0 - \sum_k p_k c_k) - \sum_k b_k \log p_k + \sum_k b_k \log b_k.$$

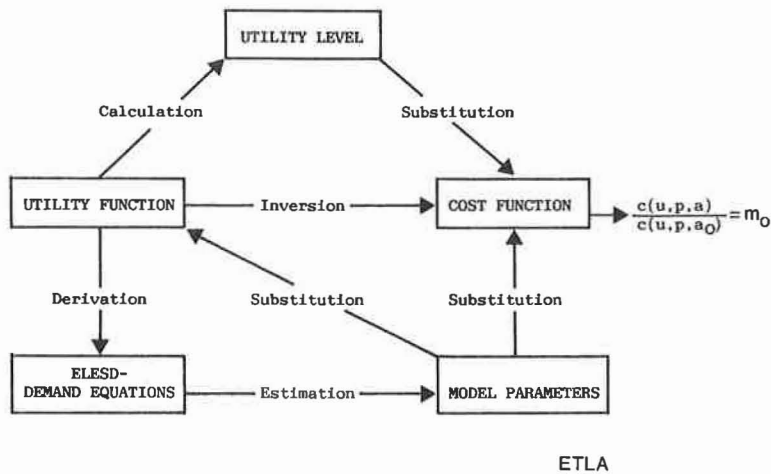
Next the functions (4) and (5) are equalized, and assuming that $p_k = 1$ for all k we can solve:

$$(6) \quad m_0 = \frac{n}{\prod_{k=1}^n m_k^{b_k}} + (y_0)^{-1} \left[\sum_{k=1}^n m_k c_k - \left(\sum_{k=1}^n m_k^{b_k} \right) \left(\sum_{k=1}^n c_k \right) \right].$$

From (6) we can see that we can get all the necessary parameters from the estimated expenditure system. An important observation is that the income level of the reference household, y_0 , appears as an argument for the equivalence scales m_0 .

While (6) in this paper gives the scale numbers only for households of different size, the second model enables a more detailed classification of household type. In the ELES application we can include such characteristics as the age of the household head and the number and age distribution of the children. The procedure is described in figure 4.

Figure 4. The procedure for estimating equivalence scales from a demand system



The equivalence scales are here defined as the minimum cost for a household to attain a utility level u with prevailing prices p compared to the minimum cost of the reference household to attain the same utility level at the same prices.

As can be seen from figure 4 the procedure begins with the estimation of the demand system. Substituting the parameters of the estimated demand system and by choosing the income level of the reference household the utility level can be calculated. Substituting the utility level and relevant parameters to the corresponding cost function, the minimum costs to the same utility level for different household types can be calculated. It is also important to notice here, as in (6), that the equivalence scales depend on the income level of the reference household.

4. DATA AND ESTIMATION

In this paper we utilize the household survey data collected by the Central Statistical Office of Finland. There were sample-based consumption studies already at the beginning of the century, but the first "complete" household survey was made in 1966. Since then a survey has been made every fifth year. The data we are using represent all households in Finland in 1976 and 1981. In the 1976 survey 3383 households kept books on their expenditures for a month and 7871 households were interviewed at a yearly level. In the 1981 survey 7368 households kept books on their expenditures for two weeks. They were then interviewed at monthly and yearly levels. In both surveys also other data registers, like the tax register, were used.

The concepts in household surveys in Finland are based on U.N. recommendations (Provisional Guidelines on Statistics of the Distribution of Income, Consumption and Accumulation, 1977, and System of National Accounts, 1968).

In this paper the ELES demand system has been estimated for households of one to six or more persons. We do not present here the regression equations in order to save space. The ELES model with dummies for household characteristics was estimated for the years 1976 and 1981.

The demand equations were estimated for the following commodity groups:

1. Food
2. Beverages and tobacco
3. Clothing and footwear
4. Dwelling, heating, light and power
5. Household furniture, fitments and services
6. Medical and health care
7. Transport
8. Recreation, education, cultural services
9. Other goods and services.

Both the 1976 and 1981 data were adjusted so that the actuarial expenditures like imputed housing expenditure or income were omitted so that the expenditures of a household were expressed in "real monetary" terms.

As the explanatory variables in regressions the following ones were used:

- y = Disposable income of the household
- L_1 = Logarithmic size of the household
- D_1 = Dummy for the head of the household, less than 45 years old
- D_2 = " " " , 45 - 65 years old
- D_3 = Dummy for a household with one child, less than 7 years old
- D_4 = " " " , more than 7 years old
- D_5 = " " " two children, less than 7 years old

D_6	=	"	"	two children, one less, one more than 7 years old
D_7	=	"	"	two children, both more than 7 years old
D_8	=	"	"	three children, less than 7 years old
D_9	=	"	"	three children, two less, one more than 7 years old
D_{10}	=	"	"	three children, one less, two more than 7 years old
D_{11}	=	"	"	three children, more than 7 years old

The estimated regression equations are presented in appendix tables A1 and A2.

In both models, ELES by the size of the household and ELES with the characteristics dummies the OLS estimation method was used. This immediately raises the question concerning the heteroskedasticity of the data. (For tests of heteroskedasticity with 1976 data, see Hagfors 1987b.) Though the parameter estimates are still unbiased, they are inefficient.

In order to make corrections for heteroskedasticity the estimations were repeated by using the GLS estimation method. In the next chapter we present commodity specific equivalence scales based on both estimation methods.

5. COMMODITY SPECIFIC SCALES

As mentioned above, the equivalence scales depend on the chosen commodity group. The reason for this is mainly that the returns to scale in consumption are different for different commodity groups. This is one of the main reasons to use a complete demand system. Here we weight dif-

ferent commodity specific scales together in one general equivalence scale.

Table 1. Commodity specific equivalence scales from the ELES demand system for households of different size in the year 1976

Commodity group	Estimation method	Scales					
		m ₁₁	m ₂₁	m ₃₁	m ₄₁	m ₅₁	m ₆₊₁
1	OLS	1.000	1.984	2.530	3.168	3.439	4.652
	GLS	1.000	1.809	2.639	3.360	3.579	4.875
2	OLS	1.000	1.988	3.260	3.423	2.980	3.886
	GLS	1.000	1.936	3.164	3.539	2.859	4.070
3	OLS	1.000	1.341	2.283	3.194	3.233	3.869
	GLS	1.000	1.379	2.440	3.455	3.094	4.141
4	OLS	1.000	1.271	1.720	2.109	1.780	2.237
	GLS	1.000	1.174	1.664	2.085	1.802	2.155
5	OLS	1.000	1.662	2.478	3.263	3.201	3.441
	GLS	1.000	1.803	2.774	3.565	3.225	3.573
6	OLS	1.000	1.634	1.558	2.093	1.960	2.518
	GLS	1.000	1.769	1.777	2.350	2.199	2.755
7	OLS	1.000	1.618	3.378	4.052	4.080	4.180
	GLS	1.000	1.496	3.287	4.325	3.932	4.274
8	OLS	1.000	1.534	3.142	4.025	3.309	4.202
	GLS	1.000	1.476	3.911	4.205	0.059	4.437
9	OLS	1.000	0.822	1.513	2.101	1.532	2.357
	GLS	1.000	0.672	1.660	2.083	1.457	2.265

In this paper we have estimated the ELES demand system for households of size 1 to 6 or more. As a first step the subsistence parameters for different commodities in different household groups were calculated from the estimated demand equations in different household groups. Also the allocation parameters were calculated. In the estimation both OLS and GLS methods were used. Commodity specific scales in commodity groups 1-9 are presented in table 1 for the year 1976.

In table 1 a household with one member is a reference household and has a scale number of 1.00 in all commodity groups. If there exist returns to scale in the consumption of different commodities, the number in the last column should be less than 6. From table 1 it can be seen that there are returns to scale in all commodity groups when the size of the household is growing. This phenomenon is strongest in the consumption of dwelling, heating, light and power, medical and health care and in the consumption of other goods and services.

The weakest returns to scale effects are in the consumption of food, transport and recreation, education and cultural services. Especially the results concerning housing expenditures and food consumption seem to be intuitively acceptable.

The growth of the scale numbers is monotonical only in commodity groups (1), (3) and (7). In other groups the scale number is decreasing when the size of the household is growing from 4 to 5. This is probably due to the fact that simultaneously with the growth of the size of the household, also the age structure of the household is changing. The effects of this are then dominating the effects of the growth in the size.

An interesting feature is exhibited by the scale numbers of the last commodity group, other goods and services. Here the scale number is decreasing when the size of the household doubles. This can be explained with the data. Take commodity group (9), for example, which includes meals outside the home. When two single persons start to live together, it is probable that the share of home made meals rises and in this way the returns to scale of the household are increasing.

In table 1 also the GLS estimation results are expressed. The general prevailing feature is that the GLS commodity specific scales behave in a way that is similar to the OLS scale numbers. They are, however, slightly bigger than the OLS numbers except in commodity groups (4) and (9).

Next we present the commodity specific equivalence scales for the year 1981 in table 2.

Table 2. Commodity specific equivalence scales from the ELES demand system for households of different size in the year 1981

Commodity group	Estimation method	Scales					
		m_{11}	m_{21}	m_{31}	m_{41}	m_{51}	m_{6+1}
1	OLS	1.000	1.970	2.634	3.234	4.019	4.646
	GLS	1.000	1.891	2.622	3.081	3.802	4.956
2	OLS	1.000	1.706	2.494	2.987	3.271	3.092
	GLS	1.000	1.803	2.514	3.065	3.330	3.534
3	OLS	1.000	1.518	2.635	3.855	4.279	5.201
	GLS	1.000	1.291	2.432	3.152	3.510	4.205
4	OLS	1.000	1.245	1.714	2.178	2.065	1.867
	GLS	1.000	1.329	1.877	2.240	2.102	1.889
5	OLS	1.000	1.739	2.335	2.933	3.182	3.650
	GLS	1.000	1.818	2.297	2.620	3.008	3.686
6	OLS	1.000	1.694	1.936	2.201	2.353	2.690
	GLS	1.000	2.005	2.375	2.515	2.829	3.248
7	OLS	1.000	1.723	3.171	3.980	4.543	5.549
	GLS	1.000	2.185	3.998	4.785	5.373	6.477
8	OLS	1.000	1.342	2.483	3.488	3.342	3.126
	GLS	1.000	1.505	2.759	3.526	3.476	3.765
9	OLS	1.000	1.097	1.705	2.121	2.147	2.239
	GLS	1.000	1.430	2.072	2.460	2.199	2.794

The commodity specific scales in table 2 reveal some changes compared to the numbers for the year 1976. The returns to scale are strongest in commodity groups (4), (6) and (9), as in 1976. In food consumption the

returns to scale are rather weak and at the same level as in 1976. The returns to scale have increased strongly in commodity group (8). The opposite development has happened in commodity groups (3) and (7). The results of the GLS estimation seem to be also similar to the 1976 results. In all commodity groups except clothing and footwear the GLS scale numbers are greater than the OLS scale numbers.

The commodity specific scales are not increasing monotonically in commodity group (2), where in the OLS estimation the scale number decreases while moving from a five-member household to a six or more member household. The same thing is happening in commodity group (4), where the decrement begins already with a household of four members. This happens for both OLS and GLS scale numbers. The decrement of the scale numbers occurs also in commodity group (8) and in the GLS numbers of the commodity group (9).

All in all it can be concluded that there exist returns to scale in consumption when the size of the household is growing and that the scale effect is different in different commodity groups. From this it follows that by basing the equivalence scale calculations on some commodity group, there is a danger that the general scale will be over or underestimated, depending on the commodity group chosen. It seems to be that at least in developed countries the equivalence scales should be based on all commodity groups. This supports the use of the complete demand systems. (See Deaton, 1981, Nicholson, 1976 and Hagfors 1988).

In the next section we present the general household equivalence scales for the years 1976 and 1981 first by household size and then for different household types.

6. GENERAL EQUIVALENCE SCALES

The general household equivalence scales in this paper are calculated by using the ELES and ELES0 models. When we concentrate only on households with different number of members, we can apply equation (6) in the calculation of equivalence scales. As is obvious, the scale numbers based on the ELES model are functions of the income level of the reference household. Therefore we have chosen a one-member household for each income level of 15000, 20000, 40000, 60000 and 80000 FIM as a reference household in the year 1976. Households are classified according to size, ranging from one to six or more. The ELES demand system was estimated for each household group.

After having substituted estimated subsistence and allocation parameters of the corresponding ELES demand system into the equation (6), the general equivalence scales were computed at the different levels of each reference household's income. The results are presented in table 3.

Table 3. General household equivalence scales from ELES demand system for households of different size and level of income in the year 1976

Level of income	Estimation method	Household size					
		1	2	3	4	5	6+
15000	OLS	1.000	1.668	2.471	3.248	3.407	4.399
	GLS	1.000	1.401	2.010	2.725	2.979	4.836
20000	OLS	1.000	1.630	2.547	3.218	3.289	4.155
	GLS	1.000	1.342	1.828	2.386	2.582	4.003
40000	OLS	1.000	1.574	2.550	3.174	3.111	3.880
	GLS	1.000	1.254	1.556	1.876	1.986	2.754
60000	OLS	1.000	1.555	2.551	3.160	3.052	3.788
	GLS	1.000	1.224	1.466	1.707	1.787	2.338
80000	OLS	1.000	1.546	2.552	3.152	3.022	3.742
	GLS	1.000	1.209	1.420	1.622	1.688	2.129

The equivalence scale numbers in table 3 are increasing monotonically except in higher income levels when the size of the household is increasing from 4 to 5. When the GLS method was used, there was strict monotonicity in all cases.

It is clearly seen that when the income level of the reference household is rising, the marginal increase in the scale number due to an additional member is decreasing. This effect is stronger in the GLS estimations. The implication of this is that if the equivalence scales based on the method described above are applied in economic welfare comparisons and the income level is ignored, the results will favor high income households with several members.

The equivalence scales for the year 1981 are computed for different income levels and the results are presented in the table 4. Here the income levels for the reference household are 25000, 50000, 75000,

Table 4. General household equivalence scales from the EIES demand system for households of different size and level of income in the year 1981

Level of income	Estimation method	Household size					
		1	2	3	4	5	6+
25000	OLS	1.000	1.630	2.481	3.214	3.982	5.367
	GLS	1.000	1.721	2.719	3.306	4.073	5.502
50000	OLS	1.000	1.571	2.444	3.143	3.654	4.507
	GLS	1.000	1.724	2.712	3.271	3.760	4.792
75000	OLS	1.000	1.552	2.432	3.119	3.544	4.220
	GLS	1.000	1.725	2.709	3.259	3.655	4.565
100000	OLS	1.000	1.542	2.426	3.107	3.490	4.077
	GLS	1.000	1.726	2.708	3.253	3.603	4.447
125000	OLS	1.000	1.536	2.422	3.100	3.457	3.991
	GLS	1.000	1.726	2.707	3.249	3.572	4.377

100000 and 125000 FIM. Again both OLS and GLS estimation methods were used.

The equivalence scales in table 4 are increasing monotonically in all cases. The economies of scale effects are also present here, but not so strongly as in the year 1976. The OLS numbers are smaller than the GLS numbers at all income levels. The effect of the income level of the reference household is much stronger in 1981 than in 1976.

When comparing the 1976 and 1981 general equivalence scales, it is rather difficult to see how the changes in the commodity specific scales in tables 1 and 2 have influenced the general scales, at least intuitively. This is because there have been several fundamental changes in the equivalence scale computations, including the following:

- Changes in the composition of households. In the years 1976-1981 the average size of households decreased and so did the overall returns to scale.
- Changes in relative prices.
- Changes in relative incomes of different household groups.
- Changes in the composition of the consumption basket in different household groups.

The results presented above imply that applying the 1976 scale in to 1981 distribution studies, or vice versa, will lead to erroneous conclusions.

The size of the household is an important characteristic in the determination of equivalence scales. There are, however, also other characteristics which are of importance when consumption-based equivalence scales are to be calculated. Especially the age of the head of the household and the number and the age distribution of children should be mentioned.

(Hagfors 1988 and Hagfors-Koljonen 1986.) Next we will present the general household equivalence scales when the second model, ELES, is used. The procedure follows the one presented in figure 4.

The household equivalence scale in this case is defined as a relation between the two cost functions:

$$(7) \quad m_0 = \frac{c(u, p, a)}{c(u, p, a_0)}, \text{ where}$$

u = the chosen utility level,

p = prices of the commodities,

a = the vector of the characteristics of household a ,

a_0 = the vector of the characteristics of a reference household.

The interpretation of (7) is that m_0 gives the relative minimum costs that household a needs in order to be at the same utility level u as the reference household when the prices p are prevailing. Here again the equivalence scale is a function of the income level of the reference household.

The cost function for a household with characteristics vector a can be written in the following way (Van der Gaag and Smolensky 1982):

$$(8) \quad c(u, a) = \sum_k c_k (1+d_k^i a) + \exp[u - \sum_k b_k \log b_k + \sum_k b_k \log (1+d_k^i a)],$$

where $(1+d_k^i a)$ represent commodity specific scales for the household a and is estimated from regression coefficients of the dummy variables.

For a reference household these are all set equal to 1.00.

The values of the cost functions (8) were computed for both 1976 and 1981 for different household types. The general household equivalence scales were then calculated by using formula (7). A single person less than 45 years old was selected as a reference household. In 1976 the income levels of the reference households were 15000, 30000 and 60000 FIM. In 1981 the income levels were 20000, 40000 and 80000 FIM respectively. The results are presented in tables 5 and 6.

It can be seen from table 5 that there exist strong returns to scale when the size of the household is growing. The age structure of children has an influence on the scale numbers. When the share of the school-aged children is increasing, while the head of the household remains in the same age group, the scale numbers are growing.

When the age of the head of the household is increasing, the equivalence scales are decreasing. It should be noticed, however, that the age classification in this application is quite rough. For instance, in the group of the old-aged households there is a difference between those people with good health and the very old ones, who are not able to consume as much anymore due to physical reasons even though they could afford it, so the scale numbers might be different with a tighter age classification. Here the large share of very old women is pressing the scale numbers of the pensioners down.

The growth of the level of the income of the reference household is decreasing the scale numbers.

Similar features in the scale numbers exist for 1981 in table 6. The age effect is not, however, so strong as in 1976. There are also relative

Table 5. The general constant utility household equivalence, scales from the ELES demand system in Finland in the year 1976. The reference household is a single person under 45 years old.

Household type	Age of the head and income level of the reference household								
	15000	<45 30000	60000	15000	45-65 30000	60000	15000	>65 30000	60000
ONE PERSON	1.000	1.000	1.000	0.843	0.778	0.735	0.560	0.424	0.333
TWO PERSONS									
1 adult, 1 child < 7	1.537	1.499	1.473	1.401	1.295	1.224			
1 adult, 1 child 7-18	1.797	1.695	1.627	1.647	1.467	1.347			
2 adults	1.618	1.512	1.440	1.462	1.275	1.150	1.163	0.890	0.708
THREE PERSONS									
1 adult, 2 children < 7	2.129	1.990	1.897	1.971	1.749	1.600			
1 adult, 1 child < 7, 1 child 7-18	2.143	2.033	1.960	1.990	1.801	1.674			
1 adult, 2 children 7-18	2.164	1.971	1.842	2.003	1.726	1.541			
2 adults, 1 child < 7	1.911	1.808	1.739	1.760	1.580	1.459			
2 adults, 1 child 7-18	2.160	1.986	1.870	2.020	1.759	1.585			
FOUR PERSONS									
2 adults, 2 children < 7	2.387	2.197	2.070	2.223	1.947	1.763			
2 adults, 1 child < 7, 1 child 7-18	2.400	2.240	2.133	2.242	2.000	1.837			
2 adults, 2 children 7-18	2.421	2.178	2.015	2.255	1.924	1.703			
FIVE PERSONS									
2 adults, 3 children < 7	2.582	2.387	2.256	2.402	2.112	1.920			
2 adults, 2 children < 7, 1 child 7-18	2.868	2.667	2.532	2.722	2.433	2.239			
2 adults, 1 child < 7, 2 children 7-18	2.498	2.247	2.079	2.379	2.058	1.844			
2 adults, 3 children 7-18	2.599	2.326	2.144	2.471	2.124	1.891			

Table 6. The general constant utility household equivalence scales from the ELES demand system in Finland in the year 1981. The reference household is a single person under 45 years old.

