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EX-DIVIDEND DAY STOCK PRICE BEHAVIOUR, TAXES AND DISCRETE PRICES; A SIMULATION EXPERIMENT

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ABSTRACT: This paper examines how accurately the tax rate implicit in the ex-day price drop can be estimated with commonly used methods when stock prices are discrete. The results of our simulation experiment suggest that the GLS-estimator first proposed by Michaely (1991) is the best statistic among the four statistics examined. It is unbiased and has the smallest variance. The traditional ratio of price drop to dividend performs the worst. The results show that tick rules are important only if the ex-day price drop needs no adjustment for overnight return. If ex-day price drops are contaminated by overnight returns, tick rules do not affect the results of ex-day studies. The effect of tick rules is more than offset by errors made in eliminating the overnight return. As long as ex-day prices must be adjusted for overnight return, it does not matter whether prices are continuous or discrete. We also find that standard errors of commonly used test statistics are high enough to make the identification of tax clientele effects very difficult.

JEL classification codes: G12, G35

KEY WORDS: Asset pricing, Dividends, Taxation, Tick size

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TIIVISTELMÄ: Tutkimuksessa selvitetään, kuinka tarkasti osinkolipun irtoamispäivän eli ex-päivän osakekurssin laskun implikoima pääomatulon veroaste voidaan estimoida tavanomaisimmin käytetyillä menetelmillä kun osakekurssien muutokset ovat epäjatkuvia. Simulointikokeiden tulokset viittaavat siihen, että Michaelyn (1991) GLS-estimaattori toimii tutkituista estimaattoreista parhaiten, sillä se on harhaton ja sillä on pienin varianssi. Perinteinen ex-päivän kurssilaskun ja osingon suhde toimii huonoiten. Mikäli kurssilaskuun sekoittuu osakkeen yön aikana ansaitsema tuotto, osakkeiden noteeraustarkkuus ei vaikuta tarkkuuteen, jolla veroaste kyetään estimoimaan. Tämä johtuu siitä, että yliyön tuoton arvioimisessa tehtävät virheet ovat paljon merkittävämpiä kuin virheet, jotka aiheutuvat noteeraustarkkuudesta. Siten veroasteen estimointitarkkuuden kannalta on samantekevää, ovatko osakkeiden kurssimuutokset jatkuvia vai ei-jatkuvia, jos ex-päivän kurssia on korjattava arviolla yliyön tuotosta. Tutkimuksessa havaitaan myös, että tarkasteltujen estimaattoreiden keskivirheet ovat niin suuria, että clientele -vaikutusten luotettava havaitseminen on hyvin vaikeaa.

1 INTRODUCTION

In their seminal paper Elton and Gruber (1970) show that ex-dividend day price drops can be used to estimate the tax rate of the marginal investor. If capital gains are taxed at a lower (higher) rate than dividend income the ex-day price drop is smaller (larger) than the dividend.[1] Tax rates on dividend income and capital gains are important in corporate finance (for example in dividend policy and capital structure issues) but information about them is difficult to get. In this respect the ex-day method, which does not assume any particular asset pricing model, can be useful. Kalay (1982) notes that estimating the implicit tax rate may be difficult if two or more groups of investors differ substantially in their tax treatment of capital income and one group of investors finds it profitable for tax reasons to either collect or avoid dividends. If trading because of the dividend is important, ex-day price drops reflect the tax rates of the investors engaged in dividend trading and not the more interesting tax rates of long-term investors. Dividend trading is expected to be more important among stocks with high yield, low bid-ask spread (transaction costs) and high liquidity.

A large number of studies has applied the ex-day methodology to data from different countries and time periods. The empirical results are mixed. Barclay (1987) examines NYSE data from a period with no income tax in the U.S., 1900-1910, and another period after the introduction of the income tax, 1962-1985. Consistent with the tax explanation in 1900-1910 investors valued (before-tax) dividends and capital gains as perfect substitutes, but in 1962-1985 the value of dividends relative to capital gains was much lower due to a tax penalty on dividend income. Poterba and Summers (1984) report similar results for two dividend tax reforms in the U.K.. Michaely (1991) and Robin (1991) examine the effect of the U.S. 1986 Tax Reform Act on ex-dividend behaviour and find conflicting results. Michaely finds abnormal return behaviour consistent with short-term trading around ex-days. Also Lakonishok and Vermaelen (1983) and Booth and Johnston (1984) obtain conflicting results with Canadian data around the 1972 tax reform. Eades, Hess and Kim (1984) report statistically significant negative abnormal ex-day returns for non-

[1] This is equivalent to saying that ex-day returns increase (decrease) with dividend yield. Early empirical studies (see Campbell and Beranek (1955) and Durand and May (1960)) report that stock prices tend to fall on ex-dividend days by less than the dividend amount.

taxable cash distributions that should have no tax consequences at all. This suggests that factors other than taxes may also influence ex-day return behaviour. One such factor may be risk premia as suggested by Grammatikos (1989) and Fedenia and Grammatikos (1993). Karpoff and Walkling (1988) find strong evidence of short-term trading among high yield stocks in the U.S. after a reduction in the costs of short-term trading following the introduction of negotiable commissions in 1975, and practically no evidence of short-term trading before.

This paper is motivated by two things. First, some ex-day studies like Lakonishok and Vermaelen (1983) reveal that different ex-day methods yield sometimes quite different results even when exactly the same data is used.[2] Yet the literature gives little guidance as to what methods to use or how to judge existing empirical results on the basis of the method used. A second motivation comes from two recent papers, Bali and Hite (1998) and Frank and Jagannathan (1998), demonstrating that discrete prices may have a considerable effect on ex-day price behaviour. Frank and Jagannathan find that discrete prices cause prices in the Hong Kong stock market to fall on ex-days on average by less than the dividend amount even though neither dividends nor capital gains are taxed at all.

We run a simulation experiment to examine how accurately the tax rate implicit in the ex-day price drop can be estimated with commonly used methods, in particular when stock prices are discrete. Unwilling to underestimate the ability of these methods we make several simplifying assumptions to generate extremely favourable conditions for an ex-day study. We assume that the ex-day price drop always reflects capital income tax rates to the extent that it is possible in a discrete price world. Taking cum-dividend prices, dividends and the tax rates as given, we compute ex-day prices so that we introduce noise from only two sources to the ex-day price. Without this noise all variation in ex-day price drops could be explained by taxes only. The first source of noise is discrete prices. If the product of the tax rate and the dividend is not a multiple of the tick the price drop reflects the tax rate with error. We use two distinct tick rules applied at the Helsinki Stock

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2 Lakonishok and Vermaelen (1983) have 555 and 671 Canadian shares in 1971 and 1972 with average dividend yields of 1.07% and 1.13%. The average ratio of ex-day price drop to dividend varies considerably from 0.26 to 0.46 in 1971 and from 0.07 to 0.33 in 1972 depending on the choice ex-day prices and whether prices are adjusted for overnight return or not. The respective ranges for the equally weighted portfolio statistic are much narrower, 0.39-0.48 in 1971 and 0.21-0.28 in 1972. Thus different methods can yield remarkably different results even with relatively large samples.

Exchange, one before 1996 and the other in 1996-98. The second source of noise is that the ex-day price drop reflects both taxes and overnight return and these two can be separated only by estimating the overnight return somehow. The error at which the overnight return is estimated depends on return volatility and has a direct impact on how accurately the tax rate can be estimated. The question we ask is how much does the noise brought by discrete prices and unknown overnight returns affect the estimate of the tax rate.

Our results show that the best statistic is the GLS-estimator first proposed by Michaely (1991). It is unbiased, also when prices are discrete, and has the smallest variance among the four candidates. The traditional average ex-ratio performs clearly the worst. It is sometimes systematically biased and has the largest variance. Results obtained with the GLS-estimator show that tick rules are an important factor in determining ex-day price behaviour and the effect depends on tick size. If the smallest price change is small the tick rule has only a negligible effect on estimator variance, while the effect is considerable if the smallest price interval is large.

