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EX-DIVIDEND DAY STOCK RETURNS AND TICK RULES

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ABSTRACT: We derive a model of ex-dividend day stock price behaviour which takes price discreteness explicitly into account. Price discreteness adds to the model a new parameter, which describes how the market rounds prices, and makes the model nonlinear. Rounding is needed, because prices must change by multiples of given increments. We show that the ex-dividend ratio is a piecewise decreasing convex function of the dividend amount. For a given cum-dividend day stock price the ex-day return is a piecewise linear function of dividend yield. The effect of price discreteness on ex-day returns decreases with stock price. Therefore the cross-sectional distribution of cum-day stock prices is important for ex-day price behaviour.

JEL classification codes: G12, G35

KEY WORDS: Asset pricing, Dividends, Taxation, Tick size

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TIIVISTELMÄ: Tutkimuksessa laajennetaan mallia, joka kuvaa osakekurssien käyttäytymistä osinkolipun irtoamispäivänä eli ex-päivänä siten, että se ottaa eksplisiittisesti huomioon hintojen noteeraustarkkuuden. Noteeraustarkkuuden huomioon ottaminen lisää malliin uuden tekijän, joka kuvaa sitä, miten osakekurssi pyöristetään markkinoilla, ja tekee samalla mallista epälineaarisen. Pyöristäminen on tarpeen, koska hintamuutosten on käytännössä oltava pienimmän sallitun muutoksen kerrannaisia. Tutkimuksessa osoitetaan, että hinnan muutoksen ja osingon suhde on osingon paloittainen ja vähenevä konveksi funktio. Ex-päivän osaketuotto on osinkotuoton paloittain lineaarinen funktio kun yhtiökokouspäivän osakekurssi otetaan annettuna. Epäjatkuvien hintamuutosten vaikutus ex-päivän tuottoon alenee osakekurssin kasvaessa. Yhtiökokouspäivän osakekurssi on siten otettava huomioon arvioitaessa kurssikäyttäytymistä osinkolipun irrotessa.

1 INTRODUCTION

In their seminal paper Elton and Gruber (1970) show that ex-dividend day price drops can be used to estimate the tax rate of the marginal investor. If capital gains are taxed at a lower (higher) rate than dividend income the ex-day price drop is smaller (larger) than the dividend.[1] Tax rates on dividend income and capital gains are important in corporate finance (for example in dividend policy and capital structure issues) but information about them is difficult to get. In this respect the ex-day method, which does not assume any particular asset pricing model, can be useful. Kalay (1982) notes that estimating the implicit tax rate may be difficult if two or more groups of investors differ substantially in their tax treatment of capital income and one group of investors finds it profitable for tax reasons to either collect or avoid dividends. If trading because of the dividend is important, ex-day price drops reflect the tax rates of the investors engaged in dividend trading and not the more interesting tax rates of long-term investors. Dividend trading is expected to be more important among stocks with high yield, low bid-ask spread (transaction costs) and high liquidity.

A large number of studies has applied the ex-day methodology to data from different countries and time periods. The empirical results are mixed. Barclay (1987) examines NYSE data from a period with no income tax in the U.S., 1900-1910, and another period after the introduction of the income tax, 1962-1985. Consistent with the tax explanation in 1900-1910 investors valued (before-tax) dividends and capital gains as perfect substitutes, but in 1962-1985 the value of dividends relative to capital gains was much lower due to a tax penalty on dividend income. Poterba and Summers (1984) report similar results for two dividend tax reforms in the U.K. Michaely (1991) and Robin (1991) examine the effect of the U.S. 1986 Tax Reform Act on ex-dividend behaviour and find conflicting results. Michaely finds abnormal return behaviour consistent with short-term trading around ex-days. Also Lakonishok and Vermaelen (1983) and Booth and Johnston (1984) obtain conflicting results with Canadian data around the 1972 tax reform. Eades, Hess and Kim (1984) report statistically significant negative abnormal ex-day returns for non-

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1 This is equivalent to saying that ex-day returns increase (decrease) with dividend yield. Early empirical studies (see Campbell and Beranek (1955) and Durand and May (1960)) report that stock prices tend to fall on ex-dividend days by less than the dividend amount.

taxable cash distributions that should have no tax consequences at all. This suggests that factors other than taxes may also influence ex-day return behaviour. One such factor may be risk premia as suggested by Grammatikos (1989) and Fedenia and Grammatikos (1993). Karpoff and Walkling (1988) find strong evidence of short-term trading among high yield stocks in the U.S. after a reduction in the costs of short-term trading following the introduction of negotiable commissions in 1975, and practically no evidence of short-term trading before.

In two recent papers Bali and Hite (1998) and Frank and Jagannathan (1998) demonstrate that discrete prices may have a considerable effect on ex-day price behaviour. Frank and Jagannathan find that discrete prices cause prices in the Hong Kong stock market to fall on ex-days on average by less than the dividend amount even though neither dividends nor capital gains are taxed at all.

This paper extends the basic ex-day model to take price discreteness into account. We show that taking the tick size into account transforms the original linear model into a nonlinear one and adds to the model a new rounding rule parameter, which can be estimated. The rounding rule ensures that price changes are multiples of given minimum increments. Discrete prices make the ex-day price drop a step function of the dividend amount, and the ex-dividend ratio of Elton and Gruber a piecewise, decreasing convex function of dividend amount. The pattern of the ex-dividend ratio is independent of stock price as long as tick size remains unchanged.

The extended model implies that ex-day returns can be systematically positive or negative even in the absence of taxes. For a given cum-dividend day price ex-day return is a piecewise linear function of dividend yield. In general, the effect of price discreteness on ex-day returns decreases with stock price. Therefore the cross-sectional distribution of cum-day stock price is important for ex-day price behaviour.