Household type	Age of the head and income level of the reference household								
	20000	<45 40000	80000	20000	45-65 40000	80000	20000	>65 40000	80000
ONE PERSON	1.000	1.000	1.000	0.987	0.961	0.943	0.775	0.723	0.689
TWO PERSONS									
1 adult, 1 child < 7	1.701	1.607	1.544	1.654	1.519	1.428			
1 adult, 1 child 7-18	1.692	1.579	1.502	1.676	1.535	1.440			
2 adults	1.665	1.543	1.460	1.648	1.498	1.396	1.443	1.270	1.154
THREE PERSONS									
1 adult, 2 children < 7	2.187	1.996	1.867	2.170	1.952	1.804			
1 adult, 1 child < 7, 1 child 7-18	2.080	1.891	1.765	2.062	1.846	1.701			
1 adult, 2 children 7-18	2.099	1.903	1.771	2.080	1.855	1.704			
2 adults, 1 child < 7	2.052	1.869	1.745	2.034	1.824	1.681			
2 adults, 1 child 7-18	2.073	1.883	1.755	2.055	1.837	1.689			
FOUR PERSONS									
2 adults, 2 children < 7	2.455	2.210	2.044	2.436	2.161	1.977			
2 adults, 1 child < 7, 1 child 7-18	2.347	2.104	1.941	2.329	2.058	1.875			
2 adults, 2 children 7-18	2.364	2.112	1.943	2.346	2.066	1.877			
FIVE PERSONS									
2 adults, 3 children < 7	2.556	2.283	2.098	2.538	2.236	2.032			
2 adults, 2 children < 7, 1 child 7-18	2.522	2.224	2.024	2.503	2.177	1.957			
2 adults, 1 child < 7, 2 children 7-18	2.613	2.319	2.122	2.515	2.180	1.955			
2 adults, 3 children 7-18	2.722	2.412	2.203	2.704	2.365	2.137			

changes in the scale numbers of some household types. On the basis of the tables 5 and 6 it is concluded, that

- the equivalence scales have changed from the year 1976 to the year 1981.

A closer analysis of the different household types would be needed to reveal the reasons behind the changes, but we are leaving this work for another paper.

7. CONCLUDING REMARKS

There are several ways to compute household equivalence scales. The scales also differ when they are derived by using different methods. Some scales do take into account the characteristics of the households from many points of view, in some cases the only possibility is to classify the households by the number of members because of the lack of data. The economic behaviour of the households is but one of the approaches. With all its weaknesses it has the remarkable feature that now it is possible to implement the theory of consumer choice and in this way get a theoretical background which can be tested.

In this paper we have chosen a demand system approach. The identification problem of the scale numbers from demand equations and the solutions available led to the choice of the ELES model. The household equivalence scales were computed with this model for the years 1976 and 1981. From the results we can conclude, that

- there exist returns to scale in consumption when the size of the household is growing,
- the returns to scale differ in different commodity groups,
- adults and children bring different additions to scales,
- additions differ by the age structure of the children,
- scale numbers decrease when the head of the household is aging,
- the growth of the income level will bring a diminishing addition to the scale numbers when the size of the household is growing,
- there are changes in the scales from the year 1976 to the year 1981.

Finally it should be noticed that there are several weaknesses in the method that has been applied above. First, the ELES model is derived from the Stone and Geary utility function, which is additive and leads to the separability between commodity groups. The restrictions of additive preferences in empirical research has a price. (See Deaton 1974). Secondly it has been found out (Hagfors 1987,1988) that the scale numbers in urban and rural areas in Finland differ from each other.

All these conclusions drawn together indicate the direction for future research:

- the implementation of a flexible functional form demand system ,
- the utilization of more current data and
- the widening of the relevant characteristics of the households.

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Appendix 1. Commodity grouping

1. Food
 - Bread and grain products
 - Meat
 - Fish
 - Milk, cheese and eggs
 - Fats and edible oils
 - Fruit and vegetables
 - Potatoes, potato products
 - Sugar
 - Coffee, tea, cocoa
 - Other foodstuffs
 - Paid-for meals and free meals
2. Beverages and tobacco
 - Non-alcoholic beverages
 - Alcoholic beverages
 - Tobacco
3. Clothing and footwear
 - Clothing
 - Footwear
4. Dwelling, heating, light and power
 - Private apartment in housing company
 - Privately owned house
 - Free dwelling
 - Rent of actual dwelling
 - Rent of other dwellings and repair of dwelling made by tenants
 - Water rates
 - Heating, light and power
 - A weekend house and heating, light and power
5. Household furniture, fitments and services
 - Furniture, works of art and carpets
 - Textiles and other fitments
 - Household machines
 - Household equipment
 - Household articles and services
 - Domestic help
6. Medical and health care
 - Drugs and pharmaceutical preparations
 - Therapeutical equipment
 - Doctor's charges, laboratory, medical examination and treatment charges
 - Hospital and sanatorium fees
7. Transport
 - Private vehicles
 - Running costs of private vehicles
 - Domestic travels
 - Journeys abroad
 - Transport and storage of household articles, free transport services
 - Communication
8. Recreation, education, cultural services
 - Radio, TV
 - Larger durable goods
 - Other hobby articles
 - Hobby articles, their parts and repair
 - Recreation and cultural services
 - Books
 - Newspapers and periodicals
 - Education
9. Other goods and services
 - Personal hygiene and care
 - Other articles
 - Restaurant, cafe and hotel expenses
 - Package tours
 - Other financial services
 - Other services

Appendixtable A1. Commodity demand equations for ELES demand system from 1976 household survey. For variables, see page 179. (t-values in parentheses)

Variables	(1) Food	(2) Beverages and tobacco	(3) Clothing and footwear	(4) Dwelling, heating etc.	(5) Household furniture	(6) Medical and health care	(7) Transport	(8) Recreation, education etc.	(9) Other good and services
Constant	2.3693	-0.0210	0.1433	1.5423	0.1831	0.3524	-0.8966	-0.1147	-0.1721
y	0.0358	0.0222	0.0379	0.0761	0.0439	0.0137	0.1080	0.0611	0.0684
	(9.608)	(12.580)	(11.612)	(24.846)	(15.227)	(15.290)	(14.163)	(13.573)	(23.178)
L ₁	5.3700	0.1339	0.5774	-0.2943	0.1815	0.1501	0.6509	0.0908	-0.8848
	(35.162)	(1.847)	(4.308)	(-2.342)	(1.533)	(4.074)	(2.081)	(0.492)	(-7.309)
D ₁ <45	-0.6275	0.5113	0.7445	0.4240	0.4879	-0.2232	1.6360	1.1195	1.2989
	(-3.590)	(6.162)	(4.853)	(2.948)	(3.601)	(-5.294)	(4.569)	(5.297)	(9.376)
D ₂ 45-65	0.1784	0.3008	0.3607	0.0138	0.1797	-0.0762	1.3901	0.3450	0.5507
	(1.060)	(3.765)	(2.442)	(0.100)	(1.378)	(-1.876)	(4.032)	(1.695)	(4.127)
D ₃ (1,0)	-1.3809	0.0531	-0.3188	0.8246	0.3651	-0.0692	0.2534	1.3255	-0.1208
	(-6.526)	(0.529)	(-1.717)	(4.736)	(2.226)	(-1.357)	(0.585)	(5.180)	(-0.720)
D ₄ (0,1)	-0.4345	-0.1579	0.3463	0.3220	-0.0797	-0.0657	0.7571	0.4890	0.3256
	(-2.330)	(-1.784)	(2.116)	(2.099)	(-0.552)	(-1.460)	(1.982)	(2.169)	(2.203)
D ₅ (2,0)	-0.9284	-0.0433	0.0735	1.5038	0.5342	-0.0233	-0.0209	0.7786	-0.0969
	(-3.251)	(-0.320)	(0.293)	(6.399)	(2.413)	(-0.338)	(-0.036)	(2.254)	(-0.428)
D ₆ (1,1)	-0.8396	0.0389	0.4757	0.5416	0.5226	-0.0598	0.4693	1.0300	0.1604
	(-2.681)	(0.262)	(1.731)	(2.101)	(2.153)	(-0.792)	(0.732)	(2.720)	(0.646)
D ₇ (0,2)	-0.0348	-0.2474	1.1739	0.7482	0.0008	0.0107	-0.7030	0.7735	-0.1366
	(-0.150)	(-2.247)	(5.765)	(3.919)	(0.005)	(0.192)	(-1.479)	(2.757)	(-0.743)
D ₈ (3,0)	-0.2413	-0.2794	0.4648	-0.4855	0.4516	0.0285	3.0928	0.2140	-0.2939
	(-0.313)	(-0.764)	(0.687)	(-0.766)	(0.756)	(0.154)	(1.960)	(0.230)	(-0.481)
D ₉ (2,1)	-0.5503	0.7162	-0.4233	2.0334	1.4009	0.0076	2.6288	-0.0732	-0.4478
	(-0.777)	(2.130)	(-0.681)	(3.488)	(2.551)	(0.045)	(1.811)	(-0.085)	(-0.797)
D ₁₀ (1,2)	-0.3838	-0.2503	0.5533	0.9695	0.2736	-0.1989	-0.8821	0.6942	-0.0247
	(-0.931)	(-1.280)	(1.530)	(2.859)	(0.857)	(-2.002)	(-1.045)	(1.393)	(-0.076)
D ₁₁ (0,3)	0.0003	-0.1131	0.5168	1.3337	-0.0051	-0.2151	0.4811	0.2316	-0.1252
	(0.001)	(-0.706)	(1.744)	(4.802)	(-0.020)	(-2.643)	(0.696)	(0.567)	(-0.468)
F	309.7411	35.7558	55.0638	102.6717	48.4833	41.1289	45.0320	48.5903	66.6057
R ²	0.5471	0.1224	0.1768	0.2859	0.1590	0.1382	0.1494	0.1593	0.2062

Appendixtable A2. Commodity demand equations for ELES demand system from 1981 household survey.
(t-values in parentheses)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Variables	Food	Beverages and tobacco	Clothing and footwear	Dwelling, heating etc.	Household furniture	Medical and health care	Transport	Recreation, education etc.	Other good and services
Constant	4162.5	-21.928	-648.655	2842.16	310.555	706.444	-4390.99	-396.479	-399.064
y	0.0259 (6.796)	0.0175 (12.321)	0.0378 (17.556)	0.0778 (34.365)	0.0381 (20.396)	0.0095 (17.681)	0.1771 (23.804)	0.0691 (22.818)	0.1492 (41.336)
L ₁	2.2365 (33.794)	0.1595 (6.463)	0.3537 (9.456)	-0.3109 (-7.897)	0.1124 (3.460)	0.0864 (9.230)	0.7723 (5.971)	-0.0206 (-0.391)	-0.4696 (-7.485)
D ₁ <45	-0.1587 (-4.698)	0.0962 (7.627)	0.1144 (5.987)	0.1202 (5.979)	0.0657 (3.958)	-0.0450 (-9.411)	0.4691 (7.101)	0.1522 (5.658)	0.3389 (10.576)
D ₂ 45-65	0.0377 (1.129)	0.0489 (3.921)	0.0834 (4.413)	0.0507 (2.546)	0.0290 (1.762)	-0.0167 (-3.533)	0.3795 (5.801)	0.0582 (2.188)	0.2406 (7.583)
D ₃ (1,0)	-0.1903 (-4.476)	-0.0337 (-2.126)	-0.0094 (-0.394)	0.2344 (9.263)	0.0660 (3.164)	-0.0178 (-2.967)	-0.1505 (-1.811)	0.2115 (6.250)	-0.0163 (-0.405)
D ₄ (0,1)	-0.0753 (-2.365)	-0.0177 (-1.491)	0.0665 (3.694)	0.1253 (11.672)	-0.0014 (-0.087)	-0.0135 (-2.994)	-0.0757 (-1.216)	0.0872 (3.437)	0.0554 (1.838)
D ₅ (2,0)	-0.0927 (-1.621)	-0.0461 (-2.160)	-0.0223 (-0.693)	0.4381 (12.871)	0.0968 (3.447)	-0.0148 (-1.834)	-0.0873 (-0.781)	0.2725 (5.986)	-0.0049 (-0.092)
D ₆ (1,1)	-0.1900 (-3.591)	-0.0371 (-1.881)	0.0438 (1.467)	0.3887 (12.346)	0.0431 (1.659)	-0.0170 (-2.280)	-0.3055 (-2.954)	0.1993 (4.733)	-0.0196 (-0.393)
D ₇ (0,2)	-0.0509 (-1.268)	-0.0545 (-3.642)	0.1248 (5.501)	0.2789 (11.672)	-0.0377 (-1.910)	-0.0212 (-3.719)	-0.2109 (-2.687)	0.1594 (4.986)	-0.0012 (-0.032)
D ₈ (3,0)	-0.3699 (-2.255)	-0.1535 (-2.508)	-0.1038 (-1.120)	0.5642 (5.779)	0.0353 (0.439)	0.0022 (0.094)	0.0542 (0.169)	0.2123 (1.625)	-0.0411 (-0.265)
D ₉ (2,1)	-0.2076 (-1.730)	-0.0566 (-1.266)	-0.0320 (-0.472)	0.5707 (7.993)	0.1210 (2.054)	-0.0405 (-2.389)	-0.6138 (-2.6170)	0.2127 (2.227)	-0.0295 (-0.2595)
D ₁₀ (1,2)	-0.0700 (-0.918)	-0.1368 (-4.811)	0.1070 (2.485)	0.3570 (7.869)	0.1412 (3.773)	-0.0280 (-2.599)	-0.1456 (-0.977)	0.1524 (2.510)	-0.0200 (-0.028)
D ₁₁ (0,3)	0.2110 (2.705)	-0.0761 (-2.168)	0.1942 (4.406)	0.3094 (6.666)	0.0355 (0.927)	-0.0368 (-3.336)	-0.2038 (-1.336)	0.2671 (4.302)	-0.0175 (-0.237)
F	248.427	46.084	123.735	192.376	80.943	76.260	112.688	99.431	200.322
R ²	0.305	0.075	0.179	0.254	0.125	0.119	0.166	0.149	0.262

ON CONCENTRATION OF SOCIAL TRANSFERS AND THE ECONOMIC WELFARE OF HOUSEHOLDS IN FINLAND

by Robert Hagfors and Risto Sullström

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1. INTRODUCTION

It has been said that Finland is a welfare state. The main objectives of the welfare state are not, however, always expressed in a very precise manner. Where the redistribution of economic resources among households is the goal of the policy adopted, the main instruments used by governments have been usually transfers from or to the households.

The main stream of distribution research in Finland has concentrated on studying these income flows. In this way it has been possible to produce information on how to affect the income formation of the households at different stages and finally the distribution of the income and purchasing power of the households.

Basically, an economist is not interested in the distribution of income, but in the distribution of the economic welfare or economic wellbeing that households can purchase with their disposable income. This means that the distribution of income is relevant from the economic welfare point of view only so far as it reflects the economic welfare of households. Strictly speaking, in theory this happens only when the preferences of the households are identical and homothetic. From the studies concerning household budgets it seems that neither of these restrictions holds empirically. (For references, see Jorgenson and Slesnick 1986). One way to adjust the disposable income by differences in the characteristics of the households and, in a way, differences in preferences, is to implement household equivalence scales. This is the approach chosen here. There have been some efforts to study the distribution of economic welfare earlier (Hagfors and Koljonen 1984, Sullström 1987, Uusitalo 1987).

In these studies some relatively rough equivalence scales are used. (For more on the equivalence scales applied in these studies, see Hagfors 1987a.) In this paper we are trying to take into account the changes in the economic circumstances which the households are facing. This is done by using year specific scales for the years 1976 and 1981.

When we are attempting to determine how social transfers affect the distribution of the economic welfare of households, we have to specify the meaning of the concept "economic welfare" or "wellbeing". As a proxy for this we use the disposable income or factor income deflated with equivalence scales. The deflator is constructed so that it reflects the minimum cost that a household needs in order to be at the same utility level as the reference household. The utility level is determined through the consumption of market goods and the budget constraint. We make here the separability assumption concerning the consumption of goods and leisure time. Also the assumption of inter-temporal separability holds here.

Economic welfare here is defined in a narrow sense. It is based on the consumption of households and should as such be separated from the more far-reaching concepts like "quality of life" and "life style". We believe, however, that it is easier to interpret and expand the concept used in this paper to a wider basis than vice versa. For the decision making of the government it may also be useful to have information on the economic framework in which the households are living.

We are proceeding so that in the next chapter we give a general description of the distributional statistics we are using later in

this paper. Especially the normative nature of some of the measures is pointed out.

In the third section we present the two household surveys applied. The income concepts are defined and in the same connection we present the equivalence scales we are using in this paper.

The difference between the distribution of income and the distribution of economic welfare and the changes that have occurred in these in the years 1976 and 1981 are studied next.

In the fifth section we present the concentration of the social transfers to the households when households are arranged according to the level of economic welfare. This is done first for transfers to all households and then at a more disaggregated level for some transfers to certain household types.

We give a summary of the results in the last section.

2. MEASURES OF DISTRIBUTION

There exist many different ways to characterize income distributions. It is a common practice to use summary statistics in the evaluation of inequality in the distribution of income or in the evaluation of the redistributive impact of policy changes. In this paper we consider only the coefficient of variation, the Atkinson inequality measure, the Theil inequality measure and the Gini and concentration coefficients, or their graphical counterparts the Lorenz and concentration curves.

The aim of this section is to show how these measures correspond to the orderings of alternative social states according to certain social welfare functions.

The Lorenz curve is usually used in comparisons of ordered distributions.

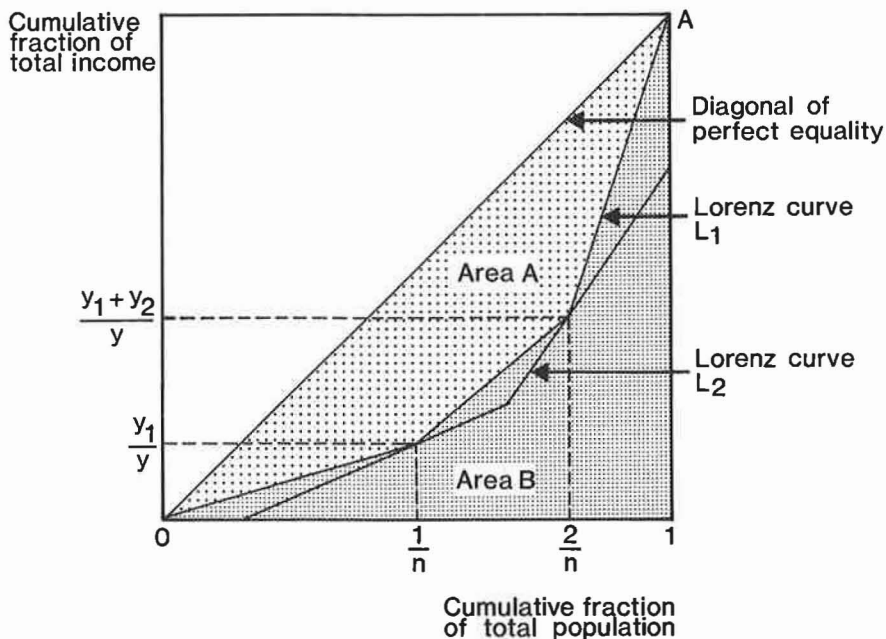
Let the income distribution be represented by a vector $(y_1, y_2, \dots, y_n)$, where y_i is the income of the i th income receiving unit ($i=1, 2, \dots, n$). If the incomes are arranged according to size,

$$(1) \quad y_1 \leq y_2 \leq \dots \leq y_n,$$

the Lorenz curve expresses how many per cent of total income (Y) a certain per cent of ranked income earners will get. In the simplified figure 1 the points of the Lorenz curve correspond to the coordinates

$$(2) \quad (0,0); (1/n, y_1/Y); (2/n, (y_1+y_2)/Y); \dots; (1,1)$$

Figure 1. The Lorenz curve



Between the points (0,0) and (1,1) the Lorenz curve is located under the diagonal. The further from the diagonal, the less equal the income distribution is said to be. The full equality prevails when $y_1=y_2=\dots=y_n$, which corresponds to the diagonal OA in the figure 1.

