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**COMPETITION IN LOCAL LOAN MARKETS.  
AN APPLICATION OF LINEAR CITY-MODEL  
WITH PRICE DISCRIMINATION\*\***

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**ABSTRACT:** In this paper I analyze the local banking markets in the spatial competition context. In the model one bank is located at the one end of the unit long line and two rival banks are located at another end. Ends are considered as centers of two towns. Borrowers are located on the line between the centers. Analysis shows that price discrimination is dominating action for the bank with local monopoly and the cooperativeness of banks in oligopoly-town depends on discount rate of banks, market size development, and development of transportation costs. If banks in oligopoly-town act cooperatively and this town is bigger enough, the monopolistic bank gets lower interest returns than its rivals. If the noncooperative equilibrium realizes in the game in oligopoly-town, for given type of loans returns are always higher for the bank with local monopoly.

**KEY WORDS:** Banking, Spatial competition, Price discrimination, local monopoly.

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**TIIVISTELMÄ:** Tässä paperissa analysoin paikallisia pankkimarkkinoita spatiaalisen kilpailun kontekstissa. Mallissa yksi pankki on sijoittunut yksikköpitkän suoran toiseen päähän ja kaksi kilpailevaa pankkia toiseen päähän. Suoran päät ovat kahden kunnan keskustat. Laina-asiakkaat ovat sijoittuneet suoralle keskusten väliin. Analyysissa näytetään, että hintadiskriminointi on paikallisen monopoliaseman omaavalle pankille dominoiva toimenpide ja kahden pankin paikkakunnan pankkien yhteistyö riippuu diskonttokoron lisäksi kuljetuskustannusten ja lainojen määrän kehityksestä. Jos pankit oligopoli-paikkakunnalla käyttäytyvät kooperatiivisesti ja paikkakunta on riittävän paljon suurempi, ovat monopolistipankin korkotuotot kilpailijoiden tuottoja pienemmät. Jos taas pankit oligopoli-paikkakunnalla käyttäytyvät kilpailullisesti, ovat monopolistipankin korkotuotot aina suuremmat tietyn tyyppisille lainoille.

**AVAINSANAT:** Pankkitoiminta, spatiaalinen kilpailu, hintadiskriminointi, alueellinen monopoli

# 1 Introduction

In local Finnish banking markets quite usual market structure is one with one bank with local monopoly and two or more banks operating in neighboring town. In this paper I analyze loan market competition in this kind of markets.

In analysis I use infinitely repeated price game in Hotelling's linear city model with given bank locations such that one bank is located at one end of the line and two rivals are located at another one. Also demand in the market is assumed to be periodically totally inelastic, but the demand can have either decreasing or increasing trend. Banks are assumed to live forever. Customers are assumed to minimize the total cost of the loan including interest costs and transportation costs, which are assumed to diminish in time. Customers are assumed to live one period. Main difference to Hotelling's (1929) basic model, and to several paper applying this celebrated model (see e.g. Thisse & Vives 1988, Aalto-Setälä 1999), is that borrowers are not necessarily distributed with density of 1, and instead of that they can be distributed with any density function. Bank with local monopoly is assumed to be able to discriminate borrowers in interest rates according to their location. Now, by price discrimination I mean that discriminator prices the loan such that borrower's total loan cost is equally high both in bank with local monopoly and in rival bank in neighboring town.

Typically in price discrimination studies it is assumed that firms are absorbing the transportation costs and including those into product price (see e.g. Hoover 1937, Norman 1981, 1983, Thisse & Vives 1988). Actually, this requires that product are home delivered and the transportation costs are lower for the firm than for the customer; otherwise customer just promises to pick-up the product from the firm and bears his own transportation costs like assumed in Hotelling's original paper. Basically, this is the case in competition in lending markets. Banks are already offering lots of services by remote access technologies, but it is still in some cases required to meet bank officials personally. In these cases for customer it is cheaper to travel to bank by his own expenses than to let bank send the officials to home and pay the expenses in loan price. Therefore in this model banks use customers' location to price discrimination and customers bear the expenses of transportation.

In their paper Thisse & Vives (1988) showed that when firms have possibility of price discrimination, the price discrimination is dominating action. As mentioned, in their analysis Thisse & Vives assumed uniform customer distribution. In this paper I show that not depending on customer distribution, the discrimination is dominating action.

The outline of the paper is following. In section 2 I present the preliminaries of the model. In section 3 I present the structure of the game

and compute the sub-game perfect equilibrium. In section 4 I show how the interest rate depends on the customer distribution and section 5 concludes.

## 2 Preliminaries

Consider a case, where there are 2 cities,  $i$  and  $j$ . In city  $i$  there is bank  $m$  with local monopoly and in city  $j$  there are two rival banks, bank 1 and bank 2. The cities are located on a line with length of 1. Locations on the line are denoted by  $x$ ,  $x \in [0, 1]$ . Location on bank  $m$  is  $x_m = 0$  and locations of bank 1 and 2 are  $x_1 = x_2 = 1$ .

