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## AN OVERLAPPING-GENERATIONS SIMULATION MODEL FOR THE LITHUANIAN ECONOMY*

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**ABSTRACT:** The paper describes a dynamic general equilibrium simulation model with an overlapping-generations structure. It was built as a part of the Phare ACE research project, whose purpose is to identify and analyse the current and potential future problems with the pension system, and analyse and propose alternatives to alleviate the problems. The model is an open-economy version of the well-known Auerbach - Kotlikoff type of models, extended to include a pension sector describing the Lithuanian public pension system. Other particularly Lithuanian features in the model are mortality rates and population structure, and taxation both from incentive and tax revenue points of view. The model has been calibrated so that public debt, net foreign debt, labour supply profiles by cohorts, pension - wage relations, and export share of output are close to those observed in Lithuania in 1997. Technically, the model is an offspring of the FOG model that has been widely used in pension, social policy and taxation analysis in Finland.

**KEY WORDS:** Dynamic computable general equilibrium model, overlapping generations

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**TIIVISTELMÄ:** Liettuan eläkejärjestelmän kehitysvaihtoehtoja tutkivan Phare ACE projektin yhteydessä on rakennettu dynaaminen yleisen tasapainon simulointimalli. Se on avotalouden versio Auerbachin ja Kotlikoffin aloittamasta limittäisten sukupolvien numeerisesta mallintamistavasta. Mallissa on merkittävänä osana Liettuan julkista eläkejärjestelmää kuvaava eläkesektori. Myös verojärjestelmä, sekä kotitalouksille että yrityksille aiheutuvien kannustinvaikutusten että fiskaalisten tekijöiden osalta, kuvaa Liettuan taloutta. Malli on kalibroitu niin, että julkinen velka, ulkomainen nettovelka, työn tarjonta kohorteittain, palkkojen ja eläkkeiden suhde ja viennin osuus taloudessa ovat lähellä Liettuan tilannetta vuonna 1997. Teknisesti malli pohjautuu Suomen eläke- ja sosiaalipolitiikan ja verotuksen analyysissa laajalti käytettyyn FOG-malliin.

**AVAINSANAT:** Dynaaminen tasapainomalli, limittäiset sukupolvet, numeerinen simulointi.

# 1. INTRODUCTION[1]

This paper describes a dynamic general equilibrium simulation model with an overlapping-generations structure. It was built as a part of the Phare ACE research project "Lithuanian Pension Systems: Alternatives and Proposals for the Future". The purpose of the project was to identify and analyse the current and potential future problems with the pension system, and analyse and propose alternatives to alleviate the problems. The results are reported in Phare Study Group (1999): Lithuanian Pension System: Alternatives and Proposals for the Future. A Summary Report. The model analysis is presented in more detail in Lassila, J. (1999): Pension Policies in Lithuania - A Dynamic General Equilibrium Analysis. ETLA Discussion Paper No. 670.

The model is an open-economy version of the type of dynamic simulation models originated by Auerbach - Kotlikoff (1987). Households maximise their lifetime utilities and firms' maximise the value of their shares, both under perfect foresight. Thus the behaviour of agents is derived from structural parameters of preferences and technology. Numerical overlapping-generations models have become a standard tool in pension policy analysis in market economies. Time-scale in these models is long, several decades. This is necessary when one tries to evaluate the effects of pension policies on different generations. Technically, the Lithuanian model is an offspring of the FOG model that has been widely used in pension, social policy and taxation analysis in Finland, see e.g. Lassila - Palm - Valkonen, 1997, and Lassila - Valkonen, 1999.

Some features, especially the pension system, are particularly Lithuanian in this model. Taxation, both from households' and firms' point of views, and from the tax revenue angle, resembles the Lithuanian situation in 1997. Mortality rates and thus the population age structure are derived from Lithuanian statistics. The model has been calibrated so that public debt, net foreign debt, labour supply profiles by cohorts, pension - wage relations, and export share of output are close to those observed in Lithuania in 1997. The parameter values were taken from studies concerning other countries, as no studies concerning crucial parameter estimates have been carried out in Lithuania.

Section 2 describes the structure of the model in general and by sectors and markets.
Section 3 discusses the choice of parameter values and considers the calibration of the model to the Lithuanian economy.

1 The model programming was done by Eija Kauppi. The author also wishes to thank Romas Lazutka, Audrone Morkuniene and Svend E. Hougaard Jensen, the partners in the Phare research project, and Tarmo Valkonen for useful comments and suggestions.

# 2. THE MODEL

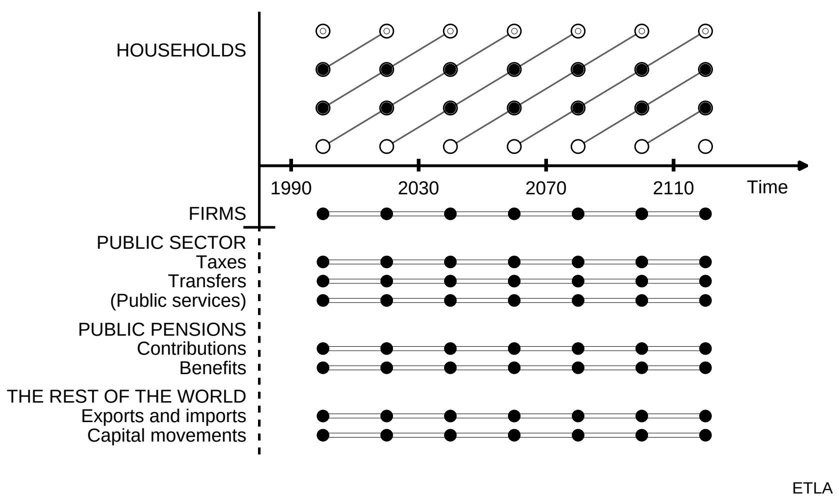
## 2.1 General

The model is an Auerbach-Kotlikoff-type, perfect foresight numerical overlapping generations general equilibrium model. It has five sectors: households, enterprises, a government, a pension sector and a foreign sector. In some simulations a sixth sector is introduced: a private pension sector with voluntary or mandatory fully funded pensions. The goods, labour and capital markets are competitive and prices balance the demand and supply period-by-period.