When the Lorenz curve of the income distribution lies everywhere inside another one it is possible to say that the first one is socially preferred. This analysis is typically concerned with income distribution comparisons which yield partial orderings of social states. Only those income distributions whose Lorenz curves do not cross are comparable.

Gini coefficient can be expressed by using the two areas A and B in figure 1 and by noticing, that $A+B=1/2$ in the unit square,

$$(3) \quad Gini = \frac{A}{A+B} = 2A .$$

The more equal the distribution to be measured is, the lower the value the Gini index obtains. When the distribution is totally equal, the value of the Gini index is zero and for a totally unequal distribution it is one.

It is possible to define the Gini coefficient as an average diversion of the absolute values of pairs of incomes (Kanbur 1984):

$$G = (1/2n^2\bar{y}) \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j| \quad , \quad (i,j=1,2,\dots,n)$$

$$= 1 + 1/n - 2/(n^2\bar{y}) \sum_{i=1}^n y_i(n+1-i) \quad , \quad (i,j=1,2,\dots,n) .$$

The Gini coefficient takes into account the diversion of every pair of incomes, so that a transfer from a higher income unit to the lower income unit always decreases the value of the coefficient.

The third expression of the Gini index implies the fact that the index depends on the number of income earners at different income levels. Here the Gini index is defined as a weighted average of the income levels of individuals, when the weights are the positions of individuals in the income ranking (Sen 1972).

$$(5) \quad G = 1 + (1/n) - (2/n^2 \bar{y})(y_2 + 2y_3 + \dots, n + ny) ,$$

where $y_1 \geq y_2 \geq \dots, \geq y_n$.

The fourth measure used in this paper, the Theil index, is defined by the following formula:

$$(6) \quad T(y;n) = (1/n) \sum_1 (y_i/\bar{y}) \ln(y_i/\bar{y})$$

The Coefficient of Variation (cv) can be defined as a square root of the relation of the variance of income and average income

$$(7) \quad cv = \sqrt{V/\bar{y}}$$

The coefficient weights transfers in different parts of the distribution equally, but gives no answer to the question of whether heavier weight should be given to a low income individual becoming richer or to a high income individual becoming even richer.

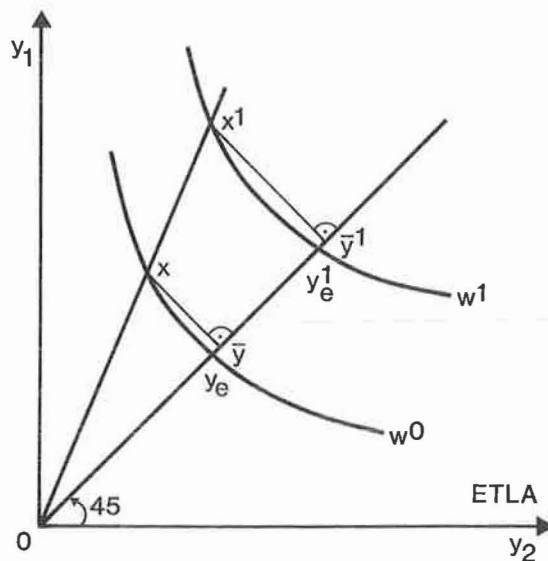
The alternative approach to the Lorenz comparisons is to derive the form of the social welfare function which corresponds exactly to the ranking of income distributions according to some of these summary statistics. However, in order to use income distribution statistics as welfare change measures, we must consider individual income as an appropriate measure of individual welfare. We assume that our income measures accurately account for the difficulties.

We proceed, following Atkinson (1970), by defining a particular measure of inequality and observing then the social welfare functions which would rank income distributions in the same order as do the Gini coefficient and the coefficient of variation.

Consider the social indifference curves (w^0 and w^1) in income space in figure 2, where a two-person case is illustrated. Suppose that income is distributed according to the point x with $y_1 > y_2$. The mean income associated with x is $\bar{y}$ along the 45° line. Let us define the equally distributed equivalent income y_e as that amount of income which, if given equally to all persons, would provide the same level of social welfare as at x , that is

$$(8) \quad W(y_e, y_e, \dots, y_e) = W(y_1, y_2, \dots, y_n)$$

Figure 2. The Atkinson index of inequality



$A(y)$ is bounded by 0 and 1, and the greater the curvature of the social indifference curve, the greater $A(y)$ will be.

Table 3. The current transfers received by households according to the 1981 Household survey

		B111.FIM
I. Social security benefits		21.2
NP	National pension	6.5
OP	Occupational pension	11.1
-	Sickness and injury insurance	1.4
CB	Child benefits	1.7
UB	Unemployment benefits	0.4
II. Social assistances		4.2
SB	Social and housing benefits	0.8
OTHER	Other social security	3.4
I+II. Current transfers received		25.3

The current transfers received by households in table 3 are divided into two main categories. Social security benefits cover 84 per cent of transfers and the rest goes to social assistance purposes. National and occupational pensions are the major groups of social security. The share of pensions is going to increase in Finland for two reasons. First, the demographic structure of the population is changing so that the share of pensioners is increasing. Secondly, the share of small national pensions of all pensions is decreasing.

In the light of the development described above it is interesting to see how the two types of pensions are distributed among households and how they affect the distribution of economic wellbeing of the households considered. We do this study for some household types introduced earlier. A similar kind of treatment is made also for other social transfers in table 3.

4. DISTRIBUTIONS IN THE YEARS 1976 AND 1981

4.1. Aggregate level

In this chapter we present some results we have arrived at by using the income concepts defined earlier and applying the distributional measures introduced in chapter 2. We have calculated some distribution measures for the years 1976 and 1981 which we introduce in table 4. The corresponding Lorenz curves of income concepts are presented in figure 1. Next we try to estimate what has happened to the distributions during the research period.

Table 4. Some distribution measures of the disposable income of the households (D1/h) and of the disposable income per equivalence scale (D1/m) for the years 1976 and 1981 in Finland

Year	Income concept	Mean	cv	G	Theil	Atkinson, value of e	
						1.5	1.1
1976	D1/h	35100	0.544	0.295	0.159	0.230	0.163
	D1/m	22717	0.447	0.223	0.090	0.143	0.095
1981	D1/h	59639	0.544	0.301	0.175	0.445	0.182
	D1/m	43493	0.380	0.207	0.088	0.352	0.099

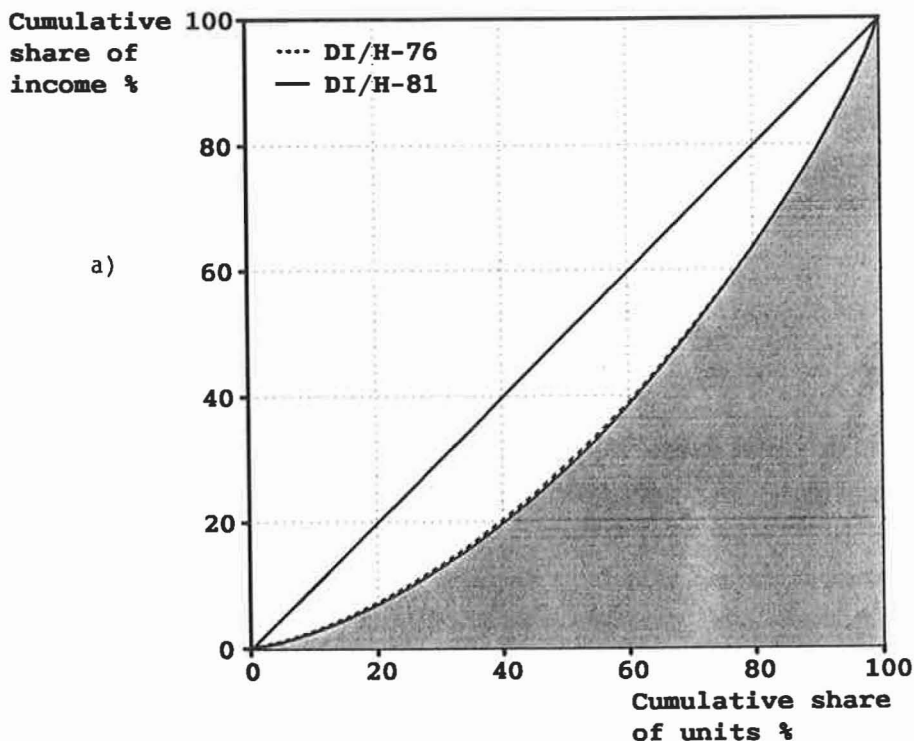
First, if we look at the coefficient of variation and the purchasing power of households, that is the disposable income per household, there seems to have happened no change at all. There is a small increase in inequality in the Gini index, and little bit stronger increase in Theil's measure. Also the Atkinson index reveals that inequality in purchasing power has increased, especially when the low income households are weighted in the index. In figure 3.a) the Lorenz curves move close to each other.

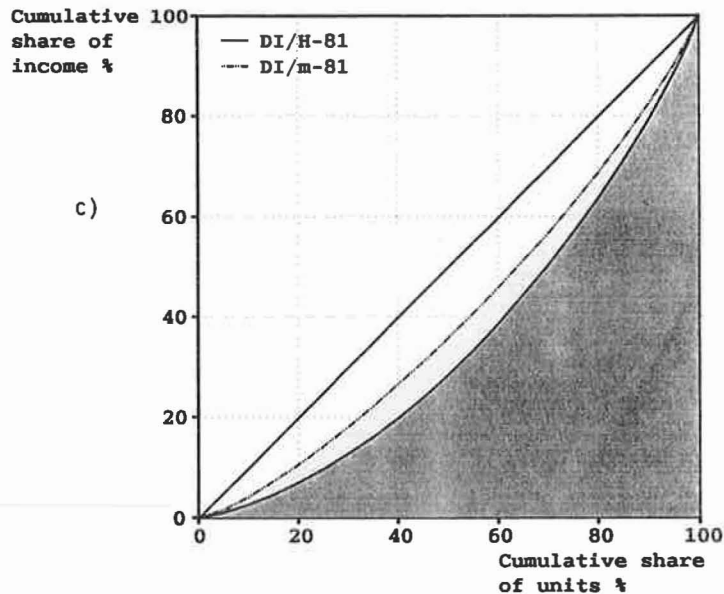
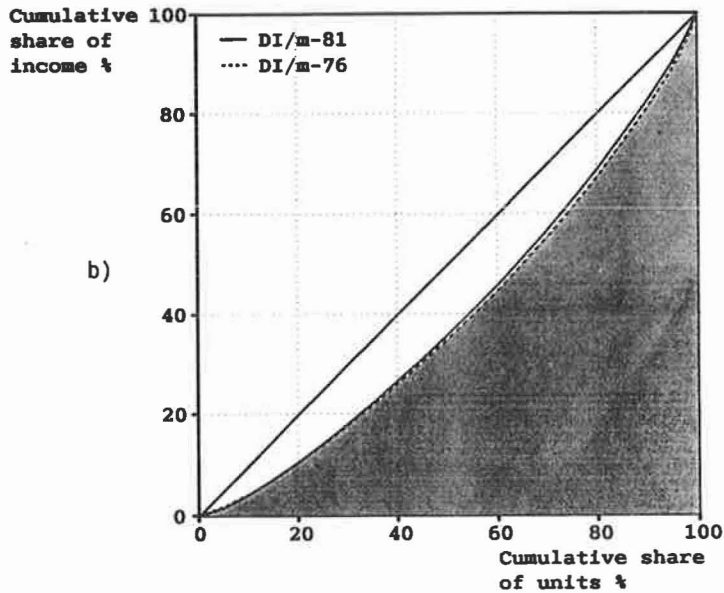
Next we move on to the concept of economic wellbeing. Let us first compare the distributions of income per equivalent member in the years 1976 and

1981. In table 4 we can see a clear decrement in the inequality of the economic wellbeing of households in measures like coefficient of variation, Gini index and to a lesser degree in Theil's index. When we weight high income households, the Atkinson index has remained approximately unchanged. When the low income households are weighted, the Atkinson index shows that the inequality of economic wellbeing did increase during the research period.

These results are consistent with the figure 3.b), where the Lorenz curves are crossing each other at the lower end.

Figure 3. a) The Lorenz curves of the disposable income per household in the years 1976 and 1981,
 b) the Lorenz curves of the disposable income per equivalence scale in the years 1976 and 1981 and
 c) the Lorenz curves of the disposable income per household and the disposable income per equivalence scale in the year 1981 in Finland





Finally we present a comparison of the two income concepts in the year 1981. It is clear from the table 4 and the figure 3.c) that we can get a rather different picture from the prevailing inequality in the society by the choice of the income concept. When we take into account the differences in the structures of the households, the Lorenz curve in figure 1.c) stays all the way inside the curve where the households are assumed to be identical. One should, however, be careful about making conclusions regarding the level of social welfare in these cases.

4.2. Household types

The income concepts we study are the disposable income and factor income of the households. In this section we are disaggregating the households into different types and calculating the distribution statistics separately in each household type. The household types follow the classification of table 2. The statistics are presented only for the year 1981. The average disposable incomes and factor incomes of different household types are presented in table 5.

Table 5. The disposable income per household (Di/h), the disposable income per equivalence scale (Di/m), the factor income per household (Fi/h) and the factor income per equivalence scale (Fi/m) in different household types from the 1981 Household survey in Finland, FIM

Household type	Income concept	Age of the head of the household			
		-24	25-44	45-64	65-
Single person	Di/h	20460	36618	32210	24821
	Di/m	20460	36618	32918	32284
	Fi/h	20957	47440	31080	4201
	Fi/m	20957	47440	31764	5464
Single provider	Di/h	31206	48451	48783	29804
	Di/m	22689	32572	32839	22904
	Fi/h	24557	45274	45792	12505
	Fi/m	17885	31389	30555	9610
Couple without children	Di/h	55196	68728	61275	44270
	Di/m	35515	44674	40606	33051
	Fi/h	64776	89969	65872	11665
	Fi/m	41679	58526	43652	8709
Couple with children	Di/h	53300	77297	81911	52397
	Di/m	27057	37865	41880	31284
	Fi/h	53474	95703	104314	19243
	Fi/m	27420	46859	52862	10487
Adult household	Di/h	70791	84181	88664	58904
	Di/m	35734	40668	44623	37307
	Fi/h	66622	92033	106726	38339
	Fi/m	31202	41380	50271	19448

There are some features which should be noticed from table 5. Though the age grouping is rather rough, the life cycle behavior of income concepts is obvious. It is clearly seen that the factor incomes in the oldest age group are lower than in other age groups, which is natural. The incomes of single persons and single providers are on the average slightly lower than in other household groups. This seems to be true even if the incomes are deflated with equivalence scales.

The functioning of equivalence scales can be seen from the figures of table 5. In the upper left corner the disposable income per household and per equivalent member are equal. This is also the case with the factor income.

For the age group 45 years old and over the figures differ from each other. This is because our equivalence scales are constructed for three age groups and here we have single persons of four ages. In all groups where there are more than one person the incomes of different definitions differ from each other.

Next we are trying to find out how these incomes are distributed inside the household groups. In all cases the households are arranged according to their disposable income per household. This means that only in the case of disposable income per household are we able to calculate the Gini index G . In all other cases we will present the concentration index G^* . (See Kakwani 1980, chapter 8.)

We present also the distribution measures for factor income of the households. This is the income concept which is usually taken as a basic definition before the government has had its own intervention. In this

Table 6. The values of the Gini index G of household disposable income in different household types from the 1981 Household survey in Finland (total=0.301)

Household type	Age of the head of the household			
	-24	25-44	45-64	65-
Single person	0.282	0.194	0.237	0.213
Single provider	0.122	0.182	0.179	0.219
Couple without children	0.164	0.170	0.221	0.215
Couple with children	0.162	0.166	0.207	0.204
Adult household	0.216	0.206	0.219	0.252

case naturally only the so-called "first round effects" are considered.

Gini indexes for relevant household types are presented in table 6.

Table 7. The concentration index G* of household disposable income per equivalence scale (D1/m), factor income (F1/h) and factor income per equivalence scale (F1/m) in different household types from the 1981 Household survey in Finland. (Total G*: D1/m=0.182, F1/h=0.335 and F1/m=0.293)

Household type	Income concept	Age of the head of the household			
		-24	25-44	45-64	65-
Single person	D1/m	0.282	0.194	0.243	0.221
	F1/h	0.425	0.230	0.466	0.467
	F1/m	0.425	0.230	0.472	0.474
Single provider	D1/m	0.133	0.174	0.177	0.196
	F1/h	0.177	0.238	0.292	0.454
	F1/m	0.188	0.230	0.298	0.439
Couple without children	D1/m	0.174	0.181	0.240	0.246
	F1/h	0.207	0.213	0.357	0.455
	F1/m	0.216	0.223	0.374	0.479
Couple with children	D1/m	0.169	0.175	0.216	0.212
	F1/h	0.244	0.214	0.276	0.448
	F1/m	0.249	0.221	0.281	0.431
Adult household	D1/m	0.166	0.160	0.182	0.210
	F1/h	0.287	0.235	0.276	0.495
	F1/m	0.233	0.212	0.258	0.462

From table 6 we find that measured with the Gini index the distribution of disposable income is most unequal in the groups of single young persons, single persons who are arriving at the pension age and adult households where the head of the household is a pensioner. The distribution of disposable income is most equally distributed among single providers and couples with children.

In order to be able to calculate the Gini indexes for other income concepts it would have been necessary to arrange the income receiving units according to the relevant income concept in each case. Therefore, we will present only the Concentration indexes of disposable income per equivalence scale, factor income per household and factor income per equivalence scale in table 7.

The absolute value of the Concentration index will tell the amount of inequality in the concentration of the income concept in question and the plus or minus sign if it is concentrated towards high income or low income households respectively. From table 7 we can see that when the households are arranged according to disposable income per household, the disposable income per equivalence scale is more equally concentrated than the two-factor income concepts.

The concentration of factor incomes in the pensioners age group is relatively high in all household types. High concentration is also present in the factor incomes of young single persons and especially among single persons who are about to reach the pension age.

5. SOCIAL TRANSFERS IN THE YEAR 1981

5.1. General picture

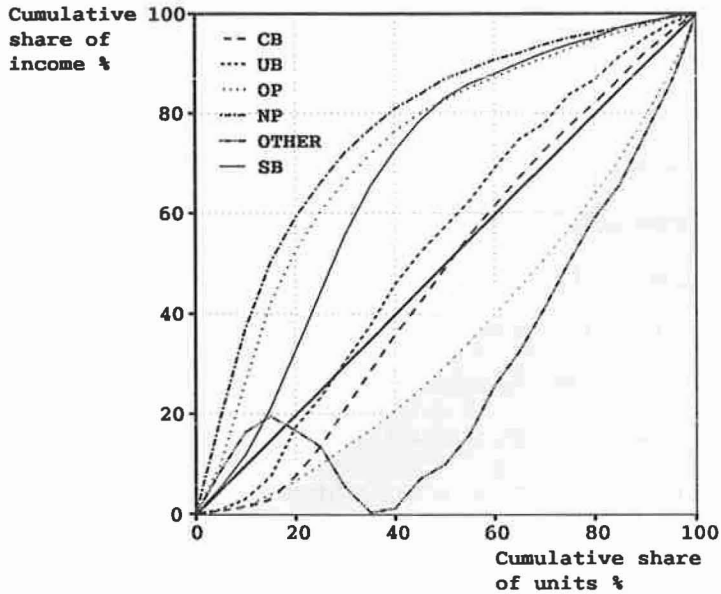
In this chapter we are taking a closer look at the social transfers of table 3. In what follows we have proceeded so that we have arranged households according to the factor income per equivalence scale in ascending order. Next we draw some concentration curves and compute concentration indexes in order to see if the social transfers redistribute economic wellbeing as they are supposed to do. We present the general behavior of the transfers to all households in table 8 and figure 4. For the purpose of comparisons we have added to all concentration figures a shaded area which is bounded by a curve. This curve is constructed by arranging the households according to the disposable income per household and then cumulating the factor incomes per equivalence scale.