On every period  $t$  there are  $N_t$  loan applicants applying for loan sized  $L$  for some similar use.<sup>1</sup> Number of borrowers is assumed to develop exogenously according to process  $N_t = \gamma N_{t-1}$ ,  $\gamma \in [0, \infty[$ . Loan applicants are assumed to be located on the line according to a continuous density function  $f(x)$ ,  $f(x) > 0 \forall x \in [0, 1]$  and are assumed to live (or be active) one period.<sup>2</sup> Banks are assumed to live forever. Borrowers are granted the loan at the beginning of the period, and at the end of the period they pay the loan capital plus interest. Borrowers are assumed to face transportation costs at size  $\tau_t$  per distance unit and transportation costs develop according to process  $\tau_t = \rho \tau_{t-1}$ ,  $\rho \in [0, 1]$ .<sup>3</sup>

Banks set the loan interest rate to maximize their profits. To define the minimum price of the loan, assume that every bank in the game face cost equal  $r^*L$  for loan sized  $L$ , which has to be paid at the end of the period. Furthermore, assume that every bank assumes that the share  $\lambda_G$  of borrowers are good quality borrowers who generate return  $r$  for the loans, and the share  $(1 - \lambda_G)$  are bad quality borrowers generating credit losses for the bank<sup>4</sup>. Average credit losses are share  $\Omega$  of loan size. Then we can write

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<sup>1</sup>By this similarity assumption, differences in riskiness of lending between the banks can be avoided and later equally high risk of credit losses can be assumed.

<sup>2</sup>Basically, it can be more intuitive to assume that borrowers, or at least most of them, stay alive after the loan period - otherwise any of them would have any incentive to pay the loan back.

<sup>3</sup>Since borrowers are active only in one given period, they cannot utilize the decline in transportation costs.

<sup>4</sup>Size of parameter  $\lambda_G$  can be seen as an efficiency of banks screening activity and that is assumed to be symmetrical. More efficient screening increases the average quality of the screening passed applicants and improves banks return (and lowers zero profit loan price). This means that screening and monitoring are important factors in bank market competition. However, purpose of this paper is not to create a screening and monitoring -model. Instead, the main purpose is to model geographic spatial competition in banking. Therefore transportation costs is the main factor of the analysis and I pass the aspect of competition in screening and monitoring technologies.

zero profit condition for a bank as

$$(1 + r^*)L = (1 + r)\lambda_G L - (1 - \lambda_G)\Omega L. \quad (1)$$

From (4) we can solve free market interest rate as

$$r = \frac{1 + r^* - \lambda_G + (1 - \lambda_G)\Omega}{\lambda_G}. \quad (2)$$

That is, free market interest rate is the interest rate of the loan generating expected zero profit and covering all the costs of the supplied loan. Therefore, total marginal cost of the loan including credit losses is  $c = rL$  for each bank. From now on, even though the banks set the loan interest rate, for simplicity I use the loan price as a notation of the banks' decision variable. That is, if bank  $k$ ,  $k = 1, 2, m$  set loan interest rate at level  $r^k$  the loan price is  $p^k = r^k L$  and when  $r^k > r$ , then  $p^k$  exceeds cost loan the loan  $c$  and bank makes profit from that loan.<sup>5</sup>

Loan applicants are assumed to try minimize overall costs of the loan. Overall costs include, in addition to interest costs of the loan, also transportation costs. Transportation costs measure the importance of the distance for the loan applicant when he chooses the bank<sup>6</sup>. Present value of overall costs of the loan can be written as

$$C_{k,t}^a = \sigma (p_{k,t}^a + \tau_t d_k^a) \quad (3)$$

where  $\sigma$  is discount factor of loan applicant,  $p_{k,t}^a$  is the loan price in period  $t$ ,  $\tau_t$  is the prolonged value of transportation costs per distance unit for loan applicants accumulated during the loan period  $t$  and  $d_k^a = |x_a - x_k|$  is loan applicants distance from bank  $k$  where  $x_a$  denotes loan applicant's location and  $x_k$  bank's location.

Loan applicant is indifferent between the offers of banks  $i$  and  $j$  when the present value of loan overall costs in banks is equally high, i.e.  $C_{i,t}^a = C_{j,t}^a$ . This is, using equation (3),  $p_{i,t}^a + \tau_t d_i^a = p_{j,t}^a + \tau_t d_j^a$ . In the model, the locations of banks and loan applicants are given by  $x_m = 0$ ,  $x_1 = x_2 = 1$  and  $x_a \in [0, 1]$ . Using these, the indifference condition can be written as

$$p_{m,t}^a + x_a \tau_t = (\min\{p_{1,t}^a, p_{2,t}^a\}) + (1 - x_a)\tau_t,$$

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<sup>5</sup>Use of price as decision variable also generalizes the results to spatial competition of other product markets.

<sup>6</sup>Basically, transportation costs of the loan reflect loan applicants willingness to use remote access technologies to handle the financial operations, i.e. in turn, how important the proximity of the bank office is for him.

which gives the location of the marginal borrower, i.e. borrower who is indifferent between cheaper bank in town  $j$  and local monopoly.<sup>7</sup> Hence

$$x_a^* = \frac{\tau_t - (p_{m,t}^a - \min\{p_{1,t}^a, p_{2,t}^a\})}{2\tau_t}. \quad (4)$$

All loan applicants located such that  $x_a \in [0, x_a^*]$  minimize their loan costs by choosing loan from bank  $m$ , and those who are located such that  $x_a \in [x_a^*, 1]$  by taking loan from cheaper bank in town  $j$ . Those intervals are banks' market areas and, as can be seen, the smaller the loan price in  $m$  the bigger its market area.