The adjacent graph describes the structure of the model. The upper part denotes overlapping generations of households. Each dot represents twenty years (actually 5 years equals a unit period in the model). A white dot denotes childhood, two black dots working years, and the circumscribed circle represents years in retirement. Each generation lives through all these phases, and lifecycles go from south-east to north-west. The lower part denotes sectors other than households.

**Figure 2.1.**

**Chart 2. Model Structure**




There are several directions in the graph. Households plan their lives looking forward in their lifecycles. Expectations concerning the future affect current decisions. The retirement age affects behaviour already in working years. Firms also behave in a forward-looking fashion, horizontally in the graph. Investment, employment and production decisions are partly based on expectations. Vertically, we notice the connections and restrictions within each period. Labour supply decisions by the households must be met by the demand for labour by firms and the public sector. Households' demand for consumer goods must be met by firms' production and imports. Tax receipts and public borrowing must cover public expenditure, etc. Market equilibrium conditions and budget restrictions are thus the things presented vertically. All these decisions and restrictions, to all directions, must be fulfilled in the model solution.

## 2.2 Households

The economy has overlapping generations of households, each maximising lifetime utility with respect to consumption and leisure. Their maximisation problems can be presented as follows:

$$(1) \qquad \max_{c,l} \qquad \sum_{t=1}^{T} \frac{1}{1-\frac{1}{\gamma}} \frac{1}{(1+\delta)^{t-1}} U_t^{1-\frac{1}{\gamma}}$$

subject to the budget constraint which equals discounted wage income, pensions and transfers to discounted consumption expenditure:

$$(2) \qquad \sum_{t=1}^{T_w} \left[(1-l_t)e_t w_t (1-s^h \tau_t^e) - \tau_t^w (1-l_t e_t w_t - D_t^0)\right] R_t + \sum_{t=T_R}^{T} s^h Z_t R_t$$
$$+ \sum_{t=1}^{T} S_t R_t = \sum_{t=1}^{T} c_t p_t^C (1+\tau_t^C) R_t$$

and subject to the determination of earnings-related pensions $Z$. The periodic utility $U_t$ depends on consumption and leisure:

$$(3) \qquad U_t = (C_t^{1-\frac{1}{\rho}} + \alpha l_t^{1-\frac{1}{\rho}})^{\frac{1}{1-\frac{1}{\rho}}}$$

$R_t$ is the discount factor:

$$(4) \qquad R_t = n_t \prod_{s=0}^{t-1} \frac{1}{1+r_s^d} \qquad\qquad t = 1,\ldots T$$

where $c_t$ is consumption, $p_t^C$ denotes consumer prices, and $l_t$ is leisure. Of the constant parameters, $\gamma$ is the elasticity of intertemporal substitution, $\delta$ is the rate of time preference and $\rho$ is the elasticity of substitution between consumption and leisure. Households leave no bequests and receive none. The household age-specific transfers $S_t$ contain all social benefits, also pensions other than earnings-related pensions Z. On average a share $s^h$ of households pay pension contributions and thus receive pensions. The parameters $e_t$ describe age-dependent work efficiency, and make the age-earnings profile hump-shaped.

The parameters $\tau^w$ and $\tau^c$ describe tax rates on wage income (above a limit) and consumption. $\tau^e$ is the employee's pension contribution rate.

A full lifecycle contains $T = 14$ five-year periods. The life-cycle plan is made at the age of 20. Households start their adult lives with zero wealth, and are liquidity constrained: wealth cannot be negative at any time. Lifetimes are uncertain but follow given constant probabilities. Households, in fact, maximise expected utility. The population is large enough so that every period a given fraction of each cohort dies with probability one, so there is no uncertainty at the aggregate level. Following Yaari (1965), we assume that insurance companies pay premiums to households on their assets and receive these assets if the household dies. With perfect competition in the insurance industry, their profits are zero. The premiums that households receive vary with age. The survival probabilities $n_t$ are included in the lifetime utility function and in the discount factor formula, and depend on dying probabilities $s_j$ as follows:

(5) $$n_t = 1 \qquad t = 1$$

$$n_t = \prod_{j=1}^{t-1} (1 - s_j) \qquad t = 2, \ldots T$$

For the calculation of the pensions, we need to define the average insured income from working (from which the contributions are collected). It is defined as follows:

(6) $$D_t = \frac{1}{\sum_i N_i} \sum_{i=1}^{T_w - 3} N_i (1 - l_t^i) e_t^i w_t$$

where $i$ refers to age groups and $N$ to the number of workers in the group. The pension formula is:

(7) $$Z_t = B_t + \Phi s k D_t$$

where $B$ stands for the basic pension, $s$ stands for the length of the person's social insurance record of working under labour contract, and $k$ is the ratio of the person's income over the country's average income. We thus model pension determination as it has been since January 1, 1994 (see Morkuniene, 1998). There is a maximum of 5 for $k$ in practise, but it is not relevant for the average household described by the model. The parameter $\Phi$ is 0.005, implying that 0.5 per cent of the wage of the employee is added annually to the supplementary component of the future pension.

The accrued pension rights are indexed to wages and employment through $D$. Also $B$ is assumed to be proportional to $D$ in time, but this indexation is changed in some simulations.

The number of working periods $T_w$ is 10; people can work until they are 70 years old. The official retirement age $T_R$ is lower than that, 57.5 years initially, and increased gradually to 65 in some simulations. People in the age groups 55-59, 60-64 and 65-69 participate in the labour market despite being past the retirement age. Labour supply in those cohorts resembles the participation rates, when supply is measured in hours. It is assumed that the elderly are not as efficient as younger cohorts, so labour supply in effective units is rather small in these older cohorts. That is why these groups are dropped from (6). The efficiency is reflected in the lower wages they receive.

