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OPTIMAL FOREST POLICIES IN AN OVERLAPPING GENERATIONS ECONOMY WITH TIMBER AND MONEY BEQUESTS****

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ABSTRACT: This paper studies optimal forest policies in an overlapping generations forest economy with one-sided altruism, where timber and monetary bequests can be made across generations and forest amenities are a public good. We extend the existing economics literature by demonstrating that timber bequests can dominate money bequests in many situations, and that timber bequests may even be operative in dynamically inefficient steady states. We also characterize the optimal policy mix, which maximizes long run social welfare. This policy mix depends on the nature of the steady state equilibrium, and whether non-landowners have full access to forest stocks to enjoy amenities. The results support the argument that bequests and forest stocks must be considered jointly in determining first best and second best policies. Although these are often analyzed in isolation, we show that combinations of taxes and subsidies on harvesting and bequests may be efficient in a range of equilibria. The optimal policy mix we characterize in several types of steady states is not consistent with the choice governments typically make in practice.

Keywords: overlapping generations, timber bequest, altruism, forest taxation.

JEL classification: D64, Q23, H21, H23

1. INTRODUCTION

Forest markets and forest stocks are linked through time due to landowners whose generations overlap. The traditional approach to long run forestry is the well-known Faustmann model and its variants. Although these are based on series of infinite rotations, they do not make explicit the preferences of generations over time for intergenerational transfers. But these issues are important to forests. Forest stocks are transferred from generation to generation either by timber bequests or by sales. In an economy with strong altruistic bequest motives, the time path of forest stocks will depend on bequests made from current generations to the next. The form of timber bequests is unlike other transferable forms of capital, like money, because forest stocks can jointly provide future harvest revenues and amenity services. Thus, standing forest stock serves a dual purpose when bequeathed, allowing for both private consumption and public goods.

Previous (non-resource) analyses of overlapping generations economies with altruistic bequest motives have established that, for an economy at the golden rule (defined as the equality between real interest rates and population growth), a market solution corresponds to the socially optimal one (see e.g. Blanchard and Fisher 1988). Moreover, despite one-sided altruism from parents to children, money bequests from the current generation to the next one are never operative in dynamically inefficient economies, i.e., an economy not satisfying the golden rule condition (see e.g. Weil 1987). However, these approaches assume that bequests always take the form of money. Unlike timber, money serves only a single purpose, consumption. Given the dual purpose of forests, one important question to ask beyond the economics literature is whether an overlapping generations forest economy can ever produce operative timber bequests with one-sided altruism.

Another important question concerns government policy design. If a forest economy does not reside at the golden rule, there is scope for government intervention to shift the economy towards it, thereby achieving the first best level of forest capital and public goods. Usually, governments have at their disposal a variety of instruments for this purpose, like taxes and subsidies, which are used throughout the world to promote long term use of forests. Governments also face constraints that prevent implementing policies to attain the first best equilibrium. By and large the most important constraint is revenue pressure, resulting from the fact that most governments fund activities through revenue collections. The most common policy mix used to collect revenues in the forest sector is a tax on both harvesting and bequests.

There is one distinguishing feature of forest policy, important to government intervention, that is not addressed in the economics literature. Within each generation (both old and young) there are individuals who do not own forests but still can obtain amenity services from them. The nature of these amenities depends on access to the resource. If these citizens, who are not landowners, do <u>not</u> have access to private forests, then any value to the unharvested stock is strictly a private good. Conversely, if non-landowner citizens have free access to private forests, then amenity services represent pure public goods. Whether or not amenities are private or public should influence government policy choice, if governments indeed care about these amenities.

There is some work that focuses on policy choice problems in economies with forests and, specifically, with bequests in the form of physical timber stocks. A few papers address timber bequest motives in overlapping generations economies (Amacher et. al 1999, Lofgren 1991, Hultkrantz 1992). However, this work has not addressed the choice between timber or money bequests, and more importantly, it has not addressed the realistic case where resources can be transferred across generations as either timber or money. More importantly, it has not addressed the design of policies to achieve efficient long run forest stocks, and amenities of both a private and public nature have not been considered. Recent work that examines tax policies derived in the presence of government revenue pressures has not allowed for linkages across generations through bequests (i.e., Amacher and Brazee 1997, Koskela and Ollikainen 1997). Thus, this body of work is not useful for understanding long run efficiency or defining policies used to achieve efficiency in forest sectors.

Our purpose in this paper is to study optimal forest policy instruments when timber bequests are operative and governments face revenue pressures. Given the importance of amenity services for forest stocks, we allow for the possibility that citizens may have access to private forests to enjoy amenities, so that harvesting imposes a negative externality upon them. We also examine policy choices for two types of governments, one that tries only to achieve a certain revenue target, and one that is rent seeking and attempts to maximize revenue collections. The ultimate goal is to determine whether the common practice of taxing both timber harvests and bequests is efficient in the long run. We show that, depending on the nature of equilibria and amenity services, the optimal policy consists of a mix of taxes and subsidies when timber bequests are present.

Our results provide answers to three questions vital in the current forest policy debate. First, although much has been argued for nondistortionary lump sum instruments, can these indeed allow the economy to achieve either first best or second best outcomes? Second, given that governments throughout much of the world tax both harvests and bequests, is this the efficient policy mix in various steady states of the forest sector; that is, does this encourage the right level of long run forest stocks? And, third, what is the most efficient policy mix for governments that, as is often the case, face constraints on behavior or preferences; a related question is how these constraints impact the realization of forest stocks and public goods for future generations? The answers to these questions will be important to the ongoing debate about using government intervention to achieve long run sustainability of forest stocks and public goods in economies with forests.

We proceed as follows. In section 2 we construct a basic overlapping model of a representative agent where both money and timber can be transferred across generations. This model is then used to examine government policy choice problems in Section 3 for both cases where amenity services provided by forests are private goods only, and where amenity services are also public goods. We also examine incentives for the government to choose policies when it seeks to maximize revenue collections. Finally, there is a brief concluding section.

2. FOREST AND MONEY BEQUESTS IN AN OVERLAPPING GENERATIONS ECONOMY

2.1 Basic Set-Up

Consider an overlapping generations forest economy where individual landowners live two periods. In period t the landowner is a member of the first generation and in period t+1 s/he is a member of the second generation. Landowners maximize the following utility function,

$$U_t = u(c_{1t}) + \beta u(c_{2t}) + v(k_{1t}) + \beta v(k_{2t}) + \alpha V_{t+1}, \tag{1}$$

which is additively separable and strictly concave in its arguments, which are current and future consumption c_{1t} and c_{2t}, and current and future flow of amenity services from standing timber stocks k_{1t} and k_{2t}, valued through the *v(.)* function, for the landowner born at time t. The time preference for personal utility is β. V_{t+1} is the indirect utility of the representative landowner in the next generation, based on the corresponding utility function U_{t+1}, and α is an intergenerational discount factor which measures the strength of the bequest motive, $0 < \alpha < 1$. The utility function assumes one-sided altruism, that is, the current generation landowner can bequest to future generations, but "young" landowners do not bequest to old landowners.[1] Equation (1) is similar to a Bellman equation given the definition of the indirect utility function of the succeeding generation.

