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ASYMMETRIC INFORMATION AND THE MARKET STRUCTURE OF THE VENTURE CAPITAL INDUSTRY

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ABSTRACT: In this paper we study a horizontally differentiated market for financial in-termediation and develop a simple explanation for concentration in the financial intermediation industry. We show that under asymmetric information, if the demand for funds is not perfectly elastic, the heterogeneity of entrepreneurs in need of financing translates into a barrier to entry. That is, we do not need to resort to learning, weak property rights or exogenous costs of entry to generate this result.

JEL: D82, G21

KEYWORDS: venture capital, asymmetric information, entry, market structure

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TIIVISTELMÄ: Tässä paperissa tarkastellaan rahoituksen välittymistä horison-taalisesti differoiduilla markkinoilla ja tutkitaan markkinoiden keskittymiseen vai-kuttavia tekijöitä. Osoitamme, että mikäli rahoituksen kysyntä ei ole täysin joustavaa, rahoitusta tarvitsevan yrittäjäjoukon heterogeenisuus voi johtaa epäsymmetrisen informaation vallitessa alalletulon esteisiin. Tutkimus tuo siten vaihtoehtoisen perusteen rahoitustoimialan keskittymiselle verrattuna aikaisempaan kirjallisuuteen, joka on osoittanut, että esimerkiksi oppiminen, heikosti määritellyt informaatioon liittyvät omistusoikeudet tai eksogeeniset alalletulon kustannukset voivat johtaa alalletulon esteisiin ja siten oligopolistiseen markkinarakenteeseen.

AVAINSANAT: pääomasijoittaminen, rahoituksen välitys, epäsymmetrinen infor-maatio, alalletulo, markkinarakenne

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1 Introduction

Our objective in this paper is to understand the limitations of competition in financial intermediation when there is asymmetric information and intermediaries are providing financing to entrepreneurs of varying quality. We study whether or not free entry leads to zero intermediary mark-ups, i.e., “first-best” efficiency.

We focus on a horizontally differentiated market for financial intermediation and develop another, simpler, informational reason for concentration. We show that under asymmetric information, if the demand for funds is not perfectly elastic, then the heterogeneity of entrepreneurs in need of financing translates into a barrier to entry. Further, as the number of venture capitalists (VCs) on the market increases as a consequence of entry, there is a decrease in the excess profit share on and above the marginal cost of providing venture capital finance that is associated with product differentiation. However, under asymmetric information, the marginal cost cannot be fully integrated into the profit shares demanded by the VCs, because good entrepreneurs’ demand is more sensitive than that of bad ones, and the VCs have to price their financing at average quality of projects. When the demand for funds is not perfectly elastic, this constraint to pricing makes the profit shares resistant to changes in the marginal cost of funds, and leads to a barrier to entry. Thus, we need not resort to learning, weak property rights, or exogenous costs of entry, to generate this result.

We use a model of venture capital finance to illustrate these phenomena. We focus on venture capital finance, as that field of finance is in relatively early stages of development in many countries, and as there has been a lot of entry into the venture capital industry recently. We focus on VCs also because they often finance small, technology-based ventures in emerging industries. Such settings are more conducive to asymmetric information than is the financing of traditional, established firms.

Venture capital firms provide their services to entrepreneurial projects in exchange for a share in the profits of the venture. They also offer active managerial assistance to help the entrepreneurs to commercialize their projects (see, e.g., Gompers and Lerner 1999; Van Osnabrugge and Robinson 2000). Although their levels of involvement may vary, venture capital firms can add value to the firms in which they invest, for example, by using their networks to create alliances and to provide access to different markets, bringing in key employees, and helping to choose appropriate strategies for future growth (see, e.g., Hellman and Puri 2001, Kaplan and Strömberg 2001). Our model captures these important aspects of venture capital, and recognizes that the provision of such advice is not free of cost (see, e.g., Kanniainen and Keuschnigg 2000).

The paper proceeds as follows. In section 2, we discuss some of the studies that form the background to our paper. In section 3 we develop the basic model. In section 4, we characterize the equilibrium for a given number of intermediaries in the market. The analysis continues in section 5 by introducing an entry stage. We show that asymmetric information may translate into a barrier to enter in the venture capital industry. Section 6 concludes.