The results show that tick rules are important for ex-day studies if we can observe an ex-day price drop that only reflects capital income taxes, dividends and tick rules, and is not contaminated by any overnight return requiring adjustment. All test statistics perform significantly better in a small tick case than in a large tick case. On the other hand, if ex-day price drops are contaminated by overnight returns which must be removed by using estimates of normal daily returns, tick rules do not matter any more. The errors made in estimating the ex-day price drop, which depend on the amount of daily stock return uncertainty, are far more fatal for ex-day studies than tick rules. In fact, as long as ex-day prices must be adjusted by normal return, it does not matter whether prices are continuous or discrete.

We find that the GLS-estimator of the ex-ratio always performs at least as well as the other statistics largely because it is the only statistic that takes both stock return volatility and dividend yield explicitly into account. The average price drop to dividend ratio takes neither dividend yield nor volatility into account and therefore has the poorest performance in the simulation experiment. The standard errors of this ratio are so large that it is doubtful whether a tax parameter of any reasonable magnitude can ever be found statistically different from one at conventional significance levels and sample sizes. Gagnon and Suret (1991) have previously argued the same.

We also apply NYSE tick size of 0.125 to low dividend yield data comparable to that used earlier in tax clientele studies. The results suggest that even if we have 500 observations per dividend yield decile, use the GLS-estimator and assume low volatility of stock returns, tax clientele effects can be identified reliably only if ex-day price behaviour in one decile is substantially different from that in other deciles.

Section 2 of the paper briefly reviews the ex-day model. The set up and details of the simulation experiment are reported in section 3. Section 4 reports the results and section 5 concludes.

2 THE FRAMEWORK

Define the before and after-tax ex-dividend day stock returns, r and r_{at}, as

(1) $$r = \frac{P_{ex} - P_{cum} + D}{P_{cum}}$$

and

(2) $$r_{at} = \frac{(1-\tau_g)(P_{ex} - P_{cum}) + (1-\tau_d)D}{P_{cum}}$$

where P_{ex} and P_{cum} are the ex-dividend and cum-dividend day stock prices, D is the dividend per share, and τ_d and τ_g are marginal tax rates on dividends and capital gains. The dividend is non-zero on ex-dividend days and zero otherwise. Manipulation of equation (2) yields

(3) $$\frac{P_{ex} - P_{cum} + D}{P_{cum}} = \frac{r_{at}}{1-\tau_g} + (1-\alpha)\frac{D}{P_{cum}}$$

where $\alpha = (1\text{-}\tau_d)/(1\text{-}\tau_g)$ measures the relative value of dividends and capital gains and D/P_{cum} is dividend yield. Equation (3) implies that the expected stock return on non-ex-days is simply the grossed-up expected after-tax return. The ex-day return depends on the tax treatment of dividends and capital gains. Ex-day returns are positively (negatively) related to dividend yield if dividends are taxed more (less) heavily than capital gains, that is, if $\alpha < 1$ ($\alpha > 1$). Only when dividends and capital gains are effectively taxed at the same rate ex-day returns are unrelated to dividend yield.

Further manipulation of (3), ignoring the non-ex-day return, yields the familiar ex-dividend ratio first derived by Elton and Gruber (1970)

$$\frac{P_{cum} - P_{ex}}{D} = \alpha . \qquad (4)$$

Equation (4) predicts that stock prices fall on ex-days by less (more) than the amount of dividend if dividends are taxed more (less) heavily than capital gains. When dividends and capital gains are taxed at the same rate we expect that the ex-day price drop equals the dividend.

In real life trading rules restrict the precision at which stock prices can be quoted (see Angel (1997) for tick sizes in different countries and Anshuman and Kalay (1998) for a discussion of optimal tick size). When prices are discrete, the results derived above may not hold. For example, Dubofsky (1992) shows that due to NYSE and AMEX tick rules ex-day returns may be positively related to dividend yield even in the absence of taxes. Sorjonen (1999) demonstrates that the tick rules in the Helsinki Stock Exchange have similar implications.

3 SIMULATION

3.1 The setup

To create favourable conditions for an ex-day study we make a number of simplifying assumptions. First, capital income tax rates are taken as given. For simplicity we assume that all investors are taxed at the same rates.[3] Marginal tax rates on dividend income and

[3] Tax clienteles are assumed away. In this paper also the distinction of long-term and short-term investors is irrelevant.

capital gains are denoted by τ_d^* and τ_g^*, respectively. Secondly, stock prices fully incorporate capital income taxes. Third, the ex-dividend day price drop takes place during the night. Prices can be observed immediately before and after the ex-day price drop takes place. Thus, stock prices fall immediately after the market closes on the cum-dividend day, exactly by $\alpha^* D$, where $\alpha^* = (1-\tau_d^*)/(1-\tau_g^*)$ is the true value of the tax parameter. We take α^* as given and refer to $\alpha^* D$ as the true ex-day price drop. Under the assumptions made above a study of stock price behaviour around ex-days should yield an estimate of the tax parameter exactly equal to α^*.

Many things contribute to the fact that we observe the ex-day price drop with some noise and may therefore estimate α^* with error. We examine two potential sources of noise. The first source of noise is the tick rule, i.e. discrete price intervals, which restricts the behaviour of stock prices in such a way that prices can not always fall exactly by $\alpha^* D$ on ex-days. If the actual price drop deviates from $\alpha^* D$, stock prices do not correctly reflect the tax rate. A second source of noise stems from the fact that the ex-day price immediately after the price drop may not be observed. What we observe, is the pure ex-day price, $P_{cum} - \alpha^* D$, plus a return earned during the night, r, so that the ex-day price, P_{ex}, which can be observed is

(5) $$P_{ex} = (1+r)\left(P_{cum} - \alpha^* D\right).$$

The pure ex-day price is then given by $P_{ex} / (1+r)$. Unfortunately, the overnight return, r, can not be observed separately from the ex-day price drop. Therefore, we generally do not observe the true price drop. This is true for continuous prices also. As an estimate for the overnight return ex-day studies usually use an average historical stock or market return, or a return given by the market model.[4] Denoting the estimated overnight return by $\bar{r}$ the ex-day price drop is then estimated as

(6) $$P_{cum} - \frac{P_{ex}}{1+\bar{r}}.$$

The error in estimating the overnight return, $r - \bar{r}$, depends on return uncertainty and the tick rule, if discrete prices must be used in estimating $\bar{r}$. The rest of the paper

4 For estimating expected returns, see Brown and Warner (1985).

concentrates on examining how well does the price drop in (6) reflect capital income taxes.

3.2 Tick rules

The simulation experiment employs the tick rules of the Helsinki Stock Exchange (HeSE). The tick size at the HeSE was a step function of stock price before 1999.[5] There were four tick categories before 1996 and two tick categories in 1996-98. In the following we refer to the rule applied before 1996 as 'rule 1' and to the rule applied in 1996-98 as 'rule 2'. Table 1 shows the stock price categories and the respective tick sizes both in Finnish markkas and in approximate U.S. dollars to allow a comparison to the New York Stock Exchange (NYSE). Under rule 2 the tick size at HeSE was smaller than at NySE at all stock price levels.[6] Under rule 1 the HeSE tick size was smaller than at NYSE for stocks selling for less than FIM 100 and larger for more expensive stocks. The simulation experiment applies both rules 1 and 2.