The landowner can harvest each period and obtain consumption while they are young or old. They can save while young and obtain higher future consumption at the market interest rate, and when they are old they may leave a timber or money bequest to the next generation. With this in mind, consumption, harvesting, and the forest stock are defined through the following equations of motion over two periods of life for the representative landowner in each generation.

$$c_{1t} = w_t - T + p_t^* x_{1t} + (1-\tau_m)m_t - \tau_b p_t b_t - s_t \tag{2}$$

$$c_{2t} = p_{t+1}^* x_{2t} + R_{t+1}s_t - (1+n)m_{t+1} \tag{3}$$

$$x_{2t} = (b_t - x_{1t}) + g(b_t - x_{1t}) - b_{t+1}(1+n) \tag{4}$$

$$k_{1t} = b_t - x_{1t} \tag{5}$$

$$k_{2t} = (1+n)b_{t+1} \tag{6}$$

[1] The specification of the utility function in (1) ensures that if one of the bequest motives is operative, the overlapping generations problem we present here is identical to an infinite time horizon utility maximization problem (see e.g. Blanchard and Fischer 1988, pp. 104-107). By and large, one-sided altruism is the more common case, especially with regards to forests.

where r_{t+1} is the market interest rate in time period t+1 and $R_{t+1} = 1+r_{t+1}$. Timber bequests made from the previous generation (who are old at time t) are represented by b_t, while the bequest of unharvested timber made from the representative landowner (who is old at time t+1) to the landowner born at time t+1 is given by b_{t+1}. Similarly, monetary bequests derived from selling land or timber are defined using m_t and m_{t+1} in (2) and (3), and n is the population growth rate. We follow the economics literature and assume that the landowner works only when young, receiving wage income from inelastically supplying one unit of labor, and retires when old as a member of the second generation. We also assume for convenience that the representative landowner has a rate of time preference equal to the market interest rate.[2]

Policy instruments exist in the common forms and include a lump sum transfer targeting income, T, distortionary instruments targeting money and timber bequests, τ_m and τ_b respectively, and a distortionary instrument targeting forest harvesting, τ_x. We do not restrict the signs of these policies; a nonnegative instrument represents a tax and a nonpositive instrument represents a subsidy. All policies are assumed to be constant over time.[3] Prices for timber harvested are given by p_t and p_{t+1} in periods t and $t+1$. In (2) and (3) we make use of net prices, $p_t^* = (1-\tau_x)p_t$ and $p_{t+1}^* = (1-\tau_x)p_{t+1}$. x_{1t} and x_{2t} are rates of harvesting in the first and second periods for the current generation landowner, respectively. Unharvested forest stock increases between periods according to a concave growth function, $g(.)$ in (4).

Finally, equations (5) and (6) define the unharvested forest stocks, which provide nontimber amenities described by the strictly concave valuation functions $v(k_{1t})$ and $v(k_{2t})$. These are equivalent to the state equations governing changes in the forest stock over time for each representative generation. Note that there is a timing assumption implicit in the presentation of these equations of motion. All activities in equations (5) and (6) are assumed to take place in the beginning of the periods. Thus, in the beginning of the first period, the landowner harvests from a stock, b_t, that was bequeathed to him/her from the last generation (eqn 5). The landowner lets the remaining stock, defined by what is harvested, grow into the second period, i.e., $k_{1t} = b_t - x_{1t}$. In the beginning of the second period, the landowner decides how much to harvest in that period and how much to give as a bequest to heirs. Hence, his ending stock (at then end of the landowner's life) is equal to $(1+n)b_{t+1}$ (this can be shown by substituting equation (4) into (6)).[4]

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[2] The assumption of an exogenous interest rate and landowners who are interest rate-takers makes sense in practice. Typically, the value of forest capital is comparatively small relative to the total value of capital in other sectors of the economy.

[3] This is used for notational simplicity. Since we focus later on the steady state equilibrium, where all time-indexed parameters are constant, it would not matter how tax policies are presented in the model. In fact, the assumption of constant policies is probably more the rule than the exception in practice. Governments would face great resistance if they attempted to adjust policies regularly, either because of administrative costs or the political process which determines tax rates in most countries. Thus, the assumption of constant policies is probably a reasonable one.

[4] Naturally, the next generation harvests a part of k_{2t}; this harvest is defined as x_{1t+1}. Therefore, the forest stock that exists at the end of the second period is actually, $k_{2t+1} = (1+n)(b_{t+1} - x_{1t+1})$. In our dynasty model the current generation takes into account the harvesting by the young in the steady state, so that we can concentrate on that generations' decisions only.

2.2 Operative Bequest Motives

The representative landowner chooses saving, harvesting, money and timber bequests so as to maximize (1) subject to (2) – (6). Assuming that saving and harvesting are at an interior solution yields the following first-order conditions[5]

$$U_s^i = -u'(c_{1t}) + R\beta u'(c_{2t}) = 0 \tag{7a}$$

$$U_{x_{1t}}^i = u'(c_{1t})p_t^* - \beta u'(c_{2t})(1+g')p_{t+1}^* - v'(k_{1t}) = 0 \tag{7b}$$

Equation (7a) is the optimal savings condition, which states that landowners save so as to equate marginal utilities of consumption across periods. Equation (7b) is the optimal harvest condition for period t. Using equation (7a) this can be re-expressed as the following harvesting rule: $Rp_t^* - (1+g')p_{t+1}^* = \dfrac{v'(k_{1t})}{\beta u'(c_{2t})}$. Hence, harvesting in period t is undertaken up to the point where the difference between the marginal revenue and opportunity costs of harvesting equals the marginal rate of substitution between amenities and consumption.

As for the conditions for money and timber bequests we obtain respectively

$$U_{m_{t+1}}^i = -(1+n)\beta u'(c_{2t}) + \alpha u'(c_{1t+1})(1-\tau_m) \le 0 \qquad (=0 \; if \;\; m_{t+1} > 0)\,. \tag{7c}$$

$$U_{b_{t+1}}^i = -\beta u'(c_{2t})p_{t+1}^*(1+n) + \beta v'(k_{2t})(1+n) \tag{7d}$$

$$+\alpha\left[u'(c_{1t+1})\tau_b p_{t+1}^* + \beta u'(c_{2t+2})(1+g')p_{t+2}^* + v'(k_{1t+1})\right] \le 0 \;\; (=0 \; if \;\; b_{t+1} > 0)$$

Equation (7c) describes the landowner’s optimal money bequest. The landowner leaves this bequest so that the marginal benefit, in terms of future consumption for the next generation, equals the marginal cost, in terms of forgone consumption for the landowner. Equation (7d) describes the optimal timber bequest left by a landowner who is a member of the second generation in period t+1. For this landowner, the marginal costs of leaving a bequest equal lost consumption, and this is equated to marginal benefits in terms of amenities to the landowner and future consumption and amenities accruing to future generations which inherit the bequest.

We follow the usual practice from the literature in this field and assume that the first-order conditions define a unique steady state equilibrium; we then can examine the conditions for operative (i.e., positive) bequests.

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[5] In what follows the derivatives are noted by primes for functions with one argument and the partial derivative by subscripts for functions with many arguments. Hence, e.g. $f'(T) = \frac{\partial f(T)}{\partial T}$ for $f(T)$, while $A_x(x,y) = \frac{\partial A(x,y)}{\partial x}$ for $A(x,y)$, etc.

Bequest Motives in the Absence of Policies

The condition for operative *money* bequests, derived from the first-order condition (7c) without taxes, is,

$$m_{t+1} > 0 \Leftrightarrow \alpha > \frac{(1+n)}{(1+r)} \qquad (8)$$

where $0 < \alpha < 1$ (see Appendix 1 for the derivation of equations (8) and (9)). Equation (8) implies money bequests are zero when the steady state is dynamically inefficient, i.e., when $r < n$.[6]

The condition for operative steady state *timber* stock bequests without taxes equals,

$$b_{t+1} > 0 \Leftrightarrow \alpha > \frac{(1+n)[1-\dfrac{v'(\hat{k}_2)}{pu'(\hat{c}_2)}]}{(1+r)A}, \qquad (9)$$

where $A = 1 - R^{-1}(1+g') + (1+g')$, and the symbol '^' refers to the steady state value of a variable in the absence of money and timber bequests. According to (9), the propensity for a landowner to leave a bequest in the steady state depends on the interest rate, r, population growth, forest growth embodied in 'A', and the marginal rate of substitution between amenities and consumption benefits, v'(.)/u'(.).