Pensions can be received while working, but working after the retirement age lowers pensions. For instance, in the age group 60 - 64 the pension is:

(8) $$Z_t = B_t + \Phi skD_t\left[1 - v_t(1 - l_t)\right]$$

where the parameter *v* determines how much working reduces the earnings-related part of the pension.

Although the model uses five-year unit periods, the retirement age can be within one such period. In that period the person has work income as usual, pensions for the part he or she is retired, and working after retirement reduces the earnings-related part of the pension. Denote the share of working in that period by g, which is between zero and one. The pension is now

(9) $$Z_t = (1 - g_t)B_t + (1 - g_t)\Phi skD_t\left[1 - v_t(1 - l_t)\right]$$

Increases in the retirement age are modelled through *g*. Initially *g = 0.5* in the age group 55 - 59, and it gradually moves to the age group 65 - 69 and gets a value *0*.

The actual equations of the model are the first-order conditions derived from the optimisation problem. In the labour-supply (or leisure) equations, the discounted effects of marginal work in a period on the pension benefits in each period are taken into account.

We have also modelled private fully funded pensions, to analyse the possible privatisation of the Lithuanian pension system. Participation in this system may be voluntary or mandatory. Contributions are collected from wage income, and the retired person will get an annuity from age 65 onwards. It is a defined contribution system, so the annuity $Z^p$ depends on the amount of contributions and the yield on these contributions, according to equation (10).

(10) $$\sum_{t=10}^{T} Z^p R_t = \sum_{t=1}^{9} c_t^Z (1 - l_t) e_t w_t R_t$$

The household sector consists of fourteen households, of different age, in each period. Total consumption, labour supply, pensions and transfers received and taxes paid are aggregated from individual household decisions.

**Household submodels**

There are two household submodels connected to the main model. The purpose is to evaluate the welfare effects of pension policies separately for taxpayers and tax evaders. The first submodel describes a normal ´honest´ household, who pays all the required taxes and contributions. It also receives pensions when old, because it has a record of working and paying contributions. The other household type is tax evaders. They pay neither contributions nor income taxes. They don´t get public pensions, either, because no working records exist. These two household types are otherwise similar: they make optimal life-cycle decisions with the same preferences, and face similar consumer prices, gross wages, interest rates and liquidity constraints. The net wages are different, and so are public pensions.

The submodel is used as follows. First, an initial steady state is created for both household types. From the initial steady state growth path of the total model, paths of variables that are required for the household´s optimal decision making are copied to both household type submodels. Then these households, cohort-wise, make their life-cycle plans. These form the initial steady states. When any policy change is analysed, the analysis is first done with the main model. Then, again, the relevant variable values are copied to the submodels, and the two household sectors reoptimise their decisions. Comparing the outcomes of this reoptimisation with the initial steady state gives the policy effects on both household types.

The households in the main model are practically weighted averages of the two types. The household submodel outcomes, however, have no effects on the main model outcomes. The aggregates of the submodel outcomes are usually, but not always, very close to the main model outcomes. Iterating between the main model and the submodels would thus sometimes yield changes to results (at the cost of a non-negligible additional computational burden). Aggregation would presume, however, that households face the same wage level. That itself is suspect. The wage rate agreed between a tax-evading employee and a tax-evading employer may yield different wage levels than those negotiated between tax-paying parties. The outcomes may be rather close also: the tax and contribution evading employer gains the contribution rate, 30 %, and the employee gains the income tax, 33 % over the threshold income, plus the one percent contribution. For big employers the gains are probably smaller, if they report, and pay contributions, for a part of the wage bill, and a corresponding part is withheld from the employee's wage for the income tax.

**Liquidity constraints**

We assume that all households are limited in their ability to borrow against future labour income. This affects the welfare losses associated with different financing arrangements of pensions. Technically, liquidity constraints are modelled as in Perraudin and Pujol (1991): the financial wealth of any household must be nonnegative at the end of each period.

Negative end-of-period wealth is restricted to zero. A zero-wealth restriction effectively divides one budget constraint into two. So the life-cycle optimisation is now done with two budget constraints instead of one. If the result yields negative wealth, we proceed in a similar fashion: divide the budget constraints again. We continue this as long as is necessary.

If the free solution has more than one negative wealth, which one should we restrict to zero? In the non-constrained solution, indifference prevails between alternative uses of marginal income. This indifference contains a time preference, and the interest effect is also taken into account. So, discounted marginal placements yield equal changes to lifetime utility. Negative end-of-period wealth is restricted to zero, which is equal to negative extra income. Roughly, that should be distributed so that the change is equal in different periods in discounted terms. So, we start from discounted minimum (negative) wealths.

The restrictions are sufficient, because they yield non-negative end-of-period wealths. To check whether or not the restrictions are necessary, we drop them out one at a time, which should yield negative wealth at least once each time. Thus far the procedure has not yet produced unnecessary restrictions.

## 2.3 Firms

A representative small firm produces the domestic good using capital inherited from the previous period, intermediate goods and labour. Infinite horizon decisions of investment, employment and use of intermediate goods are made to maximise the firm's market value. The firm takes the prices, demand of production and supply of factors at given prices, and production technology and taxation as given. Intermediate and capital goods are costs minimising CES composites of domestic and imported goods. Investments are financed by retained earnings and debt.