2 Background

The current literature on market structure in the financial services industry examines asymmetric information between market participants and the intermediaries’ role in solving the frictions that stem from asymmetric information. Dell’Ariccia, Friedman and Marquez (1999) show that because banks learn by lending, incumbent banks have an information advantage over entrant banks, a situation that creates an adverse selection problem that may block the entry of additional banks. In their model, a third bank never enters, leading to an equilibrium market structure that is duopolistic.

Dell'Ariccia (2001) also considers learning by lending. Using a dynamic model of horizontally differentiated banking industry, he shows that because of adverse selection that stems from the informational asymmetries between the incumbent and entrant banks, the market is characterized by an endogenous fixed entry costs for new potential rivals.

Anand and Galetovic (2000) develop a model of financial intermediation with weak property rights over information. They show that the equilibrium market structure is oligopolistic. In the paper, the model is used to explain the market structures that prevail in investment banking and in venture capital. In their model, entry is limited because there is a need for self-enforcing cooperation that enables a commitment by each intermediary not to free-ride on others' information gathering.

There are several theories of market concentration in the financial services industry that are not related to informational problems. In addition to the traditional explanation of scale economies, the market for intermediated finance might be (highly) concentrated because intermediaries specialize and offer differentiated financial services. This view originates from the seminal article of Salop (1979), in which the author develops a circular model of horizontal differentiation. Williamson (1987), Besanko and Thakor (1992), Chiappori, Perez-Castrillo and Verdier (1995), Dell'Ariccia (2001), and Cordella and Levy Yeyati (2002) represent some of the studies that have applied the Salop model to banking and lending markets.

Models of horizontal differentiation suggest that specialization is feasible if there are fixed or sunk costs. In the provision of financial services, such costs might relate, for example, to deal structuring, or establishing a branch network. The existence of asymmetric information may also lead to economies of scale (Boyd and Prescott 1986, Williamson 1986). In the absence of fixed costs, the only long-run equilibrium is characterized by a continuum of financial intermediaries along the product or location space (see, e.g., Williamson 1987, Chiappori et al. 1995).

3 A Model of an Imperfectly Competitive Venture Capital Market

We consider a Salop type spatial model of financial intermediation, constructed along the lines of, e.g., Williamson (1987), Besanko and Thakor (1992) and Chiappori et al. (1995), who all study banking markets. We denote VCs by *i*. The VCs are located on a circle of unit circumference, and a unit mass of entrepreneurs is distributed uniformly along the circle. We assume maximal differentiation in location, hence VCs are located symmetrically on the circle. The location of an entrepreneur is denoted *x*. When approaching a VC, an entrepreneur incurs a transportation (participation) cost. We denote the cost per unit of length as τ,[1] and the economy is universally risk-neutral.

Our interpretation of product differentiation in this model is that each entrepreneur has an ideal "type" of venture capitalist. Although all VCs are equally efficient in terms of (expected) gross profits, holding effort constant, the transaction costs of dealing with them vary. We assume that these transaction costs (for example, the ease with which the entrepreneur and VC understand each other) are known ex ante.

Entrepreneurs have access to a project that requires an initial investment (start-up cost) of size unity. We assume that entrepreneurs have no initial wealth and therefore must acquire external financing. Each entrepreneur can pursue only one project. From the perspective of the entrepreneurs, the VCs are the only source of outside financing in

[1] The VCs cannot determine the location of an entrepreneur, and therefore no location-based price-discrimination is feasible.

this economy. The VC receives a profit share s_i in return for its capital injection. In our model the marginal cost of funds for a VC is normalized to one.

Besides financing, the chosen VC provides managerial assistance a_i. This requires effort, the cost of which is modeled as $C(a_i) = \frac{1}{2}ca_i^2$. In spirit of, e.g., Repullo and Suarez (1998), we assume that the VC's advice is essential to the project's success. In other words, if the VC does not advise the project, it is worthless (see below).

There are two types of entrepreneurs, *H* and *L*. The project of an *H*-type entrepreneur is of high quality and has an expected value of $a_iV_H > 0$. Type *L*'s project is of low quality and has an expected value of a_iV_L, where $V_H > V_L > 0$ holds. In the population of entrepreneurs, the respective fractions of these two types are λ and $(1-\lambda)$. The VCs cannot determine the type of an entrepreneur, but know the fraction of each type in the population. We eliminate any ex ante screening by VCs. Although such screening is important in reality (Kaplan and Strömberg 2001), we could include it in the model without altering the conclusions, but only at the cost of adding unnecessary complexity. Our results hold provided that ex ante screening is imperfect, i.e., that a degree of asymmetric information remains.