In order to compare the condition for operative timber bequests with the one for money bequests, we consider now how (8) compares with (9). Using the natural assumption that the rate of time preference equals the interest rate, $R\beta = 1$, we can show that consumption occurs over time to equalize marginal utilities, $u'(\hat{c}_1) = u'(\hat{c}_2)$ (see 7a). Using this the timber bequest co $- R^{-1}(1+n)\left[u'(\hat{c}_2)p - v'(\hat{k}_2)\right] + \alpha\left[u'(\hat{c}_2)(1+g')p + v'(\hat{k}_1)\right] = 0$ ndition can be rewritten as, so that $0 < [1 - v'(\hat{k}_2)/pu'(\hat{c}_2)] < 1$. We can further simplify the term A in the RHS denominator of (9) using the interior solution for harvesting (7b), $u'(c_2)p\left[1 - R^{-1}(1+g')\right] = v'(k_1)$, which implies that $A = 1 - R^{-1}(1+g') + (1+g') > (1+g')$. Hence the additional term in (9) relative to (8) is less than one, i.e.,

$$\frac{[1-\dfrac{v'(\hat{k}_2)}{pu'(\hat{c}_2)}]}{1 - R^{-1}(1+g') + (1+g')} < 1. \qquad (10)$$

Therefore, comparing (8) and (9) reveals that the timber bequest condition is more easily satisfied than the money bequest condition, in the absence of policies, when there are non-timber amenities that accrue to landowners holding forest stocks represented by the second numerator term in (10). Intuitively, if there is no penalty for bequests and amenities are present, then there is less incentive for the landowner to convert timber to money (through harvesting) prior to leaving a bequest. This comes from the fact that forests are a special

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6 See Weil (1987) for a proof of a similar condition in the non-forestry case. For further an analysis of the implications of two-sided altruism when parents leave bequests and children give gifts for their parents, see Kimball (1987).

form of capital. Besides harvestable timber, forests provide amenity services, and these represent an additional benefit of giving and receiving timber bequests.

Given equation (10), it is also evident from (9) that the condition for operative timber bequests may be satisfied even in a dynamically efficient economy where r > n. Most importantly, because of (10) this may also hold for the dynamically inefficient case, where $r < n$ (e.g., see Romer 2001, pp. 85-89 for a discussion of dynamic efficiency in overlapping generations models).

Finally, we can use the Kuhn-Tucker conditions to determine when one bequest *dominates* another. The result to highlight is the case where money bequests are optimally zero but timber bequests are strictly positive in the steady state, i.e.,

$$\frac{(1+n)}{(1+r)} > \alpha > \frac{(1+n)}{(1+r)A}(1-\frac{v'(k_2)}{pu'(c)})$$

This equation shows again the importance of amenities. If amenities accrue to standing forest stock, then the landowner is more likely to bequeath this stock, rather than harvesting and bequeathing money, to future generations.[7] Thus we have established

<u>Proposition 1.</u> *When the rate of time preference is equal to the interest rate, timber bequests may dominate money bequests in the absence of policies when amenities to the standing forest stock exist. While money bequests are not operative for a dynamically inefficient economy, timber bequests may still be.*

Proposition 1 is a new result in the overlapping generations literature on bequests, and it has implications here for a government wishing to achieve the first best outcome. The economics literature demonstrates that money bequests are operative only in dynamically efficient economies, while money gifts are operative in dynamically inefficient economies. We have shown that timber bequests, however, can be operative and dominant in many equilibria, as long as there is one-sided altruism where parents transfer resources to children. This is due to the fact that, besides timber, forests provide amenities for both givers and receives of timber bequests. This results in a smaller net loss in marginal utility and a higher utility of receiving timber bequests compared to money bequests.

Bequests Motives in the Presence of Policies

Proposition 1 was derived assuming an absence of any policies. Next we examine bequest motives in the presence of policies. We will show that the threshold level of altruism depends on the tax structure, and "excessive" taxation may lead to elimination of bequest-motivated transfers.

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[7] If the timber price (and hence the price of forestland) reflects fully both timber and forest amenity values, then selling forest land and giving this as a money bequest would be equivalent to giving timber bequests to the next generation. Forest industry is, however, predominantly interested in timber services as an input to production. Consequently, in this case timber price may not reflect amenity values, and the equivalence between money and timber bequests cannot be presumed to hold.

The operative timber bequest condition in the presence of taxes can be expressed as

$$b_{t+1} > 0 \Leftrightarrow \alpha > \frac{(1+n)[1-\tau_x - \dfrac{v'(\hat{k}_2)}{pu'(\hat{c}_2)}]}{(1+r)(A(1-\tau_x)-\tau_b)} \qquad (9')$$

Using the same procedure as above, we can show that $0 < [1-\tau_x - \dfrac{v'(\hat{k}_2)}{pu'(\hat{c}_2)}] < 1$ must hold at an interior solution. The condition for operative money bequests, derived from first-order condition (7c), is

$$m_{t+1} > 0 \;\Leftrightarrow\; \frac{(1+n)}{(1+r)(1-\tau_m)} \qquad (8')$$

Therefore, if it is also true that $A(1-\tau_x)-\tau_b \geq 1-\tau_m$, then we have a sufficient condition for the timber bequest motive to be reinforced, relative to the money bequest motive, in the presence of policies (compare (9') with (8')).

An equivalent condition for $A(1-\tau_x)-\tau_b \geq 1-\tau_m$ to hold is

$$A = 1 + \frac{r(1+g')}{1+r} \geq \frac{1-\tau_m+\tau_b}{(1-\tau_x)}, \qquad (10''')$$

where $r > g'$ at the interior solution for harvesting. This shows that if $\tau_m \leq \tau_b$, then the timber bequest necessary condition is more easily satisfied than the money bequest necessary condition when there is a *subsidy* for timber bequests. Thus, a harvest tax being equal or higher than the money bequest tax, and a bequest subsidy, reinforce conditions for timber bequests. Consider, however, when there is a *tax* on timber and money bequests and a *tax* on harvesting, i.e. $\tau_b, \tau_m, \tau_x > 0$. In this case, we cannot say formally which type of bequest is reinforced, because it depends on the relative size of bequest and harvest taxes. Hence we have

<u>Corollary 1</u>: *When the rate of time preference is equal to the market interest rate, a timber bequest subsidy reinforces the condition for operative timber bequests in the presence of harvest and money bequest taxes, provided that the money bequest tax does not exceed the harvest tax. If all activities are taxed, it is unclear whether timber bequests are reinforced relative to money bequests.* [8]

By and large, the policy mix most often used by governments in North America and Europe to raise revenues is a tax on <u>both</u> bequests and harvests. But the results here argue that such a mix may not be efficient. We will return to this issue in Section 3.

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[8] In a different context Caballe (1998) has also shown that the threshold level of altruism depends on the tax structure.