Table 1: Tick size at the Helsinki Stock Exchange

Stock price		Tick size			
		RULE 1 - 31.12.1995		RULE 2 1.1.1996 - 31.12.1998	
FIM	*(USD)*	*FIM*	*(USD)*	*FIM*	*(USD)*
0.01- 10	(- 2)	0.01	(1/500)	0.01	(1/500)
10 - 100	(2 - 20)	0.10	(1/50)	0.10	(1/50)
100 - 1000	(20 - 200)	1	(1/5)	0.10	(1/50)
> 1000	(> 200)	10	(2)	0.10	(1/50)

3.3 Details

We start by creating continuous price data free of any tick rules. First, we draw a cross-section of cum-day stock prices, $P_{i,cum}$, for a sample of N stocks ($i = 1,...,N$) from a uniform

5 Since January 1999 all stocks are quoted in euros and the tick is one cent for all stocks regardless of the price.

6 At the NYSE stocks less than \$0.50 trade in multiples of 1/32, stocks between \$0.50 and \$1.00 in multiples of 1/16 and stocks priced over \$1 in multiples of 1/8 (see Angel (1997)).

distribution. Since the accuracy of price quotations is not limited in any way, these prices are continuous by nature. We make a simplifying assumption that the returns of all N stocks are normally and identically distributed. For each of the N stocks we draw a full history of daily returns, r_{it}, for 121 trading days (t = -119,...,-1, *cum*, *ex*), where the ex-day return includes only the overnight return and not the ex-day price drop. Thus a total of 121N daily returns are drawn. We interpret these returns as logarithmic price differences. Using the cum-day prices and the historic returns, we work out the continuous price history, P_{it} (t = -120,...,-1, *cum*), excluding the ex-day, for the N stocks.[7]

In practise stock prices must obey a tick rule. The stock price history, P_{it}, includes continuous fundamental prices. To obtain the prices that conform to a tick rule we round each of these prices to the closest legitimate price, and denote the stock price history, P_{it}, forced to conform to the tick rule j by P^j_{it} (t = -120,...,-1, *cum*)[8].

Dividend yields, δ_i, are randomly drawn from a uniform distribution for the N stocks. The cash dividend, DIV_i, is then given by $DIV_i = \delta_i P_{i,cum}$. In practise dividends are not continuous. Therefore we round dividends to the nearest 0.01.[9] From now on dividends are at least 0.1 and always multiples of 0.01. We denote the rounded dividend by D_i. The observed dividend yields which are needed in estimating the tax parameter are then $d_i = D_i/P_{i,cum}$ for the continuous price case and $d^j_i = D_i/P^j_{i,cum}$ for the tick rule case.

With continuous prices we observe stock returns r_{it} (t = -120,...,-1, *cum*). A different set of returns is observed when stock prices follow tick rule j. We use the legitimate discrete

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7 Normally ex-day studies examine a particular ex-dividend period including, say, 5 to 45 trading days before and after the ex-day for examining abnormal return and volume behaviour. We have no particular role for the ex-day period. Therefore, we simply take it to cover only the ex-day.

8 This means that ex-day prices 101.51 and 101.50 are rounded to 102 and 101, respectively.

9 At first glance it might seem that letting dividends be multiples of 0.01 (rather than, say, 0.05 or 0.1) almost guarantees that discrete prices practically never fall exactly by the dividend amount. This in turn would suggest that our choice of dividend accuracy by itself might lead to ex-day methods to produce biased results. A more careful look at the issue reveals that quite the opposite is true. To illustrate this, let dividends be multiples of 0.05, tick size 0.1 and $\alpha^* = 1$. Ex-day price drops should now be multiples of 0.05. Now we have a problem of how the market rounds prices. Let dividends be 0.05, 0.10, 0.15 and 0.2. If 0.05 is rounded downwards, then price drops will be 0, 0.1, 0.1 and 0.2, and ex-ratios 0, 1, 0.67 and 1. Ex-ratios would always be less than or equal to α^* and the average ex-ratio would be less than one. On the other hand, if 0.05 is rounded upwards, then price drops would be 0.1, 0.1, 0.2 and 0.2, and ex-ratios 2, 1, 1.33 and 1. Now ex-ratios would always be larger than or equal to α^*. In both cases it would appear that ex-ratios are systematically biased, the sign of the bias depending on the rounding rule. In general, similar problems will arise whenever $\alpha^* D = ½$ *tick*. Thus, even if the choice of letting dividends be multiples of 0.01 may seem exaggerated, it is done for a reason.

prices, P^j_{it} (t = -121,...,-1, *cum*), to generate a new set of returns, r^j_{it}, that are observed in the presence of tick rule j. These returns are given by

$$(7) \qquad r^j_{it} = \ln\left(\frac{P^j_{it}}{P^j_{it-1}}\right) \qquad t = -119,\ldots,-1, cum.$$

Now we have created both a continuous price history and a price history conforming to tick rule j for days -120,...,-1, *cum*, thus excluding the ex-dividend day price data. In the continuous price case, the pure ex-day price, which we can not observe, is

$$(8) \qquad P_{i,cum} - \alpha^* D_i ,$$

whereas the observable ex-day price is

$$(9) \qquad P_{i,ex} = \left(1 + r_{i,ex}\right)\left(P_{i,cum} - \alpha^* D_i\right).$$

When prices follow tick rule j, the observed ex-day price, $P^j_{i,ex}$, is obtained by rounding

$$(10) \qquad \left(1 + r_{i,ex}\right)\left(P^j_{i,cum} - \alpha^* D_i\right)$$

to the closest legitimate price.[10] For eliminating the overnight return we need an estimate of a normal daily return. We estimate the normal return as an average daily stock return using the return data from 120 days preceding the ex-dividend day (i.e days -119,...,-1, *cum*). The relevant ex-day price drops are

$$(11) \qquad P_{i,cum} - \frac{P_{i,ex}}{1 + \bar{r}_i}$$

for the continuous price case and

$$(12) \qquad P^j_{i,cum} - \frac{P^j_{i,ex}}{1 + \bar{r}^j_i}$$

for the tick rule case where the estimates of normal daily return are

$$\bar{r}_i = \frac{1}{120}\sum_{t=-119}^{cum} r_{it} \qquad \text{and} \qquad \bar{r}^j_i = \frac{1}{120}\sum_{t=-119}^{cum} r^j_{it} .$$

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[10] Note that we actually assume here that investors always observe the cum-day price and then decide what the ex-day price should be.

In the simulation experiment the true tax parameter takes values $\alpha^* = \{0.8, 1, 1.2)$, ranging from a clear capital gains preference ($\alpha^* = 0.8$) to an equally clear preference for dividend income ($\alpha^* = 1.2$). We assume that dividend yields are uniformly distributed and control the distribution by choosing the mean dividend yield from three alternatives. In the low yield case $\delta_i \sim U(0,0.02)$, in the middle yield case $\delta_i \sim U(0,0.06)$, and in the high yield case $\delta_i \sim U(0,0.10)$, so that average dividend yields are roughly 1, 3 and 5 per cent. The continuous cum-dividend stock prices at the closing are also drawn from uniform distributions. We use two alternatives, $P_{cum} \sim U(1,200)$ and $P_{cum} \sim U(1,500)$. The average cum prices are thus roughly 100 and 250. For simplicity we assume that all stocks have the same annual mean return, $\mu = 0.15$. All return uncertainty, whatever its source might be, is assumed to be captured by daily volatility, σ. We try three alternative assumptions about σ. The daily volatility is either constant for all stocks ($\sigma = 0.01$ or $\sigma = 0.02$), or is uniformly distributed ($\sigma_i \sim U(0.005,0.02)$). Daily volatility of 0.01 (0.02) corresponds to annual volatility of 16% (32%). Time varying moments of the return distribution are ruled out. Finally, we let the sample size N be 20, 50, 100 or 200. These sample sizes are very moderate from the U.S. perspective. However, they are quite realistic when price data from European stock exchanges is used. In addition, often subsamples have to be used in examining dividend clienteles.[11]

For every parameter combination $\{\alpha^*, \delta_i, \sigma, P_{cum}, N\}$, we generate 10000 data sets. Dividend yields and cum-dividend prices are drawn separately for each of the 10000 data sets, but the distribution from which they are drawn is always the same. Each of the 10000 data sets is used to estimate the tax parameter α with four methods for five cases, which we describe in detail below. The 20 estimates are recorded. There will thus be one distribution of α's, with 10000 observations, for every parameter combination, for each of the four methods and for each of the five cases. It is these distributions and especially their first two moments that we are interested in. These distributions are used to assess the unbiasedness and efficiency of the four statistics in estimating the true tax parameter, α^*, in the absence and presence of tick rules when the characteristics of the sample are taken as given.