The nonlinearity and nondifferentiability of the extended model make empirical work difficult. We use Finnish stock price data from 1989-95 to estimate the model by using a grid search method. The results suggest that the average ex-ratio is approximately 0.7, which is consistent with earlier ex-day studies that use Finnish data, and that the market rounding rule is difficult to estimate. Since we are unable to perform significance tests these results must be treated as illustrative only.

We start by briefly discussing the theoretical relationhip between ex-dividend day stock returns and dividend yield in a world of continuous prices in Section 2. Section 3 uses the tick rules of the Helsinki Stock Exchange to illustrate the effect of discrete prices on ex-day stock prices both graphically and via a numerical example. The Finnish tick rules are useful due to a relatively wide tick. We derive and discuss the implications of the extended model in Section 4. Section 5 reports results of elementary empirical analysis, and conclusions are drawn in Section 6.

2 THE FRAMEWORK

Define the expected before and after-tax returns, $E(r_t)$ and $E(r_{at})$, as

(1) $$E(r_t) = \frac{E(P_t) - P_{t-1} + D_t}{P_{t-1}}$$

and

(2) $$E(r_{at}) = \frac{(1-\tau_g)E(P_t - P_{t-1}) + (1-\tau_d)D_t}{P_{t-1}}$$

where

- $E(.)$ = the expectations operator
- P_t = day t stock price
- D_t = dividend per share
- τ_d = marginal tax rate on dividends
- τ_g = marginal tax rate on capital gains.

The dividend is non-zero on ex-dividend days and zero otherwise. Manipulating (2) yields

(3) $$E(r_t) = \frac{E(r_{at})}{1-\tau_g} + (1-\alpha)\frac{D_t}{P_{t-1}}$$

where $\alpha = (1-\tau_d)/(1-\tau_g)$ measures the relative value of dividends and capital gains and D_t/P_{t-1} is dividend yield. Equation (3) implies that the expected stock return on non-ex-days is simply the grossed-up expected after-tax return. On ex-days the expected return depends on the tax treatment of dividends and capital gains. Further manipulation of (3) yields the familiar result first derived by Elton and Gruber (1970)

(4) $$\frac{P_{t-1} - E(P_t)}{D_t} = \alpha .$$

Equation (3) predicts that the expected ex-day return is a linear function of dividend yield. In particular, ex-day returns are positively (negatively) related to dividend yield if dividends are taxed more (less) heavily than capital gains, that is, if $\alpha < 1$ ($\alpha > 1$). In terms of equation (4) this implies that stock prices fall on ex-days by less (more) than the amount of dividend. Only when dividends and capital gains are effectively taxed at the same rate expected ex-day returns are unrelated to dividend yield. This is equivalent to saying that the ex-day price drop equals the dividend.

If trading rules restrict the precision at which stock prices can be quoted, the results derived above may not hold any more. For example Dubofsky (1992) shows that due to NYSE and AMEX tick rules ex-day returns may be positively related to dividend yield even in the absence of taxes. In the following we demonstrate that the lowest tick rules in the Helsinki Stock Exchange have similar effects.

3 LOWEST TICK RULES AT THE HeSE

In real life trading rules of stock exchanges restrict the precision at which stock prices are quoted.[2] Table 1 shows the tick sizes applied at the Helsinki Stock Exchange (HeSE) before 1.1.1999. The tick size is a step function of stock price with four price (and tick) categories before 1.1.1996 and two price categories in 1.1.1996 - 31.12.1998. The table shows these price categories and the respective tick sizes in Finnish markkas and in approximate U.S. dollars, assuming that $1 = FIM 5. In the NYSE the most common tick size is $0.125, which is equivalent to FIM 0.625. Thus, before 1996 tick size in the HeSE was larger than in the NYSE for stocks selling for at least FIM 100 (or $20) and smaller for less expensive stocks. In 1996-98 the HeSE tick size was always less than one sixth of the NYSE tick size. From the beginning of 1999 HeSE quotes stock prices in euros and the tick size is always 1 cent which is approximately one seventeenth of a dollar.

This section illustrates first with a numerical example and then graphically how tick size affects ex-dividend day price behaviour. We show that the effect of tick rules on ex-day returns can be fairly large. For this illustration we need to choose a tick rule with a relatively large tick size. Therefore we choose the tick rule applied in the Helsinki Stock Exchange (HeSE) before 1.1.1996 to stocks in the third price category. The same reasoning can be applied to tick rules in other stock exchanges even though the effect of these rules on ex-day returns may be more modest.

Table 1

Tick Size at the Helsinki Stock Exchange

Stock price		Tick size			
		- 31.12.1995		1.1.1996 - 31.12.1998	
FIM	*(USD)*	*FIM*	*(USD)*	*FIM*	*(USD)*
0.01- 10	(- 2)	0.01	(0.002)	0.01	(0.002)
10 - 100	(2 - 20)	0.10	(0.02)	0.10	(0.02)
100 - 1000	(20 - 200)	1	(0.2)	0.10	(0.02)
> 1000	(> 200)	10	(2)	0.10	(0.02)

[2] See Angel (1997) for tick sizes in different countries and Anshuman and Kalay (1998) for a discussion of optimal tick size.

There are three assumptions implicit in equations (3) and (4). First, all investors have identical tax rates τ_d and τ_g, second, stock price drops on ex-dividend days exactly reflect the tax rates of the marginal investor so that the ex-day price drop is equal to αD_t, and finally, price changes of any size are possible. To illustrate the effect of tick rules we pick three cum-dividend prices from the third price category, 110, 200 and 1000, respectively, and let the dividend per share take different values. In this price category all price changes must be multiples of 1. Sometimes αD_t is not an integer, in which case the market has to round prices somehow. We assume that the market rounds 0.50 down to 0 and 0.51 up to 1. Thus, for example, ex-day prices of 101.51 and 101.50 are rounded to 102 and 101, respectively. This is equivalent to assuming that the ex-day price drop that actually takes place is an integer $\leq \alpha D_t + \frac{1}{2} \times tick$, so that for example the price drop corresponding to αD_t of 3.49 is 3 while the price drop corresponding to αD_t of 3.50 is 4. This rounding rule is chosen for its simplicity. Later in the paper we introduce a more general rounding rule.