3. THE DESIGN OF POLICY INSTRUMENTS

We now turn to the question of how the government should structure harvest and timber bequest policies to raise revenues and maximize social welfare. We do this under the relevant case that timber bequests are operative, i.e., allowing us to abstract from monetary bequests. As a precursor, in Appendix 2 we show how the effects of taxes on timber bequests and harvesting can be decomposed into income and substitution effects using the Slutsky equations. From now on we will refer to the compensated substitution effects of timber and bequest taxes on harvesting and bequests as $x^c_{\tau_x}$, $x^c_{\tau_b}$, $b^c_{\tau_b}$, and $b^c_{\tau_x}$ respectively. As we demonstrate in the Appendix 2, $x^c_{\tau_x}$, $x^c_{\tau_b}$, $b^c_{\tau_b}$ are all negative, while $b^c_{\tau_x}$ remains ambiguous a priori. The only thing, however, that matters from the viewpoint of optimal tax design is the "own effect" of taxes, i.e., $x^c_{\tau_x}$ and $b^c_{\tau_b}$. These have a natural interpretation: a compensated rise in the tax on harvesting or on timber bequests will decrease these activities ($b^c_{\tau_b} < 0$; $x^c_{\tau_x} < 0$).

We also assume that the government, when designing its policy, faces a binding revenue target, as is common. Moreover, if there is a free access to private forests and amenities are associated with the forest stock, then forest preservation and transfers across generations represent public goods that must be taken into account by the government when choosing policies. In the next two sections, we show formally how the choice of policies depends critically on the position of the economy relative to the golden rule steady state. The golden rule steady state is the one where consumption is maximized (See Blanchard and Fisher 1989, pp. 104-107). Other possible steady states are defined based on over or under-accumulation of capital relative to the golden rule.

3.1 Optimal Policy Mix when Private Amenity Services are Private Goods

We start by examining the benevolent government's choice of policy instruments. We will first assume that amenity services are private goods, i.e., that citizens (presumably recreators) do not have access to private forests and do not enjoy the corresponding amenity services.

Let the government's exogenous revenue target be denoted by *G*, which must be met in the steady state. Given that *n* is the growth rate of the population, the steady state revenue target can be derived by adding tax revenue collections over the two overlapping generations that are alive at any point in time.

$$G = T + \tau_b pb + \tau_x \frac{(2+n)}{(1+n)} px, \tag{11}$$

where x and *b* denote steady state levels of harvesting and timber bequests, respectively. Equation (11) is written in per capita form. The second term represents bequest tax revenue collections. The third RHS term represents harvest tax revenue collections from both generations. This term is multiplied by (2+n)/(1+n) to account for population growth that occurs between the old and young generation. Thus, at each point in time, the last term on the RHS of (11) reflects that fact that members of both the second generation and the first generation alive at time t have positive harvests, but only the old generation makes the timber bequest at that point in time. Recall this is consistent with one-sided altruism.

The optimal policy choice for this type of problem is obtained assuming the government credibly commits to policies and behaves as a Stackelberg leader, with the private sector following by choosing harvesting and bequest activity. The responses of private individuals to the government's policy choices are defined through the Slutsky decompositions referenced earlier (see Appendix 2). The government therefore chooses policies to maximize a welfare function given by the indirect utility of overlapping generations $V^*(T,\tau_x,\tau_b,....)$, subject to its revenue requirement (11) and the optimal behavior of the representative landowner in each generation,[9]

$$\underset{\{T,\tau_x,\tau_b\}}{Max} V^*(T,\tau_x,\tau_b,....) \qquad \text{s.t. (11).} \qquad (12)$$

The Lagrangian for the problem is $\Omega = V^* - \lambda[G - \tau_b pb - T - \tau_x \frac{(2+n)}{(1+n)} px]$, where λ is the shadow price of government budget constraint, i.e., the marginal cost of public funds.

The first order condition for the optimal lump sum tax, T is,

$$\Omega_T = V_T^* + \lambda[1 + \tau_b pb_T + \tau_x \frac{(2+n)}{(1+n)} px_T] = 0, \qquad (13a)$$

where $V_T^* = -(u'(c_{1t}) + \alpha u'(c_{2t+1})) < 0$. Equation (13a) implies that the government sets the lump sum tax rate so that the marginal welfare loss (first RHS term) equals the marginal cost of collecting revenue from the tax (second RHS bracketed term). Obviously, the revenue collection effect depends on population growth rates, timber prices across generations, as well as the response of timber bequests and harvesting to the lump sum tax (b_T and x_T). The remaining first-order conditions for the bequest and harvest policies are, respectively,

$$\Omega_{\tau_b} = V_{\tau_b}^* + \lambda[pb + \tau_b pb_{\tau_b} + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_b}] = 0, \qquad (13b)$$

$$\Omega_{\tau_x} = V_{\tau_x}^* + \lambda[\tau_b pb_{\tau_x} + \frac{(2+n)}{(1+n)} px + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_x}] = 0, \qquad (13c)$$

where, $V_{\tau_b}^* = pbV_T^* < 0$, and $V_{\tau_x}^* = (1 + R^{-1})pxV_T^* < 0$. These conditions imply, again, that the government will choose policies to balance marginal changes in revenue collections, brought about by policy-induced changes in harvesting and bequests (terms multiplying λ) with marginal welfare changes defined through the indirect utility function.

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[9] See e.g. Ihori 1996 for a discussion of this basic policy problem.

Assuming that T is set at the optimal level, denoted T^*, we can rewrite the remaining first order conditions in matrix form by rearranging and substituting using the Slutsky equations for x_{τ_x}, x_{τ_b}, b_{τ_b}, and b_{τ_x} (see Appendix 3 for the details),

$$\begin{pmatrix} b^c_{\tau_b} & \frac{(2+n)}{(1+n)} x^c_{\tau_b} \\ b^c_{\tau_x} & \frac{(2+n)}{(1+n)} x^c_{\tau_x} \end{pmatrix} \begin{pmatrix} \tau_b \\ \tau_x \end{pmatrix} = \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix}, \qquad (14)$$

where $\varepsilon = -(1/(1+n) - 1/(1+r))x$. Using (14), we can now obtain expressions for the optimal harvest and timber bequest policies by solving the problem in (14),

$$\tau^*_{x|T=T^*} = \frac{\frac{(1+n)}{(2+n)} b^c_{\tau_b} \varepsilon}{[x^c_{\tau_x} b^c_{\tau_b} - x^c_{\tau_b} b^c_{\tau_x}]} \qquad (15a)$$

$$\tau^*_{b|T=T^*} = -\frac{x^c_{\tau_b} \varepsilon}{[x^c_{\tau_x} b^c_{\tau_b} - x^c_{\tau_b} b^c_{\tau_x}]} \qquad (15b)$$

It can be shown that the denominators of the equations (15a-15b) are positive due to properties of the Slutsky equations for this problem.

Interpreting these results requires that we consider three cases; one where the economy resides at the golden rule and consumption per capita is maximized (r = n); one where the economy resides in a dynamically inefficient steady state (r<n) and there is an over-investment of capital; and one where the economy resides in a dynamically efficient steady state (r>n) and there is an under-investment of capital. [10] In addition, we will make use of Proposition 1 concerning operative bequests.

The Golden Rule

The golden rule level of forest capital (and hence bequests) occurs in the steady state only if the interest rate equals the growth rate of the population, i.e., when $r = n$. In this case, the economy is termed "dynamically efficient," and therefore ε in (15a) and (15b) equal zero (recall $\varepsilon = -(1/(1+n) - 1/(1+r))x$). This implies that τ_b^* and τ_x^* must equal zero. Intuitively, if the economy is already residing at the golden rule level of capital and consumption, the government should use only nondistortionary lump sum taxes to raise revenues. Harvesting and bequest policies are inefficient, as they would serve only to move long run forest capital away from its efficient level through distortions in bequests.