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[11] Finnish ex-day studies offer a good example of small samples. Hietala and Keloharju (1995) have a subsample of 59 unrestricted shares, the entire sample of Hedvall, Liljeblom and Löflund (1998) consists of 122 observations, Sorjonen (1988) has usually less than 30 observations annually and less than 100 observations during longer periods, and Sorjonen (1995) has less than 70 observations per subperiod.

In a simulation experiment with 10000 repetitions the variance of the estimator often becomes very small as we shall see soon. In such cases extremely small deviations of the mean from α^* can turn out to be statistically significant thus judging the estimator biased. Such precision has little relevance here. In empirical ex-day studies only the first two decimals of α are usually interesting. Therefore we take the pragmatic ad hoc view that if the average estimate falls in the range $\alpha^* \pm 0.01$ the estimator is unbiased and biased otherwise.

The five cases we examine are the following. In the first case stock prices are continuous. Pure ex-day prices can not be observed and therefore the observed ex-day price must be discounted with the overnight return estimated from past continuous prices as the average daily return. Any deviation of estimates from α^* is caused by errors in estimating the overnight return. Next the same continuous stock prices are rounded so that they conform to tick rules 1 and 2. The tick rule is the only reason why ex-dividend day stock price drops can not precisely reflect capital income taxes. In the absence of a discrete price rule we would always estimate α^* correctly. Any deviation of the estimates from α^* can be attributed to the tick rule only. The rounded cum-day prices are common knowledge. The ex-day price is computed by reducing $\alpha^* D_i$ from the known rounded cum-day price and then rounding to the nearest legitimate price.[12] Finally, the two tick rule cases are extended by assuming that the pure ex-day price drop can not be observed. To estimate the ex-day price drop the observed ex-day price must be discounted with the overnight return estimated from past discrete (or rounded) prices as the average daily return.

3.4 Methods

Four closely related models are used to estimate the tax parameter, α. We denote the four alternatives simply by α_1, α_2, α_3 and α_4, where α_1 is the average ex-dividend day ratio of Elton and Gruber (1970), α_2 is the GLS estimator of the average ex-ratio first proposed by Michaely (1991), α_3 is the equally weighted portfolio statistic of Lakonishok and Vermaelen (1983), and α_4 is the OLS estimator of the tax rate obtained from the return specification (3). The models that we estimate are (13), (15), (17) and (18) below.

[12] Since it is assumed that prices can be observed immediately after the ex-day price drop, we have no role for the price history and do not estimate the overnight return nor return volatility.

A useful starting point for deriving these models is the return specification (3). If the normal return is taken into account already in computing the ex-day price drop, as we do in equations (11) and (12), the empirical version of (3) can be written without a constant as

$$(13) \qquad \frac{P_{i,ex} - P_{i,cum} + D_i}{P_{i,cum}} = (1-\alpha_4)\frac{D_i}{P_{i,cum}} + \varepsilon_i \qquad \varepsilon_i \sim N(0,\sigma_i^2)$$

where ε_i is the unexpected ex-day return. Usually return volatilities differ accross stocks. Therefore the variance of ε_i is not constant and the model is heteroskedastic when cross-sectional data is used. The OLS estimator is still unbiased but inefficient. Lakonishok and Vermaelen (1983) suggest an alternative formulation of (13), which does not solve the heteroskedasticity problem. Rearranging terms and letting $\Delta P_i = P_{i,cum} - P_{i,ex}$ and $d_i = D_i / P_{i,cum}$ we can rewrite (13) as

$$(14) \qquad \frac{\Delta P_i}{P_{i,cum}} = \alpha d_i + \varepsilon_i .$$

Taking averages of both sides and solving for α yields the equally weighted portfolio statistic

$$(15) \qquad \alpha_3 = \overline{\left(\frac{\Delta P}{P}\right)} \Big/ \overline{d}$$

where $\overline{d}$ and $\overline{(\Delta P/P)}$ are simple averages of d_i and $\Delta P_i / P_{i,cum}$, respectively.

To correct for heteroskedasticity both sides of (13) must be divided by σ_i so that

$$(16) \qquad \frac{1}{\sigma_i}\frac{P_{i,ex} - P_{i,cum} + D_i}{P_{i,cum}} = (1-\alpha_2)\frac{d_i}{\sigma_i} + \eta_i \qquad \eta_i \sim N(0,\sigma^2).$$

Rearranging terms yields the *GLS*-estimator of the ex-dividend ratio

$$(17) \qquad \frac{d_i}{\sigma_i}\frac{\Delta P_i}{D_i} = \alpha_2 \frac{d_i}{\sigma_i} + \eta_i \qquad \eta_i \sim N(0,\sigma^2).$$

Both formulations (16) and (17) can be used to estimate α_2. Note that if we do not have information about σ_i and can not make the correction for heteroskedasticity, (17) reduces to (13), in which case α_2 is always equal to α_4. Also, if there is only little variance in σ_i, α_2 and α_4 yield similar results.

Finally, the traditional ex-day ratio of Elton and Gruber (1970), which corresponds to equation (4), is obtained by dividing both sides of (17) by d_i/σ_i

$$(18) \qquad \frac{\Delta P_i}{D_i} = \alpha_1 + \xi_i \qquad \xi_i \sim N\left(0, \sigma_i^2/d_i^2\right).$$

The average ex-dividend ratio can thus be estimated by regressing the relative price drops $\Delta P_i / D_i$ on a constant only. α_1 is an unbiased estimator of the tax parameter. The residual is heteroskedastic though, because differences in return volatilities and dividend yields are not taken explicitly into account.

When dividend yields and return volatilities of all stocks are close to one another we would expect that all four methods perform equally well. When data displays variance in dividend yields we expect that α_1 is outperformed by the three statistics, α_2, α_3 and α_4, which take dividend yield explicitly into account. Finally, if return volatilities of stocks differ we expect that α_2 outperforms all other statistics, because it is the only statistic that takes volatility into account.

4 RESULTS

4.1 Only a tick rule

We start from the simpliest case where the only noise in the observed ex-day price drop is caused by a tick rule. Under the simplifying assumptions made, any deviation of average test statistics from α^* can be interpreted as a systematic bias caused by the tick rule. In addition, the standard error can be interpreted as the risk, caused by the tick rule, of getting misleading estimates of α^*.[13]

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[13] If in our simplistic world prices were continuous ex-day prices would exactly reflect capital income taxes and each of the four test statistics would always estimate α^* correctly. Thus, in an experiment of 10 000 trials each test statistic would average α^* and have a variance of zero. Indeed, this is the ideal that our results must be compared to.

The calculations are based on the assumption that the pure ex-day drop can be observed (with noise caused by tick requirements) and therefore no correction for overnight return is required. Historic return behaviour and volatility have no role here. In GLS-estimation we simply set the volatility parameter equal to one, so that essentially α_2 is always equal to α_4. We choose to report only α_2. The means and standard errors (s_1, s_2 and s_3) of the three test statistics (α_1, α_2 and α_3) are reported in table 2 for tick rules 1 and 2 when $P_{cum} \leq 500$. The means are independent of sample size, N, and therefore we report them only once, for $N = 200$.

A general finding is that not all estimation techniques are equally reliable. The best test statistics, α_2 and α_3, are on average very accurate. For example, with rule 1 the 95 % confidence interval of α_2 is [0.757, 0.849] when $\alpha^* = 0.8$, $N = 20$ and $\delta \leq 0.02$, and much narrower with larger samples and higher dividend yields. The GLS-estimator has a lower variance. An ordinary F-test reveals that the difference between variances is statistically significant. The variances do not seem to depend on α^*.