Table 8. Concentration measures of some social transfers to households in the year 1981

	Transfer	Mean	G*
NP	National pension	3144	-0.534
OP	Occupational pension	5162	-0.452
CB	Child benefit	1234	0.056
UB	Unemployment benefits	269	-0.064
SB	Social and housing benefits	1937	-0.362
OTHER	Other social security	400	0.403

The pensions form the largest share of the social transfers. National pensions are concentrated strongly in low welfare households. This is also true with occupational pensions, but to a slightly lesser degree. In order to estimate the economic consequences of pensions a more detailed analysis should be done, with for instance intertemporal saving behavior and precautionary motives which we do not have. Social and housing benefits are also efficient economic welfare distributors when we look at all

Figure 4. Social transfers in the year 1981



households. Child benefits and unemployment benefits follow the diagonal, which represents the egalitarian line. The concentration curve of the other social security has a different behavior from the other transfers. This follows partly from the make-up of the group. Here the transfers from households to other households are included, and these are of course negative for some households. It looks like the households with negative shares are concentrated to second and third deciles in the economic welfare ranking.

Altogether the results of the table 8 and figure 4 give a consistent picture of the behavior of the social transfers to all households. Next we will see if the picture remains as clear as before when we consider some specific household types and how some social transfers are concentrated among them at a more disaggregated level.

The measure of inequality defined by Atkinson is

$$(9) \quad A(y) = 1 - y_e / \bar{y} .$$

In the case of the homothetic and anonymous social welfare function the value of W can be represented as

$$(10) \quad W(y_1, y_2, \dots, y_n) = \bar{y}(1-A(y)) ,$$

which is consistent with the inequality measure $A(y)$. When a specific form for $A(y)$ is given, the social welfare function W gives the same ordering of income distributions that can be derived from the right-hand side of formula (9) above. If the mean income is constant, the rankings are the same for $y(1-A(y))$ and $(1-A(y))$. In this sense we are moving beyond the Lorenz curve type of comparisons which give only a partial ordering of distributions of equal means.

If the Gini coefficient is substituted for $A(y)$, the social welfare function that would rank income distributions in the same order as the Gini coefficient is (Blackorby and Donaldson 1978):

$$(11) \quad W(y) = (y_1 + 3y_2 + 5y_3 + \dots + (2n-1)y_n)/n^2 .$$

Formula (10) is homothetic for income changes which do not affect the ranking of persons.

For the coefficient of variation the social welfare function which will give equivalent rankings of income distributions of identical means is

$$(12) \quad W(y) = \bar{y} - \sqrt{((y_1 - \bar{y})^2)/n} .$$

This means that the W 's are symmetric parabolas around the 45° line.

The ordinary presentation of the Atkinson measure (12) was used in computations (Atkinson 1970):

$$(13) \quad A = \begin{cases} 1 - [(1/n) \sum (y_i/\bar{y})^{1-e}]^{1/(1-e)} & , e \neq 1 \\ 1 - \exp[(1/n) \sum \log(y_i/\bar{y})] & , e = 1 \end{cases}$$

The Atkinson measure A tells how large a share of total income the society would be willing to sacrifice in order to get an equal distribution of income. If A is for instance 0.3, it means that equalizing incomes requires a 30 per cent increase in total income.

The results in this paper were computed for two values of e, 1.5 and 1.1. The interpretation of parameter e could be that it describes the valuation of the loss the society is willing to approve for the transfer of one unit of money to an individual who has half as much income. If e=1, taking one FIM from this richer individual is offset by giving 0.50 FIM to the poorer individual. When e=2, taking one FIM from the rich individual is equalized by giving 0.25 FIM to a poor individual.

In the evaluation of the redistributive effects of the social transfers in this paper also the concentration curves and indexes were determined. When the Lorenz curve is defined by the coordinates $(F(y), F_1(y))$, where $F(y)$ gives the share of those income earners who get income y or less and $F_1(y)$ the corresponding income share out of total income, the concentration curve has as ordinate axis $F_1(g(y))$, corresponding to some function $g(y)$ (Kakwani 1980). On the vertical axis in the concentration presentation there is the cumulative share of the income transfer. The value of the Concentration index (C) can be computed by subtracting from one twice the area below the concentration curve. The difference between the Concentration index and the Gini index is twice the area between the

curves $F_1(y)$ and $F_1(g(y))$, so that by making comparisons between these two coefficients it is possible to make conclusions on the distribution effects of an income transfer (Kakwani 1980).

3. DATA AND SOME DEFINITIONS

3.1. General description of household surveys

In this study household surveys from the years 1976 and 1981 are used as data. The first one of these is based on the two-stage stratified cluster sampling. The sampling units in the first stage were determined according to the regional base sample. The sampling units at the second stage were determined according to the housing units. We included into the calculations only the 3348 households who were part of the book-keeping sample.

In the household survey for 1981 the sampling unit was an individual in the register of the population. As a result the sampling probability depended on the number of the persons over 15 years old in the sampling framework. Inhabitants not registered in dwellings and those registered as being institutionalized were excluded from the sample. In the final sample of the year 1981 there were 7368 households.

The estimates we have calculated include some random errors because they are based on the sampling data. The smaller the subgroup of the population considered, the fewer the members of that group that are included in the sample and the more the results are influenced by chance. The 1981 household survey can be compared with the population and housing census of 1980. Income distribution statistics and

the national accounts imply to a good correspondence for the provinces and by age distribution. In the population and housing census there were somewhat older age groups, however. In the Uusimaa and the Turku and Pori provinces there were fewer people according to the 1981 household survey than according to the distribution statistics. (The Central Statistical Office of Finland, 1985.)

The sampling loss in the inquiry depended on the length of the book-keeping period. In the 1976 survey, where the period was one month, the loss was 31 per cent at the book-keeping stage and 11 per cent at the yearly interview level. In the 1981 household survey, where the book-keeping period was two weeks, the book-keeping period loss was about 25 per cent and at the yearly interview level the final loss remained at 28 per cent. The loss was 30 per cent in the cities and 18 per cent in rural communities. The greatest loss existed in Helsinki (40 per cent) and the smallest in the rural communities in central Finland. In the socio-economic groups the loss was largest in the group of pensioners, over 30 per cent. Among students and employers the loss was small, 14 and 19 per cent respectively. The loss in the 1981 household survey was large at both ends of the income distribution.

The loss in the sample of the household survey implies that the distributions studied are more equal than the actual ones. This is harmful because the households of the most interesting groups, from the viewpoint of social security, belong to the sampling loss group more frequently than on the average. In the distribution comparisons between the years 1976 and 1981 we have tried to approximate the correspondence between the income definitions by using adjustments recommended by the Central Statistical Office of Finland (1985).

3.2. Income definitions and equivalence scales

One useful way to study how the government is trying to distribute the economic welfare or wellbeing of the households is to concentrate on the income formation process at different stages. In table 1 the income formation of the households in the years 1976 and 1981 is presented such as it was in the household surveys from the corresponding years.

We focus our attention in this paper on three income concepts:

- factor income,
- current transfers received and
- disposable income.

When we are trying to describe how the government affects the incomes of a household, the factor income could be the best starting point. The next step would then be to study the transfers received and the transfers paid by the household. In this way we would end up with the disposable income of the household. We follow this commonly used practice in this paper with the exception that we concentrate here only on the transfers received by households. In this respect our analysis is clearly lacking. A more complete study should include, in addition to the cash transfers which we are considering, transfers in kind, price subsidies, publicly provided merit goods and pure public goods.

Though we are bypassing the problems of the effects of taxes and other transfers paid by households, upon which several differing opinions exist in public debate, we do not consider the issue unimportant. Instead, we shall return to the subject in subsequent studies.

The conclusions in this paper concern the short-run period. This means, that the reactions of the households to the actions of the government are not present. A satisfactory framework to do that kind of analysis would be a general equilibrium model. Though this approach has been little by little adopted also in Finland and the construction of models is on the way, we have to wait a few years for reasonable results and even then the cases which can be considered are probably quite rough and oversimplified.

In the following analysis we are using mainly the measures introduced in the previous section. Before that we are giving some background details for a correct scaling of the transfer problems.

In the year 1976 the disposable income of households was about 48 per cent related to GNP. By the year 1981 this increased to 51 per cent. The real growth was 21.4 per cent because of the relatively high inflation rate during the period. Current transfers remained unchanged, while the transfers paid decreased slightly.

The disposable income of households doubled nominally during the research period. The real growth per household was , however, only about 6 per cent, which means a little bit more than 1 per cent real growth per household annually. In the year 1976 the factor income of the household was on the average 40604 FIM. The households received on the average 8245 FIM transfers and the disposable income was 34746 FIM per household. The development of the income components is presented in table 1.

Table 1. The income formation process of the households in the years 1976 and 1981 in Finland, current prices

Income components	1976		1981	
	Total (B111.FIM)	Per household (FIM)	Total (B111.FIM)	Per household (FIM)
+ Wages and salaries	54.8	33477	101.2	54046
+ Entrepreneurial income	9.0	5499	14.4	7676
= Primary income	63.8	38976	115.6	61722
+ Property income	2.7	1628	4.8	2572
= Factor income	66.4	40604	120.4	64293
+ Current transfers received	13.5	8245	25.3	13513
- Current transfers paid	23.1	14102	34.0	18167
= Disposable income	56.8	34746	111.7	59639

Although the factor income is a good starting point for studying the effects of the government on the disposable income of the households, the disposable income is "per se" a good candidate for comparing the distribution of the economic wellbeing of the households. For this purpose, however, some adjustments for structural differences of the households are needed. This is done by using equivalence scales.

Income adjustments: a) disposable income per equivalence scale,
 b) factor income per equivalence scale.

We can say that the disposable income describes actually the purchasing power of the household, and as such it is a very rough measure of the economic welfare, because the households are assumed to be identical in structure. It is possible to adjust the income concept on a per capita basis. This is not satisfactory, because it ignores the existing differences between adults and children.

The economic welfare point of view is present in adjustments a) and b). This is because we have deflated the relevant income concept of a household with an index which has been constructed so that it reflects the

relative income needs of households of differing characteristics in order to be at the same utility level as the reference household.

The equivalence scales we are using are based on the computations made by Hagfors (1988). The constant utility scales were constructed by estimating cost functions for different household types from the 1976 and 1981 household surveys using econometric methods. For practical purposes the scales are constructed for households of different size, number of adults and children and age of the head of the household. The scales applied here are called RH II-scales. The equivalence scales are presented for low, medium and high income reference households in the years 1976 and 1981. The 1976 RH I-scales were introduced by Hagfors (1987b), but they were calculated only for an average income reference household. The scales applied in this paper are presented in the appendix.

There are some features in the scales which should be mentioned. First, the scale numbers are decreasing with the age of the head of the household. This happens in both years under study, but is stronger in 1976. Secondly, the income need brought by an additional member depends on the income level of the household. This effect has remained quite unchanged during the period.

3.3. Classification of households

In this paper we are applying the household concept that is used in the household surveys by the Central Statistical Office of Finland. This means that the people living in institutional care are excluded

from the household population. The total number of households increased during the research period by 15 per cent, or 242,000 households.

We are classifying the households according to five different household types. A criterion for this choice has been that the household types should be relevant from the social policy point of view. The increases in the sizes of the different household types has not been even, as can be seen from the tables 2 and 3. While in the year 1976 about 47 per cent of households were such that the head of the household was less than 45 years old, in the year 1981 the amount was 51 per cent. So the share of young households increased during the research period. The share of middle-aged households decreased somewhat while the share of the aged households remained unchanged. The largest decrement happened in the share of the group "couple with children" while the share of adult households, single persons and single providers clearly

Table 2. Number of different household types in the years 1976 and 1981 in Finland (in thousands)

Symbol	Household type	Year	Age of the head of the household				
			(-24)	-44	45-64	65-	Total
SP	Single person	1976		135.0	134.7	146.8	416.5
		1981	(77.7)	217.8	153.9	165.9	537.6
SPR	Single provider	1976		30.7	7.8	-	38.5
		1981	(3.0)	45.6	12.3	1.3	59.2
CWOC	Couple without children	1976		107.8	157.9	93.5	359.2
		1981	(29.0)	129.8	154.9	109.5	394.2
CWC	Couple with children	1976		462.0	165.1	4.5	631.6
		1981	(18.2)	450.0	106.3	4.0	560.3
AH	Adult household	1976		38.5	122.9	23.7	185.1
		1981	(10.8)	104.7	185.9	31.0	321.6
	Total	1976		774.0	588.4	268.5	1630.9
		1981	(138.7)	947.9	613.3	311.7	1872.9

increased. It is obvious that the development described above rests on the fact that the large "post bellum" age-groups still fall under the heading "less than 45 years old".

3.4. Social transfers

The effects of government policies can, of course, occur via several different ways. On the expenditure side the effects depend on the instrument chosen. In principle there are three different routes to follow, namely direct cash payments to households, in-kind gifts of commodities and subsidies of prices. The following general conclusions are derived from the theory of consumer behavior. (See Call and Holahan 1983.)

First, if the household has preferences which are weak for the in-kind commodity in question, for instance food, then the household may end up at a lower level of utility if resale of the commodity is restricted. Secondly, subsidies work basically in the same way as do indirect taxes. Therefore the effects are also here dependent on the preferences of the households.

The direct cash transfer payments to the households are basically equivalent to direct taxes. They provide larger choice sets to the households than do the other two alternatives, and in that way give possibilities to households to reach a higher utility level than before.

We are concentrating in this paper on the transfer payments. The figures for the year 1981 are presented in table 3.

5.2. The effects on some household types

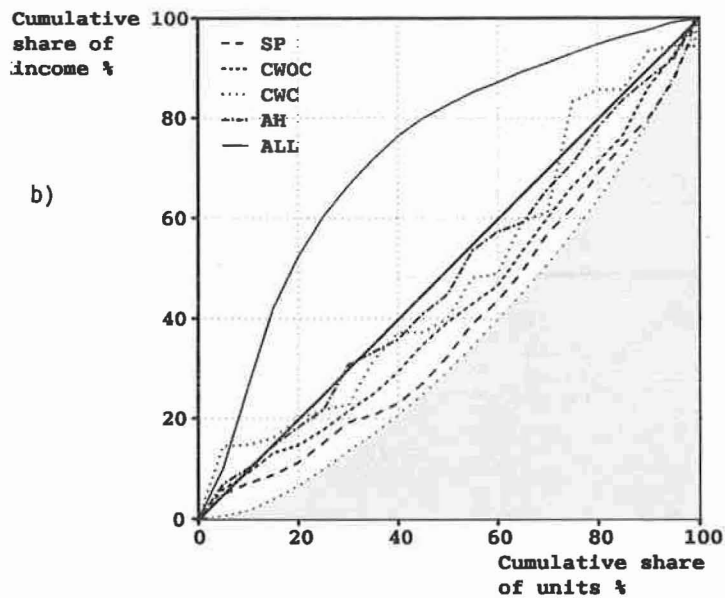
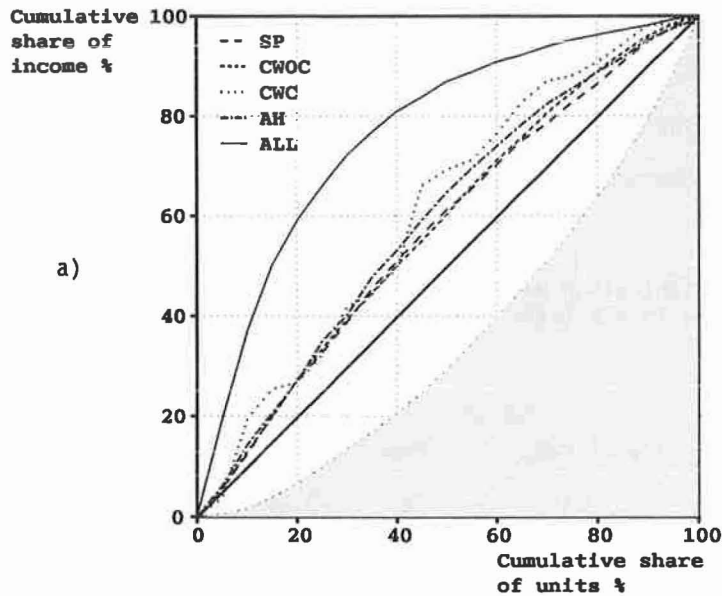
Let us first take a closer look at the pensions. In figure 5 we have the concentration curves of national and occupational pensions for some household types and for all households. In table 9 we have calculated concentration indexes for all five household types classified by the age of the household head.

Table 9. The concentration measures of national and occupational pensions in different household types in the year 1981

Household type	Age of head	National pensions		Occupational pensions	
		Mean	G*	Mean	G*
Single person	-24	203	-0.409	66	-0.025
	25-44	322	-0.856	522	-0.801
	45-64	3014	-0.613	6658	-0.306
	65-	9695	-0.140	12171	0.224
Single provider	-24	..	..	..	..
	25-44	745	-0.293	2455	-0.280
	45-64	3581	-0.347	7857	-0.279
	65-	7310	-0.416	8918	0.731
Couple without children	-24	310	-0.592	123	-0.678
	25-44	131	-0.651	586	-0.824
	45-64	2826	-0.612	11287	-0.410
	65-	12508	-0.151	24397	0.145
Couple with children	-24	175	-0.334	86	-0.390
	25-44	249	-0.369	443	-0.612
	45-64	1010	-0.605	4118	-0.568
	65-	10998	-0.177	24583	0.075
Adult households	-24	7355	-0.240	9822	-0.131
	25-44	7338	-0.245	6490	-0.151
	45-64	5086	-0.436	5841	-0.328
	65-	17083	-0.175	16835	0.050
All households		3144	-0.534	5162	-0.452

From figure 5 it can be seen that the concentration curves for national pensions and occupational pensions behave quite differently in separate household types than for all households together. One might even claim

Figure 5. The concentration curves of national pensions a) and occupational pensions b) in some household types in the year 1981



that by watching only the curves for all households the redistributive effects would become over-estimated. Anyway, the national pensions seem to be more effective in redistributing economic welfare, with the restrictions mentioned above, than do the occupational pensions. Partly this is explained by the fact that in Finland the occupational pensions are income nested.

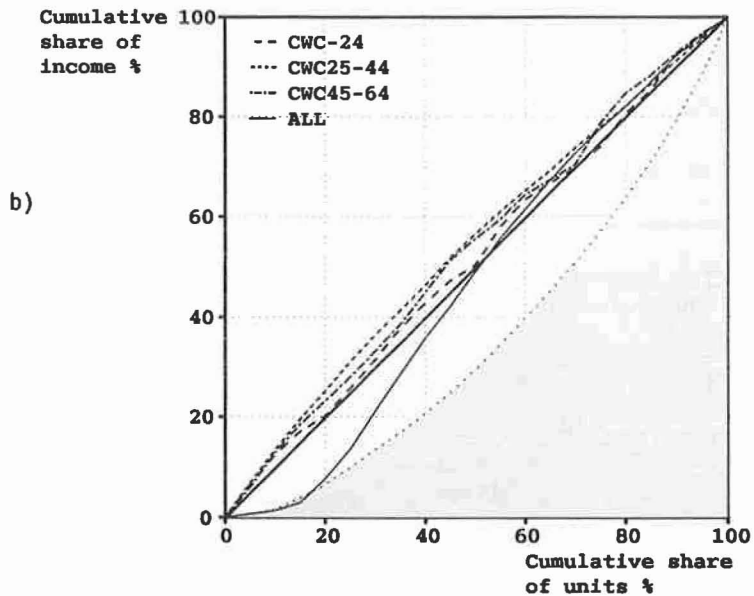
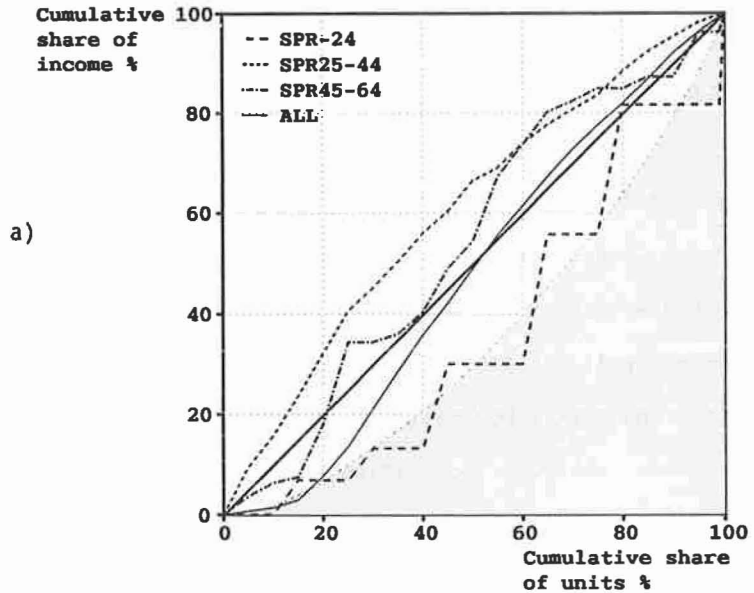
When we look at the results in table 9, the picture is still sharpening. Now we can see that though the concentration indexes for all households are rather close each other, there is a big difference in the concentration of the occupational and national pensions between the age groups "65 years old and over". For national pensions the concentration indexes are all negative, while for occupational pensions they are all positive. In spite of this, the total concentration of occupational pensions is negative.