VCs compete for entrepreneurs via profit share-offers and the level of advice. In spirit of the banking model of Cordella and Levy Yeati (2002), we assume that the profit shares chosen by VCs are observable, but the level of advice (effort) is not.[2] Before deciding which VC to approach, each entrepreneur must rationally evaluate VC *i*'s advisory decision according to the expectation $E(a_i) = a_i^e$. In a symmetric rational expectations Nash equilibrium (see Cordella and Levy Yeyati 2002, Boot and Schmeits

[2] This assumption is similar to that made in Cordella and Levy Yeati (2002), for they assume that depositors can observe banks' deposit interest rates, but cannot observe the monitoring (risk) level chosen by the banks; see also Boot and Schmeits (2000). We would like to also note here that the advisory level could be made exogenous without changing the main result of the paper.

2000), these beliefs are fulfilled, as the entrepreneurs infer that the VCs' advisory decisions are those that will prevail in the Nash equilibrium.

To capture these considerations, the timing of events is as follows: First, knowing that only the profit share requests become observable, VCs simultaneously choose the profit shares that are required by each investor in exchange for a capital injection, and the level of advice. Second, the entrepreneurs choose among VCs, and must do so without being able to observe the chosen advisory level.

4 The Short-Term, No Entry, Equilibrium

Assume *n* symmetrically located VCs on a circle. Moreover, so as to rule out exit, assume for now that the VCs are able to make positive expected profits.

We begin by deriving demand functions for capital injections and focus on the behavior of an entrepreneur (of type *t*) located at distance $x \in [0, 1/n]$ from the i^{th} VC. Approaching the VC yields an expected return of $(1-s_i)a_i^e V_t$ to the entrepreneur, gross of travel costs. Assuming that each VC's relevant competitors are right next to her, the VC is able to attract the entrepreneur only if its profit share offer is higher than those offered by the rival VCs, i.e., if $(1-s_i)a_i^e V_t - \tau x \geq (1-\bar{s})\bar{a}^e V_t - \tau(1/n - x)$ with $\bar{s} = s_j$ and $\bar{a}^e = a_j^e$ for $j \neq i$.

Provided that the inequality holds and τ is small enough, the entrepreneur chooses the i^{th} VC and the market will be covered in equilibrium. Therefore, we focus on the case of *full-scale competition* (to use the terminology of Villas-Boas and Schmidt-Mohr, 1999). Because we also assume that for each VC, the relevant competitors are right next to her, there is a lower bound for the transportation costs. That is, we can show that these costs must be large enough to partition the market in segments of size $1/n$.

Under the standard assumption of full-scale competition, the total demand for funds that the i^{th} VC faces from the *H*- and *L*-type entrepreneurs is given by the following equations:

$$D_i(H) = \lambda\left(n^{-1} + \tau^{-1}V_H((1-s_i)a_i^e - (1-\overline{s})\overline{a}^e)\right) \tag{1a}$$

$$D_i(L) = (1-\lambda)\left(n^{-1} + \tau^{-1}V_L((1-s_i)a_i^e - (1-\overline{s})\overline{a}^e)\right) \tag{1b}$$

VC *i*'s expected profits can be written as

$$\pi_i = D_i(H)A_i(H) + D_i(L)A_i(L)\,. \tag{2}$$

where $A_i(t) \equiv s_i a_i V_t - 1 - \frac{1}{2}ca_i^2$ is the expected profit per funded project of type t, $t = H, L$.

We note that in our model, unlike in studies such as, e.g., Williamson (1987), there are no (exogenous) fixed costs of financial intermediation. Were we to introduce these, in the current model they would constitute an additional barrier to entry once the entry is allowed for. To emphasize the role of asymmetric information as an entry-deterring mechanism, we exclude them by assumption. As noted earlier, we also have no ex-ante screening of projects, even though this is often regarded as one of the main features of VC financing. Such screening would not change the results provided that the screening technology is imperfect.