Next, let me define the demand functions. Bank  $m$ 's demand function<sup>8</sup> is

$$D_{m,t}(p_{m,t}, p_{1,t}, p_{2,t}) = \begin{cases} 0 & \text{if } p_{m,t} \geq \min\{p_{1,t}, p_{2,t}\} + \tau_t \\ N_t \int_0^{\frac{\tau_t - (p_{m,t} - \min\{p_{1,t}, p_{2,t}\})}{2\tau_t}} f(x) dx & \text{if } -\tau_t \leq p_{m,t} - \min\{p_{1,t}, p_{2,t}\} < \tau_t \end{cases} \quad (5)$$

bank 1's is

$$D_{1,t}(p_{m,t}, p_{1,t}, p_{2,t}) = \begin{cases} 0 & \text{if } p_{1,t} > p_{2,t} \\ & \text{or } p_{1,t} \geq p_{m,t} + \tau_t \\ \frac{N_t}{2} \int_{\frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t}}^1 f(x) dx & \text{if } p_{1,t} = p_{2,t} \text{ and} \\ & p_{m,t} - \tau_t \leq p_{1,t} < p_{m,t} + \tau_t \\ N_t \int_{\frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t}}^1 f(x) dx & \text{if } p_{1,t} < p_{2,t} \text{ and} \\ & p_{m,t} - \tau_t \leq p_{1,t} < p_{m,t} + \tau_t \end{cases} \quad (6)$$

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<sup>7</sup>Even though the banks do not absorb the transportation cost into loan price, and instead of that leave the customers bear it, for the borrower the cost minimization is same as it would be in transportation cost absorption case (see same equation in Thisse & Vives 1988). Of course, this requires that borrowers transportation costs are equally high to banks' ones. Actually, the firms' answer to question "to absorb or not to absorb" can be defined by relative transportation costs; if firm's transportation cost per distance unit is higher than customer's one, then there is no point to offer home delivery since for customer it is cheaper to go to firms door and pick the product home by own costs. Basically, that is the case in loan markets. Even though banks are already offering lots of services by remote access technologies and putting the costs in the service prices, it is still in some cases required to meet bank officials personally. In these cases for customer it is cheaper to travel to bank by his own expenses than to let bank send the officials to home and pay the expenses in loan price. As conclusion can be said that in price discrimination models the used transportation cost per unit is the smaller one when compared firm's cost and customer's cost - indifference condition is the same.

<sup>8</sup>If  $p_{a,t}^m > \min\{p_{a,t}^1, p_{a,t}^2\} + \tau_t$ , even loan applicants located at  $x = 0$  minimize the loan costs by taking a loan from cheaper bank in town  $j$ . This applies also later oppositely; higher margin to loan price in neighboring bank is  $\tau_t$ .

and bank 2's one is

$$D_{2,t}(p_{m,t}, p_{1,t}, p_{2,t}) = \begin{cases} 0 & \text{if } p_{2,t} > p_{1,t} \\ & \text{or } p_{2,t} \geq p_{m,t} + \tau_t \\ \frac{N_t}{2} \int_{\tau_t - \frac{(p_{m,t} - p_{2,t})}{2\tau_t}}^1 f(x) dx & \text{if } p_{2,t} = p_{1,t} \text{ and} \\ & p_{m,t} - \tau_t \leq p_{2,t} < p_{m,t} + \tau_t \\ N_t \int_{\tau_t - \frac{(p_{m,t} - p_{2,t})}{2\tau_t}}^1 f(x) dx & \text{if } p_{2,t} < p_{1,t} \text{ and} \\ & p_{m,t} - \tau_t \leq p_{2,t} < p_{m,t} + \tau_t \end{cases} \quad (7)$$

Demand functions of banks 1 and 2 are inter-related in the sense that if one sets the loan price below the price of the rival, it obtains all borrowers in that market area since the product in this case is perfectly homogenous, i.e. product is not differentiated by providers' locations.

### 3 The game

The sequence of the decisions made in every period  $t$  of the game is illustrated in figure 1. The first mover is the bank 1, who decides the price of the loan, then moves bank 2 giving the best response to bank 1's loan price, and finally bank  $m$  decides its action, whether to discriminate its borrowers by their location or set the same price of the loan for every borrower.