In the same way we have drawn concentration curves and calculated concentration indexes for child benefits to different household types. As relevant types we have chosen couples with children and single providers. The concentration curves are presented in the figure 6. Table 10 contains the corresponding concentration indexes.

Table 10. The concentration measures of child benefits in different household types in the year 1981

Household type	Age of head	Mean	G*
Couple with children	-24	1382	0.201
	25-44	2075	-0.193
	45-64	967	-0.051
Single provider	-24	2255	-0.015
	25-44	2877	-0.081
	45-64	1424	-0.065
All households		1234	0.056

Figure 6. The concentration curves of child benefits in some household types in the year 1981. a) single providers, b) couples with children



In the figure 6 a) some features should be noticed. The curve for young single providers is stepwise increasing, which is due to too few observations in the sample. The other curves in figure 6 a) seem to be located above the diagonal. When we look at figure 6 b) we see that the concentration curves for the couples with children closely follow the egalitarian line.

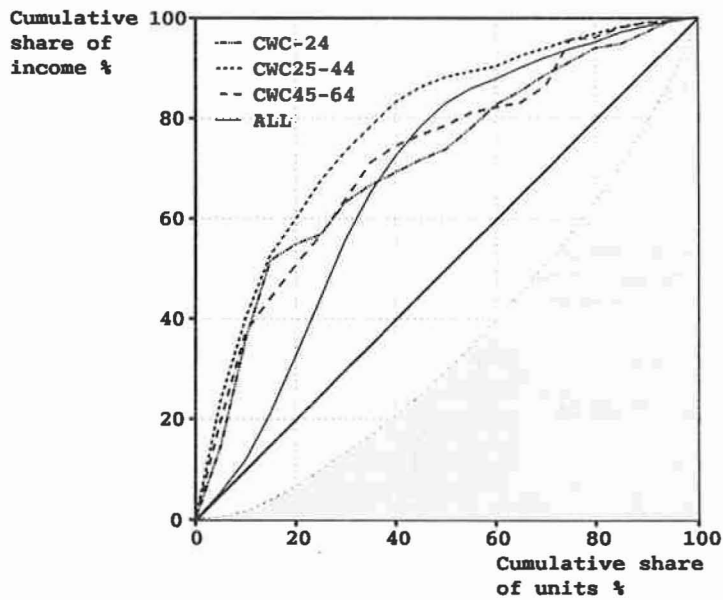
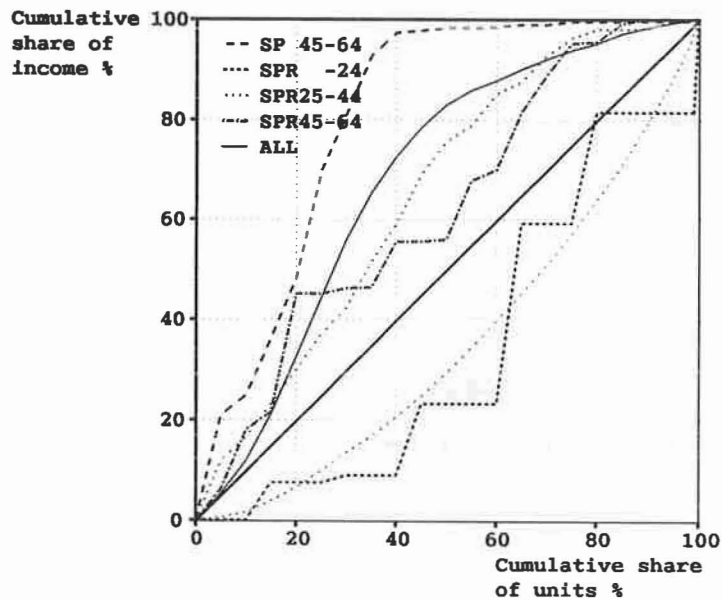
When the households are arranged according to the factor income deflated with an index which reflects the characteristics of the household, the child benefits are inefficient in redistributing the economic wellbeing among the households with children. Of course, they change the relative income levels of the households with children and other households.

The last social transfer we are considering is the social and housing benefits. This presents the social assistances side of the social transfers. The concentration indexes for some household types are presented in table 11 and the corresponding concentration curves in figure 7.

Table 11. The concentration measures of social and housing benefits in different household types in the year 1981

Household type	Age of head	Mean	G*
Couple with children	-24	7511	-0.414
	25-44	2131	-0.561
	45-64	1356	-0.443
Single provider	-24	7779	0.224
	25-44	6994	-0.285
	45-64	2702	-0.231
Single person	45-64	716	-0.619
All households		1937	-0.362

Figure 7. The concentration curves of social and housing benefits to certain household types in the year 1981



Also here the concentration curves vary significantly. The curves for couples with children follow relatively closely the form of the curve for all households. There is one clear exception: the group of elderly single persons in the upper part of figure 7. The social assistances are among those most strongly concentrated among low economic welfare persons.

6. CONCLUDING REMARKS

The development and distribution of economic welfare among households in Finland was the object of our study. We approached the issue by applying equivalence scales. These scales react to the characteristics of the households and also to the changes in relative prices and incomes. In this way we concentrated mainly on the horizontal equity point of view instead of vertical equity. Our second main interest was to see how the social transfers affect the distribution of economic welfare. We were able to draw the following conclusions:

- The distribution of disposable income of the households has remained approximately unchanged during the period 1976-81.
- The distribution of economic welfare, as we defined it, has become more equal during the same period. However, when the households at the lowest end of the welfare distribution are weighted more heavily, the inequality has increased. The Lorenz curves did cross each other, which has its own limiting effect on the conclusions.

- National pensions, occupational pensions and social and housing benefits are most effective in redistributing economic welfare. These results are conditional on the assumptions concerning inter-temporal behavior of the households.
- Child benefits are inefficient in redistributing economic welfare among households with children. However, they have an influence on the relative levels of incomes of households with children and other households.
- Single persons approaching the pension age is the group where social assistances concentrate most strongly at the lower end of the welfare distribution. Here we obviously have one of the central target groups of the social policy.
- Starting from the aggregate level we found out that by disaggregating both the income and transfer concepts on the one hand and households on the other, some new features could be found out and some conclusions based totally on aggregate observations could be called into question.
- The data did not allow us to study the complete life-cycle behavior of the households more closely.
- The results we have arrived at could be considered as useful working hypotheses for future research, keeping in mind that our results in this study are conditional on the equivalence scales which we applied and on the sampling nature of our data.

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APPENDIX. RH II-scale 1976 (General constant utility household equivalence scale for the year 1976)

		Age of head -45					45-65					65-						
House- hold size		Number of children																
		0	1	2	3	4+	0	1	2	3	4+	0	1	2	3	4+		
Low income	1	1.000					1	0.843				1	0.560					
	2	1.618	1.418				2	1.462	1.271			2	1.163	0.988				
	3	2.017	2.036	1.785			3	1.862	1.890	1.621		3	1.573	1.591	1.338			
	4	2.368	2.435	2.403	2.019		4	2.212	2.290	2.240	1.874	4	1.923	2.001	1.941	1.591		
	5	2.670	2.786	2.802	2.638	2.123	5	2.513	2.639	2.640	2.493	2.027	5	2.223	2.351	2.351	2.194	1.744
	6+	2.967	3.088	3.153	3.036	2.741	6+	2.810	2.941	2.989	2.893	2.646	6+	2.523	2.651	2.701	2.604	2.347
		0	1	2	3	4+	0	1	2	3	4+	0	1	2	3	4+		
Medium income	1	1.000					1	0.778				1	0.424					
	2	1.512	1.386				2	1.275	1.172			2	0.890	0.818				
	3	1.912	1.898	1.593			3	1.675	1.669	1.460		3	1.290	1.284	1.106			
	4	2.262	2.298	2.104	1.795		4	2.025	2.069	1.957	1.684	4	1.640	1.684	1.572	1.330		
	5	2.562	2.648	2.504	2.306	1.945	5	2.325	2.419	2.357	2.181	1.844	5	1.940	2.034	1.972	1.796	1.490
	6+	2.862	2.948	2.854	2.706	2.456	6+	2.625	2.719	2.707	2.581	2.341	6+	2.240	2.334	2.322	2.196	1.956
		0	1	2	3	4+	0	1	2	3	4+	0	1	2	3	4+		
High income	1	1.000					1	0.735				1	0.333					
	2	1.440	1.364				2	1.150	1.105			2	0.708	0.703				
	3	1.840	1.804	1.632			3	1.550	1.520	1.352		3	1.108	1.078	0.951			
	4	2.190	2.204	2.072	1.812		4	1.900	1.920	1.767	1.558	4	1.458	1.478	1.325	1.156		
	5	2.490	2.554	2.472	2.253	1.923	5	2.200	2.270	2.167	1.973	1.688	5	1.758	1.828	1.725	1.531	1.286
	6+	2.790	2.854	2.822	2.653	2.363	6+	2.500	2.570	2.518	2.373	2.103	6+	2.050	2.128	2.075	1.939	1.661

APPENDIX. RH II-scale 1981 (General constant utility household equivalence scale for the year 1981)

	House- hold size	Age of head -45					45-65					65-						
		Number of children																
		0	1	2	3	4+	0	1	2	3	4+	0	1	2	3	4+		
Low income	1	1.000					1	0.987				1	0.775					
	2	1.665	1.398				2	1.648	1.384			2	1.443	1.172				
	3	2.015	2.063	1.715			3	1.998	2.045	1.709		3	1.793	1.840	1.497			
	4	2.365	2.413	2.389	1.930		4	2.348	2.395	2.370	1.904	4	2.143	2.190	2.165	1.692		
	5	2.665	2.763	2.739	2.603	2.080	5	2.643	2.745	2.720	2.565	2.034	5	2.443	2.540	2.515	2.360	1.822
	6+	2.965	3.063	3.089	2.953	2.753	6+	2.948	3.045	3.070	2.915	2.695	6+	2.743	2.839	2.865	2.710	2.490
Medium income	1	1.000					1	0.961				1	0.723					
	2	1.543	1.333				2	1.498	1.294			2	1.270	1.056				
	3	1.893	1.876	1.599			3	1.848	1.831	1.558		3	1.620	1.603	1.320			
	4	2.243	2.226	2.142	1.767		4	2.198	2.181	2.095	1.703	4	1.970	1.953	1.867	1.465		
	5	2.543	2.576	2.492	2.310	1.887	5	2.498	2.301	2.445	2.240	1.813	5	2.270	2.303	2.217	2.012	1.575
	6+	2.843	2.876	2.842	2.660	2.430	6+	2.798	2.831	2.775	2.590	2.350	6+	2.570	2.603	2.567	2.362	2.122
High income	1	1.000					1	0.943				1	0.689					
	2	1.460	1.270				2	1.396	1.232			2	1.154	0.978				
	3	1.810	1.750	1.516			3	1.746	1.685	1.457		3	1.504	1.443	1.203			
	4	2.160	2.100	1.976	1.652		4	2.096	2.035	1.910	1.567	4	1.854	1.713	1.668	1.313		
	5	2.460	2.450	2.326	2.112	1.752	5	2.376	2.385	2.260	2.020	1.657	5	2.154	2.143	2.018	1.778	1.403
	6+	2.760	2.750	2.676	2.402	2.212	6+	2.696	2.685	2.619	2.370	2.110	6+	2.454	2.443	2.368	2.128	1.868

PART III

PERSONAL TAXATION

REAL CHANGES IN PHYSICAL PERSONS' INCOME TAXATION IN FINLAND

by Juhani Turkkila

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1. INTRODUCTION

Under Finnish tax legislation, the incomes received by an individual either in money form or in kind, as benefits having money value, are regarded as his income liable to taxation. When the individual's taxable income is determined, he is permitted to make specified deductions from his total income. The Income Wealth Tax Act currently in force classifies the permissible deductions into natural deductions, deductions to be made from the total income in both state income taxation and local (municipal) income taxation, those to be made only in state income taxation and those to be made only in local income taxation. In addition, the Act includes provisions concerning deductions to be made from the state income tax.

In Table 1, figures obtained from the official income and wealth statistics are given on physical persons' total incomes liable to state income taxation, those liable to local income taxation and on the corresponding total permissible deductions for the years 1969, 1974, 1979 and 1982.

Table 1. Physical persons' incomes and the deductions made from their incomes for taxation in selected years

Year Item	1969		1974		1979		1982	
	BILL FIM	Deductions, % of income	BILL FIM	Deductions, % of income	BILL FIM	Deductions, % of income	BILL FIM	Deductions, % of income
Total incomes								
- state taxation	21585		47957		91202		139543	
- local taxation	21526		47716		91398		139872	
Deductions								
- state taxation	6902	32	14694	31	26134	29	39410	28
- local taxation	2788	13	5747	12	10362	11	17020	12

Source: The Income and Wealth Statistics for the years concerned.

The table shows that, considered at the macro-level, the ratio of the total deductions to gross income in the years concerned was somewhat less than a third in state income taxation and somewhat more than a tenth in local income taxation.

2. THE DESIGN OF THE STUDIES

The heaviness of physical persons' income taxation has been examined so far by two studies.

The main objective of the first study was to examine year-to-year changes in the tax rate for real income of given size. First, analytic expressions were derived for calculating the income tax and the income tax rate when the taxable income is known, and the shape of the tax rate function was considered.

Following this, the adjustment of income tax scales for inflation and theoretical questions related to the measurement of real changes in the tax scales were discussed. Next, the real changes in income tax scales, by taxation classes and income brackets, in the years 1949-1979 were presented, and an endeavour was made to find a method for combining this data into yearly characteristics. Finally, the construction of an index suitable for the description of relative changes in income tax scales was considered.

The primary purpose of the second study was to examine how the system of permissible deductions in personal income taxation increased or reduced, in real terms, the heaviness of physical persons' income taxation in Finland in the years 1949-1982. The intention was to determine

the impact that the deductions applied in the various years had on the course of income taxation. The increase in the heaviness of income taxation from one point in time to another was defined here as the rise due to the system of permissible deductions in the overall income tax rate in real terms. An increase in the heaviness of taxation in real terms due to this system, or a rise in the proportion of a given real income to be paid in income taxes, may be caused by the fact, i.e., that the permissible deductions are not adjusted for inflation.

3. REAL CHANGES IN STATE INCOME TAX RATES

The tax rate function can be written, in the case of the logarithmic taxable income $z = \log y$, as

$$(1) \quad (z) = \sum_{k=0}^K [m_k - (m_k - \theta_k) e^{(z_k - z)}] \cdot \phi(z - l_k),$$

where $\theta_k = T(y_k)/y_k$ is the tax rate at the lower limit y_k of the k :th income bracket and where m_k , the marginal tax rate in the income bracket l_k , is a constant dependent on the income bracket. The phi-function on the right, $\phi(p)$, is a so-called truth function of the statement p : $\phi(p) = 1$ if p is true and $\phi(p) = 0$ if p is false. The number of income brackets is K , and l_k indicates the logarithmic income bracket.

The lower limits y_k of the income brackets in the income tax scales for 1951, their logarithms z_k and the corresponding income taxes $T(y_k)$ and tax rates θ_k , as well as the marginal tax rate m_k for each income bracket k in taxation class I (single persons)¹⁾ are set out in Table 2 on page 245.

1) The taxation class depended on the taxable person's marital status and number of dependants.

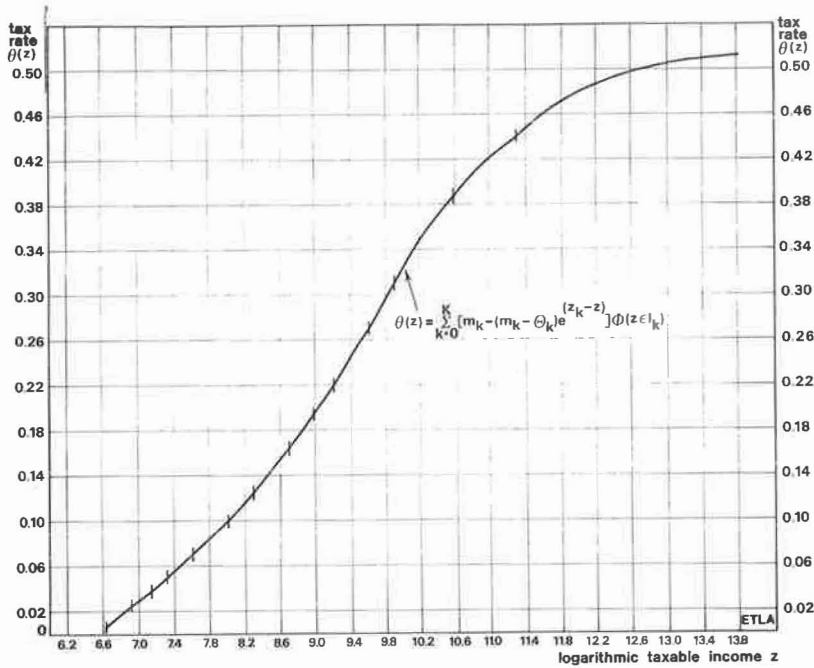
Table 2. Income tax scale for 1951.

l_k	y_k	z_k	$T(y_k)$	Θ_k	m_k
0	0	-	0	-	-
1	750	6.62	5	0.007	0.08
2	1 000	6.91	25	0.025	0.09
3	1 250	7.13	47.5	0.038	0.11
4	1 500	7.31	75	0.050	0.13
5	2 000	7.60	140	0.070	0.16
6	3 000	8.01	300	0.100	0.20
7	4 000	8.29	500	0.125	0.24
8	6 000	8.70	980	0.163	0.29
9	8 000	8.99	1 560	0.195	0.32
10	10 000	9.21	2 200	0.220	0.37
11	15 000	9.62	4 050	0.270	0.43
12	20 000	9.90	6 200	0.310	0.47
13	40 000	10.60	15 600	0.390	0.49
14	80 000	11.29	35 200	0.440	0.52

Symbols: l_k = Income bracket
 y_k = Lower limit of income bracket (FIM)
 z_k = Logarithmic taxable income
 $T(y_k)$ = Income tax at the lower limit y_k (FIM)
 Θ_k = Tax rate at the lower limit y_k , $\Theta_k = T(y_k)/(y_k)$
 m_k = Marginal tax rate (constant on the corresponding income bracket)

As appears from the table, the tax rate rises continually with increasing income. The tax rate function $\Theta(z)$ for taxation class I for the year 1951 in the interval FIM 750 - FIM 1 000 000, or when $6.62 \leq z \leq 13.82$, is represented in Figure 1 on page 5. The limits of the income brackets are indicated in the graph by small line segments. From the figure it appears, i.a., that in the highest income bracket, for very large incomes, the tax rate approaches the marginal tax rate 0.52. Though the course of the function is regular on the whole, within each income bracket it follows a complicated, winding course.

Figure 1. Income tax scale for 1951.