Conditional on other VCs' strategies, the i^{th} VC chooses s_i and a_i so as to maximize (2). In equilibrium, symmetrically distributed VCs have the same profit share request and offer the same level of advice. The entrepreneurs' beliefs regarding the levels of advice are unbiased. Symmetry and rational prior beliefs imply $s_i = \overline{s} \equiv s$ and $a_i = a_i^e = \overline{a} = \overline{a}^e \equiv a$. The first-order conditions $\partial\pi/\partial s=0$ and $\partial\pi/\partial a=0$ can therefore be simplified and solved simultaneously to yield:

$$s=\sqrt{\frac{c(1+\tau n^{-1})}{\overline{\overline{V}}-\frac{1}{2}\overline{V}^{2}}} \tag{3a}$$

$$a=\frac{\overline{V}}{c}\sqrt{\frac{c(1+\tau n^{-1})}{\overline{\overline{V}}-\frac{1}{2}\overline{V}^{2}}} \tag{3b}$$

where $\overline{V}\equiv\lambda V_{H}+(1-\lambda)V_{L}$ and $\overline{\overline{V}}\equiv\lambda V_{H}^{2}+(1-\lambda)V_{L}^{2}$. Equations (3a) and (3b) define the equilibrium profit shares and advisory intensities for a given number of VC (i.e. *n*) in the market.

To compute the VCs' expected profits, we note that at the symmetric equilibrium $D(H)+D(L)=(\lambda+(1-\lambda))n^{-1}=n^{-1}$. We have

$$\pi=\frac{\left[\frac{1}{2}\tau n^{-1}\overline{V}-\left(\overline{\overline{V}}-\overline{V}^{2}\right)\right]}{n\left(\overline{\overline{V}}-\frac{1}{2}\overline{V}^{2}\right)}. \tag{4}$$

Because $\overline{\overline{V}}-\overline{V}^{2}=(1-\lambda)\lambda(V^{H}-V^{L})^{2}>0$, the denominator in equation (4) is strictly positive. Therefore, the numerator is a key determinant of the structure of the industry under free entry.

5 The Long-Term, Free Entry, Equilibrium

Under free entry, the number of VCs ($\hat{n}$) is such that $\pi(\hat{n})=0$. The condition implies that the numerator of (4) must be equal to zero, yielding

$$\hat{n}=\frac{\frac{1}{2}\tau(\lambda V_{H}+(1-\lambda)V_{L})}{\lambda(1-\lambda)(V_{H}-V_{L})^{2}}. \tag{5}$$

The right hand side of (5) is unambiguously positive and bounded from above as long as $V_{H}>V_{L}>0$. Therefore, we have proven the following:[3]

[3] This result recalls the "natural oligopolies" result of Shaked and Sutton (1983). See also Sutton (1991).

Proposition: *With free entry, the number of entering venture capital firms remains finite.*

To understand the result, we compare the above situation of asymmetric information to that of symmetric information, where VCs can identify the type of each entrepreneur. Under symmetric information, VCs would separately optimize for *H*- and *L*-type entrepreneurs, but under asymmetric information, they are forced to optimize for the average project. This constraint implies that under asymmetric information, the equilibrium profit shares become less sensitive to changes in the marginal cost of funds than they are under symmetric information.

The profit shares become less sensitive to changes in the marginal cost of funds because for a given change in the marginal cost, the (weighted average) elasticity of the VC's profit share is greater under symmetric information than is the elasticity under asymmetric information. This difference is important because when the number of VCs increases as a consequence of entry, each VC's profit compresses through two channels: first, his market share (the equilibrium number of projects financed) decreases; second, the expected profit per project decreases. Under symmetric information, the expected profit per project remains strictly positive for any finite n (the net profits per project are τ/n as in the standard Salop model), but under asymmetric information this is not the case. There is a negative constant in the expression for the net profit per project (eq. (4) times n), indicating that the VCs cannot fully pass through increases in the marginal cost of funds. The inflexibility of pricing undermines the profitability of VCs and leads to a barrier to entry.

It is this inflexibility created by the asymmetry of information, together with the heterogeneity of entrepreneurs and product differentiation, that creates a condition sufficient for ruling out a perfectly competitive market for financial intermediation. The heterogeneity of entrepreneurs is important, because the value of (5) approaches infinity when V_H approaches V_L. The asymmetry of information matters, because if the VCs were able to identify the type of an entrepreneur and design the contracts accordingly (i.e., a separate contract for each type), our proposition would *not* hold even when $V_H > V_L > 0$. Finally, product differentiation is important, because in the absence of it, the model would collapse to a standard Bertrand game with homogenous products and the equilibrium number of VCs would be undetermined between two and infinity. Indeed, without product differentiation, all entrepreneurs would have the same elasticity of demand. In addition, even in the presence of product differentiation, we must use the standard assumption that the transportation cost is paid in advance.