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[10] The distinction of dynamic efficiency and inefficiency simply relates to whether the economy is growing faster or slower than the rate of interest. The golden rule, using the terminology of Phelps (1961), is where these rates equate. This gives the highest rate of consumption and capital stock (see also Blanchard and Fisher 1989, pp. 102-103).

Dynamic Inefficiency

When the economy is dynamically inefficient and timber bequests are operative (see Proposition 1), we have $r < n$ so that $\varepsilon > 0$. Now, notice from (15a) and (15b) that the optimal policy mix will require nonzero bequest and harvest policies. Specifically, using the results from Appendix 2 that compensated substitution effects are negative, $x^c_{\tau_b} < 0$ and $b^c_{\tau_b} < 0$, and recalling that the numerator of (15a)-(15b) is always positive, it is clear that the government should issue a *subsidy* for forest harvesting $\tau_x < 0$ combined with a *tax* on forest bequests, $\tau_b > 0$. Intuitively, when the economy is inefficient there is an overinvestment of capital in the long run. In the forestry case, this implies the steady state forest stock is too high. One way for the government to correct such an inefficiency is to subsidize the reduction of forest capital. Revenue shortfalls can be satisfied by taxing timber bequests. This dual purpose is achieved with the tax-subsidy mix. According to Corollary 1 this combination, however, makes an operative timber bequest less likely.

Dynamic Efficiency

Finally, for the case of dynamic efficiency ($r > n$), $\varepsilon < 0$, the optimal tax-subsidy mix is reversed. By taxing harvesting and subsidizing timber bequests the government can move the economy closer to the golden rule according to (15a) and (15b). Further, by Corollary 1, this instrument mix also reinforces the condition for operative timber bequests, making them more likely. Because the steady state forest stock is too low, harvest taxes and bequest subsidies encourage intergenerational transfers and long run investments in forests. Summarizing and assuming that the timber bequest is operative in the dynamically inefficient economy, we have

Proposition 2. (Amenity services are private goods)

If the economy is at the golden rule (r=n), the lump sum tax is sufficient to collect revenues and distortionary instruments should not be used; if the economy is dynamically inefficient (r<n), the optimal policy mix includes also a subsidy on harvesting and a tax on timber bequests; if the economy is dynamically efficient (r>n), the optimal policy mix includes a subsidy on timber bequests and a tax on harvesting.

Proposition 2 represents a departure from the general economics literature, where it has been argued that capital taxes must always be optimally equal to zero in the steady state of overlapping generations economies.[11] But our results establish that (timber) capital taxes may indeed be optimal for the forestry case. The proposition has practical importance as well. For forest policy design, it is critical to know where the economy resides relative to the golden rule, as the signs of the bequest and harvest taxes will reverse if the economy moves from a dynamically inefficient to an efficient equilibrium. Hence, attaining a social welfare optimum in a forest economy may require either to subsidizing or taxing bequests or taxing or subsidizing timber harvests. Generally, though, some type of subsidy will always be optimal.

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[11] See Atheson, Chari and Kehoe (1999), who have both summarized and extended the previous literature to argue that in the case without renewable resources the optimality of zero capital income taxation is a remarkably robust result.

Let us consider, finally, what happens if the timber bequest is not operative and the economy is dynamically inefficient (see Corrollary 1). It is straightforward to establish that, in addition to the lump sum tax, the government should now levy a subsidy on harvesting (see Appendix 3, eqns A3.2a and A3.2c and A3.2b). Hence, we have

Corollary 2. *If the timber bequest is inoperative in the dynamically inefficient economy, the optimal tax policy is a combination of a lump sum tax and a subsidy on harvesting.*

The intuition behind Corollary 2 follows along the same lines as above. The government has no need to subsidize timber bequests because they are no longer operative.

3.2 Optimal Policy Mix when Amenity Services are Public Goods

We now turn to the case where amenity services from unharvested timber stocks are not only a private good, but also a public good in the sense that non-landowner citizens (i.e., recreators) have access to enter private forestlands and enjoy amenities from forest stocks. These amenities could include those obtained through direct access to private forests, or those such as non-use values unrelated to access associated with forests by non-landowners, such as airshed and watershed protection as well as habitat for various species. In either case, this is an interesting and important extension because it is most relevant in practice.

In this case the government must account for how conservation of the timber stock, through bequests, brings utility to recreators in both current and future generations. A classical Pigouvian externality exists if one generation makes decisions about harvesting and bequests not accounting for the benefit future generations and recreators receive from amenity services of forest stocks. Given that the marginal welfare effect of any policy is important to its application, it is reasonable to expect that these public goods will alter the optimal mix of policies and will change the distribution of bequests and forest stocks in the long run steady state equilibrium. We now investigate these issues.

As the government values public goods from forests, its policy problem is modified accordingly,

$$\underset{\{T,\tau_b,\tau_x\}}{Max} V^*(T,\tau_b,\tau_b)+(n+2)(q-1)[v(k_{1t})+R^{-1}v(k_{2t})+\alpha(v(k_{1t+1})+R^{-1}v(k_{2t+1}))] \quad (16)$$

s.t. (11).

where $(n+2)(q-1)$ now represents the number of non-landowner recreators in the old and young generation who have free uncongested access to public goods produced by unharvested forests (set aside by each generation).[12]

Following a procedure similar to the one we used for deriving equation (15b) and (15c), we can solve for optimal policies assuming that the lump sum tax is set at its optimal level

[12] The ‘q-1’ term reflects the fact that amenities are also present in V(.) from (1). We also assume without loss that the number of recreators increases at the same rate as the population of landowners.

(denoted by T^a). Using the Slutsky decompositions, we then arrive at an expression for the modified optimal steady state timber harvesting policy (see Appendix 4),

$$\tau^a_{x|T=T^a} = \tau^*_{x|T=T^*} + \frac{(q-1)(1+n)[v'(k_{1t}) + \alpha v'(k_{1t+1})]}{\lambda p} \tag{17}$$

where the superscript 'a' refers to the public goods case. Recall that the superscript '* ' refers to the optimal policy level corresponding to the case where amenity services represent private goods (see eqns (15a) and (15b)).

By the same procedure we can obtain the optimal steady state bequest policy (see Appendix4),

$$\tau^a_{b|T=T^a} = \tau^*_{b|T=T^*} - \frac{(n+2)(q-1)R^{-1}(1+n)[v'(k_{2t}) + \alpha v'(k_{2t+1})]}{\lambda p} \tag{18}$$

Equations (17)-(18) imply that the optimal tax choice is now composed of the policy which solves the previous government's problem (first term on each RHS), plus a new additive "Pigouvian" term (second term on each RHS).[13] The Pigouvian term represents an adjustment that reflects public goods lost through harvesting of the steady state forest stock. This term ensures that the efficient level of public goods (timber stock) is achieved in the long run and that the government also achieves its binding revenue target.[14]

Golden Rule

The presence of the Pigouvian adjustment in (17) – (18) arises because the forest economy allows for bequests in the form of unharvested forest stock. If the economy resides at the golden rule steady state (r = n), recall we have shown ε = 0. Thus, the distortional tax rates derived in the absence of public goods are zero, $\tau^*_{b|T=T^*} = 0$ and $\tau^*_{x|T=T^*} = 0$ like we showed earlier. Using this, we would now find from equations (17) and (18) that $\tau^a_b < 0$ and $\tau^a_x > 0$. In this case the government corrects the externality via the Pigouvian adjustment, but raises revenues using the lump sum tax. Thus,

<u>Proposition 3.</u> (Amenity services are public goods)
If the economy resides at the golden rule, then the lump sum tax is no longer sufficient for optimal policy. In addition to the lump sum tax, the optimal policy mix includes a subsidy on timber bequests and tax on harvesting.