We let the dividend change with increments of 0.01 and for each dividend compute the ex-day price assuming that only taxes, the tick rule and the rounding rule affect price behaviour on ex-days. Let us first examine the ex-dividend day price drop. With continuous prices the ex-day price drop would be a linear function of the dividend amount with slope coefficient equal to α. Now that we have a discrete tick rule the price drop becomes a step function of the dividend amount, where the points of discontinuity occur when the direction of rounding changes. Figure 1 graphs the ex-day price drop as a function of dividend amount when α is equal to 0.8, 1 or 1.2. Table 2 shows the calculations for $\alpha = 1$. As long as the price stays within the same price category and the same tick rule is applied the ex-day price drop is independent of price. Therefore the same pattern of price drops holds for all prices in the third price category. The width of the steps in figure 1 depends on α. As α falls the steps become wider. If $\alpha = 1$ and $D_t = 0.51$ the price drop is 1, but if $\alpha = 0.8$ the price does not change at all. On the other hand, if $\alpha = 1.2$ a dividend of 0.425 is enough to make the price fall by 1. The deviation of the actual price drop from αD_t measures the noise caused by the tick size.

Figure 2 shows the ex-day ratios as functions of dividend amount. With given α and continuous prices ex-ratios would be constant for all dividends. With discrete prices ex-ratio is a piecewise convex function of dividend. Again, for a given α, this pattern is independent of stock price as long as prices stay in the same price category and the same tick size applies. A comparison of figures 2b and 2c shows that a larger α implies a wider range for the values that ex-ratios can take. Ex-day studies have traditionally recognised that small dividends are a problem and figure 2 shows why. The range of values that the ex-ratio can take is wide for small dividends. Therefore the risk of small dividend samples yielding misleading results is considerable. Figure 2 also shows that there is no single definition of a small dividend. From an ex-day point of view a small dividend is not the same as a small D_t. Whether a dividend is small depends on αD_t and the stock price which determines the tick size. The vertical difference between the ex-ratio and the constant α is the noise caused by the tick rule.

Table 2

An Illustration of the Effect of Tick Rules on Ex-Dividend Day Ratios and Returns in the Absence of Taxes

		Cum-dividend day stock price							
		P_{cum} = 110				P_{cum} = 1000			
Dividend	Ex-day price drop	Dividend yield, %	P_{ex}	Ex-Ratio	Ex-day return, %	Dividend yield, %	P_{ex}	Ex-Ratio	Ex-day return, %
0.50	1	0.45	109	2.00	-0.45	0.05	999	2.00	-0.05
0.75	1	0.68	109	1.33	-0.23	0.08	999	1.33	-0.03
1.00	1	0.91	109	1.00	0.00	0.10	999	1.00	0.00
1.25	1	1.14	109	0.80	0.23	0.13	999	0.80	0.03
1.49	1	1.35	109	0.67	0.45	0.15	999	0.67	0.05
1.50	2	1.36	108	1.33	-0.45	0.15	998	1.33	-0.05
1.75	2	1.59	108	1.14	-0.23	0.18	998	1.14	-0.03
2.00	2	1.82	108	1.00	0.00	0.20	998	1.00	0.00
2.25	2	2.05	108	0.89	0.23	0.23	998	0.89	0.03
2.49	2	2.26	108	0.80	0.45	0.25	998	0.80	0.05
2.50	3	2.27	107	1.20	-0.45	0.25	997	1.20	-0.05
2.75	3	2.50	107	1.09	-0.23	0.28	997	1.09	-0.03
3.00	3	2.73	107	1.00	0.00	0.30	997	1.00	0.00
3.25	3	2.95	107	0.92	0.23	0.33	997	0.92	0.03
3.49	3	3.17	107	0.86	0.45	0.35	997	0.86	0.05
3.50	4	3.18	106	1.14	-0.45	0.35	996	1.14	-0.05
3.75	4	3.41	106	1.07	-0.23	0.38	996	1.07	-0.03
4.00	4	3.64	106	1.00	0.00	0.40	996	1.00	0.00
4.25	4	3.86	106	0.94	0.23	0.43	996	0.94	0.03
4.49	4	4.08	106	0.89	0.45	0.45	996	0.89	0.05