The formulation of the production structure follows Keuschnigg and Kohler (1994). The structure applied in this study is essentially a one-sector version of a model intended for multisector use. The structure can be described as follows:

(11) $$F_t = A\left\{\varepsilon K_{t-1}^{1/(1-\beta)} + (1-\varepsilon)L_t^{1/(1-\beta)}\right\}^{\frac{\beta}{\beta-1}}$$

(12) $$G(I_t, K_{t-1}) = \xi \frac{I_t^2}{2K_{t-1}}$$

(13) $$Y_t = \frac{F(K_{t-1}, L_t) - G(I_t, K_{t-1})}{1-\zeta}$$

The value-added production function *F* is a CES function of capital and labour. In the process of installing new capital some production is lost due to investment adjustment costs *G*. These installation costs depend positively on investments and negatively on the amount of capital. The use of the composite intermediate good is determined as a fixed proportion $\zeta$ of gross production *Y*.

Domestic households consider bonds and firms' shares as perfect substitutes in their portfolios. The arbitrage condition between after-tax returns on bonds and shares is:

(14) $$r_{t-1}V_{t-1} = D_t + (V_t - V_{t-1})(1-\tau^g)$$

where the left-hand side describes the invested amount yielding the domestic untaxed interest rate. On the right-hand side, the first term is the untaxed dividend income and the second term the net capital gain.

The arbitrage condition can be transformed to a form where the market value of the shares equals the present value of expected future dividends:

(15) $$V_t = \sum_{s=t+1}^{\infty} \frac{1}{1-\tau^g} D_s \prod_{v=t}^{s-1} \frac{1}{1 + r_v\left(\frac{1}{1-\tau^g}\right)}$$

The dividends are a residual from the firm's cash flow identity:

(16) $$D_t = (1-\tau_t^F)\left[P_t^F(F_t - G_t) - (1 + s_s^f \tau_t^z) w_t L_t - r_{t-1}B_{t-1}^f - P_t^K I_t\right] + B_t^f - B_{t-1}^f$$

where the dividend in period t is determined by profits before depreciation minus investment expenditure plus any increase in corporate debt. Corporate debt is preferred when financing investments, but its use is limited to a fixed ratio of the replacement value of corporate capital. Notice that profits that are invested are not taxed in Lithuania.

The firm chooses the optimal path of investment, use of labour and intermediate goods to maximise the current period dividend and the firm's value at the end of the period. If there are no unexpected shocks, there is no need to revise the optimal plan and it will be followed forever. Capital depreciates at a constant annual rate of d. The constraints of the maximisation are the initial capital stock and an equation describing its dynamics:

(17) $$K_t = K_{t-1}(1-d) + I_t$$

Three of the four first order conditions of the constrained optimisation are used as model equations. The first implies that investments should be carried out until the marginal benefit from an additional unit of investment equals the marginal cost. The marginal cost includes the price of a unit of capital plus the installation cost. The condition can be transformed to a q-theory investment equation (18). The optimality condition of capital says that capital should be installed until the return of an additional unit is large enough to cover the expenses of carrying the capital to the next period. These expenses include interest, depreciation and the change in the replacement price of capital. This condition is transformed to an equation (19) describing the path of the shadow value of the capital. In a steady state this marginal productivity condition of capital can be written as (20). The terms within the brackets are the depreciation rate *d* and the interest cost of the capital stock. The two terms to the right of the brackets are based on adjustment costs linked to replacement investments.

(18) $$I_t = \frac{(q_t - \frac{1}{1-\tau^g}) K_{t-1}}{\frac{1}{1-\tau^g}(1-\tau^F)\xi \, p_t^F}$$

(19) $$q_t = (\frac{1}{1-\tau^g}\left[(1-\tau^F)(P_{t+1}^F(F_K - G_K) - rbp_t^K) - bp_t^K + \tau^F dp_t^K\right]$$
$$+ \frac{1}{1-\tau^g} bp_t^K (1 + r\frac{1}{1-\tau^g}) + q_{t+1}(1-d)\frac{1}{1+r_t\frac{1}{1-\tau^g}}$$

(20) $$F_K - G_K = \frac{p^K}{p^F}\left[d + br + (1-b)r\frac{1}{(1-\tau^g)(1-\tau^F)}\right] + r\frac{1}{1-\tau^g}\xi d + \xi d^2$$

The third condition says that the marginal benefit of an extra unit of labour should cover wage costs plus the employer's pension contribution:

(21) $$P_t^F F_L = (1 + s^f \tau_t^z) w_t$$

The fourth condition is a transversality condition ensuring that the discounted shadow value of capital goes to zero as time approaches infinity.

The market value of the firm is linked to the shadow value of the capital in the leveraged firm as follows:

(22) $$V_t = K_t q_t - B_t^f$$

where $B_t^f$ is the firm's debt. This link has been derived using the homogeneity of production and capital installation technologies. The value of the firm jumps whenever unexpected news about the firm's future profitability enters the market. Domestic households own the firms. When the value of the firms jumps, and changes the households' wealth, they reoptimise their life-cycle plans immediately.

## 2.4 Government sector

The government collects revenue from wage income taxes, consumption taxes, and other taxes *O*, consisting of taxes on corporate income and capital gains, and uses the proceeds to pay transfers *S* to households, to pay interest on outstanding debt *B* and to employ civil servants to produce public services. These services are provided free of charge and are not taken into account in individual utility considerations. The government also gives transfers *T* to the social insurance fund.

(23) $$S_t^G + L_t^G w_t (1 + s^f \tau_t^z) + r_{t-1} B_{t-1}^G + T_t = L_t w_t \tau_t^w + C_t P_t^C \tau^c + O_t$$

The share of public employment is 0.3 of the total employment both in steady states and in dynamic simulations. Usually the government holds tax rates constant and balances the budget using transfers to households.

## 2.5 State Social Insurance Fund

The State Social Security Fund pays public old-age pensions *Z* and transfers *S*, consisting of disability and widows' pensions and sickness, maternity and unemployment benefits. The finance comes mainly from employers' and employees' contributions, and transfers from the government. The *s* parameters describe the shares of employers and employees who pay contributions. Notice that the share is the same for public and private employers.