Since the game is repeated, banks 1 and 2 are assumed to play "trigger strategies" to ensure cooperative behavior of each other, i.e. if one of the banks deviates from joint profit maximization price rival will retaliate it by pricing the loans to marginal costs for every subsequent periods.<sup>9</sup> Bank 1 has two alternative actions; either cooperate ( $C$ ) or set the price at the noncooperative level ( $NC$ ). Bank 2 has three alternative actions; cooperation ( $C$ ), cheating ( $Ch$ ) or to set the price at noncooperative level ( $NC$ ). If bank 2 deviates it prices a loan at level  $p_{2,t} = p_{1,t} - \varepsilon$  and captures all the borrowers in markets near to town  $j$  and since it knows bank 1's response to deviation the best response price in subsequent periods is  $p_{2,t+i} = 0, \forall i \in \{1, \dots, T\}, T \rightarrow \infty$ . If bank 2 sets loan price at cooperative level in period  $t$  then bank 1 also acts cooperatively in period  $t + 1$  and if bank 2 deviates at period  $t$  then  $p_{1,t+i} = 0, \forall i \in \{1, \dots, T\}, T \rightarrow \infty$ . Game structure and demand functions give  $p_{1,t} \leq p_{2,t}$  for every period  $t$ . Thus bank 2 never offers loan at a higher price than first mover.<sup>10</sup>

<sup>9</sup>If the game was played only once, then in the equilibrium of the game banks 1 and 2 would behave noncooperatively and both of them would earn zero-profits.

<sup>10</sup>That is the case when banks play trigger strategy. If banks played, for example tit-for-tat strategy that would not always hold.

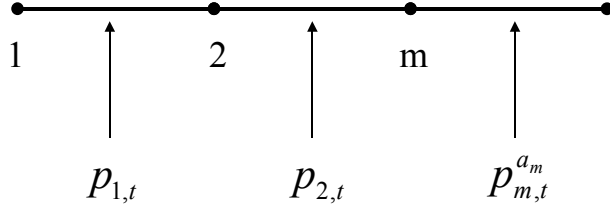


Figure 1: Timeline of the decision making in the game.

In the game, bank  $m$  has two alternative actions; to set uniform price to each borrowers ( $U$ ) maximizing profit of the bank with given actions of rival banks, or to price discriminate the borrowers according to their location ( $D$ ). By price discrimination bank tries to capture borrowers' surplus in transportation costs by setting loan prices such that all borrowers are (almost) indifferent to best offer given by rival bank. Since the loan price in bank  $k$  for loan applicant  $a$  is  $p_{k,t} = C(r_{k,t}^a, L)$ ,  $k = m, 1, 2$  the indifference condition can be written as

$$p_{k,t}^a + \tau_t d_m^a = \min\{p_{1,t}^a, p_{2,t}^a\} + \tau_t d_j^a,$$

where  $d_m^a$  is loan applicant  $a$ 's distance to bank  $m$ ,  $\min\{p_{1,t}^a, p_{2,t}^a\}$  is the minimum price of the loan in town  $j$  and  $d_j^a$  loan applicant  $a$ 's distance to town  $j$ . Since location of bank  $m$  is given  $x_m = 0$  and the locations of banks 1 and 2 are  $x_1 = x_2 = 1$ , loan applicant  $a$ 's distance to bank  $m$  is  $d_m^a = x_a$  and distance to banks 1 and 2 is  $d_1^a = d_2^a = 1 - x_a$ . Then the discriminating pricing scheme of bank  $m$  is  $p_{m,t}^a = \min\{p_{1,t}^a, p_{2,t}^a\} + (1 - 2x_a)\tau_t - \varepsilon$ , i.e. bank  $m$  captures borrower's surplus in transportation costs. Figure 2 presents the sequential price setting and the outcome profits.



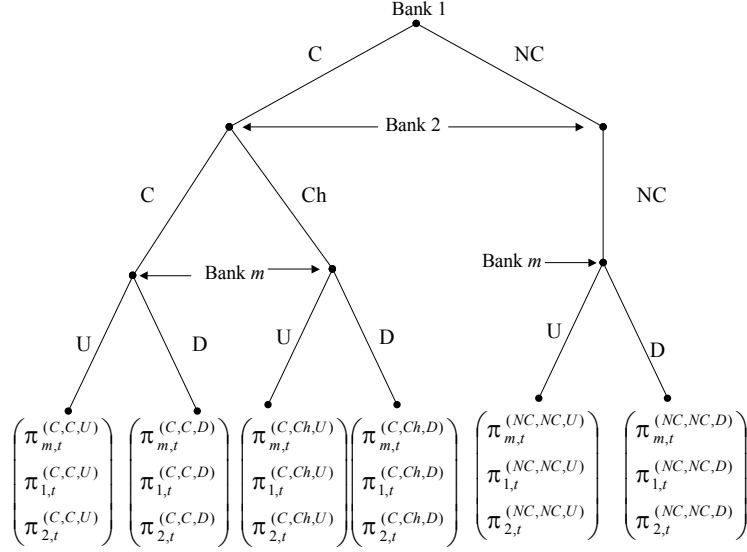


Figure 2: The sequence of moves of the game in period  $t$ .

### 3.1 Profit maximization of the banks, outcome prices and profits.

Banks maximize their profits by setting the loan price at optimal level given the optimal price of the rivals. For a local monopoly we get

$$\max_{p_{m,t}} \pi_{m,t}^U = N_t \int_0^{\frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t}} (p_{m,t} - c) f(x) dx \quad (8)$$

when it uses uniform pricing, and

$$\max_{p_{m,t}} \pi_{m,t}^D = N_t \int_0^{\frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t}} (p_{2,t} + (1 - 2x)\tau_t - c) f(x) dx \quad (9)$$

when it discriminates its borrowers by their locations. Term  $p_{m,t}$  denotes loan price for the marginal borrower.