Figure 1: Dividend and Ex-Day price Drop

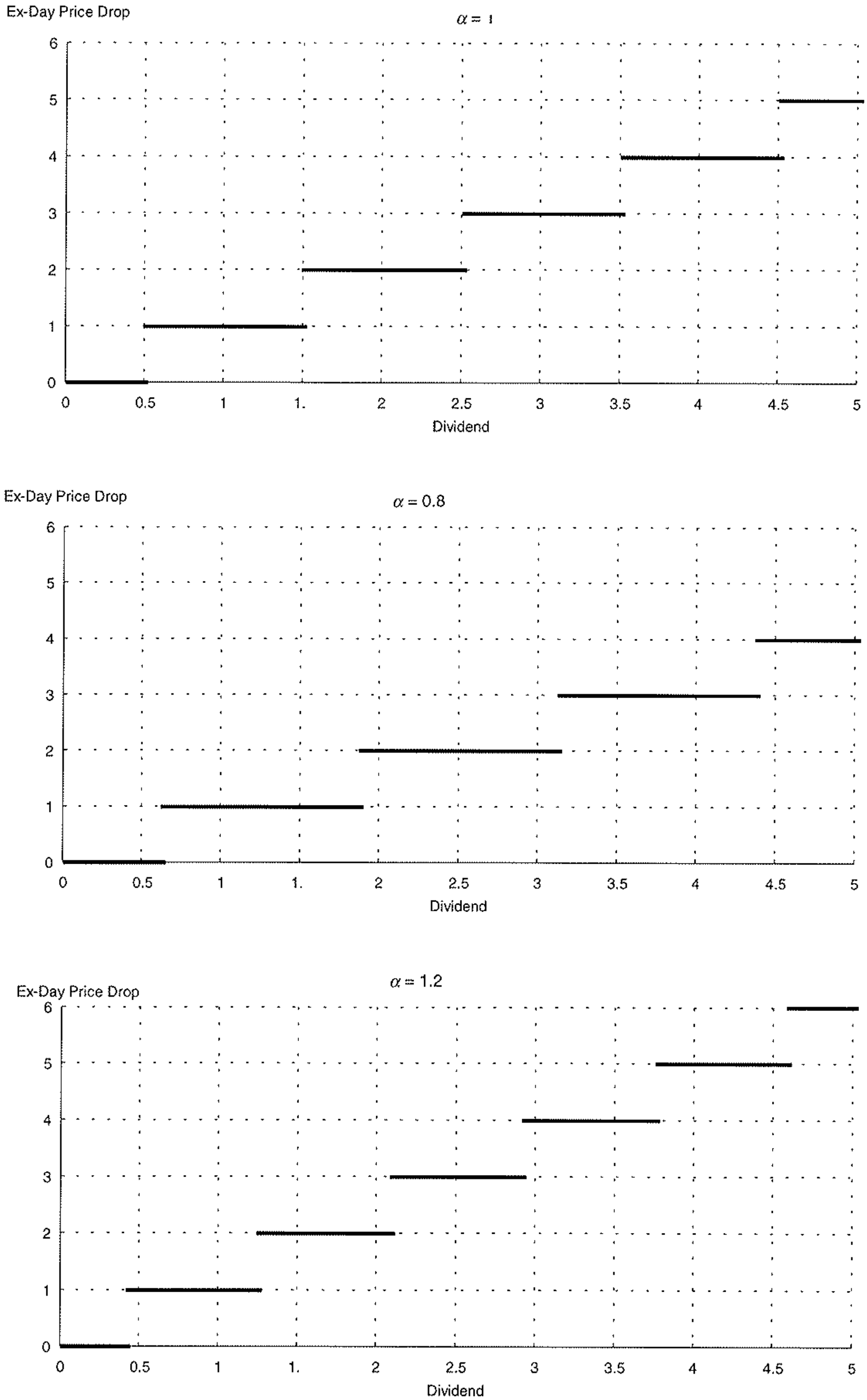


Figure 2: Dividend and Ex-Day Ratio