(24) $$T_t + L_t w_t (s^f \tau_t^z + s^h \tau_t^e) = Z_t + S_t^s$$

Transfers are paid to all age groups equally (per capita) and are not taken into account as earnings-related benefits in individual decisions, but are of course taken into account as income items.

## 2.6 Private pension funds

In some simulations, private pension funds receive contributions from employees and pay defined-contribution benefits in the form of annuities. The budget constraint is

(25) $$H_t = H_{t-1}(1 + r_{t-1}) + c_t^Z L_t w_t - Z_t^P$$

where *H* denotes the fund, $c^Z$ the contribution rate and $Z^P$ total private pensions. See also section 2.2.

## 2.7 The consumption fund

The consumption fund is a calibration device. One calibration difficulty in numerical OLG models concerns household wealth and net foreign debt. As there is no precautionary saving, household wealth is low, especially if they can borrow against future income. The counterpart in a financially open economy is net foreign debt. The interest flow abroad reduces domestic consumption and distorts the tax receipt side. Closing the economy financially would disturb the rate of return side and probably exaggerate the yield of e.g. pension funds. One possible solution is to assume negative rates of time preference for the households, as in Auerbach et al. (1989). That would increase saving, as future consumption would be favoured vis-a-vis current consumption. As such, negative rates of time preference are rather implausible. The solution to this calibration problem here is called the Consumption Fund. It is an exogenous stock of wealth. Its interest receipts are used to buy consumption goods, with prices including the VAT. It can be interpreted as households' total precautionary savings, and consumption may be interpreted as minimum consumption, so these consumption amounts can be neglected in utility calculations. In the model, the consumption fund appears in the total wealth equation and in the goods market equations. The consumption goods bought by the fund are baskets of foreign goods and domestically produced goods, exactly like the consumption baskets of the households and with the same price elasticities.

The fund's consumption is determined by the fund's interest income and consumer prices:

(26) $$C_t^{CF} = A_{t-1}^{CF} r_{t-1} / P_t^C (1+\tau_t^c)$$

where $A^{CF}$ is the size (value) of the fund.

## 2.8 Foreign sector

The model imitates a small open economy, where the export share of total demand is large. The amount exported depends on the price elasticity of foreign demand:

(27) $$X_t = x\left(\frac{P_t^d}{P_t^M}\right)^{\sigma^E}$$

A large negative value for the elasticity implies that a small country has to adjust to the price level of international markets. The basic parameter values are: .$x$ = 0.6, $\sigma^E = -10$.

The imported good is used in consumption, investments and as an intermediate good in production. Its price is determined in the international markets. It is an imperfect substitute for the home-made good. The demand conditions are described with a CES structure.

Domestic interest rate deviates from the international rate, if net foreign assets exceed or fall short of a given value (see Lassila et al. 1997).

(28) $$r_t = r_t^f + \frac{A_t^f - A_0^f}{\varpi}$$

## 2.9 Markets

The model includes four markets, which balance every period. Wages adjust to equate supply and demand in the labour markets. Total employment equals the sum of private employment and public employment (equation 29). In the domestic good markets, firms are the sole supplier. The product is used by other firms as a part of the composite intermediate and investment goods, by households as a part of the composite consumption good and by foreign agents. The demand of domestic agents is determined by a cost minimising CES structure. The equilibrium condition which determines the price of the domestic good is equation (30). Domestic demand for the fixed-price imported good is also determined by minimising costs of the composite goods (the price of the imported good serves as a numeraire in the model). The perfectly elastic supply adjusts to demand in these markets (equation 31). The fourth market is the capital market. In this market, savings and investment are balanced. The arbitrage condition of domestic households ensures that they are ex ante indifferent between investing their savings in bonds and in firms' shares. Total savings are a sum of domestic savings and foreign portfolio investments. Equation (32) describes the parallel stock equilibrium.

(29) $$L_t^T = L_t + L^G$$

(30) $$Y_t = \zeta Y_t v_t^d + I_t i_t^d + (C_t + C_t^{CF})\, c_t^d + X_t$$

(31) $$M_t = \zeta Y_t v_t^M + I_t i_t^M + (C_t + C_t^{CF})\, c^M t$$

(32) $$W_t = V_t + B_t^f + B_t^G - A_t^{CF} - H_t + A_t^f$$

where the unit demands are

(33) $$v_t^M = \left(\frac{mP_t^C}{P_t^M}\right)^{\sigma^M} = i_t^M = c_t^M\ ,\ v_t^d = \left(\frac{(1-m)P_t^C}{P_t^d}\right)^{\sigma^M} = i_t^d = c_t^d$$

The price of the domestic good $P^d$ is endogenous and the price of the imported good serves as the numeraire in the model. Other prices are linked to them according to the following equations. The model facilitates the use of different share parameters $m$ and price elasticities $\sigma$ to consumption, investment and intermediate goods, but in this study we use the common values $m$ = 0.3 and $\sigma^M$ = 0.99 to all goods, which explains equations (33) and (36). $P^H$ is the price of the intermediate good.

(34) $$P_t^F = (P_t^d - \zeta P_t^H)/(1-\zeta)$$

(35) $$P_t^C = \left[m^{\sigma^M}(P_t^M)^{1-\sigma^M} + (1-m)^{\sigma^M}(P_t^d)^{1-\sigma^M}\right]^{1/(1-\sigma^M)}$$

(36) $$P_t^C = P_t^K = P_t^H$$

## 2.10 Population

The model has two alternative population blocks. The first corresponds to the actual and projected development in Lithuania. The size of the population changes in time, and the relative size of different age groups varies. This block is used in estimating the burden of ageing to the pension system and the public sector.

The second population block describes a stationary population with survival probabilities set to current Lithuanian levels. This block is used in most pension policy analysis. This choice is made because running the model with varying population is often very slow difficult. Finding the solution depends much more on good starting values with the varying population block. Running the model with stationary population block is usually easier and the solution can in most cases be found. The choice of population blocks has usually very little effects on policy results, unless the ageing phenomena is directly analysed.