In the previous case without public goods accruing to the forest stock (Proposition 2), if the economy was already at the golden rule steady state, optimal harvest and bequest policies equaled zero and all revenue was to be raised with nondistortionary lump sum taxes. However, when public goods are lost from harvesting timber, the government should employ both lump sum and distortional instruments. This result is another case where capital

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[13] The additive form of the solution is similar to what Sandmo (1975) found for optimal taxes in the presence of externalities.

[14] Of course, we speak of efficiency in the second best sense because of the exogenous revenue target. Further, the level of resulting public goods is therefore a second best one as well.

taxes, in the form of timber, are optimally positive in the long run, even though the capital stock equates with its golden rule level. In contrast to the standard economics literature, our new result occurs because overaccumulation of forest capital relative to the golden rule might actually be *efficient* if public goods are conserved from the stock.

Dynamic Efficiency

Consider the opposing case of dynamic inefficiency, where r < n. Here, there is an overinvestment of capital in the long run, so that in a forestry context the long run forest stock is too high. From Section 3 we know $\tau^*_{b}\big|_{T=T^*} > 0$ and $\tau^*_{x}\big|_{T=T^*} < 0$ when r < n and amenities are not public goods. These policies were required to decrease the forest stock, which is too high from Society's viewpoint.

The new Pigouvian components in equation (17) and (18) reverse this previous result, so that a harvest subsidy and bequest tax are efficient. The different policy mix with public goods allows the economy to adjust toward a higher level of forest stock and public goods, given that there is utility obtained from existence of preserved forest stock. Hence, when amenity services are public goods and the government responds to these, the government might behave differently and the efficient policy mix might be different than the one described in Proposition 2.

Dynamic Inefficiency

Finally, for the case where the economy is dynamically efficient in the sense that r > n, recall the optimal policy choice is a subsidy on bequests and tax on harvests, $\tau^*_{b}\big|_{T=T^*} < 0$ and $\tau^*_{x}\big|_{T=T^*} > 0$. From (17) and (18) this implies when amenities exist that the efficient mix is now a bequest subsidy and a harvest tax, $\tau^a_b < 0$ and $\tau^a_x > 0$.This serves to increase the long run forest stock and correct the equilibrium underinvestment of capital. Now, the revenue collection and externality correction aspects of the policy instruments reinforce each other. We can summarize these observations in:

Proposition 4. (Amenity services are a public good)
In the dynamically inefficient economy, the optimal policy mix for harvesting and timber bequests is a priori ambiguous, while in the dynamically efficient economy the optimal policy mix includes a subsidy on timber bequests and tax on harvesting.

3.3 Optimal Policies and Rent Seeking

The assumptions concerning government preferences towards agents in the economy and revenue collection might be important to the results above. Thus far we have assumed that the government acts as a benevolent maximizer of landowners' and citizens' welfare subject to the revenue target. In many cases, though, governments have been criticized for seeking to maximize revenue collections. Such a government in our model would choose policies to maximize its tax collections without regard to the distortions in public and private goods that would accompany such behavior. The question now becomes: how does such behavior effect the optimal long run policy mix?

A government seeking to maximize revenue has been called a Leviathan government in the public choice literature. The Leviathan government's policy problem for our problem is written,

$$\underset{\{T,\tau_b,\tau_x\}}{Max}\, G = T + \tau_b pb + \tau_x \frac{(2+n)}{(1+n)} px \tag{19}$$

We can show two results with such an objective.[15] First, the lump sum tax will be set at a higher level compared to the case where the government does not maximize revenue collections, i.e., $T^l > T^*$. This is because the government does not care about the negative welfare effect of higher taxes, given by V_T^* in (13a). Second, following a similar procedure as before, the solution to the optimal policy mix can be obtained as,

$$\tau^*_{x|T=T^l} = \frac{\frac{(1+n)}{(2+n)} b^c_{\tau_b}\varepsilon}{[x^c_{\tau_x} b^c_{\tau_b} - x^c_{\tau_b} b^c_{\tau_x}]} \tag{20a}$$

$$\tau^*_{b|T=T^l} = -\frac{x^c_{\tau_b}\varepsilon}{[x^c_{\tau_x} b^c_{\tau_b} - x^c_{\tau_b} b^c_{\tau_x}]} \tag{20b}$$

where ε is defined as before. Hence, the Leviathan government ends up with qualitatively similar policies as the benevolent government does when amenities are private goods. The only difference is that the levels of taxes and subsidies are higher.[16]

Proposition 5. *If the government is Leviathan in that it seeks to maximize revenue collections, the optimal structure of policy instruments is equivalent to the benevolent government case when amenity services are private goods, but all taxes and subsidies are higher.*

If public goods are present and the government maximizes revenues, the optimal bequest and harvest policies will continue to be given by (20a) and (20b). This is because in this case the government ignores the positive welfare effect of forest stocks on future generations. As a result, for a Leviathan government there is no longer a distinction between policy instruments that depends on amenities.[17]

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15 A full set of result is available from the authors upon request.

16 The qualifications of Proposition 5 for the case of inoperative timber bequest motives remain the same as given in Corollary 2.

17 Our analysis can be generalized for a competitive equilibrium in a small open economy, where the output is produced using two inputs, capital K_t and labor L_t, according to a constant returns to scale neo-classical production function. Capital and labor are supplied by the saving and labor decision of the representative landowner in each generation. If capital depreciates fully, the marginal product of labor is equal to wage rate w_t, and the marginal product of capital is equal to the given world interest rate $1+r_t$. Therefore, the economy's capital-labor ratio k_t and the wage rate w_t would be determined independently of domestic conditions. In this case, our results continue to hold. A proof of this is available from the authors upon request. The fact that we obtain such a result in our forest economy is consistent with what Weil (1987), Thibault (2000) and Vidal (2000) find for non-forest economies. For example, Weil shows how the conditions for operative bequests can be generalized to cover the case of production economies. Thibault (2000) shows the conditions for the existence of a nontrivial steady state equilibrium in a closed OLG economy with altruistic agents, production and capital market equilibrium, and Vidal analyzes welfare effects of capital mobility in a two-country model with altruistic preferences.

4. CONCLUSIONS

In this paper we studied the optimal design of forest policies in an overlapping generation economy with intergenerational transfers. We allow for amenity services of forest stocks to exist as either private or public goods depending on whether citizens have access to private forests or not. The government is initially assumed to behave like a benevolent planner which maximizes social welfare subject to revenues and the behavior of generations. But we relax this to study the rent seeking Leviathan government, which tries to maximize its tax revenue and neglects welfare impacts. Allowing for a production sector does not affect the results we derive as long as the economy is assumed to be small and facing world output prices and interest rates.

Our results extend the existing literature on bequests by first demonstrating that timber bequests can dominate money bequests under a variety of situations. Unlike money bequests, timber bequests can be operative even in dynamically inefficient steady states. We also show that a combination of harvest taxes and timber bequest subsidies further reinforces the condition for operative timber bequests. However, a combination of harvest subsidies and timber taxes makes timber bequests less likely in the long run. If timber bequests are <u>not</u> operative in a dynamically inefficient economy, then the optimal policy mix is a combination of a lump sum tax and subsidy on harvesting. However, when timber bequests are operative, then given the optimal lump-sum tax, the optimal tax-subsidy mix between harvesting and timber bequests depends on both the nature of the steady state equilibrium, and on the question of whether amenity services of forest stocks are a public good or not.