The average ex-ratio, α_1, has the largest variance among the three test statistics and is systematically biased when rule 1 is applied. For example, in the low yield case the average α_1 is 0.754 when $\alpha^* = 0.8$ and 1.14 when $\alpha^* = 1.2$. In the high yield case α_1 performs much better but is nevertheless somewhat biased. The bias falls with dividend yield but is, surprisingly, unaffected by sample size. By using α_1 one can obtain misleading results even with relatively large samples especially when dividend yields are small.[14] [15]

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[14] A finding that α_1, the most often used statistic in ex-day studies, is biased when the only market imperfection is a tick rule, is a strong one. It immediately raises the question whether this result is an artifact. It turns out that the result can be replicated in repeated experiments. Our computer runs produce as a byproduct several tick rule experiments with the same parameter values. The results are almost identical, the difference in means and standard deviations of α_1 always being of magnitude 0.001.

[15] Problems may arise when two periods are compared according to estimated α_1's. For example, assume rule 1 and two periods with 50 shares and α^*s of 1 and 1.2. When $\alpha^* = 1$ and dividend yield ≤ 2 per cent we obtain an estimate larger than 1.042 in 2.5% of cases while $\alpha^* = 1.2$ is estimated less than 1.045 in another 2.5% of cases. There is a fair chance that a comparison of these two periods reveals no changes in the ex-day ratio even though there is a large one. Other comparisons might suggest that there are changes in the ex-day ratio when there in fact are none.

When rule 2 is applied the biases are negligible and the tax parameter can be estimated with much greater precision than with the more restrictive rule 1. If rule 1 is replaced by rule 2, s_2 and s_3 fall to approximately one third, and s_1 falls to one fifth or less. The change in variances is statistically significant at high significance levels. In general, both the bias and the standard errors tend to decrease with dividend yield. This is not a new result. The ex-dividend literature has long ago recognized that small dividends may distort results.

4.2 Tick rule and overnight return adjustment

Our results suggest that the average ex-ratio is biased under the simplifying and maybe somewhat unrealistic assumption that a tick rule is the only disturbing factor in estimating α^*. To introduce more noise we next assume that the pure ex-day price can not be observed. To estimate it we have to correct the observable ex-day price with an estimate of overnight return (see (6)), which is approximated by an average historic return. The ex-day price drop will be estimated with error if the realized overnight return deviates from the one estimated from price history. We employ exactly the same price and dividend data as before and therefore the new results can be compared to those in table 2. We find that all test statistics are unbiased for all parameter combinations examined, and the standard errors of all statistics are largely independent of α^*. Therefore we report the standard errors only for $\alpha^* = 0.8$, and do not report the average test statistics at all. Table 3 shows the results for $P_{cum} \leq 500$.

The standard errors in table 3 are considerably higher than those in table 2. The differences of variances in the two tables are statistically significant at high significance levels. Standard errors in table 3 do not depend on the tick rule used at all. Thus, tick rules have no effect on the performance of the four test statistics. Only the amount of return uncertainty is important. Increasing daily return volatility from 0.01 to 0.02 doubles the standard errors.

The standard errors of α_1 are so large that a tax parameter of 0.8 can never be found statistically different from one. The best performing test statistics are α_2 and α_4. Due to

Table 2: Simulation results with tick rules 1 and 2 ($P_{cum} \leq 500$)

Panel (a): Means

			Rule 1			Rule 2		
α^*	$\delta(\%)$	N	α_1	α_2	α_3	α_1	α_2	α_3
	≤ 2	200	0.753	0.803	0.800	0.808	0.801	0.802
0.8	≤ 6	200	0.785	0.801	0.801	0.803	0.800	0.800
	≤ 10	200	0.791	0.800	0.800	0.802	0.800	0.800
	≤ 2	200	0.949	1.002	0.999	1.004	1.002	1.003
1	≤ 6	200	0.983	1.001	1.001	1.002	1.001	1.001
	≤ 10	200	0.990	1.001	1.001	1.001	1.001	1.001
	≤ 2	200	1.146	1.202	1.198	1.192	1.199	1.198
1.2	≤ 6	200	1.183	1.201	1.201	1.197	1.200	1.200
	≤ 10	200	1.190	1.201	1.200	1.198	1.200	1.200

Panel (b): Standard deviations

			Rule 1			Rule 2		
α^*	$\delta(\%)$	N	s_1	s_2	s_3	s_1	s_2	s_3
		20	0.0656	0.0230*	0.0269	0.0134	0.0076*	0.0080
	≤ 2	50	0.0417	0.0141*	0.0167	0.0087	0.0047*	0.0050
		100	0.0293	0.0097*	0.0117	0.0060	0.0032*	0.0035
		200	0.0207	0.0069*	0.0084	0.0043	0.0023*	0.0025
		20	0.0390	0.0078*	0.0092	0.0081	0.0026*	0.0030
0.8	≤ 6	50	0.0245	0.0049*	0.0057	0.0051	0.0016*	0.0018
		100	0.0171	0.0034*	0.0040	0.0037	0.0012*	0.0013
		200	0.0122	0.0024*	0.0028	0.0026	0.0008*	0.0009
		20	0.0304	0.0047*	0.0055	0.0064	0.0016*	0.0018
	≤ 10	50	0.0188	0.0029*	0.0034	0.0041	0.0010*	0.0011
		100	0.0135	0.0021*	0.0024	0.0028	0.0007*	0.0008
		200	0.0096	0.0015*	0.0017	0.0020	0.0005*	0.0006
		20	0.0727	0.0226*	0.0267	0.0118	0.0076*	0.0078
	≤ 2	50	0.0466	0.0136*	0.0164	0.0074	0.0045*	0.0048
		100	0.0329	0.0095*	0.0116	0.0053	0.0032*	0.0034
		200	0.0231	0.0067*	0.0082	0.0036	0.0022*	0.0024
		20	0.0430	0.0077*	0.0090	0.0072	0.0026*	0.0029
1	≤ 6	50	0.0272	0.0048*	0.0056	0.0045	0.0016*	0.0018
		100	0.0191	0.0034*	0.0040	0.0032	0.0011*	0.0013
		200	0.0136	0.0024*	0.0028	0.0022	0.0008*	0.0009
		20	0.0334	0.0046*	0.0055	0.0055	0.0015*	0.0018
	≤ 10	50	0.0211	0.0029*	0.0033	0.0035	0.0010*	0.0011
		100	0.0151	0.0020*	0.0024	0.0025	0.0007*	0.0008
		200	0.0106	0.0014*	0.0017	0.0017	0.0005	0.0005
		20	0.0816	0.0228*	0.0272	0.0153	0.0077*	0.0084
	≤ 2	50	0.0511	0.0139*	0.0167	0.0098	0.0047*	0.0053
		100	0.0359	0.0096*	0.0116	0.0067	0.0032*	0.0037
		200	0.0253	0.0068*	0.0083	0.0048	0.0023*	0.0026
		20	0.0480	0.0078*	0.0092	0.0091	0.0026*	0.0030
1.2	≤ 6	50	0.0301	0.0048*	0.0056	0.0058	0.0016*	0.0019
		100	0.0212	0.0034*	0.0040	0.0041	0.0012*	0.0013
		200	0.0149	0.0024*	0.0028	0.0029	0.0008*	0.0009
		20	0.0365	0.0046*	0.0054	0.0071	0.0016*	0.0018
	≤ 10	50	0.0235	0.0028*	0.0034	0.0045	0.0010*	0.0011
		100	0.0165	0.0020*	0.0023	0.0032	0.0007*	0.0008
		200	0.0117	0.0014*	0.0017	0.0023	0.0005*	0.0006

* The estimator is unbiased and none of the other estimators has as a lower variance at 1% risk level.