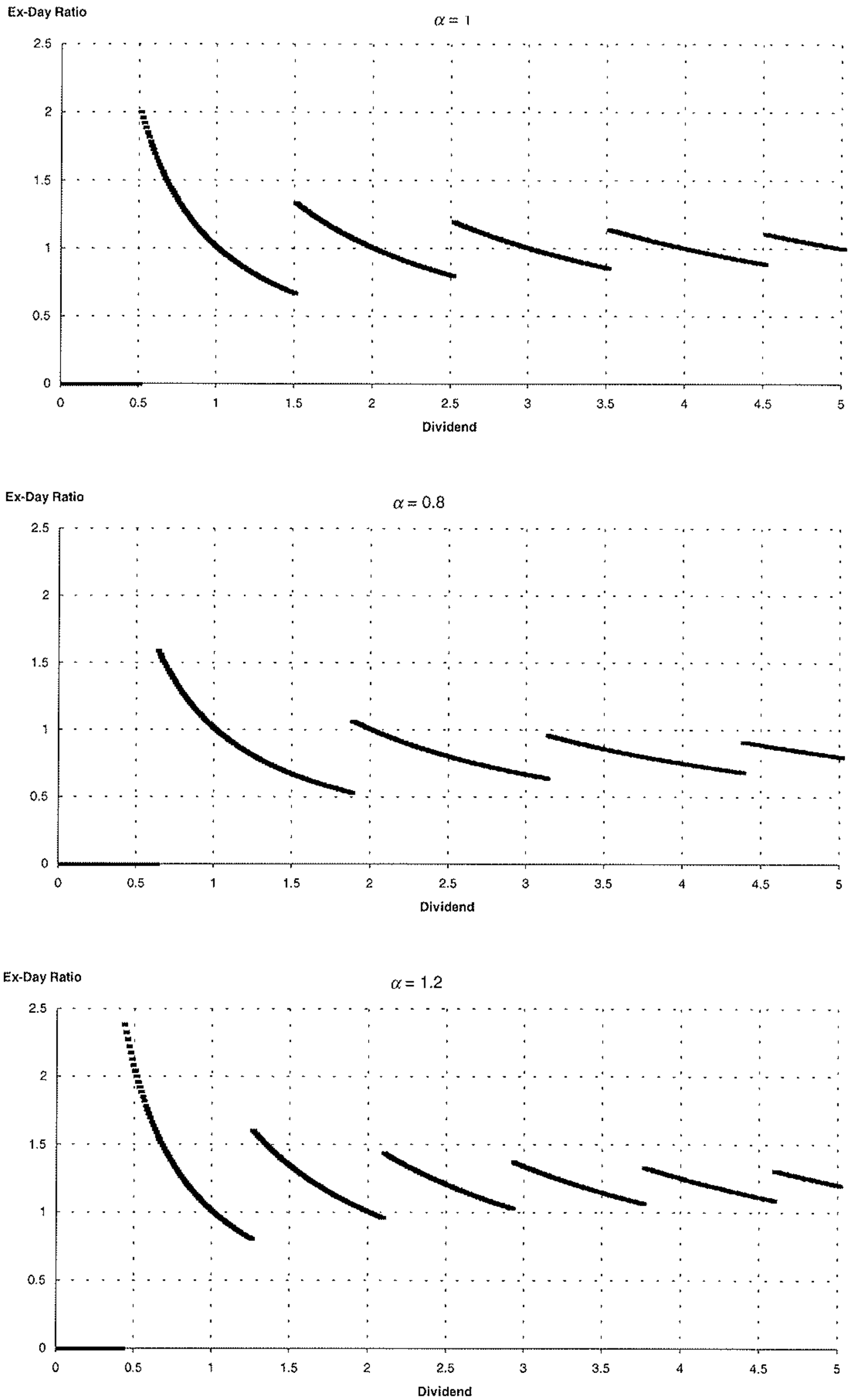


Figure 3: Dividend and Ex-Day Return

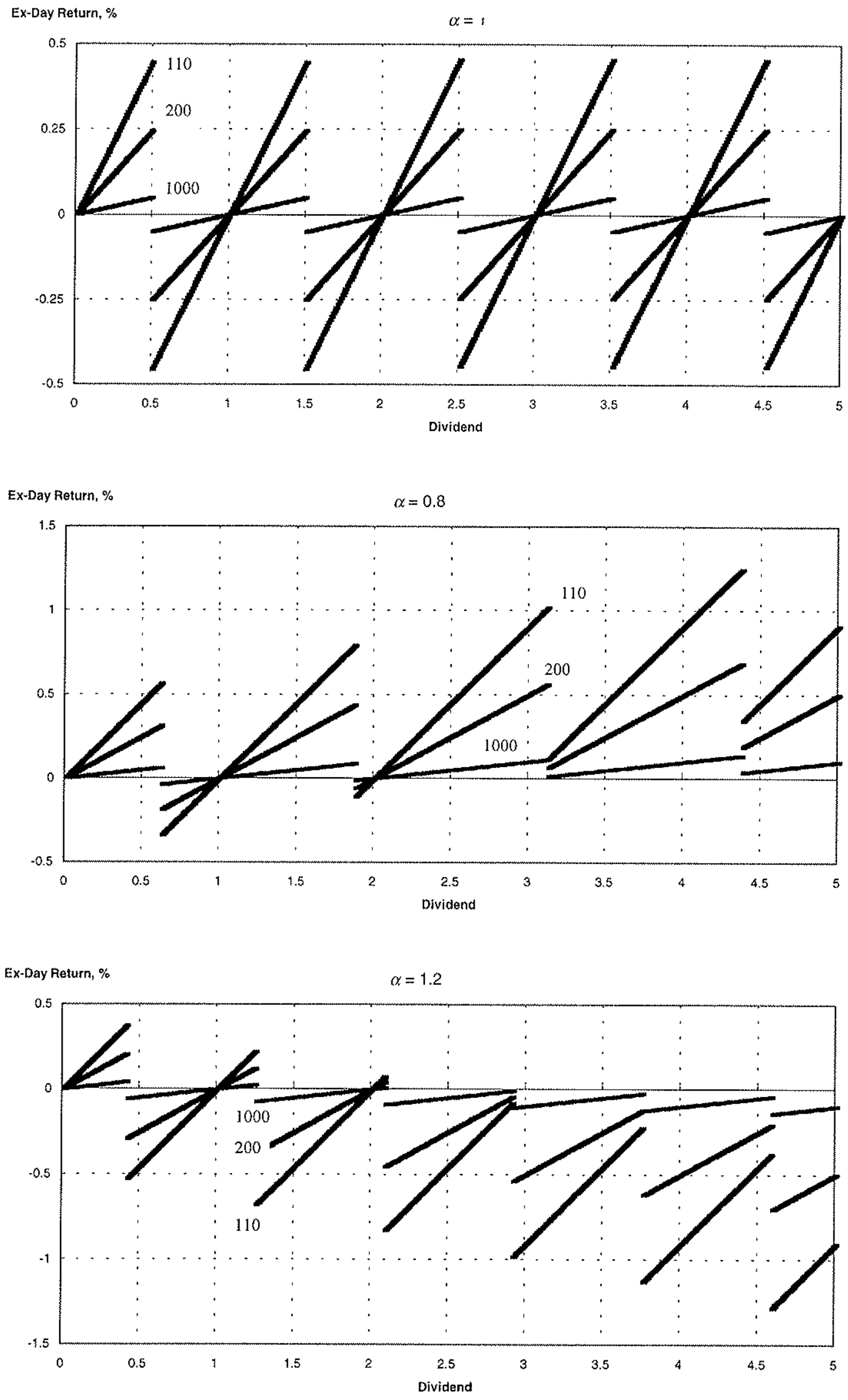
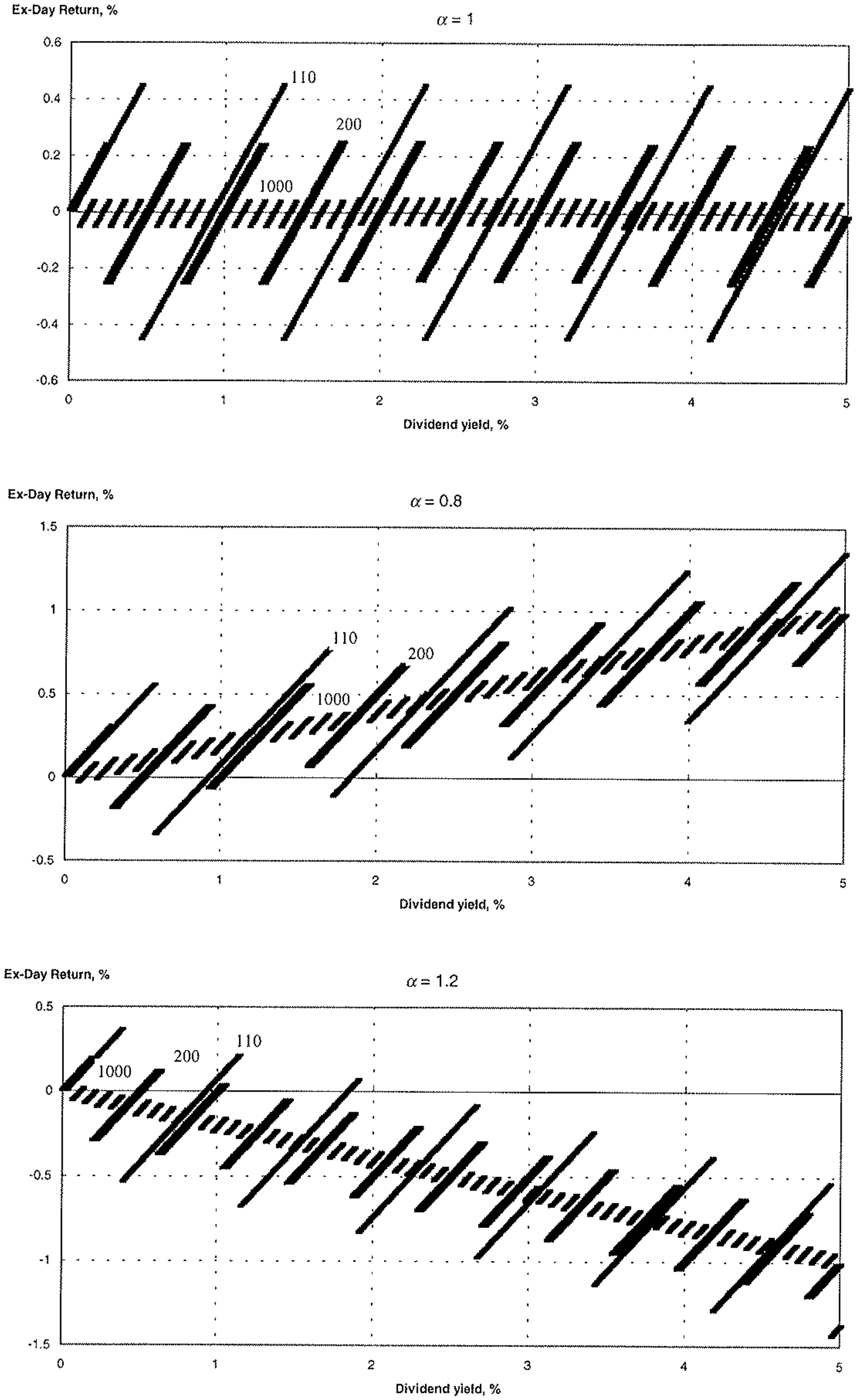