Clearly, we need not resort to learning by lending or weak property rights, as has been done in the previous papers, to provide an explanation for concentration in the financial intermediation industry. Nor are exogenous fixed costs needed for the explanation.

Substituting (5) into (3a) and (3b) gives the profit share request and level of advice in the long run equilibrium. They are

$$s = \frac{\sqrt{2c}}{\lambda V_H + (1-\lambda)V_L} \quad (6a)$$

$$a = \sqrt{\frac{2}{c}} \quad (6b)$$

respectively.

Corollary: *In the long-term the profit share of VCs is inversely (directly) related to the average quality of ventures (cost of advice). Only the marginal benefits and costs of advice determine the level of advice.*

This result demonstrates that in the long-term equilibrium of this model, the level of advice is not related to the average quality of entrepreneurs in the economy. Rather, it is incorporated directly into the equilibrium profit share request.

6 Conclusions

In this paper, we have developed a rationale for concentration in the market for intermediated finance. Under asymmetric information, the heterogeneity of entrepreneurs creates an endogenous barrier to entry, as the profit share requests of venture capital firms cannot fully adjust to reflect the average quality of ventures applying for a capital injection. Therefore, the organizational structure of the venture capital industry remains oligopolistic even with free entry.

As we noted in the introduction, this result is not specific to the VC construction of our model. Indeed, it is possible to replicate the result in a model of bank lending, where banks decide loan terms (loan size and interest rate). Although loan contracts typically have non-state-specific interest rates, the repayment of a loan depends on a borrower's probability of default. Therefore, our result can be shown to hold in a model of asymmetric information and borrower heterogeneity.

Our result is also robust to the introduction of ex ante screening; something that both banks and VCs are thought to practice. Together with previous results indicating the same through different mechanisms, our paper may provide a partial answer to the widely observed empirical fact of relatively concentrated market structures in financial intermediation.

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Appendix 1. Second-Order Conditions

In this appendix, we briefly consider the second-order sufficient conditions for the VCs maximization problem (cf. equation (2)).

Differentiating the first-order conditions $\partial\pi/\partial s=0$ and $\partial\pi/\partial a=0$ gives:

$$\frac{\partial^2 \pi_i}{(\partial s_i)^2} \equiv \pi_{ss} = \frac{-2a_i a_i^e}{\tau}\overline{V} \tag{A1}$$

$$\frac{\partial^2 \pi_i}{(\partial a_i)^2} \equiv \pi_{aa} = -c\left(D_i(H) + D_i(L)\right) \tag{A2}$$

$$\frac{\partial^2 \pi_i}{\partial s_i \partial a_i} = \frac{\partial^2 \pi_i}{\partial a_i \partial s_i} \equiv \pi_{as} = \frac{a_i a_i^e}{\tau} c\overline{V} + V_H D_i(H) + V_L D_i(L) \tag{A3}$$

The values of *a* and *s* solving the first-order conditions are a strict local maximum of the objective function provided that the Hessian of the problem is negative definite. By (A1), the first leading principal minor of the Hessian is negative. The remaining condition to be checked is that the second leading principal minor of the Hessian is positive:

$$\pi_{ss}\pi_{aa} - \pi_{as}\pi_{sa} > 0 \tag{A4}$$

In the long run equilibrium with free entry, this condition reduces to:

$$\left(\overline{V} + Z\right)^2 - 2Z < 0 \tag{A5}$$

where $Z \equiv (1-\lambda)\lambda(V_H - V_L)^2$. The left-hand side of (A5) is a rather complicated non-linear polynomial function of λ, V_H, and V_L. The most straightforward way to show that condition (A5) is satisfied for a range of parameter values is to resort to a numerical exercise. To this end, let $V_H = 1.4$ and $V_L = 0$. When evaluated at these values, the left-hand side of (A5) is increasing in λ. Thus, there is a critical point for λ below which the condition is satisfied. For example, if $\lambda = 0.1$, we get –0.11 for the left-hand side of (A5). To get an understanding of the magnitude of these numerical values, it is useful to note that if, e.g., $c = 0.0001$ and $\tau = 100$, they imply that $s = 10\%$ and $\hat{n} = 5.55$. Finally, increasing for example V_L to 0.05 and keeping the other values unchanged (including $\lambda = 0.1$) would give –0.08 for the left-hand side of (A5), and $s = 8\%$ and $\hat{n} = 10.43$.

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