Table 3: Simulation results with tick rules and return adjustment ($P_{cum} \leq 500$)

Panel (a): $\sigma = 0.01$

			Rule 1				Rule 2			
α^*	$\delta(\%)$	N	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
		20	1.100	0.193*	0.222	0.192*	1.090	0.192*	0.220	0.191*
	≤ 2	50	0.701	0.119*	0.139	0.118*	0.697	0.118*	0.138	0.117*
		100	0.501	0.083*	0.099	0.083*	0.499	0.083*	0.098	0.082*
		200	0.355	0.058*	0.068	0.058*	0.352	0.058*	0.068	0.058*
		20	0.642	0.064*	0.075	0.063*	0.640	0.063*	0.074	0.063*
0.8	≤ 6	50	0.415	0.040*	0.047	0.040*	0.413	0.040*	0.046	0.040*
		100	0.288	0.029*	0.033	0.028*	0.287	0.029*	0.033	0.028*
		200	0.204	0.020*	0.023	0.020*	0.203	0.020*	0.023	0.020*
		20	0.496	0.038*	0.045	0.037*	0.494	0.037*	0.044	0.037*
	≤ 10	50	0.320	0.024*	0.028	0.023*	0.318	0.024*	0.028	0.023*
		100	0.227	0.017*	0.020	0.017*	0.226	0.017*	0.019	0.016*
		200	0.161	0.012*	0.014	0.012*	0.160	0.012*	0.014	0.012*

Panel (b): $\sigma = 0.02$

			Rule 1				Rule 2			
α^*	$\delta(\%)$	N	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
		20	2.260	0.389*	0.449	0.386*	2.260	0.389*	0.448	0.386*
	≤ 2	50	1.390	0.236*	0.275	0.234*	1.390	0.236*	0.274	0.234*
		100	1.000	0.167*	0.196	0.165*	1.000	0.167*	0.196	0.165*
		200	0.707	0.117*	0.136	0.116*	0.706	0.116*	0.136	0.115*
		20	1.270	0.129*	0.149	0.128*	1.260	0.128*	0.148	0.127*
0.8	≤ 6	50	0.830	0.080*	0.092	0.079*	0.829	0.080*	0.092	0.079*
		100	0.570	0.058*	0.066	0.057*	0.570	0.057*	0.066	0.057*
		200	0.415	0.039*	0.046	0.039*	0.414	0.039*	0.046	0.039*
		20	1.040	0.075*	0.088	0.075*	1.030	0.075*	0.087	0.075*
	≤ 10	50	0.636	0.047*	0.055	0.046*	0.635	0.047*	0.055	0.046*
		100	0.452	0.033*	0.039	0.033*	0.451	0.033*	0.039	0.033*
		200	0.321	0.023*	0.028	0.023*	0.320	0.023*	0.028	0.023*

Panel (c): $0.005 \leq \sigma \leq 0.02$

			Rule 1				Rule 2			
α^*	$\delta(\%)$	N	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4
		20	1.490	0.198*	0.290	0.250	1.480	0.196*	0.289	0.249
	≤ 2	50	0.938	0.120*	0.183	0.155	0.935	0.119*	0.183	0.155
		100	0.666	0.083*	0.131	0.109	0.663	0.083*	0.130	0.109
		200	0.466	0.059*	0.092	0.077	0.464	0.059*	0.091	0.076
		20	0.860	0.067*	0.099	0.083	0.857	0.066*	0.098	0.083
0.8	≤ 6	50	0.556	0.041*	0.062	0.053	0.554	0.041*	0.062	0.053
		100	0.386	0.029*	0.043	0.037	0.384	0.028*	0.043	0.037
		200	0.274	0.020*	0.031	0.026	0.273	0.020*	0.030	0.026
		20	0.652	0.039*	0.059	0.050	0.648	0.039*	0.059	0.050
	≤ 10	50	0.434	0.024*	0.037	0.031	0.433	0.024*	0.037	0.031
		100	0.301	0.017*	0.026	0.022	0.300	0.017*	0.026	0.022
		200	0.209	0.012*	0.018	0.015	0.208	0.012*	0.018	0.015

* The estimator is unbiased and none of the other estimators has as a lower variance at 1% risk level.

the return adjustment the 95% confidence intervals are wide. With $N = 200$ and $\delta \leq 0.06$ the interval for α_2 is [0.76, 0.84] when $\sigma = 0.01$ and [0.722, 0.878] when $\sigma = 0.02$. When daily stock return volatility is allowed to vary between 0.5% and 2% (see panel (c)), α_2 has the lowest standard errors. This is not surprising, considering that α_2 is the only statistic that takes volatilities explicitly into account. In panels (a) and (b) volatility is basically constant and therefore α_4 performs equally well as α_2. Taking the volatilities into account reduces estimator variances by statistically significant amounts.

4.3 Continuous prices and overnight return adjustment

Next we ask how do the results (in table 3) change if we remove tick rules 1 and 2 so that prices become essentially continuous. We still assume that the pure ex-day price is unobservable. Table 4 shows both the means and standard errors of the test statistics in the continuous price case. α_3 is biased at low dividend yields. The size of the bias does not depend on sample size. The variance of α_1 is large and exactly the same in tables 3 and 4. Thus, the performance of α_1 does not depend at all on tick rules as long as overnight return adjustment is necessary.

Under the assumption of constant return volatility the best performing statistics are again α_2 and α_4. With non-constant volatility the best statistic is α_2 alone. At low dividend yields s_2 and s_4 are higher with continuous prices than with tick rules and the difference of variances is statistically significant. At higher dividend yields there is no difference in standard errors. The conclusion is that as long as it is necessary to adjust the ex-day price drop by an estimate of overnight return, it does not matter whether stock prices are discrete or not. The effect of errors made in estimating the ex-day price drop is far more important than the effect of any tick rules.

4.4 The effect of prices

We expect that the smaller the tick size, the smaller the error in estimating α^*. With tick rule 1 and 2 the average percentage tick sizes are 0.48% and 0.16% when $P_{cum} \leq 200$, and

0.37% and 0.08% when $P_{cum} \leq 500$.[16] Due to a lower average tick size, we expect rule 2 to be less restrictive than rule 1 and thus have a smaller effect on the estimation results of the simulation experiment. To test this hypothesis we let $\alpha^* = 0.8$, $N = 200$ and $\sigma_i \sim U(0.005,0.02)$, repeat the simulation experiment for two cum-day price ranges, $P_{cum} \leq 200$ and $P_{cum} \leq 500$, and use an ordinary F-test to test the equality of the variances of each statistic in the two price ranges.

Table 5 reports the p-values for the F-tests. The results show the following. First, with only tick rules 1 and 2, the variances of all statistics are always larger when the narrower price range is used, and the F-test is highly significant. This is consistent with our hypothesis that the relative tick size is important. Second, always when overnight return adjustment is necessary, the variance of α_1 is larger for the wider price range. The difference in variances is highly significant. The explanation is the following. The size of the ex-day price drop caused by dividend stripping depends only on taxes and the dividend amount but is independent of the cum-day price. Taking tax rates and the dividend as given, the higher the cum-day price the higher the ex-day price. The higher the ex-day price, the larger the overnight return adjustment to the price drop, for a given ex-day price drop and for a given normal daily return. Therefore the variance of α_1 is higher for $P_{cum} \leq 500$ than for $P_{cum} \leq 200$.

Third, when prices are continuous, the variances of α_2, α_3 and α_4 do not depend on the range of P_{cum}. This is consistent with our prediction. Cum-day prices are not supposed to affect the variances when there are no restrictions on the smallest price increment. Fourth, with tick rules and return adjustment, the variances of α_2, α_3 and α_4 are independent of the cum-day price change except when dividends are small. When dividends are small the variances are higher for the wider price range. The explanation is the same as for α_1 above. With small dividends the overnight adjustment has a larger effect on the ex-day price drop if prices are high. For higher dividend yield classes the variances of α_2, α_3 and α_4 do not depend on the level of stock prices.