**Proposition 1** *Bank  $m$ 's profit maximizes when it offers loan to marginal borrower at price  $p_{m,t} = c$ .*

**Proof.** In profit function  $\pi_{m,t}^D = N_t \int_0^{\frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t}} (p_{2,t} + (1 - 2x)\tau_t - c) f(x) dx$   $(p_{2,t} + (1 - 2x)\tau_t - c)$  expresses the profit of the loan given to borrower

located in  $x$  and can be denoted by  $\kappa$ . Substituting  $x = \frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t}$  into it, we get  $\kappa = p_{m,t} - c > 0$  if  $p_{m,t} > c$ . Since  $f(x) > 0, \forall x \in [0, 1]$ , decrease in  $p_{m,t}$  increases banks total profit when  $p_{m,t} > c$  and banks profit maximizes when  $p_{m,t} = c$ . ■

When banks 1 and 2 behave cooperatively, bank 1 set the loan price  $p_{1,t}$  at profit maximizing level with given loan price at bank  $m$  and bank 2 set the loan price  $p_{2,t} = p_{1,t}$ . Thus, bank 1 sets the loan price  $p_{1,t}$  according to

$$\max_{p_{1,t}} \pi_{1,t} = \frac{N_t}{2} \int_{\frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t}}^1 (p_{1,t} - c) f(x) dx. \quad (10)$$

The first order condition of (8) is

$$0 = \frac{\partial \pi_{m,t}^U}{\partial p_{m,t}} = N_t \left[ F \left( \frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t} \right) - \frac{p_{m,t} - c}{2\tau_t} f \left( \frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t} \right) \right] \quad (11)$$

where  $F \left( \frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t} \right) = \int_0^{\frac{\tau_t - (p_{m,t} - p_{2,t})}{2\tau_t}} f(x) dx$ , i.e. the share of borrowers in town  $i$ 's market area. When banks 1 and 2 cooperate we get

$$0 = \frac{\partial \pi_{1,t}}{\partial p_{1,t}} = \frac{N_t}{2} \left[ 1 - F \left( \frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t} \right) - \frac{p_{1,t} - c}{2\tau_t} f \left( \frac{\tau_t - (p_{m,t} - p_{1,t})}{2\tau_t} \right) \right]. \quad (12)$$

If bank  $m$  uses price discrimination, then it offers loan to marginal borrower at price  $p_{m,t} = c$ . If bank 2 cheats in period  $t$  it offers loan at price  $p_{2,t}^{(C,Ch,U)} = p_{1,t}^{(C,C,U)} - \varepsilon$  if it knows that bank  $m$  uses uniform pricing, and at price  $p_{2,t}^{(C,Ch,D)} = p_{2,t}^{(C,C,D)} - \varepsilon$  if it knows that bank  $m$  discriminates, where  $\varepsilon$  is infinitesimal constant.<sup>11</sup>

As assumed banks 1 and 2 are playing trigger strategy. Then, if bank 2 cheats in period  $t$ , it earns profit  $\pi_{2,t}^{(C,Ch,a_{m,t})}$  and makes zero-profit in all subsequent periods. If it acts cooperatively, banks 1 and 2 maximizes profits jointly and bank 2 earns profit  $\pi_{2,t+i}^{(C,C,a_{m,t})}$  in all periods  $i \in [0, \infty[$ . Bank 2 has thus incentive to cheat if

$$\delta \pi_{2,t}^{(C,Ch,a_{m,t})} > \sum_{i=0}^{\infty} \delta^{1+i} \pi_{2,t+i}^{(C,C,a_{m,t})}. \quad (13)$$

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<sup>11</sup>Since  $\varepsilon$  is assumed to be infinitesimal constant, its effect on bank  $m$ 's loan price and the profits is so close to zero that it can be left out of analysis.

Equation (13) demonstrates that if the present value of the future profits by cooperation is lower than one with cheating, bank 2 have no incentive to cooperate.<sup>12</sup>

### 3.2 Sub-game perfect equilibrium

The game has five proper sub-games; three where bank  $m$  chooses its pricing action and optimal price with given actions of banks 1 and 2, and two where bank 2 chooses its actions. Let me now compute the sub-game perfect equilibrium (SPE) of the game.