Figure 4: Dividend Yield and Ex-Day Return



Next we turn to examine ex-day returns. Figure 3 shows ex-day returns as a function of dividend amount. The usage of dividend amount in the horizontal axis is somewhat unorthodox but it illustrates the effect of tick rules on ex-day returns very nicely. With continuous prices and $\alpha = 1$ we would expect that ex-day returns are zero. Figure 3 shows that with discrete prices, for a given cum-day stock price, ex-day return is a piecewise linear function of the dividend amount. The slope of the linear segments falls with stock price. The segments are steep for $P_{cum} = 110$ and flat for $P_{cum} = 1000$. This suggests that the noise caused by the tick rule on ex-day returns is smallest for stocks with a small relative tick size. Table 2 shows that the noise is only 0.05% for $P_{cum} = 1000$ but almost 0.5% for P_{cum} close to 100. The (cross-sectional) cum-day price distribution is therefore important even though it is usually ignored in ex-day studies.

To transform figure 3 from a plot of returns and dividends to one of returns and dividend yields we only have to rescale the horizontal axis by dividing dividend amounts by the relevant (cum-day) stock prices. This means that the relatively flat linear segments in figure 3 of expensive shares will be packed more tightly than the steeper segments of less expensive shares. The result is shown in Figure 4. For a given cum-price ex-day returns are a piecewise linear function of dividend yield and the linear segments are different for stocks with different prices. Figure 4 shows that the tick rule adds a regular sawtooth pattern around the linear relationship predicted by equation (3).

Every point in figure 4 shows one possible combination of ex-day return and dividend yield for a given cum-day stock price. There is a different pattern of ex-day returns for every price. A sample of real data is simply a collection of these points with some additional noise. For example, assume that $\alpha = 1$ as in figure 4a. One can imagine several different samples of points taken from the plot in 4a. Because of the tick rule some of the samples may give the impression that ex-day returns increase or decrease with dividend yield even though ex-day returns are not related to dividend yield for tax reasons. Furthermore, because of the tick rule the data could suggest that $\alpha < (>) 1$ even though quite the opposite is true.

The regular ex-day return patterns in figures 3 and 4 show that the effect of tick rules over and above the effect of taxes on ex-day returns is regular and therefore predictable. To estimate α properly we must explicitly model the tick size. Section 4 shows how it can be done.

4 EX-DAY RETURN MODEL IN THE PRESENCE OF TICK RULES

In the absence of tick rules the ex-day price drop is exactly αD_t and the expected ex-day price (dropping the expectation operator) is

(5) $$P_t = P_{t-1} - \alpha D_t .$$

With a discrete price rule prices can not always fall exactly by αD_t. For simplicity, assume that prices have to be integers so that the smallest price change is ± 1 [3] and denote the integer part of P_t by $P_t^* = \langle P_t \rangle$.[4] The ex-day price with discrete prices is then given by

(6) $$P_t^* = \langle P_{t-1} - \alpha D_t + \delta \rangle$$

where δ is the rule that the market uses to round prices to satisfy tick rules. For example, let P_{t-1} = 105 and αD_t either 2.99 or 3.01. With continuous prices P_t would be either 102.01 or 101.99. If ex-day price is always rounded down (and the price drop up), $\delta = 0$ and P_t^* is either 102 or 101. If prices are rounded up (and the price drop down) δ is slightly smaller than the tick size. With tick size of 1 we can set $\delta = 0.99$, in which case P_t^* is either 103 or 102. Finally, if 0.5×*tick* is rounded down and 0.51×*tick* up, as in our examples in section 3, δ is slightly smaller than ½×*tick*. If for example $\delta = 0.49$, we obtain P_t^* = 102 for both dividends.