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[16] Let the price, x, ranging from a to b, follow a univariate distribution so that $x \sim U(a,b)$. The density function of x is $f_x(x) = 1/(b-a)$ when $x \in [a,b]$ and $f_x(x) = 0$ otherwise. Assume the tick size is, say, 1, so that the relative tick size is given by $y = 1/x$. The density function of y is $f_y(y) = f_x(x)/y^2$ when $y \in [1/b,1/a]$ and $f_y(y) = 0$ otherwise. The average relative tick is the expectation of y. The numbers above are obtained by properly weighting the average relative ticks of the relevant price categories.

Table 4: Simulation results with continuous prices and return adjustment ($P_{cum} \leq 500$)

Panel (a): $\sigma = 0.01$

α^*	$\delta(\%)$	N	α_1	α_2	α_3	α_4	s_1	s_2	s_3	s_4
		20	0.790	0.798	0.826	0.794	1.090	0.197*	0.235	0.197*
	≤ 2	50	0.804	0.801	0.829	0.797	0.697	0.124*	0.149	0.123*
		100	0.793	0.799	0.826	0.795	0.499	0.087*	0.105	0.087*
		200	0.798	0.800	0.827	0.796	0.352	0.061*	0.072	0.061*
		20	0.799	0.800	0.804	0.799	0.640	0.063*	0.076	0.063*
0.8	≤ 6	50	0.800	0.800	0.804	0.800	0.413	0.040*	0.047	0.040*
		100	0.804	0.800	0.804	0.800	0.287	0.029*	0.034	0.028*
		200	0.798	0.800	0.804	0.799	0.203	0.020*	0.024	0.020*
		20	0.808	0.799	0.801	0.799	0.494	0.037*	0.045	0.037*
	≤ 10	50	0.798	0.800	0.802	0.800	0.318	0.024*	0.028	0.023*
		100	0.804	0.800	0.802	0.800	0.226	0.017*	0.020	0.016*
		200	0.799	0.800	0.801	0.800	0.160	0.012*	0.014	0.012*

Panel (b): $\sigma = 0.02$

α^*	$\delta(\%)$	N	α_1	α_2	α_3	α_4	s_1	s_2	s_3	s_4
		20	0.766	0.795	0.820	0.792	2.260	0.399*	0.465	0.399*
	≤ 2	50	0.790	0.799	0.824	0.795	1.390	0.244*	0.285	0.243*
		100	0.793	0.798	0.825	0.794	1.000	0.173*	0.204	0.173*
		200	0.787	0.797	0.824	0.794	0.707	0.122*	0.142	0.121*
		20	0.782	0.801	0.804	0.801	1.260	0.129*	0.150	0.128*
0.8	≤ 6	50	0.798	0.801	0.805	0.800	0.829	0.080*	0.093	0.080*
		100	0.803	0.802	0.805	0.801	0.569	0.058*	0.066	0.057*
		200	0.800	0.800	0.804	0.799	0.414	0.039*	0.046	0.039*
		20	0.793	0.798	0.799	0.798	1.030	0.075*	0.088	0.075*
	≤ 10	50	0.793	0.800	0.802	0.800	0.635	0.047*	0.055	0.046*
		100	0.796	0.800	0.801	0.800	0.451	0.033*	0.039	0.033*
		200	0.800	0.800	0.802	0.800	0.320	0.023*	0.028	0.023*

Panel (c): $0.005 \leq \sigma \leq 0.02$

α^*	$\delta(\%)$	N	α_1	α_2	α_3	α_4	s_1	s_2	s_3	s_4
		20	0.804	0.801	0.832	0.799	1.480	0.201*	0.304	0.257
	≤ 2	50	0.798	0.799	0.827	0.795	0.935	0.123*	0.192	0.161
		100	0.799	0.801	0.828	0.797	0.663	0.086*	0.137	0.115*
		200	0.801	0.800	0.827	0.796	0.464	0.062*	0.096	0.081
		20	0.801	0.800	0.803	0.798	0.856	0.066*	0.100	0.083
0.8	≤ 6	50	0.795	0.800	0.805	0.800	0.554	0.041*	0.063	0.053
		100	0.799	0.800	0.805	0.800	0.384	0.028*	0.044	0.037
		200	0.801	0.800	0.804	0.799	0.273	0.020*	0.031	0.026
		20	0.807	0.800	0.801	0.800	0.648	0.039*	0.059	0.050
	≤ 10	50	0.800	0.800	0.801	0.800	0.433	0.024*	0.037	0.031
		100	0.807	0.800	0.802	0.800	0.300	0.016*	0.026	0.022
		200	0.802	0.800	0.802	0.800	0.208	0.012*	0.018	0.015

* The estimator is unbiased and none of the other estimators has as a lower variance at 1% risk level.

Table 5: p-values for the F-test that variances of test statistics are the same for $P_{cum} \leq 200$ and $P_{cum} \leq 500$

($\alpha^* = 0.8$, $N = 200$, $0.005 \leq \sigma \leq 0.02$)

Tick rule 1

δ (%)	α_1	α_2	α_3
≤ 2	0.000	0.000	0.000
≤ 6	0.000	0.000	0.000
≤ 10	0.000	0.000	0.000

Tick rule 2

δ (%)	α_1	α_2	α_3
≤ 2	0.000	0.000	0.000
≤ 6	0.000	0.000	0.000
≤ 10	0.000	0.000	0.000

Continuous prices and return adjustment

δ (%)	α_1	α_2	α_3	α_4
≤ 2	0.000	0.435	0.073	0.085
≤ 6	0.000	0.309	0.054	0.352
≤ 10	0.000	0.500	0.293	0.500

Tick rule 1 and return adjustment

δ (%)	α_1	α_2	α_3	α_4
≤ 2	0.000	0.000	0.000	0.000
≤ 6	0.000	0.500	0.162	0.351
≤ 10	0.000	0.500	0.292	0.500

Tick rule 2 and return adjustment

δ (%)	α_1	α_2	α_3	α_4
≤ 2	0.000	0.000	0.000	0.000
≤ 6	0.000	0.308	0.161	0.500
≤ 10	0.000	0.199	0.291	0.500

These results suggest that the level of stock prices may be an important factor for ex-day price behaviour. Cum-day prices appear to be important when low dividend yield data is studied and always when the test statistic used is α_1.

4.4 Clientele tests with NYSE tick rules

Tables 3 and 4 demonstrate that even with 200 observations standard errors of ex-day test statistics are fairly large when dividend yield is at most 2%. Since low dividend yield data is very common in ex-day studies, and especially in tax clientele studies, the reliability of ex-day tests using low yield stocks deserves some further investigation. For example in the U.S. average dividend yields in the lowest yield decile can be lower than 0.3% and typically exceed 1% only in 3 or 4 highest yield deciles, implying that at least 60% of shares have dividend yields less than 1%. [17]

Table 6 shows the results of a clientele test simulation. The set up of the experiment is exactly as before except that this time the smallest price increment is 0.125 as at the NYSE. The average dividend yields in dividend yield deciles are taken from Michaely (1991). Using the decile means we construct the lower and upper bounds of the yield classes and for each class draw the dividend yields from a uniform distribution. There are 500 stocks in each decile and the true tax parameter is $\alpha^* = 0.8$. Cum-day stock prices are drawn from a uniform distribution with prices ranging from 5 to 100 dollars. Dividends are computed as the product of dividend yield and cum-day price and rounded so that they are always at least 0.10 and always multiples of 0.01. Stock return volatilities are drawn from a uniform distribution with $\sigma_i \sim U(0.005,0.02)$. The experiment is repeated 10000 times. Table 6 reports the results for only two statistics, α_1, which is commonly used, and α_2, which had the best performance in earlier simulations.