**Proposition 2** *The game has unique SPE, which is: (i)  $(C, (C, NC), (D, D, D))$  when  $\delta\gamma\rho \geq \frac{1}{2}$ , and (ii)  $(NC, (Ch, NC), (D, D, D))$  when  $\delta\gamma\rho < \frac{1}{2}$ .*

**Proof.** Bank  $m$  takes loan price in town  $j$ , denote by  $p_{2,t}$ , as given and choose the profit maximizing action. By price discrimination  $m$  gets

$$\pi_{m,t}^D = N_t \int_0^{\frac{\tau_t + p_{2,t} - p_{m,t}^D}{2\tau_t}} (p_{2,t} - c + (1 - 2x)\tau_t) f(x) dx$$

and by uniform pricing it gets

$$\pi_{m,t}^U = N_t \int_0^{\frac{\tau_t + p_{2,t} - p_{m,t}^U}{2\tau_t}} (p_{m,t} - c) f(x) dx.$$

Above  $p_{m,t}^D$  is loan price for marginal borrower. For  $p_{m,t}^D$  holds  $p_{m,t}^D = c$ , if  $p_{2,t} \leq \tau_t + c$  and  $p_{m,t}^D = p_{2,t} - \tau_t$ , if  $p_{2,t} > \tau_t + c$ . In latter case  $m$  can profitably capture all the borrowers in markets. Since it is obvious that  $p_{m,t}^U \in [p_{m,t}^D, p_{2,t} + \tau_t]$ ,  $\pi_{m,t}^D$  can be written as

$$\begin{aligned} \pi_{m,t}^D &= N_t \int_0^{\frac{\tau_t + p_{2,t} - p_{m,t}^U}{2\tau_t}} (p_{2,t} - c + (1 - 2x)\tau_t) f(x) dx \\ &\quad + N_t \int_{\frac{\tau_t + p_{2,t} - p_{m,t}^U}{2\tau_t}}^{\frac{\tau_t + p_{2,t} - p_{m,t}^D}{2\tau_t}} (p_{2,t} - c + (1 - 2x)\tau_t) f(x) dx. \end{aligned}$$

Consider the revenue generated by borrower located at  $x^* = \frac{\tau_t + p_{2,t} - p_{m,t}^U}{2\tau_t}$ , when  $m$  discriminates. Loan revenue from this borrower is  $p_{m,t}^D(x^*) = p_{2,t} + (1 - 2\frac{\tau_t + p_{2,t} - p_{m,t}^U}{2\tau_t})\tau_t = p_{m,t}^U$ , and loan revenues from the borrowers located on

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<sup>12</sup>For the cooperation in repeated games, see e.g. Shy (1996).

interval  $x \in [0, x^*[$  are above  $p_{m,t}^U$ . Therefore, not depending on the distribution of the customers or on given  $p_{2,t}$ , discrimination dominates uniform pricing and  $(D, D, D)$  is dominating strategy for  $m$ .

Bank 2 knows that bank  $m$  discriminates and if it cheats bank 1 will play noncooperative action forever. Denote bank 2's present value of the profits at period  $t$  by  $\Pi_{2,t}^{(a_1,t,a_2,t,D)} = \sum_{i=0}^{\infty} \delta^i \pi_{2,t}^{(a_1,t+i,a_2,t+i,D)}$ . Since bank 1 plays trigger strategy, if bank 2 cheats, present value of its profits is  $\Pi_{2,t}^{(C,Ch,D)} = 2\pi_{2,t}^{(C,C,D)}$  and if it behaves cooperatively,  $\Pi_{2,t}^{(C,C,D)} = \sum_{i=0}^{\infty} \delta^i \pi_{2,t+i}^{(C,C,D)}$ . Bank 2 has incentive to cheat, if  $\Pi_{2,t}^{(C,Ch,D)} > \Pi_{2,t}^{(C,C,D)}$ . As can be seen in equation (10), the cooperative profit in period  $t$  can be written in form  $\Pi_{2,t}^{(C,C,D)} = AN_t\tau_t$ , where  $A$  is non-negative real number constant presenting bank 2's market share multiplied by price-cost margin, and depending on the shape of the density function. Using this inequality  $\Pi_{2,t}^{(C,Ch,D)} > \Pi_{2,t}^{(C,C,D)}$  simplifies to

$$2 > \frac{1}{1 - \delta\gamma\rho}$$

when  $N_t = \gamma N_{t-1}$ , and  $\tau_t = \rho\tau_{t-1}$ . Therefore, if  $\delta\gamma\rho \geq \frac{1}{2}$  and bank 1 prices loan at cooperative price level, bank 2 plays cooperative action and if  $\delta\gamma\rho < \frac{1}{2}$  and bank 1 prices loan at cooperative price level, bank 2 cheats. If bank 1 plays  $NC$ , then bank 2 replies always by  $NC$ . Then bank 2's strategy can be written as  $(Ch, NC)$ , if  $\delta\gamma\rho < \frac{1}{2}$  and as  $(C, NC)$ , if  $\delta\gamma\rho \geq \frac{1}{2}$ .

Since bank 1 knows the consequences of its actions, it will play  $(NC)$ , if  $\delta\gamma\rho < \frac{1}{2}$  and  $(C)$ , if  $\delta\gamma\rho \geq \frac{1}{2}$ . Therefore, if  $\delta\gamma\rho < \frac{1}{2}$  holds then SPE of the game is  $(NC, (Ch, NC), (D, D, D))$  and if  $\delta\gamma\rho \geq \frac{1}{2}$  holds then SPE of the game is  $(C, (C, NC), (D, D, D))$ . ■

As can be seen, if the markets and/or the importance of the distance are declining the collusion forced by trigger strategy can be broken (in addition to typical effect of the discount factor). Then, for bank 2 the profit in the present period dominates the smaller future profits. This result can be also generalized such that when there where  $n$  banks in town  $j$ , then the cooperation will be broken, if  $\delta\gamma\rho < \frac{n-1}{n}$  (for derivation, see appendix).