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3 This assumption is not restrictive. For example, if the smallest price change is ± 0.1, we can always multiply the prices and dividends by 10 after which prices must be integers with smallest price change equal to ± 1.

4 The integer part of P_t is the largest integer $\leq P_t$. Respectively P_t - $\langle P_t \rangle$ is called the fractional part of P_t (see Kolmogorov and Fomin (1975)).

Note that we can not take P_{t-1} in (6) out of the brackets, because generally

$$P_{t-1} + \langle \alpha D_t + \delta \rangle \neq \langle P_{t-1} - \alpha D_t + \delta \rangle.$$

In the left hand side the integer is taken of the price drop and not the price, as in the right hand side, and these two are generally not the same thing. The ex-dividend day price drop is simply

(7) $$P_t^* - P_{t-1} = \langle P_{t-1} - \alpha D_t + \delta \rangle - P_{t-1}.$$

We can write the ex-dividend day stock return in the presence of tick rules, which we denote by r_t^*, as a sum of two components, the ex-day return with continuous prices and a correction term, that is

(8) $$\frac{P_t^* - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} + \frac{P_t^* - P_t}{P_{t-1}}.$$

Letting γ denote the normal return on non-ex-days equation (3) can be rewritten as

(9) $$r_t = \gamma + (1-\alpha)\frac{D_t}{P_{t-1}}.$$

Inserting equations (5), (6) and (9) into (8) we can write the ex-day return in the presence of the tick rule and the rounding rule, δ, as

(10) $$r_t^* = \gamma + (1-\alpha)\frac{D_t}{P_{t-1}} - \frac{(P_{t-1} - \alpha D_t) - \langle P_{t-1} - \alpha D_t + \delta \rangle}{P_{t-1}}.$$

Further manipulation of (10) yields the formula for the ex-ratio

(11) $$\frac{P_{t-1}(1+\gamma) - P_t^*}{D_t} = \alpha + \frac{(P_{t-1} - \alpha D_t) - \langle P_{t-1} - \alpha D_t + \delta \rangle}{D_t}.$$

A number of implications arise. First, the tick rule adds to the ex-day return and ratio models (3) and (4) an additional term, P_t - P_t^*, which has two components. The first term, P_{t-1} - αD_t, is the ex-day stock price with continuous prices assuming that ex-day price drops reflect the value of dividends. The second term, $\langle P_{t-1}$ - $\alpha D_t + \delta \rangle$, is the same ex-day stock price with discrete prices. The difference of the two terms is the effect of tick rules on ex-day price when the cum-day price, tax rates, the dividend and the rounding rule, δ, are taken as given. It is this difference that generates the zigzag pattern of ex-day returns in figure 4. If the discrete price rule has no effect on the ex-day price, the additional term vanishes and (10) and (11) reduce to equations (3) and (4) again. Secondly, P_t - P_t^* is always smaller than the tick. Assume that the tick is 1 as in the Finnish case. If $\delta = 0.499$, the maximum value $|P_t - P_t^*|$ can take is 0.5. The maximum effect of tick size on ex-day returns is 0.05% when $P_{t-1} = 1000$ and almost 0.5% when P_{t-1} is close to 100. If $\delta = 0$ (the market rounds prices down) or $\delta = 0.999$ (the market rounds prices up) $|P_t - P_t^*|$ can not exceed 1. The maximum contribution of tick size on ex-day returns is therefore 0.1% when $P_{t-1} = 1000$ and almost 1% when P_{t-1} is close to 100. Since the dividend amount is always smaller that the cum-day price, the effect of tick size is larger on ex-ratios than ex-day returns. Third, as figure 4 demonstrates, if $\alpha = 1$ ex-day returns can be systematically positive or negative. If $\alpha > 1$ equation (3) predicts negative ex-day returns, but (10) can explain also positive returns. Respectively, equation (3) predicts positive ex-day returns if $\alpha < 1$, while equation (10) implies that negative returns are not excluded with discrete prices. Finally, accounting for the tick changes the original linear model into a nonlinear one, which due to the integer term is not differentiable everywhere. The nonlinearity and nondifferentiability make empirical testing of (10) and (11) difficult.

5 EMPIRICAL ANALYSIS

To get some idea of the effect of tick rules on ex-day returns we perform a simple empirical exercise. We collect stock price and dividend data from the HeSE in 1989-95.[5] The data consists of restricted stocks before 1993 and unrestricted after that. We require that stocks in the sample have closing quotes on cum-days and opening quotes on ex-days. No previous price history is required.

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5 The results of Sorjonen (1995) and Sorjonen (1999) suggest that in spite of changes in capital income taxation ex-day behaviour during this time period is sufficiently homogeneous to warrant this combined time period.

Table 1 shows the tick rule applied during the data period. It also shows that after excluding stocks priced less than 1 markka[6], we obtain, by multilying the price limits and tick size of one category by ten, the price limits and tick size of the next category. We can therefore scale all prices and dividends so that all stocks belong to the same price category and have the same absolute tick size. The scaling of prices and dividends does not change anything in terms of equations (3) and (4), neither does it change the relative tick size. We transform stocks in the two lowest price categories into third price category stocks by multiplying the prices and dividends of lowest price category stocks by 100 and those of second price category stocks by 10. After that the prices of all stocks are between 100 and 1000. Finally, we remove one stock with an ex-day price less than 100. There are no highest price category stocks in our sample.