## 2.11 Growth version of the dynamic model

The equations above describe the economy without any other economic growth than a possible growth of population. The model has also an option for exogenous labour-saving technical progress. We denote its rate by $\eta$. It affects the labour input, which becomes $(1+\eta)^t L_t$ instead of $L_t$ .

Some easily understandable changes are required in the growth version. Exogenous variables such as public debt and the consumption fund, which are normally held constant, are now held constant in relation to total production. This changes the respective budget constraints. The scale parameter of the export demand equation, and other similar parameters, now grows at the rate of technical progress. On the household side the periodic utility function changes.

(37) $$U_{t,i} = (c_{t,i}^{1-\frac{1}{\rho}} + \alpha(\theta_t l_{t,i})^{1-\frac{1}{\rho}})^{\frac{1}{1-\frac{1}{\rho}}}$$

where, as in Broer et al. (1994),

(38) $$\theta_t = (1+\psi)^i (1+\eta)^{t-i}$$

with $t$ referring to time and $i$ to the age group. The formulation makes balanced growth possible. It also prevents the effect that households, knowing that their real wages in the latter part of working period will be high, postpone labour supply excessively to that part of their lifecycles.

These changes remove most of the effects from economic growth. Something remains, however. Temporary increases in the pension level and temporary changes in indexation, e.g., can now be done, and the welfare calculations take into account that transfers to current old may increase their welfare significantly while the costs to much richer younger generations are relatively small.

## 2.12 Steady state model

The dynamic model has a steady-state counterpart. It is mostly used to obtain initial values for dynamic analysis and in sensitivity analysis. The model is small. In a steady state everything remains constant in time, so all time notations in the equations above, except for households, can be abandoned and the model gets very much simpler. The household sector has only one full life-cycle, within which the time aspect remains.

There is also a steady-growth version of the steady-state model. The growth comes in the form of exogenous labour-saving technical progress. Instead of staying constant, every quantity measured in money grows at a common rate. All prices remain constant except the wage rate which also grows at the common growth rate. This model is also used for initial values and sensitivity analysis.

## 2.13 Solving the model

The numerical model is solved in two steps. The first step is to find the initial steady growth path. This is done with the steady-growth version of the model. The results are then transferred to the main dynamic model to describe the time path of the economy before the policy measure.

The second step is to implement the pension policy measure and to solve the main dynamic model. This is done by iterating the two segments of the model. First households' life-time labour supply and consumption are solved, keeping prices, wages and interest rates fixed. The aggregated numbers are transferred from the household block to the other segment, which consists of firms, public and pension sectors, foreign agents and the market equilibria. This segment is solved with a Fair-Taylor algorithm. That gives new prices, wages, interest rates and the balancing variables for the public sector. The iteration between the segments is continued until the equilibrium has been found. The segmental solution procedure is adopted from Broer and Westerhout (1993). The model uses Gauss software.

## 3. NOTES ON CALIBRATION AND PARAMETER VALUES

No studies concerning crucial parameter estimates have been carried out in Lithuania. We thus take the parameter values from studies concerning other countries. If Lithuania integrates well with other European countries, the model may describe the future, developed market economy of Lithuania much more than the past Lithuania, and in some respects also more than the current Lithuania.

### Households:

*Labour supply*: The labour force participation rates are available in Statistical yearbook. Lithuanians also work after the retirement age. To make the model mimic the data, the efficiency coefficients *e* can be used. As there is a negative pension effect on labour supply after retirement, changes in wages can cause quite large changes in the price of leisure. We could increase  so that old people would value leisure more, to get the basic supply right. We also lower the amount of time available, which usually is 1. This is done for the age group 65 - 69.

The labour supply plan of households has both inter- and intratemporal dimension. The intratemporal choice is between consuming leisure and goods. We have chosen a value of 0.75 for the elasticity of substitution. This value is again somewhat lower than the one used by Kenc and Perraudin (1996), but more in line with some other Finnish and international studies.

*Saving*: As explained in section 2.7, one calibration difficulty concerns household wealth and net foreign debt. As there is no precautionary saving, household wealth is low, and the counterpart in a financially open economy is net foreign debt. The solution here is called the Consumption Fund. It is a stock of wealth, given as an exogenous ratio to GDP. Its interest receipts are used to accumulate the stock to keep the ratio to GDP constant in the long term, and the excess yield is used to buy consumption goods, with prices including the VAT.

The intertemporal elasticity of consumption determines the sensitivity of household saving to the net yield. The general opinion about the likely value of the parameter is that it is close to zero largely because of the contribution of Hall (1988). On the other hand, cross-sectional studies like Blundell et al. (1994) generate markedly higher values than time-series studies. Our choice was to use a value of 0.5. The intratemporal elasticity between consumption and leisure is chosen to be 0.75, which is close to the value used by Auerbach and Kotlikoff (1987), but somewhat lower than the unitary elasticity estimated for Finland by Törmä (1989).

### Firms:

In the CES production function of firms the central parameters are the share parameter and the elasticity of substitution between capital and labour. The value of the share parameter has been adjusted to 0.35 in order to yield the observed value-added shares of labour and capital incomes. With respect to the elasticity of substitution between the production factors, cross-sectional data generates again higher values than time series data. The Finnish studies (e.g. Törmä 1989 and Tarkka et al. 1990) yield somewhat higher values for the elasticity than the median value of 0.58 found by Rowthorn (1996), who surveys 33 studies. We chose the value of 0.7.

The model has one type of capital, which is assumed to depreciate yearly by 9 per cent. The value of the investment adjustment cost parameter is set to be 2, which corresponds to the lower end of the available estimates.

**Macroeconomic features:**

We have assumed a balanced growth path, where labour productivity grows 4 % annually. The real interest rate is 6 % on average; changes in net foreign debt cause slight variations in the interest rate according to the specification similar to Perraudin and Pujol.