## 4 Customer distribution and returns of lending

One main question of this paper was, when the bank with local monopoly can get lower loan returns than banks in neighboring town. In this section I present an illustrative example how the population distribution affects on loan returns.

As previously assumed, there are in the market  $N_t$  loan applicants, each one trying to borrow money for similar type of use. Also, the only differentiation of the loans is the suppliers' location. In this illustration customers are located in the line according to linear density function with non-negative slope; non-negative since bank with local monopoly is located at  $x_m = 0$  and it is more intuitive to assume that town of two banks has more population than town with only one bank. Even though the linear density function does not fit very well in real world's population distribution, it can illustrate well enough the effect of the size difference on the equilibrium prices and on the banks' profits and returns.

Denote density function of customer distribution by  $f(x, a, b) = ax + b$ , where  $a$  and  $b$  are non-negative real value constants. Since  $f(x)$  is density function and all the borrowers are located on the interval  $[0,1]$ ,  $b$  can be solved as a function of  $a$ . Then  $b$  can be written as  $b = 1 - \frac{1}{2}a$  and furthermore, density function as  $f(x, a) = \frac{2x-1}{2}a + 1$ .

At first, let me analyze the case when  $\delta\gamma\rho \geq \frac{1}{2}$ , i.e. bank 2 has no economic incentive to cheat and the equilibrium of the game is  $(C, (C, NC), (D, D, D))$ . Profit function of  $m$  is now

$$\pi_{m,t} = N_t \int_0^{\frac{\tau_t - (p_{m,t} - p_{j,t})}{2\tau_t}} (p_{j,t} + (1 - 2x)\tau_t - c) \left( \frac{2x-1}{2}a + 1 \right) dx \quad (14)$$

and profit function of the banks in town  $j$  is

$$\pi_{j,t} = \frac{N_t}{2} \int_{\frac{\tau_t - (p_{m,t} - p_{j,t})}{2\tau_t}}^1 (p_{j,t} - c) \left( \frac{2x-1}{2}a + 1 \right) dx. \quad (15)$$

As previously showed, profit maximizing loan price of bank  $m$  for marginal borrower is  $p_{m,t} = c$ . Then the first order condition of banks in town  $j$  can be written as

$$0 = \frac{\partial \pi_{j,t}}{\partial p_{j,t}} = \frac{8c\tau_t - 8\tau_t p_{1,t} + 6acp_{1,t} + 4\tau_t^2 - 3ac^2 + a\tau_t^2 - 3ap_{1,t}^2}{16\tau_t^2}$$

and the equilibrium loan price in town  $j$  is then

$$p_{j,t}^* = c + \frac{1}{3a} \left( -4 + \sqrt{16 + 12a + 3a^2} \right) \tau_t \quad (16)$$

which is also the average loan price. Average loan price of bank  $m$  in equilibrium can be written as

$$p_{m,t}^{AVE} = p_{j,t}^* + \frac{\int_0^{\frac{\tau_t + p_{j,t}^* - c}{2\tau_t}} (1 - 2x)\tau_t \left( \frac{2x-1}{2}a + 1 \right) dx}{\int_0^{\frac{\tau_t + p_{j,t}^* - c}{2\tau_t}} \left( \frac{2x-1}{2}a + 1 \right) dx} \quad (17)$$

Unfortunately this price equation cannot be written as shortly as equation (16) since integrals in the last term generate pretty complex result. One can calculate the exact functional form of it by any mathematical software. Loan returns of banks 1 and 2 are then

$$r_{j,t} = \frac{c + \frac{1}{3a} (-4 + \sqrt{16 + 12a + 3a^2}) \tau_t}{L}. \quad (18)$$

and return of the average loan

$$r_{m,t} = \frac{p_{m,t}^{AVE}}{L}. \quad (19)$$

Since both equation (18) and equation (19) have same divisor  $L$ , if  $p_{m,t}^{AVE} - p_{j,t}^* > 0$  then  $r_{m,t} > r_{j,t}$ . Using equation (17) this price inequality can be straightforwardly written as

$$\begin{aligned} \Delta p_t &= p_{m,t}^{AVE} - p_{j,t}^* \\ &= \frac{\int_0^{\frac{\tau_t + p_{j,t}^* - c}{2\tau_t}} (1 - 2x) \tau_t \left(\frac{2x-1}{2}a + 1\right) dx}{\int_0^{\frac{\tau_t + p_{j,t}^* - c}{2\tau_t}} \left(\frac{2x-1}{2}a + 1\right) dx} \end{aligned} \quad (20)$$

Again, writing the final form of this (a function of  $a$  multiplied by  $\tau_t$ ) required so much space, and is not so illustrative as the figure 3 presenting the difference.