When forest amenities are private goods and the economy resides at the golden rule, only a lump sum tax suffices. In dynamically efficient economy, the lump sum policy is complemented by a harvest tax and bequest subsidy, because the steady state forest stock is too low. In a dynamically inefficient economy a harvest subsidy and bequest tax becomes optimal. If amenity services are public goods due to access, then a bequest subsidy and harvest tax are needed, both in the golden rule and in the dynamically efficient economy, to raise the equilibrium forest stock to an efficient level. However, the efficient policy mix is ambiguous if the economy is dynamically inefficient. Finally, if the government acts to maximize revenues, then the optimal policy mix will be qualitatively similar to that of benevolent government in the absence of public goods, but the tax and subsidy rates will be higher.

These results offer several practical policy implications. For most governments, which have the dual purpose of raising revenue and providing public goods, our work suggests that policies targeting harvesting should not be adopted without simultaneous consideration of policies targeting bequests within the forest sector. Yet in practice and in the literature these policies are often analyzed in isolation. Moreover, one seldom finds subsidies for timber bequests in practice. Rather the usual case is for governments (at least in North America and Europe) to impose taxes on both timber harvesting and bequests. We demonstrated that there is no case where such a policy mix is an efficient means to raise revenue and provide public forest goods, for a variety of equilibria.

The policy design issues we address are not discussed in the current debate on sustainability. But our results suggest they should be part of this debate, at least for economies with altruistic motives. Failure to consider bequest and timber policy instruments jointly, and failure to predict the impacts of constraints governments face in applying them, will lead to potentially inefficient distributions of timber stocks in the long run. This in turn will lead to inefficient distributions of public goods in the long run. In any case, the usual practice of taxing without accompanying subsidization, or vice versa, will not work.

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Appendix 1. Conditions for operative money and timber bequests

From the consumption-saving choice (7a) we can define the implicit real interest rate in the absence of money and timber bequests as

A1.1 $1+\bar{r} = u'(\hat{c}_{1t}) / \beta u'(\hat{c}_{2t})$, with consumption defined as

A1.2 $\hat{c}_{1t} = w_t + p_t x_{1t} - s_t$

A1.3 $\hat{c}_{2t} = p_{t+1} x_{2t} + R_{t+1} s_t$.

Equations A1.2 and A1.3 represent the value of consumption over both periods of life for a landowner in the first and second generation, respectively. The first-order condition for money bequest (7d) can be rewritten

A1.4 $U^i_{m_{t+1}}\Big|_{\substack{m_{t+1}=0 \\ b_{t+1}=0}} = \left[-(1+n) + \alpha(1+\bar{r})\right]\beta u'(c_{2t}) \leq 0 \;(=0 \; if \;\; m_{t+1} > 0)$,

which gives the condition for operative money bequests

A1.5 $m_{t+1} > 0 \Leftrightarrow \alpha > \dfrac{(1+n)}{(1+\bar{r})}$.

In the steady state $r = \bar{r}$, and this gives equation (8) in the text.

By a similar procedure, combined with the assumption that $R\beta = 1$, the first-order condition for operative timber bequests in the steady state can be written

A1.6 $U^i_{b_{t+1}}\Big|_{\substack{m_{t+1}=0 \\ b_{t+1}=0}} = -(1+n)\left[u'(\hat{c}_2)p - v'(\hat{k}_2)\right] + \alpha u'(\hat{c}_2)p(1+\bar{r})\left[1 - R^{-1}(1+g') + (1+g')\right]$

$$= \left\{-(1+n)\left[1 - \frac{v'(\hat{k}_2)}{u'(\hat{c}_2)p}\right] + \alpha(1+\bar{r})A\right\} u'(\hat{c}_2)p \leq 0 \;(=0 \; if \;\; b_{t+1} > 0),$$

where $A = 1 - R^{-1}(1+g') + (1+g')$.

This yields equation (9) of the text,

A1.6 $b_{t+1} > 0 \Leftrightarrow \alpha > \dfrac{(1+n)}{(1+\bar{r})A}\left[1 - \dfrac{v'(\hat{k}_2)}{u'(\hat{c}_2)p}\right]$

Appendix 2. Derivation of Slutsky equations for tax changes

Define the indirect utility function of the current generation as a function of taxes as $V^*(T,\tau_x,\tau_b,....)$. Given that $V_T^* = -(u'(c_{1t}) + \alpha u'(c_{2t+1})) < 0$ using the envelope theorem (see e.g. Mas-Colell, Whinston and Green 1995, pp. 964-966), we can invert V^* to obtain an expression for T,

A2.1 $\quad T = f(\tau_x,\tau_b,....U^0)$,

where U^0 is the maximum utility level for given taxes T, τ_x and τ_b. Substituting this for T on the indirect utility function gives

A2.2 $\quad V^*(f(\tau_x,\tau_b,....U^0),\tau_x,\tau_b,....) = U^0$,

which is the compensated indirect utility function (see e.g. Diamond and Yaari 1972).

Differentiating A2.2 with respect to τ_x and τ_b, respectively, produces $V_{\tau_x}^* + V_T^* f_{\tau_x} = 0$ and $V_{\tau_b}^* + V_T^* f_{\tau_b} = 0$, where $V_{\tau_x}^* = (1+R^{-1})pxV_T^*$ and $V_{\tau_b}^* = pbV_T^*$. From these we can solve the compensation needed to keep the current generation's utility constant, i.e.,

A2.3 $\quad f_{\tau_x} = -\dfrac{V_{\tau_x}^*}{V_T^*} = -(1+R^{-1})px$; and $f_{\tau_b} = -\dfrac{V_{\tau_b}^*}{V_T^*} = -pb$

From consumer theory we know that, for harvesting and timber bequests, the utility maximization problem implies,

A2.4a $\quad x(\tau_x, f(\tau_x,...),...) = x^c(\tau_x,....U^0)$; $\quad x(\tau_b, f(\tau_b,...),...) = x^c(\tau_b,....U^0)$

A2.4b $\quad b(\tau_x, f(\tau_x,...),...) = b^c(\tau_x,....U^0)$; $\quad b(\tau_b, f(\tau_b,...),...) = b^c(\tau_b,....U^0)$,

where x and b describe the ordinary behavioral functions, and x^c and b^c are defined as compensated behavior, derived subject to a given level of utility. Differentiating A2.4a and A2.4b produces $x_{\tau_x} + x_T f_{\tau_x} = x_{\tau_x}^c$; $x_{\tau_b} + x_T f_{\tau_b} = x_{\tau_b}^c$; and $b_{\tau_x} + b_T f_{\tau_x} = b_{\tau_x}^c$; $b_{\tau_b} + b_T f_{\tau_b} = b_{\tau_b}^c$, respectively. Recalling A2.3 then gives the Slutsky decompositions for the effects of taxes on harvesting and timber bequests.

A2.5 $\quad x_{\tau_x} = x_{\tau_x}^c + (1+R^{-1})pxx_T$; and $x_{\tau_b} = x_{\tau_b}^c + pbx_T$

A2.6 $\quad b_{\tau_x} = b_{\tau_x}^c + (1+R^{-1})pxb_T$; and $b_{\tau_b} = b_{\tau_b}^c + (1+R^{-1})pxb_T$

The income effects $(1+R^{-1})pxx_T$ and pbb_T can be solved by applying Cramer's Rule to obtain expressions for x_T and b_T. The substitution effects can be solved from equations A2.5 and A2.6, or by formulating the cost minimization problem and deriving the substitution effects from this. It is straightforward to show that the following signs hold for the substitution effects[18]

A2.7 $x^c_{\tau_x} < 0$; $x^c_{\tau_b} < 0$

A2.8 $b^c_{\tau_x} = ?$; $b^c_{\tau_b} < 0$

Even though $b^c_{\tau_x}$ is ambiguous a priori, it has no qualitative bearing on optimal tax or subsidy policy, as is shown in the text.