When the pure ex-day price can be observed, both α_1 and α_2 perform well only in the two highest deciles. In these deciles they are both unbiased and have the same standard errors. However, in the lowest seven deciles both estimators are severely biased. Since both

[17] Examples of clientele studies include Booth and Johnston (1984), Grammatikos (1989), Michaely (1991) and Robin (1991), among others. In these studies stocks are grouped according to dividend yield and the tax parameter is estimated separately for each group.

estimators have very small standard errors, the true tax parameter, α^*, falls outside the 95% confidence intervals. In addition, the average estimates give the impression that α falls with dividend yield even though α^* is constant at all dividend yields. This finding shows that tick rules can have a major effect on estimating α^*.

The poor results for the lowest yield deciles are partly explained by our requirement that dividends are not smaller than 0.10. To see that consider decile 1. Dividend yields range from 0.09 % to 0.25% and cum-day prices from 5 to 100. The highest possible dividend is then 0.25. With the average dividend yield of 0.17% the cum day price would have to be 59 to make the dividend higher than 0.10, and 74 to make the dividend 0.125. The majority of dividends are thus rounded to 0.10. With $\alpha^* = 0.8$ the true price drop would be 0.08, implying that the majority of ex-day prices are rounded down in the experiment. This explains why average ex-ratios are so large in the lowest deciles. Note, however, that setting the minimum dividend requirement to 0.125 would not solve this problem. With $\alpha^* = 0.8$ the true price drop would be 0.10 and the majority of ex-day prices would still be rounded down. The average ex-ratios would still overestimate the true tax parameter, but less than reported in table 6.

The results change dramatically when discrete ex-day prices must be adjusted for the overnight return. Both methods are unbiased at all dividend yields. However, standard errors are much larger now, and α_1 has a significantly higher variance than α_2 in all yield deciles. When prices are continuous both α_1 and α_2 perform well in all yield deciles. The standard errors of α_1 are the same for continuous and discrete prices (with return adjustment). Therefore it does not matter whether prices are continuous or discrete. This result holds for α_2 only in deciles 5 to 10. In deciles 1 to 4 the variances are higher with continuous prices than with NYSE tick, suggesting that at low dividend yields tick rules may actually be helpful.

Table 6 demonstrates that identifying a tax clientele effect is very difficult even if one has 5000 ex-day observations available (500 observations in each decile). Standard errors of the most commonly used test statistics are large and their confidence intervals wide. The evidence suggests that one can identify a tax clientele effect reliably only if average ex-ratios in the yield deciles are substantially different from one another. We get this result even when the estimator with the smallest variance is used. Further note that this result is obtained assuming a fairly modest level of stock return volatility, ranging from only 8% to 32% annually. When the annual return volatility of all stocks is assumed to be 32%, the standard errors of α_1 are almost two times as high as those in table 6.

Table 6: Simulation results with NYSE tick rules ($P_{cum} \leq 100$)

($\alpha^* = 0.8$, $N = 500$, $0.005 \leq \sigma \leq 0.02$)

		NYSE tick rule		*NYSE tick rule + return adjustment*		*Continuous prices + return adjustment*	
Decile	*Dividend yield, %*	α_1	α_2	α_1	α_2	α_1	α_2
1	0.09 - 0.25	1.117 (0.009)	1.222 (0.004)	0.799 (0.277)	0.801 (0.110)	0.799 (0.277)	0.798 (0.202)
2	0.25 - 0.39	0.947 (0.011)	1.122 (0.013)	0.802 (0.170)	0.803 (0.098)	0.801 (0.169)	0.800 (0.125)
3	0.39 - 0.51	0.893 (0.0010)	1.028 (0.017)	0.801 (0.125)	0.802 (0.085)	0.800 (0.125)	0.800 (0.095)
4	0.51 - 0.62	0.868 (0.009)	0.964 (0.017)	0.800 (0.100)	0.800 (0.071)	0.800 (0.100)	0.799 (0.074)
5	0.62 - 0.73	0.854 (0.008)	0.919 (0.015)	0.800 (0.086)	0.801 (0.064)	0.800 (0.085)	0.800 (0.065)
6	0.73 - 0.85	0.843 (0.007)	0.887 (0.013)	0.801 (0.074)	0.801 (0.055)	0.801 (0.074)	0.800 (0.054)
7	0.85 - 1.06	0.831 (0.006)	0.855 (0.010)	0.800 (0.061)	0.800 (0.048)	0.799 (0.061)	0.799 (0.048)
8	1.06 - 1.43	0.819 (0.005)	0.827 (0.007)	0.800 (0.048)	0.800 (0.036)	0.800 (0.048)	0.800 (0.036)
9	1.43 - 2.05	0.807 (0.004)	0.807 (0.004)	0.800 (0.035)	0.800 (0.026)	0.800 (0.034)	0.800 (0.025)
10	2.05 - 2.85	0.799 (0.003)	0.799 (0.003)	0.800 (0.0243)	0.800 (0.019)	0.800 (0.024)	0.800 (0.019)

Standard errors in parentheses.

5 CONCLUSIONS

We conduct a simulation experiment which grants favourable conditions for an ex-day study and examines how well do commonly used ex-day methods estimate a given implicit tax rate. Ex-day prices are assumed to reflect capital income taxes as far it is possible when prices are discrete. We examine three cases. In the first case the true ex-day price drop caused by dividend stripping can be observed and there is no need to worry about the return earned by the stock overnight. In the second case we assume that the true ex-day price can not be observed. To estimate it the overnight return earned by the stock must be estimated from past returns. In the third case stock prices are continuous and the overnight return must be estimated. We use two different tick rules. The first was used at the Helsinki Stock Exchange before 1996 and the second in 1996-98. Tick size during the former period was larger than that of the NYSE for stock whose prices exceeded an equivalent of 20 dollars, while the tick size during the latter period was always smaller than the NYSE tick of 12.5 cents.

The results show that the importance of tick rules for ex-day studies depends on the nature of the ex-day price drop that we observe. First, we find that tick rules are important if we can observe an ex-day price drop that only reflects capital income taxes, dividends and tick rules, and is not contaminated by any overnight return requiring adjustment. The standard errors of all test statistics are significantly smaller in a small tick case than in a large tick case. Therefore a smaller tick allows us to estimate the implicit tax rate more accurately than a larger tick. Furthermore, when the larger tick is used some test statistics, most notably the average ex-ratio, are systematically biased. The size of bias falls with dividend yield but is unaffected by sample size. Second, if the ex-day price drop is contaminated by overnight return which must be removed by using an estimate of normal daily return, tick rules do not matter any more. Only the amount of return uncertainty is important. The effect of errors made in estimating the ex-day price drop is far more important than the effect of any tick rules. The standard errors of the test statistics do not depend on the tick rules at all. Furthermore, removing tick requirements altogether have hardly any effect on standard errors. Thus from an ex-day study point of view it does not matter whether prices are continuous or discrete as long as ex-day prices must be adjusted by normal return. Tick rules are not important.

Not all ex-day methods are equally reliable. Knowing this can be very useful in interpreting empirical results when alternative statistics are used. We find that the GLS-estimator of the ex-ratio always performs at least as well as the other statistics largely because it is the only statistic that takes both stock return volatility and dividend yield explicitly into account. If return volatility is the same for all stocks, the model that explains ex-day stock returns by dividend yields performs equally well. The traditional ex-dividend day ratio of Elton and Gruber (1970) takes neither dividend yield nor volatility into account and therefore has the poorest performance in the simulation experiment. The standard errors of the average ex-ratio are so large that it is doubtful whether a tax parameter of any reasonable magnitude can ever be found statistically different from one at conventional significance levels and sample sizes. Gagnon and Suret (1991) have previously argued the same. The equally weighted portfolio statistic of Lakonishok and Vermaelen (1983) performs almost as well as the ex-day return model, but has higher standard errors.

We further investigated the performance of ex-day methods when NYSE tick size of 12.5 cents was applied to low dividend yield data comparable to that used earlier in tax clientele studies. The evidence suggests that even if one has access to 5000 ex-day observations, tax clientele effects can be identified reliably only if ex-day price behaviour in one decile is substantially different from that in other deciles. We get this result even when the GLS-estimator is used and when only modest level of stock return volatility is assumed.

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