As can be seen in figure 3, more the population concentrated in the town  $j$ , smaller the loan return for bank  $m$  in equilibrium. If the concentration in town  $j$  was high enough, the bank  $m$ 's loan return was lower than one of banks 1 and 2. Therefore, even when the bank  $m$  has local monopoly and can discriminate its borrowers by their location, if banks 1 and 2 act cooperatively, they can get higher loan return than bank  $m$ .

In a case where  $\delta\gamma\rho < \frac{1}{2}$ <sup>13</sup> banks 1 and 2 act noncooperatively and the sub-game perfect equilibrium of the game is  $(NC, (Ch, NC), (D, D, D))$ . Now the equilibrium loan price in town  $j$  is  $p_{j,t} = c$ . Substituting this into equation (20) gives  $\Delta p_t = \frac{2a-6}{3a-12}\tau_t > 0, a \in [0, 2]$ . Therefore in this equilibrium bank  $m$ 's loan return is always higher than loan return of banks in town  $j$ .

## 5 Concluding remarks

In this paper I analyzed the competition in local lending market. Product (loan) was differentiated only by the location of the supplier and the game

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<sup>13</sup>This is the case where market is declining and/or the transportation costs are diminishing. In real life reason for these could be migration and technological development.

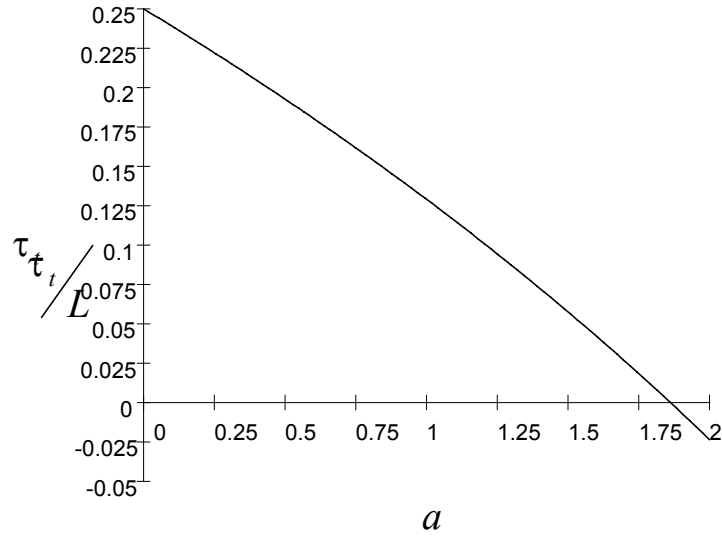


Figure 3: Price difference on average loan prices

was repeated spatial price game with totally inelastic periodic demand and given locations for both of customer and supplier. Also, the marginal cost was equally high for each bank.

In analysis I showed that for monopolist bank discrimination is dominating action with any customer distribution. Collusiveness in oligopoly-town depends inversely, in addition to usual discount factor, on the development of transportation costs and on the development of the market size measured by number of borrowers. If market size and transportation costs decline fast enough, banks in multibank town behaved noncooperatively. Also, it was shown that more banks, easier the collusion breaks.

If the banks in oligopoly town behave cooperatively, it is possible that their loan returns are higher than one of bank with local monopoly. That requires that previous town is bigger enough than latter one. If the banks in oligopoly town act noncooperatively then local monopoly's loan return is lower than in last case but always higher than returns of its rivals. Also then the market share of monopolists is smaller.

On results of the game must be said the following. Retaliation of discrimination of monopolist bank can also be action alternative for banks in oligopoly, and this can lead kind of semi-collusive outcome where bank with local monopoly price loan price at uniform pricing -outcome level and discriminates borrowers located "left" from location of marginal borrower. Also,

in the model loan size was same for every borrower. However, the loan price was defined by transportation costs, i.e. the distance gave the pricing power. If the average loan is lower in one banks market are the profit is as high as it used to be but the loan capital is smaller and then the returns of lending increases. Also, in model the market size changes but not the customer distribution.

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## A Collusiveness and number of banks in oligopoly town

The number of operating banks in town  $j$  does not affect on behavior of  $m$ , i.e. discrimination is dominating action for  $m$ . Consider the behavior of bank  $n$ , i.e. the bank last in the sequence of decision making in town  $j$ . Bank  $n$  has incentive to cheat, if

$$\delta\pi_{n,t}^{(C,Ch,D)} > \sum_{i=0}^{\infty} \delta^{1+i} \pi_{n,t+i}^{(C,C,D)} \quad (\text{A1})$$

As previously,  $\pi_{n,t}^{(C,Ch,D)} = n\pi_{n,t}^{(C,C,D)}$  and profits per period in cooperation are of type  $\pi_{n,t+i}^{(C,C,D)} = AN_{t+i}\tau_{t+i}$ , where  $A$  is a constant. Using these, equation (A1) simplifies to

$$n > \frac{1}{1 - \delta\gamma\rho}$$

and using this we can conclude that bank  $n$  has incentive to cheat if

$$\delta\gamma\rho < \frac{n-1}{n}.$$

Since every bank  $b \in \{1, 2, \dots, n-1\}$  know that, they will play  $NC$  if  $\delta\gamma\rho < \frac{n-1}{n}$  and  $C$  if  $\delta\gamma\rho \geq \frac{n-1}{n}$ .

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