The price elasticity of exports for the five-year model period is -4. This is based on the value of -3.8 estimated by Tarkka and Willman (1990). The scale parameter describing export demand has been adjusted to generate the required exports/GPD ratio. The elasticity of substitution between the domestic and the imported good has been chosen to be 0.99 also when firms consider the composition of investment and intermediate goods. It can be contrasted to the elasticity of aggregate imports (0.78) estimated by Tarkka and Willman.

**The public pension sector:**

State social insurance fund's budget in 1997 was as follows, according to the Statistical yearbook: Revenues came almost entirely (98.8 %) from contributions (employers 94.8%, employees 3.1%, farmers etc. 0.9 %). Social insurance payments formed 92.2 % of expenditure, consisting of pensions 71.1 % (old age pensions 54.6 %, disability pensions 11.8 %, widows etc.pensions 4.8 %), sickness and maternity payments took 10.1 %, unemployment benefits 3.3 %, health benefits 7.6 %, expenditure of the Board 3.9 %, and transfers to the Compulsory Health Insurance Fund budget 3.9 %. The fund's deficit was 0.8 %.

In the model, the revenues come entirely from contributions based on wages. Expenditure is divided into old-age pensions and other expenditure. Other expenditure is 83 % of old-age pensions in the base run. In simulations, other expenditure is kept at the base-run level. This other expenditure is modelled as transfers to households, and paid equally (per capita) to all age groups. Transfers are not taken into account as earnings-related benefits in individual decisions, but are of course taken into account as income items.

**Taxation:**

The tax system has been changed quite often, and many expect it to remain fairly unstable. From a modelling point of view it is rather simple.

Personal income tax: There is a flat tax rate of 33% of wage income above a fixed limit. Pensions are not taxed. Social benefits (transfers) are also mostly untaxed.

Personal capital income taxes: Rents are taxed, but dividends and interest income are not. Capital gains are taxed.

There is a uniform VAT rate of 18 %, with some items exempted.

Social security contributions: 30 % employer, 1 % employee. For a part of the public sector the rates are 22.5 + 1 %. There are also health contributions, but they are mostly taken from the income tax receipts. Employers pay about 1 per cent.

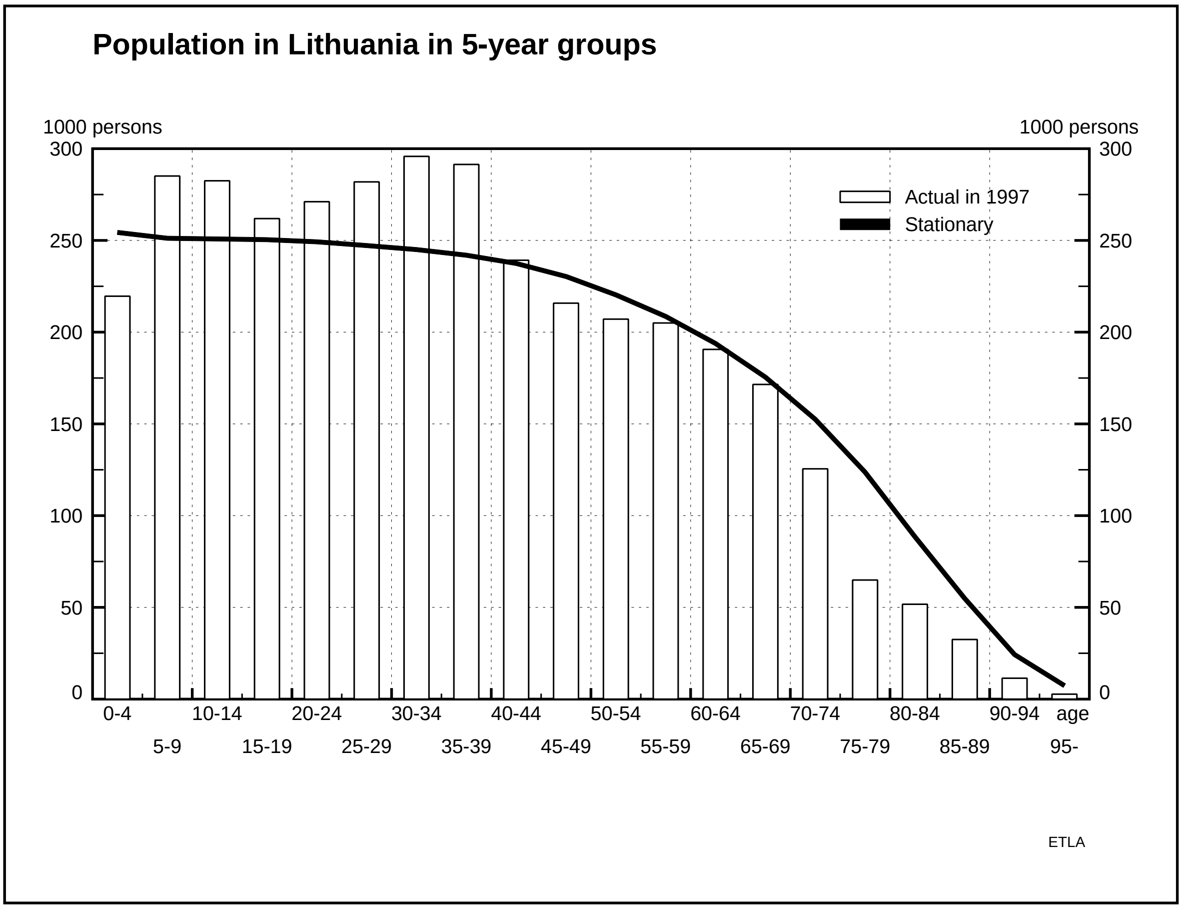
Corporate taxes: The corporate income tax is 29 % of gross income minus costs. If this income is invested, it is not taxed.