Once the tax function is known, we are able to consider the inflation adjustment of the tax scales. The starting point for this adjustment is the principle that only a real increase in income is permitted to increase the share of the income tax in taxable income. The real increase in income, or the increase in real income, is here understood to mean the increase in taxable income as deflated by the cost of living index. When the tax scales are adjusted for inflation, this means that taxation will not grow heavier as a result of inflation.

In the following, a kind of taxation in which real income is subject to taxation and in which only a rise in real income is permitted to raise the tax rate is referred to as hypothetical taxation. On the other hand, taxation in accordance with the provisions of the law, as it is carried out for each particular year of assessment, will be referred to as actual

taxation. Thus, for instance, by the hypothetical tax rate function of an assessment year is meant that year's tax rate function corrected for inflation, in contradistinction to the same year's actual tax rate function.

Generally, the hypothetical tax rate function for the year t is obtained from the actual tax rate function for the year 1951 as follows:

$$(2) \quad \theta_t^{51}(z) = \theta^{51}(z - \dot{p}_{51}^t) = \theta^{51}(\tilde{z}) .$$

The hypothetical tax rate $\theta_t^{51}(z)$, corresponding to any arbitrarily chosen logarithmic income $z = \log y$ in the year t , has to be set equal to the actual tax rate $\theta^{51}(z)$ of the logarithmic income z reduced by the rate of inflation, $\dot{p}_{51}^t$, between the years 1951 and t ; i.e., $z = z - \dot{p}_{51}^t = \log y - \log(p_{t0}^t / p_{t0}^{51})$. Had this correction been made, in the year t taxation in real terms would have remained at the level of 1951.

The real changes in income tax scales between 1951 and the year t at various levels of taxable income are indicated by the difference between the actual and the hypothetical tax rate function. The difference function, $\xi(z)$, which indicates the real change in taxation, in terms of tax rates, is

$$(3) \quad \xi_{51}^{t,1}(z) = \theta^{t,1}(z) - \theta_t^{51,1}(z),$$

where 1 refers to the taxation class.

If the value of the difference function is positive at a given logarithmic income $z = \log y$, the actual tax rate exceeds the hypothetical one and, thus, at this point, taxation had become heavier compared with

the year 1951. A negative difference, on the other hand, indicates that taxation had been reduced.

The difference function for taxation class I in 1952, denoted by $\xi^{52,I}(z)$, at the nominal logarithmic income level 8 is considered by way of example. The value of the difference function is then

$$\begin{aligned}
 (4) \quad \xi^{52,I}(8) &= \theta^{52,I}(8) - \theta_{52}^{51,I}(8) \\
 &= 0.0763 - 0.0971 \\
 &= -0.0208.
 \end{aligned}$$

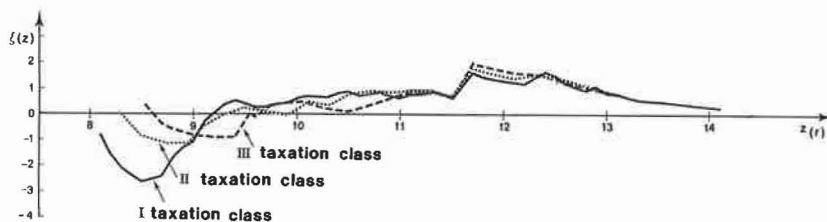
Thus, in terms of tax rates, there was a reduction of about 2 percentage points in the taxation of the taxable logarithmic income $z = 8$ ($y = \text{FIM } 2980$) between the years 1951 and 1952.

In the study, the difference functions were computed for the years 1949-79 so that, in each case, two successive years were compared. The main interest of ours centred upon the values the difference functions obtained at various levels of taxable income. Attention was paid, in addition, to questions such as: Was the course of the difference function relatively steady or oscillating, and how did the difference functions for the various taxation classes differ from one another in each particular year? The general shape of the difference functions for one and the same taxation class was similar. However, the value of the difference function corresponding to the same taxable income sometimes varied considerably from one taxation class to another. Because of the changes that had taken place in the parameters of the tax function, there were even marked differences in shape between the difference functions for various years.

Next we can examine the behaviour of the function for one year as an example. The graphs of the difference functions according to the taxation class in 1974 are presented in Figure 2. On the horizontal axis we have the logarithmic taxable real income, i.e. income in 1974 prices.

The real change of income tax scales is presented in percentage points on the vertical axis. If the tax rate for a given real income has risen with respect to the previous year, this tightening will be represented by a positive number and vice versa.

Figure 2. The graphs of the difference functions according to taxation classes in 1974

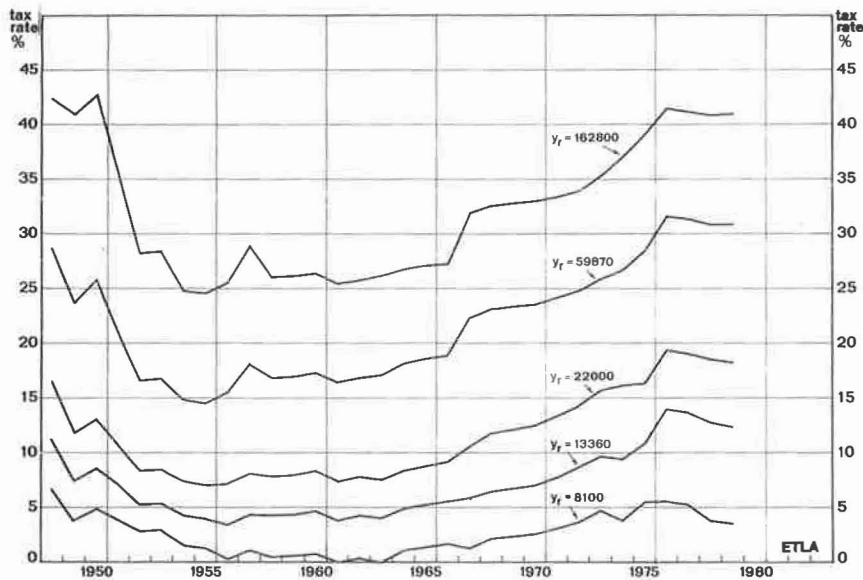


From the shapes of the 1974 difference functions one can conclude that the behaviour is quite similar. Between 8-9.2 of logarithmic real income ($3000 \text{ FIM} \leq y \leq 12\,000 \text{ FIM}$) the taxation decreased, but differently for different incomes and taxation classes. Beginning from about 13 000 FIM of taxable income the tax rates increased in all taxation classes and the growth was largest, 2.5 percentage points, at the taxable income of 120 000 FIM.

Let us consider next the shape of the income tax rates at some real income levels.

In Figure 3. there are the tax rates in 1948-74 for the III taxation class corresponding to 1974 taxable incomes of 8100, 13 360, 22 000, 60 000 and 163 000 FIM.¹⁾ The graphs have been extended to the years 1975-79 with the relevant values of the difference function.

Figure 3. The development of the state income tax rate in 1948-79 at certain real income levels



The following conclusions can be made about the development of the income tax rate at certain real income levels.

At the lowest taxable income in question the tax rate was highest in 1948. After that the tax rate decreased and in the first years of the 60's one had to pay on this income only a small amount of taxes or none at all. After the middle of the 60's the tax rate grew and was at its highest in 1976, 5,6 %.

1) The price level is now (1989) 3.5 times higher than in 1974, so by multiplying figures by 3.5 you can get the income levels in current prices.

At the other real income levels in question the taxation by tax rate has been heaviest in the beginning of the period and in the middle of the 70's. The taxation in real terms was lowest in the middle of the 50's. It appears from the changes in the tax rates from the middle of the 50's to the middle of the 70's at different real income levels that the tax rate rose at the lowest real income level about 5 percentage points, for medium incomes 11-12 percentage points and for large incomes about 16 percentage points.

When an effort is made to construct an aggregate indicator of changes in income tax rates, designed to show how much the taxation of income has been increased or reduced from one year to the next on average, the following questions, among others, are met:

- 1) How should the changes in tax scales be described (by taxation classes) when the real income varies?
- 2) How should the data relating to various taxation classes be combined?
- 3) How should aggregation over all income levels be performed?
- 4) On what kind of scale (in terms of tax rates, relative changes in taxes, or the like) should the changes be measured?

In order to determine the average real changes in the tax scales, the difference functions were aggregated over tax classes and income brackets, by using the 1964 distribution of taxable income as weights. The results thus obtained answered the question: What would the annual absolute change in the average tax rate have been if the nominal incomes corresponding to the real incomes of the year 1964 had been subjected to taxation in accordance with the actual tax scales?

Finally, relative changes in the tax scales were investigated. The changes that occur in the income tax scales will also of course be reflected as changes in taxes. For example, if the income tax scales are not adjusted upwards according to inflation, this is reflected as an increase in the taxes in the new situation, as compared with the old situation. Besides the difference functions, the relative changes in real taxes can be used as indicators of change. In this case the ratio between the taxes computed according to the new and the old situation indicates whether the taxation has become heavier or easier, i.e., the ratio

$$(5) \quad \frac{T^1(y^0)}{T^0(y^0)} = \frac{\sum_{j=1}^n T^1(y_j^0)}{\sum_{j=1}^n T^0(y_j^0)}$$

where $y^0 = (y_1^0, \dots, y_n^0)$ is the vector of the taxable income of the persons $a_1, \dots, a_n$ in the old real-income situation, and T^1 refers to the new tax scale and T^0 refers to the old tax scale.

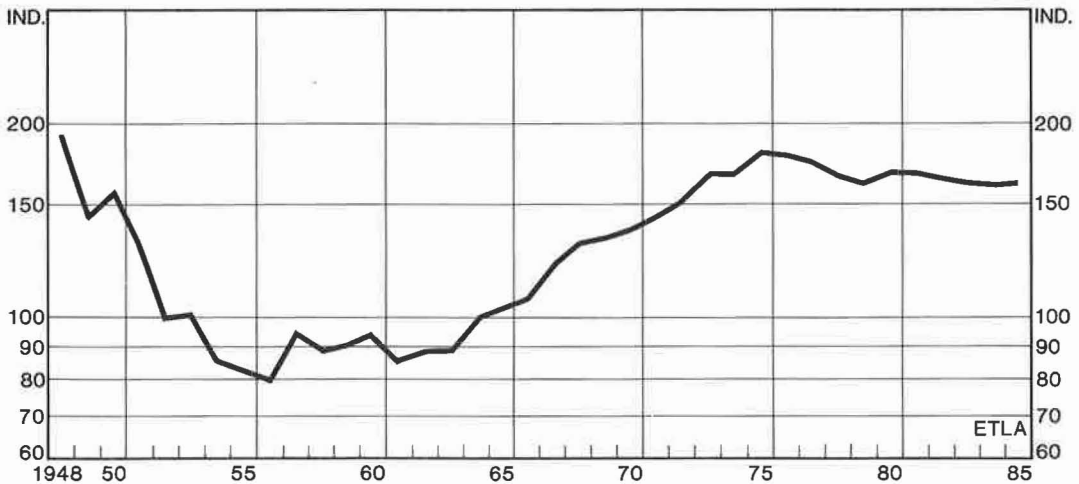
The relative changes in tax scales can be described in a way similar to that used in describing the course of, say, the cost of living, or by means of price indices. The analogy to a price index is arrived at as follows. If the tax charged on a given amount of real income, or the tax rate function $\Theta(y)$, is compared to the price (p) of a good and the real income y to the quantity (q) of the good, the relative changes in the tax scales can be computed making use of this analogy. For example, the formula for computing a tax scale index corresponding to Laspeyres' price index is

$$(6) \quad T_0^1(La) = \frac{\Theta^1(y^0) \cdot y^0}{\Theta^0(y^0) \cdot y^0} = \frac{\sum_{j=1}^n \Theta^1(y_j^0) y_j^0}{\sum_{j=1}^n \Theta^0(y_j^0) y_j^0},$$

which indicates the ratio between taxes, as determined in accordance with the new tax scales, and those as determined in accordance with the old ones in the real income situation y^0 . To put it somewhat differently, this formula answers the question: How much more (or less) will be paid in taxes according to the new tax scales than according to the old ones when the real income situation corresponds to the old situation?

In the following Figure 4, the course of the tax scales index "1964=100", based on (6), is represented.

Figure 4. The course of the tax scale index "1964=100" in 1948-85



4. THE SYSTEM OF DEDUCTIONS

In order to examine the bearing of changes in the system of deductions on the heaviness of taxation, in real terms, two alternative taxation (for one and the same year) may initially be compared. It can be thought, in other words, that the incomes of the same persons are taxed in accordance with the actual deductions system and, on the other hand, in accordance with a hypothetical system chosen for the purpose and that the results are compared. The actual deductions system consists of the deductions actually applied in the year concerned, in their legislatively provided form. The hypothetical system may be any other system chosen - say, the one applied in the preceding year but altered in such a way that the deductions are adjusted for inflation.

In addition to the income situation, all external factors affecting the size of the deductions - such as family relations, health, etc. - are kept unchanged. In the case of each kind of deduction, only its size - in money terms or as a percentage, etc. - is allowed to differ from one system to the other. The purpose of the calculations is to reveal the extent to which a real change, in contradistinction to a nominal one, made exclusively in the deductions system will lead to changes in taxation (i.e., to changes in the change in taxes); this is why all other factors with an impact on taxation are kept unchanged.

When two alternative deductions systems are compared, gross income is kept the same in the two situations concerned. The hypothetical tax rate forms a basis of comparison. If the tax rate, i.e., the total income taxes/gross income ratio, is higher in the hypothetical than in the actual taxation, the hypothetical system is conducive to heavier

taxation in comparison with the actual system. Considered from the standpoint of change, the actual system means a shift toward reduced income taxation, in comparison with the hypothetical system.

Next, the link with a given year is severed, and all money values are expressed in real terms, in order to make comparisons over time possible. The deductions systems of two different years are then compared. A given base year is chosen, and the deductions system for it then represents hypothetical taxation, and this deductions system is compared with the actual deductions systems applied in each of the years to be considered. The deductions system has grown conducive to heavier taxation, in real terms, if the total collected in taxes is greater according to the actual deductions system, compared with what it would have been according to the hypothetical system.

Typically, this may be the case when, under inflationary conditions, the permissible deductions have not been adjusted for inflation, so that their real value has declined. On the other hand, if the deductions system actually applied in a given year leads to collecting a smaller amount of taxes, in real terms, the deductions system has been conducive to reduced income taxation, in comparison with the base year. An example of this is provided by the introduction of a new kind of deduction, which some individual or individuals may use, whereas no other changes, in real terms, have taken place.

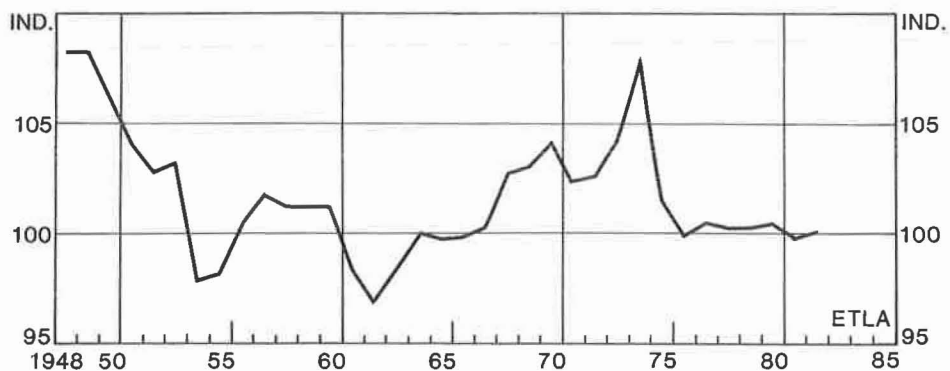
In the calculations, the taxation effected in accordance with the actual deductions system was compared with taxation consistent with a hypothetical deductions system. The calculations, made by income brackets and tax classes, related to the years 1948-1976. For the years

1977-1982, macro-level calculations were performed. The comparisons were based on a deductions system consistent with the one applied in 1964.

On the basis of this data, an index was constructed, which may be termed, in accordance with its base year, the deduction index "1964=100". For each particular year, the total that should have been paid in income taxes if the base year's deductions system had been applied was compared with the total actually collected.

The deduction index shows how much more or how much less taxes were collected, in real terms, from the same real income liable to taxation in each particular year, in comparison with the total that would have been collected according to the base year's deductions system. The year-to-year changes in the deductions index show how far the deductions system had become conducive to heavier or to lighter taxation, compared with the preceding year. The course of the index is shown in the following chart.

Figure 5. The course of the deduction index "1964=100" in 1948-82



As appears from the figure, the deductions system reduced, in real terms, income taxation in the early years of the period under consideration. In the end of the 1950s the deduction system was heavier. In the early 1960s the system slightly reduced income taxation in real terms. From the year 1962 on, until the mid-1970s, income taxation grew heavier, in real terms, owing to the deductions system. Over that period of time, the real rise due to the deductions system averaged one percentage point per year.

In the middle of the 1970s the deductions index dropped rather sharply. It should be pointed out, however, that in 1975 the state income tax scales, for instance, were changed in such a way that taxation grew heavier in real terms, and this was partly compensated for by changing the permissible deductions. Since 1977, the impact of the deductions system on the heaviness of taxation has been more or less neutral, as a result of the adjustments for inflation that have been made in the deductions.

In our next study we will try to combine all the factors affecting real changes in physical persons' income taxation, that is to say:

- state income tax rates,
- municipal income tax rates,
- the system of deductions.

THE EFFECTS OF TAXATION ON INCOME DISTRIBUTION

by Christian Edgren

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1. INTRODUCTION

The discussions on wage policy in Finland have been divided mainly into two types of nominal wage increases, namely the solitary wage policy giving absolutely equal wage increases over the income range and the equal percentual wage increase. In this study we intend to show that due to the system of progressive income taxation the discussions should rather be centred on two other types of income growth patterns which, from the distributional point of view, are more relevant.

How taxation effects the income formation and its distribution is a complicated problem. In this paper we shall be limited to the study of the direct effects concerning the income taxation. The traditional way to examine the problem is to apply a cross-sectional approach, whereby cross-section data comparisions of the distribution of the tax burden relative to the gross income between two points of time are made.

As an alternative to the traditional static comparative analysis, we try to formulate some explicite statements of the role of the tax system in the process behind income distributional changes. The question we shall address is: "Under which conditions regarding changes in pre-tax incomes and changes of the tax rules can we expect to have decreasing or increasing relative income differences of post-tax incomes?"

The study is based on some mathematical formulas concerning aggregation of tax progressions. The exercise gives us two main solutions regarding the pre-tax income growth pattern:

The equiproportional pre-tax income growth results in constant pre-tax income distribution, but relatively decreasing post-tax income differences, while a pre-tax income growth pattern, which can be derived by the formulas in the study - the standardized gross income deflator solution - results in constant post-tax income distribution but increasing relative pre-tax income differences.

On the basis of these two borderline solutions it is possible to find out the distributional consequences of pre-tax income growth patterns which diverge from these borderline cases.

Because there is much confusion concerning the problem of change in progression, tightening of taxation and the income distributional consequences of taxation, we suggest a standardized comparison for them all. This means that changes in them should be decomposed into an income effect, with constant pre-tax distribution, and into a reallocation effect due to change in pre-tax income distribution.

2. THE TAX INSTRUMENTS AND THE INCOME DISTRIBUTION

The analysis in this paper is quite different from the common type of income distributional studies. Due to the need of operationality and due to the individual-unit based tax system, the units used in the study are individuals and not households as is common in income distributional analysis. The specified tax function here gives individual taxes as a function of individual pre-tax incomes.

The main tax instruments considered in the study are:

- rules defining incomes liable to taxation
- rules of deductions from income

- the tax schedule in state income taxation
- proportional tax rates (municipal tax rate, insured persons social security fees)
- deductions from the tax

The study is based on an examination of the tax instruments, the equal or almost incrementally equal individual tax functions, the income distribution and a consistent aggregation of the individual tax functions, both over individuals (tax paying units) and over different types of taxes (progressive and proportional taxation). These elements are necessary for the construction of the overall measures of the progression.

The progression is defined at the micro level in three ways. They are the sensitivities of the tax, the residual income and the average tax rate with respect to an infinitesimal change in the pre-tax income. The aggregation of these local progression measures requires, for the sake of consistency, an assumption of infinitesimal relative income changes for every individual at his income point. This is a necessary condition because only a discrete equiproportionate income growth can converge to zero with the same speed at every income level.