[18] The precise formulas are available from the authors upon request.

Appendix 3. Optimal tax formulas, when amenity services are private goods

The Lagrangian corresponding to the planner's problem is

A3.1 $\underset{\{T,\tau_b,\tau_x\}}{Max}\,\Omega = V^*(T,\tau_b,\tau_x) - \lambda[G^0 - \tau_b pb - \tau_x px]$

The first-order conditions for optimal choice of taxes are

A3.2a $\Omega_T = V_T^* + \lambda[1 + \tau_b pb_T + \tau_x \dfrac{(2+n)}{(1+n)} px_T] = 0$

A3.2b $\Omega_{\tau_b} = V_{\tau_b}^* + \lambda[pb + \tau_b pb_T + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_b}] = 0$

A3.2c $\Omega_{\tau_x} = V_{\tau_x}^* + \lambda[\tau_b pb_{\tau_x} + \dfrac{(2+n)}{(1+n)} px + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_x}] = 0$,

where $V_T^* = -(u'(c_{1t}) + \alpha u'(c_{2t+1}))$, $V_{\tau_b}^* = pbV_T^*$, and $V_{\tau_x}^* = (1+R^{-1})pxV_T^*$

By applying the Slutsky decompositions we can re-express A3.2b and A3.2c as

A3.3a $\Omega_{\tau_b} = pb\Omega_T + \lambda[\tau_b pb_{\tau_b}^c + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_b}^c] = 0$

A3.3b $\Omega_{\tau_x} = (1+R^{-1})px\Omega_T + \lambda[\tau_b pb_{\tau_x}^c + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_x}^c + p\varepsilon] = 0$,

where

$$pb\Omega_T = pb[V_T^* + \lambda(1 + \tau_b pb_T + \tau_x \frac{(2+n)}{(1+n)} px_T)],$$

$$(1+R^{-1})px\Omega_T = (1+R^{-1})px[V_T^* + \lambda(1 + \tau_b pb_T + \tau_x \frac{(2+n)}{(1+n)} px_T)]$$

and $\varepsilon = (\dfrac{1}{1+n} - \dfrac{1}{1+r})x$.

If the lump sum tax has been set at its optimal level, T=T*, then the first RHS terms are zero in A3.3a and A3.3b, because $pb\Omega_T = 0$ and $(1+R^{-1})px\Omega_T = 0$. Hence, when the government revenue constraint is binding, so that $\lambda > 0$, we obtain the following expressions given in the text:

A3.4a $\Omega_{\tau_b|T=T^*} = \tau_b pb_{\tau_b}^c + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_b}^c = 0$

A3.4b $\Omega_{\tau_x|T=T^*} = \tau_b pb_{\tau_x}^c + \tau_x \dfrac{(2+n)}{(1+n)} px_{\tau_x}^c - \dfrac{(2+n)}{(1+n)} - (1+R^{-1}) = 0$

Equation A3.4a and A3.4b define a system of two equations with optimal tax rates as endogenous variables

A3.5a $\tau_b pb_{\tau_b}^c + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_b}^c = 0$

A3.5b $\tau_b pb_{\tau_x}^c + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_x}^c = -p\varepsilon$,

which was given in matrix form as equation (15) in the text.

Appendix 4. Optimal tax formulas, when amenity services are public goods

The Lagrangian to the government's maximization problem is now

A4.1 $$\underset{\{T,\tau_b,\tau_x\}}{Max}\,\Omega^A = V^*(T,\tau_b,\tau_b) + (n+2)(q-1)[v(k_{1t}) + R^{-1}v(k_{2t}) + \alpha(v(k_{1t+1}) + R^{-1}v(k_{2t+1}))]$$
$$-\lambda[G^0 - \tau_b pb - T - \tau_x \frac{(2+n)}{(1+n)} px]$$

For expositional convenience, define the derivatives of the amenity valuation functions as $D = (v'(k_{1t}) + \alpha v'(k_{1t+1}))$ and $D^0 = R^{-1}(v'(k_{2t}) + \alpha v'(k_{2t+1}))$. Using this we can express the first-order conditions as follows

A4.2a $$\Omega^A_T = V^*_T + (n+2)(q-1)(Dx_T - D^0 b_T) + \lambda[1 + \tau_b pb_T + \tau_x \frac{(2+n)}{(1+n)} px_T] = 0$$

A4.2b $$\Omega^A_{\tau_b} = V^*_{\tau_b} + (n+2)(q-1)(Dx_{\tau_b} - D^0 b_{\tau_b}) + \lambda[pb + \tau_b pb_{\tau_b} + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_b}] = 0$$

A4.2c
$$\Omega^A_{\tau_x} = V^*_{\tau_x} + (n+2)(q-1)(Dx_{\tau_x} - D^0 b_{\tau_x}) + \lambda[\tau_b pb_{\tau_x} + \frac{(2+n)}{(1+n)} px + \tau_x \frac{(2+n)}{(1+n)} px_{\tau_b}] = 0$$

By applying the Slutsky decompositions derived in Appendix 2, we can now re-express the first-order conditions A4.2b and A4.2c as

A4.3a $$\Omega^A_{\tau_b} = pb\Omega^A_T + (n+2)(q-1)(D^0 b^c_{\tau_b} - Dx^c_{\tau_b}) + \lambda[\tau_b pb^c_{\tau_b} + \tau_x \frac{(2+n)}{(1+n)} px^c_{\tau_b}] = 0$$

A4.3b
$$\Omega^A_{\tau_b} = (1+R^{-1})px\Omega^A_T + (n+2)(q-1)(D^0 b^c_{\tau_x} - Dx^c_{\tau_x}) + \lambda[\tau_b pb^c_{\tau_x} +_x \frac{(2+n)}{(1+n)} px^c_{\tau_x} + p\varepsilon] = 0$$

Hence, when the lump sum tax has been set at the optimal level, $T = T^A$, we have

A4.4a $$\Omega^A_{\tau_b|T=T^A} = (n+2)(q-1)(D^0 b^c_{\tau_b} - Dx^c_{\tau_b}) + \lambda[\tau_b pb^c_{\tau_b} + \tau_x \frac{(2+n)}{(1+n)} px^c_{\tau_b}] = 0$$

A4.4b $$\Omega^A_{\tau_b|T=T^A} = (n+2)(q-1)(D^0 b^c_{\tau_x} - Dx^c_{\tau_x}) + \lambda[\tau_b pb^c_{\tau_x} +_x \frac{(2+n)}{(1+n)} px^c_{\tau_x} + \varepsilon] = 0$$

These two optimal conditions define the optimal harvest and bequest tax rates as the two equation system:

A4.5a $$\tau_b pb^c_{\tau_b} + \tau_x \frac{(2+n)}{(1+n)} px^c_{\tau_b} = \lambda^{-1}(n+2)(q-1)(D^0 b^c_{\tau_b} - Dx^c_{\tau_b})$$

A4.5b $$\tau_b pb^c_{\tau_x} + \tau_x \frac{(2+n)}{(1+n)} px^c_{\tau_x} = \lambda^{-1}(n+2)(q-1)(D^0 b^c_{\tau_x} - Dx^c_{\tau_x}) + p\varepsilon\,,$$

From which equations (19) and (20) in the text can be derived.

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