The starting points for empirical analysis are equations (10) and (11). For simplicity we assume that $\gamma = 0$. We also simplify notation by dropping the time subscripts so that P_i denotes the cum-day price of stock *i*. The empirical models can now be written as

$$r_i^* = (1-\hat{\alpha})\frac{D_i}{P_i} + \varepsilon_i \tag{12}$$

and

$$r_i^* = (1-\hat{\alpha})\frac{D_i}{P_i} - \frac{(P_i - \hat{\alpha}D_i) - \langle P_i - \hat{\alpha}D_i + \hat{\delta}\rangle}{P_i} + \varepsilon_i . \tag{13}$$

We first estimate (12), without a constant term, with ordinary least squares. The residuals are both heteroskedastic and non-normal. Heteroskedasticity is expected because different stocks have different volatilities. To reduce non-normality we remove eight observations for which the standardised residual exceeds 2.6. After that we estimate (12) again. We obtain the following result:

$$\hat{\alpha} = 0.6676, \quad s(\hat{\alpha}) = 0.0464, \quad R^2 = 0.270, \quad N = 210$$

where $s(\hat{\alpha})$ is the standard error of $\hat{\alpha}$. The tax parameter is smaller than one and statistically significant, which suggests a preference for capital gains. The estimate is

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6 This is not a very restrictive assumption. In the HeSE there are very few stocks in the lowest and highest price category.

reasonably close to estimates of earlier studies that use Finnish data. The model is still heteroskedastic, but residual non-normality can be rejected at 5% risk level.[7]

Model (13) is nonlinear and not differentiable everywhere. Also both parameters that we want to estimate are inside the integer term. Since analytic solutions for the estimators of α and δ are difficult to obtain, we use a grid search method. We let δ obtain values from 0 to 0.99 with steps of 0.001 and α obtain values from 0 to 1.5 with steps of 0.0001. We repeat the search with smaller step sizes. For each parameter combination we compute the sum of squared errors, given by

$$\sum_{i=1}^{N}\left[r_i^* - (1-\hat{\alpha})\frac{D_i}{P_i} - \frac{(P_i - \hat{\alpha}D_i) - \langle P_i - \hat{\alpha}D_i + \hat{\delta}\rangle}{P_i}\right]^2, \tag{14}$$

where N is sample size. The parameter combination that minimises this sum is chosen. We obtain the following parameter estimates:

$$\hat{\alpha} = 0.6821, \quad \hat{\delta} = 0.981, \quad N = 210.$$

The estimate of α is very close to that of the linear model. The estimate of δ, 0.981, is very high. It suggests that the market on average tends to round ex-dividend day stock prices up. This in turn implies that the tick size potentially has a large impact on ex-day returns. However, since we are unable to compute standard errors and conduct standard significance tests, we can not say whether the estimate of delta is statistically different from say, 0.49. Therefore the empirical results have mostly illustrative value.

We re-estimate both models for a subsample of stocks whose dividends are multiples of tick size. The results of the linear model are

$$\hat{\alpha} = 0.7244, \quad s(\hat{\alpha}) = 0.0564, \quad R^2 = 0.286, \quad N = 137$$

and the results of the nonlinear model

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7 The value of the Breusch-Pagan test statistic is 19.64, which is highly significant, while the value of the Bera-Jarque test statistic is 5.892.

$$\hat{\alpha} = 0.7093, \quad \hat{\delta} = 0.301, \quad N = 137.$$

Again the tax parameter in the linear model is significantly less than one. The tax parameter estimates of the linear and the non-linear models are very close to one another, and also very close to the ones obtained for the entire sample. However, the estimate of the rounding rule parameter is only 0.3 for the subsample, which is considerably less than 0.98 of the entire sample. The huge gap between the estimates suggests that the rounding rule is probably very sensitive to small changes in the tax parameter and therefore potentially very difficult to estimate.

6 CONCLUSIONS

Elton and Gruber (1970) were the first to show that stock prices on ex-dividend days can be used to infer the relative tax treatment of dividends and capital gains. They and subsequent research has shown that for given tax rates the ex-dividend day price drop is a linear function of the dividend amount. It follows that, first, the ex-dividend ratio, that is, the ex-dividend day price drop divided by the dividend, is constant for all dividend amounts and independent of stock price, and second, ex-dividend day stock returns are a linear function of dividend yield. This model implies that if dividends are taxed more (less) heavily than capital gains, stock prices on ex-days fall by more (less) than the dividend amount, and ex-day stock returns are positively (negatively) related to dividend yield.

This paper extends the basic ex-day model to take tick rules into account. We show that taking the tick size into account adds to the model a new parameter, which describes how the market rounds prices. Rounding is needed, because prices must change by multiples of given increments. At the same time the original linear model transforms into a nonlinear one.

The extended model yields a number of predictions. The ex-day price drop becomes a step function of the dividend amount. The points of discontinuity occur when the direction of rounding changes. It follows that the ex-dividend ratio is a piecewise, decreasing convex function of dividend amount. For given tax rates this pattern is independent of stock price as long as the same tick size applies. For small dividends the ex-ratio can take a wide range of values, which suggests that the odds of getting misleading results must not be ignored. This is not a new result; ex-day studies have traditionally recognised that small dividends are a problem. What is a new result, however, is that there is no simple definition of a small dividend. We demonstrate that whether a dividend is small depends not only on the dividend amount, but also on tax rates and the cum-day stock price that determines the tick size.

For a given cum-day price ex-day return is a piecewise linear function of dividend yield. The effect of tick size on ex-day returns is measured by the height of the linear segments. In general, the tick effect decreases with stock price. Therefore the (cross-sectional) cum-day price distribution is important for ex-day price behaviour, even though it has been ignored in earlier ex-day studies.

The extended ex-day return model implies that ex-day returns can be systematically positive or negative even in the absence of taxes. If dividends are taxed more (less) heavily than capital gains, the basic model predicts that ex-day returns are positive (negative), but the extended model predicts that when tick size is taken into account the possibility of negative (positive) ex-day returns can not be excluded.

The nonlinearity and nondifferentiability of the extended model make empirical work difficult. We use Finnish stock price data from 1989-95 to estimate the model by using a grid search method. The results suggest that the average ex-ratio is approximately 0.7 and that the market rounding rule is difficult to estimate. Since we are unable to perform significance tests these results must be treated as illustrative only.

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