The aggregation formula, introduced later on, says that individual local marginal tax rates, average tax rates and average tax progression should be aggregated using individual pre-tax income shares, tax liability progressions using individual tax shares and residual income progressions using residual income shares at the point of measurement. The aggregation turns out to be consistent when the aggregated marginal and average tax rates are sufficient for deriving the other measures

at the macro level. The different progression measures indicate both at the micro and macro level the same phenomenon but looked at from different points of view.

We shall in the progression study distinguish the cross-sectional analysis giving the distribution of the individual tax parameters and progressions relative to the respective macro level averages at a point in time from time series analysis based on macro level parameter averages. The change of the macro level marginal or average tax rates in time series analysis should be decomposed into the change of the parameters under a given constant pre-tax distribution and into the change in them due to a change in the distribution of pre-tax incomes. The same distinction has to be done when analyzing changes of the post-tax income distribution.

3. THE TAX FUNCTION CONCEPT

A short description of the tax function in the analysis.

The state income tax schedule (tax scale) is a real function, determining the tax as a function of the post-deduction income. The system of deductions contains many items, which are partly determined directly or indirectly from the income level but also partly on the basis of different socio-economic characteristics. To integrate the deductions into the tax function, an average function has been used, which gives the post-deduction income as a function of income before deductions (gross income). The system of deductions from income is integrated in the state income tax function. The municipal tax and

the insured persons social security fees are considered as proportional taxes. The proportional tax rate, calculated as a proportion of gross income, can further be added to the state income tax function to form together the total income tax function.

The proportional taxes added to the state income tax, increase the marginal and average tax rates by the same proportion on every gross income level. The proportional taxes increase the tax levels, but do not necessarily make the total tax function more or less progressive than the state income tax function. The proportional taxation does not affect the average rate progression but instead the other progression measures.

4. THEORETICAL CONSIDERATIONS OF THE TAX FUNCTION

4.1. Individual tax functions are not necessarily identical

The tax function can be considered identical for every individual along the whole income range. The tax scale in state income taxation and the proportional tax rates are strictly identical for every individual. The system of deductions from income make the individual tax functions a priori non-identical, because some of the deduction items are determined by the individual socio-economic status.

The identical-statement is, however, not necessary in the case of aggregation of the individual tax functions at a certain point in time. In this respect there exists the possibility of tax functions to be individually different. When analyzing the effects of a marginal change of income on taxes it also happens in the neighborhood of every

individual income level point, which means that we consider only an infinitesimal part of the different hypothetical individual tax functions as relevant for the individual in question.

This objection is based on the fact that the effects of a marginal change on the tax in an income point and on average for the whole income range is a sensitivity analysis. More important for the aggregation and its outcome is in fact the assumed pattern for marginal income growth even in the case of infinitely small changes.

4.2. The individual income level tax parameters

There is hardly a unique measure for progression. We therefore introduce the different tax parameters on the micro level, which are derivable from the tax function. It applies to the whole income range and the income concept need not for the moment be defined.

The progression measures proposed by Musgrave and Thin (1948) are:

$$(1a) \quad \pi(y) = \frac{d\theta(y)}{d \log y} = m(y) - \theta(y) \quad (\text{average rate progression})$$

$$(1b) \quad e(y) = \frac{d \log T}{d \log y} = \frac{m(y)}{\theta(y)} = 1 + \frac{\pi(y)}{\theta(y)} \quad (\text{liability progression})$$

$$(1c) \quad \beta(y) = \frac{d \log (y - T(y))}{d \log y} = \frac{1 - m(y)}{1 - \theta(y)} \quad (\text{residual income progression})$$

These measure the sensitivity of the average tax rate, tax revenue and net income with respect to a marginal change in income at different income points of the tax function. In equations (1a-1c) T stands for tax and y for income. Tax function parameters are the marginal tax

rate $m(y)$, the average tax rate $\theta(y)$ and the difference between the marginal and average tax rate $\pi(y)$. The measures are related to each other, and can be used to calculate whether the taxation at a specific income point is progressive, proportional or regressive.

4.3. The consistent aggregation of tax parameters and progression measures - the arc-elasticity

The relative change of the tax liability at the macro level is

$$\begin{aligned}
 (2) \quad \Delta \log T &= \frac{\sum \frac{\hat{y}_i}{\hat{T}} m(\bar{y}_i)}{\hat{T}/\hat{y}} \Delta \log y + \frac{1}{\hat{T}/\hat{y}} \left(\sum \frac{\hat{y}_i}{\hat{y}} \right) ((m(\bar{y}_i) - \hat{m})(\Delta \log y_i - \Delta \log y)) \\
 &= \frac{\hat{m}}{\hat{\theta}} \Delta \log y + \frac{1}{\hat{\theta}} \text{cov}(m(\bar{y}_i), \Delta \log y_i),
 \end{aligned}$$

where $m(\bar{y}_i)$ is the marginal tax rate for the individual in the midpoint $\bar{y}_i$ of incomes $[y_i^0, y_i^1]$, $\hat{y}_i$ is his incomes $[y_i^0, y_i^1]$ logarithmic average $L(y_i^0, y_i^1)$, $\hat{T}$ and $\hat{y}$ are the logarithmic total averages $L(T^0, T^1)$ and $L(y^0, y^1)$ of tax liability respective income in situations 0 and 1, and $\hat{m}$ is the marginal tax rate on average and $\hat{\theta}$ the average tax rate.

The relative change of the tax liability is decomposed into two parts. The first part is the change in tax liability due to the relative change of total incomes and the second is the covariance part, which takes care of the growth pattern in our study.

The covariance is the weighted sum of the products of variations from the averages and it is negative if the growth in incomes is in favour

of relatively low income earners, positive if it is in favour of relatively high income earners and zero if the growth is equiproportionate. This can be proved and follows from the fact that in progressive taxation the income, which corresponds to the average marginal tax rate, i.e. $y(\bar{m})$, is on a higher level than the average income $\bar{y}$, and the marginal tax rate, which corresponds to the average income level $m(\bar{y})$, is on a lower level than the average marginal tax rate $\bar{m}$.

As shown later on, the conclusions of the sign of the covariance-term can be used when concluding the post-tax income distributional changes for different pre-tax income growth patterns.

4.4. The progression defined at a point in time

If relative income growth is equal for every individual, the covariance term in equation (2) disappears. If income growth approaches zero, we receive the tax liability progression in the individuals income points y in the 0-situation, i.e.

$$(3) \quad \bar{e} = \frac{\sum \frac{y_i}{y} m(y_i)}{T/y} = \frac{\bar{m}}{\bar{\theta}} = 1 + \frac{\sum \frac{y_i}{y} \pi(y_i)}{T/y} = 1 + \frac{\bar{\pi}}{\bar{\theta}} = \sum \frac{T_i}{T} e(y_i) ,$$

where $\bar{\pi}$ is the sensitivity of the average tax rate (the average rate progression). The residual income progression is on average

$$(4) \quad \bar{\beta} = \frac{1-\bar{m}}{1-\bar{\theta}} = \sum w_i^d \beta(y_i)$$

where w_i^d is the net income weight for individual i on income level y_i .

From formulas (1a-1c), (3) and (4) we can see that the different progression measures belong both at the micro and macro levels to the same family. The aggregations from micro to macro are consequently consistent with each other.

The average marginal tax rate and the overall progression change from one year to the next both due to changes in tax rules and changes of individual incomes (or changes of incomes relative to the changes of tax rules) and due to changes in the distribution of pre-tax income.

The marginal tax rate and the progression change at the macro level when pre-tax incomes grow both equiproportionately and non-equiproportionately and tax rules are kept constant. The same happens when tax rules are changed and incomes are kept constant. The marginal tax rate and the progressions are, however, stable if the rate of indexation of tax rules equals the equiproportionate income growth of the individuals.

If the rate of indexation of tax rules is lower than the income growth rate, which is the usual case, the average marginal tax and average tax rates increase, the overall tax liability progression and the residual income progression in turn decrease on average. If the rate of indexation of tax rules, however, is higher than the income growth rate, we have the reverse effects.

The average rate progression is more sensitive than the other measures and the changes in it depend on the overall profile of marginal tax rates of the tax function and from it derived average tax rates and the income distribution of pre-tax incomes. As expected, its change depends on the changes of marginal and average tax rates in relation to each other (see figure 6 on page 277).

5. THE GROSS INCOME DEFLATOR

The tax liability progression is greater than one for different income points if the tax function is progressive, one if proportional and less than one if regressive. If the tax function is progressive the liability progression on average decreases with the income level. The liability progression can, however, increase at lower income levels due to the specified deduction function in the progressive-proportional tax function case (see figure 7 on page 278).

The residual income progression is for the progressive tax function less than one and decreases with the income level. The gross income deflator, which is the reciprocal of the residual income progression is therefore greater than one, and on average increasing with the income level. On the individual income level at an income point, the deflator is a sensitivity measure indicating the compensating pre-tax income growth for a one-percent increase in the post-tax income. The average gross income deflator indicates the compensating equiproportionate pre-tax income growth for a one-percent average increase in post-tax incomes.

As we shall see later on, the use of the gross income deflator gives the possibility to standardize the individual pre-tax income growth in relation to the total income growth, in order to at the same time keep the post-tax income distribution constant.

6. EMPIRICAL CONSIDERATIONS AND RESULTS

6.1. The tax structure

In the empirical part of the study we concentrate on the tax paying individuals. The limitation of the study can be clarified by table 1. From it we can see the different circumstances why some individuals are excluded from the study. By taking all income receivers into consideration we would miss much of the reality with respect to the actual debate on the reformation of the tax system.

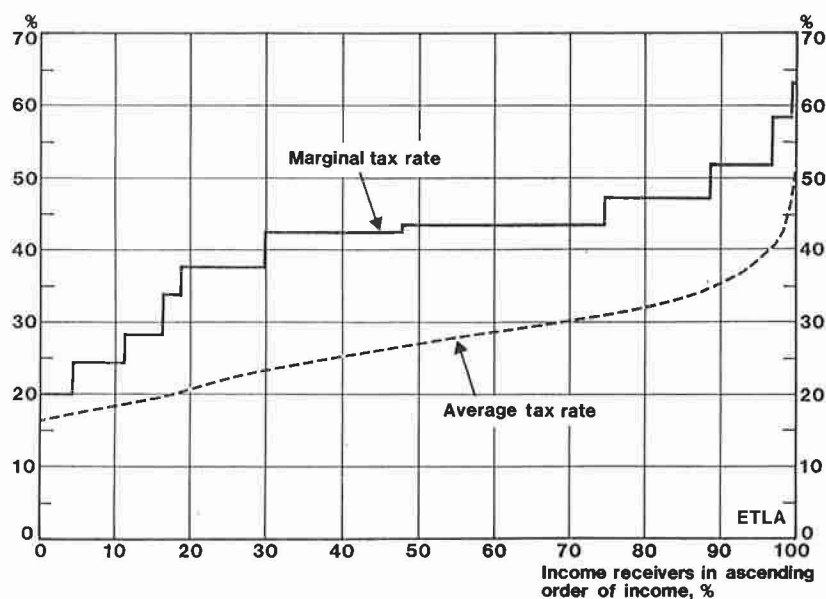
In time series analysis of the overall progression of all income receivers, for example, much of the historical trend would be explained by the increase in the share of tax paying people of the total population.

Table.1. The population and the taxpayers balance in years 1983-1985, 1000 persons

	1983	1984	1985
The whole population	4856	4882	4902
-Not liable to taxation	264	237	194
=Persons liable to taxation	4592	4645	4708
-Persons without incomes	698	744	719
=Income receivers	3893	3901	3989
-Persons with zero post-deduction incomes	367	302	288
=Persons with positive post deduction incomes	3526	3599	3701
-Persons not paying taxes due to the lower limit of the tax schedule and due to deductions from taxes	1218	1146	1095
=Persons paying taxes in state income taxation	2308	2453	2606

The results and conclusions of the study are, however, very general and applicable also to other populations shown in table 1.

Figure 1. Marginal and average tax rate according to income receivers in ascending order of income, percent.



The whole tax structure characterized by marginal and average tax rates is summarized in table 2. The structure of individuals paying both progressive state income tax and proportional taxes is described in figure 1, where we can see that the marginal tax rate increases very fast for the first thirty percent of the individuals resulting in increasing differences between marginal and average tax rates. For the following forty percent the marginal tax rate is relatively stable and connected with a decreasing rate of progression.

In figure 2 we find the concentration curves for pre-tax incomes, net incomes and tax revenue and the respective concentration index numbers. The distribution of gross income behind the concentration curve for pre-tax incomes is shown in figure 3.

Figure 2. Lorenz-curves for gross income, net income and taxes, Concentration indices: Gross Income = 0.261, Net income = 0.206, Taxes = 0.389.

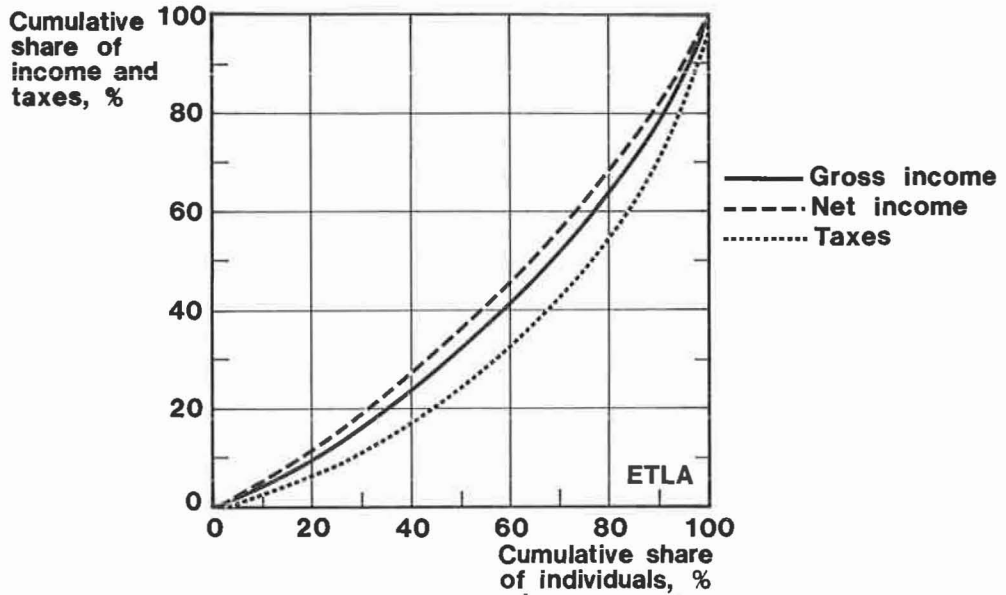


Figure 3. The distribution of taxpayers and their gross income according to income level, percent.

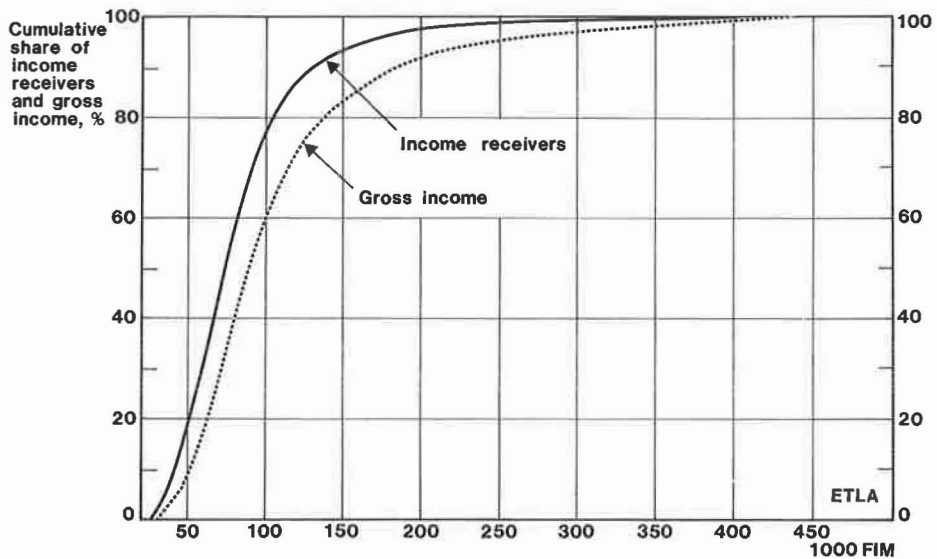


Table 2. The distribution of all income receivers according to marginal and average tax rates¹⁾

Marginal tax rate %	Percent of income re- ceivers	Average tax rate %	Percent of income re- ceivers
0	20.8	0	20.8
>16 ²⁾	79.2	>16 ²⁾	79.2
>16	65.3	>16	65.3
>20	62.7	>20	53.9
>30	54.9	>30	19.6
>40	45.7	>40	1.9
>50	7.8	>50	0.3
>60	0.6		

- 1) An approximation for the year 1987 on the basis of the balance in table 1 for the year 1985 and the distribution in figure 1
 2) The number of individuals paying only proportional taxes were 554 thousand in the year 1985

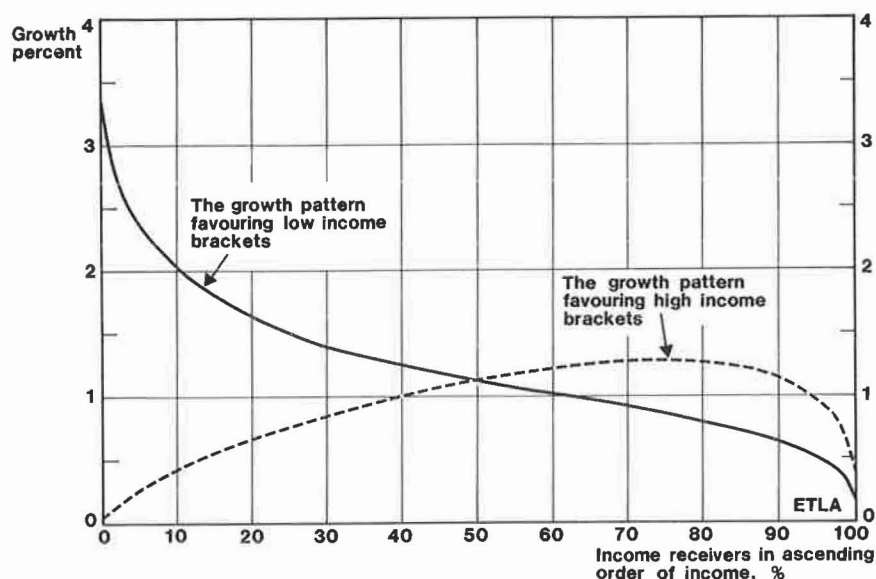
6.2. The sensitivity of the average tax parameters and the overall progressions

In order to test the sensitivity of the different average measures of taxation and the change of the relative income distribution, we have simulated four different growth patterns of the gross income. As a starting point we have taken the situation in the year 1987 with tax rules and the distribution of incomes for taxpayers as given. Using the aggregation formula (3), we derive values for the aggregate and average tax parameters in four hypothetical situations. These situations are generated by an average (and total) increase of gross incomes adding up to one percent in each of the four different cases. The growth patterns behind the income increase are:¹⁾

- 1) The increase by equal amounts of money (case AEI) has been transformed into percentual changes as shown in figure 4. The growth patterns for the FHIC case is shown in the same figure 4. The income growth for the SGID case is given in figure 5 as the relative variation from the average one-percent growth. In the figure equiproportionate growth is marked at 100.