A difficult choice in these types of models is whether to use nominal rates or effective rates. The former may be better with regard to incentives, the latter with fiscal issues. We have mostly used nominal rates.

The **grey economy** is an integral part of the Lithuanian economy, as in all transition economies. The size of the problem is unknown. In the model a part of households does not pay income taxes and pension contributions, and an equal part of employers doesn't pay contributions.

**Population:**

The figure describes both population blocks in the model. The stationary population block is based on current Lithuanian survival probabilities. The other block corresponds to the actual and projected development in Lithuania. The size of the population changes in time, and the relative size of different age groups varies. The time paths for the future are obtained by chaining the 1997 data with the survival probabilities and the average 1990 - 1996 fertility rates.



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# Appendix. List of model variables and parameters

## Endogenous variables

| | |
|---|---|
| | FIRMS |
| $K$ | capital stock of the firms |
| $Y$ | gross production of the domestic good |
| $G$ | installation costs |
| $F$ | value added |
| $p^F$ | price of the value added |
| $V$ | value of the firms |
| $D$ | dividends |
| $B^F$ | firms' debt |
| $V^N$ | value of the new shares of the firm |
| $I$ | investments |
| | PRODUCT MARKETS |
| $i^d$ | demand of the domestic good in investment use |
| $i^m$ | demand of the imported good in investment use |
| $p^K$ | price of the composite investment good |
| $c^d$ | demand of the domestic good in consumption use |
| $c^m$ | demand of the imported good in consumption use |
| $p^C$ | price of the composite consumption good |
| $v^d$ | demand of the domestic good in intermediate use |
| $v^m$ | demand of the imported good in intermediate use |
| $p^v$ | price of the composite intermediate good |
| $p^d$ | price of the domestic good |
| $p^m$ | price of the imported good |
| | FOREIGN TRADE AND INTEREST RATE |
| $X$ | exports |
| $m$ | imports |
| $A^f$ | net foreign assets |
| $r^d$ | domestic interest rate (yearly) |
| $C^f$ | current account surplus |
| | LABOUR MARKETS |
| $L^F$ | private employment |
| $L^G$ | public employment |
| $L$ | aggregate labour supply |
| $w$ | wage rate |
| | HOUSEHOLDS |
| $c$ | consumption of one household |
| $C$ | aggregate consumption |
| $l$ | leisure of one household |
| $U$ | utility of one household |
| $S$ | transfer received by one household |
| $W$ | aggregate household wealth |

| | |
|---|---|
| | PENSION SYSTEMS |
| D | average insured income from working |
| Z | public pension |
| $Z^p$ | private pension |
| B | basic part of public pension |
| $S^s$ | transfers from social insurance fund to households |
| H | value of the private pension fund's assets |
| s | length of person's social insurance record |
| k | ratio of person's income over the country's average income |
| | GOVERNMENT |
| $S^G$ | transfers from government to households |
| $B^g$ | public debt |
| $\tau^C$ | value added tax rate |
| $D^0$ | minimum taxable income |
| | OTHERS |
| $A^{CF}$ | value of the consumption fund's assets |
| $C^{CF}$ | consumption by the consumption fund |

## Eksogenous variables and parameters

| | | |
|---|---|---|
| personal income tax | $\tau^w$ | 0.33 |
| dividend income tax | $\tau^D$ | 0 |
| interest income tax | $\tau^r$ | 0 |
| capital gains tax | $\tau^g$ | 0.1 |
| corporate income tax | $\tau^F$ | 0.29 |
| employee's pension contribution | $\tau^e$ | 0.01 |
| employer's pension contribution | $\tau^z$ | 0.3 |
| employee's private pension contribution rate | $c^Z$ | 0.1 |
| depreciation rate (yearly) | d | 0.09 |
| installation cost parameter | $\xi$ | 2 |
| share of the value of firms' capital financed by debt | $b$ | 0.6 |
| dividend distribution parameter | $a$ | 1 |
| input-output coefficient for the composite intermediate good input | $\zeta$ | 0.1 |
| labour share parameter of the value added production fuction | $\varepsilon$ | 0.35 |
| elasticity of substitution between labour and capital | $\beta$ | 0.7 |
| rate of labour-saving technical progress (yearly) | $\eta$ | 0.04 |
| life-cycle labour supply adjustment parameter | $\psi$ | 0.04 |
| scale parameter for value added | $A$ | 1 |
| share parameter of domestic good for consumption | $\nu^C$ | 0.7 |
| share parameter of domestic good for investment | $\nu^K$ | 0.7 |
| share parameter of domestic good for intermediate use | $\nu^v$ | 0.7 |
| elasticity of substitution between imported and domestic good in consumption | $\sigma^C$ | 0.99 |
| elasticity of substitution between imported and domestic good in investment | $\sigma^K$ | 0.99 |
| elasticity of substitution between imported and domestic good in intermediate use | $\sigma^v$ | 0.99 |

| | | |
|---|---|---|
| scale parameter of export demand | $x$ | 0.6 |
| price elasticity of export demand | $\sigma^X$ | -4 |
| foreign interest rate (yearly) | $r^f$ | 0.04 |
| sensitivity parameter of capital movements | $\varpi$ | 3 |
| | | |
| elasticity of intertemporal substitution of consumption | $\gamma$ | 0.5 |
| elasticity of substitution between consumption and leisure | $\rho$ | 0.75 |
| rate of time preference (yearly) | $\delta^n$ | 0.01 |
| leisure preference parameter | $\alpha^n$ | 0.81 |
| age-dependent working efficiency | e | 0.4 - 1.5 |
| public earnings-related pension accrual rate, annual | $\phi$ | 0.5 % |
| survival rate | $n$ | |
| mortality rate | $s$ | |
| share of employers paying pension contributions | $S^f$ | 0.8 |
| share of employees paying pension contributions | $S^h$ | 0.8 |
| retirement age parameter | $g$ | 0 - 1 |
| pension reduction parameter | $v$ | 1 |