- 1) equiproportionate income growth (EPI)
- 1i) increase by absolutely equal amounts (AEI)
- 1ii) increase according to the scaled gross income deflator (SGID) and
- 1v) increase of incomes favouring higher income classes (FHIC)

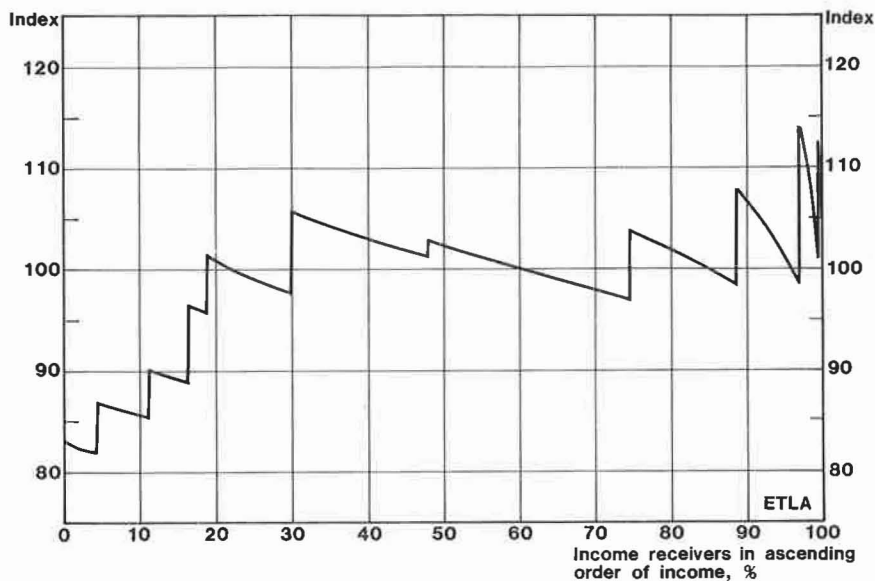
Figure 4. The growth patterns for income increases favouring individuals in low and high income brackets, percent.



The EPI case guarantees a constant pre-tax distribution, while the others generate diverging pre-tax distributions. The cases 1i) - 1v) can therefore be analyzed as income transfers with respect to the EPI case. It gives the reallocation effects or consequences on the tax and the residual income side. The AEI is here the only clear income transfer case (a transfer from the rich to the poor) because the pre-tax income concentration curve has only one point of intersection with the original curve. The pre-tax income concentration curve of SGID, which is a solution resolved implicitly to generate the same individual residual income shares as in the original situation, has 9 points of intersection with the original pre-tax income concentration curve. The FHIC has respec-

tively two points of intersection and should be taken primarily as an income transfer from the poor to the rich.

Figure 5. Relative variation of gross income deflator from the average according to income receivers in ascending order of income, (100 = one percent growth).



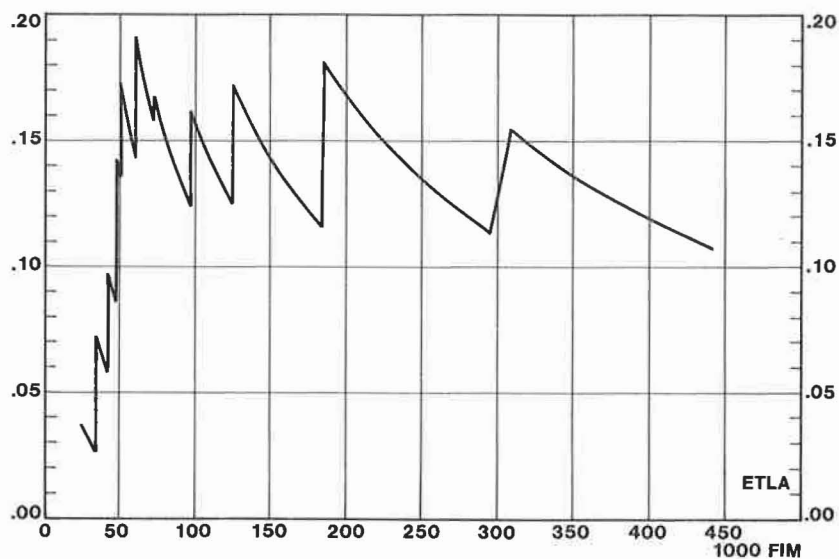
In table 3 the average tax parameters, overall progression measures in the original situation and for the four hypothetical situations are listed. The parameter values can be compared with each other because the calculation has been standardized by assuming equiproportionate (but an infinitely small one) income growth for every individual at the different hypothetical points. Both marginal and average tax rates are higher in the four different situations than in the original situation, i.e. because the tax rules are assumed to be constant. The overall tax liability progression and the residual income progression have decreased. The average rate progression has increased or remained stable. The gross income deflator is bigger than in the original situation. All of the changes in the averages are the effects of the average income increase plus the distributional effects.

Table 3. AVERAGE VALUES OF TAX PARAMETERS AND PROGRESSION

	Original situation (1987)	After 1 % increase in total Gross Income ¹⁾			
		equipropor- tionate increase	absolutely equal in- crease in FIM	according to scaled GID	favouring relatively high income classes
		EPI	AEI	SGID	FHIC
Marginal tax rate, %					
- total taxation	44.752	44.903	44.881	44.901	44.900
- state income taxation	28.612	28.763	28.741	28.761	28.760
Average tax rate, %					
- total taxation	30.370	30.513	30.476	30.517	30.520
- state income taxation	14.230	14.373	14.336	14.377	14.380
Tax liability progression					
- total taxation	1.474	1.472	1.473	1.471	1.471
- state income taxation	2.011	2.001	2.005	2.000	2.000
Average rate progression					
- total taxation	0.1438	0.1439	0.1441	0.1438	0.1438
- state income taxation	0.1438	0.1439	0.1441	0.1438	0.1438
Residual income progression	0.7935	0.7929	0.7928	0.7930	0.7930
Gross income deflator	1.2603	1.2612	1.2613	1.2611	1.2610
Concentration index numbers					
- pre-tax incomes	0.26142	0.26142	0.25883	0.26159	0.26208
- post-tax incomes	0.20595	0.20579	0.20358	0.20595	0.20654
- tax revenue	0.38859	0.38808	0.38486	0.38827	0.38853

1) Approximately also for an additional one-percent increase in income above the tax rules indexation

Figure 6. Average tax rate progression according to income level.



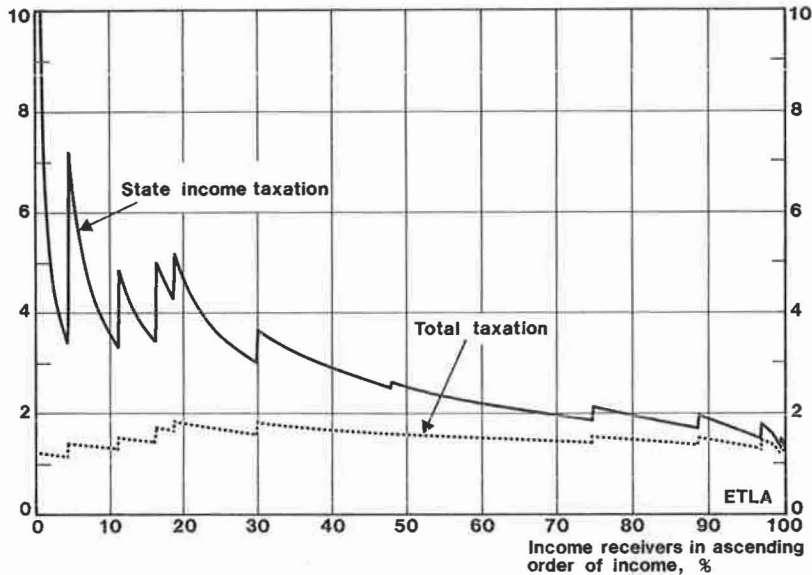
6.3. The relative income differences

From the concentration index numbers (CI) in table 3, we note that in the EPI case the relative pre-tax income differences remain stable, decrease in the AEI and increase in the SGID and FHIC cases. The post-tax relative income differences decrease in the EPI and AEI cases and increase in the FHIC case. CI remain stable in the SGID case, because the pre-tax income growth pattern has been resolved to give this solution.

6.4. The redistributive effects of taxation

It is hard to believe that the distributional changes can reflect changes in progression. Like in the case of progression, which should be calculated at a point with the standardized assumption of infinitesimal

Figure 7. Tax liability progression in state income taxation and total taxation according to income receivers in ascending order of income.



equiproportionate growth, we have to standardize the analysis of effects of the taxation of the distribution to make a meaningful comparison. In the case of fixed tax rules we have in table 3 the EPI case as a standardized solution. In this case the pre-tax distribution is kept constant with respect to the original situation. The other cases are income transfer situations with respect to the EPI case.

6.4.1. Effects on the average tax rate

In table 4 we see that the income reallocation effect in the AEI case is negative, indicating a decreasing macro average tax rate. This result is also shown in equation (2). In the AEI case namely the covariance is also negative and the tax revenue increase is relatively less than in the EPI case, for which the covariance is zero. The average tax therefore increases less in the AEI case than in the EPI case.

In the SGID case the income reallocational effect is positive, which means an increasing average tax rate. Due to the fact that the GID is on the average higher for incomes in the upper part and on average lower in the lower part of the income range, we should expect positive covariances in equation (2), a result which is parallel to the reallocational effect in table 4.

Table 4. The change of the marginal and average tax rate at the macro level due to changes in pre-tax incomes under constant distribution and due to the reallocation of pre-tax incomes (income transfers)

	Original situation	Increase due to the average income increase	Re-allocation effect	Final situation
marginal tax rate				
EPI	44.752	0.151	0.000	44.903
AEI	44.752	0.151	0.022	44.881
SGID	44.752	0.151	0.002	44.901
FHIC	44.752	0.151	0.003	44.900
average tax rate				
EPI	30.370	0.143	0.000	30.513
AEI	30.370	0.143	0.037	30.476
SGID	30.370	0.143	0.004	30.517
FHIC	30.370	0.143	0.007	30.520

Both EPI and SGID cases are of great significance for the conclusions on the taxation. In growth pattern cases like EPI the real gross wage increases with the same speed for every individual, but the net real wage increases somewhat faster in the lower tail and slightly more slowly in the upper tail of the distribution than on average, because the progression bites relatively more in the upper than in the lower tail of the distribution.

In the SGID growth type, the real net wage increases at the same speed on every income level, because the residual income shares are the same as in the original situation. The real gross wages increase, however, relatively faster in the upper tail and relatively more slowly in the lower tail of the distribution.

The increase of net wages on average is both in nominal and in real terms less in the SGID case than in the EPI case, because the reallocational effect according to the average tax rate is positive. The covariance in equation (2) is also positive (the tax liability increases faster in SGID than in EPI from which follows the conclusion regarding the residual income).

6.4.2. Effects on the residual income distribution

In table 5 we present the income distributional effects of taxation, decomposed into the income effect and the income transfer effect. The results are quite in conformity with the conclusion from table 4. The income effect calculated according to constant pre-tax distribution is negative, which means decreasing relative post-tax income differences. In the AEI case both the reallocation of pre-tax incomes and its effect on post-tax income distribution are negative. The relative post-tax income differences have thus decreased.

In the SGID case there is an increase of relative pre-tax income differences, which compensate the decreasing income effect on the post-tax income side. In the FHIC case the decreasing income effect more than compensates for the even bigger pre-tax income differences.

Table 5. THE CHANGES OF CONCENTRATION INDICES DECOMPOSED INTO THE INCOME AND THE INCOME DISTRIBUTIONAL EFFECTS OF TAXATION

	EPI		AEI		SGID		FHIC	
	pre-tax	post-tax	pre-tax	post-tax	pre-tax	post-tax	pre-tax	post-tax
The original situation	0.26142	0.20595	0.26142	0.20595	0.26142	0.20595	0.26142	0.20595
The income effect	0.00000	-0.00016	0.00000	-0.00016	0.00000	-0.00016	0.00000	-0.00016
The trans-fer effects	0.00000	0.00000	-0.00259	0.00221	+0.00017	+0.00016	+0.00066	+0.00075
The final situations	0.26142	0.20579	0.25883	0.20358	0.26159	0.20595	0.26208	0.20654
The difference of the area between pre- and post-tax incomes in the original and the final situations	+ 0.00016		-0.00022		+ 0.00017		+ 0.00007	

As we see in table 5 the area between pre- and post-tax incomes have decreased in the AEI case and increased in the other cases. The tax function has been kept constant and the average rate of progression has increased faster in the AEI case than in the other cases. The liability progression and the residual progression have decreased, because the gravity point of taxation has moved upwards along the tax function as the income grows.

We can conclude that the change of the area between the pre- and post-tax incomes can not reveal the change in direction of the progression. Therefore, we should separate the concept of progression from the concept of the distributional effects of taxation.

To clarify what has been done, we may look at the progression measure of Kakwani (Kakwani, 1984)

$$(5) P = ((1-ATR)/ATR) * (CI_{pre} - CI_{post}),$$

where ATR is the average tax rate for all tax paying units, and CI_{pre} and CI_{post} are the concentration indices for pre- and post-tax income respectively.

Given the original situation we have examined the change of P:

- 1) by keeping CI_{pre} constant and calculating the change of CI_{post} and ATR due to a one-percent change of incomes (EPI case)
- 2) by letting CI_{pre} and CI_{post} decrease, decomposed into two terms, i.e. the change in CI_{post} with constant CI_{pre} and the change due to changes in CI_{pre} ; there is also given the respective decomposed changes of the ATR (AEI case)

- 3) by keeping CI_{post} constant and calculating the required change in CI_{pre} with the same decomposition as in point 2) (SGID case)
- 4) by letting both CI_{pre} and CI_{post} increase (FHIC case).

Formula (5) is, however, by itself not very informative for figuring out the reason for changes in the progression. The formula of Kakwani does not say anything explicit about the functional relation between the change in the area between the pre- and post-tax incomes and the change in the average tax rate. The formula does not contain an assumption which could standardize the comparison of the progression at two different points in time.

We have solved the problem here by aggregating the progression characteristics of the individuals at the two points in time by assuming infinitesimal income growth for every individual at the respective points of time. In this way we can measure the overall progression irrespective of the discrete change of the incomes. The change of the income distribution is taken into account by the aggregation weights at respective points in time.

For the discrete case we have the arc-elasticity of the type given in formula (2) and the covariance term, which take care of the change in the pre-tax distribution. Formula (2) can also be rewritten for residual incomes.

7. CHANGES OF THE TAX FUNCTION

A new tax function may be considered as some systematical change in relation to the previous function. As a standard solution to the problem we

have the case where the rate of indexation of the tax function is the same as the income growth rate. This trivial but very useful assumption implies a scaling of the tax rules and incomes. The result in case of equiproportionate income growth is that the progression and distribution measures remain stable, because marginal and average tax rates remain stable both at the micro and macro levels.

If, however, the rate of indexation of the tax rules is less than the income growth, the examination can be returned to the cases already analyzed. The income growth for the different cases is then considered as marginal increases above the indexation rate. The original situation is then converted into the situation where no distributional changes have happened. Our conclusions made earlier are therefore, in general, valid for these types of tax changes.

A tax reform which tends to broaden the tax base by eliminating deductions is in a sense almost only a technical reformulation of the tax function if the deduction function happens to be a real function. This is perhaps the case if at the same time the aim is to keep the tax revenue and the individual average tax rates constant. Then also the marginal tax rates must be the same as before, although, defined with respect to gross income.

A more relevant tax reform is one in which the aim is to reallocate the tax burden by, for example, increasing the average tax rate in high income brackets and decreasing it in low income brackets. Then the marginal tax rate in the tax function rises at an increasing rate with the income level. The marginal and average tax rates at the macro level and also the average tax rate progression should, however, remain stable if the total tax revenue has to be kept constant.

If the tax reform also includes enlarging of the tax base by income elements, which according to the tax law earlier are stated as tax free, much of the outcome depends on the distribution of the tax free incomes relative to the incomes liable to taxation.

Table 6. Effects on relative post-tax income differences due to different growth path.

TYPES OF GROWTH PATHS	post-tax relative income differences: increase (in) decrease (de) no change (nc)
1. No changes in tax rules	
- equiproportionate changes of pre-tax income	de
- changes by absolutely equal amounts	de
- changes of pre-tax incomes according to individual gross income deflators	nc
- changes of pre-tax incomes favouring individuals in relatively high income brackets	in
2. Indexation of tax rules	
- an equiproportionate rate of change of pre-tax incomes as big as the rate of indexation of tax rules	nc
- the rate of indexation of tax rules is less than the average rate of growth of pre-tax incomes	
- growth rate equiproportionate	de
- growth path favouring on the margin individuals in relatively low income brackets	de
- growth path according to GID on the margin	nc
- growth path favouring on the margin individuals in relatively high income brackets	in
Conclusions:	
- no change in relative post-tax income differences require increasing relative pre-tax income dif- ferences.	
- to secure stabile relative post-tax income dif- ferences, pre-tax incomes have to change non- equiproportionately or tax rules should be indexed less at the bottom of the income range and cor- respondingly more at the top of the income range.	

8. THE MAIN RESULTS

Our main task was to examine the relation between the taxation and the income distribution. Our interest was focused especially on the connection between the income growth pattern (changes of the pre-tax structure) and changes of the post-tax relative income differences both for a given tax function and for a systematical change of that function.

Using the formula (2) we have proved that the change in the tax revenue depends on the growth pattern and especially on growth patterns diverging from the equiproportionate case. The borderline cases EPI and SGID and the sign of their covariance terms display the direction of the structural change of the post-tax income differences for income solutions whatever they happen to be.

The arc-elasticity, which in concrete situations depends on the growth pattern of the income and its growth rate, is not suitable as a progression measure. We suggest for the measuring of the progression equal relative growth standardized point estimates of the sensitivity at the macro level. These are the liability progression, the residual income and average rate progression.

To analyze the tightening of the taxation, we have to standardize the procedure to the equal growth case. There is no sense in speculating, for example, why the average tax rate has increased more for the rich than for the poor if in the same time the pre-tax income has happened to grow faster for the rich than for the poor. For this purpose we have introduced the partial reallocation effect of the pre-tax income.

To put these three above-mentioned aspects together we consider the neutrality concept. As already seen, the equiproportionate growth is neutral from the pre-tax income distribution, but not from the post-tax income distribution point of view. On the other hand, the gross income deflator solution is neutral from the post-tax income distribution, but not from the pre-tax income distribution point of view. Both neutralities can not be reached at the same time in a progressive tax system.

The indexation of the tax function is sometimes said to be neutral, because if the pre-tax incomes grows at the same rate as the scale indexation, every individual pays exactly the same share of tax on his income as before. This is true whatever the income distribution happens to be. In this special case both pre- and post-tax distributions remain stable and also the overall progression remains the same as before the indexation.

In progressive taxation, given the tax rules, equiproportionate growth of the pre-tax incomes tightens the taxation on average and relatively more for individuals at the top of the income scale and relatively less for individuals at the bottom of the income scale. Although the relative income differences of pre-tax incomes are unchanged, the post-tax income differences are declining.

If the growth of pre-tax incomes in non-equiproportionate is favouring relatively poor individuals, the outcome is decreasing relative income differences both for pre- and post-tax incomes. The pre-tax reallocation effect is negative both on the average tax rate and the post-tax income distribution index.

In order to keep relative post-tax income differences unchanged, the pre-tax incomes must grow non-equiproportionately favouring rich ones,

which means increasing pre-tax income differences. The relevant growth rates can be derived by the individual gross income deflators. These can be aggregated to an average weighted deflator. The average value of parameters in the deflator is determined by tax instruments and the income distribution.

Income growth patterns which favour higher income brackets by "overshooting" the deflator allocation, result in increasing relative post-tax income differences.

If the tax rules are adjusted (a form of indexation) with the same rate as pre-tax incomes grows, and the growth pattern is equiproportionate, each individual has the same marginal and average tax rate as before. Both pre- and post-tax relative income differences remain unchanged.

Because the rate of indexation of tax rules (deductions from income and the tax schedule) usually is smaller than the growth rate of pre-tax incomes, we generally expect decreasing relative post-tax income differences.

If the income solutions are of the equal percentual increase of wages type, the indexation of tax rules is non-neutral from the point of view of post-tax income distribution. On the other hand there are different income items, for example, income from property, which fluctuate over the time, and therefore we can expect pre-tax income differences to vary more than post-tax income differences.

If the income solutions, for instance, lead to absolutely equal wage increases - percentually high at the bottom of the income scale and

percentually low at the top of the income scale, but on average percentually higher than the tax rules indexation rate - then we have declining post-tax income differences.

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Figures:

- Fig. 1. Marginal and average tax rate according to income receivers in ascending order of income, percent.
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